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SELECTING A BOND-PRICING MODEL FOR TRADING: BENCHMARKING, POOLING, AND OTHER ISSUES

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Selecting a bond-pricing model for trading: benchmarking, pooling, and other issues*

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Preliminary – but comments (and references to this paper) are very welcome

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Abstract

Does one make money trading on the deviations between observed bond prices and values proposed by bond-pricing models? We extend Sercu and Wu (1997)'s work to more models and more data, but we especially refine the methodology. In particular, we provide a normal-return benchmark that markedly improves upon the Sercu-Wu ones in terms of noisiness and bias. We also obtain an impression as to how much of the typical deviation consists of mispricing and how much is model mis-estimation or mis-specification. Lastly, we find that pooled time-series and cross-sectional estimation, as applied by e.g. De Munnik and Schotman (1994), does help in stabilizing the parameter, but hardly improves the trader's profits. In terms of performance for trading purposes there is little difference across models, at least when one re-estimates and trades daily, and we observe no relation with various measures of fit in the estimation stage.

Keywords: Term Structure of Interest Rates, Bonds
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Selecting a bond-pricing model for trading: benchmarking, pooling, and other issues

Introduction

Since the late 1970s, term-structure (TS) theory has evolved from qualitative propositions about shapes of interest-rate curves to very specific, non-linear models that price both bonds and derivatives. Following Sercu and Wu (1997), our test of eight such models center on the question how much money can be made by trading on the deviations between observed bond prices and values proposed by bond-pricing models. Sercu and Wu (SW) report that such trading generates abnormal returns. Our original objective, in the present paper, was to extend their work to more data and more models, including especially two-factor models. However, when we also applied the SW trading-rule to a-select portfolios (like buying short-term bonds only, or long-term bonds only), we found that some of these naive buy-and-hold strategies seemed to provide abnormal returns too. This prompted us to come up with a new benchmark-return strategy that avoids such biases and minimizes noise. Another difference relative to SW is that estimation is not necessarily based on a single cross-section, but alternatively also on five or twenty pooled cross-sections, following De Munnik and Schotman (1994). We find that there are moderate abnormal profits to be made from using formal models, of the order of two to four percent per year. Pooled estimation does help in stabilizing the parameters, but hardly improves the trader's profits. In terms of performance for trading purposes there is little difference across models, at least when one re-estimates and trades daily, and we observe no relation between usefulness as a trading tool and statistical performance measured by the goodness of fit obtained at the estimation stage.

In the remainder of this introduction we position our work relative to other empirical TS work, we justify some fundamental choices in the research design, and we outline the paper.

In general, the empirics spawned by (and providing feedback to) theoretical work relate to either the appropriateness of the models' assumptions, or the prices it produces, or its delta's or hedge ratios. In the first category one strand of studies, illustrated by Chan, Karolyi, Longstaff and Sanders (1992), attempts to accept or reject the stochastic form of the factors put forward in TS theory. Others pragmatically let the data decide on the data-generating process for often-used factors in TS modelling like the short term interest rate, and also try out

additional features like non-linearities (Aït-Sahalia, 1996a, 1996b; Stanton, 1997; Chapman and Pearson, 1999) or volatility clustering (Bali, 2003) or regime shifts (Ang and Bekaert, 2002a, 2002b). Work related to the prices produced by these models, rather than to the underlying processes, ranges from purely fitting the term structure to bond prices (as do *e.g.* De Munnik and Schotman, 1994, and Brown and Dybvig, 1985) to determining whether derivative prices calculated from estimated TSs are close to observed market prices. In practice, deriving sound prices for all types of instruments at the same time proves to be a difficult task. Lastly, rather than studying underlying processes or fitted prices, one can also verify the correctness of the delta or hedge ratios proposed by these models, like *e.g.* Driessens, Klaassen and Melenberg (2000), Gupta and Subrahmanyam (2000), and Sercu and Vinaimont (2003). Our work fits in the second category—prices—but has linkages to the third strand of empirical work: like studies of hedge ratios, it has a dynamic, intertemporal flavor and adopts the professional user's point of view. So while we do look at the static goodness of fit within cross-sections (and even provide a new measure of flexibility), this is mainly to see whether statistical goodness of fit bears any relation to price-change predictability and practical use.

The raw material we work with is straight government bonds, in particular Belgium's "OLO" bonds. The advantage of a simple instrument is that there is absolute clarity with respect to terms and conditions. Also, turnover in OLOs is high. True, we could also include derivative products in the analysis. However, the BEF market was entirely OTC; thus, there is no organized market, no records of transaction prices nor a coherent data set of quotes; and the terms and conditions are not standardized.

The models we select are all closed-form as far as zero-bond bond prices are concerned. This does limit the range of the work. However, selecting these models makes the estimation procedure in essence straightforward, as there is no need for numerical approximations. A concomitant advantage is that all models can be estimated in essentially the same way, non-linear least squares. While an assessment of whether the estimation procedure influences the performance can be interesting as well, we prefer to keep this outside this particular paper. Within the range of closed-form models we limit our selection to a few one- and two-factor models. Our aspiration is not to cover all possible specifications, but to sample a range of models that differ in terms of complexity and ability to fit the data.

We close our introduction with an outline of the paper. In Section 1 we present our shortlist of TS models; we describe the data; and we provide some statistical measures on how each of the models fit the bond prices cross-sectionally. In Section 2, we determine whether models are

able to detect mispricing. This consists of a review and validity check of various measures of normal returns, a regression analysis of abnormal returns, and the implementation of various trading strategies. Section 3 discusses the question whether anything is gained by doing the estimation in pooled cross-sections. Section 4 connects the results from Sections 1 and 2, and concludes.

1 Statistical Fit

1.1 The models

The models we work with are, in order of complexity, (*i*) the cubic spline; (*ii*) two seminal one-factor models, (*iii*) four two-factor models. Most of these are widely known, but to identify the parameter estimates presented below we nevertheless need to agree on a notation. Thus, the key factor processes or equations are presented below.

The Vasicek model. Vasicek(1977) assumes a mean-reverting Gaussian process for the instantaneous interest rate,

$$dr(t) = \alpha\left(\beta - r(t)\right)dt + \sigma dW(t), \tag{1.1}$$

where $\alpha > 0$ is the mean reversion parameter, β the unconditional mean of r(t), σ the volatility of the spot rate, and $W(t)$ a standard Brownian motion. The price of risk is assumed to be constant.

The Cox-Ingersoll-Ross Model. The second model, by Cox, Ingersoll and Ross (1985), is general-equilibrium in nature. It assumes log-utility investors facing a mean-reverting square-root process for output, and from these derives a mean-reverting square-root process for the instantaneous rate and an endogenous price of risk. The process for r is

$$dr(t) = \alpha\left(\beta - r(t)\right)dt + \sigma\sqrt{r(t)}dW(t) \quad , \tag{1.2}$$

where $\alpha > 0$ is the mean reversion parameter, β the unconditional mean of $r(t)$, σ a measure of volatility of the spot rate, and $W(t)$ a standard Brownian motion.

The Richard Model. Starting from the Fisher equation, Richard (1978) assumes that the instantaneous real interest rate (R) and the expected inflation rate (π) each follow a mean-reverting squareroot process:

$$dR(t) = a(R^* - R)dt + \sigma_R\sqrt{R}\, dZ_R(t), \text{ and} \tag{1.3}$$

$$d\pi(t) = c(\pi^* - \pi)dt + \sigma_\pi\sqrt{\pi}\, dZ_\pi(t). \tag{1.4}$$

The correlation between Z_R and Z_π is assumed to be zero. Actual inflation is expected inflation plus noise, and the nominal rate is the real rate plus expected inflation:

$$dP(t)/P(t) = \pi(t)dt + \sigma_P(\pi, R)dZ_P(t), \text{ and} \tag{1.5}$$

$$r(t) = R(T) + \pi(t)(1 - \sigma_P^2). \tag{1.6}$$

The Longstaff and Schwartz model. Longstaff and Schwartz (1992) develop a two-factor general equilibrium model of the term structure that builds upon CIR. They take the short-term interest rate and the instantaneous variance of the short-term interest rate as the two driving factors. The mathematical structure is very smilar to Richards', though. Initially, Longstaff and Schwartz assume two unobservable state variables, X and Y, which follow squareroot processes,

$$dX = (a - bX)dt + c\sqrt{X}dW_2(t), \text{ and} \tag{1.7}$$

$$dY = (d - eY)dt + f\sqrt{Y}dW_3(t), \tag{1.8}$$

and which affect expected returns on investment as follows:

$$\frac{dQ}{Q} = (\mu X + \theta Y)dt + \sigma\sqrt{Y}dW_1(t), \tag{1.9}$$

where W_2 is assumed to be uncorrelated with W_1 and W_3. Assuming log utility, expected growth in marginal utility—the instantaneous interest rate—is expected output minus variance of output. Thus,

$$r(t) = \alpha x + \beta y \tag{1.10}$$

where $\alpha = \mu c^2$, $\beta = (\theta - \sigma^2)f^2$, $x = X/c^2$, $y = Y/f^2$, $\gamma = a/c$, $\delta = b$, $\eta = d/f^2$. The variance of changes in the short-term interest rate is

$$V(t) = \alpha^2 x + \beta^2 y. \tag{1.11}$$

The Balduzzi, Das, Foresi and Sundaram model. Balduzzi, Das, Foresi and Sundaram (2000) develop a two-factor model where the first factor r is the short rate and the second factor, θ, is the mean level of the short rate (in the sense of the long-run level to which the rate is attracted, everything else being the same). The short rate follows the same process as in the Vasicek setting,

$$dr(t) = \kappa(\theta - r)dt + \sigma dW_1(t). \tag{1.12}$$

except that it is attracted not to a constant mean but to a moving target, with

$$d\theta(t) = (a + b\theta)dt + \eta dW_2(t), \tag{1.13}$$

with a, b and η constants. The two processes can be correlated: $dW_1 dW_2 = \rho dt$. The prices of risk are assumed to be constant.

