Dynamic Order Submission Strategies with Competition between a Dealer Market and a Crossing Network[1]

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Abstract

We analyze a dynamic microstructure model in which a dealer market (DM) and a crossing network (CN) interact for three informational settings. A key result is that coexistence of trading systems generates systematic patterns in order flow, which depend on the degree of transparency. Further, we study overall welfare, measured by the gains from trade of all agents, and compare it to the maximum overall welfare. The discrepancy between both measures is attributable to two inefficiencies. Due to these inefficiencies, introducing a CN next to a DM, as well as increasing the transparency level not necessarily produce greater overall welfare.

JEL Codes: G10, G20

Keywords: Alternative Trading Systems, Crossing Network, Order Flow, Transparency, Welfare

1 Introduction

An open issue in market microstructure is how investors behave when an asset trades simultaneously on several markets that may show a different degree of transparency. The topics of competition between markets and the optimal degree of transparency have become even more relevant in recent years, with the emergence of Alternative Trading Systems (ATSs). These ATSs operate next to traditional exchanges and exhibit distinct institutional characteristics. Therefore, traders face the decision where to trade, taking into account the advantages and disadvantages of each trading venue.

In this paper, we deal with Crossing Networks (CNs), which are one specific type of ATS. CNs are defined by the SEC (1998) as "systems that allow participants to enter unpriced orders to buy and sell securities. Orders are crossed at a specified time at a price derived from another market" (i.e. the continuous market). A pioneering CN, both in the US and on a worldwide level, is ITG's POSIT.[1] However, crossing applications could also be found in different environments. Already in 1990, the NYSE had introduced post-close crossing sessions. Currently, other traditional markets are adding crossing facilities into their market structure as well; see e.g. Deutsche Börse's Xetra XXL in September 2001 or the NASDAQ Crossing Network in May 2007. Further, also investment banks now opt to pool institutional order flow into CNs as a response to new regulatory initiatives, such as Regulation NMS in the US and MiFID in Europe (see e.g. the Block Interest Discovery Service (BIDS)).

Despite the prevalence of CNs next to continuous markets, the dynamic aspects and welfare implications of the coexistence of these systems have not yet been well explored. This paper aims to fill this gap and addresses two important policy questions that also relate to long-standing issues within the market microstructure literature. First, where do investors trade when there are multiple trading venues for a single asset? More specifically, we consider the choice between a CN and a continuous dealer market (DM) under different degrees of transparency. Second, what is the optimal organization and structure of financial markets? In particular, we study whether coexistence of a CN and a DM creates added value in terms of welfare and investigate the optimal transparency level when markets coexist.

Our model for studying traders' order submission strategies starts from that in Parlour (1998). While she models a limit order market, we deal with sequentially arriving traders who are able to choose between a CN and a DM. When both trading systems coexist, traders can

[1] ITG's POSIT is used by approximately 550 major institutions and broker/dealers and crossed about 35 million US shares per day in November 2005, according to Towergroup. The total market volume amounted to 98 million shares per day in the same period, as compared to almost 1.8 billion for the NYSE.

obtain guaranteed execution in the DM, opt for cheaper but (possibly) uncertain execution on the CN, or refrain from trading. An important feature of the competition between CNs and traditional markets is that they offer a different degree of transparency (see e.g. Bloomfield and O'Hara (2000)). Whereas traditional markets may vary in their degree of mandated transparency, Regulations ATS and NMS in the US and MiFID in Europe do not require CNs to provide information about their order book. We therefore investigate how different degrees of transparency at both markets influence traders' order submission strategies and affect welfare. More specifically, we develop the analysis for three different informational settings: transparency, "partial" opaqueness and "complete" opaqueness. Transparency occurs when traders are fully informed about the past order flow at both markets. Hence, before determining their trading strategy, they observe the prevailing CN net order imbalance, i.e. the difference between the number of buy and sell orders in the CN order book. In reality, however, CNs are rather opaque. We incorporate this by analyzing partial opaqueness: traders only observe previous trades at the DM. Complete opaqueness implies that both markets are opaque such that traders are uninformed about past CN and DM order flow.

We summarize our main findings around the two policy questions that we address. The first set of results relates to order submission strategies and order flow patterns. Common to the three informational settings, CN and DM are shown to cater for different types of traders: investors with a higher willingness to trade are more inclined to trade at a DM. Further, we find that the CN's order flow increases when an asset exhibits a higher "relative spread" (i.e. a higher bid-ask spread in proportion to its underlying value). Moreover, the existence of a CN results in "order creation" due to the participation of "CN-only traders": investors who have a relatively lower willingness to trade submit orders to a CN, whereas they would never trade at a DM. We also find a "trade diversion" effect, which occurs because the introduction of a CN causes some trades to be diverted away from the DM to the CN. A key result of our paper is that the transparency and partial opaqueness settings generate systematic patterns in order flow. In particular, current CN order flow stimulates the arrival of future CN counterparties. In addition, current CN order flow hinders future CN orders on the same market side. The intuition for our key result is that the net order imbalance created by the current order is more (less) favorable for future orders on the opposite (same) market side, which is reflected in their respective execution probabilities. Under complete opaqueness, these patterns do not arise because traders do not observe any order flow. Although this result for transparency is reminiscent of the findings in Parlour (1998) for a limit order market, two major differences exist. First, in Parlour's model both market and limit orders have implications for future order flow. In our model, by contrast, only CN orders produce systematic patterns in order flow. Secondly, we show that the transparency level of the CN and the DM affects the nature of the order flow patterns. The result that order flow is informative about execution probabilities is novel to the market microstructure literature. The reasoning for this informativeness of order flow is that, when markets are partially opaque, observing no order flow relative to a DM trade may be perceived as good news for a successive CN order as it increases the likelihood of a counterparty in the book. However, no order may also be perceived as bad news when it entails the preemption of a successive CN order. Overall, these theoretical insights point to a time-varying order flow at a CN and trade flow at a DM, even in the absence of asymmetric information. This has important policy implications for supervisory authorities that are attempting to correctly infer the presence of informed trading. Further, these insights need to be accounted for when measuring "normal" order flow.

Our second set of results concerns welfare. We build on previous work that studies welfare and the optimal degree of transparency (see e.g. Pagano and Roëll (1996), Glosten (1998), Bloomfield and O'Hara (2000), Viswanathan and Wang (2002), Parlour and Seppi (2003), Goettler, Parlour and Rajan (2005), and Rindi (2007)). Our paper complements this literature by considering the impact of transparency on welfare when coexisting trading systems compete for uninformed order flow. We compare welfare for a CN and DM in isolation, as well as for coexisting markets to that of a fictitious market without inefficiencies, producing maximum welfare. The employed measure is "overall" welfare, which accounts for the gains from trade of all parties involved (including dealers). A first result is that a CN in isolation offers greater overall welfare than a DM in isolation if the execution probability at the CN and the relative spread are high. Second, coexistence of a CN and a DM only generates higher overall welfare than a DM in isolation for assets with a high relative spread. Third, under coexistence, transparency outranks both opaqueness settings for assets with a high relative spread. However, the opposite result is obtained with a low relative spread.

Focusing next on the discrepancy with the maximum overall welfare benchmark, we find it is driven by two main inefficiencies, which in turn hinge on coexistence and the level of transparency. The first inefficiency concerns the foregone welfare potential from trades by "CN-only traders", labelled as the "CN-only-traders inefficiency". This inefficiency is largest for a DM in isolation where these CN-only traders do not participate, lowest for a CN in isolation where all traders can only participate in the CN, and intermediate for coexistence.

Transparency reduces the CN-only-traders inefficiency as counterparties are more attracted to the CN. The second inefficiency stems from traders diverting to the CN while also being willing to trade at the DM, and is labelled "CN-diverted-traders inefficiency". Diverting traders do not take into account all gains from trade as they maximize their individual trading gains only. With a DM in isolation, this inefficiency is zero, whereas it is at its maximum for a CN in isolation. The CN-diverted-traders inefficiency increases in the degree of transparency only for liquid stocks, i.e. stocks with a low relative spread. Our welfare findings have important policy implications. Coexistence and greater transparency do not necessarily improve overall welfare. In addition, asking for more transparency may be beneficial for one set of traders but not for others.

Further, our paper is related to two recent strands of research. A first line of work develops dynamic microstructure models for a limit order market (see e.g. Harris (1998), Foucault (1999), Goettler, Parlour and Rajan (2005), Foucault, Kadan and Kandel (2005), and Rosu (2005)).[2] Our paper contributes to this line of research as we introduce a dynamic microstructure model to study (partly, at least) endogenous liquidity supply when two different trading venues compete. The limit order market model in Parlour (1998) is positioned closest to ours. However, a number of important differences exist. First, we analyze the optimal order submission strategies and the consequences for welfare when traders choose between two trading venues that have different institutional characteristics, whereas Parlour (1998) considers the choice between market and limit orders within a single market. Second, given that the cross in the CN occurs at the DM midquote, our model allows for submitting orders "within the spread". Third, while Parlour (1998) deals with transparency (which is the case for most limit order markets), we also consider two opaqueness settings. Finally, the models' resulting dynamics feature some important differences. In our model, only a CN order generates systematic patterns in order flow, whereas in Parlour (1998) both market and limit orders have an impact.

A second line of recent work models competition between financial markets when assets trade at multiple markets (see e.g. Glosten (1994), Parlour and Seppi (2003), and Foucault and Menkveld (2007)).[3] Recently, a few papers have studied explicitly the interaction between

[2] Note that static equilibrium models of the limit order book are much more common. Examples include Glosten (1994), Chakravarty and Holden (1995), Rock (1996), and Seppi (1997).

[3] In particular, the seminal contribution is provided by Glosten (1994), who considers the design of pure limit order markets to analyze their competitive viability. Parlour and Seppi (2003) extend this model by focusing on competition between a pure limit order market and a hybrid market. Foucault and Menkveld (2007) deal with order submission at two pure limit order markets when a fraction of brokers apply Smart Order Routing Technologies (SORT). Other work on the competition between trading systems includes Glosten (1998), Santos and Scheinkman (2001), Di Noia (2001), Viswanathan and Wang (2002), Chemmanur and Fulghieri (2006),

a CN and a DM. Existing models, however, consider a static environment to analyze this competition. Hendershott and Mendelson (2000) develop a model in which informed and uninformed traders decide simultaneously to submit orders to one of the two markets in order to analyze the effect of the introduction of a CN next to a DM. Expanding on this paper, Dönges and Heinemann (2006) focus on game-theoretic refinements to accommodate the multiplicity of equilibria in the coordination game. We contribute to this line of work because we explicitly introduce dynamics into the analysis. These dynamics are important: a typical characteristic of a CN is that it "matches" orders at a specified time during the trading day, while the other market operates simultaneously in a continuous fashion. In particular, traders arrive sequentially, and their submission strategy is determined both by the current CN book (when transparent) and by their expectations of the behavior of future traders.

There are by now a substantial number of empirical papers that analyze the interaction between trading systems (for an overview, see Biais, Glosten and Spatt (2005)). However, papers that investigate empirically the impact of a CN on other trading systems are still rather scarce. Gresse (2006) studies the impact of ITG's POSIT on the DM segment of the London Stock Exchange. She finds that POSIT has a share of the total trading volume of about one to two percent in these stocks, but that its probability of execution is still low (2-4%). Conrad, Johnson and Wahal (2003) use proprietary data of US institutional investors who choose between trading platforms. They find that realized execution costs are generally lower on CNs. Næs and Skjeltorp (2003) and Næs and Ødegaard (2006) focus on orders from the Norwegian Government Petroleum Fund that are first sent to a CN and then, in the case of non-execution, to brokers. They also find lower CN trading costs but argue that these may be offset fully by the non-trading costs due to adverse selection, which are implicitly present at the CN. Finally, Fong, Madhavan and Swan (2004) focus on the impact of block trades on different trading venues. They find that competition from a CN imposes no adverse effect on the liquidity of the limit order book.

The remainder of the paper is organized as follows. Section 2 presents the setup of the model. Section 3 provides an analysis of the two markets in isolation. Next, Section 4 studies their coexistence. In this section, we first consider transparency and then turn to two degrees of opaqueness: partial and complete. Section 5 offers a discussion of the welfare implications of our model. Section 6 concludes. All proofs are given in Appendix A.

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and Foucault and Parlour (2004).

2 Setup of the Model

The model we develop adapts that in Parlour (1998) to analyze dynamic competition between two trading systems. In our economy, there are two goods: consumption on day 1 and on day 2, denoted by C_1 and C_2, respectively. Agents are risk neutral and differ in their preferences over consumption of these two goods. These preferences are given by the following utility function:

$$U\left(C_1, C_2; \beta\right) = C_1 + \beta C_2 \tag{1}$$

where β is the subjective preference or type of the agent reflecting her personal trade-off between current and future consumption. Next to these two goods, C_1 and C_2, an asset exists that pays out V units of C_2 on day 2. Thus, V can be interpreted as the underlying or fundamental value of the asset. During the first day, the trading day, claims to the asset can be exchanged for C_1. Prices in the market are exchange ratios C_1/C_2. Agents can then construct their preferred consumption path by trading claims to this asset. The trading day consists of T periods, indexed by $t = 1, ..., T$. In each period, exactly one agent (also referred to as trader) arrives in the market, and each agent arrives at most once. The arriving agent at time t is characterized by two elements. First, her initial endowment determines her trading orientation. With probability π_b, she is a buyer and has a unit of the asset she can buy, which we denote by 1. With probability $\pi_s = 1 - \pi_b$, she is a seller and has a unit of the asset she can sell, -1. Second, the agent arriving at t has a type β_t, which is drawn from an i.i.d. continuous distribution $F(.)$ with a corresponding density function $f(.)$ and support $\left[\underline{\beta}, \bar{\beta}\right]$, where we assume $0 \leq \underline{\beta} \leq 1 \leq \bar{\beta}$. This β_t, which appears as well in the utility function above, could also be seen as a reflection of the time t agent's willingness to trade.[4] In particular, if the agent is a buyer, she will be more eager to buy if she has a high beta. Conversely, a seller will be more eager to sell if she has a low β_t. In order to see this, assume that the arriving agent is a buyer. Buying the asset yields $\beta_t V$. She compares this value with the price in the market and buys if the price (which is denominated in units of C_1) is lower than the value she attaches to the asset. If β_t is high, she attaches more weight to consumption on the second day and hence will be more eager to trade than if β_t is low. The reasoning is that the trading gains are higher in the former case. Similarly, a seller with a low beta will be more eager to sell, because she prefers consumption on the first day.

