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Ayesha, Nazuk; Sadia, Nadir and Javid, Shabbir
NUST University, Islamabad, Pakistan. , Riphah International University, Islmabad, Pakistan. , Quaid e Azam University, Islamabad, Pakistan.

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Adjustment of the Auxiliary Variable(s) for Estimation of a Finite Population Mean

Ayesha Nazuk[1], Sadia Nadir[2] and Javid Shabbir[3]

1*: NUST Business School, National University of Sciences and Technology, H. 12, Islamabad, Pakistan. Email: ayesha.nazuk@nbs.edu.pk,*

2: Department of Basic Sciences, Rippah International University, Islamabad, Pakistan.

3: Department of Statistics, Quaid-i-Azam University 45320, Islamabad, Pakistan.

Abstract

In this paper we have worked to weight and transform various estimators by Prasad (1986) and Lui (1991). We have introduced some ratio and ratio type estimators under weighting, transformation and model based approach, environment. We have introduced estimators efficient than estimators proposed by Chakrabarty (1979), Singh and Singh (1997), Singh (2002) and Singh et al. (2006).

Keywords:

model based approach, percent relative efficiency, product estimator, ratio estimator, regression estimator, simple mean unit estimator,

1. Introduction

Many researchers have worked on improving the efficiency of estimation of population mean of the study variable Y, when an auxiliary variable X, correlated with Y, is observable. Not only researchers formulated the ratio and product estimators but introduced several variants of these in order to improve the efficiency of estimators. Some of the researchers who introduced several variants of the ratio and product estimators include Bandyopadhyay (1994), Singh and Singh (1997), Singh (2000, 2002).

We propose new estimators by the procedure of (i) idyllic weighting of existing estimators, (ii) transformation of the variables involved in ratio and regression type estimators (iii) imposing a model based approach. Many researchers have worked on weighting two or more estimators so as to improve the efficiency of estimators of population mean, some of these are Upadhyaya et al. (1985), Singh (2002) and Singh et al. (2006). Some of the researchers who employed the transformation technique, on ratio and regression type estimators, include Chakrabarty (1979), Srivenkataramana and Tracy (1980), and Sahoo and Jena (2000).

Durbin (1959) used the following model to estimate the population mean $\bar{Y}$.

$$\left.\begin{array}{l} y_i = \alpha + \beta x_i + e_i\,;\ \beta < 0, \\ \text{with the assumptions} \\ (i)\ \ E(e_i / x_i) = 0, \\ (ii)\ \ E(e_i\, e_j / x_i\, x_j) = 0 \text{ for i} \neq \text{j}, \\ (iii)\ \ V(e_i / x_i) = n\delta,\ \delta \text{ is a constant of order } n^{-1}, \\ (iv)\ \text{The variate } x_i / n \text{ have a gamma distribution with the parameter } m = n\,h. \end{array}\right] \quad (1)$$

where

$$\left.\begin{array}{l} e_i \text{ is white noise,} \\ \alpha = \bar{Y}\left[\ K - \rho\ \ / K\right], \\ \beta = \bar{Y}\left[\ \rho\ \ / Km\right], \\ \delta = \bar{Y}^2\left[\ 1 - \rho^2\ \ / K^2 m\right],\ K = C_{x.} / C_y. \end{array}\right] \quad (2)$$

In this paper we propose a new weighted estimator of the population mean. The proposed estimator is compared with the estimators proposed by Chakrabarty (1979), Singh and Singh (1997), Singh (2002) and Singh et al. (2006). We also present some transformed estimators of $\bar{Y}$.

2.1 The proposed estimator for weights summing to unity

We propose a new weighted estimator, whose weights sum up to one, with the aim to obtain more precise estimates. The proposed estimator is compared with the estimators proposed by Chakrabarty (1979), Singh and Singh (1997), Singh (2002) and Singh et al. (2006).

Durbin’s (1959) model has been utilized to proceed further with the estimation process.

Consider the estimator suggested by Prasad (1986), given by

$$t_{pra} = \bar{y} - \frac{\bar{x}}{\bar{X}} + 1, \tag{3}$$

where $\bar{y}$ and $\bar{x}$ are usual sample means corresponding to population means $\bar{Y}$ and $\bar{X}$, respectively.

Later on this estimator was modified by Lui (1991), who used it in design based approach, as follows

$$t_L = \alpha\,\bar{y} + (1-\alpha)\left(\bar{y} - \frac{\bar{x}}{\bar{X}} + 1\right), \tag{4}$$

where α is a constant.

Now we use the estimator t_L, given in (2.2) under model based approach as follows,

$$\bar{y}_{prop} = d_1 t_{pra} + d_2 \bar{y}, \tag{5}$$

subject to $d_1 + d_2 = 1$, where d_1 and d_2 are weights.