The Baz and Das model. Baz and Das (1996) extend the Vasicek model by adding a Poisson jump process N(t) with intensity rate λ. The process for the short-term rate in the extended Vasicek jump-diffusion process then becomes:

$$dr(t) = \alpha(\beta - r(t))\,dt + \sigma dW(t) + JdN(t). \tag{1.14}$$

with α the mean reversion coefficient, β the long-term mean of the short interest rate, and σ the instantaneous volatility. The intensity of the jump is defined by J, which is assumed to be a normal variable with mean θ and a standard deviation of δ. This one-factor model jump-diffusion model can be easily extended when one assumes two orthogonal factors. To that end two similar processes can be defined:

$$\begin{aligned} dy_1(t) &= \alpha_1(\beta_1 - y_1(t))\,dt + \sigma_1 dW_1(t) + J_1 dN_1(t) && (1.15)\\ dy_2(t) &= \alpha_2(\beta_2 - y_2(t))\,dt + \sigma_2 dW_2(t) + J_2 dN_2(t), && (1.16)\\ r(t) &= y_1(t) + y_2(t) && (1.17) \end{aligned}$$

where $dy_1(t)$ and $dy_2(t)$ are independent.

The Cubic Spline. McCulloch (1975) uses the cubic spline to curve-fit the TS. The price of a discount bond with remaining life τ is then given by

$$P(\tau) = a_1\tau + a_2\tau^2 + a_3\tau^3 + \sum_{j=1}^{K} d_i\,[max(\tau - k_j, 0)]^3 \tag{1.18}$$

where k_i are the K knot points or knots. These divide the maturity range into $K+1$ distinct sections, within each of which the TS follows a cubic and where the cubics smoothly join at the knots. The choice of the number of knots and their values is rather arbitrary. For comparability with Sercu and Wu, we set two knots, at 2 and 7 years. The parameters a_1, a_2, a_3, d_1 and d_2 can be estimated by an ordinary linear regression.

This finishes our presentation of the models and their notation; the data to which these models are taken come next.

1.2 Data

The test ground for our selection of term structure models are a class of Belgian government bonds called *Obligations Linéaires / Lineaire Obligaties* (OLOs). OLOs have many advantages

relative to ordinary Belgian government bonds. OLO maturities nowadays run up to thirty years and contain no embedded option features. Being registered bonds rather than bearer securities, OLOs are mainly held by corporations, making tax clientele effects less likely. Furthermore, Belgian OLOs are actively traded each working day: there are about twenty market makers obliged to quote on request, with a legal bound on the spread. Transaction costs are therefore low and comparable between bonds. We obtain the OLO mid-prices from the Central Bank of Belgium. As Belgian government bonds are mainly traded off the exchange, the Central Bank of Belgium, comparable to practices by the Fed in the US, carries out a daily survey at 3 pm. The quotes represent the view of the biggest market makers on the fair value of each bond and are as such not transaction prices.[1] Our sample contains 29 OLOs in total. We choose our sample period to include all trading days between June 1, 1992 to December 13, 1998. The decision is based on minimum cross-sectional sample size and an even maturity range. Before June 1992, too few OLOs were traded to meaningfully fit the different models. Secondly, on December 13, 1998, for the first time a 30-year OLO gets introduced. Before that date, the longest issues were OLOs with 20 years to maturity. The first issue of the 30-year OLO creates a serious gap in time to maturity/duration between the 30-year bond and the bond with the next-longest time to maturity (then 18 years). Limiting the sample to December 1998 also reduces potential influence from the introduction of the common currency in the Euro-zone. We also include T-bill data for six maturities (two and four weeks; and 2, 3, 6 and 12 months) to enhance the estimation of the short end of the term structure. The T-bills, however, do not enter the performance tests.

1.3 Estimation of the Term Structure Models

Note that we estimate directly from all available raw coupon-bond data, not from a few zero-coupon interest rates or swap quotes. That is, each coupon-bond price is written as the sum of the present values of its pay-outs, each of these present values being specified as the zero-coupon-bond pricing equation of the model that is being considered. In the unpooled estimation, our base case, the procedure is that for each day in the sample we estimate the models cross-sectionally by non-linear least squares, that is, by minimizing the sum of squared errors between observed bond prices and fitted values. The optimization method used is a

[1] The advantage of using mid-prices instead of transaction prices is that we need to worry less about bid-ask bounces, non-synchronized data or temporal liquidity shocks creating extra noise in transaction data.

Table 1: Cross-sectional estimation of the Vasicek model, estimated and derived parameters

	Vasicek																
	ϕ_0	α	ϕ_1	ϕ_2	r	R_L	μ	σ^2									RMS
average	0.20483	0.37432	0.37575	-0.01756	0.05288	0.08631	-0.00786	-0.00193									0.0018
Median	0.15103	0.31329	0.26753	-0.02941	0.05281	0.08591	-0.00518	0.00436									0.0015
	Cox-Ingersoll-Ross																
	θ_1	θ_2	θ_3	r		R_L	σ^2	μ									RMSE
average	0.31491	0.24019	1.20002	0.04859	0.08255	0.05943	0.0074										0.0021
Median	0.22633	0.10522	1.08310	0.05036	0.08298	0.01885	0.0081										0.0020
	Richard																
	a	c	σ_R	ϕ_R	σ_π	ϕ_π	R^*	π^*	σ_P	R	π	r					RMSE
Average	-0.00027	0.23	0.030	0.54	0.2900	-35.60	573.36	0.245	0.907	0.069	-0.042	0.059					0.0016
Median	0.00000	0.21	0.001	0.54	0.0005	-0.00012	0.00088	0.244	0.919	0.064	-0.000	0.062					0.0015
	Longstaff-Schwartz																
	α	β	δ	γ	ν	η	r	V	R_L								RMSE
average	0.1055	0.2142	0.0272	1.1847	1.1711	-0.4185	0.0547	0.0096	0.0866								0.0015
Median	0.0535	0.0937	0.0728	0.2563	0.2793	0.0510	0.0472	0.0044	0.0870								0.0013
	Balduzzi-Das-Foresi-Sundaram																
	κ	σ	a	b	η	ρ	λ	r	θ								RMSE
Average	0.1673	0.0012	0.0603	-1.3691	0.0943	2.8690	-1.123	0.0529	0.0337								0.0012
Median	0.1302	-0.0001	0.0636	-.5533	0.0744	0.3071	0.4021	.04507	.03927								0.0010
	Baz-Das																
	α_1	α_2	ε_1	ε_2	σ_1	σ_2	β_1	β_2	θ_1	θ_2	δ_1	δ_2	λ_1	λ_2	Y_1	Y_2	RMSE
Average	1.2402	0.2694	0.5199	-3.7014	-0.0063	0.0054	0.0495	0.0410	-0.0461	0.2756	0.0114	0.1741	0.3384	0.0757	0.1188	0.0618	0.0017
Median	1.2634	0.2531	0.0999	-3.3513	0.0000	0.0051	0.0410	0.0138	-0.0117	0.0168	0.0000	0.0001	0.1659	0.0065	0.0364	0.0333	0.0016
	Cubic Spline																
	a_1	a_2	a_3	d_1	d_2												RMSE
average	-0.05086	-0.00061	-0.00002	0.00018	-0.00018												0.0014
Median	-0.04600	-0.00333	0.00030	-0.00009	-0.00020												0.0013

Marquardt procedure.

Averages and medians of the coefficients and of the implied numbers with a ready economic interpretation, like the long-run asymptotic interest rate, produce acceptable values, as can be seen in table 1.

As expected from pure cross-sectional regressions, and as documented before by *e.g.* Brown and Dybvig (1985) and De Munnik and Schotman (1994), parameters occasionally turn nonsensical for some subperiods and some models. For instance, estimated implicit variances can be negative. The alternative would be to force specific parameters to behave within theoretical constraints—for instance, ≥ 0 for the variance. However, all too often the solution then is to set the parameter at the bound. Also, estimation then often turns unstable or the models show absolute inability in fitting the term structure. Nonsensical estimates and unstable solutions tend to mean that the objective function is hardly affected by the parameter. By allowing the parameters to free-range, we are mainly assessing whether the functional form of the model provides a good tool to summarize the term structure. Violations of theoretical constraints do not necessarily mean that this specific model is less useful. Indeed, one of the themes in this paper is to investigate the link between practical usefulness, complexity and fit.

