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2009

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Abstract: The recent literature on Purchasing Power Parity (PPP) has emphasized the role of two phenomena that may lead to the rejection of the PPP hypothesis: structural breaks and nonlinear adjustment induced by transaction costs. These two hypotheses are analyzed separately in the literature. We develop tests for unit roots that account jointly for structural breaks and nonlinear adjustment. Structural breaks are modeled by means of a Fourier function that allows for infrequent smooth temporary mean changes and is hence compatible with long-run PPP. Nonlinear adjustment is modeled by means of an ESTAR model. Our tests present good finite sample properties. The tests are applied to a set of 15 OECD countries' RERs and are able to reject the null of a unit root in 14 cases. The breaks are usually associated with the great appreciation and later depreciation of the dollar in the 1980s and the ESTAR adjustment appears to play an important role.

JEL codes: C22; F40

Keywords: Fourier model; ESTAR; nonlinear adjustment; PPP;

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1. Introduction

Purchasing Power Parity (PPP) remains one of the core assumptions of long-run equilibrium in a wide range of open economy macroeconomic models. Its importance has generated hundreds of empirical and theoretical papers.[1] The resiliency of the random walk model for the real exchange rate (RER) has attracted wide attention because its incompatibility with PPP. To date, the consensus view is that, for very long time spans, PPP appears to hold although the speed of mean reversion of the RER is slow. Although the evidence regarding the recent historical period of floating exchange rates since the break-down of the Bretton-Woods system increasingly supports mean-reversion, this evidence is still not as conclusive. This is especially the case when using US-dollar based bilateral RERs.

The failure to find evidence in favor of PPP in the literature developed during the 1980s and early 1990s led to several new technical developments that attempted to correct the shortcomings of previous studies. Tests of the PPP hypothesis are commonly based on unit root tests on the RER. A RER that reverts to a constant mean is compatible with PPP, whereas a non-stationary RER would violate the hypothesis. The main problem of the initial studies based on standard unit root tests is the widely reported lack of power of these tests for finite samples. This problem is exacerbated in the typical sample periods used for tests based on post-Bretton-Woods data that usually span for 20-30 years. Attempts at circumventing this problem led to four main developments in the

[1] Excellent overviews of the PPP literature can be found in the seminal papers of Rogoff (1996), Sarno and Taylor (2002) and Taylor and Taylor (2004).

PPP literature. The first one is the use of historical datasets that substantially increase the sample period of analysis hence increasing the power of the tests as pioneered by Lothian and Taylor (1996). These tests, however, do not account for the fact that exchange rate regimes have experienced several important changes in the last century. The observed increased volatility of the RER under the floating period (see Frankel and Rose, 1995) that led the way to the development of sticky price models of the nominal exchange rate, bears the question of whether PPP is a valid explanation of exchange rate determination under floating exchange rate regimes.

The second development is the use of panel techniques such as in, for instance, Coakley and Fuertes (1997) and Papell (1997). Panel unit root tests can increase the power of unit root tests by making use of cross-sectional information. Although there are important merits to this approach, it has also been criticized on several basis. For instance, the existence of cross-sectional correlation that may lead to size distortions was first pointed out in O'Connell (1998). A second criticism from Sarno and Taylor (1998) is that in many of the panel unit root tests the null hypothesis is such that we could reject it if only one of the cross-sectional units is a stationary process.[2] This criticism is also important because it points out that panel methods can give a general picture of the stationarity properties of RERs, but fail to give an answer on a case by case basis. Finally, as pointed out by Banerjee et al (2005), the potential presence of cross-unit cointegration relations may lead to size distortions that may account for the higher likelihood of rejecting the null of a unit root.

A third important innovation was made possible by the appearance of unit root tests that allow for breaking deterministic components as pioneered by Perron (1989). When RER deviate persistently from their equilibrium value due to long-lived events such as bubbles the mean to which they revert presents a temporary break which, if not accounted for, can also lead to spurious acceptance of the unit root null. PPP tests that allow for one or multiple structural breaks were developed, for instance, in Hegwood and Papell (1998). They apply unit root tests that allow for multiple breaks on a historical dataset of five US dollar-based RERs and find evidence of mean reversion. However, as these changes appear to be permanent, they emphasize that this is not support for the standard but rather a qualified version of PPP which they term quasi-PPP. For PPP to hold structural breaks in the series have to be temporary, so that the mean toward which the RER reverts at the start and end of the sample is the same. Recognizing this point, Papell and Prodan (2006) test for PPP using a restricted structural change model where long-run PPP is imposed.[3] They use historical data for seven countries' RER against the US dollar for which no previous evidence of PPP is found.[4] Their results, when using a structural change model where the change is temporary, reject the null of a unit root in only in two cases. Regarding the scarce evidence on post-Bretton-Woods PPP, especially for US-dollar bilateral exchange rates, Papell (2002) argues that an important event that may have driven the rejection of PPP is the "Great Appreciation" of the US dollar in the early 1980s and its subsequent depreciation. This bubble-like behaviour of the dollar generated breaks in the series that may induce acceptance of the null of a unit root in RERs. In order to test this hypothesis, he proposes modelling breaks in the slope of the mean. He then applies univariate unit root tests to the RER series where these breaks have been accounted for using 20 countries and post-Bretton-Woods quarterly data. His findings show that, using univariate tests only, we cannot reject the unit root null for 19 cases. He then applies a panel test and finds that, for those countries that show a common pattern of breaks associated with the 1980s dollar movements, the unit root null can be rejected. This test, however, shares some of the shortcomings of panels tests discussed above and also

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[2] See also Taylor and Sarno (1998) for Monte Carlo evidence.

[3] They also test for a version of PPP allowing for a time trend justified on the basis of Balassa-Samuelson-type effects.

[4] Recently, Prodan (2008) developed tests for restricted structural change with better size properties that help reconcile contradictions between unit root and structural change tests in historical RER data.

requires a common pattern of breaks across cross-sectional units. Following a similar line of argument, Gadea et al (2004) test for unit roots on post-Bretton-Woods quarterly RER data for 14 EU countries using multiple breaks. They reject the unit root null in six countries using the 1974-1996 sample but only when the structural break is allowed to be permanent (quasi-PPP). When the sample is extended to 2001 no evidence of PPP or quasi-PPP is found in any country. In all, although it is clear that structural breaks are present in the RER series, the evidence in support for PPP during the recent float using tests allowing for breaks is still very scarce.