The proposed estimator, $\bar{y}_{prop}$, is unbiased and its variance is given by

The variance of the proposed estimator $\bar{y}_{prop}$ is

$$V(\bar{y}_{prop}) = E\left(\bar{y}_{prop} - \bar{Y}\right)^2$$

$$= (d_2)^2 E\left(\bar{y} - \bar{Y}\right)^2 + d_1^2 E\left[\bar{y} - \frac{\bar{x}}{\bar{X}} + 1 - \bar{Y}\right]^2$$

$$+ 2 d_1 d_2 E\left[\left(\bar{y} - \bar{Y}\right)\left[\bar{y} - \frac{\bar{x}}{\bar{X}} + 1 - \bar{Y}\right]\right]. \tag{6}$$

Now consider (2.4) for minimizing $V(\bar{y}_{prop})$, with respect to d_1 and d_2, the minimum variance of $\bar{y}_{prop}$ is as follows

$$V(\bar{y}_{prop})_{\min}$$

$$= \frac{(\beta^2 m + \delta)\left(\dfrac{\beta m(\beta m - 2) + m\delta + 1}{m}\right) - \left(\beta(\beta m - 1) + \delta\right)^2}{\left((\beta^2 m + \delta) + \left(\dfrac{\beta m(\beta m - 2) + m\delta + 1}{m}\right) - 2\left(\beta(\beta m - 1) + \delta\right)\right)}, \tag{7}$$

2.1 The proposed estimator when weights are not summing to unity

We propose a new weighted estimator, whose weights do not necessarily sum up to one. The target has been to improve the efficiency of estimation of the population mean, in model based approach. Here we compare the proposed estimator with the estimators proposed by Chakrabarty (1979), Singh and Singh (1997), Singh (2002) and Singh et al. (2006).

The proposed estimator is given by

$$\bar{y}_{new} = h_1 t_{pra} + h_2 \bar{y},\ h_1 + h_2 \neq 1, \text{ where } h_1 \text{ and } h_2 \text{ are weights.} \quad (8)$$

The bias of $\bar{y}_{new}$ given by

$$B(\bar{y}_{new}) = E(\bar{y}_{new} - \bar{Y})$$

$$= E\left[h_1\left(\bar{y} - \bar{Y}\right) + h_2 E\left(t_{pra} - \bar{Y}\right) + \left(h_1 + h_2 - 1\right)\bar{Y}\right]$$

$$= \left(h_1 + h_2 - 1\right)\left(\alpha + \beta m\right), \quad (9)$$

The MSE of the proposed estimator $\bar{y}_{new}$ is given by

$$MSE(\bar{y}_{new}) = E\left(\bar{y}_{new} - \bar{Y}\right)^2$$

For the sake of convenience, let us define

$$\left.\begin{array}{l} A = E\left(\bar{y} - \bar{Y}\right)^2,\ B = E\left(t_{pra} - \bar{Y}\right)^2 \\ C = E\left(\bar{Y}^2\right),\ D = E\left(\bar{y} - \bar{Y}\right)\left(t_{pra} - \bar{Y}\right), \\ E = E\left(t_{pra} - \bar{Y}\right)\bar{Y},\ h_3 = h_1 + h_2 - 1. \end{array}\right\} \quad (10)$$

Now minimizing $MSE(\bar{y}_{new})$ with respect to $h_i,\ i = 1, 2,$ we get the minimum MSE of $\bar{y}_{new}$ as given by

$$MSE(\bar{y}_{new})_{\min} = h_{1(opt)}^2 A^* + h_{2(opt)}^2 B^* + h_{3(opt)}^2 C^* + 2h_{1(opt)} h_{2(opt)} D^*, \quad (11)$$

Where $h_3 = \left(h_1 + h_2\right) - 1$

3.1 Efficiency Comparison when $h_1 + h_2 \neq 1$

Let us define the following expression for obtaining the percentage relative efficiency PRE

$$PRE_j = \frac{MSE\left(\bar{y}\right)_{\min}}{MSE\left(i\right)_{\min}} \times 100, \text{ where } i = T_r,\ T_p,\ \bar{y}_{new}. \qquad (12)$$

Note that $T_p = W_{1p}\bar{y} + W_{2p}\bar{y}_p$,

Where $\bar{y}$ and $\bar{y}_p$ are respectively sample mean estimator and usual product estimator. Also (W_{1r}, W_{2r}) and (W_{1p}, W_{2p}) are suitably chosen scalar whose sums need not be unity.

Also $T_r = W_1\bar{y} + W_2\bar{y}_r$, where W_1 and W_2 are unknown weights, whose sum is not necessarily one, which are either specified or estimated and $\bar{y}$ and $\bar{y}_r$ are respectively sample mean estimator and usual ratio estimator.

In Table 1, we have compared the proposed estimator with the simple mean per unit estimator $\bar{y}$ and T_r and T_p, proposed by Singh (2002). The proposed estimator $\bar{y}_{new}$ is more efficient than $\bar{y}, T_p$ and T_r.

