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Dynamic Lot-Sizing models with Limited Resources

I. INTRODUCTION

This paper considers the lot-size scheduling problem from the production planner's point of view. Lot-sizing is the process of determining how much of each product to make and when to make it. The lot-size decision considers the trade-off between lost productivity from frequent set-ups and short runs and the higher inventory costs arising from longer runs. When the decision has to consider shared limited production resources, the problem becomes complex.

A short review of the lot-sizing literature will reveal the numerous shortcomings with respect to the stated problem. The traditional Wilson EQQ formula, has the inconvenience of spreading the lot-size decisions over a time continuum when in reality, most manufacturing decisions are more easily made at discrete points in time. The decision-maker usually decides at the beginning of a period whether to schedule a product or not. Moreover demands and costs are assumed to be constant. The issue of discrete time periods and variability of demands and costs was resolved by Wagner and Whitin [9]. Given a time-varying demand over a finite horizon, their model searches for the optimal trade-off between set-up and inventory costs. Besides the optimal solution algorithm a number of well-known heuristics were developed such as periodic order quantity, least unit cost, part period balancing,

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Silver-Meal heuristic, etc... A description and comparison of these models are given in [7]. The issue of capacity limits, however, is handled neither by the EQQ formula nor by the Wagner-Whitin lot-size algorithm.

The literature dealing with the capacity problem can be divided into two parts namely the static and the dynamic lot-sizing models (for single and multi-item problems). With static we mean that the demand and the costs for each product are constant through the (infinite) time horizon. The dynamic models, on the other hand recognize the fact that production requirements tend to be anything but uniform, they usually occur in discrete « lumps ». The static models are dealing with the determination of lot-sizes for different products under a common constraint such as a limitation on the number of set-up hours, a constraint on the total amount invested in inventory or capacity constraints derived from the medium range or aggregate planning. They usually make use of the lagrangean multiplier techniques. For a review see [8]. Recently the dynamic constrained lot-sizing problem received more attention. This very interesting topic will be discussed in this paper. In section II, a review will be given of the models searching for optimal solutions. They make use of linear programming or some more complex non-linear programming techniques. As already mentioned the recognition of the capacity problem in a dynamic environment was trivial, the solution was not. Indeed, none of the proposed algorithms are fully satisfactory from a theoretical or practical point of view. Some of the more practical oriented authors came up with interesting heuristic approaches. These will be discussed and criticized in section III. Finally section IV reports on our experience in an industrial production system.

II. ALGORITHMS FOR THE MULTI-PRODUCT CONSTRAINED DYNAMIC LOT-SIZING PROBLEM

Consider the problem of finding lot-sizes (production quantities) for a number of products whose requirements are assumed to be known over a finite planning horizon. The discrete time periods can be months or weeks. Aggregate resource constraints (e.g. derived from medium range planning) are taken into consideration.

These aggregate constraints can be workforce availabilities, limited production time, etc... The manufacturing process is characterized by batch-type production operations, a cost is incurred when setting up the production facilities for a given run. The resulting inventory at the end of each period for each product is penalized.

The existence of a fixed charge (the set-up cost) creates many problems. Every item that generates a set-up must be treated as independent, this expands the dimension of the model. The inclusion of a set-up cost produces a problem of indivisability, in this sence that if one unit is produced in a certain period the complete set-up cost is incurred introducing integer variables in the model. The fixed cost in the objective function introduces non-linearities in the cost functions. The resulting large scale, integer, non-linear programming model is hard to solve computationally.

Manne [5] was the first author providing a means for handling the problem. His formulation takes into account a characteristic of the Wagner-Whitin algorithm for the unconstrained single product problem, namely that in an optimal unconstrained production schedule production will not take place if the ending inventory of the previous period is positive. That simply means that the order quantity must satisfy production requirements over an integral number of periods. Manne's formulation goes as follows. Consider a single product, with a time horizon of 3 periods ($T = 3$). Then 4 production policies (in general 2^{T-1}) are considered :

— produce in period 1, to satisfy production for periods 1, 2 and 3.
— produce in period 1, to satisfy production for periods 1, and 2 and start up in 3.
— produce in period 1, to satisfy the production requirements for period 1 only, and start-up production in period 2, to satisfy the requirements for period 2 and 3.
— produce in every period its own production requirements.

