Spectral analysis of stochastic and analytic simulation results for a nonlinear model for the Italian economy

Bianchi, Carlo; Calzolari, Giorgio and Cleur , Eugene M.
IBM Scientific Center, Pisa, Italy, Universita' di Pisa, Italy

1978

Spectral Analysis of Stochastic and Analytic Simulation Results for a Nonlinear Model for the Italian Economy

C. Bianchi, G. Calzolari and E.M. Cleur, Pisa

When dealing with nonlinear econometric models, resort is often made to simulation techniques for the investigation of their dynamic properties. A spectral analysis using stochastic and analytic simulation is carried out on a nonlinear model of the Italian economy. The two approches are empirically compared.

KEYWORDS: stochastic simulation, nonlinear econometric models, analytic simulation, spectral analysis, Monte Carlo methods.

1. *Introduction*

It is a well known result that an analytical description of the properties of a system of simultaneous difference equations can be obtained in a straightforward manner by the use of the spectral representation of a stochastic process. Such an approach is also known to have several advantages over a direct estimation of the spectra of the endogenous variables generated by a stochastic simulation of the system. Unfortunately it is not possible to obtain such a description in the nonlinear case.

In order to overcome this problem alternative estimates of the spectra may be obtained through a linearization of the nonlinear model as in Howrey(1971). However, apart from the difficulties implicit in such an approach, if the model is highly nonlinear the linear approximation could be misleading. Hence it is often necessary to resort to a stochastic simulation approach, as in several contributions in Hickman(1972).

Alternatively Howrey and Klein(1972) have proposed overcoming this difficulty by means of a simulation approach which, strictly speaking, is not stochastic, but is a combination of numerical simulation and an analytic evaluation of the spectrum; they often refer to this approach as an "analytic simulation", whereas Klein(1973) terms it as an "evaluation of the spectrum by empirical nonstochastic simulation".

After a short description, the stochastic and analytic simulation approaches will be applied to a nonlinear macroeconometric model for Italy. It should be remembered that the main purpose of this paper is to compare the performance of the two approaches just mentioned rather than make inferences on the cyclical properties of the model or on the economic meaning of the results.

2. Spectrum evaluation by means of stochastic simulation

The basic idea underlying this method is that of applying spectral analysis to the "time series" or "observations" generated by means of a stochastic simulation of the model beyond the sample period. The reason for introducing stochastic shocks over a long period is that of determining, as suggested by Adelman and Adelman(1959), whether business cycles with realistic characteristics are found in the simulated results. As in other Monte Carlo experiments, the simulation (e.g. each stochastic simulation run) is replicated many times over the same period; the spectrum is computed for each run and averaged over the number of replications.

Even though this methodology is not new in applied econometrics, there is no uniqueness of consensus about the way in which such an experiment should be performed.

Several problems arise which are related to the use of a short-term model for long-run simulations, so that it is often necessary to modify the structure of the model in order to take into account long-run factors. In such a context there is no direct interest in the results of the long-run forecast itself, but a reasonable forecast should in any case be produced, and hence, in order to obtain a control deterministic solution which could be regarded as a base-line solution for the stochastic simulation runs, a realistic projection of all the exogenous variables is required.

Other problems arise due to the very nature of spectral analysis. For instance, spectral analysis generally requires very long time-series, so that, at least in principle, more reliable spectral estimates can be obtained only from a very long simulation period; this could raise the difficulty of obtaining a realistic control solution especially for large dynamic nonlinear econometric models, in which convergence and stability problems can be present. Further, classical spectral analysis can only be applied to stationary series, and hence, for each shocked run, it is necessary to filter any trend; generally in these experiments the detrending is performed by subtracting out the control deterministic solution, but in some cases the paths of the control and stochastic solutions suggest the use of other filters.

The way in which all these aspects have been faced in our experiment on the ISPE model will be briefly discussed in Section 4. It must be recalled here that, for each replication, the random shocks have been generated by means of McCarty's(1972) procedure and, after control solution detrending, spectrum densities have been computed using the formula:

$$f(\lambda_j) = 1/(2\pi)\left\{C_0 + 2\sum_{s=1}^{M-1} C_s w_s \cos(\lambda_j s)\right\} \tag{1}$$

where:

$C_s = 1/(N-s)\sum_{t=1}^{N-s}(x_t-\bar{x})(x_{t+s}-\bar{x})$ is the autocovariance function,

w_s is the Parzen window, M the truncation point and $\lambda_j = \pi j/M$.