3.2 Efficiency Comparison when $d_1 + d_2 = 1$

The following expression is used to obtain the percent relative efficiency PRE.

$$PRE\left(l\right) = \frac{V\left(\bar{y}\right)_{\min}}{MSE\left(l\right)_{\min}} \times 100, \text{where } l = \bar{y}_{prop},\ \bar{y}_a,\ \bar{y}_{rc1} \qquad (13)$$

Where $\bar{y}_a = (1-w)\bar{y} + w\bar{y}_{pi}$; $0 < w < 1$, $\bar{y}_{pi} = (p^*/\bar{X})$,

$p^* = \left[2p - 0.5\left(p_1 + p_2\right)\right]$,

$p = \bar{y}\,\bar{x}$ and $p_i = \bar{y}_i\,\bar{x}_i$, $r_{C1} = \left(1-W\right)\bar{y} + W\bar{y}_r$; $W \geq 0$, W is a constant weight and

$$r^{*} = 2r - 0.5\ (r_1 + r_2)\ ,\ \ r_j = \frac{\bar{y}_j^{'}}{\bar{x}_j^{'}} = \frac{(n\bar{y} - p\bar{y}_j)\ /\ (n-p)}{(n\bar{x} - p\bar{x}_j)\ /\ (n-p)},\ j = 1, 2.$$

In Table 2, we compared proposed estimator $\bar{y}_{prop}$ with $\bar{y}$ as well as the estimator, $\bar{y}_a$ proposed by Singh et al. (2006). Note that the proposed estimator, $\bar{y}_{prop}$, is more efficient than $\bar{y}$ for different values of m, k and ρ. Scrutinizing Table 3, one can easily see that the numerical supremacy of the proposed estimator $\bar{y}_{prop}$ over r_{C1}.

3.3 Comparison of the Proposed Estimator $\bar{y}_{new}$ with Chakrabarty (1979), under varying weights

In Table 4 and 5, all the comparisons are done under the varying weights situation. We have chosen different values of m, K and ρ under the varying weights of the proposed estimator with the r_{C1} under its varying weights. Also following the convention by many researchers like Rao and Webster (1966) we have taken $\bar{Y} = 6$ across all the numerical computations.

Analyzing the numerical results we can easily conclude that under non-optimum weights, the proposed estimator $\bar{y}_{prop}$, in which sum of the weights in assumed to be equal to one, is efficient than r_{C1} and $\bar{y}$.

4.1 Transformed Estimator

In this section we introduce some variants of the proposed estimator with the, well met, aim of increasing the efficiency of the estimation of the population mean of a quantitative variable. By using the transformed auxiliary variable, many researchers such as Chakrabarty (1979), Srivenkataramana and Tracy (1986), and others have discussed that the transformation of auxiliary variable reduces the bias and may or may not increases the efficiency of the estimators. Mohanty and Sahoo (1995) presented a new transformation of the auxiliary variable by using its minimum and maximum variables.

So we present here the setup for the transformation and apply it in our scenario. Let us have a finite population of N, represented by $\Omega = \{(X_1, Y_1), (X_2, Y_2), (X_3, Y_3), ...(Y_N, Y_N)\}$. Let X, Y be two positively correlated random variables and let (x_i, y_i), $i = 1, 2, ...N$ also $1 \le i \le N$ be a simple random sample of size n. Using the transformation presented by Mohanty and Sahoo (1995) we have

$$\left.\begin{array}{l} u_i = \dfrac{x_i + x_m}{x_M + x_m},\ z_i = \dfrac{x_i + x_M}{x_M + x_m}, \\ \text{then we have } \bar{u} = \dfrac{\bar{x} + x_m}{x_M + x_m},\ \bar{Z} = \dfrac{\bar{X} + x_M}{x_M + x_m},\ \bar{z} = \dfrac{\bar{x} + x_M}{x_M + x_m}, \text{ and } \bar{U} = \dfrac{\bar{X} + x_m}{x_M + x_m}. \end{array}\right\} \quad (14)$$

where x_M and x_m are respectively the minimum and maximum values of x. Also $\bar{z}, \bar{u}$ and $\bar{Z}, \bar{U}$ are the sample and population means of transformed variables, respectively.

Now we present the two transformed estimators of $\bar{Y}$.

$$\left.\bar{y}_{tran(1)} = \bar{y} - \frac{\bar{u}}{\bar{U}} + 1 \text{ and } \bar{y}_{tran(2)} = \bar{y} - \frac{\bar{z}}{\bar{Z}} + 1\right\}. \quad (15)$$

Now applying the transformation given in (4.1) we have,

$$\bar{y}_{tran(1)} = \bar{y} - \frac{(\bar{x} + \bar{X} - \bar{X} + x_m)/\bar{X}}{\dfrac{\bar{X}}{\bar{X}} + \dfrac{x_m}{\bar{X}}} + 1, \quad (16)$$

$$\bar{y}_{tran(1)} = \bar{y} - \frac{\bar{u}}{\bar{U}} + 1.$$

$$\bar{y}_{tran(1)} = \bar{y} - \frac{e_1}{c_1}, \text{ where } e_1 = \frac{\bar{x} - \bar{X}}{\bar{X}}, c_1 = 1 + \frac{x_m}{X}.$$

$$\bar{y}_{tran(1)} - \bar{Y} = \bar{y} - \frac{e_1}{c_1} - \bar{Y}, \qquad (17)$$

Substitution of the values of $\bar{Y}$ and $\bar{y}$ from Durbin (1959) model, given in (1.1), we get

$$\bar{y}_{tran(1)} - \bar{Y} = \beta\left(\bar{x} - m\right) + \bar{u} - \frac{e_1}{c_1}. \qquad (18)$$

The bias and variance of $\bar{y}_{tran(1)}$ are, respectively given by

$$B\left(\bar{y}_{tran(1)}\right) = 0. \qquad (19)$$

$$V\left(\bar{y}_{tran(1)}\right) = \beta^2 m + \delta + \frac{1}{c_1^2 m} - \frac{2\beta}{c_1}. \qquad (20)$$