For each product there are 2^{T-1} sequences or in total $N.2^{T-1}$, where N is the number of products.

Let us define :

X_{ijt} : amount produced by means of production sequence j, product i, in period t.

$i = 1, \dots N$

$j = 1, \dots J$ where $J = 2^{T-1}$

$t = 1, \dots T$

d_{it} : requirements in period t, product i.
s_{it} : set-up cost in period t, product i.
v_{it} : variable production cost in period t, product i.
h_{it} : inventory holding cost in period t, product i.
I_{it} : inventory at the end of period t, product i.
t_{ij} : total production, inventory holding and set-up costs for sequence j, product i.
l_{ijt} : usage of the resource for a production quantity X_{ijt}.
$l_{ijt} = k_{it} \,.\, X_{ijt}$
k_{it} : factor that translates the amount produced into the usage of the resource.
L_t : resource constraint in period t.
Θ_{ij} : a variable indicating production sequence j for product i.

The great advantage of introducing variables Θ_{ij} is that for each Θ_{ij} it is easy to compute t_{ij} :

$$t_{ij} = \sum_{t=1}^{T} [s_{it} \,.\, \delta(X_{ijt}) + v_{it} \,.\, X_{ijt} + h_{it} \,.\, I_{it}]$$

$$\left|\begin{array}{l} \delta\,(X_{ijt}) = 0 \text{ if } X_{ijt} = 0 \\ \delta\,(X_{ijt}) = 1 \text{ if } X_{ijt} > 0 \end{array}\right.$$

The usage of the resource per period can be found very easily, since with each Θ_{ij} a known production sequence is attached.
The model (A) :

$$\text{Minimize } Z = \sum_{i=1}^{N} \sum_{j=1}^{J} t_{ij} \,.\, \Theta_{ij} \qquad (1)$$

s. t.

$$\sum_{i=1}^{N} \sum_{j=1}^{J} l_{ijt} \,.\, \Theta_{ij} \leqslant L_t \quad t = 1,\ldots T \qquad (2)$$

$$\sum_{j=1}^{J} \Theta_{ij} = 1 \qquad i = 1,\ldots N \qquad (3)$$

$$\Theta_{ij} \geqslant 0 \qquad (4)$$

Expression (1) states the objective of the model, namely, minimize variable production, set-up and inventory holding costs. Constraint (2) represents the constrained resources and (3) indicates that for each i at least one production sequence Θ_{ij} must be selected.

Manne [5] suggested to solve problem (A) by means of linear programming. If it turns out that the Θ_{ij} are all integer in an optimal solution to model (A), then the resulting solution is optimal for the original integer problem formulated below (model B).

Model (B) :

$$\text{Minimize } Z^* = \sum_{i=1}^{N} \sum_{t=1}^{T} [s_{it} Y_{it} + v_{it} x_{it} + h_{it} I_{it}]$$

Subject to :

$$I_{i,t-1} + x_{it} - I_{it} = d_{it} \qquad \left| \begin{array}{l} i = 1, \dots N \\ t = 1, \dots T \end{array} \right.$$

$$\sum_{i=1}^{N} k_{it} \cdot x_{it} \leqslant L_t \qquad t = 1, \dots T$$

$$x_{it} \leqslant m_{it} \cdot Y_{it} \qquad \left| \begin{array}{l} i = 1, \dots N \\ t = 1, \dots T \end{array} \right.$$

$$Y_{it} = 0 \text{ or } 1$$

where :

x_{it} = production period t, product i.

m_{it} = maximum production period t, product i.

Y_{it} = integer variable : if $x_{it} = 0$, then Y_{it} will be zero.
$x_{it} > 0$, then Y_{it} will be one.