3. Spectrum evaluation by means of analytic simulation

As mentioned above, this technique was suggested by Howrey and Klein(1972). Even if it can be proved to be asymptotically exact only for linear systems, from a computational point of view it can be applied exactly in the same way also to nonlinear systems. Briefly the method is as follows. Let

$$A(L)Y(t) = B(L)X(t) + U(t) \tag{2}$$

be a linear econometric model in structural form, where $A(L)$ and $B(L)$ are matrices of polinomials in the lag operator L; $Y(t)$, $X(t)$ and $U(t)$ are, respectively, the vectors of endogenous and exogenous variables and of the structural disturbances at time t. Making explicit $Y(t)$ on the left hand side provides:

$$Y(t) = A^{-1}(L)B(L)X(t) + A^{-1}(L)U(t). \tag{3}$$

We are interested in computing the spectral matrix of the second term on the right hand side, which represents the deviations from the control solution corresponding to the random disturbance process $U(t)$. The straightforward solution is:

$$S_y(\lambda) = A^{-1}(e^{-i\lambda})S_u(\lambda)A^{-1*}(e^{-i\lambda}) \tag{4}$$

where $*$ means conjugate transpose and $S_u(\lambda)$ is the spectrum matrix of $U(t)$.

The method proposed by Howrey and Klein starts from the representation of the vector $A^{-1}(L)U(t)$ as a weighted average of the residuals, that is:

$$D_0U(t) + D_1U(t-1) + \ldots = \sum_{s=0}^{\infty} D_sU(t-s) \tag{5}$$

where D_s is the matrix of the partial derivatives of the elements of $Y(t)$ with respect to the elements of $U(t-s)$. These derivatives can be computed by means of numerical simulation for both linear and nonlinear models by computing deviations of disturbed solutions from a control solution. More exactly, the procedure for computing the elements of D_s (for any s) is the following (see also Chow(1975),pp.134-136).

1) A dynamic simulation run must be performed for $s+1$ periods with all the values of $U(t)=0$ $(t=t_0,t_0+1,\ldots,t_0+s)$; this is taken as the control solution.
2) The j-th component of the vector $U(t_0)$ is set equal to ϵ, an arbitrarily small number, while all the other components are again set to zero; to zero are also set all the components of the vectors $U(t)$ at time $t\neq t_0$.
3) The dynamic simulation run is performed as above; the difference between the values of the endogenous variables computed at time t_0+s and the corresponding values of the control solution supplies the desired deviations. These deviations, divided by ϵ, provide the values of the partial derivatives of the endogenous variables with respect to the j-th element of the disturbance vector lagged of s periods.

Steps 2 and 3 must be repeated for all the values of j (all the components of the $U(t_0)$ vector) completing, in this way, the computation of the matrix D_s.

The process must be repeated from step 1 for values of s=0,1,2,... up to a reasonable truncation point in the sum (5).

An optimal choice of the value for ϵ can be made in such a way as to guarantee at least 3-4 exact decimal digits for each derivative in the case of nonlinear model (if the model is linear, it is well known that any value of ϵ leads always to the same result).

The analytic simulation approach can be usefully used in place of equation (4) even in linear models when the dimensions of the matrix A(L) are such that it could be difficult to enter the correct input, i.e. to write the coefficients in the right position. On the other hand the Gauss-Seidel solution procedure generally also used in the simulation of large linear models can be easily adapted to handle this problem, rather than have the computing burden involved in inverting large complex-valued matrices at each frequency.

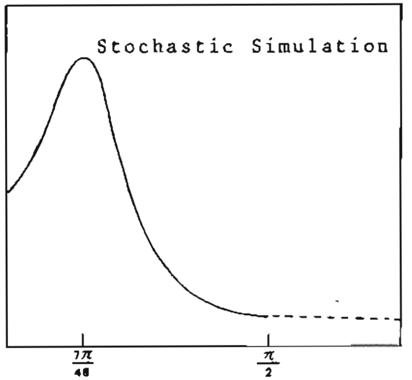
4. *Spectral analysis of the ISPE model*