[4] Alternatively, Parlour (1998) argues that β_t can be interpreted as a subjective valuation of the asset, or an agent-specific prior over the next day's asset value V. Hence, the market becomes a private values auction, as in Glosten and Milgrom (1985).

Traders can choose between submitting an order to a dealer market (DM) or to a crossing network (CN), or not submit an order at all. We assume that competition between dealers on the DM is sufficiently intense such that the spread is one tick, that is $A - B = 1$, with A the ask price and B the bid price and $A > V > B$. The one-tick assumption allows us to focus on the interaction between markets, abstracting from strategic interactions between dealers. At the same time, a one-tick spread represents the most competitive position for the DM when competing with a CN.[5] Dealer bid and ask quotes do not move during the trading day. The implication is that buyers can always buy at a price A, the price at which a dealer is willing to sell. Sellers who are looking for immediacy in the DM obtain B.

On the CN, we assume that the matching of orders (the "cross") takes place at the end of the trading day (that is, after the action of the agent arriving in period T) and that this matching occurs according to time priority. The price of the cross is derived from the bid and ask in the DM and equals the midquote $\frac{A+B}{2}$. Given our assumptions, orders at the CN face no price uncertainty. All orders submitted to the CN are stored in the CN book. Due to time priority, the only variable relevant to traders for this book is the amount of net unfulfilled orders or the "net order imbalance", n_t, i.e. the difference between the amounts of previously submitted buy and sell orders. When $n_t > 0$ (< 0) there are more buy (sell) orders than sell (buy) orders in the CN book before the order at time t. After the action of the trader at time t, there are three possible evolutions of the net order imbalance:

$$n_{t+1} = \left| \begin{array}{ll} n_t + 1 & \text{trader } t \text{ submits a buy order to the CN} \\ n_t - 1 & \text{trader } t \text{ submits a sell order to the CN} \\ n_t & \text{trader } t \text{ submits no order to the CN} \end{array} \right. . \tag{2}$$

Note that $n_{t+1} = n_t$ may stem from a trade at a dealer or from not trading at all at time t. Once submitted, orders cannot be modified or cancelled. This means that orders remain in the CN book until the cross. Order execution is determined by the final net imbalance between the queue of buy orders and the queue of sell orders. Define n_{T+1} as the CN book at the time of the cross, then if $n_{T+1} = 0$, meaning no imbalance, all submitted orders are executed. If $n_{T+1} < 0$, given time priority, the last $|n_{T+1}|$ submitted sell orders remain

[5] Bessembinder (2003) finds average (volume-weighted) quoted spreads on NASDAQ equal to 1.77 cents (with the tick size being 1 cent), which is relatively close to our one-tick assumption. More generally, this "one tick" should not be interpreted literally, but rather taken as a metaphor for the most competitive situation in which competition between dealers have driven the inside spread to its minimum level. As will be shown later, it is the relative spread that matters for submission strategies of agents. For example, saving the half-spread in the CN is more valuable when the bid is $1 and the ask $1.01, than if they are $100 and $100.01, respectively (assuming that the common tick size is one cent).

unexecuted. If $n_{T+1} > 0$, the last n_{T+1} buy orders remain unexecuted. It goes without saying that time priority influences the order submission strategies of the traders. In practice, some CNs indeed implement a time priority rule. Examples include the Crossing Session I at the NYSE (rule 904 of SR-NYSE-90-52), ITG's POSIT-Now which offers a continuous intraday CN (implicitly granting time priority) and Xetra XXL employing a volume/time priority rule. Other CNs are often reluctant to share information on their matching procedure and may use different matching procedures like pro-rata systems.

We consider three informational settings: transparency, complete opaqueness and partial opaqueness. Common to the three settings, a trader arriving at time t knows her trading orientation (buyer or seller), her own β_t, the bid and ask price of the dealer (B and A), the distribution of β ($F(\beta)$), the distribution of buyers and sellers (π_b and π_s), and the length of the trading day (T). With transparency, traders also observe past CN order flow and thus know the net order imbalance (n_t) in the CN, as well as past DM trades. In contrast, with complete opaqueness traders do not observe any order flow, whereas with partial opaqueness, traders only observe DM trades. In the two opaqueness settings, traders therefore need to form expectations about n_t.

3 Markets in Isolation

Within this section, we successively consider the equilibria for a DM and a CN in isolation. This approach allows us to gain insight into the model and the structure and functioning of each market. Do note that the informational setting does not influence the outcomes of the derived equilibria. Therefore, there is no need to explicitly indicate the informational setting with markets in isolation.

We first consider a *DM in isolation.* In this case, a trader submits an order to the DM as long as this yields a positive profit; otherwise, she prefers not to trade. The trader's profit of a buy order is the difference between her valuation $\beta_t V$ and the price paid A, i.e. $\beta_t V - A$. Similarly, for a sell order, the profit is $B - \beta_t V$. From these profits, the cutoff values, i.e. the values for β_t at which a trader is indifferent between submitting no order and trade at the DM, are computed as $\frac{A}{V}$ for a buyer and $\frac{B}{V}$ for a seller. These cutoff values could be interpreted in the following way. A buyer arriving at t who has a β_t higher than $\frac{A}{V}$ will buy at the DM; all the others will not. In turn, when the trader at t is a seller, she will only sell at the DM if her β_t is smaller than $\frac{B}{V}$. The order submission strategies are depicted in Panel

A of Figure 1. Note that traders who have a β_t between $\frac{B}{V}$ and $\frac{A}{V}$ never submit an order, regardless of their trading orientation.

We now turn to a *CN in isolation*. A trader submits a CN order as long as this results in a positive *expected* profit. We need to consider expected profits as the execution of a CN order may not be certain. If the order executes, the trader's profit is the difference between her valuation and the price paid (the midquote). The expected profit of a CN buy order is $p_t^{b,CN}\left(\beta_t V - \frac{A+B}{2}\right)$, with $p_t^{b,CN}$ the expected probability of execution. The first and second superscript denote the trading orientation (b) and the considered market in isolation (CN), respectively. The subscript indicates the period of arrival t. For a CN sell order, the expected profit is $p_t^{s,CN}\left(\frac{A+B}{2} - \beta_t V\right)$, with $p_t^{s,CN}$ the probability of execution of a sell order submitted to the CN at t. These probabilities hinge on the net order imbalance in the CN (i.e. $p_t^{b,CN}(n_t)$ and $p_t^{s,CN}(n_t)$), but for notational convenience we suppress this dependence.[6] The reasoning behind this dependence is that if a trader makes the net order imbalance more pronounced by joining the "longer" side of the book, enough future orders need to arrive at the "shorter" side of the book to obtain execution. This is more likely earlier on the trading day, when there are still a lot of periods to come. When the expected profit of a CN order is negative, the trader chooses to abstain, which results in zero profits. Solving for β_t, both for a buyer and a seller, we find that the cutoff value - the value of β_t at which a trader is indifferent between submitting a CN order and no order - equals $\frac{A+B}{2V}$. Hence, a buyer (seller) arriving at t will submit a CN buy (sell) order if her β_t is higher (lower) than $\frac{A+B}{2V}$. To be complete, these cutoff values hold if the respective execution probability is strictly positive. If it is zero, a trader is indifferent between a CN order and no order, because both yield zero profit. If this occurs, we assume that traders prefer to abstain.[7] The order submission strategies are summarized in Panel B of Figure 1. Note that, in contrast with a DM in isolation, there is no range of betas where neither a buyer nor a seller submits an order. The reasoning is that a CN does not have a spread, whereas a DM is characterized by a one-tick spread.

Please insert Figure 1 around here.

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[6] Note that, next to the state variable n_t, execution probabilities also depend on $F(.)$, π_b, π_s and the time left until the end of the trading day $T-t$.

[7] Under complete or partial opaqueness, traders may not know that the execution probabilities are zero. However, whether or not they participate does not influence the number of trades on the CN. Therefore, the informational setting does not affect the equilibrium outcome.

4 Coexistence of Markets

In this section, we characterize the equilibrium order submission strategies when a CN and a DM coexist. We first consider transparency, i.e. traders observe past CN order flow (and thus know the net order imbalance in the CN), as well as past DM trades. Subsequently, we analyze opaqueness, i.e. when traders do not observe any order flow (complete opaqueness) or only DM trades (partial opaqueness). The methodology is identical in all settings. For a trader arriving at time t we calculate a cutoff β_t at which she is indifferent between two strategies, rationally anticipating the impact of her order on execution probabilities. Furthermore, we develop empirical predictions on order flow dynamics.

For all variables, we denote the informational setting by the subscript $i = tr, co$ or po, indicating transparency, complete or partial opaqueness, respectively. Adding a superscript reflecting coexistence of a CN and a DM, as was done for the isolation cases, now becomes redundant because the informational setting's subscript sufficiently indicates that coexistence of markets is considered.

4.1 Transparency

4.1.1 Equilibrium

The time t trader chooses between three possible strategies. First, she can initiate a trade at the dealer; such an order has a guaranteed, immediate execution. Second, she could opt for submitting an order to the CN. This would yield a better price as it allows the trader to save the half-spread. With such an order, however, she might face the risk of non-execution. Execution is certain when, upon arrival, she faces a favorable net order imbalance; otherwise, the probability that the order will be executed is lower than one. Third, she can refrain from trading when it yields a negative (expected) profit. Denote the strategy of a buyer who arrives at time t under transparency (tr) by $\phi^b_{t,tr}(n_t, \beta_t)$ and of a seller by $\phi^s_{t,tr}(n_t, \beta_t)$ where the notation stresses that the strategy depends on the time t CN's net order imbalance, n_t, and the trader's type β_t. Note that these strategies hinge on time and are non-stationary.

The setup of this model can be seen as a stochastic sequential game. Moreover, due to the recursive nature of the game, an equilibrium is guaranteed to exist and this equilibrium is unique (because traders are indifferent between choices with zero probability). Applying the approach introduced above to solve the trader's choice problem, i.e. computing cutoff values for β_t at which traders are indifferent between two strategies, Proposition 1 states the resulting equilibrium strategies of a trader arriving at t.

Proposition 1 *If the time t trader is a buyer, there exist cutoff values such that*

$$\beta_t \in \left| \begin{array}{lll} \left[\underline{\beta}, \underline{\beta}^b_{t,tr}\left(p^b_{t,tr}\right)\right] & \phi^b_{t,tr}\left(n_t, \beta_t\right) = 0 & \textit{(no order)} \\ \left(\underline{\beta}^b_{t,tr}\left(p^b_{t,tr}\right), \bar{\beta}^b_{t,tr}\left(p^b_{t,tr}\right)\right] & \phi^b_{t,tr}\left(n_t, \beta_t\right) = 1^{CN} & \textit{(buy order to CN)} \\ \left(\bar{\beta}^b_{t,tr}\left(p^b_{t,tr}\right), \bar{\beta}\right] & \phi^b_{t,tr}\left(n_t, \beta_t\right) = 1^{DM} & \textit{(buy at DM)} \end{array} \right. . \quad (3)$$

Similarly, if the time t trader is a seller, there exist cutoff values such that

$$\beta_t \in \left| \begin{array}{lll} \left[\underline{\beta}, \underline{\beta}^s_{t,tr}\left(p^s_{t,tr}\right)\right) & \phi^s_{t,tr}\left(n_t, \beta_t\right) = -1^{DM} & \textit{(sell at DM)} \\ \left[\underline{\beta}^s_{t,tr}\left(p^s_{t,tr}\right), \bar{\beta}^s_{t,tr}\left(p^s_{t,tr}\right)\right) & \phi^s_{t,tr}\left(n_t, \beta_t\right) = -1^{CN} & \textit{(sell order to CN)} \\ \left[\bar{\beta}^s_{t,tr}\left(p^s_{t,tr}\right), \bar{\beta}\right] & \phi^s_{t,tr}\left(n_t, \beta_t\right) = 0 & \textit{(no order)} \end{array} \right. . \quad (4)$$

Proof. See Appendix A. ■

In this proposition, "1^{DM}" denotes a buy at the DM (which transacts at the ask), and "-1^{DM}" a sell at the DM (transacting at the bid). Similarly, "1^{CN}" and "-1^{CN}" stand for a buy and sell order to the CN, respectively. Employing our one-tick spread assumption, $A - B = 1$, we find that the cutoff β_t of a buyer who is indifferent between an order to the CN and a DM trade, $\bar{\beta}^b_{t,tr}\left(p^b_{t,tr}\right)$, is given by

$$\bar{\beta}^b_{t,tr}\left(p^b_{t,tr}\right) = \min\left[\frac{\frac{A+B}{2}}{V} + \frac{1/2}{V\left(1 - p^b_{t,tr}\right)}, \bar{\beta}\right]. \quad (5)$$

When indifferent between CN and DM, we assume traders opt for the CN. Furthermore, $\underline{\beta}^b_{t,tr}\left(p^b_{t,tr}\right)$, the cutoff β_t at which a buyer is indifferent between a CN buy order and no order, is equal to

$$\underline{\beta}^b_{t,tr}\left(p^b_{t,tr}\right) = \left| \begin{array}{ll} \frac{\frac{A+B}{2}}{V} & \text{if } p^b_{t,tr} > 0 \\ \frac{A}{V} & \text{otherwise} \end{array} \right. . \quad (6)$$

Similarly, $\underline{\beta}^s_{t,tr}\left(p^s_{t,tr}\right)$ is the cutoff β_t of a seller who is indifferent between a CN order and a DM trade:

$$\underline{\beta}^s_{t,tr}\left(p^s_{t,tr}\right) = \max\left[\frac{\frac{A+B}{2}}{V} - \frac{1/2}{V\left(1 - p^s_{t,tr}\right)}, \underline{\beta}\right], \quad (7)$$

whereas $\bar{\beta}^s_{t,tr}\left(p^s_{t,tr}\right)$ holds for a seller at t who is indifferent between a CN order and no order, with

$$\bar{\beta}^{s}_{t,tr}\left(p^{s}_{t,tr}\right) = \left| \begin{array}{ll} \frac{\frac{A+B}{2}}{V} & \text{if } p^{s}_{t,tr} > 0 \\ \frac{B}{V} & \text{otherwise} \end{array} \right. . \tag{8}$$

The equilibrium order submission strategies are summarized in Figure 2 (for $i = tr$). Comparing this graph with Panels A and B in Figure 1, do note that due to altering execution probabilities some cutoff values become *dynamic* and may change every period t. Further, compared to the DM in isolation, order creation occurs stemming from CN-only traders: buyers with $\beta_t \in \left(\frac{A+B}{2V}, \frac{A}{V}\right]$ and sellers with $\beta_t \in \left[\frac{B}{V}, \frac{A+B}{2V}\right)$ now submit orders to the CN, whereas they would never participate at the DM. Such order creation, induced by the CN, is confirmed empirically by Gresse (2006). The CN also introduces competition for the DM as it may *divert trades* away from the DM.[8] The welfare implications of both order creation and trade diversion will be discussed in Section 5.