The fourth relevant development in empirical studies of the PPP hypothesis is the potential existence of nonlinear mean reversion in RERs. The main idea is that RERs may revert to their mean only when they are sufficiently far away from it. When they are close to their mean, RERs may behave as non-stationary processes. This kind of nonlinear behaviour is compatible with a stationary RER and PPP but with a "band of inaction." This has been justified on theoretical basis as the consequence of transaction costs that make it unprofitable to arbitrage goods across the world unless price differentials are, in absolute terms, above the cost of shipping the goods, which would generate a threshold-like behaviour. This argument has been put forward by Michael et al (1997) and Taylor et al (2001) amongst others.[5] Due to the heterogeneity of transaction costs, it is likely that this threshold differs for each traded good and at the aggregate level RERs would behave as a smooth threshold autoregressive process. The initial evidence pointed towards the importance of these nonlinearities, but formal tests of unit roots against the alternative of nonlinear mean reversion were developed later. Sollis et al (2002) develop a formal test and find support for PPP in 6 out of 17 countries using post-1972 monthly RER data against the US-dollar. This evidence appears stronger for European countries against the DM, where they reject the unit root null in 8 out of 14 cases. Kapetanios et al (2003) also find more supporting evidence for PPP as they reject the unit root null in 6 out of 10 countries, but their sample period is 1957-1998 which also includes observations for the fixed exchange rate system. Bahmani-Oskooee et al (2007), on the other hand, only find evidence in favour of PPP in 8 out of 23 countries using a version of the Kapetanios et al (2003) test allowing only for a constant.[6] Overall, although these results broadly support the existence of nonlinear mean reversion, the evidence on PPP for US-dollar based RERs in the last 30 years is not as yet as robust as for longer historical time spans.[7]

In this paper we take stock these two latter innovations in the literature and develop tests that allow simultaneously for the existence of temporary structural breaks and nonlinear mean reversion in the RER. In principle, although the literature has treated both problems separately, there is no reason to assume that each explanation is exclusive. For instance, as emphasized by Papell (2002), the events that led to the appreciation and subsequent depreciation of the dollar in the early 1980s may have generated large equilibrium exchange rate swings, but mean reversion towards this value can still take a nonlinear form if agents' beliefs are heterogeneous. If this is the case, taking into account either breaks or nonlinear adjustment separately is likely to yield tests with low power and hence over-acceptance of the null of non-stationarity. Our objective in this paper is to re-visit the evidence on PPP by allowing for both temporary breaks that are compatible with long-run PPP and nonlinear mean reversion. By doing so, we are able to bring together these two separate strands of the literature. We hence develop unit root tests that allow for multiple endogenous temporary (smooth) breaks and nonlinear mean reversion of the form emphasized by the transaction costs literature. We then apply these tests to a set of 15 US-dollar based

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[5] Another justification for this behavior can be found in Killian and Taylor (2003) where they argue that heterogeneous agents' beliefs in the foreign exchange market may also lead to nonlinear adjustment.

[6] See also Chortareas and Kapetanios (2004) for evidence related to the Japanese yen and Imbs et al (2003) for evidence on sectoral real exchange rate dynamics amongst many others.

bilateral RERs for the post-Bretton-Woods floating period. Our findings show very strong support in favor of PPP as we are able to reject the null of a unit root in 14 cases (the exception being Canada).

The paper is organized as follows. In the next Section we present the unit root tests, develop critical values, and analyze their small sample properties. Section 3 presents and discusses the results and Section 4 concludes.

2. Unit root tests

The basic idea behind the tests developed in this section is to use trigonometric variables that capture large changes in the mean of the RER together with smooth transition functions that allow capturing nonlinear adjustment to this deterministic component. These tests can be considered as alternatives to Perron (1989), Zivot-Andrews (1992), and Bai and Perron (2003) for modeling breaks that also allow for asymmetries in the speed of mean reversion. The trigonometric function modeling breaks is built so that changes in the mean are temporary and hence the start and end values are restricted to be the same. The breaks modeled using this function are smooth changes rather than jump functions. This is especially appropriate in our application. In fixed exchange rate regimes revaluations or devaluations of the currency may lead to immediate jumps in the mean of the RER. But in a regime of floating exchange rates, mean changes are likely to take time as the exchange rate adjusts to its new level hence making a smooth break function more appropriate than a mean shift function as in previous applications. Nevertheless, the interpretation of the breaks in the mean of the series remains the same

[7] Recently, Amara and Papell (2006) and Elliott and Pesavento (2006) have also found broad support for PPP by making use of more powerful until root tests that exploit the information contained in stationary covariates.

as in previous studies. Changes in productivity, fiscal policy, current account positions and tastes can all have important impacts on equilibrium RERs.[8]

Consider the following model for a stochastic variable y_t

$$y_t = \delta(t) + v_t, \tag{1}$$

where $v_t \sim N(0,\sigma)$ and $\delta(t)$ is a time-varying deterministic component. Following Becker et al. (2004), Becker et al (2006) and Enders and Lee (2004) we use a Fourier series expansion to approximate the unknown number of breaks of unknown form $\delta(t)$ as

$$\delta(t) = \delta_0 + \sum_{k=1}^{G} \delta_1^k \sin(\frac{2\pi kt}{T}) + \sum_{k=1}^{G} \delta_2^k \cos(\frac{2\pi kt}{T}) \tag{2}$$

where k is the number of frequencies of the Fourier function, t is a trend term, T is the sample size and $\pi = 3.1416$.

When G is large, then the unknown functional form $\delta(t)$ can be approximated very well. In the case where the null hypothesis $\delta_k \neq 0$ is rejected for at least one frequency $k = 1, 2, ..., G$ then the nonlinear function can explain adequately the deterministic component of y_t and at least one structural change is present in the DGP. Otherwise the linear model without any structural change emerges as a special case.

A specification problem related with model (2) is to identify the appropriate number of frequencies (G) to include in the fitted model. In dealing with this issue we follow Ludlow and Enders (2000) who showed that a single frequency is enough to

[8] Because of similar reasons, Sollis (2008) considers the possibility that mean reversion of the RER can be subject to smooth breaks that take the form of a Time Varying ESTAR model.

approximate the Fourier expansion in empirical applications. Thus, equation (2) can be written as

$$\delta(t) = \delta_0 + \delta_1 \sin(\frac{2\pi kt}{T}) + \delta_2 \cos(\frac{2\pi kt}{T}) \qquad (3)$$

According to Becker *et al*. (2004) equation (1) under specification (3) has more power to detect several smooth breaks of unknown form in the intercept than the standard Bai and Perron (2003) multi-break tests.[9] This test has the advantage that not only the breaks are detected endogenously, but the form of the break does not need to be specified a priori.[10]

If the appropriate frequency k was known then we would be able to test for the presence of unknown structural breaks in the baseline equation (1). However, the true value of k is typically unknown. A standard way to find out the most appropriate frequency k is to estimate equation (1) under definition (3) for each integer value of k in the interval 1 to 5. According to Becker et al (2006), since the breaks shift the spectral density function towards frequency zero, the most appropriate frequency for a break is likely to be at the low end of the spectrum. Thus, it is the low frequencies that are the most appropriate to use for a test of unit root versus stationarity, as these would represent structural breaks rather than short-run cyclical behavior. Hence, the value of k is then chosen as the k yielding the smallest residual sum of squares. It is worth noting at this stage that using integer values of k ensures that these breaks are temporary as the start and end values of the Fourier function are the same when k is not fractional. Hence, by making use of an integer k the break function is compatible with PPP. To illustrate this point, **Figure 1**

[9] See also Bierens (1997) for a similar approach based on Chevishev polynomials.
[10] Following Becker et al. (2006) and Enders and Lee (2004), the use of appropriate frequencies for break detection ensures that these methods substantially differ from standard smoothing techniques such as HP and BP filters that are specifically designed to transform integrated series into stationarity ones hence leading to over-fitting and size distortions.

shows two Fourier functions as in (3) with $\delta_0 = 1$ and $\delta_1 = \delta_2 = 0.5$, for a sample period T = 150. For the first function we use a frequency k = 1 and for the second k = 0.6.

