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Dependency of Risks and Stop-Loss Order

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Abstract

The correlation order, which is defined as a partial order between bivariate distributions with equal marginals, is shown to be a helpfull tool for deriving results concerning the riskiness of portfolios with pairwise dependencies. Given the distribution functions of the individual risks, it is investigated how changing the dependency assumption influences the stop-loss premiums of such portfolios.

Keywords: dependent risks, bivariate distributions, correlation order, stop-loss order.

1 Introduction

Consider the individual risk theory model with the total claims of the portfolio during some reference period (e.g. one year) given by

$$S = \sum_{i=1}^{n} X_i \tag{1}$$

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where X_i is the claim amount caused by policy i $(i = 1, 2, \ldots, n)$. In the sequel we will always assume that the individual claim amounts X_i are nonnegative random variables and that the distribution functions F_i of X_i are given.

Usually, it is assumed that the risks X_i are mutually independent because models without this restriction turn out to be less manageable. In this paper we will derive results concerning the aggregate claims S if the assumption of mutually independence is relaxed. More precisely, we will assume that the portfolio contains a number of couples (e.g. wife and husband) with non-independent risks. Therefore, we will rearrange and rewrite (1) as

$$S = \sum_{i=1}^{m} (X_{2i-1} + X_{2i}) + \sum_{i=2m+1}^{n} X_i \tag{2}$$

with m the number of coupled risks. For any i and j $(i,j = 1, 2, \ldots, n;\ i \neq j)$ we assume that X_i and X_j are independent risks, except if they are members of the same couple (X_{2k-1}, X_{2k}), $(k = 1, 2, \ldots, m)$. The class of all multivariate random variables $(X_1, \ldots, X_n)$ with given marginals F_i of X_i and with the pairwise dependency structure as explained above, will be denoted by $R(F_1, \ldots, F_n)$.

It is clear that for any $(X_1, \ldots, X_n)$ belonging to $R(F_1, ..., F_n)$, the riskiness of the aggregate claims $S = X_1 + ... + X_n$ will be strongly dependent on the way of dependency between the members of couples.

In order to compare the riskiness of the aggregate claims of different elements of $R(F_1, \ldots, F_n)$, we will use the stop-loss order.

Definition 1 *A risk S_1 is said to precede a risk S_2 in stop-loss order, written $S_1 \leq_{sl} S_2$, if their stop-loss premiums are ordered uniformly:*

$$E(S_1 - d)_+ \leq E(S_2 - d)_+$$

for all retentions $d \geq 0$.

Let $(X_1, \ldots, X_n)$ and $(Y_1, \ldots, Y_n)$ be two elements of $R(F_1, \ldots, F_n)$ and denote their respective sums by

$$S_1 = \sum_{i=1}^{m} (X_{2i-1} + X_{2i}) + \sum_{i=2m+1}^{n} X_i$$

and

$$S_2 = \sum_{i=1}^{m} (Y_{2i-1} + Y_{2i}) + \sum_{i=2m+1}^{n} Y_i$$

We want to find ordering relations between the corresponding couples of S_1 and S_2 which imply a stop-loss order for S_1 and S_2. More precisely, we are looking for a partial order $\leq_{ord}$ between bivariate distributed random variables which has the following property:

$$(X_{2k-1}, X_{2k}) \leq_{ord} (Y_{2k-1}, Y_{2k}) \qquad (k = 1, 2, \ldots, m) \tag{3}$$

implies

$$S_1 \leq_{sl} S_2 \tag{4}$$

A well-known property of stop-loss ordering is that it is preserved under convolution of independent risks, see e.g. Goovaerts et al.(1990). Hence, a sufficient condition for (4) to be true is

$$X_{2k-1} + X_{2k} \leq_{sl} Y_{2k-1} + Y_{2k} \qquad (k = 1, 2, \ldots, m) \tag{5}$$

So it follows immediately that we can restrict ourselves to the following problem: Find a partial order $\leq_{ord}$ between bivariate distributed random variables (X_1, X_2) and (Y_1, Y_2) with the same marginal distributions, for which the following property holds:

$$(X_1, X_2) \leq_{ord} (Y_1, Y_2) \tag{6}$$

implies

$$X_1 + X_2 \leq_{sl} Y_1 + Y_2 \tag{7}$$

It is clear that an ordering $\leq_{ord}$ for which (6) implies (7) will immediately lead to a solution of the problem described by (3) and (4).