Please insert Figure 2 around here.

It is clear that if the execution probability at the CN is larger, an arriving trader is more likely to opt for a CN order. This execution probability is a crucial element in the choice between a CN order and a DM trade as it determines expected profits. When trader t submits a CN order, she changes the net order imbalance in the CN. This affects the execution probabilities of future CN orders and hence also the strategies chosen by future traders. When determining her optimal strategy, trader t must take these effects of her order into account. Proposition 2 shows how the CN's net order imbalance influences execution probabilities.

Proposition 2 *In equilibrium, at any time t for any net order imbalance n_t, if the CN's net order imbalance is one unit higher, then the probability of execution of a buy (sell) order will be lower (higher). If the CN's net order imbalance is one unit lower, then the probability of execution of a buy (sell) order will be higher (lower). Hence, $\forall\ n_t$, t,*

$$\begin{array}{ll} (i) & p^{b}_{t,tr}(n_t - 1) \geq p^{b}_{t,tr}(n_t) \geq p^{b}_{t,tr}(n_t + 1) \\ (ii) & p^{s}_{t,tr}(n_t - 1) \leq p^{s}_{t,tr}(n_t) \leq p^{s}_{t,tr}(n_t + 1) \end{array} . \tag{9}$$

Proof. See Appendix A. ■

[8] Note that both effects could lead to overall trade creation but also to overall trade reduction. The reason that there might be trade reduction is that some of the investors who would choose to trade at the DM, were it to operate in isolation, might now opt for the CN, at which their order may remain unfilled.

Proposition 2 argues that a trader faces intertemporal competition from earlier traders of the same orientation already present in the CN book. The reasoning is as follows. Suppose that a buyer arrives at time t and $n_t \geq -1$.[9] Then, if the net order imbalance is one unit higher, i.e. n_t+1, because there is either one additional buy order or one less sell order in the CN book, an additional CN order at the sell side must arrive in order to obtain execution. This lowers the execution probability compared to when there is a smaller imbalance.

Further, do note that execution probabilities only depend on the net order imbalance, and not on the individual length of the buy and sell queue in the CN book. This is in contrast with a limit order market as presented in Parlour (1998). In such a market, execution probabilities are influenced even when both queues are one unit longer or shorter.

4.1.2 Empirical Predictions on Order Flow Dynamics

We now analyze order flow patterns and present them in two propositions. In each proposition, we depart from a given n_t, and from a specific order, a DM trade or a CN order submitted by the time t trader. We investigate the effect on the order flow to the DM and the CN in the subsequent period. We first assume that the current order (at time t) is a DM trade. Denote by $\Pr[.|.]$ the conditional probability. Proposition 3 then states that the probability of occurrence of any type of order at $t+1$ (i.e. a DM buy, DM sell, CN buy or CN sell) does not depend on whether the current transaction is a DM buy or a DM sell.

Proposition 3 *Under transparency, trades at the DM do not generate systematic patterns in order flow. In particular, the probabilities of occurrence of buy orders on the DM and the CN at time $t+1$ are independent of whether the trade at time t was a DM buy or a DM sell:*

$$\Pr\left[1^{DM} \ \ at\ t+1|\phi^b_{t,tr}\left(n_t,\beta_t\right)=1^{DM},n_t\right] \\ =\Pr\left[1^{DM} \ \ at\ t+1|\phi^s_{t,tr}\left(n_t,\beta_t\right)=-1^{DM},n_t\right], \tag{10}$$

$$\Pr\left[1^{CN} \ \ at\ t+1|\phi^b_{t,tr}\left(n_t,\beta_t\right)=1^{DM},n_t\right] \\ =\Pr\left[1^{CN} \ \ at\ t+1|\phi^s_{t,tr}\left(n_t,\beta_t\right)=-1^{DM},n_t\right]. \tag{11}$$

A symmetric result holds for the other side of the market.

Proof. Contained in the discussion below. ■

[9] If $n_t < -1$, the execution probability of a CN buy order with a net order imbalance of n_t and n_t+1 both equal one.

The results in Proposition 3 are driven by the following intuition. The CN's net order imbalance n_{t+1} remains n_t, independently of whether at time t there was a DM buy or a DM sell. Therefore, for any trader at $t+1$ with type β_{t+1}, the probabilities of this trader submitting a buy or sell order to the DM or the CN at time $t+1$ is independent of whether the trade at time t was a DM buy or DM sell. As this applies to any trader of a specific type, it also holds when aggregating over all types β_{t+1}.

However, the conclusions change when we assume that the order at t was a CN order instead of a DM trade. In this case, we obtain systematic patterns in order flow although buyers and sellers arrive randomly. Part a of Proposition 4 shows that a CN buy order at t is more likely to invite a CN sell order at $t+1$, but is more likely to hinder the submission of a CN buy order at $t+1$, compared to when trader t did not submit a CN buy order. In turn, part b of Proposition 4 shows it becomes more likely that the time $t+1$ trade will be a DM buy if the order at t was a CN buy order than if it was another type of order.

Proposition 4 *Under transparency, CN orders generate systematic patterns in order flow. In particular, it holds that:*

a) the probability of a CN buy order occurring at time $t+1$ is smaller if the order submitted at time t was a CN buy order than if it was a DM trade (buy or sell). This, in turn, is smaller than the probability of a CN buy order at $t+1$, conditional upon a CN sell order at t:

$$\begin{aligned}&\Pr\left[1^{CN} \ \textit{at}\ t+1|\phi^b_{t,tr}(n_t,\beta_t)=1^{CN},n_t\right] \\ &\leq \Pr\left[1^{CN} \ \textit{at}\ t+1|\phi^b_{t,tr}(n_t,\beta_t)=1^{DM} \ \textit{or}\ \phi^s_{t,tr}(n_t,\beta_t)=-1^{DM},n_t\right] \\ &\leq \Pr\left[1^{CN} \ \textit{at}\ t+1|\phi^s_{t,tr}(n_t,\beta_t)=-1^{CN},n_t\right];\end{aligned} \tag{12}$$

b) the probability of a DM buy occurring at time $t+1$ is greater if the order submitted at time t was a CN buy order than if it was a DM trade (buy or sell). This, in turn, is greater than the probability of a DM buy at $t+1$, conditional upon a CN sell order at t:

$$\begin{aligned}&\Pr\left[1^{DM} \ \textit{at}\ t+1|\phi^b_{t,tr}(n_t,\beta_t)=1^{CN},n_t\right] \\ &\geq \Pr\left[1^{DM} \ \textit{at}\ t+1|\phi^b_{t,tr}(n_t,\beta_t)=1^{DM} \ \textit{or}\ \phi^s_{t,tr}(n_t,\beta_t)=-1^{DM},n_t\right] \\ &\geq \Pr\left[1^{DM} \ \textit{at}\ t+1|\phi^s_{t,tr}(n_t,\beta_t)=-1^{CN},n_t\right].\end{aligned} \tag{13}$$

For both a and b, a symmetric result holds for the other side of the market.

Proof. See Appendix A. ■

The reasoning behind part a of Proposition 4 is as follows. Assume that the $t+1$ trader is a buyer with type β_{t+1} and start from a net order imbalance at t, n_t. As argued above, this trader arriving at $t+1$ decides where to trade based upon the net order imbalance n_{t+1}, which determines the expected execution probability at the CN. Clearly, the time t trader's decision potentially affects n_{t+1} and resultingly the execution probability of a CN buy order submitted at $t+1$. More specifically, submitting a CN sell order at t renders n_{t+1} more favorable for a buyer arriving at $t+1$, whereas submitting a CN buy order at t has the reverse effect. Performing a DM buy or sell trade, however, does not affect n_{t+1} at all (i.e. $n_t = n_{t+1}$). As this reasoning applies to a trader of any specific type, it also holds when aggregating over all types β_{t+1}. A similar intuition holds for part b of Proposition 4.

In general, Proposition 4 demonstrates the existence of systematic patterns in order flow. This finding is of importance to empirical researchers. The literature tends to attribute such patterns to informed trading, whereas our model shows that they can also stem from the interaction between two trading venues. For example, a series of consecutive buy trades at the DM need not imply that some traders have private information; it might result from an unfavorable imbalance in the CN book for the buyer. Thus, empirical research focusing on patterns in DM order flow (while neglecting the CN) might yield incorrect conclusions. An interesting empirical application of our model would, therefore, be to determine the importance of this interaction effect in explaining order flow patterns relative to other factors.

Furthermore, note that the CN in our model also exhibits two opposing externalities, as in Hendershott and Mendelson (2000). On the one hand, a positive (liquidity) externality prevails on the CN because adding an order is beneficial to counterparties arriving at later times; as such, liquidity attracts additional liquidity. On the other hand, a negative (crowding) externality exists as early arriving low liquidity value traders may preempt higher liquidity value traders arriving later in the trading day. Hence, these externalities, identified by Hendershott and Mendelson, also hold in a dynamic context in which traders arrive sequentially instead of simultaneously.

Finally, it is worth stressing that the underlying dynamics behind the patterns outlined in Propositions 3 and 4 are very different from those behind the propositions on order flow patterns in Parlour (1998). As argued above, in the case of a limit order market, the individual lengths of the queues at bid and ask are important. Both market and limit orders alter the available depth and resultingly the lengths of the queues at these quotes, thus affecting the execution probabilities of subsequent limit orders. As such, both order types influence the probabilities of occurrence of subsequent orders and cause systematic patterns in order flow to arise. In contrast, within our model, which has both a DM and a CN, it is the net order imbalance in the CN that is relevant and this imbalance is influenced only by CN orders, not by DM trades.

4.2 Opaqueness

With transparency, traders conditioned their strategies on past order flow and the resulting observable net order imbalance in the CN book. In reality, however, some CNs are rather opaque and do not actively disseminate information on their order book. In this subsection, we adapt our model to capture two degrees of opaqueness, and contrast them to transparency. Other models of CNs, such as Hendershott and Mendelson (2000) or Dönges and Heinemann (2006), cannot compare different informational settings, because they deal with simultaneous order submissions.

4.2.1 Complete Opaqueness

Complete opaqueness implies that a trader no longer observes past DM trades and past CN order flow. In order to condition her strategy on the net order imbalance in the CN book, n_t, she needs to form expectations about it. She does so in two steps. First, she is able to solve any past trader's optimization problem. Combining this with the common knowledge on $F(\beta)$, π_b and π_s in a second step allows her to compute the expected net order imbalance in the CN book at t, formally denoted by $E_{t,co}(n_t)$ (where the subscript "*co*" indicates complete opaqueness). On the basis of this $E_{t,co}(n_t)$, she computes the expected execution probability of a CN order submitted at t, which we denote by $p^b_{t,co}\left(E_{t,co}(n_t)\right)$ if she is a buyer, and by $p^s_{t,co}\left(E_{t,co}(n_t)\right)$ if she is a seller.[10] As in Subsection 4.1, we suppress the dependence for notational convenience and denote them in short by $p^b_{t,co}$ and $p^s_{t,co}$, respectively. Using these calculations as a basis, she determines the time t cutoff betas: i.e. $\underline{\beta}^b_{t,co}\left(p^b_{t,co}\right)$, $\bar{\beta}^b_{t,co}\left(p^b_{t,co}\right)$ for a buyer and $\underline{\beta}^s_{t,co}\left(p^s_{t,co}\right)$, $\bar{\beta}^s_{t,co}\left(p^s_{t,co}\right)$ for a seller. Now denote the optimal strategy of a buyer arriving at t as $\phi^b_{t,co}\left(E_{t,co}(n_t), \beta_t\right)$, and that of a seller by $\phi^s_{t,co}\left(E_{t,co}(n_t), \beta_t\right)$. It is then possible to reformulate Proposition 1 for the complete opaqueness setting by replacing the cutoff betas and strategies with their respective counterparts, defined in the current subsection. This modified Proposition 1 characterizes traders' equilibrium order submission

[10] Clearly, to derive these probabilities, as in the transparency case, she needs to solve the choice problems for traders who will arrive at later periods.

strategies in a setting of complete opaqueness. These strategies are shown in Figure 2 (for $i = co$).

