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THE BEHAVIOR OF FUZZY IMPLICATIONS IN A FUZZY KNOWLEDGE BASE

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Abstract

More and more companies today discover the advantages of using knowledge bases for their processes and services. Recently, fuzzy set theory has also captured the attention due to good performances within control systems. Therefore, it is very appealing to combine the advantages of these two areas into a fuzzy knowledge base. However, obtaining the results of control systems in a knowledge based environment is not so straightforward. This paper will investigate one aspect of the reasoning process, namely the behavior of the implication. From the different tests performed, four main behaviors of implications can be found. First of all, there are the implications not always resulting in a convex set. A second class - the so-called impotent implications- doesn't change the predefined set at all. A third grouping reveals always a constant value portion, that rises or falls according to the changed input. A final division shifts the complete set in its whole conformably the intuition.

Keywords

fuzzy set theory, fuzzy KBS, fuzzy implication operators

1. Introduction

Today the number of companies interested in knowledge-based systems is continuously increasing. More and more firms discover the advantages of using knowledge bases for their processes and services. Several companies have already build a system giving financial advice, determining the insurance premium, controlling processes in chemical plants, etc.

Together with this growing interest for knowledge bases, fuzzy set theory gets also more and more attention of businesses. For example, fuzzy set theory was used in an automatic transmission system, in washing machines, in vacuum cleaners and also in a H_2-leakage system [7]. The general conclusion in this paper was that 'fuzzy control appears to be very useful when applied to the identification and control of ill-structured systems, where e.g. linearity and time invariance cannot be assumed, the process is characterized by significant transport lags, and is subject to random disturbances.' They also stated that 'fuzzy control systems are often characterized by their robustness, easy maintainability, and their ability to achieve good controls with comparatively low development and implementation efforts and costs'.

With this increasing interest for knowledge bases and fuzzy set theory, it is challenging to try to combine both concepts into a fuzzy knowledge base. However, both concepts have their critical points which have to be overcome if the integration wants to be successful. A first critical point when developing a knowledge base is the acquisition of the knowledge. Knowledge can either be extracted out of the underlying processes or out of the expert. However, the difficulty is that the expertise which the specialist has build up over the years has become more like an intuition to him. It is therefore not so easy for the expert to explicitly give all the relevant data and rules. Also when examining the processes, it is possible that not all important elements are noted and that some critical aspects which are implicitly embedded into the process, are not observed. So, it can well be that after the consultation of either the processes or the expert, the acquired knowledge is incorrect. Above that, a second critical point when building a knowledge base is, that the number of rules in reality can amount to several hundreds or even thousands. In such a case, testing becomes enormous and it is very difficult to keep an overview of the reasoning process. To overcome these problems, certain methodologies have been developed. Hwang [8] for example has developed the Knowledge Acquisition tool for Fuzzy Expert Systems (KAFES) to extract the knowledge and build the fuzzy knowledge base.

A critical point for the fuzzy side is, that the results of a fuzzy system are hard to predict. The deduction of new knowledge -also called the inference process- needed to determine the necessary action, is still not so predictable as with classical knowledge bases. Fuzzy control systems, don't experience this as a great drawback since they can adjust their action by the use of feedback loops. Opposed to fuzzy control systems, fuzzy knowledge bases cannot make use of those feedback loops to fine-tune the outcome. So the requirement that the first result has to be correct, puts a lot of emphasis on the inference process. Since the deduction of the new knowledge is done by the implication, it is very important to be able to determine the behavior of implications. That is what this paper is investigating.

This paper is organized as follows : the first section starts with an short introduction of fuzzy set theory. Subsequently, the importance of the implication in the inference process of a fuzzy knowledge base is explained. Then, the behavior of several fuzzy implication functions is studied. Finally, some concluding remarks are given.