1.4 Goodness of fit, cross-sectional and longitudinal

In this section, we explore the characteristics of the regression residual and proceed by ranking the models according to their ability to fit the coupon bond prices in the market. Summary statistics on the bond-price residuals can be found in Table 2. The tables are not set up per individual bond because of the changing time to maturity as time passes on. Instead, we package the bonds into six simple time-to-maturity portfolios. For every day in the sample, the first portfolio combines residuals from bonds with time-to-maturity not exceeding 1 year at that time. The other groups similarly contain bonds from 1 to 2, 2 to 4, 4 to 8, 8 to 15 and over 15 years time to maturity. A puzzling feature of the average errors per bracket is that, for all time-to-maturity brackets, all models are unanimous about the direction of mispricing; even the *ad hoc* spline, with the fewest restrictions on the TS shape, perfectly agrees with the average errors of the more structured models. Especially CIR has problems with estimating the shortest end of the TS, with an average error of minus 13.3 and and average absolute error of 18 basis points. Our concern, initially, had been that no model would be able to capture the short end of the TS well, characterized as it was by a sharp hump during the 1992 and 1993 turmoil in the Exchange Rate Mechanism. However, the period was, apparently, too short to

Table 2: Summary numbers on bond-price residuals from pure cross-sectional estimation, grouped by time-to-maturity.

Key: Bond-price residuals for each model are grouped into time-to-maturity brackets. The summary statistics we show are the Average Error (Avg) and the Average Absolute Pricing Error (AAE) per time-to-maturity bracket. All numbers are in basis points and par value for bonds equals 100.

		vasicek	cir	rich	ls	bdfs	b-d	spline
>3m ≤1y	avg	-2.9	-13.3	-4.4	-3.7	-3.8	-1.4	-4.1
	AAE	8.3	18.0	9.3	7.3	7.3	7.8	6.9
>1y ≤2y	avg	3.1	2.6	0.3	-0.8	0.7	2.7	2.0
	AAE	8.6	11.4	7.1	6.4	6.9	8.2	8.2
>2y ≤4y	avg	1.0	2.1	1.6	1.4	1.3	-4.3	1.9
	AAE	10.5	10.9	9.1	7.4	8.6	14.2	8.7
>4y ≤8y	avg	-4.6	-1.7	0.3	-1.2	-2.2	-4.3	-2.5
	AAE	14.2	16.5	14.3	13.5	13.6	14.2	12.5
>8y ≤15y	avg	7.4	5.3	2.7	3.2	4.1	8.1	2.8
	AAE	24.0	24.2	21.6	21.0	21.6	24.3	18.6
>15y	avg	-10.3	-9.0	-5.4	-5.0	-5.6	-12.4	-1.2
	AAE	16.1	13.5	12.6	12.0	11.1	19.8	7.5
overall	avg	0.19	0.30	0.58	0.17	0.18	0.33	0.26
	AAE	15.6	16.9	14.3	13.3	13.7	15.8	12.4

matter in the average. Instead, across all models, the highest Average Absolute Errors (AAEs) are actually found in the bracket containing bonds with time-to-maturity between eight and fifteen years.

The ranking for overall AAEs and average RMSEs is summarized in table 3, alongside other rankings that will be introduced later. The top three models in terms of fit for both criteria are BDFS, Longstaff-Schwartz and the spline. The worst model with respect to these criteria is CIR's, even though the differences are never staggering. Yet that ranking is totally overturned as soon as we adopt two other measures of goodness of fit that have to do with longitudinal properties. These measures are (*i*) the autocorrelation in the residuals, extracted from the daily cross-sectional regressions and grouped bond by bond into time series; and (*ii*) the average run length, *i.e.* the average number of consecutive days where the residuals of a given bond all have the same sign. Both are measures of persistence of unexplained bond values. Of course, these numbers do not tell us whether any high persistence is due to market inefficiency (the market is slow to realize and correct its mistakes) or model mis-specification (some TS shapes cannot be captured, and since the shapes persist, the apparent pricing errors persist too) or persistent mis-estimation; but the same ambiguity reigns with respect to the

Table 3: Size and persistence of errors, across models

Key: We show two measures of unexplained variability in prices, the Average Absolute Error (AAE) and the Average Root Mean Square, the average standard deviation of the residuals. Both are measured in basis points. Also shown are the autocorrelation, averaged across bonds, of the time series of residuals per bond extracted from each cross section, and the average run length (in days), where a run is defined as a sequence of days where the residuals have the same sign.

	vasicek	cir	rich	ls	bdfs	b-d	spline
				statistics			
AAE	15.6	16.9	14.3	13.3	13.7	15.8	12.4
ARMSE	17.5	20.5	16.0	14.6	12.0	17.1	13.9
autocorr	0.94	0.85	0.74	0.85	0.86	0.73	0.93
avg runl	17.6	12.2	7.7	14.9	13.9	7.4	17.7
				ranking of models			
AAE	5	7	4	2	3	6	1
RMSE	6	7	4	3	1	5	2
autocorr	7	3	2	4	5	1	6
avg runl	6	3	2	5	4	1	7

MSE of a regression.

The autocorrelations are unexpectedly high, ranging from 0.74 (Baz-Das) to 0.93-0.94 (Vasicek and the spline), with most other models hovering around 0.85. In the same vain, correcting a mistake requires on average anywhere between 7 days (Baz-Das) and 18 (Vasicek). Equally unexpectedly, there is little connection between size and persistence of pricing errors. The spline, which does well in terms of cross-sectional fitting, produces quite persistent errors while CIR, rather bad at fitting across bonds, does relatively better in terms of persistence. The distinct performance of Baz-Das in terms of persistence—the difference with the second best is quite marked—is likewise hard to explain from the MSEs. Still, the size of the autocorrelations and the lengths of runs of same-sign residuals is disconcerting. It is hard to believe that all of this would be pure market efficiency; rather, inability to capture twists in the TS seems to be at least as plausible an explanation. This second view could fit in with our earlier observation that all models seem to produce similar errors for bonds in the same time-to-maturity bracket. Tests of what part of the error is market mistakes versus model misspecification are the main subject of the next section.

2 Market errors *v* model errors

In the previous section we established a ranking of the competing models based on the natural belief that smaller errors between model and observed prices translate in better pricing capabilities of that model. Especially for the purpose of pricing options, many potential users of a

model would balk if that model misprices the underlying. The fact that there still is a residual would be acceptable if these were random and shortlived deviations caused by, say, transaction costs or stale data and causing, in turn, some random estimation error in the coefficients too. The high persistence of the residuals we observed belies this: there must be a market error or inefficiency and/or a model error. In this section we attempt to quantify these components.

2.1 Decomposing scaled residuals into abnormal returns and model errors

Denote observed and fitted prices by $P_{i,t}$ and $P^*_{i,t}$, respectively, and use $V_{i,t}$ to denote the unobservable true value. Ideally we would decompose the residual into a market error (the pricing mistake) and a model error, *i.e.* mis-specification and -estimation:

$$RES_{i,t} \stackrel{\text{def}}{=} P_{i,t} - P^*_{i,t} \tag{2.19}$$

$$= \underbrace{P_{i,t} - V_{i,t}}_{\text{market error at } t} + \underbrace{V_{i,t} - P^*_{i,t}}_{\text{model error at } t} . \tag{2.20}$$

The obvious problem is that the true value is not observable. We proceed initially under the testable null that the previous price is correct and the fitted price useless. (We later generalize.) If so, then $V_{i,t} = P_{i,t-1}\left[1 + \widehat{HP}_{i,t-1}\right]$, with $\widehat{HP}_{i,t-1}$ the normal holding-period return for a bond with the terms and conditions of i between days $t-1$ and t. The normal return should also take into account market movements in the TS during that day. Leaving aside, for a moment, the question of how the normal return should be identified, we proceed as follows. Under the null, the decomposition becomes

$$RES_{i,t} = \underbrace{P_{i,t} - P_{i,t-1}\left[1 + \widehat{HP}_{i,t-1}\right]}_{\text{market error at } t, \text{ under } H_0} + \underbrace{P_{i,t-1}\left[1 + \widehat{HP}_{i,t-1}\right] - P^*_{i,t}}_{\text{model error at } t, \text{ under } H_0} . \tag{2.21}$$

In the previous section we looked at the autocorrelation in the price-scaled residuals. These variables can now be decomposed into

$$\frac{RES_{i,t}}{P_{i,t-1}} = \left[\frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}} - \widehat{HP}_{i,t-1}\right] + \left[1 + \widehat{HP}_{i,t-1} - \frac{P^*_{i,t}}{P_{i,t-1}}\right],$$

$$= \underbrace{HP_{i,t-1} - \widehat{HP}_{i,t-1}}_{\text{abnormal return}} + \left[\widehat{HP}_{i,t-1} - \left[\frac{P^*_{i,t}}{P_{i,t-1}} - 1\right]\right] \tag{2.22}$$

Thus, first, the components of the scaled residual are (*i*) the abnormal return, and (*ii*) the difference between the normal return and the percentage price-change needed to wipe out yesterday's apparent mispricing; and, second, the abnormal-return component is directly linked

to the percentage market error.[2] The component we chose to study is the familiar abnormal return. Under the null $P_{i,t-1} = V_{i,t-1}$, yesterday's apparent mispricing is a fiction created by bad models, so it can affect neither the normal return nor the actual one. Thus, we should find no link between abnormal return and yesterday's deemed mispricing. In the regression below,

$$\begin{aligned} AR_{i,t} &\stackrel{\text{def}}{=} HP_{i,t-1} - \widehat{HP}_{i,t-1}, \\ &= a_{i,t} + b_{i,t}\frac{RES_{i,t-1}}{P_{i,t-1}} + \varepsilon_{i,t} \end{aligned} \tag{2.23}$$

the slope b should be zero. This testable proposition can be weakened to situations where yesterday's price is not fully correct, but still correct only up to random noise. Then an errors-in-variables problem biases the slope towards zero but as the prediction is a zero slope anyway, this does not matter under the Null.