4.2.2 Partial Opaqueness

Under "partial" opaqueness, traders only observe previous DM trades, but not CN order flow or the net order imbalance in the CN book. This informational setting corresponds closest to reality as DMs in general exhibit (mandated) transparency, while CNs are rather opaque. A time t trader thus observes in each past period either a DM buy, a DM sell, or no trade at the DM. In the latter case, she does not know whether a CN buy, a CN sell, or no order was submitted. Her information is now clearly richer than under complete opaqueness, allowing her to form more precise expectations about n_t. Denote by $E_{t,po}(n_t)$ the expected net order imbalance for the trader at time t under partial opaqueness (subscript "po"). This $E_{t,po}(n_t)$ allows her to compute the expected execution probability of a CN order, $p^b_{t,po}\left(E_{t,po}(n_t)\right)$ for a buy order and $p^s_{t,po}\left(E_{t,po}(n_t)\right)$ for a sell order. In short, we denote these probabilities as $p^b_{t,po}$ and $p^s_{t,po}$, respectively. Again, she can determine her cutoff betas: $\underline{\beta}^b_{t,po}\left(p^b_{t,po}\right)$ and $\bar{\beta}^b_{t,po}\left(p^b_{t,po}\right)$ if she is a buyer, or $\underline{\beta}^s_{t,po}\left(p^s_{t,po}\right)$ and $\bar{\beta}^s_{t,po}\left(p^s_{t,po}\right)$ if she is a seller. Note that, in contrast to complete opaqueness, these cutoff betas exhibit path dependency.[11] In other words, they hinge on past traders' decisions, which are now partly observed. The resulting optimal strategy for the time t trader is denoted by $\phi^b_{t,po}\left(E_{t,po}(n_t), \beta_t\right)$ for a buyer and $\phi^s_{t,po}\left(E_{t,po}(n_t), \beta_t\right)$ for a seller. Reformulating Proposition 1 for partial opaqueness characterizes the traders' equilibrium order submission strategies within this setting. These strategies are summarized in Figure 2 (for $i = po$).

4.2.3 Empirical Predictions on Order Flow Dynamics

With complete opaqueness, later arriving traders do not observe previous traders' strategies and also anticipate that their own decision will not be revealed to subsequent traders. Resultingly, traders' decisions become independent of past orders. Each arriving trader will make decisions using general predictions about past and future traders' behavior and about the resulting expected net order imbalance in the CN book. Therefore, and in contrast to the transparency setting, empirical work that ex post observes the different decisions of investors should not find path dependency.

[11] Clearly, however, the cutoff betas under transparency and partial opaqueness are in general not equal, because the information of the trader at time t differs in both settings.

The question then arises whether the systematic patterns in order flow under transparency extend to the partial opaqueness setting. Proposition 5 reveals that the answer is ambiguous. Systematic patterns do also arise under partial opaqueness, but their nature may be different from those under transparency. More specifically, Proposition 5 shows that the probability of occurrence of a CN buy order at $t+1$, after observing a DM trade at t, can be smaller, equal to, or larger than that after observing no DM trade. As shown in the proof of this proposition, the resulting patterns depend on the probabilities the trader at $t+1$ assigns to a CN buy, CN sell or no order at t and on her expectations of the CN's net order imbalance. Complementary, it can become more, equally, or less likely that the trader at $t+1$ submits a DM buy if the time t order was a DM trade, than if no DM trade was observed.

Proposition 5 *With partial opaqueness, order flow patterns are ambiguous. In particular,*

a) the probability of a CN buy order occurring at $t+1$ *if the order at* t *was an observed DM trade (buy or sell) can be smaller, equal to, or larger than that if no DM trade was observed at* t *(i.e.* $\phi^s_{t,po}(E_{t,po}(c_t), \beta_t) = -1^{CN}$ *or* 0*, or* $\phi^b_{t,po}(E_{t,po}(c_t), \beta_t) = 1^{CN}$ *or* 0*):*

$$\begin{gathered}\Pr\left[1^{CN} \text{ at } t+1 | \phi^b_{t,po}\left(E_{t,po}(n_t), \beta_t\right) = 1^{DM} \right. \\ \left. \text{or } \phi^s_{t,po}\left(E_{t,po}(n_t), \beta_t\right) = -1^{DM}, E_{t,po}(n_t)\right] \\ < \text{ or } = \text{ or } > \\ \Pr\left[1^{CN} \text{ at } t+1 | \text{no DM trade observed at } t, E_{t,po}(n_t)\right];\end{gathered} \tag{14}$$

b) the probability of a DM buy occurring at $t+1$ *if a DM trade (buy or sell) was observed at* t *can be smaller, equal to, or larger than that if no DM trade was observed at* t *(i.e.* $\phi^s_{t,po}(E_{t,po}(c_t), \beta_t) = -1^{CN}$ *or* 0*, or* $\phi^b_{t,po}(E_{t,po}(c_t), \beta_t) = 1^{CN}$ *or* 0*):*

$$\begin{gathered}\Pr\left[1^{DM} \text{ at } t+1 | \phi^b_{t,po}\left(E_{t,po}(n_t), \beta_t\right) = 1^{DM} \right. \\ \left. \text{or } \phi^s_{t,po}\left(E_{t,po}(n_t), \beta_t\right) = -1^{DM}, E_{t,po}(n_t)\right] \\ < \text{ or } = \text{ or } > \\ \Pr\left[1^{DM} \text{ at } t+1 | \text{no DM trade observed at } t, E_{t,po}(n_t)\right].\end{gathered} \tag{15}$$

For both a and b, a symmetric result holds for the other side of the market.

Proof. See Appendix A. ■

Proposition 5 thus shows that systematic patterns in order flow under partial opaqueness may change over time and work in two directions. The intuition for this finding is that observing no DM trade relative to a DM trade may be "good" news or "bad" news for a successive CN order. It may be "good" news because observing no DM trade may reveal the addition of counterparties. It is "bad" news when the observation of no DM trade suggests that an interesting opportunity at a CN may have been preempted by a time t trader. This result is in contrast with the patterns in the transparency setting, which were determined unambiguously. In other words, we find that the CN's transparency level plays an important role for order flow patterns to both the CN and the DM. Changing this institutional property of the CN may, therefore, affect order flow.

5 Welfare Analysis

We organize our welfare analysis as follows. Subsection 5.1 offers a formal definition of our overall welfare concept and the inefficiencies we consider. In Subsection 5.2, we discuss overall welfare for the isolation cases and the coexistence of markets. Next, in Subsection 5.3, the different sources of inefficiencies are further analyzed. Finally, in Subsection 5.4, we highlight some further considerations regarding welfare and propose some extensions to our analysis.

Throughout all illustrations in this section, we assume that β is uniformly distributed over $[0.8, 1.2]$, $\pi_b = 0.5$ and $\frac{A+B}{2} = V$.[12] Further, we assume $T = 3$, i.e. a three-period trading day. However, our results remain robust when $T = 2$ or 4; adding more periods only makes the calculations more complex and does not alter our conclusions.[13]

5.1 Overall Welfare and Inefficiencies: Definitions

Our *ex ante* welfare measure builds on rational trader behavior. It is therefore identical to the "mean" realized *ex post* welfare. More specifically, the welfare measure we employ is overall welfare, $\overline{OW}$, that is, the sum of all agents' expected gains from trade (see Glosten (1998), Goettler, Parlour and Rajan (2005), or Hollifield, Miller, Sandas and Slive (2006) for a similar approach in defining welfare). In our model, $\overline{OW}$ consists of both trader welfare and dealer welfare. The upper bar denotes we consider *average per-period* overall welfare, but from now on we use the term "overall welfare" for brevity. We compute $\overline{OW}$ for the different cases and settings and compare it to the *maximum overall welfare* $\overline{OW}^{\max}$, i.e. the overall welfare when all buy and sell orders execute with probability one at the midquote.

[12] Thus, the one-tick spread is assumed to be positioned symmetrically around V. We checked the robustness of our results by investigating different ranges for beta.

[13] The results of these robustness tests are available upon request from the authors.

Implicitly, this benchmark measure assumes a counterparty that always provides liquidity at the midquote and makes zero profit. Hence, $\overline{OW}^{\max}$ represents a financial market without inefficiencies; for example, a DM with dealers who are willing to trade at the midquote, or a CN with certainty of execution. Formally, the maximum overall welfare is defined as

$$\overline{OW}^{\max} = \pi_b \int\limits_{\frac{A+B}{2V}}^{\overline{\beta}} \left(\beta_t V - \frac{A+B}{2}\right) f(\beta_t)\, d\beta_t + \pi_s \int\limits_{\underline{\beta}}^{\frac{A+B}{2V}} \left(\frac{A+B}{2} - \beta_t V\right) f(\beta_t)\, d\beta_t. \qquad (16)$$

In this equation, we measure the gains from trade of a buyer and a seller[14] trading at the midquote with execution probability one, and integrate over all participating traders. In fact, equation (16) clearly indicates why overall welfare in practice may be lower than $\overline{OW}^{\max}$. First, an order at the CN may execute with a probability less than 1, which will lower the expected gains from trade. Secondly, the set of traders that submit an order may differ from that in $\overline{OW}^{\max}$. We use these insights to define the two inefficiencies that drive the wedge between $\overline{OW}^{\max}$ and the overall welfare in our different settings. Note that Figure 2 is instructive while reading through this subsection because it highlights the three groups of traders we are distinguishing under coexistence of a DM and CN, as well as the related inefficiencies. These three groups are: *(i)* buyers with $\beta_t \in \left(\frac{A+B}{2V}, \frac{A}{V}\right]$ and sellers with $\beta_t \in \left[\frac{B}{V}, \frac{A+B}{2V}\right)$, *(ii)* buyers with $\beta_t \in \left(\frac{A}{V}, \overline{\beta}^b_{t,i}\right]$ and sellers with $\beta_t \in \left[\underline{\beta}^s_{t,i}, \frac{B}{V}\right)$, and *(iii)* buyers with $\beta_t \in \left(\overline{\beta}^b_{t,i}, \overline{\beta}\right]$ and sellers with $\beta_t \in \left[\underline{\beta}, \underline{\beta}^s_{t,i}\right)$, with $i = tr$, *co* or *po*. For the DM or CN in isolation, we refer to Figure 1.

The first inefficiency stems from buyers with $\beta_t \in \left(\frac{A+B}{2V}, \frac{A}{V}\right]$ and sellers with $\beta_t \in \left[\frac{B}{V}, \frac{A+B}{2V}\right)$. We label this inefficiency *"CN-only-traders inefficiency"*: both under isolation and coexistence, those traders only participate in the CN, but never in the DM. In the maximum overall welfare case, the CN-only-traders inefficiency is, by definition, zero because the orders of these CN-only traders execute with probability one. We will show that the CN-only-traders inefficiency becomes positive because the average execution probability of the orders of CN-only traders is smaller than one. Traders can even opt not to participate if the execution probabilities are zero or if there is no CN.

The second inefficiency relates to those traders opting for the CN when given the choice, even though they would in fact also be willing to trade at a DM, i.e. buyers with $\beta_t \in \left(\frac{A}{V}, \overline{\beta}^b_{t,i}\right]$ and sellers with $\beta_t \in \left[\underline{\beta}^s_{t,i}, \frac{B}{V}\right)$. We label this inefficiency *"CN-diverted-traders inefficiency"*. These traders' orders create an inefficiency because the average CN's execution probability

[14] Recall that we used similar expressions when deriving Proposition 1.

is less than one. This occurs because, given the choice between DM and CN, CN-diverted traders may opt for the CN even when the expected execution probability of their order is less than one. That is, CN-diverted traders trade off the half-spread with the lower execution probability and opt for the latter. Overall welfare, however, also includes dealers' profits, which are now unrealized for this beta range. By definition, for the overall welfare maximum, this inefficiency is zero because all orders execute with probability one. The formal definitions of both inefficiencies are provided in Appendix B.

Finally, notice that buyers with $\beta_t \in \left(\overline{\beta}^b_{t,i}, \overline{\beta}\right]$ and sellers with $\beta_t \in \left[\underline{\beta}, \underline{\beta}^s_{t,i}\right)$ opt for the DM in all settings. All potential gains from trade for these trades are realized such that there is no overall welfare inefficiency. Prices are simply transfers between traders and dealers.

5.2 Overall Welfare

5.2.1 Maximum Overall Welfare

We present the maximum overall welfare, $\overline{OW}^{\max}$, in Figure 3. We observe that $\overline{OW}^{\max}$ increases linearly in V because the gains from trade increase linearly in V and the set of participating traders is independent of V. It can be shown that $\overline{OW}^{\max}$ does not hinge on T as per-period welfare is constant, i.e. adding more periods does not affect the maximum overall welfare

5.2.2 Markets in Isolation

With a *DM in isolation*, all submitted orders result in trades. However, buyers with β_t lower than $\frac{A}{V}$ and sellers with β_t higher than $\frac{B}{V}$ do not submit orders (see Panel A of Figure 1). The overall welfare at the DM in isolation, $\overline{OW}^{DM}$, consists of all gains from trade, including dealer profits, from all trades. Prices are simply transfers between traders and dealers. However, they do affect the participation of traders. A major benefit of a DM, which drives $\overline{OW}^{DM}$, is that traders who have a higher willingness to trade always participate and that all gains from trade are realized for these trades. Further, $\overline{OW}^{DM}$ can be shown to be independent of the number of periods T. $\overline{OW}^{DM}$ is shown in Figure 3, where it is assumed that the dealer has a β equal to one. Clearly, $\overline{OW}^{DM}$ increases in V as more traders participate and their realized gains per trade at the DM increase with larger V.