2. Fuzzy set theory and knowledge bases

2.1 Introduction to fuzzy set theory

Fuzzy set theory was founded by Zadeh in 1965 [9] and can be seen as an extension of classical reasoning in such a way that, while classical logic only works with values of zero (the statement is false) or one (the statement is true), this new kind of reasoning allows also values between 0 and 1. By doing so, a better representation of the information is possible. If one has to rank cars by their volume and if one isn't very sure whether the considered car is big or small, the car gets a value of 0.5. This figure has to be interpreted as follows : the car belongs to the set of big cars to the degree of 50 percent. Another car could for example have a value of 0.7. These values given to the statement 'this car belongs to the set of big cars' are called membership values, because they express the degree by which an item is member of a specified set. If we denote this membership value by $\mu_A(m)$, where m is a specific car and A the set of big cars. When all possible volumes are placed on one axis and the according membership values on the other, μ_A represents the membership function, defined as follows: $\mu_A : U \rightarrow [0,1]$, where U represents the universe of discourse, containing all possible values. The membership function gives thus for each value of the fuzzy set the according membership value. While for each object of the universe a value can be determined, a fuzzy set considers only those objects that are relevant. So a fuzzy set is a subset of the universe.
Typical membership functions are the Gamma function, the Lambda function and the bell-shaped function (see Figure A).

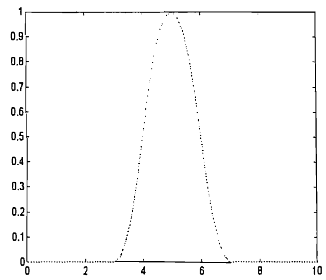

Figure A: Bell function

These functions are used to define several states of a certain condition. When, for example, a condition *amount of rain* is needed, the state '*low rain*' can be modeled by the Lambda function, the state '*medium rain*' by the bell-shaped function and the state '*high rain*' by the function. In that way, three 'variables' of the condition *amount of rain* can be modeled and used.

2.2 Fuzzy expert systems and inferencing

As already stated in the introduction, the result of the reasoning process of a fuzzy expert system is not so easy to predict as the one of a classical system. The reason therefore is twofold. First, fuzzy set theory is a generalization of classical logic. So, instead of working with only two possible values (0 and 1), a continuous range of values between zero and one is to be considered. Because the combination of two values through the use of a certain implication can now result in any value between zero and one, it is clear that this dilation yields a higher complexity and makes is hard to predict. The second reason is that one can easily create an implication himself, since the only condition that an implication has to fulfill is to generate values between one and zero. Hence, the same two values but combined with another implication can result in a different value. To better understand these reasons, it is useful to have an insight in the inference process.

The inference process is based on a generalization of the 'modus ponens':

If X = A then Y = B
X = A'
Y = B',

which means that when *A'* is entered as input, the rule comes up with *B'* as the result. Suppose that the following rules are in a knowledge base:

If rain is low then harvest is high;
If rain is medium then harvest is medium;
If rain is high then harvest is low;

When during consultation the state *more or less high (=A')* is entered for the condition *rain(=X)*, the knowledge base has to know which rule to fire because a value can now belong to different states of the same condition. Therefore, a method is used to compare two fuzzy sets, called the closeness measure, introduced by Zadeh, is defined as

$$\text{SUP MIN}_{x \in X} (A(x), B(x))$$

This means that, for each x, the minimum of the function value for A and B is taken. When this is done for each x, the maximum of those minima is taken as a measure of the similarity for A and B. So using this measure, we find a closeness measure of 0.32 between *medium* and *more or less high* and a closeness measure of 1 for *high* and our new input. Graphically interpreted, this measure calculates the highest value of the intersection of the two sets. Hence the new input matches best with the condition state *high* and the last rule is fired.

Now that one of the three rules is chosen, the firing of that rule consists of four stages which are given in Figure B. First, there has to be a relation between *rain* and *harvest* to be able to alter the set of *harvest* to *B'* when the set *high* of *rain* changes. This relation between *A* and *B* is defined by the implication, shown in the first quadrant of Figure B. Secondly, to deduct the new result, the changed set has to be presented in three dimensions. Therefore a technique, called cylindrical extension, is used and does nothing else than elongating the set over the new dimension, as illustrated in the second quadrant. The third step compares the former two graphs by taking the minimum. The result is pictured in the third quadrant. Finally, the resulting set has to be retranslated to a two-dimensional set. For that purpose, the chart is projected on the dimensions of *harvest*. This projection, together with the old set *B*, is already shown in the lower right corner of Figure B, but is repeated in the fourth quadrant with the stars representing the old set *B*. In this example, Kleene-Dienes was used as the implication.