The solution space of model (B) is not restricted to Wagner-Whitin sequences, and therefore guarantees the optimal solution if the model is solved as an integer program. There is a great deal of confusion about these two models in the literature up to now. The L.P. results of Manne have been interpreted incorrectly by many authors. They say that if model (A) is solved as an integer program the optimal solution will be found. In other words, they consider the production sequences represented by the variables Θ_{ij} as the only candidate schedules for an optimal solution to the original integer program formulated as model (B). All research is concentrated on model (A), which is optimal only if the L.P. solution turns out to be integer. In this case $Z = Z^{*}$. If N is large in comparison with T, we may expect that model (A) offers a good approximation to the optimal solution for the following reason : there are at most $T + N$ positive variables in the L.P. solution of model (A), and at least one of these variables will be associated with each of the N constraints (3). Thus, there are at most T instances for which more than one Θ_{ij} is positive. If N is much greater than T, the Θ_{ij} fractional problem is reduced. Some authors extended model (A) by including hiring, firing and overtime decisions to make the problem more realistic. Different solution techniques were applied such as column generation techniques [2], lagrangean relaxation methods [4] and decomposition programming [2].

None of the proposed algorithms are satisfactory from a theoretical and practical point of view. The heuristic approaches discussed in the next section are also based on model (A). It is our belief that this is not the best direction for further research, but for many practical considerations it may be the easiest way.

III. HEURISTIC APPROACHES

The objective of heuristic approaches is to find solutions for complex decision problems with less computational effort compared with the optimum seeking algorithms, and at the same time to approach the optimal solution as close as possible. A good heuristic tries to incorporate in its procedures knowledge about the characteristics of the optimal solution. The candidate schedules for an optimal solution are Wagner-Whitin schedules and convex combinations of these pure strategies. This last category of sequences makes the solution to the stated problem very hard to solve. Two heuristic will now be discussed, first, the Eisenhüt [3] heuristic and second, the Newson [6] approach. Both are concerned with the search for combinations of Wagner-Whitin sequences, no convex combinations are considered. This is of course a severe limitation for the heuristics available in the literature up to now.

A. Eisenhüt heuristic [3].

The Eisenhüt heuristic starts from a requirement matrix for each product in each discrete time period, and then tries to group requirements of different periods in the same lot without exceeding the capacity constraints. For each requirement a coefficient is calculated which indicates if cost reductions are possible by including the requirement in a lot. The derivation of this « *appreciation factor* » goes as follows :
Single product :

$$C(T) = \frac{s + I(T)}{T} \quad \text{where } I(T) = h \sum_{t=1}^{T} (t-1) . d_t \qquad (5)$$

s	:	set-up cost.
h	:	inventory holding cost per unit, per period.
d_t	:	demand in period t, $t = 1, \ldots T$.
$I(T)$	:	the cumulative inventory holding cost for a lot including requirements from period 1 up to T.
$C(T)$	:	combined set-up and inventory holding cost per period for an order cycle of length T.

Note that we assume constant costs through time, which we allow here only for simplicity of the exposition.

$$C'(T) = \frac{I'(T)}{T} - \frac{I(T)}{T^2} - \frac{s}{T^2} \qquad (6)$$

It is clear that

$$I'(T) = h . (T-1) . d_T$$

$$\sum_{t=1}^{T} (t-1) = \frac{(T-1) . T}{2} \quad \text{or} \quad (T-1) = \frac{2}{T} \sum_{t=1}^{T} (t-1)$$

then

$$I'(T) \text{ is replaced by : } \frac{2.h}{T} \sum_{t=1}^{T} (t-1) . d_t = \frac{2}{T} . I(T) \qquad (7)$$

This relation is an equality if $d_t = d$, for all t.
Substituting (7) in (6), we obtain :

$$C'(T) = \frac{I(T) - s}{T^2} \qquad (8)$$

Let $C'(T) \rightarrow 0$, and since T has discrete values we find the unconstrained order cycles T* if : $\left| \begin{array}{l} I(T^*) \leqslant s \\ I(T^* + 1) > s \end{array} \right.$

This is the well known « *Part Period Balancing* » criterion.

$C'(T)$ can be interpreted as follows :

for $T < T^*$ the term $C'(T)$ will be negative indicating a cost reduction,

for $T > T^*$ $C'(T)$ will be positive indicating a cost increase,

$C'(T)$ is in other words the change in cost by including the requirement of period T in the existing lot.