The Null, in the above, was that prices are correct possibly up to random noise; the fitted model value had, by assumption, no information content. Suppose, more generally, that the true price is a convex combination of the model's fitted value and the observed price. (Below, we will call the weight of the model price in the convex combination—w, in the first equation below—the "relevance" coefficient for the model.) It is easy to show that, in that case, the slope should be between zero and -1:

$$\begin{aligned} \text{if } V_{i,t} &= (1-w)P_{i,t-1} + w\ P^*_{i,t-1} \\ &= P_{i,t-1} - w\ RES_{i,t-1} \\ \text{and } HP_{i,t-1} &= \alpha\left[\frac{V_{i,t-1} - P_{i,t-1}}{P_{i,t-1}}\right] + news \\ \text{then } AR_{i,t-1} &= -\alpha\ w\ RES_{i,t-1} + (news - \widehat{HP}_{i,t-1}) \end{aligned} \tag{2.24}$$

In general, therefore, the size of the slope b in (2.23) depends on w (the relevance of the fitted price in the convex combination) and α (the adjustment speed between actual and true value). The limit case is $b = -1$: the fitted price fully captures yesterday's true value, and all of yesterday's pricing error is set right overnight (up to random noise). If the first equation contains noise, the estimates of $\alpha \cdot w$ are biased towards zero.

From the length of the runs in the time series of residuals studied in the previous section, the adjustment speed over one single day may be quite low. To have an idea of the relevance of yesterday's residual, w, separately, we extend the holding period from one day to two and

[2]This assumes that the previous price is correct; but since deviations are a matter of a few basis points, the inaccuracy introduced by an incorrect lagged price is very much second order of smalls.

four weeks—a horizon that encompasses the longest average run lengths documented in the previous sections, so that the adjustment can be assumed to be reasonably complete. Then b gives an indication of the long-run relevance w, probably still biased towards zero because of an errors-in-variables problem.

To make all this operational, we need an acceptable measure of normal return. This is discussed in the next section.

2.2 Normal returns and abnormal returns

We need to know what the normal holding-period return is between days $t-1$ and t on a bond like i, given what happened in the market during that period. By definition there are no individual bonds with matching characteristics in the market,[3] and if there had been one, it would have been impossible to determine whether that matching bond is priced correctly on days $t-1$ and t. We have considered five models for the normal return, three of them already adopted in SW (1997) and two new ones.

The own-model fitted return. From the model and the daily parameter estimates SW compute fitted prices at times $t-1$ and t, and from those they compute the expected return. This approach does take into account market movements as well as the i-th bond's terms and conditions. But the fitted prices are subject to model-specification and -estimation errors. More fundamentally, if we want to distinguish the performance of various models, each model will have its own benchmark and each model is temporarily presumed to be the correct one. We abandoned this approach on these grounds.

A delta-neutral zero-investment portfolio. A rather different approach would be to discard the use of benchmarks completely and construct portfolios with the allegedly underpriced bonds held long and the overpriced bonds short, setting the weights such that the entire position is delta-neutral. This appraoch is subject to estimation errors in levels and deltas and does not allow a separate analysis of under- and overpriced bonds.[4] The interpretation is ambiguous, too: we are testing, at the same time, the model's ability to get the bond price right *and* its delta(s). Finally, comparison across models becomes harder, as for each test the framework is again model dependent.

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[3]Subsequent bond issues with the same terms and conditions are assimilated with the original "line", hence the same "linear bond".

[4]Shortsell restrictions may reduce the market's ability to reduce overpricing.

The duration-based market model. This benchmark return, proposed by Elton and Gruber (1991) and adopted by SW, is based on the "market mode" familiar from stock-market studies. In the bond-market version the bond's market sensitivity or beta is not estimated but computed, notably as the ratio of the duration of the target bond to the duration of the market as a whole. This approach does take into account the market movements and the i-th bond's terms and conditions, at least in sofar as they affect duration. Estimation errors in the duration are minimal, and pricing errors are largely diversified away by taking a wide portfolio as the basis. By construction, this approach generates a zero abnormal return for the market as a whole, that is, it is correct on average, across all bonds. Long and short positions can be studied separately. The drawback is that it only works under the well-known duration-model assumptions. Non-parallell shifts, like rotation, may (and do) induce serious errors in the estimated normal returns for short or long bonds separately.

The duration-and-convexity matching portfolio. In this benchmark, proposed by SW, one constructs a mimicking portfolio from three equally weighted subportfolios, each consisting of all available short, middle and long bonds, respectively, that are in the market. The mimicking consists of matching price, duration and convexity of the target bond. Because three subportfolios are used and the problem is linear, the weights for each of the subportfolios are uniquely defined. This model has similar pros and cons as the duration market-model. One difference is that, being a quadratic approximation rather than a linear one, this model is better suited to deal with large shifts. Also, since it uses three portfolios, it will price correctly, on average, each of the three subclasses of bonds rather than just the market-wide average bond. However, it may (and does) still misprice the very short or very long bonds. Also, the three benchmark portfolios, consisting of just one third of the (limited) market, are less well diversified than the market portfolio and, therefore, more subject to measurement error.

The minimum-variance duration-and-convexity matching portfolio. In this last approach in our list we form a matching portfolio not from three pre-determined portfolios but from all individual bonds (except, of course, for the bond that is being studied). The weights x_i for each traded bond i are chosen so as to minimize the variance of the portfolio subject to the constraint that the portfolio weights sum to unity and that the portfolio has the same duration and convexity of the bond that is to be matched.[5] To estimate the covariance matrix of the

[5] We also stop bonds from taking up more than one quarter of the portfolio, so that the mimicking portfolios are well-diversified; but it turns out that this restriction is never binding.

bonds that enter the portfolio, we use 60 trading days of historical returns. When a bond does not trade over the 60-day estimation period (e.g. when it was issued very recently) it is not included as a possible candidate for the portfolio. Relative to the previous normal-return model, this approach does look for a portfolio that best resembles the bond to be studied. Duration and convexity are just taken as two conveniently familiar characteristics that heavily rely on time to maturity, and also the minvar approach helps guaranteeing that we pick bonds with similar characteristics.

We have validity-tested the three methods that avoid the circularity of using the model to be evaluated.[6] Which of these three return benchmarks is most reliable can be determined by examining the "abnormal" returns realized by holding static, a-select portfolios (e.g. an equally weighted portfolio of short-lived bonds). Abnormal returns on such test portfolios measured against the benchmark candidates should on average be close to zero.

Figure 1 provides plots of the time series of accumulating abnormal returns generated by the candidate benchmarks for four equally weighted portfolios: the total sample, and the bonds in the 4-8, 8-15, and >15 year brackets. The duration ratio model does well for the all-bond portfolio, by construction, but rather badly fails the test for subportfolios: after 6 years, the cumulative "abnormal" return on this simple investment strategy peaks at 8% for the short bonds and drops to minus 14% for the long bonds. The results for Sercu and Wu's three-portfolio duration-convexity matched investments are only marginally better. The minimum-variance benchmark, by contrast, prices all time-to-maturity-bracket portfolios correctly and performs equally well for the all-sample portfolio, never drifting farther than one percent from the zero line. We therefore use, in what follows, the minimum-variance benchmark to calculate abnormal returns.

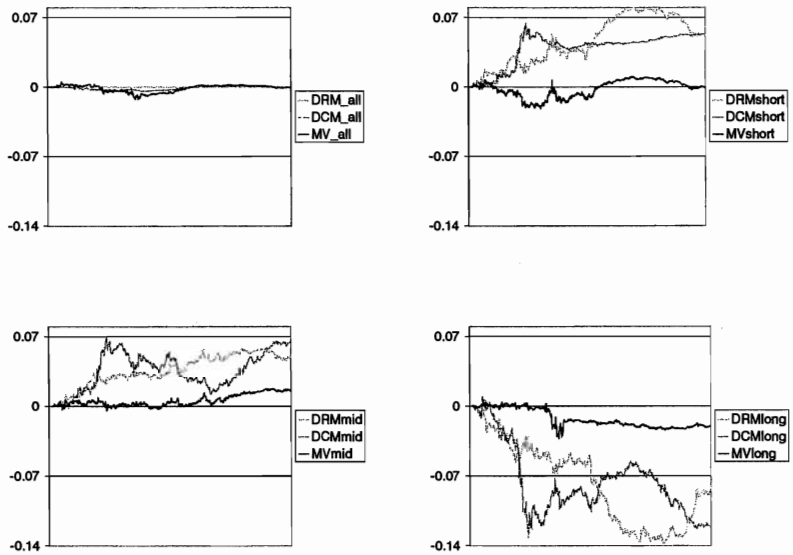
2.3 Regression Tests

Using the normal-return model validated in the preceding section we can now compute abnormal returns on each bond. The next step is to regress the abnormal return for bond i between t-1 and t, $AR_{i,t}$, on $RES_{i,t-1-L}/P_{i,t-1-L}$, the relative pricing error observed at the beginning

[6]The portfolios used to construct Duration-and-Convexity matches are the T-bills, the 1-to-3-year bonds, and the >3-year issues, as in SW.

Figure 1: Benchmark Tests.