A formal test for the presence of unknown breaks in the DGP of y_t can then be carried out by testing the null hypothesis $H_o : \delta_1 = \delta_2 = 0$ against the alternative $H_1 : \delta_1 = \delta_2 \neq 0$. A $F-$statistic, $F_\mu(\hat{k})$, can be employed to test this null hypothesis. Monte Carlo simulations that approximate the empirical distribution for this test are tabulated in Becker et al (2006). This test for restricted (temporary) structural breaks performs especially well relative to other tests when breaks are temporary and when breaks tend to happen in opposite directions (see evidence in Becker et al, 2004). It should be noted that since the $F-$statistic has low power if the data are non-stationary this could be used only when the null of a unit root is rejected.

Within this context, given the model

$$y_t = \delta_0 + \delta_1 \sin(\frac{2\pi kt}{T}) + \delta_2 \cos(\frac{2\pi t}{T}) + v_t, \tag{4}$$

the null unit root hypothesis which is the focus of our interest can be stated as follows:

$$H_0 : v_t = \mu_t, \quad \mu_t = \mu_{t-1} + h_t$$

where h_t is assumed to be a stationary process with zero mean. The test statistics we propose are then calculated *via* a three step procedure. The procedure is implemented as follows:

Step 1. The first step involves finding the optimal frequency k^*. We estimate non-linear deterministic component in model (4) by OLS for values of *k* between 1 and 5 and select the one that minimizes the residual sum of squares. We then compute the OLS residuals

$$\hat{v}_t = y_t - \hat{\delta}_0 + \hat{\delta}_1 \sin(\frac{2\pi k^* t}{T}) + \hat{\delta}_2 \cos(\frac{2\pi k^* t}{T}).$$

Step 2. In the second step we test for a unit root on the OLS residuals of *step one*. Given that, as discussed above, mean reversion may be nonlinear due to transaction costs or heterogeneous agents' beliefs, we propose the following three linear and non-linear models:

$$\Delta v_t = \alpha_1 v_{t-1} + \sum_{j=1}^{p} \beta_j \Delta v_{t-j} + u_t \tag{5}$$

$$\Delta v_t = \rho v_{t-1}(1 - \exp(-\theta \Delta v_{t-i}^2)) + \sum_{j=1}^{p} \alpha_j \Delta v_{t-j} + u_t \qquad i = 1,2,\ldots..L \tag{6}$$

$$\Delta v_t = \lambda_1 v_{t-1}^3 + \sum_{j=1}^{p} \beta_j \Delta v_{t-j} + u_t \tag{7}$$

where $\theta > 0$ and u_t is a white noise error term.

Step 3. If we reject the null of a unit root in step two, the third step consists of testing for $H_o : \delta_1 = \delta_2 = 0$ against the alternative $H_1 : \delta_1 = \delta_2 \neq 0$ in (4) using the F-test $F_\mu(\hat{k})$. If the null hypothesis is rejected, we can conclude that the variable is stationary around a breaking deterministic function.

Model (5) is a standard ADF regression – we call this test Fourier-ADF (FADF) test – that assumes linear adjustment towards equilibrium. Models (6) and (7) assume that the adjustment speed is nonlinear and follows an Exponential Smooth Transition

Autoregressive (ESTAR) process as those used in Michael et al (1997) and Taylor et al (2001). These correspond to the unit root tests developed by Kilic and de Jong (2006) [model (6)] and Kapetanios et al (2003) [model (7)]. All models allow for testing for a unit root in the original series after removing the breaks in the deterministic component. In the linear case the null unit root hypothesis $H_0 : \alpha_1 = 0$ is tested against the alternative $H_0 : \alpha_1 \neq 0$. Models (6) and (7) allow, in addition to temporary breaks, testing for a unit root against a non-linear alternative. In particular in the model suggested by Kilic and de Jong (2006) the transition parameter θ determines the speed of transition between two extreme regimes. The exponential transition function $F(\theta, \Delta v_{t-i})$ is bounded between zero and unity with Δv_{t-i} being the transition variable that determines the regime. The use of Δv_{t-i} as a transition variable ensures that the transition variable is not highly persistent[11]. At the extremes of $F(\theta, \Delta v_{t-i}) = 0$ and $F(\theta, \Delta v_{t-i}) = 1$ the smooth transition model (6) is linear and the corresponding AR(1) models are given by $v_t = v_{t-1} + u_t$ and $v_t = (1+\rho) v_{t-1} + u_t$ respectively. This exponential function implies that that the speed of mean reversion is faster when the transition variable is sufficiently far away from zero. In other words, mean reversion will be faster when the RER is far from its equilibrium value determined by the Fourier function, whereas it behaves as a unit root process when it is close to it.[12]

To test the null unit root hypothesis we follow Kilic and de Jong (2006) and use the following t-statistic:

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[11] In the empirical application the test statistic is computed over a range $i \in [1, L]$. The i that yields the model with the smallest SSR is selected.

[12] Lundbergh et al (2003) suggest a different way of specifying jointly smooth time-dependent breaks and nonlinear adjustment using a Time Varying STAR (TV-STAR) model. Their model, however, only allows for a one-time smooth break in the series that is a specific function of time. The Fourier model allows for an unspecified number of breaks that can also encompass a wider variety of functions (see Enders and Lee, 2004).

$$\text{F-Sup-}t_{iN} = \sup_{(\theta)\in\Theta} \left\{ \frac{\tilde{\rho}(\theta)}{s.e.(\tilde{\rho}(\theta))} \right\}_{\rho=0},$$

where $\dfrac{\hat{\rho}(\theta)}{s.e.(\hat{\rho}(\theta))}$ is the t-ratio test for the null hypothesis H_o: $\rho = 0$ for the range of values of θ defined as $\Theta = [\underline{\theta}, \overline{\theta}]$ and $0 < \underline{\theta} < \theta < \overline{\theta}$. This corresponds to the values of θ yielding the smallest sum of squared residuals. The initial value of θ is estimated using a grid search method over the range [0.1,0.2,……300].