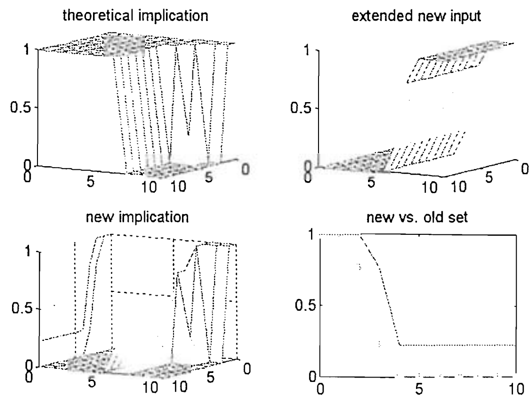

Figure B: Rule firing in a fuzzy environment

2.3 The Behaviour of Fuzzy Implications

In the former section, it is illustrated that the choice of the implication can have a significant impact on the deduction of new knowledge. Consequently, it is important to be aware of the behavior of the chosen implication. In particular the implications of Table A will be examined on some -at first sight- very logical characteristics. Although, several of them don't even fulfill these intuitive requirements.

Early Zadeh	MAX(1-x, MIN(x,y))
Lukasiewicz	MIN(1, 1-x+y)
Mamdani	MIN(x,y)
Standard Strict	1 if $x \leq y$, else 0
Gödel (Standard Star)	1 if $x \leq y$, else y
Standard Strict-Star	MIN(Strict(x,y), Gödel(1-x,1-y))
Standard Star-Strict	MIN(Gödel(x,y), Strict(1-x,1-y))
Standard Star-Star	MIN(Gödel(x,y), Gödel(1-x,1-y))
Standard Strict-Strict	MIN(Strict(x,y), Strict(1-x,1-y))
Kleene-Dienes	MAX(1-x, y)
Gaines	1 if $x \leq y$, else y/x
modified Gaines	MIN(1, y/x, (1-x)/(1-y) if $x > 0$ and $y < 1$)
Kleene-Dienes-Lukasiewicz	1 - x + xy
Willmott	MIN(Max(1-x, y), Max(x, 1-x), Max(y, 1-y))
Standard Sharp	1 if $x < 1$ or y=1, else 0
Wu1	1 if $x \leq y$, else MIN(1-x,y)
Wu2	0 if $x < y$, else y
Yager	y^x

Table A: Fuzzy implication operators

The following tests, proposed in [3] and [4], will be conducted:

- A' = A
- A' = more or less A
- A' = unknown
- A' = very, very low, i.e. no rain

2.3.1 A' = A

Intuitively, one could expect that when the input is exactly the same as the predefined conditions, the resulting action remains also the same. Surprisingly, several implications of Table A don't fulfill this requirement. The results can be divided in three categories. The first class contains all implications which come up with an horizontal line. This is what Chang *et al.* in [3] calls a 'constant value portion'. One example of that class is Figure C, showing the inference process of the Early Zadeh implication. To this class also belongs Lukasiewicz, Kleene-Dienes, Kleene-Dienes-Lukasiewicz, Willmott and Standard Sharp.

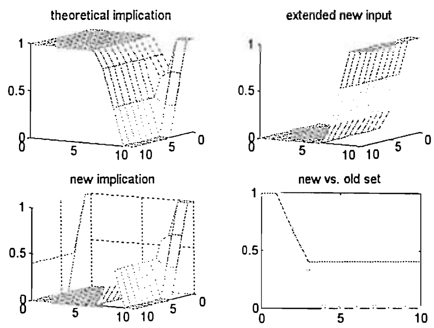

Figure C: Early-Zadeh implication

A second class gives a very similar fuzzy set as the theoretical action. The idea of what the fuzzy set stands for, stays the same. Figure D illustrates the inference of the Standard Strict implication. Other operators with the same behavior are Standard Strict-Star, Standard Strict-Strict, Gaines, modified Gaines, Wu1 and Yager. A third group comes up with exactly the same theoretical action and are actually the only ones who fulfill this requirement. This category consists of Mamdani, Gödel, Standard Star-Strict, Standard Star-Star and Wu2.