Key: Three control strategies are tested for producing (near-)zero abnormal returns. These control strategies are (*i*) a duration-matching combination of the T-bills and the OLO bond market portfolio (DRM); (*ii*) a duration-and-convexity-matching combination of the T-bills, the OLO bond market below 3 years, and the market above 3 years (DCM); and (*iii*) a minimum-variance duration-and-convexity-matching combination of all bonds expect the one to be matched (MV). The four test portfolios used below are the market portfolio and three time-to-maturity-bracket funds: ≤3y, 3-8y, and >8y; in the graphs below they are labeled "all", "short", "medium", and "long".



of the holding period ($L = 0$) or the day before ($L = 1$):

$$AR_{i,t} = a_{i,t} + b_{i,t}\frac{RES_{i,t-1-L}}{P_{i,t-1-L}} + \varepsilon_{i,t} \tag{2.25}$$

Recall from (2.24) that $b = -\alpha \cdot w$, where α is the adjustment speed and w the relevance of the model's fitted price, both numbers between 0 and 1. The version with $L = 0$ was discussed before. SW also work with $L = 1$ (and higher, in fact). One reason is that their prices are transaction prices, thus inducing bid-ask bounce correlated with the next-day return.[7] Our data being midpoint quotes, the problem does not arise here. Nevertheless, the trader interested in the information content cannot instantaneously import the data, run a complicated non-linear regression, and still buy or sell the very moment a quote is given. Even though building-in a full 24-hour delay vastly exxagerates in the other direction, it is the best we can do with daily data. In addition to experimenting with $L = 1$, we vary the holding period in the abnormal return from 1- to 10- and 20-day periods; at 20 days, the degree of adjustment towards the correct price, α, should be close to unity and the corresponding b therefore gives an idea of the total relevance of the observed price discrepancy. We run these 2×3 regressions for each individual bond and test two specific hypotheses H_1: $b = 0$ and $a = 0$ (that is, no relevance); and H_2: $b = -1$ and $a = 0$ (perfect relevance, and full adjustment within the holding period).

Table 4 summarizes the regression results (average, mean, significance and sign of the estimates) for 1-, 10- and 20-day holding periods, and for $L = 0$ (top part) and $L = 1$ (bottom part). For virtually all regressions with respect to one-day holding periods we see negative estimates of b for both immediate and one-day-lagged trading. Most of these are also significant; the rare positive estimates, in contrast, are never significant. Thus, statistically there is an information content and the market does react to it. Algebraically, however, the average immediate one-day reaction coefficients are low—between -0.052 (CIR) and -0.083 (LS)—and the next-day reactions are up to one-half lower again.

If these low one-day immediate reaction coefficients reflect sluggishness in the market rather than a low relevance coefficient, then a low b is good news for a trader. In an attempt to extract from this b coefficient the relevance coefficient w, we increase the holding period for AR to 10 and 20 days. Average slope coefficients for a two-week holding period are now much more seizable, ranging between -0.20 (BDFS) and -0.28 (LS); and adding another 2 weeks further

[7] If the last trade at $t-1$ is at the *bid*, then the residual tends to be low, while the subsequent return starting from that low price tends to be high. This biases b negatively.

Table 4: Regression tests on abnormal returns: market v model errors

Key: We regress $AR_{i,t} = a_{i,t} + b_{i,t}[RES_{i,t-1-L}/P_{i,t-1-L}] + \varepsilon_{i,t}$ with $AR_{i,t}$ = abnormal return for bond i between $t-1$ and $t-1+\Delta$, $\Delta = \{1, 10, 20\}$ days; and $RES_{i,t-1-L}/P_{i,t-1-L}$ = the bond's L-days-lagged relative pricing error, $L = \{1, 2\}$ days. Entries like "pos 27(19)" mean that 27 coefficients were positive, whereof 19 significantly so.

Panel A: Instant Reaction ($L = 0$)

	Vasicek (1F)		CIR (1F)		Richard (2F)		LS (2F)		BDFS (2F)		Baz-Das (2F)		Spline	
1 day	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*
average	-0.058	5.30E-07	-0.052	1.60E-05	-0.062	2.22E-05	-0.083	6.70E-06	-0.064	1.70E-05	-0.056	9.82E-06	-0.076	1.69E-05
median	-0.037	8.60E-06	-0.040	8.70E-06	-0.041	7.94E-06	-0.039	3.20E-06	-0.041	9.70E-06	-0.038	8.62E-06	-0.054	5.60E-06
# neg	27(19)	12(4)	27(19)	8(1)	27(21)	9(1)	27(19)	11(3)	25(19)	8(1)	26(19)	9(2)	27(20)	12(3)
# pos	0(0)	15(2)	0(0)	19(2)	0(0)	18(2)	0(0)	16(4)	2(0)	19(3)	1(0)	18(2)	0(0)	15(4)
10 day	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*
average	-0.232	1.60E-05	-0.249	2.77E-05	-0.223	5.07E-05	-0.281	3.74E-05	-0.208	4.15E-05	-0.216	3.24E-05	-0.275	1.21E-05
median	-0.226	5.51E-05	-0.273	7.22E-05	-0.213	6.05E-05	-0.240	5.64E-05	-0.218	8.84E-05	-0.213	6.68E-05	-0.281	3.07E-05
# neg	24(20)	11(5)	27(26)	12(3)	26(21)	11(4)	25(19)	11(3)	23(20)	11(3)	25(20)	9(4)	24(21)	12(5)
# pos	3(0)	16(8)	0(0)	15(10)	1(1)	16(10)	2(0)	16(7)	4(0)	16(10)	2(0)	18(10)	3(0)	15(9)
20 day	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*
average	-0.311	3.83E-05	-0.362	3.80E-05	-0.311	7.63E-05	-0.353	5.86E-05	-0.298	5.62E-05	-0.279	5.45E-05	-0.373	2.4E-06
median	-0.319	1.47E-04	-0.369	1.50E-04	-0.270	1.38E-04	-0.367	8.73E-05	-0.358	1.64E-04	-0.293	1.47E-04	-0.394	7.3E-05
# neg	23(19)	12(5)	27(21)	12(3)	23(20)	11(6)	22(20)	11(5)	22(20)	10(5)	22(20)	9(4)	23(20)	12(7)
# pos	4(1)	15(11)	0(0)	15(12)	4(1)	16(11)	5(1)	16(10)	5(3)	17(11)	5(1)	18(11)	4(2)	15(10

Panel B: Delayed Reaction ($L = 1$)

	Vasicek (1F)		CIR (1F)		Richard (2F)		LS (2F)		BDFS (2F)		Baz-Das (2F)		Spline	
1 day	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*
average	-0.028	3.20E-06	-0.032	7.30E-06	-0.029	8.27E-06	-0.042	8.40E-06	-0.028	8.70E-06	-0.028	2.87E-06	-0.037	4.90E-06
Median	-0.027	8.60E-07	-0.027	5.30E-06	-0.025	7.25E-06	-0.027	4.10E-06	-0.025	6.80E-06	-0.027	5.41E-06	-0.034	6.50E-07
# neg	23(11)	13(1)	27(17)	9(1)	25(8)	10(1)	27(11)	12(2)	25(9)	11(1)	26(11)	9(1)	24(12)	13(2)
# pos	4(0)	14(0)	0(0)	18(1)	2(0)	17(1)	0(0)	15(3)	2(0)	16(3)	1(0)	18(1)	3(0)	14(2)
10 day	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*
average	-0.181	2.09E-05	-0.209	1.21E-05	-0.170	3.09E-05	-0.206	3.75E-05	-0.154	2.74E-05	-0.163	2.49E-05	-0.209	5.97E-07
Median	-0.210	6.65E-05	-0.226	6.39E-05	-0.184	6.30E-05	-0.211	4.46E-05	-0.145	7.84E-05	-0.188	6.16E-05	-0.240	4.09E-05
# neg	23(18)	12(4)	27(21)	12(3)	24(18)	11(2)	23(16)	11(3)	22(17)	10(3)	23(19)	9(3)	23(18)	12(5)
# pos	4(0)	15(7)	0(0)	15(10)	3(1)	16(7)	4(0)	16(8)	5(1)	17(8)	4(0)	18(9)	4(1)	15(8)
20 day	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*
average	-0.260	4.27E-05	-0.322	3.09E-05	-0.256	5.98E-05	-0.269	6.15E-05	-0.242	4.51E-05	-0.229	4.79E-05	-0.303	-8.2E-06
Median	-0.301	1.13E-04	-0.324	1.51E-05	-0.273	1.23E-04	-0.315	8.04E-05	-0.297	1.27E-04	-0.254	1.07E-04	-0.338	5.7E-05
# neg	23(18)	11(5)	27(21)	13(3)	23(18)	11(5)	22(18)	10(5)	21(20)	11(6)	22(20)	11(4)	23(19)	12(7)
# pos	4(3)	16(11)	0(0)	14(12)	4(2)	16(10)	5(1)	17(10)	6(4)	16(11)	5(2)	16(11)	4(3)	15(10)

boosts the coefficients to at least -0.28 (Baz and Das) and occasionally even -0.37 (spline). Thus, the news is good from the trader's point of view. First, 30 percent or more of the observed price discrepancy is relevant in the sense that it gets reflected in the price within one month. And second, the adjustment seems to be slow: even a trader that has to wait a full day before reacting loses a mere 3-5 percent of that 30-plus. On the downside, note that the 20-day return is noisier, too: the relative importance of the initial mispricing shrinks because, over a longer horizon, there are so many other influences affecting the price. This noisiness is reflected in the variability of the 20-day b coefficients across bonds, an indicator of which is the number of instances with the wrong (positive) sign for the 20-day-AR regressions.