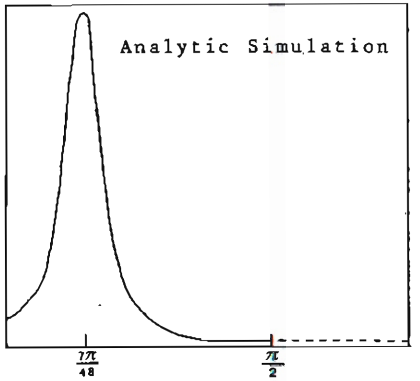
The nonlinear model used in the experiment is the annual model of the real and fiscal sector of the Italian economy developed by a team led by ISPE (Istituto di Studi per la Programmazione Economica), and described in Sartori(1977). The version of the model used in both the stochastic and analytic simulation experiments consists of 34 equations, 19 of which are stochastic; the model was estimated by two stage least squares with principal components according to method 4 of Kloek and Mennes(1960) for the sample period 1955-1976. The choice of this method was based on the dynamc simulation behaviour of the model in the sample period, as described in Bianchi, Calzolari and Sartori(1978).

As carried out by Fitzgerald(1973), and in order to avoid the heteroschedasticity induced by the nonlinearities of the model, the stochastic simulation was undertaken by keeping the exogenous variables fixed, and the spectra were calculated from the residuals from the control (deterministic) solution after having eliminated the first 20 observations; such a procedure helps eliminate respectively trend and the transitory component. The simulation period, after several trials, was set at 110 years into the future. This was suggested by the fact that, in longer simulation periods, the control solution path reached values unreasonable from an economic point of view.

The analytic simulation procedure, after previous tests on linear models (Klein-I and Samuelson-Hicks), was applied to the ISPE model. The simulation run, that is the truncation point in the Sum (5), was 100 periods; in fact after 90 periods, the results remain constant up to the first 2-3 significant digits. Even here the exogenous variables were kept fixed over all the simulation period. An optimal choice of ϵ was made for each component of the vector $U(t_0)$; in order of magnitude, its value was approximately equal to the standard deviation of the corresponding element of $U(t_0)$ divided by 10^4, hence requiring a great precision (small tolerance at the convergence point) in the Gauss-Seidel iterative solution algorithm.

The figures below display, for the Total Private Production, respectively the average power spectrum computed on 50 replicated stochastic simulations and the power spectrum computed by means of analytic simulation.





All the endogenous variables have well defined peaks at frequency 0.073 cycles/year, when using the analytic simulation, thereby indicating the generation of an overall 14 year cycle by the model. A few of the variables also contain substantial power at zero frequency, which means that trend or very long cyclical components are also present. On the other hand, the spectra of the series obtained from the stochastic simulation procedure have slightly higher power at zero frequency and the peaks are still well defined in the neighbourhood of frequency 0.073 cycles/year even if slightly less pronounced.

From a qualitative point of view, the two approaches give the same information and, on the basis of this concordance, we may conclude that the ISPE model helps generate a reasonably long cyclical pattern, but does not reproduce the well known business cycles which are generally said to have periodicity of between 7 and 10 years.

We have also calculated cross-spectra between some of the endogenous variables with the intention of analysing the relationship between the 14 year cycles previously identified. We may thus note that the 14 year cycles which dominate the stochastic components of Employment in the Industrial Sector and Industrial Production, of Private Consumption Deflator and Wages per Employee in the Industrial Sector, and of Private Investment and Net Private Consumption are all very highly correlated (the coherence in all three cases is higher than 0.95), with the following lead-lag relationships:

1) Private Consumption Deflator lags Wages by a little less than one year;
2) Investment lags Consumption by about half a year;
3) Industrial Employment lags Production by about one year.

5. *Concluding remarks*

With respect to the stochastic simulation approach, the analytic simulation was empirically found to have the following advantages.

The experimental error due to random number generation is avoided.

It is unnecessary to replicate the stochastic solutions of the model many times; more exactly, we require only one control solution and as many disturbed solutions as the number of stochastic equations (19, in our case), instead of at least fifty replications as was found necessary in our case to get reliable results from the stochastic simulation approach. Therefore the cost of computation was smaller (5 minutes of CPU time for the ISPE model, instead of 10 minutes).