Finally, the Kapetanios' et al (2003) unit root test is in fact a linearized version of the Kilic and de Jong (2006) test that uses, unlike Kilic and de Jong (2006), v_{t-1} as transition variable. Specifically, the Kapetanios et al (2003) test consists of testing for the null of a unit root against a non linear alternative not on the original model (6) but on an auxiliary model which is obtained by approximating the transition function around the origin, that is $\theta = 0$. Then the test for the unit root null $\lambda_1 = 0$ against the alternative $\lambda_1 < 0$ is obtained with the following t-statistic:[13]

$$F - t_{NL} = \frac{\hat{\lambda}_1}{s.e.(\hat{\lambda})_1}$$

As shown by Becker et al (2006) the asymptotic distribution of any of the derived test statistics depends only on the frequency k of the Fourier series. We hence tabulated critical values for the three tests via Monte Carlo simulations under the null of a random walk for values of *k* between 1 and 5 and sample sizes of 100, 250 and 500 observations. The critical values were obtained from 10,000 replications using a pseudo-random number generator. These critical values are reported in **Tables 1 to 3**.

[13] In essence, this is a simultaneous test for nonlinearity and a unit root.

2.1 Size and power properties

We carry out a Monte Carlo experiment in order to investigate the small sample size and power properties of the tests suggested in the previous Section. We first consider the size of the tests using the following DGP

$$y_t = \delta_0 + \delta_1 \sin(\frac{2\pi k^* t}{T}) + \delta_2 \cos(\frac{2\pi k^* t}{T}) + v_t \qquad (8)$$

$$v_t = v_{t-1} + \varepsilon_t$$

where ε_t is a sequence of standard normal errors and k^* stands for the optimal frequency.

The empirical size is considered for each test for sample sizes $T = \{100\ 250\}$, values of $k^* = \{1\ 2\ 3\}$, and $\delta_1 = \delta_2 = \{1\ 0.5\ 0.1\}$ with a nominal size of 5%. The small value of 0.1 for $\delta_1 = \delta_2$ would correspond to an almost linear process.[14] We then applied the first two steps of our procedure selecting *k* and then testing for unit roots using the three models (5)-(7). The results are displayed in **Table 4**. The following conclusions can be extracted from the size analysis:

(a) All tests display empirical sizes that are very close to the nominal.
(b) The F-Sup-t_{iN} test performs better than the other two when $T = 100$, $\delta_1 = \delta_2 = 1$ and $k^* = 1,2,3$. In these cases the FADF and $F - t_{NL}$ tests tend to over-reject very slightly.
(c) The $F - t_{NL}$ test has less size distortions than the other two when $\delta_1 = \delta_2 = 0.5, 0.1$.

[14] We also considered a sample size of 50 and values for $\delta_1 = \delta_2 = 0.05$ and the results did not change in any significant way.

(d) When the sample size is increased to $T = 250$ all tests show only small distortions.

To check the robustness of our findings, the empirical size of the three tests was also simulated for two cases with non-normal errors. In particular, we considered errors drawn form both the $\chi^2(1) - 1$ distribution and the $t(6)$ distribution. In both cases the simulation results indicated that all three tests are robust against both types of non-normal errors. In all the cases the empirical sizes were very close to the nominal for all sample sizes using the critical values of Tables 1 to 3.[15] Overall, the three tests present very good size properties that ensure that the model is not over-fitting and hence leading to over-rejections of the unit root null.

Next, we investigate the power properties of the unit root tests against globally stationary process using the following Fourier-ESTAR model as a DGP:

$$y_t = \delta_0 + \delta_1 \sin(\frac{2\pi k^* t}{T}) + \delta_2 \cos(\frac{2\pi k^* t}{T}) + v_t \tag{9}$$

$$v_t = v_{t-1} + \rho v_{t-1}(1 - \exp(-\theta \Delta v_{t-i}^2)) + \sum_{j=1}^{p} \alpha_j \Delta v_{t-j} + u_t$$

All combinations of the following parameter values and frequencies were used: $T = \{100\ 250\}$, $\rho = \{-1.5, -1, -0.5, -0.1\}$, $\theta = \{0.01, 0.5, 1\}$, $\delta_1 = \delta_2 = \{1, 0.1\}$ and $k^* = \{1,2,3\}$. The results from these power experiments for a sample size of 250 are shown in **Table 5**.[16] The general outcome is that for values of $\delta_1 = \delta_2 = 1$ the $F - t_{NL}$ test is more powerful than the simple ADF test only for $k^* = \{1,2\}$ while the F-Sup-t_{iN} test is

[15] These findings are not reported here to save space but are available from the authors upon request.

[16] The results for T = 100 did not change the ranking of the results reported and are available on request.

more powerful than all other tests for high values of $\rho = \{-1.5, -1\}$ and regardless of the values of k^* and θ. The power of the F-Sup-t_{iN} test decreases for low values of $\rho = \{-0.5, -0.1\}$ and high values of $\theta = \{0.5, 1\}$. This is not unexpected since as θ becomes larger, $E(\exp(-\theta \Delta v_{t-i}^2))$ decreases and the process tends to be less persistent. This might occur in a situation where most realizations of the process occur in the neighborhood of the middle regime. For high values of $\rho = \{-1.5, -1\}$ and $\theta = \{0.5, 1\}$ the power of all tests approaches unity as expected. The above conclusions are also confirmed in the case where $\delta_1 = \delta_2 = 0.1$. The only difference that emerges is that for high and medium values of $\rho = \{-1.5, -1, -0.5\}$ and for values of $\theta = \{0.5, 1\}$ the power of all three tests is equal to unity. This is not surprising since now y_t does not contain very dominant trigonometric terms and the transition is still smooth but relatively short in duration. On the other hand, for very low values of $\theta = \{0.1\}$ the F-Sup-t_{iN} test performs better than the other two.

Finally, to further investigate the power properties of the proposed unit root tests we conduct an experiment where the process is locally explosive but globally stationary. To this end we used a DGP as (9) but now v_t follows

$$v_t = (1 + 0.1)v_{t-1} + \rho v_{t-1}(1 - \exp(-\theta \Delta v_{t-i}^2)) + \sum_{j=1}^{p} \alpha_j \Delta v_{t-j} + u_t$$

where $\rho = \{-0.25, -0.5, -1\}$, $\theta = \{0.01, 0.5, 1\}$, $\delta_1 = \delta_2 = \{1, 0.1\}$ and $k = \{1, 2, 3\}$. The results from these power experiments for a sample size of 250 are presented in **Table 6**. According to these findings the F-Sup-t_{iN} test is more powerful than the FADF and the $F - t_{NL}$ tests. This happens irrespective of the values of ρ, θ, δ_i and k. The FADF test performs better than the $F - t_{NL}$ test when $\delta_1 = \delta_2 = 1$ in all the cases except for $\rho = \{-0.25, -0.5\}$ and $k = 1$. A similar situation is observed when $\delta_1 = \delta_2 = 0.1$. Overall we can conclude that the F-Sup-t_{iN} test is more powerful than the other two when the process is locally explosive. On the contrary, the $F - t_{NL}$ test only yields power gains in a small number of cases relative to the simple FADF test.