The economic importance of all this is still unclear as the initial signals are quite small: 30 percent of a 15-bp mispricing is not a large gain. Thus, we need to know how often large gains occur, whether it is wortwhile focusing on large gains only, and so on. These issues are addressed in the next section.

2.4 Base-Case Trading Rules: set-up and results

We construct contrarian portfolios by buying underpriced bonds and selling overpriced bonds. Contrarian strategies are based on the deviation of observed asset prices from their fundamental values. The further an observed asset deviates from its fundamental value, the larger should be the correction and, therefore, the higher the weight that should be assigned to the asset in the contrarian portfolio. In implementing this trading strategy, we set up two basic portfolios, a "buy" portfolio, where weights are assigned to undervalued assets, and a "sell" portfolio that contains overpriced assets. When we construct such a time-$(t-1)$ short or long portfolio p (where $p = s$ (sell) or b (buy)) on the basis of the pricing errors observed at $t-1-L$, with $L = 0$ for instant trading and $L = 1$ for delayed trading, then we set the weight for bond i as follows:

$$w_{p,i,t-1-L} = \frac{RES_{i,t-1-L} D_{p,i,t-1-L}}{\sum_{i=1}^{N_{p,t}} RES_{i,t-1-L} D_{p,i,t-1-L}}, p = b, s \quad , \tag{2.26}$$

where $RES_{i,t-1-L}$ is the residual for bond i as estimated from the time-$(t-1-L)$ cross-section; $D_{b,i,t-1-L} = 1$ if $RES_{i,t-1-L}$ is positive and 0 otherwise; $D_{s,i,t-1-L} = -1$ if $RES_{i,t-1-L}$ is negative and 0 otherwise; and $\mathrm{N}_{p,t}$ the number of assets in portfolio p. Note that $w_{p,i,t-1-L} \geq 0$ and $\sum_{i=1}^{N_t} w_{p,i,t-1-L} = 1$. The abnormal return of a contrarian strategy can then be measured as

$$AR_{p,t,L} = \sum_{i=1}^{N_t} w_{p,i,t-1-L} D_p AR_{i,t}, \quad , \tag{2.27}$$

where $p = (b, s)$, D_p is equals 1 when $p = b$ and −1 when $p = s$. This is our base-case setup. In variants discussed in the next section we ignore the smaller signals *RES* and/or trade less frequently than daily.

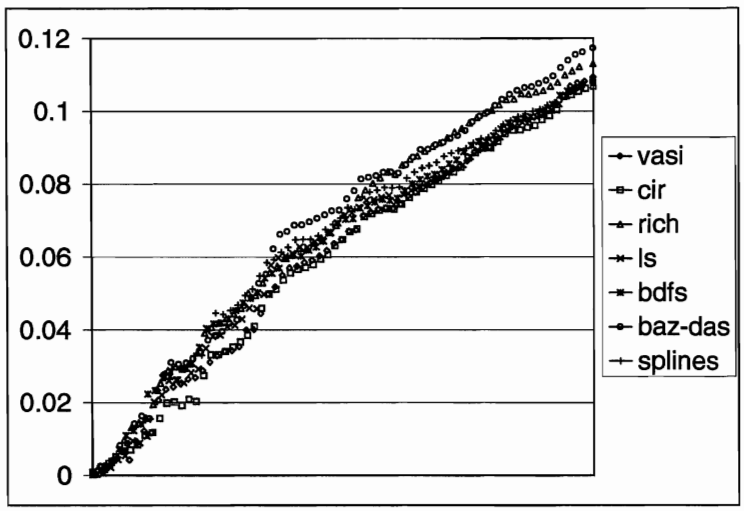
Table 5 displays percentage profits from contrarian strategies, cumulative over 6.5 years, for $L = 0$ or 1. All the outcomes are statistically very significant, so t-stats are omitted. Our discussion is centered on the combined payoffs from buying and selling ("$b + s$", in the table), which are obtained by adding the accumulated gains from the long and short positions and expressing them as a fraction of the initial notional value. (Since "$b + s$" is a zero-investment strategy, the resulting percentages are not returns in the usual sense.) The table also provides cumulative abnormal returns for buy and sell separately, but there is little to say about these except that they are usually quite similar, and always statistically indistinguishable.[8]

At this stage we are interested in the base-case numbers only, starting with one-day holding periods and instantaneous trading. Although the pricing models seemed rather different in terms of in fit, persistance of mispricing, and reaction coefficients, all models produce very similar "$b + s$" CARs, ranging from 21% to 23% over 6.5 years—about 3% *per annum*. The results are not due to one or two freak episodes; rather, they accumulate steadily over time throughout the period, as can be seen from Figure 2 where the evolution of contrarian profits over time is illustrated for immediate trading. Nor are the results due to a few bonds or to one or two maturity classes: when we group the *CAR*s of individual bonds into the six time-to-maturity brackets used before, we find that each of the brackets contributes positively.

When introducing a one-day lag between signal recognition and the actual trading, CARs drop markedly, by about 11% cumulative: a one-day interval between the signal and the execution of trades yields CARs between 11% and 12.5% in total, *i.e.* about 1.5% *p.a.* True, it is unlikely that professional investors need 24 hours to import the data and run a regression, so that the realistically feasible profits are probably closer to the no-lag profits than to the once-lagged result. Still, the "$L = 0$" results are too optimistic. In the next tests, we try and jazz up the base case by being more selective: should we react to each signal, no matter how small? Also, how much is lost if we trade every 10 or 20 days rather than daily? It turns out that a good dose of selectivity recuperates half of the revenue that would be lost by waiting a full day.

[8]The buy results do dominate the sell returns in most cases, but in view of the enormous dependencies across the experiment it is probably dangerous to attach much importance to this observation.

Figure 2: Evolution of CARs over time. Pure cross-sectional estimation. Minimum variance portfolio used as Benchmark.



The results are before costs. There are no records of detailed spreads per market maker or best quotes at any moment, but in those days spreads were of the order of magnitude of 6 bp (of the price). Given an annual churn rate of about 25, two-costs would amount to about 1.5% *p.a.* for a buy or a sell strategy and 3% for $b+s$, which would reduce the base-case strategies to mere break-even propositions at $L = 0$, and loss proposisions at $L = 1$. However, the selective applications return far more, as documented below. More fundamentally, many banks trade for liquidity reasons. So their transaction costs are inevitable and, therefore, irrelevant for our purpose. Given that they have to buy or sell, the message is that it is worth pausing two seconds to run a simple spline regression before the trade. For a portfolio manager who faces random in- and outflows every day, a quick look at the residuals would have added about 1.5% to the annual return.

2.5 Filtering out the smaller discrepancies or revising less often

In the preceding section, the bond weights were proportional to the estimated discrepancy; still, we might be able to improve the results by altogether eliminating the bonds with the smallest residuals. Two obvious reasons are that the expected gain is small anyway (a relevant

Table 5: Cumulative Abnormal Returns for trading Strategies, in percent

Key: In the base case, all bonds are held (short or long depending on the sign of the initial or lagged mispricing), while in the filtered versions only the top 50% or 25% of the mispricing signals are acted upon, the rest is ignored. The best among the buy strategies and the best among the sell strategies of a given row are indicated by sharps ($^\sharp$); the worst buy and sells are indicated by flats ($^\flat$).

Instant Reaction (L=0)

	One-factor models						Two-factor models												(n.a.)		
	Vasicek			CIR			Richard			LS			BDFS			Baz-Das			Spline		
	b+s	buy	sell	b+s	buy	sell	b+s	buy	sell	b+s	buy	sell	b+s	buy	sell	b+s	buy	sell	b+s	buy	sell
	Panel A: One-day holding period																				
base case	21.8	11.5	10.3	21.4	11.8	$^\flat$9.5	22.8	$^\sharp$13.0	9.8	21.6	11.6	9.9	21.6	$^\flat$11.3	10.3	23.4	12.7	$^\sharp$10.8	21.6	11.6	9.9
50% biggest	20.0	$^\flat$10.3	9.8	25.0	13.4	11.6	28.0	$^\sharp$15.4	12.6	25.8	13.0	12.8	25.4	12.4	13	27	13.8	$^\sharp$13.2	20.8	11.2	$^\flat$9.6
25% biggest	34.8	$^\flat$16.4	$^\sharp$18.5	33.8	18.4	15.3	34.6	$^\sharp$20.1	$^\flat$14.5	32.8	17.4	15.5	35.6	17.7	18	34.8	17.5	17.3	33.2	17.4	15.7
	Panel B: Two-week Holding Period																				
base case	11.2	6.5	4.6	11.0	6.4	4.7	10.8	$^\sharp$6.8	4.1	10.8	6.4	4.4	10.2	$^\flat$5.6	4.7	12.4	6.6	$^\sharp$5.9	9.8	5.9	$^\flat$4.0
50% biggest	10.0	5.6	4.5	12.6	6.7	5.9	13.0	7.5	5.5	12.8	7.0	5.9	12.0	6.6	5.4	14.6	$^\sharp$8.1	$^\sharp$6.5	9.2	$^\flat$4.9	$^\flat$4.4
25% biggest	17.0	10.6	6.5	15.2	9.1	$^\flat$6.0	17.0	$^\sharp$10.9	6.2	17.8	10.2	7.6	16.6	9.9	6.7	18.4	10.5	$^\sharp$7.8	14.2	$^\flat$7.7	6.6
	Panel C: One-Month Holding Period																				
base case	8.0	4.9	3.1	8.8	5.2	3.6	8.4	$^\sharp$5.4	3.1	7.8	4.7	3.1	7.8	$^\flat$4.2	3.5	9.0	4.9	$^\sharp$4.0	7.4	4.6	$^\flat$2.7
50% biggest	7.2	4.1	$^\flat$3.1	10.2	5.6	$^\sharp$4.6	10.2	5.9	4.3	9.2	4.8	4.4	9.2	5.1	4.0	10.6	$^\sharp$6.1	4.4	6.8	$^\flat$3.6	$^\flat$3.1
25% biggest	11.8	7.2	4.5	11.4	7.2	$^\flat$4.1	13.0	$^\sharp$8.3	4.7	12.4	6.7	$^\sharp$5.7	12.4	7.6	4.8	13.2	7.9	5.3	10.4	$^\flat$5.4	4.9