The length of the simulation period in the analytic simulation depends only from the choice of a reasonable truncation point in the sum (5), while in the stochastic simulation approach, to improve the resolution of the spectrum estimator, the simulation period and the truncation point in (1) must be increased (see, for example, the considerations by Jenkins and Watts(1968) on the problems of bias and inconsistency of the spectrum estimates (pp.245-247)). Using 100 periods in the analytic simulation approach we obtained results that were exact up to 2-3 significant digits (in the sense that they did not change when the number of periods was further increased); in order to get the same precision with the stochastic simulation approach, in previous experiments with linear models several hundreds of replications had to be performed on a simulation period of a few thousand years and truncation point of about 400, requiring, in this way, more than one hour of CPU time for the Klein-I model. For the ISPE model it was practically impossible to get the same precision in the stochastic simulation results, since, as already mentioned, simulation runs longer than 110-120 periods led to meaningless results.

All these considerations seem to stress, in the particular case considered here, a preference in the use of analytic simulation. In a more general case, given a larger model with, say, one hundred stochastic equations, the number of necessary (disturbed) solutions in the analytic simulation approach should be one hundred, i.e. greater than the number of replications required to get reliable results with the stochastic simulation, which would then turn the balance in favour of stochastic simulation. However, this may only be true in qualitative terms since, from our above mentioned experiment on linear models, in order to obtain the same resolution in the spectral estimates from the stochastic simulation approach, the simulation period would probably have to be much longer and the number of replications much greater.

A possible drawback in the use of the analytic simulation approach however, could be found in what Howrey and Klein point out (p.600) on the fact that such a method can be easily proved to be exact in the case of linear models, while "no attempt is made at this point to justify this approach to the analysis of nonlinear systems beyond the analogy with linear systems". Chow also notes that (p.136): "It seems reasonable to suppose that the method will work well only if the system, net of the effects of random disturbances and of the exogenous variables, has a stable equilibrium"; this however is not a drawback, as the same criticism can be made of the stochastic simulation approach.

All the simulation experiments have been carried out by means of the program by Bianchi, Calzolari and Corsi(1978) on a computer IBM/370 model 168; some special features have been added to this program to allow analytic simulation as well.

References

Adelman,I. and F.L.Adelman, "The Dynamic Properties of the Klein-Goldberger Model", Econometrica, 28, 596-625, 1959.

Bianchi,C., G.Calzolari and P.Corsi, "A Program for Stochastic Simulation of Econometric Models", Econometrica, 46, 235-236, 1978.

Bianchi,C., G.Calzolari and F.Sartori, "Stima e Simulazione di un Sistema Nonlineare di Equazioni Simultanee: una Applicazione al Modello ISPE del Metodo dei Minimi Quadrati a Due Stadi con Componenti Principali", Quaderni ISPE, Roma, 1978, (in Italian, forthcoming).

Chow,G.C., "Analysis and Control of Dynamic Economic Systems" John Wiley & Sons, New York, 1975.

Fitzgerald,V.W., "Dynamic Properties of a Non-Linear Econometric Model", in "Econometric Studies of Macro and Monetary Relations", ed. by A.A.Powell and R.A.Williams, 169-193, North Holland, Amsterdam, 1973.

Hickman,B.G., editor, "Econometric Models of Cyclical Behavior", Studies in Income and Wealth n.36, NBER, New York, 1972.

Howrey,E.P., "Stochastic Properties of the Klein-Goldberger Model", Econometrica, 39, 73-87, 1971.

Howrey,E.P. and L.R.Klein, "Dynamic Properties of Nonlinear Econometric Models", International Economic Review, 13, 599-618, 1972.

Jenkins,G.M. and D.G.Watts, "Spectral Analysis and its Applications", Holden-Day, San Francisco, 1968.

Klein,L.R., "Dynamic Analysis of Economic Systems", Int. J. Math. Educ. Sci. Technol., 4, 341-359, 1973.

Kloek,T. and L.B.M.Mennes, "Simultaneous Equations Estimation Based on Principal Components of Predetermined Variables", Econometrica, 28, 45-61, 1960.

McCarthy,M.D., "Some Notes on the Generation of Pseudo-Structural Errors for Use in Stochastic Simulation Studies", in "Econometric Models of Cyclical Behavior", ed. by B.G.Hickman, Studies in Income and Wealth n.36, 185-191, NBER, New York, 1972.

Sartori,F., "An Economic Model for Italy", presented at the IBM UK/IIASA Seminar on "Models for Regional Planning and Policy Making", Laxenburg, Vienna, 1977.