Conversely, the overall welfare at a *CN in isolation,* $\overline{OW}^{CN}$, only stems from trading gains realized by traders. Whereas all buyers with β_t higher than $\frac{A+B}{2V}$ and all sellers with β_t lower than $\frac{A+B}{2V}$ submit orders to the CN (see Panel B of Figure 1), not all submitted orders will result in trades when execution probabilities are smaller than one.[15]

Please insert Figure 3 around here.

Comparing $\overline{OW}^{CN}$, $\overline{OW}^{DM}$ and $\overline{OW}^{\max}$ in Figure 3, we observe that, as expected, both isolation cases perform worse than the overall welfare maximum, i.e. $\overline{OW}^{CN} \leq \overline{OW}^{\max}$ and $\overline{OW}^{DM} \leq \overline{OW}^{\max}$, for all V. The underperformance of the DM in isolation decreases in V because the smaller relative spread, defined as the bid-ask spread divided by the underlying value of the asset, induces more traders to participate when V becomes larger. In contrast, the underperformance of the CN in isolation increases in V because the execution probabilities are independent from V, implying that more and more gains from trade are unrealized when V increases and orders are not executed.[16]

Further, we find that in terms of overall welfare, the DM performs better than the CN for higher V, i.e. for assets that have a higher underlying value (or a lower relative spread). $\overline{OW}^{DM}$ is 0 for $V < 2.5$ as the spread prices the DM out of the market within this region, whereas the CN still generates welfare.

5.2.3 Coexistence of Markets

We now analyze overall welfare relative to $\overline{OW}^{\max}$ when traders endogenously route their orders to the system which maximizes their individual expected gains from trade. The analysis below explores whether coexistence of both trading systems improves overall welfare as compared to the isolation cases, and further provides insights on the impact of a change in the informational setting. We denote overall welfare when the CN and the DM coexist under transparency, complete opaqueness and partial opaqueness by $\overline{OW}_{tr}$, $\overline{OW}_{co}$ and $\overline{OW}_{po}$, respectively. Figure 3 does not include $\overline{OW}_{co}$ because it visually almost coincides with $\overline{OW}_{po}$.

In Figure 3, we observe three main results. First, as expected, coexistence always generates lower overall welfare than the maximum overall welfare, $\overline{OW}^{\max}$. Second, the overall welfare of a DM in isolation, $\overline{OW}^{DM}$, outranks coexistence for the three informational settings, except for lower values of V. Coexistence of trading systems therefore does not necessarily improve overall welfare. Third, transparency yields higher overall welfare than partial

[15] Interestingly, $\overline{OW}^{CN}$ can be shown to increase in the number of periods T because a higher number of periods produces positive liquidity externalities on the expected execution probabilities.

[16] As $\overline{OW}^{CN}$ increases in T, the difference between $\overline{OW}^{\max}$ and $\overline{OW}^{CN}$ could be expected to decrease should the trading day consist of more than three periods. Given that $\overline{OW}^{DM}$ is independent of T, the difference between $\overline{OW}^{\max}$ and $\overline{OW}^{DM}$ remains constant in T.

opaqueness only for low values of V. Indeed, we notice in Figure 3 that $\overline{OW}_{po}$ and $\overline{OW}_{tr}$ intersect for V around 7.5. A similar result applies when comparing $\overline{OW}_{co}$ to $\overline{OW}_{po}$ where $\overline{OW}_{co}$ is smaller than $\overline{OW}_{po}$ for small V but larger for high V. This confirms the result that greater opaqueness results in increased overall welfare for higher V. In the next subsection, we study the major factors that drive these results.

5.3 The Role of Inefficiencies

5.3.1 Markets in Isolation

With a DM in isolation, the CN-only-traders inefficiency is at its maximum because these traders do not participate. Panel A of Figure 4 shows that the CN-only-traders inefficiency with a DM in isolation decreases in V. The reasoning is that there are less CN-only traders and that these buyers' (sellers') beta is lower (higher) with greater V. In turn, with a CN in isolation, the CN-only-traders inefficiency is positive and takes intermediate values: the CN-only traders participate but the average execution probability is less than one. The lower the average execution probability, the greater the CN-only-traders inefficiency.[17]

Consider now the CN-diverted-traders inefficiency as depicted in Panel B of Figure 4. This inefficiency equals zero for a DM in isolation. This is because the set of CN-diverted traders by definition is empty. In contrast, with a CN in isolation, all buyers with a $\beta_t \in \left(\frac{A}{V}, \bar{\beta}\right]$ and sellers with $\beta_t \in \left[\underline{\beta}, \frac{B}{V}\right)$ are evidently CN-diverted and induce an overall welfare inefficiency because their orders might not find a counterparty. This inefficiency increases in V for two reasons. First, the set of CN-diverted traders relatively increases for larger V. Second, traders' welfare losses from non-execution increase when V becomes larger.

The sum of both inefficiencies (CN-only-traders inefficiency and CN-diverted-traders inefficiency) constitutes the difference between $\overline{OW}^{CN}$ and $\overline{OW}^{\max}$, and $\overline{OW}^{DM}$ and $\overline{OW}^{\max}$, respectively, as depicted in Figure 3. Given that the CN-diverted-traders inefficiency is zero for a DM in isolation, the difference between $\overline{OW}^{DM}$ and $\overline{OW}^{\max}$ is driven solely by the CN-only-traders inefficiency. In contrast, the difference between $\overline{OW}^{\max}$ and $\overline{OW}^{CN}$ is attributed to both inefficiencies. A comparison of Panels A and B in Figure 4 shows that for a CN in isolation, the CN-diverted-traders inefficiency is higher than the CN-only-traders inefficiency for sufficiently high V.

Please insert Figure 4 around here.

[17] The average execution probability increases when more counterparties are present in the system. This happens, for example, when T increases.

5.3.2 Coexistence of Markets

Figure 3 revealed that coexistence and transparency not necessarily improve overall welfare. To improve our understanding of this result, we study how coexistence and transparency affect the CN-only-traders inefficiency and the CN-diverted-traders inefficiency.

The impact of coexistence We first discuss the impact of coexistence on the CN-diverted-traders inefficiency. This inefficiency is strictly positive when the markets coexist and is, therefore, larger than the inefficiency for a DM in isolation (which was zero). In turn, with coexistence, the CN-diverted-traders inefficiency is lower than for a CN in isolation. Indeed, when the two trading systems coexist, high valuation traders (buyers with $\beta_t \in \left(\bar{\beta}^b_{t,i}, \bar{\beta}\right]$ and sellers with $\beta_t \in \left[\underline{\beta}, \underline{\beta}^s_{t,i}\right)$) are not forced to go to the CN and opt for the DM, which lowers the created welfare inefficiency. Further, we observe that the CN-diverted-traders inefficiency under coexistence exhibits an inverse U-shape in V. Before explaining this non-linearity, note that the CN-diverted-traders inefficiency is zero both in the absence of trade diversion and with "complete" trade diversion featuring execution probabilities equal to one. The CN-diverted-traders inefficiency is positive for any degree of trade diversion featuring execution probabilities lower than one. Panel B of Figure 4 displays the inverse U-shape in V. The CN-diverted-traders inefficiency is zero when $V < 2.5$ because there is no trade diversion in this region.[18] Next, we notice an increase of the CN-diverted-traders inefficiency for higher V: there is complete trade diversion with relatively low execution probabilities. The CN-diverted-traders inefficiency declines again when V is sufficiently high because more traders tend to prefer the DM at higher values of V. In sum, we find that due to trade diversion, the CN-diverted-traders inefficiency is greater under coexistence than in a DM in isolation. However, this inefficiency is less under coexistence than in a CN in isolation, due to less traders being diverted.

Next, the CN-only-traders inefficiency is quite high for low values of V and coincides with that of the CN in isolation. It tends towards zero for higher V as this set of traders becomes relatively less important and their willingness to trade decreases. When the two trading systems coexist, the CN-only-traders inefficiency is lower than for a DM in isolation, where this inefficiency was at its maximum, but greater than for a CN in isolation. This latter result stems from the fact that now less trade diversion of interesting counterparties occurs than in a CN in isolation, which lowers the execution probability of CN-only traders.

[18] With $V < 2.5$, the DM prices itself out of the market, which entails that no traders can be diverted from the DM to the CN.

The sum of the effects of both inefficiencies determines the impact of coexistence on overall welfare. Coexistence only leads to higher welfare than a DM in isolation for low values of V: the decrease in the CN-only-traders inefficiency then dominates the modestly positive CN-diverted-traders inefficiency since execution probabilities at the CN are quite high. The opposite holds for higher V. As compared to a CN in isolation, the CN-only-traders inefficiency increases when the two trading systems coexist, but the CN-diverted-traders inefficiency falls. However, the sum of the two inefficiencies is less than in a CN in isolation.

The impact of opaqueness Figure 3 showed that transparency does not necessarily outperform opaqueness when trading systems coexist. Indeed, transparency is only better at low values of V. We now analyze this result further by studying how transparency affects the two inefficiencies, as illustrated in the two Panels of Figure 4.

Panel A of Figure 4 shows that the CN-only-traders inefficiency decreases as the degree of transparency increases. This is because transparency invites counterparties to the CN as more traders divert to the CN (i.e. the liquidity externality of the CN). This increased order flow leads to the orders created by the CN having a greater execution probability, thus reducing the CN-only-traders inefficiency. Hence, completely transparent markets are most beneficial for this segment of traders.

The CN-diverted-traders inefficiency, in contrast, does not decrease unambiguously as the degree of transparency increases. This inefficiency falls as the degree of transparency increases only when V is small. While diverted trades imply with certainty that dealers do not earn the spread, they only yield welfare for the executed part of the CN order flow. With complete and partial opaqueness and small V, traders may expect attractive execution probabilities of their orders and therefore still divert to the CN even though no orders are present in the CN book (which was invisible to these traders). Transparency prevents these ex post wrong paths from taking place, which entails that the CN-diverted-traders inefficiency is lower as trade diversion occurs more when execution probabilities are very large. That is, for low V, transparency induces either complete trade diversion with high execution probabilities in certain contingencies or no trade diversion at all in other contingencies, whereas opaqueness tends to induce intermediate trade diversion combined with lower execution probabilities. However, for higher V, in the initial period, transparency induces relatively large trade diversion because diverted traders know that their orders are visible to arriving counterparties.

More trade diversion combined with relatively low execution probabilities, however, may be harmful for overall welfare. This happens with transparency and at large values of V.

5.4 Further Welfare Considerations and Extensions

We first decompose overall welfare into trading gains realized by traders and by dealers, as shown in Figure 5. It is clear that in contrast to the overall welfare case, now prices do not simply cancel out as they affect both trader and dealer welfare, more specifically in an opposite way.

Panel A in Figure 5 presents the average per-period trader welfare for the two isolation cases and two informational settings under coexistence (again complete opaqueness is visually close to partial opaqueness and therefore omitted). We obtain two main results. First, coexistence always produces greater trader welfare than the DM in isolation as it widens traders' opportunity sets. Second, a greater degree of transparency increases trader welfare unambiguously because traders anticipate their orders will be revealed to potential counterparties.[19] This contrasts with the finding for overall welfare, where a larger degree of transparency only resulted in higher $\overline{OW}$ at low values of V.

Next, Panel B of Figure 5 displays average per-period dealer welfare. First, note that dealer welfare is obviously highest in the case of the DM in isolation because trading at the DM is then at its maximum. Under coexistence, part of the order flow is diverted to the CN, which results in lower dealer welfare. Secondly, dealer welfare is higher under transparency for relatively low values of V only. Then, trade diversion is relatively minor, because traders opt for the DM when they are informed about the lack of counterparties at the CN.

In sum, while traders jointly prefer more transparency, dealers do not necessarily agree because they only desire a higher degree of transparency for low values of V.

Please insert Figure 5 around here.

Finally, our main model can be extended in many ways. A particularly interesting extension concerns traders who may have single or multiple units to trade. Two cases can be considered. The first is where the traders' choice of trading system is made before knowing their order size (as in Viswanathan and Wang (2002)). This implies that orders cannot be

[19] This finding implies that a CN that maximizes the welfare of those trading on its system should become more transparent. While CNs are typically quite opaque in practice, in line with this result, we recently observed that some CNs have become more transparent. Examples include BIDS and ITG's POSIT-Now with BLOCKalert, which reveal the arrival of potential counterparties to the investor community.

split such that our main findings are only affected in a rather straightforward way (i.e. larger orders affect execution probabilities and therefore the choice between CN and DM). The second case is where traders who have a multiple-unit order can engage in order splitting. In our analysis, order splitting implies choosing for different trading systems. This case becomes increasingly more complex as the state space expands, because some traders may have one unit whereas others have multiple units. To identify the impact of trading multiple units and the potential issue of order splitting, consider a stylized two-period trading day in which all traders are exposed to identical multiple-unit orders. We argue that most of our results remain unaffected for transparency and complete opaqueness as traders are risk neutral. More specifically, in transparent markets, traders' order submission strategies will not change compared to our previous analysis. This is because it is always optimal for a first-period trader to put the entire multiple-unit order either on the CN or the DM. Her decision is unaffected as the order is revealed publicly and the "good" counterparty will find it optimal to take either the entire order or no unit at all. With complete opaqueness, equivalently, the traders' strategies are unaffected by order size as traders are risk neutral. Technically, the system of equations to be solved is independent of the size of trades. With partial opaqueness, however, traders might find it interesting to submit one order to the DM and route the remainder to the CN. In this way, they "reveal" that orders were submitted to the CN. The results of partial opaqueness could then be expected to become closer to those for transparency.