Delayed Reaction (L=1)

	Vasicek			CIR			Richard			LS			BDFS			Baz-Das			Spline		
	b+s	buy	sell	b+s	buy	sell	b+s	buy	sell	b+s	buy	sell	b+s	buy	sell	b+s	buy	sell	b+s	buy	sell
	Panel A: One-day holding period																				
base case	13.0	6.9	6.0	13.2	7.6	$^\flat$5.6	13.6	$^\sharp$8.0	$^\flat$5.6	13.4	7.0	6.4	12.8	$^\flat$6.4	6.4	14.6	7.9	$^\sharp$6.8	13.4	7.0	6.4
50% biggest	10.8	5.7	$^\flat$5.2	16.2	$^\sharp$9.3	6.9	15.8	8.8	7.1	15.6	7.5	8.0	15.4	7.8	7.6	17.2	8.8	$^\sharp$8.5	10.8	$^\flat$5.5	5.3
25% biggest	19.0	10.8	8.1	17.8	12.0	$^\flat$5.8	22.0	$^\sharp$12.5	9.5	21.8	10.0	$^\sharp$11.8	17.2	9.4	7.9	17.8	9.4	8.3	14.8	$^\flat$8.8	6.1
	Panel B: Two-week Holding Period																				
base case	9.6	5.8	3.8	9.8	5.7	4.1	9.4	$^\sharp$5.9	3.4	9.4	5.6	3.7	8.8	$^\flat$4.8	4.0	10.6	5.6	$^\sharp$4.9	8.2	5.0	$^\flat$3.3
50% biggest	8.6	4.8	$^\flat$3.7	11.0	5.9	5.2	11.2	6.5	4.7	11.2	6.1	5.1	10.4	5.8	4.6	12.4	$^\sharp$7.0	$^\sharp$5.4	7.8	$^\flat$4.0	3.8
25% biggest	14.4	$^\sharp$9.5	5.0	12.6	7.9	$^\flat$4.7	14.4	9.3	5.1	15.4	8.9	$^\sharp$6.5	14.2	8.6	5.6	15.6	9.2	$^\sharp$6.5	11.8	$^\flat$6.2	5.6
	Panel C: One-Month Holding Period																				
base case	7.2	4.5	2.8	8.0	4.8	3.3	9.4	$^\sharp$5.0	3.4	7.0	4.3	2.7	7.0	$^\flat$3.8	3.1	8.0	4.4	$^\sharp$3.6	6.4	4.1	$^\flat$2.3
50% biggest	6.6	3.8	$^\flat$2.7	9.4	5.2	4.2	9.2	5.4	3.7	8.2	4.3	$^\sharp$3.9	8.2	4.7	3.5	9.6	$^\sharp$5.7	$^\sharp$3.9	6.0	$^\flat$3.2	$^\flat$2.7
25% biggest	10.6	6.7	3.8	10.2	6.7	$^\flat$3.4	11.6	$^\sharp$7.6	4.1	11.0	6.0	$^\sharp$5.1	11.0	7.0	4.0	11.8	7.3	4.5	9.0	$^\flat$4.7	4.3

consideration when trading is costly) and that noise is probably important relative to the signal. More subtly perhaps, if mispricing takes time to disappear, mispricing may also take time to build up; if so, it is better for the trader to wait until the discrepancy is peaking before moving in.

When building our selective portfolios, we again construct two groups, one containing bonds with negative residuals and one including bonds with positive residuals. In each group and for each trading day, we now rank the bonds in terms of the size of the absolute residual. We try out two variants of filtering: the first rule keeps only the bonds with the 50% biggest absolute pricing errors in each group, while the second filter is even more selective and considers only bonds in the top quartile of absolute pricing errors. Individual bond weights are then again weighted as indicated in equation (2.26), except that, of course, we are more picky when setting the Ds.

Panel A in table 5, the second and third lines in each cell provide the CARs from the contrarian strategy based upon the 50% and the 25% loudest signals of each day. Introducing the mild filter has a positive but unspectacular effect for most models; there are, even slightly negative effects for the Vasicek model and the spline. The Richard model benefits most (at lag 0), with CARs increasing 5%, but for $L = 1$ the effect is far smaller. The jump model by Baz and Das still remains the best performing model, with CARs now up to 27% for lag 0 and 17% for lag 1. When introducing the strong filter, in contrast, outcomes do change dramatically, in some instances almost doubling the CARs for the base case. CARs are shown in panel C. In contrast to the introduction of a weaker filter, now also the Vasicek model and spline function benefit from using the stronger filter. The BDFS model and Richard model benefit most: CARs increase with 14% for immediate trading and now attain a level of 36% (almost 5% *p.a.*). Note as well that CARs remain on average very high even for longer lags.

Many private investors would not bother to evaluate and rebalance their portfolios each and every day. Thus, in this section we also investigate to what extent a reduction in the frequency of trading erodes the abnormal returns of the contrarian strategies and filter rules. In a first experiment we consider a holding period of two weeks. After the trade is made based on the contrarian strategy weights, the portfolio holding remains unchanged for two weeks. At the end of the two-week period, we then identify the then prevailing over- and underpricing and adjust the portfolio accordingly. In a second variant, we consider a holding period of one month. As in the previous sections, we investigate, next to immediate trading, the influence of a one-day difference (lag 1) between the mispricing signal and the actual trade.

Earlier, we showed that mispricing tends to gradually disappear, but with the largest adjustments in the days immediately after the detection of the pricing errors. By rebalancing only one every tenth trading day, for instance, we miss nine out of the ten best days; and in a filtered version of the trading rule, we also hold on to positions that would have been liquidated already if rebalancing had been done on a daily basis. Thus, when considering longer holding periods, and therefore less frequent rebalancing, CARs must inevitably erode. The good news, as shown in Table 5, is that the effects of rebalancing every two weeks and each month are not dramatic: for the base case without filter, CARs remain positive, in the 8-10% range. Predictably, CARs for monthly revisions are lower than for two-week periods. The difference between starting the period immediately (L=0) and leaving one day in-between (L=0) is relatively small. Again, introducing filters seriously enhances the CARs. By and large, the best performing models are the two-factor models. The spline comes out a clear last, this time.

2.6 To pool or not to pool?

A last variant we discuss is about the estimation stage rather than the trading rule itself. Schotman (1996) remarks that day-by-day cross-sectional regressions generate a lot of variability in the parameters and hence in the implied deltas, which would trigger many (probably pointless) trades for the derivatives desk. One recommended solution is to combine several consecutive cross sections. We implement this with 5- and 20-day pooling. In the economic models we constrain the parameters to be equal across cross-sections if they are assumed to be intertemporally constant. The risk-free rate, an implied number, notably is left to vary from day to day, and so is the other factor in the two-factor models. For the spline, there is no good theoretic reason to fix some parameters; indeed, when we fix all parameters the results are so atrocious that we do not bother to show them. Lastly, the pooled estimations for the Baz-Das model usually failed utterly to converge. So we are now down to five competing models.

The results, as summarized in Table 6, are not encouraging. The general rule is that pooling worsens the results, and pooling 20 days is worse than 5. There are a few exceptions: BDFS tends to improve marginally, and the combination of filtering 50% with pooling 5 days beats the base-case estimation about half of the time. But in the absence of a good reason why these exceptions would be externally valid, the general conclusion seems to be that pooling does not help for current purposes.