Similarly one may develop the expressions for bias of $\bar{y}_{tran(2)}$. The bias and variance of $\bar{y}_{tran(2)}$ are, respectively, given as follows

$$B\left(\bar{y}_{tran(2)}\right) = 0. \qquad (21)$$

$$V\left(\bar{y}_{tran(2)}\right) = \beta^2 m + \delta + \frac{1}{c_2^2 m} - \frac{2\beta}{c_2}, \text{ where } c_2 = 1 + \frac{x_M}{\bar{X}}. \qquad (22)$$

Now we propose a weighted estimator $\bar{y}_{tran(f)}$, where sum of weights is equal to one. $\bar{y}_{tran(f)}$ is as under,

$\bar{y}_{tran(f)} = f_2\,\bar{y} + f_1\,\bar{y}_{tran(1)}$, f_1 and f_2 are weights such that $f_1 + f_2 = 1$.

(23)

The bias of $\bar{y}_{tran(f)}$ is

$$B\left(\bar{y}_{tran(f)}\right) = E\left(\bar{y}_{tran(f)} - \bar{Y}\right) = f_2 E\left(\bar{y} - \bar{Y}\right) + f_1 E\left(\bar{y}_{tran(1)} - \bar{Y}\right) = 0.$$
(4.11)

The variance of $\bar{y}_{tran(f)}$ is as under,

$$V\left(\bar{y}_{tran(f)}\right) = E\left(\bar{y}_{tran(f)} - \bar{Y}\right)^2 = E\left[f_2\left(\bar{y} - \bar{Y}\right) + f_1\left(\bar{y}_{tran(1)} - \bar{Y}\right)\right]^2,$$

$$= \left(1 - f_1\right)^2 A + f_1^2 B + 2 f_1\left(1 - f_1\right) D. \qquad (24)$$

From (2.10) one can easily see that $A = E\left(\bar{y} - \bar{Y}\right)^2 = \beta^2 m + \delta$. Similarly from (4.7), one may substitute the values of $B = V\left(\bar{y}_{tran(1)} - \bar{Y}\right)^2$.

The value of D is given by

$$D = E\left[\left(\bar{y} - \bar{Y}\right)\left(\bar{y}_{tran(1)} - \bar{Y}\right)\right] = \beta^2 m + \delta - \frac{\beta}{c_1}. \qquad (25)$$

The proposed weighted estimator under condition that, $h_1 + h_2 = 1,$ is as under,

$$\bar{y}_{tran(h)} = h_2 \bar{y} + h_1 \bar{y}_{tran(2)},\ h_1 + h_2 = 1 \text{ where} \qquad (26)$$

Bias and mean square error of $\bar{y}_{tran(h)}$ are as under

$$B\left(\bar{y}_{tran(h)}\right) = 0. \qquad (27)$$

and

$$MSE\left(\bar{y}_{tran(h)}\right) = h_2^2\left(\beta m + \delta\right) + h_1^2\left\{\beta^2 m + \delta + \frac{1}{m c_2^2} - \frac{2\beta}{c_2}\right\} + 2 h_1 h_2\left\{\beta^2 m + \delta - \frac{\beta}{c_2}\right\}. \qquad (28)$$

4.2 Comparison of the proposed estimator, $\bar{y}_{tran(f)}$, with $\bar{y}_{prop}$

In this section we shall see whether the proposed estimator with transformation $\left(\bar{y}_{tran(f)}\right)$ performs better than the proposed estimator without transformation $\left(\bar{y}_{prop}\right)$. In Table 6, we present the numerical comparison of $\bar{y}_{tran(f)}$ with $\bar{y}_{prop}$. Numerical computations show that $\bar{y}_{tran(f)}$ performs better than $\bar{y}_{prop}$.

5. Conclusions

We have concentrated on model based approach which is actually a strategy where for more than one variable; one being the study variable and the rest being auxiliary closely correlated with the study variable. We have worked on introducing new estimators of population mean by using weighting and transformation technique in model based approach. We have successfully improved the efficiency of estimation of population mean. Proposed estimators are efficient under optimum as well as non-optimum weights conditions.

TABLE 1 ***PRE*** **comparison of competitive estimators in Data Set 1- 36**

Data Set	*PRE* T_r	*PRE* T_p	*PRE* $\bar{y}_{new}$	*Data Set*	*PRE* T_r	*PRE* T_p	*PRE* $\bar{y}_{new}$
1	224.375	205.798	318.750	19	629.737	478.420	704.142
2	611.595	346.581	914.062	20	2216.00	913.580	2504.00
3	1266.61	432.891	1913.19	21	4853.34	1125.34	5504.01
4	167.187	167.105	225.000	22	363.105	316.325	404.733
5	351.293	268.750	515.625	23	1156.64	679.751	1304.16
6	676.974	360.042	1013.88	24	2477.39	936.230	2804.09
7	153.492	156.250	197.916	25	274.490	251.382	305.325
8	265.969	229.263	383.854	26	802.858	549.989	904.320
9	481.080	313.521	714.583	27	1683.65	805.819	1904.16
10	427.338	337.539	506.920	28	830.592	620.775	902.938
11	1416.25	627.241	1706.24	29	3013.38	1201.13	3302.94
12	3061.33	777.206	3706.16	30	6643.28	1474.31	7302.97
13	261.681	235.346	308.304	31	464.117	399.006	503.265
14	754.596	470.530	906.611	32	1557.25	890.511	1703.03
15	1577.53	645.549	1906.33	33	3376.00	1228.03	3703.02
16	207.494	195.418	243.021	34	341.888	309.441	370.258
17	534.050	385.372	640.312	35	1070.59	716.268	1169.78
18	1082.40	556.548	1306.49	36	2283.84	1056.44	2503.06

TABLE 2 *PRE* **comparison for estimators based on data sets 37-84 given in Appendix- 1, for different values of** m, k and ρ.

