Algorithm for payoff calculation for option trading strategies using vector terminology

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Abstract

The aim of this paper is to develop an algorithm for calculating and plotting payoff of option strategies for a portfolio of path independent vanilla and exotic options. A general algorithm for calculating the vector matrix for any arbitrary combination strategy is also developed for some of the commonly option trading strategies.

1.0 Introduction

Hull [1] discusses the payoffs for long and short positions in Call and Put options by using algebraic techniques. J.S. Chaput & L.H. Ederington [3] , Natenberg[2] and Hull[1] contain the bibliographies and survey of literature on the theoretical background of option strategies for path independent vanilla and exotic options such as European , Bermuda , Forward Start , Digital/Binary and Quanto options. There are various open source option strategy calculators like “Option” [4] that only rely on algebraic analytical and graph superposition techniques to plot graphs for overall profit/loss. We in this paper develop an algorithm using vector terminology to plot final profit/loss graph of various option strategies.

2.1 Option strategies using vector notation

For a spot price S_T at time T and a strike price K , the payoff for a long position in call option is given by Max(S_T-K,0) and the payoff is Min(S_T-K,0) for the short position in the call option. Similarly the payoff for a long position in put is Max (K-S_T, 0) whereas it is Min (S_T-K, 0) for a short position in the put option. We can represent a vector payoff matrix for any option strategy as a 2xN matrix.

Vector	V_1	V_2	….	V_n
Strike Price	K_1	K_2	…..	K_n

In the above matrix the strike prices K_1,K_2, …..K_n for combination of options are in the ascending order, i.e., $K_1<K_2<\ldots<K_n$. The vector V_i can be interpreted as slope of the payoff graph of option strategy.By default the smallest strike price is always taken to be zero i.e. K_1=0.The vector is always an integer in the interval $(-\infty, \infty)$ .We can interpret the above matrix in terms of slope of the profit/loss curve obtained for option strategies.

$$slope = \begin{cases} \mathrm{V_i}\ \text{, for } K_i < K < K_{i+1} \text{ and i < n} \\ \mathrm{V_i}\text{, for K > K}_\mathrm{i} \text{ and i = n} \end{cases}$$

Vector matrix for long and short position is given by

Long Position

V_1	V_2	….	V_n
K_1	K_2	…..	K_n

Short Position

$-V_1$	$-V_2$	….	$-V_n$
K_1	K_2	…..	K_n

Using the above vector notation we can represent long and short position in call option as under

Long call

0	+1
0	K_1

Short Call

0	-1
0	K_1

For long position in call, profit/loss curve has two slopes 0 and +1 whereas for a short position the slope of profit/loss curve has two slopes 0 and –1.

Figure1:Long Position in Call Option

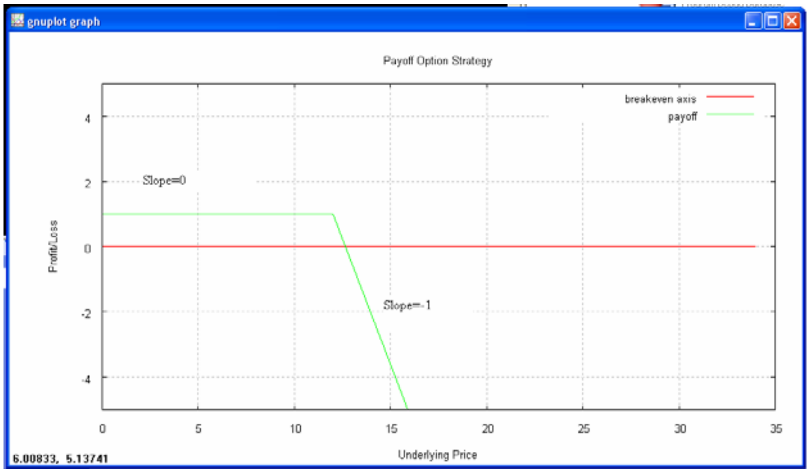

Figure 2:Short position in Call Option

Similarly, the vector matrix for long and short position in put options are:

Long Put

-1	0
0	K_1

Short Put

+1	0
0	K_1

For long position in stock, the slope of profit/loss curve is +1 and strike price is assumed to be zero whereas for short position in stock, the slope of profit/loss curve is -1 and strike price is assumed to be zero. The vector matrix notation is given as:

Long Stock

+1
0

Short Stock

-1
0

When we trade in n units of options using a particular option strategy, the entire vector row is multiplied by n.