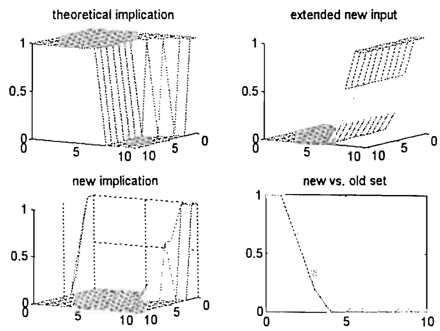

Figure D: The inference of the Standard Strict implication

2.3.2 A' = more or less A

The next step in the research consists in slightly shifting the input set to see whether the action also alters in the same degree. Chang *et al* [3] have found in previous research that a shift towards lower values didn't have the same effect as a shift in the opposite direction. When the input function is moved to the right of the graph (towards higher values of 'rain'), it is clear that the action doesn't change. When shifted to the left although, the intersection determined by the MIN-operator between the new input function and the theoretical inference graph reveals a constant value portion. This portion rises the greater the shift to the left is. This does not mean that shifting to the right can always reduce the constant value portion for each implication. Under the former requirement, the Early Zadeh operator gave already such a high constant value portion that a small move won't reduce it. The reason seems to be the presence of a frontal plane, whereby frontal has to be seen in the direction of the projection. Due to the cylindrical extension, the new input will likewise be extrapolated, i.e. frontal to the projection dimension. These two frontal planes will certainly create a 'hill' in the third quadrant and likewise a constant value portion. Such a new action is not the result which one would intuitively expect. Based on the rule 'if rain is high then harvest is low' and knowing that the amount of fallen rain is a bit lower then what is defined as 'high', the new action becomes 'a bit higher then low'. When this fuzzy set is to be displayed, one will draw an L-shaped function shifted to the right of 'low'.

To exclude this constant value portion, Chang *et al* [3] proposed to give each function a 'tail'. This means that, instead of going immediately to zero, the function drops first to a value pretty close to zero (let's say 0.0001) and then flattens out to zero. So, the 'insignificant' part of the function changes from zero to 'close to zero'. The effect on the meaning of the fuzzy set due to the redefinition is nil. By using the 'tail' definition for the condition as for the action, the constant value portion is omitted when using the Gödel-implication.

Surprisingly, this redefinition is not only helpful when using the Gödel implication but also works -in the meaning of giving intuitive expected results- for Standard Strict and Wu1. The implications where this redefinition doesn't help are Early Zadeh, Lukasiewicz, Kleene-Dienes, Kleene-Dienes-Lukasiewicz and Willmott. After taking a closer look at the function prescription, they all seem to have an (1-x) in some way or the other, causing a frontal plane. This founding confirms again the statement that frontal planes, combined with the cylindrical extension, have a great chance of giving a constant value portion which is hard to get rid of.

A different class are the combinations of Gödel and Standard Strict, producing problems with the tail definition. The redefinition fails to alter the summit on the coordinates (0,0) and these combinations will be treated differently in the following tests. Other implications where the new definition doesn't succeed are Gaines, modified Gaines, Standard Sharp and Yager. Standard Sharp reveals to be very hard to influence. Concerning Gaines and modified Gaines, redefinition cuts only a sphere out of the theoretical function, so that only for large left shifts, the result is low-alike (see Figure E).

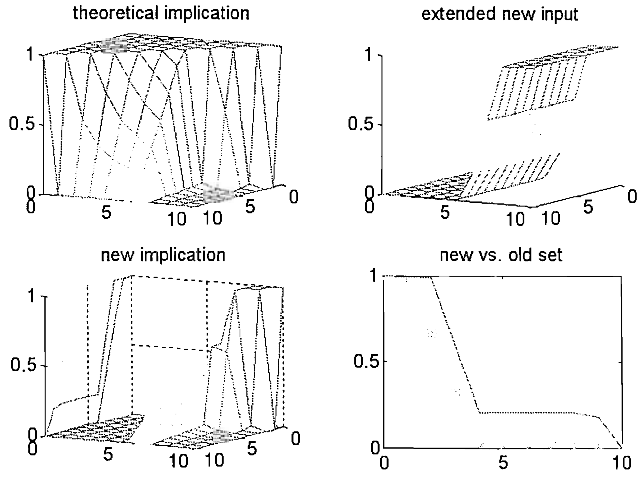

Figure E: Gaines implication

Now that we have discussed the effect of a redefinition with a tail in order to omit the constant value portion, let us reconsider the issue of a shift in the input data. It appears that the classes deducted above are still useful. First, Mamdani and Wu2 still come up with the same theoretical set as the new result. Early Zadeh, Lukasiewicz, Kleene-Dienes, Gaines, modified Gaines, Kleene-Dienes-Lukasiewicz, Willmott and Yager allow the constant value portion to move upwards. For a knowledge base, this kind of behaviour is contra-intuitive and therefore unacceptable. The third category, lets the new set shift to the right which is a comportment fully compatible to intuition. In this last group reside Standard Strict, Gödel and Wu1.