Data Set	*PRE* $\bar{y}_{prop}$	*PRE* $\bar{y}_a$	*Data set*	*PRE* $\bar{y}_{prop}$	*PRE* $\bar{y}_a$
37	104.166	102.000	64	104.166	104.056
38	104.166	103.278	65	133.333	120.000
39	104.166	103.719	66	133.333	128.571
40	104.166	103.902	67	133.333	131.034
41	133.333	112.500	68	133.333	132.000
42	133.333	123.529	69	526.315	188.621
43	133.333	128.125	70	526.315	318.328
44	133.333	130.188	71	526.315	399.507
45	526.315	140.500	72	526.315	444.314
46	526.315	226.000	73	104.166	103.278
47	526.315	307.102	74	104.166	103.902
48	526.315	367.216	75	104.166	104.044
49	104.166	102.702	76	104.166	104.097
50	104.166	103.669	77	133.333	123.529
51	104.166	103.930	78	133.333	130.188
52	104.166	104.030	79	133.333	131.858
53	133.333	118.181	80	133.333	132.487
54	133.333	127.586	81	526.315	226.070
55	133.333	130.508	82	526.315	367.216
56	133.333	131.683	83	526.315	437.109
57	526.315	173.972	84	526.315	471.080

58	526.315	294.594
59	526.315	378.776
60	526.315	428.517
61	104.166	102.907
62	104.166	103.759
63	104.166	103.975

TABLE 3 PRE comparison of $\bar{y}_{prop}$ with r_{C1}, under optimum weights

Data Set	$PRE\ \ \bar{y}_{prop}$	$PRE\ \ r_{C1}$
85	104.17	100.14
86	104.17	100.4
87	104.17	100.99
88	109.89	102.55
89	109.89	105.24
90	109.89	106.24
91	119.05	108.71
92	119.05	112.97
93	119.05	114.19
94	133.33	119.59
95	133.33	125.7
96	133.33	127.06
97	156.25	139.62
98	156.25	147.78
99	156.25	148.99

Table 4 PRE **comparison of** $\bar{y}_{prop}$ **with** r_{C1}**, under varying weights** $d_1 = 0.4$

Data set	$PRE\ \bar{y}_{prop}$	$PRE\ r_{C1}$	*Data set*	$PRE\ \bar{y}_{prop}$	$PRE\ r_{C1}$
100	101.2373	89.67001	123	105.2632	90.52984
101	102.2727	81.43575	124	102.6226	108.2882
102	103.0928	66.97454	125	105.1402	111.7998
103	101.9253	94.30912	126	107.5269	103.2659
104	103.6866	89.67001	127	101.2373	100.8159
105	105.2632	75.61791	128	102.2727	95.66547
106	102.6226	99.33399	129	103.0928	83.87131
107	105.1402	99.63562	130	101.9253	105.3779
108	107.5269	86.73099	131	103.6866	104.2889
109	101.2373	97.29573	132	105.2632	94.12864
110	102.2727	91.13444	133	102.6226	110.3456
111	103.0928	78.34044	134	105.1402	114.5921
112	101.9253	101.8879	135	107.5269	107.2191
113	103.6866	99.65983	136	101.2373	99.66217
114	105.2632	88.10026	137	102.2727	94.17752
115	102.6226	106.8794	138	103.0928	82.04015

116	105.1402	109.886	139	101.9253	104.2346
117	107.5269	100.5886	140	103.6866	102.7713
118	158.6183	151.2097	141	105.2632	92.13616
119	102.2727	92.97116	142	102.6226	109.2106
120	103.0928	80.56613	143	105.1402	113.0522
121	101.9253	103.3058	144	107.5269	105.0322
122	103.6866	101.5391			

Table 5 *PRE* **comparison of** $\bar{y}_{prop}$ **with** r_{C1}**, under varying weights** $d_1 = 0.6$

Data set	$PRE\ \bar{y}_{prop}$	$PRE\ r_{C1}$	*Data set*	$PRE\ \bar{y}_{prop}$	$PRE\ r_{C1}$
145	101.7812	80.51159	175	102.8278	104.3057
146	103.0928	63.92461	176	105.2632	93.11681
147	103.8961	44.85339	177	107.2386	71.44385
148	102.8278	86.73672	178	103.8961	111.9114
149	105.2632	72.5058	179	107.5269	106.1714
150	107.2386	51.32567	180	110.8033	83.28032
151	103.8961	93.76339	181	101.7812	95.77181
152	107.5269	83.55615	182	103.0928	80.76008
153	110.8033	59.88229	183	103.8961	60.44857
154	101.7812	92.03551	184	102.8278	102.4152
155	103.0928	76.51812	185	105.2632	90.82111
156	103.8961	56.36724	186	107.2386	69.07556
157	102.8278	98.58933	187	103.8961	109.9663
158	105.2632	86.23068	188	107.5269	103.6722
159	107.2386	64.43679	189	110.8033	80.53057
160	103.8961	106.0246			
161	107.5269	98.66047			
162	110.8033	75.14084			