Multi-product case

Up to now the $C'(T)$-criterion was based on a single product. Eisenhüt extends this coefficient to the multi-product case in the following way :

$$U(T)_j = \frac{C'(T)_j}{(d_T)_j} = \frac{[(s - I(T))/T^2]_j}{(d_T)_j} \qquad j = 1, \dots J \qquad (9)$$

j stands for the different products.

$(d_T)_j$ for the demand in period T of product j.

The cost reduction $C'(T)_j$ for product j is divided by the requirements of the last period included in the lot. This factor must be calculated for each product and for each period. If $U(T)_j$ is positive a cost reduction is expected whereas for negative coefficients no rèduction in cost is possible.

Constrained multi-product case

If capacity constraints are active Eisenhüt proposes the following rule : the order size is increased to include the requirement for a period T for the product which shows the greatest potential reduction per unit. This is repeated until the capacity constraint is violated or until no additional cost reduction is possible for any product.

The different steps in the Eisenhüt heuristic can be summarized as follows :

Step 1 : construct the requirements matrix, the rows $j, j = 1, \ldots J$, stand for the different products and the columns $i, i = 1, \ldots N$, for the different periods, the elements (i, j) indicate the requirements for product j, period i.

Step 2 : compute for each product, and for each time period, except the first, $U(T)_j$, negative values need not to be taken in consideration.

Step 3 : fill up the available capacity with the requirements of the first period (no back-orders are allowed). Units of future periods are pulled into the order in chronological sequence according to decreasing values of $U(T)_j$. This procedure is followed until either the capacity for period 1 is exceeded or until no cost reductions are possible.

Step 4 : establish a new matrix by deleting the requirements of period 1 and all requirements pulled into period 1. It is clear that in this matrix old period 2 is now period 1, etc... or in other words the matrix contains now $N - 1$ periods.

Step 5 : repeat steps 2 to 4 until the requirement matrix only contains the requirements for period N.
Example : Consider a 3-product, 3-period lot-size problem. The set-up cost is assumed to be 50 for each product in each period, and the inventory holding cost is 1 per period per product. (We assume constant costs only for simplicity of the example). One unit requires one capacity unit.

Step 1 :

	1	2	3
A	20	30	40
B	40	25	20
C	10	20	20
capacity constraint	120	40	90

Step 2:

	1	2	3
A	20*	30 [0,16]	40 [—]
B	40*	25* [0,25]	20 [—]
C	10*	20* [0,37]	20 [—]
cap const	120	40	90

$$\text{e.g. } U(2)_A = \frac{(50 - 30)/4}{30} = 0{,}166$$

Step 3: The cells indicated with * are allocated to the lots of the different products. The capacity absorption in period 1 is 115 units.

Step 4:

	1	2
A	30	40 [0,06]
B	0	20 [0,37]
C	0	20 [0,37]
	40	90

Step 5: repeat the procedure for the new requirement matrix. No improvements can be obtained compared with the already feasible requirements matrix.

Solution:

	1	2	3
A	20	30	40
B	65	0	20
C	30	0	20
cap. used	115	30	80
constraint	120	40	90

Comments :

The heuristic of Eisenhüt has some important advantages. First, the decision rule based on a marginal anlysis is logic and very easy to apply in practice. Second, the allocation of capacity occurs period after period, so that only near future (accurate) data are needed to make decisions. However, our experience in validating the heuristic on a sample of artificial and real life test problems [1], shows the method to have some serious disadvantages :

1. The coefficients $U(T)_j$ are not always nicely decreasing over time. It is clear that this causes difficulties in the allocation process. This unstable behavior of the coefficient is due to the division of $C'(T)$ by the requirement of period T (cfr formula 9). Instead of using formula (9), we propose to use the following formula :

$$U(T)_j = -C'(T)_j = \left(\frac{s - I(T)}{T^2}\right)_j \qquad (10)$$

This coefficient decreases always over time and facilitates the implementation.