$n*V_1$	$n*V_2$	….	$n*V_n$
K_1	K_2	…..	K_n

The data set for a portfolio using n option strategies can be represented as

Strategy 1

V_{11}	V_{12}
K_{11}	K_{12}

Strategy 2

V_{21}	V_{22}
K_{21}	K_{22}

…

…

…

Strategy i

V_{i1}	V_{i2}	….	V_{ij}	….
K_{i1}	K_{i2}	…..	K_{ij}	….

….

….

….

Strategy n

V_{n1}	V_{n2}	…	V_{nm}
K_{n1}	K_{n2}	….	K_{nm}

Note that the number of columns in each option strategy can be different. We can use the above derived vector matrices to form profit/loss function for any combination of option strategies using the following algorithm:

Algorithm

To plot the overall payoff strategy we need the initial Y intercept of the strategy apart from the resultant vector matrix. This Y intercept can be calculated using matrices of length greater than one using the formula

```
Yint = ∑ ( -1*Vector(A[j])*Strike_price(A[j+1]) )
Yint = Yint + Net_Premium_Paid
```

Step 1

```
For I ← 1 to no_of_options
      For j ← 1 to length_of_option_matrix
            Insert A[j] in Result_matrix in sorted increasing order on
            the basis of  Strike_price(A[j]).
```

Step 2

```
For k ← 1 to length_of_Result_matrix
      Vector(B[k])=0
      For I ← 1 to no_of_options
            For j ← 1 to length_of_option_matrix
                  If Strike_price(B[k]) = Strike_price(A[j])
                        Vector(B[k]) = Vector(B[k])+ Vector(A[j])
                  ElseIf j < length_of_option_matrix
                        If Strike_price(A[j]) < Strike_price(B[k]) <
                        Strike_price(A[j+1])
                        Vector(B[k]) = Vector(B[k])+ Vector(A[j])
                  Else
                        Vector(B[k]) = Vector(B[k])+ Vector(A[j])
```

Step 3

```
For I ← 1 to no_of_options
      j=1
      If length_of_option_matrix > 1
            Yint = Yint + -1 * Vector(A[j]) * Strike_price(A[j+1])
Yint = Yint + NetPremium
```

Step 4

```
For k ← 1 to length_of_Result_matrix - 1
      Plot line with slope Vector(B[k]) & Y Intercept Yint
      between points Strike_price(B[k]) & Strike_price(B[k+1])
      ypoint=Vector(B[k])*( Strike_price(B[k+1]) - Strike_price(B[k]) )
      + Yint
      Yint = ypoint -  Vector(B[k+1])* Strike_price(B[k+1]
k = length_of_Result_matrix
Plot line with slope Vector(B[k]) between points Strike_price(B[k]) &
infinity
```

The source code for the above algorithm is written and implemented on VC++.Net 2005 using open source graph plotting utility Gnuplot.