3. Post-Bretton-Woods RER properties

We now proceed to analyze the stationarity properties of US-dollar based bilateral RERs for a set of 15 OECD countries. We use quarterly data for the sample period 1974:1-2006:4 for all countries except those that entered the European Monetary Union (EMU) for which we use the 1974:1-1998:4. All the data was obtained from IMF's IFS database. We define the log of the RER as:

$$q_t = e_t + p_t - p_t^* \tag{10}$$

where e_t is the log of the nominal exchange rate defined as US-dollars per unit of domestic currency, p_t is the log of the domestic CPI and p_t^* is the log of the US CPI. An increase in q_t is then interpreted as a depreciation of the dollar against the domestic currency.

We first applied a ADF and DF-GLS tests to the RER series as a preliminary step in our analysis. We chose the optimal lag augmentation using the Ng and Perron (2001) Modified Akaike Information Criterion (MAIC). The results are reported in **Table 7**. As it is standard in the literature, we were unable to reject the null of a unit root using the ADF test in the majority of the countries. The only exception was New Zealand. Using the DF-GLS test, the number of rejections increases to five, including the case of New Zealand, as we would expect with a test with better power properties. We hence proceed to apply our unit root tests to our set of countries with the exception of New Zealand as in this case the RER clearly behaves as a stationary process.

Following the 3-step procedure described in the previous Section, we first fitted a Fourier model to the RER series and found the optimal frequency k^* that minimized the sum of squared residuals. The second column in **Table 8** presents the estimated optimal *k*. We can see that for the majority of the countries, the frequency is found to be 1 or 2. The only exception to this pattern is the UK where we found a high value of 4. The q_t series is plotted against the estimated Fourier function in **Figure 2**. For the great majority of the countries in the sample, the breaks follow a similar pattern. The first break is associated with the large appreciation of the US-dollar in the early 1980s induced by the monetary and fiscal policy mix in the US. This appreciation leads subsequently to a strong depreciation following the Louvre and Plaza agreements in the mid-1980s. The peak of this depreciation period differs across countries, but for most of them the US-dollar starts a process of slow appreciation during the 1990s as its output growth outperforms that of the rest of the OECD. For the countries that did not join the EMU, the dollar appreciation continues until the early 2000s when the US-dollar starts depreciating against other major currencies except the Japanese Yen. This explains the second break found for Australia, Canada, Denmark and Switzerland. This is not the case for Japan, whose currency follows a real depreciation against the dollar due to its sluggish growth performance in the last 15 years. For the UK a similar pattern to that of non-EMU countries is found but with an extra frequency change during the late 1990s and early 2000s, which is possibly associated with the successful inflation stabilization policies linked to the change in monetary policy stance. Overall, the "Great Appreciation-Depreciation" episode of the US dollar in the 1980s is driving the majority of the breaks in our sample. In this respect, our results are not dissimilar to those of Papell (2002). For countries where data until 2006:4 is available, the start of the 2000s decade also marks an important change in the mean of the series.

We then proceed to Step 2 of our testing procedure and obtained the OLS residuals from the Fourier function and applied the three unit root tests proposed above. The results from these tests are presented in columns 4 to 6 in **Table 8**. The first one is the Fourier-ADF (FADF) where we assume that mean reversion follows a linear process. For this test we can reject the null of a unit root at the 5% level for Denmark, Finland and Switzerland and for Japan at the 10% level. That is, although the breaks alone can account for increased rejection of the unit root null, the evidence in favor of PPP is not sufficiently strong. The results from the F-t_{NL} test indicate that we can reject the null in six cases. These are the countries for which we rejected a unit root using the FADF test plus Portugal and the UK. When we turn to the F-Sup$-t_{iN}$ test, though, rejection of the null of a unit root occurs in all cases with the only exception of Canada. According to the results of the Monte Carlo analysis (Table 5) this finding is not surprising given that F-Sup$-t_{iN}$ is more powerful relative to the other two tests for high values of $(1-\hat{\rho})$ and irrespective of the values of k^* and θ. The final step in our procedure consists of testing for the significance of the breaks by using the $F_\mu(\hat{k})$ test presented in the previous Section. This result, reported in column 3 of **Table 8**, shows that the null of a constant mean can be rejected in all cases.[17]

Column 5 in **Table 8** also reports the speed of transition θ, normalized by the sample standard deviation of the transition variable, and the estimated ρ coefficient. We can see that the speed of transition between the inner and outer states is quick for the majority of countries, making it close to a threshold process for countries like Denmark, Ireland, The Netherlands, and Japan. The estimated autoregressive coefficient ρ is very high in absolute terms. It averages -0.26 and ranges from -0.68 for Germany to -0.06 for Canada. The estimated roots allow us to calculate half-lives of deviations from equilibrium for each of the models. These are reported in **Table 9**. The half-lives for the

[17] Although we report this test for Canada, it has to be noted that, given that we cannot reject the null of a unit root, we cannot conclude in favor of the existence of breaks.

two nonlinear ESTAR models were calculated following Taylor (2001). In particular, Taylor (2001) calculates the half-life of a threshold autoregressive model by making use of the autoregressive coefficient in the outer regime regardless of the size of the threshold parameter or the time spent in the inner regime. We can see that the largest half-lives always correspond to those of the ADF model with the only exception of the UK for which the FADF model yields the largest half-life. This is consistent with Taylor's (2001) results where he shows that large estimated half-lives can be the result of incorrectly specifying a linear model when the true DGP is a threshold model with nonlinear adjustment. The average half-life for the ADF model is 10.11, followed by the FADF (6.58), the F-t_{NL} (4.69) and the $\text{F-Sup}-t_{iN}$ (3.22). In the latter case, half-lives range from 4.9 quarters for Australia to a short half a quarter for Germany. These half-lives would imply that the linear model yields an estimated half-life of about 2.5 years as opposed to the two Fourier-nonlinear models with half-lives of just under or just over a year.

Finally, in order to analyze the adequacy of the models, we plotted the transition functions of models (6) and (7) against the transition variable.[18] These plots are reported in **Figures 3 and 4**. The transition functions appear to be well defined for the range of values of the transition variable in most cases. However, the F-t_{NL} model for the cases of Italy, Portugal, and the UK does not contain enough observations in the inner (Italy and Portugal) or the outer (UK) regimes. A similar problem arises with the $\text{F-Sup}-t_{iN}$ model for Denmark and The Netherlands, which can be explained as a result of the very large speed of transition coefficient found for these two countries.

The results from the application of our tests, thus, present strong evidence in favor of the PPP hypothesis. Out of the 15 OECD countries in our sample, we are able to reject the unit root null in 14 cases. The results are also similar to those found in Amara and Papell (2006) who are able to reject the null of a unit root in the RER of a set of 20 OECD countries using a series of tests with stationary covariates. The only country in common with our sample for which they cannot reject the null is also Canada.[19] Our evidence shows that RERs can be represented as mean reverting processes subject to infrequent temporary smooth breaks where mean reversion occurs at a faster speed when far from equilibrium. The mean breaks are associated with the large US dollar swings in the first half of the 1980s and also with the inflexion point in the early 2000s.