2. The Eisenhüt heuristic goes « uni-directional » through the requirements matrix without feedback to fill up unused capacity. As a result, the heuristic will end up frequently with an infeasible solution. This is certainly the greatest disadvantage of the method. The only way out is to incorporate feed-back mechanisms. The following rule was tested : if for period t, $1 \leqslant t \leqslant N$, infeasibility was encountered, reallocate the unused capacity in periods 1, ... t by including requirements in a lot with less promising appreciation factors but decreasing the unused capacity.

3. According to the Eisenhüt heuristic it is possible to start a production run in a period with zero requirements. That means that the production run satisfies next periods requirements. This will cause, depending on the cost structure, too high inventory holding costs.

In summary we can say that fairly good results were obtained by introducing the following modifications in the heuristic :

— one, use formula (10).

— two, do not start a production run if the period requirement is zero.

— three, incorporate feedback mechanisms to avoid infeasibility.

Note that the Eisenhüt appreciation factor was based on the Part Period Balancing lot-sizing technique. Alternative rules can be found based on other unconstrained lot-sizing heuristics. We dirived an appreciation factor based on the Silver-Meal heuristic, which has been found to be the best heuristic [7].

$$C'(T) = \frac{I'(T)}{T} - \frac{s + I(T)}{T^2} \qquad \text{(see equation 5)}$$

$$C'(T) = \frac{I'(T)}{T} + \frac{s + I(T)}{T} - \frac{s + I(T)}{T} - \frac{s + I(T)}{T^2}$$

let $M(T) = s + I(T)$
then $M(T + 1) = s + I(T) + I'(T)$

$$C'(T) = \frac{M(T + 1)}{T} - \frac{(T + 1).M(T)}{T^2}$$

$$C'(T) = \frac{\dfrac{M(T + 1)}{T + 1} - \dfrac{M(T)}{T}}{T/T + 1} \qquad (11)$$

Let $C'(T) \rightarrow 0$, then the order cycle T* is found if :

$$\frac{M(T^*)}{T^*} \leqslant \frac{M(T^* + 1)}{T^* + 1}$$

or the average cost per period for an order cycle of length T* is smaller than the average cost for an order cycle of length $T^* + 1$. This is in fact the Silver-Meal heuristic [7].

The appreciation factor is :

$$U(T + 1)_i = \left(\frac{\dfrac{M(T)}{T} - \dfrac{M(T + 1)}{T + 1}}{T/T+1}\right)_i \qquad (12)$$

A positive $U(T+1)_j$ means that cost reductions may be expected by including the requirements of period $T+1$ in the lot. For $T=1$ there is no coefficient, note however that this is not a disadvantage since first period requirements must always be satisfied. The disadvantage of this coefficient is that for highly fluctuating requirements the coefficient becomes unstable. The advantage of the proposed coefficient (12) is that the underlying Silver-Meal procedure is better than the Part Period Balancing method [7]. If the capacity constraints are not too restrictive better results are obtained.

B. *Capacitated lot-size problem (C L S P) — P. Newson* [6]

Newson [6] also developed a heuristic for a production facility with limited fixed resources. He first considers the single product case and next the multi-item problem.

The single-product lot-sizing scheduling problem (with no capacity constraints) may be represented as a shortest route problem. Consider the three-period single product model of figure 1.