2 A partial order for bivariate distributions

2.1 Correlation order

Let $R(F_1, F_2)$ be the class of all bivariate distributed random variables with given marginals F_1 and F_2 . For any $(X_1, X_2) \in R(F_1, F_2)$ we have

$$F_1(x) = Prob(X_1 \leq x) \qquad F_2(x) = Prob(X_2 \leq x)$$

We also introduce the following notation for the bivariate distribution function:

$$F_{X_1,X_2}(x_1, x_2) = Prob(X_1 \leq x_1, X_2 \leq x_2)$$

In the sequel we will always restrict ourselves to the case of non-negative risks. Further, if we use stop-loss premiums or covariances, we will always silently assume that they are well-defined.

Now let (X_1, X_2) and (Y_1, Y_2) be two elements of $R(F_1, F_2)$. In order to investigate an order between these bivariate distributed random variables which implies stop-loss order for $X_1 + X_2$ and $Y_1 + Y_2$, we could start by comparing $Cov(X_1, X_2)$ and $Cov(Y_1, Y_2)$. At first sight, one could consider the following inequality

$$Cov(X_1, X_2) \leq Cov(Y_1, Y_2) \tag{8}$$

and investigate wether this implies

$$X_1 + X_2 \leq_{sl} Y_1 + Y_2 \tag{9}$$

Although it is customary to compute covariances in relation with dependency considerations, one number alone cannot reveal the nature of dependency adequately, and hence (8) will not imply (9) in general, a counterexample is given in Dhaene et al.(1995). However, in the special case that F_1 and F_2 are two-point distributions with zero and some positive value as mass points, (8) and (9) are equivalent, see also Dhaene et al.(1995).

Instead of comparing $Cov(X_1, X_2)$ and $Cov(Y_1, Y_2)$ one could compare $Cov(f(X_1), g(X_2))$ with $Cov(f(Y_1), g(Y_2))$ for all non-decreasing functions f and g, see e.g. Barlow et al.(1975).

Definition 2 *Let (X_1, X_2) and (Y_1, Y_2) be elements of $R(F_1, F_2)$. Then we say that (X_1, X_2) is less correlated than (Y_1, Y_2), written $(X_1, X_2) \leq_c (Y_1, Y_2)$, if*

$$Cov(f(X_1), g(X_2)) \leq Cov(f(Y_1), g(Y_2)) \tag{10}$$

for all non-decreasing functions f and g for which the covariances exist.

The correlation-order is a partial order over joint distributions in $R(F_1, F_2)$ and expresses the idea that two random variables with given marginals are more 'positively dependent' or 'positively correlated' when they have some joint distribution than some other one.

2.2 An alternative definition

In this subsection we will derive an alternative definition for the correlation order introduced above. First, we will recall and prove a lemma contained in Hoeffding(1940), which we will need for the derivation of the alternative definition. The proof will be repeated here because it is instructive for what follows.

Lemma 1 *For any $(X_1, X_2) \in R(F_1, F_2)$ we have*

$$Cov(X_1, X_2) = \int_0^\infty \int_0^\infty (F_{X_1,X_2}(u, v) - F_1(u) F_2(v))\, dudv \tag{11}$$

Proof: Let I denote the indicator function, then we have

$$x - z = \int_0^\infty \{I(z \leq u) - I(x \leq u)\}\, du \qquad (x, z \geq 0) \tag{12}$$

Hence, for $x_1, x_2, z_1, z_2 \geq 0$ we find

$$\begin{aligned}(x_1 - z_1)(x_2 - z_2) = \\ \int_0^\infty \int_0^\infty \{I(z_1 \leq u)I(z_2 \leq v) + I(x_1 \leq u)I(x_2 \leq v) \\ - I(z_1 \leq u)I(x_2 \leq v) - I(x_1 \leq u)I(z_2 \leq v)\}\, dudv\end{aligned} \tag{13}$$

Now let $(X_1,\, X_2)$ and $(Z_1,\, Z_2)$ be independent identically distributed pairs, then we have

$$2\,Cov(X_1,\, X_2) \;=\; E\,((X_1 \,-\, Z_1)(X_2 \,-\, Z_2))$$

so that we find (11) from (13). $\square$

Now we are able to state an equivalent definition for the correlation order considered in definition 2.