6 Conclusion

We have presented a dynamic microstructure model to study the interaction between two trading systems. We studied the competition between a crossing network (CN) and a dealer market (DM) within three different informational settings. In particular, a transparency setting in which agents have full information on the CN's order book and DM trades was contrasted with two opaqueness settings. Under "complete" opaqueness, traders have information neither on the CN book, nor on DM trades, while with "partial" opaqueness they observe only DM trades and not order flow to the CN. CNs are, in practice, quite opaque in that they often prevent traders from observing the CN book.

We found that introducing a CN next to a DM generates two effects on order flow. First, it leads to "*order creation*", as the CN attracts investors who would refrain from trading in the absence of a CN. This is because traders can save the half-spread when they trade at the

CN, which makes the submission of a CN order profitable for these investors. Second, some orders by relatively low willingness to trade agents trading on the DM are now diverted to the CN. This "*trade diversion*" induces competition for the DM.

We also showed that the execution probability at a CN is endogenous. It depends on the (observed or expected) net order imbalance in the CN book, the observed order flow, and the expectation of past and future orders. Thus, although we start from dealers willing to provide liquidity at exogenously given bid and ask prices, we partly endogenize liquidity supply and demand by looking at traders submitting orders for potential execution at a CN. Our dynamic model displays two externalities on the CN, as documented in Hendershott and Mendelson (2000), who consider competition between a DM and a CN in a static game with simultaneous order submissions. On the one hand, the CN is characterized by a positive (liquidity) externality as adding a CN buy (sell) order is beneficial to future CN sellers (buyers). On the other hand, a CN exhibits a negative (crowding) externality as investors with a low willingness to trade who arrive early in the trading day may preempt investors with a higher willingness to trade who arrive later.

Our welfare results can be summarized as follows. First, when comparing markets in isolation, we find that a CN provides greater overall welfare than a DM when the execution probability at the CN and the relative spread (the bid-ask spread divided by the underlying value) are high, i.e. when the time to a cross is long and the underlying value of the asset is low. Second, order creation and trade diversion determine the impact of coexistence of trading systems and the degree of transparency on welfare. For assets with a high relative spread, overall welfare increases with coexistence of trading venues as compared to a DM in isolation and with transparency as compared to both opaqueness settings. The positive contribution to welfare of order creation is then substantial enough to compensate for the limited negative impact of trade diversion. For low values of the relative spread, overall welfare decreases when there is coexistence and transparency compared to a DM in isolation. The negative impact of trade diversion then outweighs the positive impact of order creation.

Finally, our model offers a number of empirical predictions on order flow patterns. In particular, we find systematic patterns in order flow under transparency and partial opaqueness. These patterns stem from changes in the net order imbalance at the CN's order book. With transparency, we found that the probability that the next order is a CN order at the same side of the market is smaller after such an order than after any other order. In addition, the probability of a DM sell decreases and the probability of a DM buy increases when the previous order was a CN buy order. Only CN orders generate time-varying order flow on both trading systems as DM trades leave the CN's order book unaffected. Systematic patterns also arise with partial opaqueness. Although traders now only observe past DM trades and no CN orders, they use this information to form expectations on the CN's order book imbalance and to determine their trading strategy. We have shown that the degree of transparency at the CN has important implications for order flow. More specifically, compared to a transparent CN, order flow patterns may reverse when the CN is opaque. In general, our empirical predictions demonstrate that it is important to take the interaction between trading systems, as well as their institutional characteristics, into account when measuring "normal" order flow. Some order or trade flow sequences, when analyzed in individual markets, could be interpreted incorrectly as being driven by informed trading, whereas they are actually caused by the interaction of trading systems. An interesting empirical application of our model would, therefore, be to determine the importance of this "interaction effect" in explaining observed order flow patterns relative to other factors, such as private information or dealers' inventory management. We leave this for future research.

Appendix A: Proofs

Proof of Proposition 1. Suppose that the trader at time t is a buyer. She selects her strategy to maximize her profits, i.e. max $\left[\beta_t V - A, p^b_{t,tr}\left(\beta_t V - \frac{A+B}{2}\right), 0\right]$. Now define

$$\bar{\beta}^b_{t,tr}\left(p^b_{t,tr}\right) = \left| \begin{array}{ll} \bar{\beta} & \text{if } p^b_{t,tr} \geq \frac{\bar{\beta}V - A}{\bar{\beta}V - \frac{A+B}{2}} \\ \text{solves } p^b_{t,tr}\left(\bar{\beta}^b_{t,tr}V - \frac{A+B}{2}\right) = \bar{\beta}^b_{t,tr}V - A & \text{otherwise} \end{array} \right. .$$

This implies that $\bar{\beta}^b_{t,tr}\left(p^b_{t,tr}\right)$ is an upper bound on CN buying, because in the second case $p^b_{t,tr}\left(\bar{\beta}^b_{t,tr}V - \frac{A+B}{2}\right)$ increases in β at rate $p^b_{t,tr}V$, whereas $\bar{\beta}^b_{t,tr}V - A$ increases at rate V. The condition

$$p^b_{t,tr} \geq \frac{\bar{\beta}V - A}{\bar{\beta}V - \frac{A+B}{2}}$$

can be interpreted as follows. If this condition is fulfilled, then $p^b_{t,tr}\left(\bar{\beta}V - \frac{A+B}{2}\right) \geq \bar{\beta}V - A$, which implies that even for $\bar{\beta}$, the profit of an order to the CN is higher than the profit of a DM trade. In that case, traders always choose to submit a CN order and the region of βs for which investors submit DM trades is empty. After solving and some rewriting, we find that

$$\bar{\beta}^b_{t,tr}\left(p^b_{t,tr}\right) = \min\left[\frac{\frac{A+B}{2}}{V} + \frac{1/2}{V\left(1 - p^b_{t,tr}\right)}, \bar{\beta}\right].$$

The cutoff βs between submitting a CN order and remaining out of the market are determined by how large the trader's valuation of the asset is, relative to its price. The lowest β-type who would buy at the CN is the one who values the asset at $\frac{\frac{A+B}{2}}{V}$. Hence, define

$$\underline{\beta}^b_{t,tr}\left(p^b_{t,tr}\right) = \left| \begin{array}{ll} \frac{\frac{A+B}{2}}{V} & \text{if } p^b_{t,tr} > 0 \\ \frac{A}{V} & \text{otherwise} \end{array} \right. .$$

Consequently, also $\bar{\beta}^b_{t,tr}\left(p^b_{t,tr}\right) \geq \underline{\beta}^b_{t,tr}\left(p^b_{t,tr}\right)$.

A similar proof can be constructed for a seller at time t. ■

Proof of Proposition 2. First, from the definitions of $\bar{\beta}^b_{t,tr}$ and $\underline{\beta}^s_{t,tr}$, the proposition can also be formulated as follows (with e.g. $\bar{\beta}^b_{t,tr}(n_t)$ shorthand for $\bar{\beta}^b_{t,tr}\left(p^b_{t,tr}(n_t)\right)$):

$$\begin{array}{ll} (i) & \bar{\beta}^b_{t,tr}(n_t - 1) \geq \bar{\beta}^b_{t,tr}(n_t) \geq \bar{\beta}^b_{t,tr}(n_t + 1) \\ (ii) & \underline{\beta}^s_{t,tr}(n_t - 1) \geq \underline{\beta}^s_{t,tr}(n_t) \geq \underline{\beta}^s_{t,tr}(n_t + 1) \end{array} .$$

We prove the proposition in a recursive way and by contradiction. As a starting point, it can be seen that the proposition holds for the terminal period T. At time T, the execution probability of a CN order is either one (if a trader can join the side where the net order imbalance, n_T, is favorable) or zero otherwise. Then the proposition holds as:

$$\bar{\beta}^b_{T,tr} = \bar{\beta} \text{ and } \underline{\beta}^b_{T,tr} = \frac{\frac{A+B}{2}}{V}; \qquad \underline{\beta}^s_{T,tr} = \underline{\beta} \text{ and } \bar{\beta}^s_{T,tr} = \frac{\frac{A+B}{2}}{V}$$

when the execution probability of a CN order equals one, and

$$\bar{\beta}^b_{T,tr} = \underline{\beta}^b_{T,tr} = \frac{A}{V}; \qquad \underline{\beta}^s_{T,tr} = \bar{\beta}^s_{T,tr} = \frac{B}{V}$$

when the execution probability of a CN order equals zero.

Suppose now that the proposition is false. However, since it is true at T, there must exist a period τ such that for $t > \tau$, all parts of the proposition hold, but at τ at least one part does not hold. In what follows, we posit the existence of such a τ and show that for each of the possible ensuing states the action is not optimal.

Assume the right-hand side of the inequality in statement (ii) does not hold at τ. Then:

$$\underline{\beta}^s_{\tau,tr}(n_\tau) < \underline{\beta}^s_{\tau,tr}(n_\tau + 1).$$

This means that a seller that has a $\beta_\tau \in \left[\underline{\beta}^s_{\tau,tr}(n_\tau), \underline{\beta}^s_{\tau,tr}(n_\tau + 1)\right)$ submits a DM trade when the CN's net order imbalance is $n_\tau + 1$ and a CN order when the CN's net order imbalance is n_τ. In contrast, suppose that the seller would opt for a CN order in the former case (i.e. when the CN's net order imbalance is $n_\tau + 1$). The CN's net order imbalance at $\tau + 1$ then becomes n_τ. If the CN's net order imbalance at τ is n_τ, the trader submits a CN sell order, which results in the CN's net order imbalance at $\tau + 1$ being $n_\tau - 1$. The next trader, arriving at $\tau + 1$, can be either a buyer or a seller.

1. *A seller arrives at* $\tau + 1$

   In this case, we know that by assumption statement (ii) holds for all periods $t > \tau$. Therefore

$$p^s_{\tau+1,tr}(n_\tau - 1) \leq p^s_{\tau+1,tr}(n_\tau).$$

   Moreover, an order submitted at $\tau + 1$ will only be executed if the previous order in the queue has been executed. This means, conditional on a seller arriving at $\tau+1$:

$$p^s_{\tau,tr}\left(n_\tau|\text{seller arrives at } \tau+1\right) \leq p^s_{\tau,tr}\left(n_\tau+1|\text{seller arrives at } \tau+1\right).$$

Then, it follows that

$$\begin{aligned}
&\left(\frac{A+B}{2}-\beta_\tau V\right) p^s_{\tau,tr}\left(n_\tau+1|\text{seller arrives at } \tau+1\right) \\
&\geq \left(\frac{A+B}{2}-\beta_\tau V\right) p^s_{\tau,tr}\left(n_\tau|\text{seller arrives at } \tau+1\right) \\
&\geq B-\beta_\tau V.
\end{aligned}$$

Thus, conditional on the trader arriving at $\tau+1$ being a seller, the payoff of a CN sell order is higher when the net order imbalance of the CN book is relatively higher in period τ. Hence, in this case it cannot be optimal for an investor to submit a DM trade at τ when the net order imbalance is relatively higher.

2. *A buyer arrives at* $\tau+1$

We know that by assumption statement (i) is true at $\tau+1$. This means that either buyers do not change their behavior or buyers with

$$\beta_{\tau+1} \in \left[\bar{\beta}^b_{\tau+1,tr}\left(n_\tau\right), \bar{\beta}^b_{\tau+1,tr}\left(n_\tau-1\right)\right]$$

submit a DM trade when the CN's net order imbalance is n_τ, which results in a net order imbalance of n_τ at $\tau+2$, and submit a CN order when the net order imbalance is $n_\tau-1$, which yields a net order imbalance at $\tau+2$ of n_τ. Thus, the net order imbalance is back at n_τ at $\tau+2$ for both possibilities. Since execution probabilities only hinge on the net order imbalance, the execution probability for a seller arriving at $\tau+2$ equals $p^s_{\tau+2,tr}\left(n_\tau\right)$ for both possibilities. Further, since statement (ii) holds at $\tau+2$:

$$p^s_{\tau+2,tr}\left(n_\tau+1\right) \geq p^s_{\tau+2,tr}\left(n_\tau\right).$$

Given that an order submitted at $\tau+2$ can only be executed if an order submitted at

τ has been executed, it follows that:

$$\begin{aligned}&\left(\frac{A+B}{2}-\beta_\tau V\right)p^s_{\tau,tr}\left((n_\tau+1)\,|\text{buyer arrives at }\tau+1\right)\\ &\geq\left(\frac{A+B}{2}-\beta_\tau V\right)p^s_{\tau,tr}\left((n_\tau)\,|\text{buyer arrives at }\tau+1\right)\\ &\geq B-\beta_\tau V.\end{aligned}$$

Hence, conditional upon a buyer arriving at $\tau+1$, there is a contradiction.

The right-hand side of the inequality in statement (ii) is therefore true. A similar proof can be constructed for the left-hand side of the inequality in (ii) and for statement (i). ■

Proof of Proposition 4. Part a) With transparency, the time t probability of occurrence of a CN buy at $t+1$ is

$$\Pr\left[1^{CN}\text{ at }t+1\right]=\pi_b\left[F\left(\bar{\beta}^b_{t+1,tr}\left(n_{t+1}\right)\right)-F\left(\underline{\beta}^b_{t+1,tr}\left(n_{t+1}\right)\right)\right].$$

Suppose that $n_t<T-(t+1)$, such that the probability of execution of a CN buy at $t+1$ is not zero. Then $\underline{\beta}^b_{t+1,tr}\left(n_{t+1}\right)$ is independent of the net order imbalance. If the order at t was a CN buy, then the net order imbalance at $t+1$ is n_t+1, if it was a CN sell the net order imbalance becomes n_t-1 and if the order was a DM trade, the net order imbalance does not change: $n_t=n_{t+1}$. From the proof of Proposition 2, we know that

$$\bar{\beta}^b_{t+1,tr}\left(n_t+1\right)\leq\bar{\beta}^b_{t+1,tr}\left(n_t\right)\leq\bar{\beta}^b_{t+1,tr}\left(n_t-1\right).$$

Given that $F\left(.\right)$ is monotonically nondecreasing in β, the result follows.