Illustration 1: An investor buys $3 put with strike price $35 and sells for $1 a put with a strike price of $30.
(Example 10.2, page 224 given in Hull [1])
The above data can be represented as

Buy Put

-1	0
0	35

+

Sell Put

+1	0
0	30

=

Payoff(Bear Spread)

0	-1	0
0	30	35

Initial Y intercept is -1*(-1*35) + -1*(1*30) – 3 + 1 = 35 -30 -3 + 1 = 3

One can use the following form to input the data of his/her option strategy:

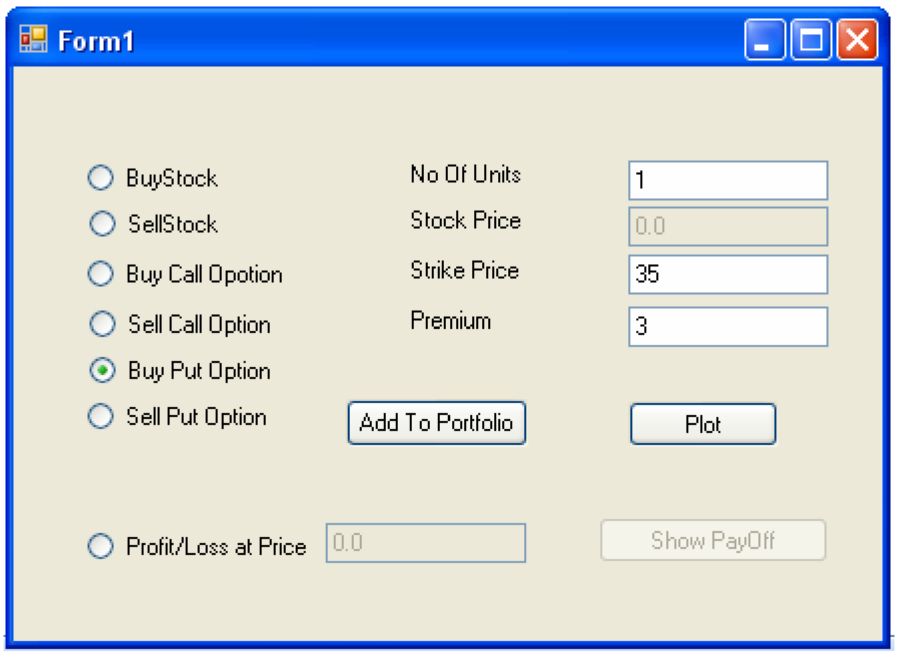

Figure 3: Input Screen

The following is the output of the final payoff of combination of option strategy in vector notation as discussed above.

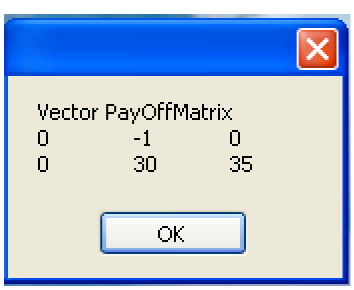

Figure 4: Vector Payoff Matrix

The algorithm gives the following resultant profit/loss graph of the above combination of option strategies in the form of a bear put spread.