163	101.2373	67.2043
164	103.0928	79.06125
165	103.8961	58.80168
166	102.8278	100.8921
167	105.2632	88.98464
168	107.2386	67.2043
169	103.8961	108.398
170	107.5269	101.6695
171	110.8033	78.35698
172	101.7812	97.62082
173	103.0928	82.88699
174	103.8961	62.53389

Table 6 comparison of $\bar{y}_{tran(f)}$ with $\bar{y}_{prop}$.

Data Set	m	k	ρ	w	*PRE* $\bar{y}_{tran(f)}$	*Data Set*	m	k	ρ	w	*PRE* $\bar{y}_{tran(f)}$
190	8	0.5	-0.5	0.25	100.7086	222	8	0.5	-0.75	0.75	104.2123
191	8	1	-0.5	0.25	101.4441	223	8	1	-0.75	0.75	108.4884
192	8	1.5	-0.5	0.25	102.2047	224	8	1.5	-0.75	0.75	112.7929
193	8	2	-0.5	0.25	102.9883	225	8	2	-0.75	0.75	117.0962
194	16	0.5	-0.5	0.25	100.7086	226	16	0.5	-0.75	0.75	104.2123
195	16	1	-0.5	0.25	101.4441	227	16	1	-0.75	0.75	108.4884
196	16	1.5	-0.5	0.25	102.2047	228	16	1.5	-0.75	0.75	112.7929
197	16	2	-0.5	0.25	102.9883	229	16	2	-0.75	0.75	117.0962
198	20	0.5	-0.5	0.25	100.7086	230	20	0.5	-0.75	0.75	104.2123
199	20	1	-0.5	0.25	101.4441	231	20	1	-0.75	0.75	108.4884
200	20	1.5	-0.5	0.25	102.2047	232	20	1.5	-0.75	0.75	112.7929
201	20	2	-0.5	0.25	102.9883	233	20	2	-0.75	0.75	117.0962
202	24	0.5	-0.5	0.25	100.7086	234	24	0.5	-0.75	0.75	104.2123
203	24	1	-0.5	0.25	101.4441	235	24	1	-0.75	0.75	108.4884
204	24	1.5	-0.5	0.25	102.2047	236	24	1.5	-0.75	0.75	112.7929
205	24	2	-0.5	0.25	102.9883	237	24	2	-0.75	0.75	117.0962

206	8	0.5	-0.6	0.5		101.9195	238	8	0.5	-0.9	0.9	106.341
207	8	1	-0.6	0.5		103.9177	239	8	1	-0.9	0.9	112.554
208	8	1.5	-0.6	0.5		105.98	240	8	1.5	-0.9	0.9	118.6153
209	8	2	-0.6	0.5		108.0932	241	8	2	-0.9	0.9	124.5091
210	16	0.5	-0.6	0.5		101.9195	242	16	0.5	-0.9	0.9	106.341
211	16	1	-0.6	0.5		103.9177	243	16	1	-0.9	0.9	112.554
212	16	1.5	-0.6	0.5		105.98	244	16	1.5	-0.9	0.9	118.6153
213	16	2	-0.6	0.5		108.0932	245	16	2	-0.9	0.9	124.5091
214	20	0.5	-0.6	0.5		101.9195	246	20	0.5	-0.9	0.9	106.341
215	20	1	-0.6	0.5		103.9177	247	20	1	-0.9	0.9	112.554
216	20	1.5	-0.6	0.5		105.98	248	20	1.5	-0.9	0.9	118.6153
217	20	2	-0.6	0.5		108.0932	249	20	2	-0.9	0.9	124.5091
218	24	0.5	-0.6	0.5		101.9195	250	24	0.5	-0.9	0.9	106.341
219	24	1	-0.6	0.5		103.9177	251	24	1	-0.9	0.9	112.554
220	24	1.5	-0.6	0.5		105.98	252	24	1.5	-0.9	0.9	118.6153
221	24	2	-0.6	0.5		108.0932	253	24	2	-0.9	0.9	124.5091