Suppose now that $n_t\geq T-(t+1)$, meaning either no CN orders are submitted, and the proposition holds trivially; or at t the extra CN order submitted changes the execution probability to zero. Then $\bar{\beta}^b_{t+1,tr}\left(n_t+1\right)=\frac{A}{V}$; hence the result follows since also $\underline{\beta}^b_{t+1,tr}\left(n_{t+1}\right)=\frac{A}{V}$.

Part b) The time t probability of occurrence of a DM buy at $t+1$ is

$$\Pr\left[1^{DM}\text{ at }t+1\right]=\pi_b\left[F\left(\bar{\beta}\right)-F\left(\bar{\beta}^b_{t+1,tr}\left(n_{t+1}\right)\right)\right].$$

$F\left(\bar{\beta}\right)$ is fixed and independent of the net order imbalance. If the order at t was a CN buy, then the net order imbalance at $t+1$ is n_t+1, if it was a CN sell the net order imbalance becomes n_t-1, and if the order was a DM trade the net order imbalance does not change:

$n_t = n_{t+1}$. From the proof of Proposition 2, we know that

$$\bar{\beta}^b_{t+1,tr}(n_t+1) \le \bar{\beta}^b_{t+1,tr}(n_t) \le \bar{\beta}^b_{t+1,tr}(n_t-1).$$

Given that $F(.)$ is monotonically nondecreasing in β, the result follows.

For both a and b, similar proofs could be constructed for sell orders. ■

Before starting the proof of Proposition 5, we first state the following corollary:

Corollary 1 *In equilibrium, $\forall\ E_{t,po}(n_t)$, t, if the expected net order imbalance is one unit higher, then the expected probability of execution of a buy (sell) order will be lower (higher). If the expected net order imbalance is one unit lower, then the expected probability of execution of a buy (sell) order will be higher (lower). Hence,*

$$\begin{array}{ll} \textit{(i)} & p^b_{t,po}\left(E_{t,po}(n_t-1)\right) \ge p^b_{t,po}\left(E_{t,po}(n_t)\right) \ge p^b_{t,po}\left(E_{t,po}(n_t+1)\right) \\ & or: \bar{\beta}^b_{t,po}\left(E_{t,po}(n_t-1)\right) \ge \bar{\beta}^b_{t,po}\left(E_{t,po}(n_t)\right) \ge \bar{\beta}^b_{t,po}\left(E_{t,po}(n_t+1)\right) \\ \textit{(ii)} & p^s_{t,po}\left(E_{t,po}(n_t-1)\right) \le p^s_{t,po}\left(E_{t,po}(n_t)\right) \le p^s_{t,po}\left(E_{t,po}(n_t+1)\right) \\ & or: \underline{\beta}^s_{t,po}\left(E_{t,po}(n_t-1)\right) \ge \underline{\beta}^s_{t,po}\left(E_{t,po}(n_t)\right) \ge \underline{\beta}^s_{t,po}\left(E_{t,po}(n_t+1)\right) \end{array}.$$

(both formulations, in terms of probabilities and in terms of betas, are equivalent)

Proof of Corollary 1. The proof proceeds along the same lines as the one of Proposition 2 and is omitted for brevity. ■

Proof of Proposition 5. Part a) With partial opaqueness, the time t probability of occurrence of a CN buy at $t+1$ is

$$\Pr\left[1^{CN} \text{ at } t+1\right] = \pi_b\left[F\left(\bar{\beta}^b_{t+1,po}\left(E_{t+1,po}(n_{t+1})\right)\right) - F\left(\underline{\beta}^b_{t+1,po}\left(E_{t+1,po}(n_{t+1})\right)\right)\right].$$

Suppose $E_{t,po}(n_t) < T-(t+1)$, such that the expected probability of execution of a CN buy at $t+1$ is not zero. Then $\underline{\beta}^b_{t+1,po}\left(E_{t+1,po}(n_{t+1})\right)$ is independent of the CN's net order imbalance. If at t a DM trade was observed, the expected net order imbalance does not change: $E_{t+1,po}(n_t) = E_{t+1,po}(n_{t+1})$. If instead no order was observed at t, then the expected net order imbalance at $t+1$ is one of the following three cases: $E_{t+1,po}(n_t)+1$ or $E_{t+1,po}(n_t)-1$ or $E_{t+1,po}(n_t)$, depending on whether at t a CN buy, CN sell, or no order was submitted, respectively. The probabilities of occurrence of each case are computed by the trader from $F(.), \pi_s, \pi_b$ and the expected net order imbalance at t. Attach to these three possibilities the probabilities π_1, π_2 and π_3, respectively. Then, the expected net order imbalance is:

$$E_{t+1,po}(n_t) + \pi_1 - \pi_2$$

since $\pi_1 + \pi_2 + \pi_3 = 1$. Hence, comparing the expected net order imbalance after observing a DM trade and observing no DM trade, we get

$$E_{t+1,po}(n_t) \text{ versus } E_{t+1,po}(n_t) + \pi_1 - \pi_2.$$

Three cases can occur:

1. $\pi_1 < \pi_2$: from Corollary 1 it then follows that

$$\bar{\beta}^b_{t+1,po}\left(E_{t+1,po}(n_t)\right) \leq \bar{\beta}^b_{t+1,po}\left(E_{t+1,po}(n_t) + \pi_1 - \pi_2\right)$$

   such that, given that $F(.)$ is monotonically nondecreasing in β, it holds that the left-hand side of equation (14) $\leq$ the right-hand side of equation (14).

2. $\pi_1 > \pi_2$: from Corollary 1 it then follows that

$$\bar{\beta}^b_{t+1,po}\left(E_{t+1,po}(n_t)\right) \geq \bar{\beta}^b_{t+1,po}\left(E_{t+1,po}(n_t) + \pi_1 - \pi_2\right)$$

   such that the left-hand side of equation (14) $\geq$ the right-hand side of equation (14).

3. $\pi_1 = \pi_2$: it then follows that

$$\bar{\beta}^b_{t+1,po}\left(E_{t+1,po}(n_t)\right) = \bar{\beta}^b_{t+1,po}\left(E_{t+1,po}(n_t + \pi_1 - \pi_2)\right)$$

   such that the left-hand side of equation (14) $=$ the right-hand side of equation (14).

Suppose $E_{t,po}(n_t) \geq T - (t+1)$. This means that either no CN orders are expected to be submitted, in which case the proposition holds trivially, or at t the extra CN order submitted changes the expected net order imbalance such that the expected execution probability becomes zero. In this case $\bar{\beta}^b_{t+1,po}\left(E_{t+1,po}(n_t+1)\right) = \frac{A}{V}$; hence the result follows because $\underline{\beta}^b_{t+1,po}\left(E_{t+1,po}(n_{t+1})\right) = \frac{A}{V}$.

Part b) This proof proceeds along the same lines as part a) and is omitted for brevity. For both parts a and b, similar proofs can be constructed for the other market side. ■

Appendix B: Formal Definitions of Inefficiencies

The "CN-only-traders inefficiency" and the "CN-diverted-traders inefficiency" fully explain the difference between $\overline{OW}^{\max}$ and overall welfare for each of the considered isolation cases and coexistence settings. By formally defining those two inefficiencies, we also implicitly define overall welfare. Indeed, simple subtraction of the sum of these two inefficiencies from $\overline{OW}^{\max}$ provides the overall welfare for each case and setting that we consider.

The CN-only-traders inefficiency stems from buyers with $\beta_t \in \left(\frac{A+B}{2V}, \frac{A}{V}\right]$ and sellers with $\beta_t \in \left[\frac{B}{V}, \frac{A+B}{2V}\right)$ that do not realize any gains from trade. We first describe the expected inefficiency under coexistence for traders arriving at time t given a (expected) net order imbalance, time to cross, ...:

$$\pi_b \int_{\frac{A+B}{2V}}^{\frac{A}{V}} \left(1 - p_{t,i}^b\right) \left(\beta_t V - V\right) f\left(\beta_t\right) d\beta_t + \pi_s \int_{\frac{B}{V}}^{\frac{A+B}{2V}} \left(1 - p_{t,i}^s\right) \left(V - \beta_t V\right) f\left(\beta_t\right) d\beta_t.$$

The first term accounts for the unrealized trading gains from buyers as orders do not execute with probability $\left(1 - p_{t,i}^b\right)$, with $i = tr, co$ or po. In a similar fashion, the second term presents the unrealized gains for sellers. Notice that these probabilities hinge on (i) the informational setting and (ii) the (expected) net order imbalance and time to the cross. To obtain the ex ante average per-period CN-only-traders inefficiency, we need to take into account the expected probability of ending up with a particular state at time t and compute the appropriate averages for t and average over $t = 1, ..., T$. For the isolation cases, the CN-only-traders inefficiency is computed in a similar way, with the probability of non-execution equal to 1 for a DM in isolation and equal to $\left(1 - p_t^{b,CN}\right)$ and $\left(1 - p_t^{s,CN}\right)$ for buyers and sellers in a CN in isolation, respectively.

Under coexistence, the CN-diverted-traders inefficiency measures the impact on overall welfare of buyers with $\beta_t \in \left(\frac{A}{V}, \bar{\beta}_{t,i}^b\right]$ and sellers with $\beta_t \in \left[\underline{\beta}_{t,i}^s, \frac{A}{V}\right)$ submitting orders to the CN. Formally, for a given setting at time t implying a (expected) net order imbalance, it can be defined as follows

$$\pi_b \int_{\frac{A}{V}}^{\bar{\beta}_{t,i}^b} \left(1 - p_{t,i}^b\right) \left(\beta_t V - V\right) f\left(\beta_t\right) d\beta_t + \pi_s \int_{\underline{\beta}_{t,i}^s}^{\frac{B}{V}} \left(1 - p_{t,i}^s\right) \left(V - \beta_t V\right) f\left(\beta_t\right) d\beta_t.$$

The first term accounts for the unrealized trading gains from buyers with $\beta_t \in \left(\frac{A}{V}, \bar{\beta}_{t,i}^b\right]$ as buy orders do not execute with probability $\left(1-p_{t,i}^{b}\right)$. Similarly, the second term represents the unrealized trading gains for sellers with $\beta_t \in \left[\underline{\beta}_{t,i}^{s}, \frac{B}{V}\right)$ as sell orders do not execute with probability $\left(1-p_{t,i}^{s}\right)$. Notice that these probabilities hinge on (i) the informational setting and (ii) the (expected) net order imbalance and time to the cross. Summing over all possible states, applying the appropriate weights for time t, and taking the average over $t = 1, ..., T$ yields the average per-period CN-diverted-traders inefficiency. For the isolation cases, the CN-diverted-traders inefficiency is computed in a similar way. Note further that the set of such traders is empty for a DM in isolation, as no traders are diverted. For a CN in isolation, the two integrals are taken between the boundaries $\frac{A}{V}$ and $\bar{\beta}$, and $\underline{\beta}$ and $\frac{B}{V}$, with the probability of non-execution equal to $\left(1-p_{t}^{b,CN}\right)$ and $\left(1-p_{t}^{s,CN}\right)$ for buyers and sellers, respectively.