Theorem 1 *Let* $(X_1\,, X_2)$ *and* $(Y_1\,, Y_2)$ *be elements of* $R(F_1, F_2)$ *. Then the following statments are equivalent:*

(a) $(X_1\,, X_2) \leq_c (Y_1\,, Y_2)$

(b) $F_{X_1,X_2}(x_1\,, x_2) \leq F_{Y_1,Y_2}(x_1\,, x_2) \qquad for\ all\ x_1,\, x_2 \geq 0$

Proof: Assume that (a) holds and choose $f(u) = I(u > x_1)$ and $g(u) = I(u > x_2)$. Then we find from (10) that

$$E\,(I(X_1 \,>\, x_1,\, X_2 \,>\, x_2)) \leq E\,(I(Y_1 \,>\, x_1,\, Y_2 \,>\, x_2))$$

or equivalently

$$Prob\;(X_1 \,>\, x_1,\, X_2 \,>\, x_2) \leq Prob\;(Y_1 \,>\, x_1,\, Y_2 \,>\, x_2)$$

from which (b) can easily be derived.

Now, suppose that (b) holds. It follows immediately that, for non-decreasing functions f and g,

$$Prob\;(f(X_1) \,\leq\, x_1,\, g(X_2) \,\leq\, x_2) \leq Prob\;(f(Y_1) \,\leq\, x_1,\, g(Y_2) \,\leq\, x_2)$$

for all $x_1,\, x_2 \geq 0$, so that (a) follows as an immediate consequence of Lemma 1 and Definition 1. $\square$

Statement (b) in Theorem 1 asserts roughly that the probability that X_1 and X_2 both realize 'small' values is not greater than the probability that Y_1 and Y_2 both realize 'equally small' values, suggesting that Y_1 and Y_2 are more positively interdependent than X_1 and X_2. The statement (b) is equivalent with each of the following statements, each understood to be valid for all x_1 and x_2 :

(c) $Prob\,(X_1 \le x_1, X_2 > x_2) \ge Prob\,(Y_1 \le x_1, Y_2 > x_2)$

(d) $Prob\,(X_1 > x_1, X_2 \le x_2) \ge Prob\,(Y_1 > x_1, Y_2 \le x_2)$

(e) $Prob\,(X_1 > x_1, X_2 > x_2) \le Prob\,(Y_1 > x_1, Y_2 > x_2)$

Each of these statements can be interpreted similarly in terms of 'more positively interdependence' of Y_1 and Y_2. Hence, the equivalence of (a) and (b) in Theorem 1 has some intuitive interpretation.

The partial order between bivariate random variables which is defined by requiring equal marginals and by requiring statement (b) in Theorem 1 to be true, was introduced by Cambanis et al.(1976), and in the economic literature by Epstein et al.(1980). For economic applications, see also Aboudi et al.(1993) and Aboudi et al.(1995).

2.3 Correlation order and stop-loss order

In this subsection we will prove that the correlation order between bivariate distributions implies stop-loss order between the distributions of their sums.

Lemma 2 *For any* $(X_1, X_2) \in R(F_1, F_2)$ *we have*

$$E(X_1 + X_2 - d)_+ = E(X_1) + E(X_2) - d + \int_0^d F_{X_1,X_2}(x, d - x)\,dx$$

Proof: We have that

$$E(X_1 + X_2 - d)_+ = E(X_1) + E(X_2) - d + E(d - X_1 - X_2)_+$$

For non-negative real numbers x_1 and x_2 the following equality holds

$$(d - x_1 - x_2)_+ = \int_0^d I(x_1 \le x, x_2 \le d - x)\,dx$$

so that

$$E(d - X_1 - X_2)_+ = \int_0^d E(I(X_1 \leq x, X_2 \leq d - x)\, dx$$

which proves the lemma. □

Now we are able to state the following result.

Theorem 2 *Let* (X_1, X_2) *and* (Y_1, Y_2) *be two elements of* $R(F_1, F_2)$. *Then*

$$(X_1, X_2) \leq_c (Y_1, Y_2)$$

implies

$$X_1 + X_2 \leq_{sl} Y_1 + Y_2$$

Proof: The proof follows immediately from Theorem 1 and Lemma 2. □

From Theorem 2 we conclude that the correlation order is a usefull tool for comparing the stop-loss premiums of sums of two non-independent risks with equal marginals.