Standard Sharp still gives a horizontal line on 1, meaning that each value of the universe belongs to the new set to the same degree. The interpretation behind this is, that nothing can be concluded from this solution. The different combinations of Gödel and Standard Strict come up with strange and complex responses because the new sets are not convex.

2.3.3 A' = unknown

Here, the following requirement is tested :

if X is A then Y is B
A' is unknown
B' is unknown

where unknown stands for a membership value of 1 over the whole universe. Remark that the minimum of a plane on membership value 1 and a theoretical inference graph, is again the theoretical graph. Since every implication has somewhere a little plane lying on membership value 1, the projection will result in a set on 1. So the majority of the implications will fulfill this requirement. The only ones who don't satisfy this test are Madman, Wu2 (both giving again the identical set) and the combinations of Gödel and Standard Strict. Willmott also belongs to the last group, not meeting this demand.

2.3.4 A' = no rain

This last characteristic comes out of the paper of Chang *et al* [3] :

if X is A then Y is B
A' is very very A
B' is very very B

This paragraph tests with what kind of result the inferencing process will come up with when the set '(virtually) no rain' is entered as input. This final test yields two categories. Early Zadeh, Lukasiewicz, Standard Star-Strict, Standard Star-Star, Kleene-Dienes, Kleene-Dienes-Lukasiewicz, Willmott and Yager all give the identical theoretical set 'low' as the result. The second class transforms the theoretical set to 'very very high'. Only Standard Strict, Standard Strict-Star, Standard Strict-Strict, modified Gaines and Wu1 belong to that class and fulfill at the same time the characteristic. Figure F illustrates the inference process when Standard Strict is used as the implication. Mamdani and Wu2, together with Gödel and Gaines who normally belong to the second category, present just like the first grouping also the theoretical set 'low'.

From a paper of Theunissen [5], the following requirement is found :

> *The conclusion B' may never be more precise then the fuzzy set B. Or put differently, the support of B' is supposed to be equal to or greater than the one of B.*

After the testing, it turns out that only the first class meets this characteristic. It is clear that from an intuitive point of view, it cannot be accepted that when the input is transformed to 'very very A', the result B' can only reach the theoretical set B.

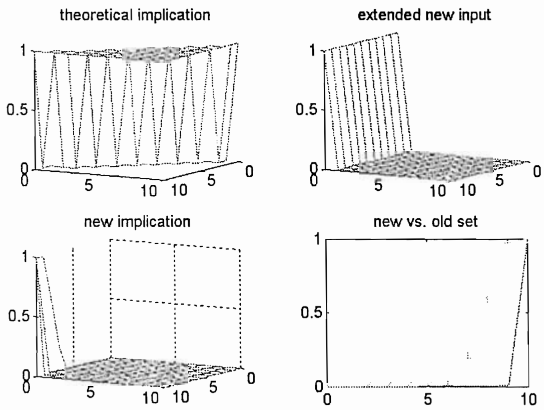

Figure F: Standard strict implication

2.4 Recapitulation of the test results

To conclude, the considered implications can be categorized as follows:

	Category 1 'constant value portion'	Category 2 'intuitive'	Category 3 'impotent'	Category 4 'not always convex'
Early Zadeh	x			
Lukasiewicz	x			
Mamdani			x	
Standard Strict		x		
Gödel		x		
Standard Strict-Star				x
Standard Star-Strict				x
Standard Star-Star				x
Standard Strict-Strict				x
Kleene-Dienes	x			
Gaines		x		
modified Gaines		x		
Kleene-Dienes-Lukasiewicz	x			
Willmott	x			
Standard Sharp			x (nearly always 1)	
Wu1		x		
Wu2			x	
Yager	x			