Table 6 continued

Data Set	m	k	ρ	w	PRE	$\bar{y}_{tran(f)}$	*Data Set*	m	k	ρ	w	PRE $\bar{y}_{tran(f)}$
254	8	0.5	-0.5	0.25		101.2757	286	8	0.5	-0.75	0.75	108.7904
255	8	1	-0.5	0.25		102.6017	287	8	1	-0.75	0.75	118.1853
256	8	1.5	-0.5	0.25		103.9762	289	8	1.5	-0.75	0.75	128.1667
257	8	2	-0.5	0.25		105.3974	290	8	2	-0.75	0.75	138.7172
258	16	0.5	-0.5	0.25		101.2757	291	16	0.5	-0.75	0.75	108.7904
259	16	1	-0.5	0.25		102.6017	292	16	1	-0.75	0.75	118.1853
260	16	1.5	-0.5	0.25		103.9762	293	16	1.5	-0.75	0.75	128.1667
261	16	2	-0.5	0.25		105.3974	294	16	2	-0.75	0.75	138.7172
262	20	0.5	-0.5	0.25		101.2757	295	20	0.5	-0.75	0.75	108.7904
263	20	1	-0.5	0.25		102.6017	296	20	1	-0.75	0.75	118.1853
264	20	1.5	-0.5	0.25		103.9762	297	20	1.5	-0.75	0.75	128.1667
265	20	2	-0.5	0.25		105.3974	298	20	2	-0.75	0.75	138.7172
266	24	0.5	-0.5	0.25		101.2757	299	24	0.5	-0.75	0.75	108.7904
267	24	1	-0.5	0.25		102.6017	300	24	1	-0.75	0.75	118.1853
268	24	1.5	-0.5	0.25		103.9762	301	24	1.5	-0.75	0.75	128.1667

269	24	2	-0.5	0.25	105.3974	302	24	2	-0.75	0.75	138.7172
270	8	0.5	-0.6	0.5	104.0090	303	8	0.5	-0.9	0.9	113.2962
271	8	1	-0.6	0.5	108.2517	304	8	1	-0.9	0.9	127.5213
272	8	1.5	-0.6	0.5	112.7183	305	8	1.5	-0.9	0.9	142.6557
273	8	2	-0.6	0.5	117.3990	306	8	2	-0.9	0.9	158.6799
274	16	0.5	-0.6	0.5	104.0090	307	16	0.5	-0.9	0.9	113.2962
275	16	1	-0.6	0.5	108.2517	308	16	1	-0.9	0.9	127.5213
276	16	1.5	-0.6	0.5	112.7183	309	16	1.5	-0.9	0.9	142.6557
277	16	2	-0.6	0.5	117.3990	310	16	2	-0.9	0.9	158.6799
278	20	0.5	-0.6	0.5	104.0090	311	20	0.5	-0.9	0.9	113.2962
279	20	1	-0.6	0.5	108.2517	312	20	1	-0.9	0.9	127.5213
280	20	1.5	-0.6	0.5	112.7183	313	20	1.5	-0.9	0.9	142.6557
281	20	2	-0.6	0.5	117.399	314	20	2	-0.9	0.9	158.6799
282	24	0.5	-0.6	0.5	104.0090	315	24	0.5	-0.9	0.9	113.2962
283	24	1	-0.6	0.5	108.2517	316	24	1	-0.9	0.9	127.5213
284	24	1.5	-0.6	0.5	112.7183	317	24	1.5	-0.9	0.9	142.6557
285	24	2	-0.6	0.5	117.3990	318	24	2	-0.9	0.9	158.6799

Appendix-I

Comparison of the proposed estimator with Singh's (2002) estimator

Table A.1 Descriptive Statistics for Data Set 1- 9

Set values	Data 1	Data 2	Data 3	Data 4	Data 5	Data 6	Data 7	Data 8	Data 9
n	2	2	2	2	2	2	2	2	2
h	4	4	4	4	4	4	4	4	4
α	0	0	0	0	0	0	0	0	0
β	0.5	1.0	1.5	0.5	1.0	1.5	0.5	1.0	1.5
δ	1	1	1	2	2	2	3	3	3

Table A.2 Descriptive Statistics for Data Set 10- 18

Set values	Data 10	Data 11	Data 12	Data 13	Data 14	Data 15	Data 16	Data 17	Data 18
n	4	4	4	4	4	4	4	4	4
h	4	4	4	4	4	4	4	4	4
α	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
β	0.5	1.0	1.5	0.5	1.0	1.5	0.5	1.0	1.5
δ	1	1	1	2	2	2	3	3	3

Table A.3 Descriptive Statistics for Data Set 19- 27

Set values	Data 19	Data 20	Data 21	Data 22	Data 23	Data 24	Data 25	Data 26	Data 27
n	3	3	3	3	3	3	3	3	3
h	8	8	8	8	8	8	8	8	8
α	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
β	0.5	1.0	1.5	0.5	1.0	1.5	0.5	1.0	1.5
δ	1	1	1	2	2	2	3	3	3

Table A.4 Descriptive Statistics for Data Set 28- 36

Set values	Data 28	Data 29	Data 30	Data 31	Data 32	Data 33	Data 34	Data 35	Data 36
n	8	8	8	8	8	8	8	8	8
h	4	4	4	4	4	4	4	4	4
α	1	1	1	1	1	1	1	1	1
β	0.5	1.0	1.5	0.5	1.0	1.5	0.5	1.0	1.5
δ	1	1	1	2	2	2	3	3	3

Comparison of the proposed estimator with Singh's (2006) estimator

Table A.5 Descriptive Statistics for Data Set 37- 44

Set values	Data 37	Data 38	Data 39	Data 40	Data 41	Data 42	Data 43	Data 44
m	8	8	8	8	8	8	8	8

k	0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0
ρ	-0.2	-0.2	-0.2	-0.2	-0.5	-0.5	-0.5	-0.5