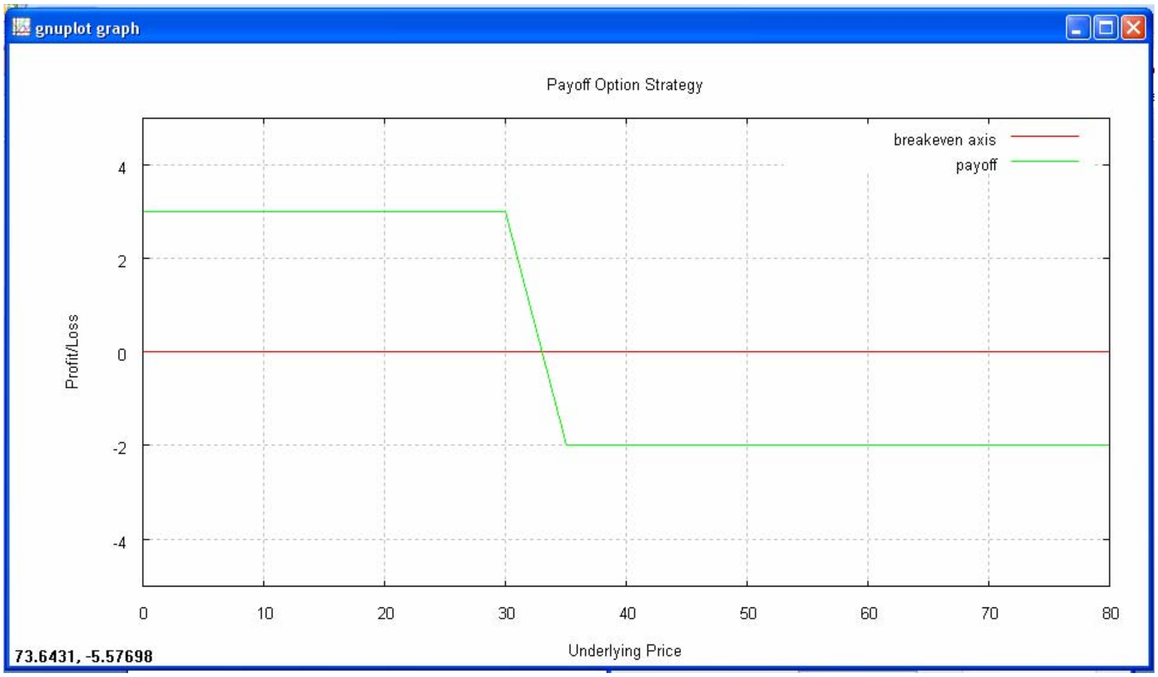

Figure 5: Payoff Graph

The loss is $2 if stock price is above $35 and the profit is $3 if stock price below $30.

2.2 Some More Complex Strategies

The following are the vector matrices for some of the commonly traded strategies:

Long Combo

($0<K_1<K_2$)

Sell Put + Buy Call = Long Combo

Sell Put

+1	0
0	K_1

Buy Call

0	+1
0	K_2

Long Combo

+1	0	+1
0	K_1	K_2

Long Straddle

Buy Put + Buy Call = Long Straddle

Buy Put

-1	0
0	K_1

Buy Call

0	+1
0	K_1

Long Straddle

-1	+1
0	K_1

Short Straddle

The vector matrix of short straddle is negative of that of long straddle

+1	-1
0	K_1

Strip

Buy call + Buy 2 puts = Strip

0	+1
0	K_1

-2	0
0	K_1

-2	+1
0	K_1

Strap

Buy 2 calls + Buy put = Strap

0	+2
0	K_1

-1	0
0	K_1

-1	+2
0	K_1

Long Strangle

($0<K_1<K_2$)

Buy put + Buy call = Long Strangle

-1	0
0	K_1

0	+1
0	K_2

-1	0	+1
0	K_1	K_2

Short Strangle

The vector matrix of short strangle is negative of that of short strangle. ($0<K_1<K_2$)

+1	0	-1
0	K_1	K_2

Collar

($0<K_1<K_2$)

Long Stock + Buy Put + Sell call =

+1
0

-1	0
0	K_1

0	-1
0	K_2

Collar

0	+1	0
0	K_1	K_2

Box Spread

($0<K_1<K_2$)

Buy Call + Sell call + Sell Put + Buy Put =

0	+1
0	K_1

0	-1
0	K_2

+1	0
0	K_1

-1	0
0	K_2

Box Spread

0	0	0
0	K_1	K_2

Long Call Butterfly

($0<K_1<K_2<K_3$)

Buy Call + Sell 2 call + Buy Call =

0	+1
0	K_1

0	-2
0	K_2

0	+1
0	K_3

Long Call Butterfly

0	+1	-1	0
0	K_1	K_2	K_3

Short Call Butterfly

The vector matrix of short call butterfly is negative of that of long call butterfly ($0<K_1<K_2<K_3$)

0	-1	+1	0
0	K_1	K_2	K_3

Long Call Condor

($0<K_1<K_2<K_3<K_4$)

Buy Call + Sell call + Sell Call + Buy Call

0	+1
0	K_1

0	-1
0	K_2

0	-1
0	K_3

0	+1
0	K_4

Long Call Condor

0	+1	0	-1	0
0	K_1	K_2	K_3	K_4

Short Call Condor

The vector matrix of short call condor is negative of that of long call condor ($0<K_1<K_2<K_3<K_4$)