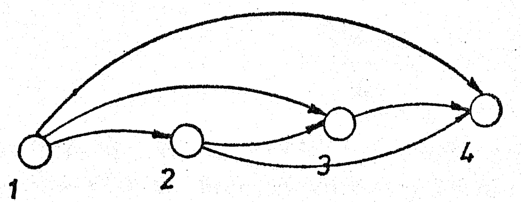


fig 1

There are $2^{(T-1)}$ or for our example 4 paths through the network of our three period problem, corresponding to the 4 pure production plans (see section II). Each arc b_{ij} represents the amount produced in period $i+1$, to satisfy production requirements for period $i+1, \ldots j$, the corresponding cost (set-up and holding costs) is denoted by c_{ij}. There are always $T(T+1)/2$ arcs in the network. The minimum cost path through the network is the optimal unconstrained production plan. Denote this cost by c^0. Under the assumption of limited resources a Next Best Path (NBP) will be defined as the path which relieves some infeasibility at the least marginal cost. It is an optimal path through the network with the currently infeasible arcs deleted. Suppose e.g. that the optimal unconstrained solution is infeasible in period $t+1$, which means that the capacity limitation is exceeded in period $t+1$. The arcs which are responsible for this infeasibility, for example arc b_{tj} (production in period $t+1$, to satisfy requirements up to j) must of course be eliminated from the network as well as the arcs of higher order namely arcs b_{tl}, $l>j$, since they represent an even greater amount of production and hence infeasibility.

The optimal path through this reduced network is the Next Best Path, with total cost equal to c^{New}. The decrease in infeasibility in period $t+1$, is denoted by

$$\Delta w_{t+1} = w^{0}_{t+1} - w^{New}_{t+1}$$

where w_{t+1} denotes the capacity absorption in period $t+1$.

The difference in cost between the old and new production plan is equal to

$$\Delta c = c^{New} - c^{0}$$

The coefficient $DJ = \dfrac{\Delta c}{\Delta w_{t+1}}$

is the implied marginal cost of capacity in the selected infeasible period. This procedure continues until all infeasibilities are eliminated. We will now develop the extension of the *NBP*-concept to multi-product problems. If more products are involved, the problem becomes more complex. Indeed, arcs to be deleted must now be carefully chosen so that the deletion is least likely to effect adversely the final feasible solution. For that reason a criterion which selects among *NBP*'s (for each product) should be a reliable measure of present and future efficiency of that NBP. A NBP must be selected for each infeasible period and for each product. For the derivation of a multi-dimensional DJ, Newson reasons as follows.

Define τ as any period t $(t = 1, 2, \ldots T)$ for which the available capacity is exceeded.

Define $\delta(f_t) = 0$ if the capacity is not exceeded in period t.
$= 1$ if the capacity is exceeded in t.

$$DJ_{i\tau} = \frac{\Delta c_i}{\Sigma_t \left[\delta(f_t) . \Delta w_{i\tau_t}\right]} \qquad \begin{array}{l} i = 1, \dots, N \\ \tau \text{ as defined} \end{array} \qquad (13)$$

The NBP to enter solution has the minimum non-negative DJ, or

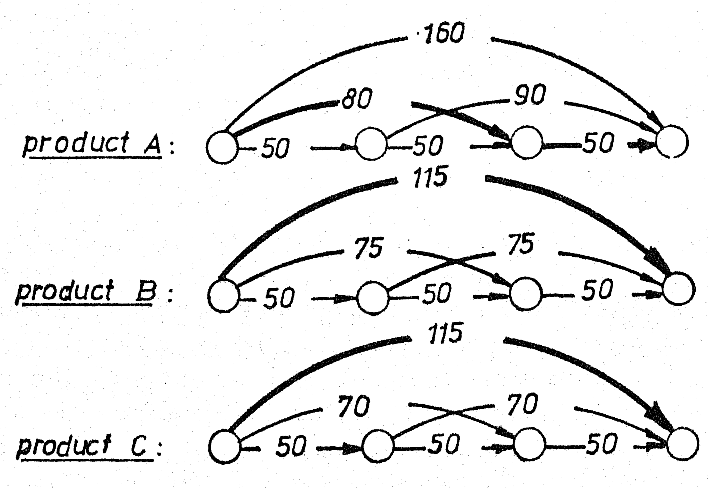
$$min_{i,\tau}\ DJ \geqslant 0$$

The nominator gives the difference in cost between the previous and the new defined NBP for product i, if period τ was the selected infeasible period. The denominator is an identification of the change in infeasibility over the horizon, since $\Delta w_{i\tau_t}$ gives the change in capacity absorption between the old and new program (for product i) and $\delta(f_t)$ indicates whether period t is feasible or not. We are assured of a decrease in infeasibility in period τ by construction (we eliminated the necessary arcs), but the NBP may have decreased infeasibilities elsewhere (as we sum over all t). It is also possible that the denominator is negative indicating an overall increase in infeasibility. Therefore we only select positive $DJ_{i,\tau}$ coefficients. For each τ there will be N NBP's. If there are Q infeasible periods, there will be $N.\ Q$ NBP's among which a selection is done.