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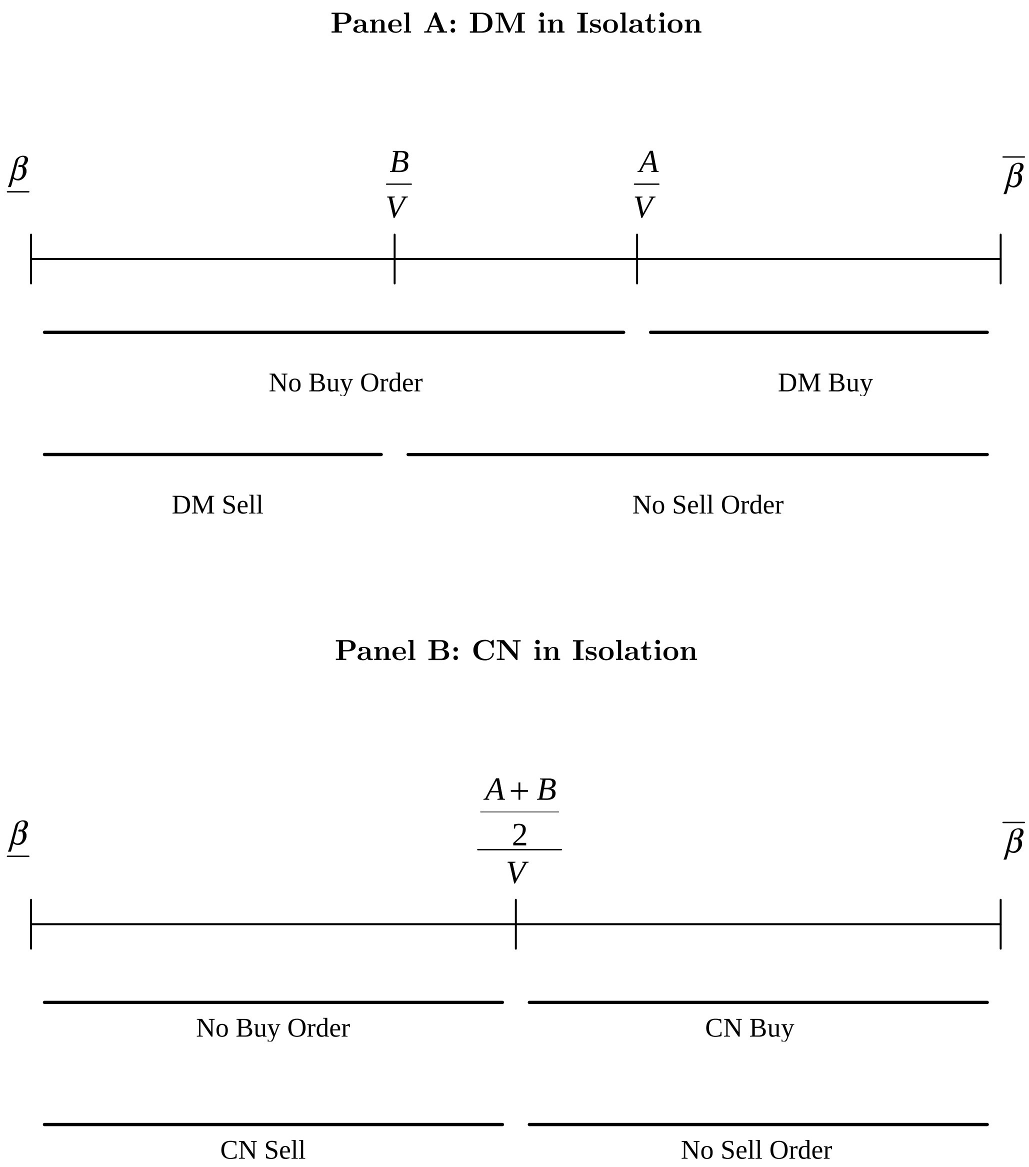
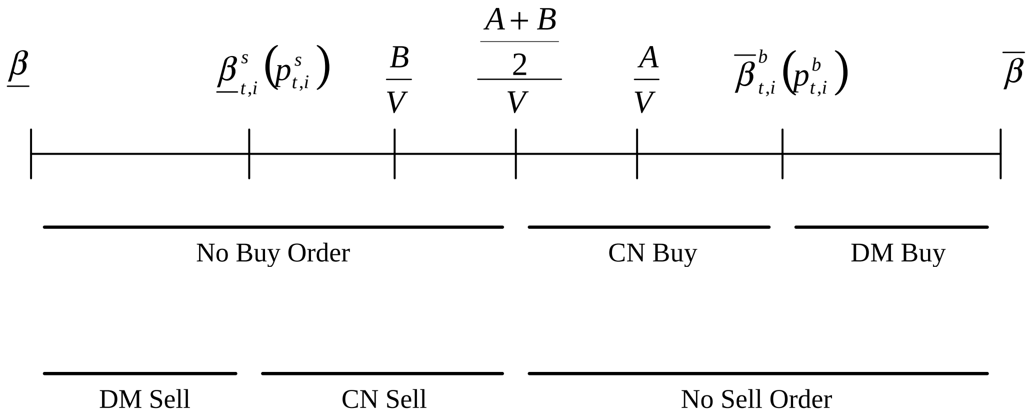


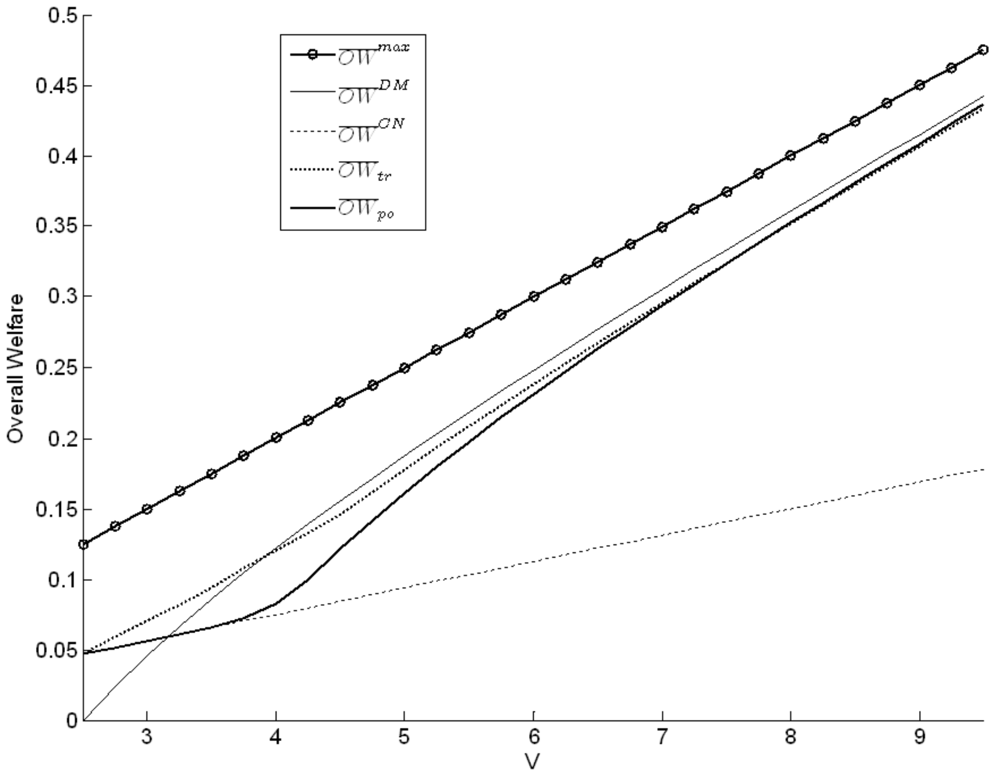
Note: This figure depicts the equilibrium of our model with only a dealer market (Panel A) or only a crossing network (Panel B). The optimal strategies of agents are drawn, conditional upon their β and trading orientation.

Figure 1: **Order Submission Strategies with a Dealer Market or a Crossing Network in Isolation**



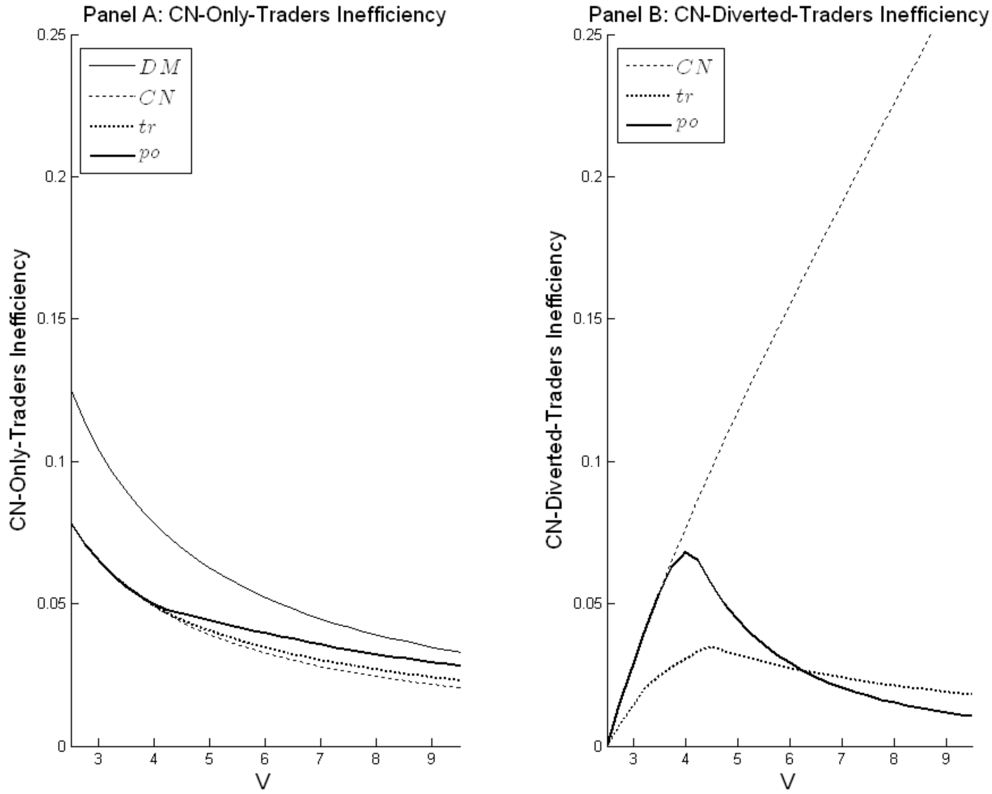
Note: This figure depicts the equilibrium of our model with a dealer market (DM) and a crossing network (CN) in coexistence with $i = tr, co$ or po (denoting transparency, complete opaqueness and partial opaqueness, respectively). The optimal strategies of agents are drawn, conditional upon their β and trading orientation. Note that $\underline{\beta}^s_{t,tr}\left(p^s_{t,tr}\right)$ and $\overline{\beta}^b_{t,i}\left(p^b_{t,i}\right)$ may differ for $i = tr, co$ or po.

Figure 2: **Order Submission Strategies with Dealer Market and Crossing Network in Coexistence**



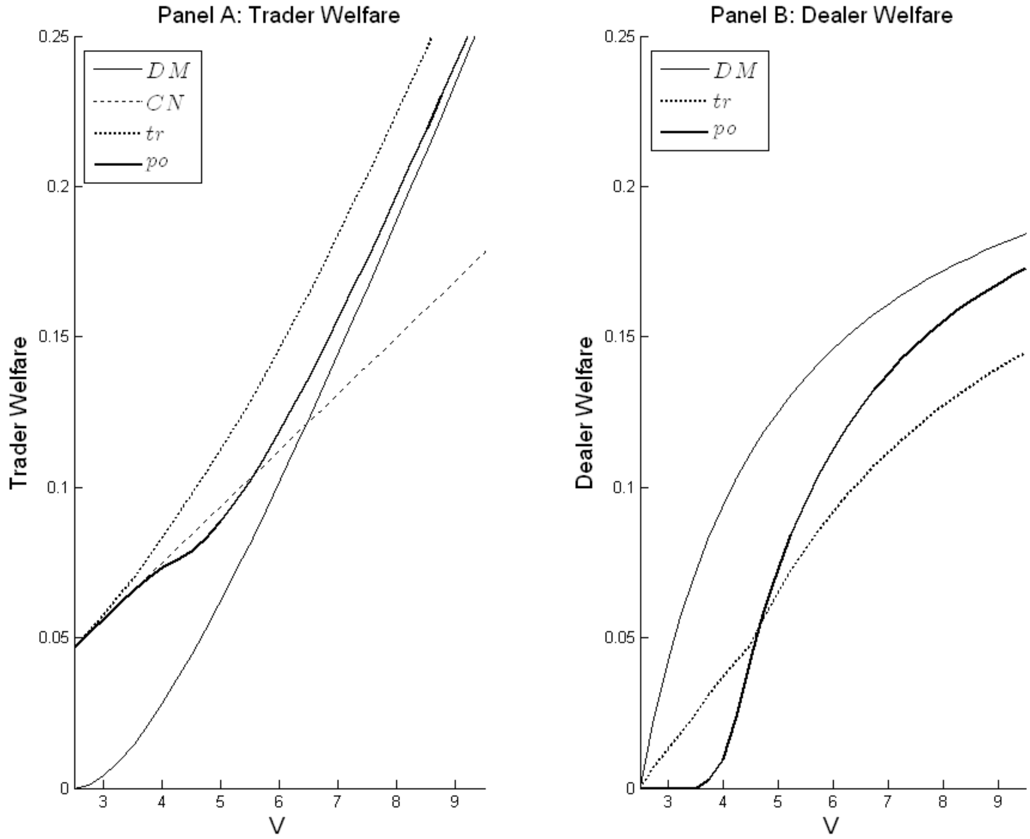
Note: This figure presents average per-period overall welfare for coexistence of a CN and a DM under transparency, $\overline{OW}_{tr}$, and partial opaqueness, $\overline{OW}_{po}$. Transparency is presented as a thick dotted line, partial opaqueness with a thick full line. Also average per-period overall welfare for the DM in isolation, $\overline{OW}^{DM}$, and for the CN in isolation, $\overline{OW}^{CN}$, are included for comparison as a thin full line and a thin dashed line, respectively. The results for complete opaqueness are very close to those of partial opaqueness and are therefore not shown. The thick full line with circles represents the maximum average per-period overall welfare $\overline{OW}^{\max}$. In the computations, we assume that β is uniformly distributed over $[0.8, 1.2]$, $\pi_b = 0.5$, $\frac{A+B}{2} = V$ and $T = 3$.

Figure 3: **Average Per-Period Overall Welfare**



Note: This figure displays the average per-period inefficiencies that drive a wedge between maximum and realized overall welfare. The left panel presents the CN-only-traders inefficiency. The right panel shows the CN-diverted-traders inefficiency. These inefficiencies are defined in Appendix B. In both panels thin full (dashed) lines represent inefficiencies for the DM (CN) in isolation. Note that the CN-diverted-traders inefficiency is zero for the DM in isolation. Thick dotted (full) lines represent the coexistence of DM and CN for transparency (partial opaqueness). The results for complete opaqueness are very close to those of partial opaqueness and are therefore not shown. In the computations, we assume that β is uniformly distributed over $[0.8, 1.2]$, $\pi_b = 0.5$, $\frac{A+B}{2} = V$ and $T = 3$.

Figure 4: **Inefficiencies**



Note: Panel A and B of this figure display average per-period trader welfare and average per-period dealer welfare. The DM in isolation and the CN in isolation are depicted as a thin full line and a thin dashed line, respectively. Note that dealer welfare for the CN in isolation is zero. Transparency for coexistence of a CN and a DM (tr) is presented as a thick dotted line, partial opaqueness (po) as a thick full line. The results for complete opaqueness are very close to those of partial opaqueness and are therefore not shown. In the computations, we assume that β is uniformly distributed over $[0.8, 1.2]$, $\pi_b = 0.5$, $\frac{A+B}{2} = V$ and $T = 3$.

Figure 5: **Average Per-Period Trader and Dealer Welfare**