Table A.6 Descriptive Statistics for Data Set 45- 52

Set values	Data 45	Data 46	Data 47	Data 48	Data 49	Data 50	Data 51	Data 52
m	8	8	8	8	16	16	16	16
k	0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0
ρ	-0.9	-0.9	-0.9	-0.9	-0.2	-0.2	-0.2	-0.2

Table A.7 Descriptive Statistics for Data Set 53- 60

Set values	Data 53	Data 54	Data 55	Data 56	Data 57	Data 58	Data 59	Data 60
m	16	16	16	16	16	16	16	16
k	0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0
ρ	-0.5	-0.5	-0.5	-0.5	-0.9	-0.9	-0.9	-0.9

Table A.8 Descriptive Statistics for Data Set 61- 68

Set values	Data 61	Data 62	Data 63	Data 64	Data 65	Data 66	Data 67	Data 68
m	20	20	20	20	20	20	20	20
k	0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0
ρ	-0.2	-0.2	-0.2	-0.2	-0.5	-0.5	-0.5	-0.5

Table A.9 Descriptive Statistics for Data Set 69- 76

Set values	Data 69	Data 70	Data 71	Data 72	Data 73	Data 74	Data 75	Data 76
m	20	20	20	20	32	32	32	32
k	0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0
ρ	-0.9	-0.9	-0.9	-0.9	-0.2	-0.2	-0.2	-0.2

Table A.10 Descriptive Statistics for Data Set 77- 84

Set values	Data 77	Data 78	Data 79	Data 80	Data 81	Data 82	Data 83	Data 84
m	32	32	32	32	32	32	32	32
k	0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0
ρ	-0.5	-0.5	-0.5	-0.5	-0.9	-0.9	-0.9	-0.9

Comparison of the proposed estimator Chakrabarty (1979) estimator

Table A.11 Descriptive Statistics for Data Set 85- 91

Set values	Data 85	Data 86	Data 87	Data 88	Data 89	Data 90	Data 91	Data 92
m	8	8	8	16	16	16	20	20
k	0.5	1.0	1.5	2.0	0.5	1.0	0.5	1.0
ρ	0.2	0.2	0.2	0.3	0.3	0.3	0.4	0.4

Table A.12 Descriptive Statistics for Data Set 93- 99

Set values	Data 93	Data 94	Data 95	Data 96	Data 97	Data 98	Data 99
m	20	24	24	24	32	24	24
k	1.5	2.0	0.5	1.0	0.5	1.0	1.5
ρ	0.4	0.5	0.5	0.5	0.6	0.6	0.6

Note: from data set 100-144 the value of $w = 0.4$.

Table A.13 Descriptive Statistics for Data Set 100-106

Set values	Data 100	Data 101	Data 102	Data 103	Data 104	Data 105	Data 106
m	8	8	8	8	8	8	8
k	0.5	1.0	1.5	0.5	1.0	1.5	0.5
ρ	0.2	0.2	0.2	0.3	0.3	0.3	0.4

Table A.14 Descriptive Statistics for Data Set 107-113

Set values	Data 107	Data 108	Data 109	Data 110	Data 111	Data 112	Data 113
m	8	8	16	16	16	16	16
k	1.0	1.5	0.5	1.0	1.5	0.5	1.0
ρ	0.4	0.4	0.2	0.2	0.2	0.3	0.3

Table A.15 Descriptive Statistics for Data Set 114-120

Set values	Data 114	Data 115	Data 116	Data 117	Data 118	Data 119	Data 120
m	16	16	16	16	20	20	20
k	1.5	0.5	1.0	1.5	0.5	1.0	1.5
ρ	0.3	0.4	0.4	0.4	0.2	0.2	0.2

Table A.16 Descriptive Statistics for Data Set 121-127

Set values	Data 121	Data 122	Data 123	Data 124	Data 125	Data 126	Data 127
m	20	20	20	20	20	20	32
k	0.5	1.0	1.5	0.5	1.0	1.5	0.5
ρ	0.3	0.3	0.3	0.4	0.4	0.4	0.2

Table A.17 Descriptive Statistics for Data Set 128-135

Set values	Data 128	Data 129	Data 130	Data 131	Data 132	Data 133	Data 134	Data 135
m	32	32	32	32	32	32	32	32
k	1.0	1.5	0.5	1.0	1.5	0.5	1.0	1.5
ρ	0.2	0.2	0.3	0.3	0.3	0.4	0.4	0.4

Table A.18 Descriptive Statistics for Data Set 136-144

Set values	Data 136	Data 137	Data 138	Data 139	Data 140	Data 141	Data 142	Data 143	Data 144
m	20	20	20	20	20	20	20	20	20
k	0.5	1.0	1.5	0.5	1.0	1.5	0.5	1.0	1.5
ρ	0.2	0.2	0.2	0.3	0.3	0.3	0.4	0.4	0.4

Note: the Data Sets 145-189 are same as Data Sets 100-144, except that $w = 0.6$, in them.

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Corresponding Author

Ayesha Nazuk

NUST Business School, National University of Sciences and Technology, H. 12, Islamabad, Pakistan. Phone +92-51-9085-3267, Cell Phone +92-331-5263-973.

Email: ayesha.nazuk@nbs.edu.pk.