4. Conclusions

Despite increasing evidence in favor of the PPP hypothesis in recent years, support for PPP for the post-Bretton-Woods floating exchange rates period has so far been sparse. This is especially the case for US-dollar-based bilateral RERs. Given the lack of power of standard unit root tests, attempts to solve this problem have ranged from the use of historical datasets to the use of panel methods. In this paper, however, we focus on the potential effect that structural breaks and nonlinear mean reversion have on tests of the PPP hypothesis. We present tests that, far from considering these two features separately, model both breaks and nonlinear adjustment jointly. We argue that, even in the presence of temporary breaks in the mean of the RER, transaction costs or heterogeneous agents' opinions can lead to a faster adjustment of the RER when it is far from its (possibly breaking) equilibrium.

We develop a set of unit root tests that account for the presence of multiple smooth temporary breaks compatible with long-run PPP by means of a Fourier function. We

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[18] We also compared the models using the Schwarz Information Criterion (SIC), and found that the F-$\text{Sup} - t_{iN}$ outperforms the rest of the models for all cases except for Canada, where the ADF outperforms the rest, and Japan, where the F-t_{NL} minimizes the SIC.

[19] Similar results are found in Elliott and Pesavento (2006).

then model reversion to this mean using a linear and two nonlinear specifications of the ESTAR form following the lead of previous literature. Our tests are easy to implement present good size and power properties.

We applied the tests to a set of 15 quarterly OECD RERs for the 1974:1-2006:4 period and we are able to reject the null of a unit root for 14 of them with the only exception of Canada. Both breaks and nonlinear adjustment appear to be important features driving the behavior of these series. The majority of the breaks are associated to the "Great Appreciation-Depreciation" of the US-dollar in the 1980s but also with changes in the behavior of the US-dollar at the turn of the new century. Although these breaks alone account for part of the increased rejection of the unit root null, modeling nonlinear mean reversion in conjunction with the breaks yields the strongest results in favor of PPP. Our calculated half-lives are also much shorter than in previous studies and average about 1 year when both breaks and nonlinear adjustment are modeled jointly.

Tables

Table 1: Null critical values for unit root tests against stationarity for the FADF statistic

T=100	*K*	1%	5%	10%
	1	-4.43	-3.85	-3.52
	2	-3.95	-3.28	-2.91
	3	-3.70	-3.06	-2.71
	4	-3.60	-2.93	-2.59
	5	-3.55	-2.90	-2.56
T=250	1	-4.36	-3.78	-3.48
	2	-3.88	-3.28	-2.95
	3	-3.68	-3.03	-2.71
	4	-3.54	-2.93	-2.64
	5	-3.51	-2.90	-2.61
T=500	1	-4.40	-3.78	-3.46
	2	-3.87	-3.27	-2.93
	3	-3.64	-3.05	-2.72
	4	-3.54	-2.97	-2.64
	5	-3.53	-2.93	-2.59

Table 2: Null critical values for unit root tests against stationarity for the F-Sup-t_{iN} statistic

T=100	*k*	1%	5%	10%
	1	-4.56	-3.92	-3.58
	2	-4.18	-3.35	-3.01
	3	-3.83	-3.18	-2.82
	4	-3.78	-3.09	-2.75
	5	-3.70	-3.03	-2.71
T=250	1	-4.39	-3.82	-3.54
	2	-3.96	-3.36	-3.02
	3	-3.78	-3.17	-2.83
	4	-3.70	-3.03	-2.76
	5	-3.62	-3.00	-2.73
T=500	1	-4.41	-3.86	-3.54
	2	-4.02	-3.36	-3.02
	3	-3.80	-3.15	-2.84
	4	-3.70	-3.13	-2.81
	5	-3.70	-3.07	-2.76

Table 3: Null critical values for unit root tests against stationarity for the F- t_{NL} statistic

T=100	*k*	1%	5%	10%
	1	-4.14	-3.59	-3.26
	2	-3.84	-3.25	-2.96
	3	-3.61	-3.06	-2.75
	4	-3.52	-2.99	-2.71
	5	-3.52	-2.92	-2.65
T=250	1	-4.19	-3.60	-3.29
	2	-3.86	-3.26	-2.99
	3	-3.65	-3.11	-2.86
	4	-3.58	-3.04	-2.77
	5	-3.51	-3.01	-2.74
T=500	1	-4.19	-3.64	-3.32
	2	-3.82	-3.28	-2.99
	3	-3.67	-3.11	-2.82
	4	-3.66	-3.06	-2.77
	5	-3.55	-3.00	-2.74

Table 4: Empirical size of the tests

	FADF			**F-** t_{NL}			**F-Sup-** t_{iN}		
	$k=1$	$k=2$	$k=3$	$k=1$	$k=2$	$k=3$	$k=1$	$k=2$	$k=3$
	T=100								
$\delta_i=1$	0.042	0.033	0.039	0.046	0.041	0.036	0.048	0.045	0.042
$\delta_i=0.5$	0.043	0.041	0.043	0.047	0.050	0.046	0.054	0.054	0.054
$\delta_i=0.1$	0.043	0.043	0.043	0.051	0.052	0.053	0.057	0.057	0.057
	T=250								
$\delta_i=1$	0.048	0.045	0.042	0.050	0.045	0.044	0.049	0.046	0.045
$\delta_i=0.5$	0.047	0.048	0.045	0.053	0.052	0.050	0.052	0.052	0.050
$\delta_i=0.1$	0.047	0.048	0.047	0.052	0.052	0.050	0.052	0.052	0.055

Note: Nominal size is 5% and number of replications = 2500.

Table 5: Empirical powers of unit root tests for a globally stationary ESTAR process at the 5% nominal level.

		FADF			**F-** t_{NL}			**F-Sup-** t_{iN}		
		$k=1$	$k=2$	$k=3$	$k=1$	$k=2$	$k=3$	$k=1$	$k=2$	$k=3$
ρ	θ	$\delta_1 = \delta_2 = 1$								
-1.5	0.1	0.271	0.615	0.816	0.389	0.630	0.759	0.392	0.702	0.820
-1.5	0.5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-1.5	1.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-1.0	0.1	0.143	0.381	0.606	0.219	0.399	0.547	0.247	0.521	0.663
-1.0	0.5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-1.0	1.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.5	0.1	0.039	0.131	0.265	0.064	0.139	0.209	0.085	0.263	0.365
-0.5	0.5	0.998	1.000	1.000	1.000	1.000	1.000	0.935	0.991	0.999
-0.5	1.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.1	0.1	0.002	0.005	0.009	0.003	0.006	0.009	0.001	0.011	0.027
-0.1	0.5	0.091	0.285	0.483	0.155	0.299	0.427	0.086	0.229	0.332
-0.1	1.0	0.108	0.316	0.523	0.177	0.328	0.456	0.087	0.234	0.338
ρ	θ	$\delta_1 = \delta_2 = 0.1$								
-1.5	0.1	0.487	0.846	0.952	0.621	0.857	0.930	0.756	0.951	0.983
-1.5	0.5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-1.5	1.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-1.0	0.1	0.240	0.595	0.783	0.343	0.606	0.726	0.492	0.751	0.901
-1.0	0.5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-1.0	1.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.5	0.1	0.048	0.199	0.370	0.085	0.211	0.307	0.145	0.412	0.562
-0.5	0.5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.5	1.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.1	0.1	0.001	0.007	0.018	0.001	0.008	0.014	0.0003	0.016	0.037
-0.1	0.5	0.167	0.452	0.650	0.245	0.466	0.592	0.138	0.341	0.458
-0.1	1.0	0.193	0.493	0.683	0.277	0.509	0.629	0.139	0.342	0.459

Note: sample size is 250 and number of replications =2500.