Table B: Categorization of the implications

Standard Sharp actually belongs also to the group of impotent implications, but differs from the other two in that a horizontal line close to 1 is continuously the result instead of the theoretical set. Similarly the combinations of Standard Strict and Gödel are left out of the conclusion because they often come up with a non-convex result. These non-convex results not only give difficulties in the interpretation but also in the fulfillment of basic conditions for a fuzzy set. From the tests performed the following conclusions can be made:

Some implications don't even meet the identity condition that if A' equals A, B' should equal B. More specific, these operators are Early Zadeh, Lukasiewicz, Kleene-Dienes, Kleene-Dienes-Lukasiewicz and Willmott. When using one of these implications, one has to be fully aware of the fact that this very intuitive requirement is not met. Although Yager belongs to the first category, it corresponds to the intuitive behaviour.

The solution of Chang *et al* in [3] to reduce the constant value portion via the tail-redefinition for the Gödel-implication, is also useful for other operators, such as Standard Strict, Standard Star-Strict, Standard Star-Star and Wu1. These are almost the same as the ones from the second class except for Gaines and modified Gaines. The effect of a redefinition by those two, results only in a decline of the constant value portion at the point (10,0).

The effect of slightly shifting the fuzzy input set, depends on the category to which the implication belongs. Shifting in the first class results merely in a shift of the constant value portion. The second grouping gives only a horizontal shift and fits intuition. Again Gaines and modified Gaines are exceptions of the second class : the fact that they couldn't get rid of their constant value portion by use of redefinition, causes both a horizontal as a vertical shift.

The implications of the first category meet the requirement of Theunissen, stating that under a large shift, a fuzzy set B' can at best only equal the predefined set B. No additional precision is added. Although Gaines and Gödel don't belong to this class, they correspond to the same behavior. As stated earlier, this manner isn't conform the intuition and can therefore not be accepted. The other category gives intuitive results and transforms the predefined set to 'very very high'. Mamdani and Wu2 keep giving the theoretical predefined set B.

The above conclusions are summarized in the next table, where the implications in italic belong to the first category, the bold ones to the second class, ✘ and ✔ stand respectively for not fulfilling and fulfilling the requirement, ↑ indicates a horizontal shift and → a vertical shift. From the tests and using intuition as a reference, only Standard Strict and Wu1 with redefinition are recommended.

	A=A'	tail-redefinition	more or less A	very very A
Early Zadeh	✘	✘	↑	identical
Lukasiewicz	✘	✘	↑	identical
Mamdani	✔	identical	identical	identical
Standard Strict	✔	✔	→	✔
Gödel	✔	✔	→	identical
Standard Strict-Star	✔	✘	not convex	✔
Standard Star-Strict	✔	✔	not convex	identical
Standard Star-Star	✔	✔	not convex	identical
Standard Strict-Strict	✔	✘	not convex	✔
Kleene-Dienes	✘	✘	↑	identical
Gaines	✔	✘	→and ↑	identical
modified Gaines	✔	✘	→and ↑	✔
Kleene-Dienes-Lukasiewicz	✘	✘	↑	identical
Willmott	✘	✘	↑	identical
Standard Sharp	✘	✘	always 1	constant portion
Wu1	✔	✔	→	✔
Wu2	✔	identical	identical	identical
Yager	✔	✘	↑	identical

Table C: Summarizing table

3. Conclusions and future research

From the different tests performed, four main types of behavior of implications can be found. First of all, there are the implications not always resulting in a convex set. They add extra complexity in satisfying requirements and their results are very difficult to interpret. A second class doesn't change the predefined set at all - the so-called impotent implications. A third grouping reveals always a constant value portion, that rises or falls according to the shift of the input. A final division shifts the complete set in its whole conformably intuition.

Further research is necessary to confirm the validity of the classification scheme. Considering the fact that it is very easy to create new implications, not all operators are included in the research. The utility of the classification is that it provokes reflections on how a specific input is translated by different implications and on whether that behavior is conform with intuition. This way of judging the behavior is more appropriate for fuzzy knowledge bases where a human has to interpret the results than as for fuzzy control systems, where the result can be corrected by a feedback loop.

Additional research is also needed to check the behavior in presence of several conditions. The example used in this paper was a rather simple one, with only one condition and action for each rule. Furthermore, no crisp values are included. So, a more elaborated example, including several conditions and crisp values, needs to be tested and the results have to be verified against these in this paper.

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