0	-1	0	+1	0
0	K_1	K_2	K_3	K_4

Illustration 2: Let a certain stock is selling at $77. An investor feels that significant change in price is un-likely in the next 3 months. He observes market price of 3 month calls as

Strike Price($)	Call Price($)
75	12
80	8
85	5

The investor decided to go long in two calls each with strike price $75 and $85 and writes two calls with strike price $80. Payoff for different levels of stock prices is given as

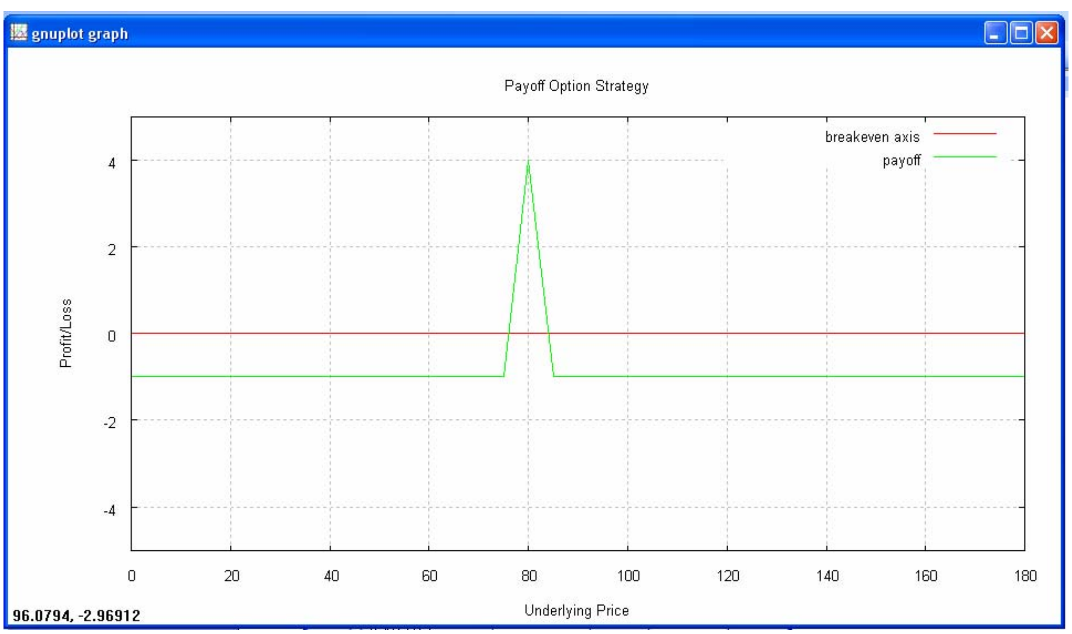

Figure 6: Payoff Graph

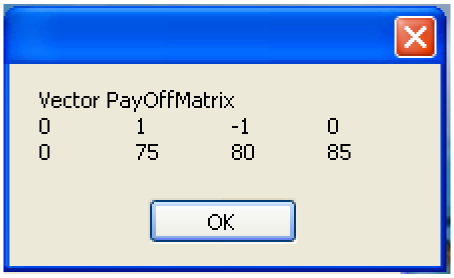

Figure 7: Vector Payoff Matrix

The profit /loss when stock price is at maturity is

Stock Price($)	Profit/Loss($)
65	-1
68	-1
73	-1
78	2
83	1

References

[1] Hull, J.C.(2009) *Options, Futures, and Other Derivatives* ,Prentice Hall .
[2] Natenberg,S.(1994) *Option Volatility and Pricing Strategies: Advanced Trading Techniques for Professionals* McGraw-Hill Professional Publishing .
[3] Chaput, J. S. and Ederington L. H., “Option Spread and Combination Trading” Journal of Derivatives, 10, 4(Summer 2003):70-88.
[4] http://sourceforge.net/projects/option