Table 6: Empirical powers of unit root tests for a locally explosive ESTAR process at the 5% nominal level

		FADF			F-t_{NL}			F-Sup-t_{iN}		
		$k=1$	$k=2$	$k=3$	$k=1$	$k=2$	$k=3$	$k=1$	$k=2$	$k=3$
ρ	θ	$\delta_1 = \delta_2 = 1$								
-0.25	0.1	0.028	0.105	0.195	0.139	0.292	0.397	0.093	0.304	0.441
-0.25	0.5	0.145	0.435	0.656	0.218	0.405	0.499	0.321	0.638	0.787
-0.25	1.0	0.189	0.507	0.733	0.215	0.405	0.500	0.345	0.647	0.795
-0.50	0.1	0.197	0.512	0.735	0.457	0.687	0.799	0.678	0.928	0.969
-0.50	0.5	0.628	0.942	0.993	0.596	0.825	0.897	0.918	0.995	0.995
-0.50	1.0	0.719	0.967	0.997	0.605	0.753	0.893	0.909	0.989	0.995
-1.00	0.1	0.538	0.903	0.978	0.780	0.921	0.967	0.923	0.995	0.997
-1.00	0.5	0.934	0.998	0.978	0.922	0.984	0.992	0.988	0.999	1.000
-1.00	1.0	1.000	1.000	1.000	0.897	0.981	0.993	0.977	0.999	1.000
ρ	θ	$\delta_1 = \delta_2 = 0.1$								
-0.25	0.1	0.035	0.157	0.276	0.265	0.504	0.614	0.277	0.670	0.816
-0.25	0.5	0.315	0.672	0.822	0.373	0.577	0.665	0.669	0.898	0.949
-0.25	1.0	0.397	0.749	0.884	0.363	0.566	0.641	0.639	0.883	0.935
-0.50	0.1	0.419	0.759	0.907	0.851	0.951	0.978	0.991	1.000	1.000
-0.50	0.5	0.956	0.998	1.000	0.947	0.983	0.992	1.000	1.000	1.000
-0.50	1.0	0.984	0.999	1.000	0.942	0.982	0.988	0.996	1.000	1.000
-1.00	0.1	0.913	0.994	0.999	0.998	1.000	1.000	1.000	1.000	1.000
-1.00	0.5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-1.00	1.0	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000

Note: sample size is 250 and number of replications =2500.

Table 7: ADF unit root tests

	Period	ADF	DF-GLS	*p*
Australia	1974:1-2006:4	-2.24	-1.05	1
Canada	1974:1-2006:4	-2.09	-1.17	3
Denmark	1974:1-2006:4	-2.21	-2.22*	1
Finland	1974:1-1998:4	-2.25	-2.15*	1
France	1974:1-1998:4	-2.19	-2.16*	1
Germany	1974:1-1998:4	-1.74	-1.63	0
Ireland	1974:1-1998:4	-2.05	-1.86	0
Italy	1974:1-1998:4	-2.26	-2.23*	1
Japan	1974:1-2006:4	-2.27	-1.53	3
Netherlands	1974:1-1998:4	-2.11	-1.80	1
New Zealand	1974:1-2006:4	-2.91*	-1.98*	1
Portugal	1974:1-1998:4	-1.76	-1.64	3
Spain	1974:1-1998:4	-1.82	-1.71	1
Switzerland	1974:1-2006:4	-2.71	-1.93	1
UK	1974:1-2006:4	-1.85	-1.30	0

Note: The optimal lag *p* was determined using the Ng and Perron (2001) MAIC. A * shows rejection of the null hypothesis at 5% statistical level. Critical values are also from Ng and Perron (2001)

Table 8: Unit root tests based on the Fourier function

	$\hat{k}$	$F_{\mu}(\hat{k})$	FADF	F-t_{NL}	θ	ρ	F-Sup-t_{iN}
Australia	2	<u>55.81</u>	-2.57 [p=3]	-2.42 [p=3]	116.40	-0.12	-3.14* [p=3]
Canada	2	76.58	-1.84 [p=3]	-2.59 [p=3]	101.01	-0.06	-2.27 [p=3]
Denmark	2	<u>63.79</u>	-3.56** [p=3]	-3.77** [p=3]	717.50	-0.15	-3.68** [p=3]
Finland	2	<u>62.51</u>	-3.57** [p=3]	-3.14* [p=3]	82.90	-0.27	-4.37** [p=3]
France	2	<u>27.11</u>	-2.66 [p=3]	-2.28 [p=3]	20.70	-0.24	-3.13* [p=3]
Germany	1	<u>24.97</u>	-2.74 [p=3]	-2.26 [p=3]	3.21	-0.68	-4.14** [p=3]
Ireland	1	<u>22.17</u>	-3.30 [p=3]	-2.10 [p=3]	154.00	-0.22	-4.03** [p=3]
Italy	1	<u>21.84</u>	-3.01 [p=3]	-2.68 [p=3]	9.20	-0.44	-3.97** [p=3]
Japan	1	<u>97.33</u>	-3.81* [p=3]	-3.77** [p=3]	150.70	-0.18	-4.13** [p=3]
Netherlands	2	<u>26.49</u>	-2.59 [p=3]	-2.33 [p=3]	771.00	-0.14	-3.29* [p=3]
Portugal	1	<u>102.16</u>	-2.89 [p=3]	-3.50* [p=3]	42.60	-0.27	-3.93** [p=3]
Spain	2	<u>37.74</u>	-1.94 [p=1]	-2.24 [p=1]	8.81	-0.37	-3.60** [p=1]
Switzerland	2	<u>28.44</u>	-3.43** [p=3]	-3.06* [p=7]	35.80	-0.22	-3.91** [p=3]
UK	4	<u>42.56</u>	-2.23 [p=7]	-3.62** [p=7]	13.30	-0.28	-3.28** [p=7]

Notes: (**) and (*) denote rejection of the null unit root hypothesis at the 5% and 10% significance level respectively. The underlined figures indicate rejection of the null of linearity at conventional significance levels. The $F_{\mu}(\hat{k})$ test is distributed as a $F-$statistic under the null hypothesis with two degrees of freedom. The critical values are taken from Table 1 of Becker et al. (2006). Both the optimal lag in the transitional variable for the F-Sup-t_{iN} and the lag augmentation for the unit root tests were determined using the SBIC.