3 Riskiest and safest dependency between two risks

Consider again the class $R(F_1, F_2)$ of all bivariate distributed random variables with given marginals F_1 and F_2 respectively. For every (X_1, X_2) and $(Y_1, Y_2) \in R(F_1, F_2)$ we will compare their respective riskiness by comparing the stop-loss premiums of $X_1 + X_2$ and $Y_1 + Y_2$. More precisely, we will say that (X_1, X_2) is less risky than (Y_1, Y_2) if

$$X_1 + X_2 \leq_{sl} Y_1 + Y_2$$

In this section we will look for the riskiest and the safest elements of $R(F_1, F_2)$. Use will be made of the following well-known result which is due to Fréchet(1951).

Lemma 3 *For any* $(X_1, X_2) \in R(F_1, F_2)$ *we have that*

$$max\,[F_1(x_1) + F_2(x_2) - 1\,;\,0] \le F_{X_1,X_2}(x_1, x_2) \le min\,[F_1(x_1), F_2(x_2)] \tag{14}$$

The upper and lower bounds are themselves bivariate distributions with marginals F_1 *and* F_2 *respectively.*

Now we can state the following result concerning the riskiest and the safest element of $R(F_1, F_2)$.

Theorem 3 *Let* (Y_1, Y_2) *and* (Z_1, Z_2) *be the elements of* $R(F_1, F_2)$ *with distribution functions given by*

$$F_{Y_1,Y_2}(x_1, x_2) = max\,[F_1(x_1) + F_2(x_2) - 1\,;\,0]$$

and

$$F_{Z_1,Z_2}(x_1, x_2) = min\,[F_1(x_1), F_2(x_2)]$$

respectively. Then for any $(X_1, X_2) \in R(F_1, F_2)$ *we have that*

$$Y_1 + Y_2 \le_{sl} X_1 + X_2 \le_{sl} Z_1 + Z_2$$

Proof: The inequalities follow immediately from Theorems 1 and 2 and from Lemma 3. □

From Theorem 3 we can conclude that the random variables (Y_1, Y_2) and (Z_1, Z_2) are the safest and the riskiest element of $R(F_1, F_2)$ respectively.

Let us now look at the special case that the two marginal distributions are equal. From Theorem 3, we find that the most risky element in $R(F, F)$ is (Z_1, Z_2) with

$$F_{Z_1,Z_2}(x_1, x_2) = min\,[F(x_1), F(x_2)] \tag{15}$$

which leads to

$$F_{Z_1,Z_2}(x, d - x) = \begin{cases} F(x) & \text{if } x \le d/2 \\ F(d - x) & \text{if } x > d/2 \end{cases}$$

From Lemma 2 we find

$$\begin{aligned} E(Z_1 + Z_2 - d)_+ &= E(Z_1) + E(Z_2) - d + \int_0^{d/2} F(x)\,dx + \int_{d/2}^{d} F(d - x)\,dx \\ &= E(Z_1) + E(Z_2) - 2\int_0^{d/2} (1 - F(x))\,dx \\ &= 2\,E(Z_1 - d/2)_+ \end{aligned}$$

so that we find the following corollary to Theorem 3.

Corrolary 1 *For any* $(X_1, X_2) \in R(F, F)$ *we have that*

$$E(X_1 + X_2 - d)_+ \le 2\,E(X_1 - d/2)_+$$

Furthermore, the upperbound is the stop-loss premium of $Z_1 + Z_2$ *with retention* d *where* $(Z_1, Z_2) \in R(F, F)$ *with distribution function (15).*

Now assume that F is an exponential distribution with parameter $\alpha \ge 0$, i.e.

$$F(x) = 1 - e^{-\alpha x} \qquad x > 0$$

Then we obtain from Corollary 1 that for any $(X_1, X_2) \in R(F, F)$, we have

$$E(X_1 + X_2 - d)_+ \le 2\int_{d/2}^{\infty} (1 - F(x))\,dx = \frac{2}{\alpha}\,e^{-\alpha d/2} \qquad (16)$$

This upperbound for the exponential case can be found in Heilmann(1986). He derived this result by using some techniques described in Meilijson et al.(1979). Heilmann also considers the riskiest element in $R(F_1, F_2)$ where F_1 and F_2 are exponential distributions with different parameters. This result can also be found from our Lemma 2 and Theorem 3.

4 Positive dependency between risks

In a great many situation, certain insured risks tend to act similarly. For instance, in group life insurance the remaining life-times of a husband and his wife can be shown to possess some "positive dependency". Several concepts of bivariate positive dependency have appeared in the mathematical literature, see Tong(1980) for a review, for actuarial applications see Norberg(1989) and Kling(1993). We will restrict ourselves to positive quadrant dependency.