Table 6: Cumulative Abnormal Returns for trading Strategies, in percent: pooled versus unpooled

Key: In the base case, all bonds are held (short or long depending on the sign of the initial or lagged mispricing), while in the filtered versions only the top 50% or 25% of the mispricing signals are acted upon, the rest is ignored. The best among the buy strategies and the best among the sell strategies of a given row are indicated by sharps ($^\sharp$); the worst buy and sells are indicated by flats ($^\flat$).

filter	pooling	Vasicek			CIR			Richard			Longstaff-Schwartz			BDFS		
		b+s	buy	sell	b+s	buy	sell	b+s	buy	sell	b+s	buy	sell	b+s	buy	sell
Panel A: One-day holding period																
base case	none	21.8	11.5	10.3	21.4	11.8	9.5	22.8	$^\sharp$13.0	9.8	21.2	11.6	9.9	21.6	11.3	10.3
	5 days	18.9	9.1	9.8	19.0	9.4	9.6	21.3	12.0	9.3	21.2	10.6	10.6	20.8	9.9	10.9
	20days	16.8	$^\flat$7.6	9.3	14.4	$^\flat$7.6	$^\flat$6.8	22.9	11.8	$^\sharp$11.1	19.3	10.0	9.4	21.2	10.7	10.5
50% biggest	none	20.0	10.3	9.8	25.0	13.4	11.6	28.0	$^\sharp$15.4	12.6	25.8	13.0	12.8	25.4	12.4	13.0
	5 days	23.9	11.1	12.8	22.9	10.7	12.2	24.9	14.1	10.8	26.8	13.4	13.4	26.1	12.2	$^\sharp$13.9
	20days	20.7	$^\flat$8.4	12.3	18.8	11.1	$^\flat$7.8	26.7	13.6	13.1	24.4	12.5	11.9	25.5	12.5	13.0
25% biggest	none	34.8	16.4	18.5	33.8	18.4	15.3	34.6	$^\sharp$20.1	14.5	32.8	17.4	15.5	35.6	17.7	18.0
	5 days	29.6	13.0	16.6	25.4	13.7	11.7	32.1	18.5	13.6	33.6	16.0	17.6	35.4	16.4	$^\sharp$19.0
	20days	24.8	$^\flat$10.0	14.8	25.4	15.4	$^\flat$10.0	28.6	16.3	12.3	28.5	16.0	12.6	33.6	17.6	15.9
Panel A: two-week holding period																
base case	none	11.2	6.5	4.6	11.0	6.4	4.7	10.9	$^\sharp$6.8	4.1	10.8	6.4	4.4	10.2	5.6	4.7
	20days	9.1	$^\flat$3.9	5.3	7.6	$^\flat$3.9	$^\flat$3.7	11.3	5.7	5.5	9.8	4.7	5.1	12.5	6.2	$^\sharp$6.3
50% biggest	none	10.0	5.6	4.5	12.6	6.7	5.9	13.0	7.5	5.5	12.8	7.0	5.9	12.0	6.6	5.4
	5 days	12.6	6.6	6.1	10.8	5.6	5.3	12.6	6.8	5.8	13.3	6.7	6.6	13.3	6.9	6.5
	20days	11.4	$^\flat$4.8	6.6	9.7	5.5	$^\flat$4.1	13.5	7.0	6.5	12.2	6.0	6.2	15.4	$^\sharp$7.9	$^\sharp$7.6
25% biggest	none	17.0	10.6	6.5	15.2	9.1	6.0	17.0	$^\sharp$10.8	6.2	17.8	10.2	7.6	16.6	9.9	6.7
	5 days	16.0	8.6	7.5	11.6	7.1	4.5	15.6	8.6	7.0	16.9	8.5	8.4	17.2	9.4	7.8
	20days	13.2	$^\flat$4.9	8.2	12.1	7.7	$^\flat$4.4	15.8	8.6	7.3	14.9	7.0	8.0	19.6	10.3	$^\sharp$9.3
Panel A: One-month holding period																
base case	none	8.0	4.9	3.1	8.8	5.2	3.6	8.5	$^\sharp$5.4	3.1	7.8	4.7	3.1	7.8	4.2	3.5
	5 days	7.6	3.9	3.6	7.1	3.6	3.4	8.5	4.7	3.8	8.0	4.1	3.8	8.1	4.2	3.9
	20days	7.1	$^\flat$3.4	3.7	6.1	$^\flat$3.4	$^\flat$2.7	8.4	4.3	4.1	6.8	3.7	3.1	9.1	4.7	$^\sharp$4.4
50% biggest	none	7.2	4.1	$^\flat$3.1	10.2	5.6	4.6	10.3	$^\sharp$5.9	4.3	9.2	4.8	4.4	9.2	5.1	4.0
	5 days	9.2	5.0	4.2	8.3	4.5	3.8	9.7	5.4	4.3	9.5	5.2	4.4	9.9	5.3	4.6
	20days	8.5	$^\flat$4.0	4.5	7.7	4.5	3.2	9.7	5.2	4.5	8.6	4.9	3.7	11.0	5.9	$^\sharp$5.1
25% biggest	none	11.8	7.2	4.5	11.4	7.2	4.1	13.0	$^\sharp$8.3	4.7	12.4	6.7	5.7	12.4	7.6	4.8
	5 days	11.3	6.3	5.0	8.8	5.7	$^\flat$3.0	11.9	6.7	5.2	11.9	6.7	5.2	13.0	7.3	5.7
	20days	10.0	$^\flat$4.4	5.5	9.8	6.3	3.5	11.0	6.2	4.8	10.7	6.2	4.5	14.0	7.7	$^\sharp$6.3

Table 7: Various measures of performance, across models

Key: We show two measures of unexplained variability in prices, the Average Absolute Error (AAE) and the Average Root Mean Square, the average standard deviation of the residuals. Both are measured in basis points. Also shown are the autocorrelation, averaged across bonds, of the time series of residuals per bond extracted from each cross section, and the average run length (in days), where a run is defined as a sequence of days where the residuals have the same sign. Next come the regression coefficients of abnormal returns on initial mispricing, for 1- or 20-day holding periods and with or without lag ($L = (1, 0)$. Lastly we show some CARs, for daily and monthly revision frequencies and for trading rules where we act only upon the 50 or 25 percent strongest signals. In the second part of the table we show the ranks of the models rather than the statistics.

	vasicek	cir	rich	ls	bdfs	b-d	spline
	statistics						
AAE	15.6	16.9	14.3	13.3	13.7	15.8	12.4
ARMSE	17.5	20.5	16.0	14.6	12.0	17.1	13.9
autocorr	0.94	0.85	0.74	0.85	0.86	0.73	0.93
avg runl	17.6	12.2	7.7	14.9	13.9	7.4	17.7
b, 1d, $L = 0$	-0.058	-0.052	-0.062	-0.083	-0.064	-0.056	-0.076
b, 20d, $L = 0$	-0.311	-0.362	-0.311	-0.353	-0.298	-0.279	-0.373
b, 20d, $L = 1$	-0.260	-0.322	-0.256	-0.269	-0.242	-0.229	-0.303
CAR, daily, 50%, $L = 0$	20.0	25.0	28.0	25.8	25.4	27.0	20.8
CAR, monthly, 25%, $L = 0$	11.8	11.4	13.0	12.4	12.4	13.2	10.4
CAR, daily, 50%, $L = 1$	10.8	16.2	15.8	15.6	15.4	17.2	10.8
CAR, monthly, 50%, $L = 1$	10.6	10.2	11.6	11.0	11.0	11.8	9.0
	ranking of models						
AAE	5	7	4	2	3	6	1
RMSE	6	7	4	3	1	5	2
autocorr	7	3	2	4	5	1	6
avg runl	6	3	2	5	4	1	7
b, 1d, $L = 0$	5	7	4	1	3	6	2
b, 20d, $L = 0$	3	2	4	3	6	7	1
b, 20d, $L = 1$	4	1	5	3	6	7	2
CAR, daily, 50%, $L = 0$	7	5	1	3	4	2	6
CAR, monthly, 25%, $L = 0$	5	6	2	3	3	1	7
CAR, daily, 50%, $L = 1$	6	2	3	4	5	1	6
CAR, monthly, 50%, $L = 1$	5	6	2	3	4	1	7

3 Conclusion

In this paper we fit a set of term structure models to government bonds.[9] The central question is whether a fixed-income-desk trader who faces an in- or outflow can more or less randomly pick a bond in a desirable time-to-maturity bracket, or instead should take a few minutes or seconds to run a cross-sectional regression. We find she should. A trader who wants to swap an overpriced bond for an underpriced one should be selective and heed only clear signals, because for these non-liquity-driven trades transaction costs are not irrelevant. Still, also for this purpose the regression residuals are useful. Another reliable finding is that there is no good case to be made for pooling, at least for our purpose; rather, the indications are mostly against such pooling. A third result is that duration- or duration-and-convexity matched control strategies are not reliable, at least when they work with pre-set portfolios covering a wide time-to-maturity spectrum. What is needed, instead, is a control portfolio with similar bonds, like the minimum-variance portfolio we adopt here.

Which model to select, if profitability is the criterion? The models are conspicuous in the similarity of their cumulative abnormal returns, at least for the base case of daily rebalancing. For filtered applications and less frequent revisions the results are more divergent, but it remains unclear to what extent this is a reliable result or just a reflection of the higher randomness one expects when there are far fewer trades. While applications in other data may shed light on this, we think that, for anyone hoping for a reliable ranking, the omens are not good. Table 7 summarizes some performance measures, both statistical and economic ones, along with the models' rankings for each of the criteria. A comparison of the spline and the Baz-Das model serves to make the case. In terms of MSE the spline looks near-perfect and Bas-Daz way below average; yet these rankings switch almost perfectly when we look at another measure of (in)flexibility, the persistence of the deviations between observed and fitted values. The same happens when we consider economic content rather than statistical fit. On the basis of the regressions one would have anticipated a great future for the spline-based trading rule, as the spline's residuals seemed to come out way ahead in terms of predicting subsequent abnormal returns. Yet the spline does bad in the trading experiments. And Bas-Daz does very well there, even though its regression coefficients were about the worst among all models. Thus,

[9]The Belgian data set is not particular in any way, but allows to work with a comfortable number of bonds. We do not believe this choice is driving any particular results. The findings in Sercu and Wu, using a similar set of Belgian data, have been confirmed by German data.

one may be better off chasing wild geese than clearly outstanding models.

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