Algorithm :

1. For each infeasible period τ, determine a NBP for each product (remember that the arcs which are responsible for infeasibility must be eliminated ; it is clear that at least one path through the network must exist).
2. Evaluate the DJ for all NBP's according to formula (13), and select the minimum $DJ_{i,\tau} \geqslant 0$. For the selected product that enters the solution, the eliminated arcs are definitely cancelled, whereas for the non-selected products the old network is restored.
3. If the solution is still infeasible go to 1, otherwise stop.

An example will illustrate the procedure. Consider the same example as in section III.A. Before starting the Newson heuristic, we will determine the unconstrained lot-sizes by means of a shortest route algorithm. The networks for the 3-products 3-periods example are as follows, with the shortest route indicated by means of heavy arrows :

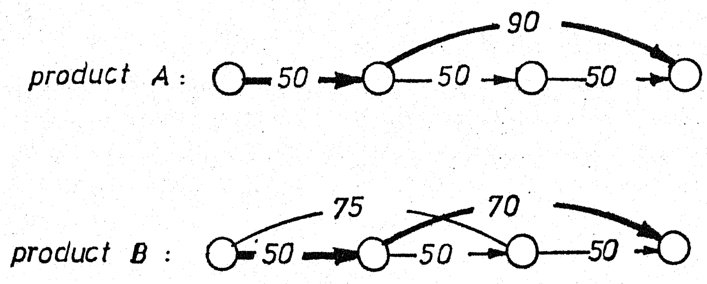


The resulting optimal unconstrained lot sizes are:

	1	2	3
A	50	0	40
B	85	0	0
C	50	0	0
total	185	0	40
Avail cap.	120	40	90

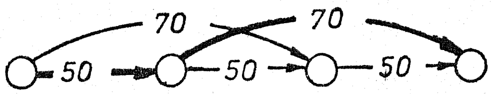
According to the Newson heuristic:

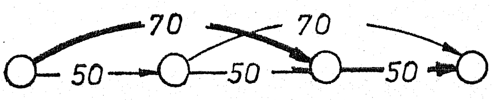
Step 1: the capacity limit is exceeded in the first period. This means $\tau = 1$ or $\delta(f_1) = 1$, $\delta(f_2) = 0$, $\delta(f_3) = 0$.
A NBP must be determined for each product. The resulting networks after the elimination of arcs responsible for the infeasibilities are given below. The corresponding NBP is indicated by heavy lines.



and the two possible NBP's for product C :

product C :





Step 2 :

calculation of the DJ-coefficients :

product A :

$$\begin{array}{lcl|l} \Delta c_A & = & 140 - 130 = 10 & \\ \Delta w_{A11} & = & 50 - 20 = 30 & \times 1 = 30 \\ \Delta w_{A12} & = & 0 - 70 = -70 & \times 0 = 0 \\ \Delta w_{A13} & = & 40 - 0 = 40 & \times 0 = 0 \\ \hline \end{array}$$

$$\sum_t \delta(f_t) . \Delta w_{i\tau_t} = 30$$

$$DJ_{A1} = + 10/30 = 0{,}33$$

product B :

$$\begin{array}{lcl|l} \Delta c_B & = & 120 - 115 = 5 & \\ \Delta w_{B11} & = & 85 - 40 = 45 & \times 1 = 45 \\ \Delta w_{B12} & = & 0 - 45 = -45 & \times 0 = 0 \\ \Delta w_{B13} & = & 0 - 0 = 0 & \times 0 = 0 \\ & & & \overline{45} \end{array}$$

$$DJ_{B1} = 5/45 = 0{,}11$$

product C :

solution C_1

$$\begin{array}{lcl|l} \Delta c_C & = & 120 - 110 = 10 & \\ \Delta w_{C11} & = & 50 - 10 = 40 & \times 1 = 40 \\ \Delta w_{C12} & = & 0 - 40 = -40 & \times 0 = 0 \\ \Delta w_{C13} & = & 0 - 0 = 0 & \times 0 = 0 \\ & & & \overline{40} \end{array}$$