Definition 3 *The random variables X_1 and X_2 are said to be positively quadrant dependent, written $PQD(X_1, X_2)$, if*

$$Prob(X_1 \leq x_1, X_2 \leq x_2) \geq Prob(X_1 \leq x_1)\, Prob(X_2 \leq x_2)$$

for all $x_1 \geq 0$, $x_2 \geq 0$.

It is clear that $PQD(X_1, X_2)$ is equivalent with saying that X_1 and X_2 are more correlated (in the sense of Definition 2) than if they were independent.

Positive quadrant dependency can be defined in terms of covariances, as is shown in the following lemma, see also Epstein et al.(1980).

Lemma 4 *Let X_1 and X_2 be two random variables. Then the following statements are equivalent :*

(a) $PQD(X_1, X_2)$

(b) $Cov(f(X_1), g(X_2)) \geq 0$ *for all non-decreasing real functions f and g for which the covariance exists*

Proof: The result follows immediately from Theorem 1 with (Y_1, Y_2) a bivariate random variable with the same marginals as (X_1, X_2) and where Y_1 and Y_2 are mutually independent. □

Remark that $PQD(X_1, X_2)$ implies that $Cov(X_1, X_2) \geq 0$. Equality only holds if X_1 and X_2 are independent.

As is shown in the following theorem, the notion of positive quadrant dependency can be used for considering the effect of the independence assumption, when the risks are positively dependent actually.

Theorem 4 *Let* (X_1, X_2) *and* (Y_1^{ind}, Y_2^{ind}) *be two elements of* $R(F_1, F_2)$ *with* $PQD(X_1, X_2)$ *and where* Y_1^{ind} *and* Y_2^{ind} *are mutually independent. Then*

$$Y_1^{ind} + Y_2^{ind} \leq_{sl} X_1 + X_2$$

Proof: The result follows immediately from Theorems 1 and 2. □

Theorem 4 states that when the marginal distributions are given, and when $PQD(X_1, X_2)$, then the independence assumption will always underestimate the actual stop-loss premiums.

Let us now consider the special case that F_i is a two-point distribution in 0 and $\alpha_i > 0$ $(i = 1, 2)$. For any $(X_1, X_2) \in R(F_1, F_2)$ with $Cov(X_1, X_2) \geq 0$, we have that

$$Pr(X_1 = \alpha_1, X_2 = \alpha_2) \geq Pr(X_1 = \alpha_1)\, Pr(X_2 = \alpha_2)$$

This inequality can be transformed into

$$Pr(X_1 = 0, X_2 = 0) \geq Pr(X_1 = 0)\, Pr(X_2 = 0)$$

from which we find

$$Pr(X_1 \leq x_1, X_2 \leq x_2) \geq Pr(X_1 \leq x_1)\, Pr(X_2 \leq x_2) \qquad x_1 \geq 0,\, x_2 \geq 0$$

We can conclude that in this special case $PQD(X_1, X_2)$ is equivalent with $Cov(X_1, X_2) \geq 0$.

From Theorem 4 we find that when the marginal distributions F_i are given two-point distributions in 0 and $\alpha_i > 0$ $(i = 1, 2)$ and when $Cov(X_1, X_2) \geq 0$, making the independence assumption will underestimate the actual stop-loss premiums. This result can also be found in Dhaene et al.(1995).

5 Concluding remarks

As stipulated in Section 1 the results that we have derived for two risks can also be used for considering the riskiness of portfolios where the only non-independent risks can be classified into couples. Several theorems, together with the stop-loss preservation property for convolutions of independent risks, immediately lead to statements about the stop-loss premiums of such portfolios.

Take Theorem 5 as an example. Consider a portfolio with given distribution functions of the individual risks where the only non-independent risks appear in couples and where the risks of each couple are positive quadrant dependent. Then we find from Theorem 5 that taking the independence assumption will always lead to underestimated values for the stop-loss premiums of the portfolio under consideration.

Finally, we remark that we have only considered bivariate dependencies in this paper.The special, but important bivariate case will often be sufficient to describe dependencies in portfolios but it also provides a theoretical stepping stone towards the concept of dependence in the multivariate case. Some notions of dependence in the multivariate case can be found in Barlow et al.(1975). One of the notions of multivariate dependency which is often used in actuarial science is the exchangeability of risks, see e.g. Jewell(1984). It is remarkable that the usefulness of other notions of multivariate dependency has hardly been considered in the actuarial literature.

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