$$DJ_{C1} = 10/40 = 0{,}25$$

solution C_2 $\Delta w_{C11} = 50 - 30 = 20$ | $\times 1 = 20$
$\Delta w_{C12} = 0 - 0 = 0$ | $\times 0 = 0$
$\Delta w_{C13} = 0 - 20 = -20$ | $\times 0 = 0$

$$\overline{20}$$

$$DJ_{C2} = 10/20 = 0{,}5$$

The minimum $DJ_i \geqslant 0$ is $DJ_{B1} = 0{,}11$.

Therefore the reduced product B network will enter the solution and the arc b_{03} is definitely cancelled. The result of this first run is :

	1	2	3
A	50	0	40
B	40	45	0
C	50	0	0
total	140	45	40
Avail cap.	120	40	90

Although we reduced the infeasibility in period 1, we introduced an infeasibility in period 2. Therefore τ will now be equal to 1 and 2. And we start the previous procedure again.
After four additional runs through the algorithm we obtain the same constrained solution as the one obtained by the Eisenhüt method, illustrated in section III.3.
The Newson heuristic has a number of interesting advantages :

1. If the sum of capacities is greater than the sum of the requirements for each product, for all periods, then the heuristic will always find a feasible solution.
2. The algorithm can easily be extended to include more than one constraint.

The disadvantage is that for highly constrained resources in growing industries, several runs through the algorithm are necessary in order to find the first feasible solution. The computer time is much larger compared with the Eisenhüt heuristic.

IV. IMPLEMENTATION - CASE STUDY [1]

The production system we analyzed can be characterized as a job-shop or intermittent system according to a sequential 3 machine- N product production process. The finishing operations (cutting, printing of a tradename or reference number, packaging,...) were excluded from the planning process. To provide effective managerial support to the decision making in production planning, it is useful to partition these decisions in a hierarchical framework (long range, medium range and short range planning). Aggregate decisions are made first and impose constraints within which more detailed decisions have to be made such as the lot-size decisions discussed in this paper. In turn, detailed decisions provide the feedback to evaluate the quality of aggregate decision making. Each hierarchical level has its own characteristics such as the length of the planning horizon, the level of detail of the required information and the managers in charge of the execution of the plans. What is really needed is an interaction mechanism among the different hierarchical levels in order to improve the decisions at all levels. We consider this problem as being of the utmost importance in the design of integrated production systems. To assure this linkage the intervention of the decision maker is necessary and techniques embedded in the process must be flexible enough to meet changing business circumstances.

Our integrated production system goes along the following lines :

Step 1 : The existing and forecasted order portfolio was broken down into its components through the use of a bill of materials file. This process gives the requirements structure for the different operations departments.

Step 2 : Infinite loading. A time phased production program is constructed in the absence of capacity constraints.

Step 3 : Aggregate production model. The infinite loading program of step 2 is used as input for a graphical aggregate production model that results in a capacity plan for the major bottleneck machine groups.

The cumulative production program is projected against the cumulative effective working time in different shift regimes with and without overtime. The purpose is to find a capacity smoothing plan that balances the overtime-undertime costs and the costs associated with inventory build-up.

Step 4: Finite loading. The capacity plan gives the constraints for our constrained lot-size submodel. At this stage of the procedure the lot-size heuristics discussed in this paper were tested. Our modified Eisenhüt heuristic was used because it requires less computational effort. In order to avoid infeasibilities, feedback mechanisms had to be incorporated and manual intervention was still necessary to find feasible plans. An eight week planning horizon was used. The capacity constraints were those of the major bottleneck machine group. The resulting lot-sizes were compared with the capacity limits of the other two machine groups and adjustments were made when necessary. The set-up and change-over time were also introduced in the lot-size submodel.

Step 5: Operations scheduling. As a result of step 4 a week-to-week production plan is obtained, these plans are the input for the day-to-day sequencing problem for the different machine groups. Existing scheduling algorithms were used for this sequencing problem.

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