Table 9: Half-lives

	ADF	FADF	F- t_{NL}	F-Sup- t_{iN}
Australia	12.02	8.65	7.26	4.93
Canada	na	na	na	na
Denmark	9.55	4.97	3.97	3.97
Finland	7.34	3.49	2.94	2.11
France	7.35	5.42	4.59	2.47
Germany	8.31	5.42	4.97	0.59
Ireland	5.95	3.72	3.49	2.78
Italy	8.31	5.33	3.97	1.16
Japan	11.20	4.27	2.76	3.49
Netherlands	8.31	6.37	4.97	4.59
Portugal	16.98	4.97	4.59	2.11
Spain	11.20	11.20	5.95	1.16
Switzerland	7.34	5.42	4.49	2.65
UK	4.97	5.94	2.50	2.07

Figures

Figure 1: Fourier functions with different frequencies

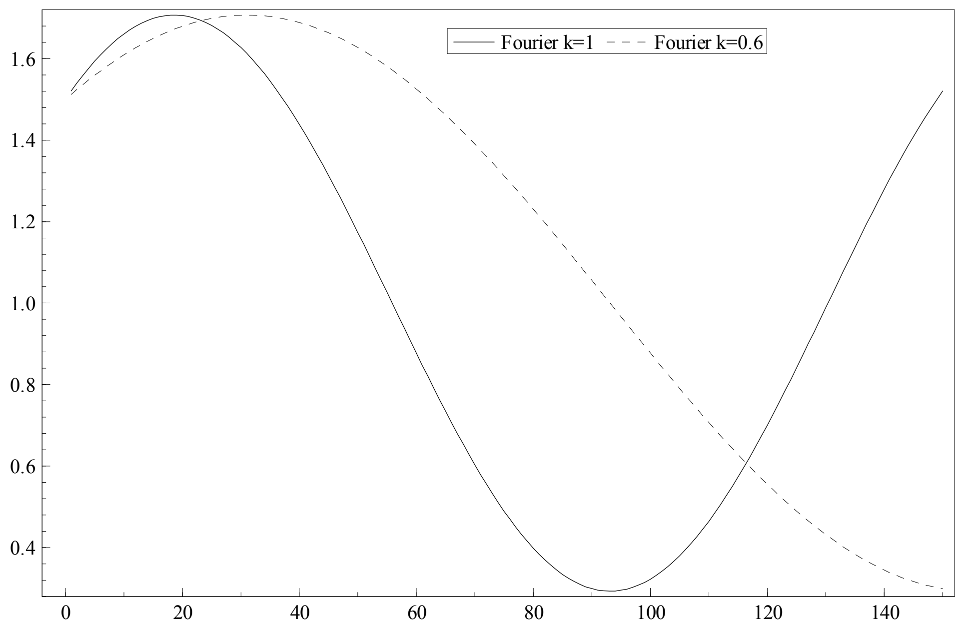


Figure 2: RER and the Fourier Function

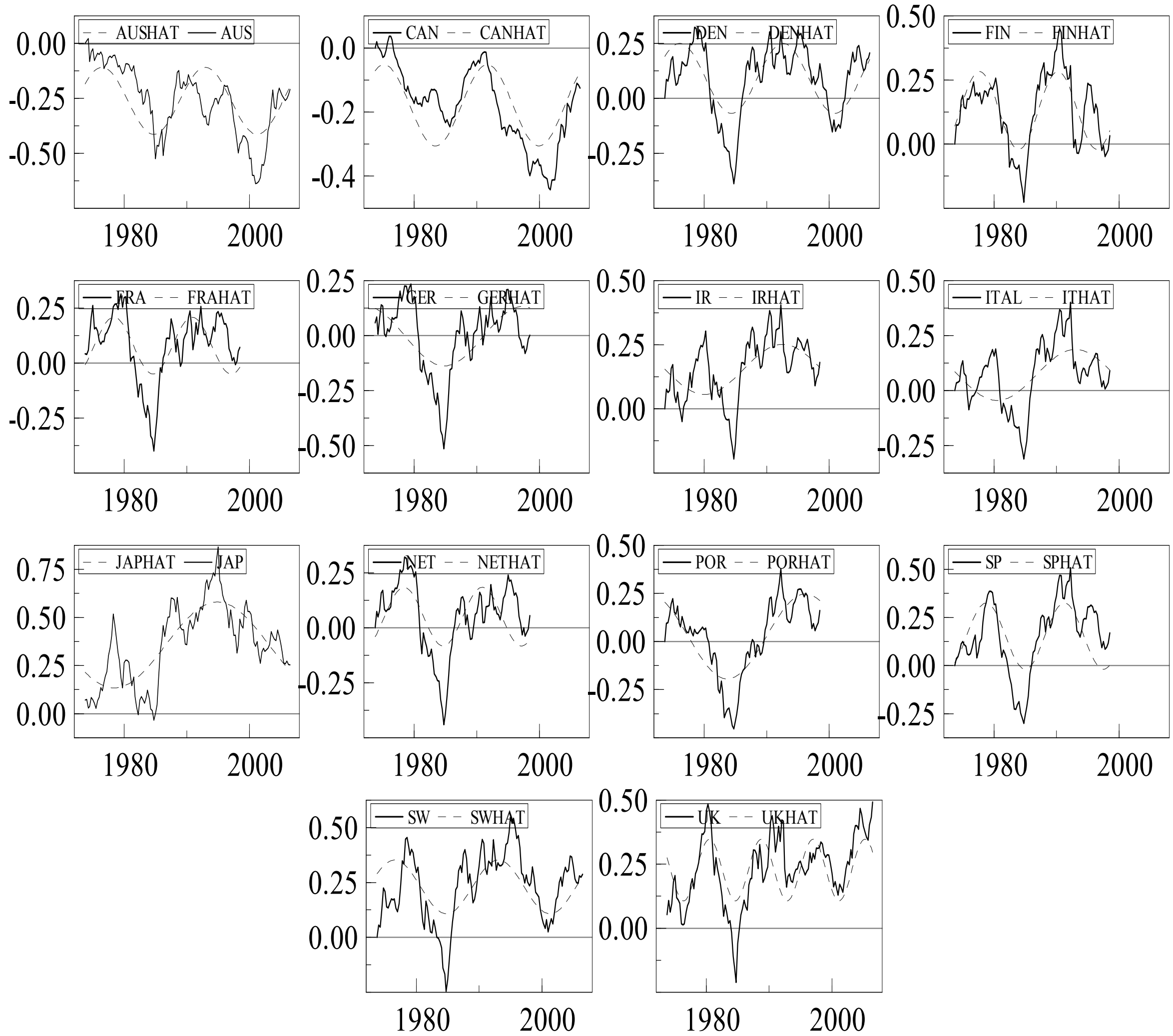


Figure 3: Transition function vs. residuals $\text{F-Sup} - t_{iN}$ **test**

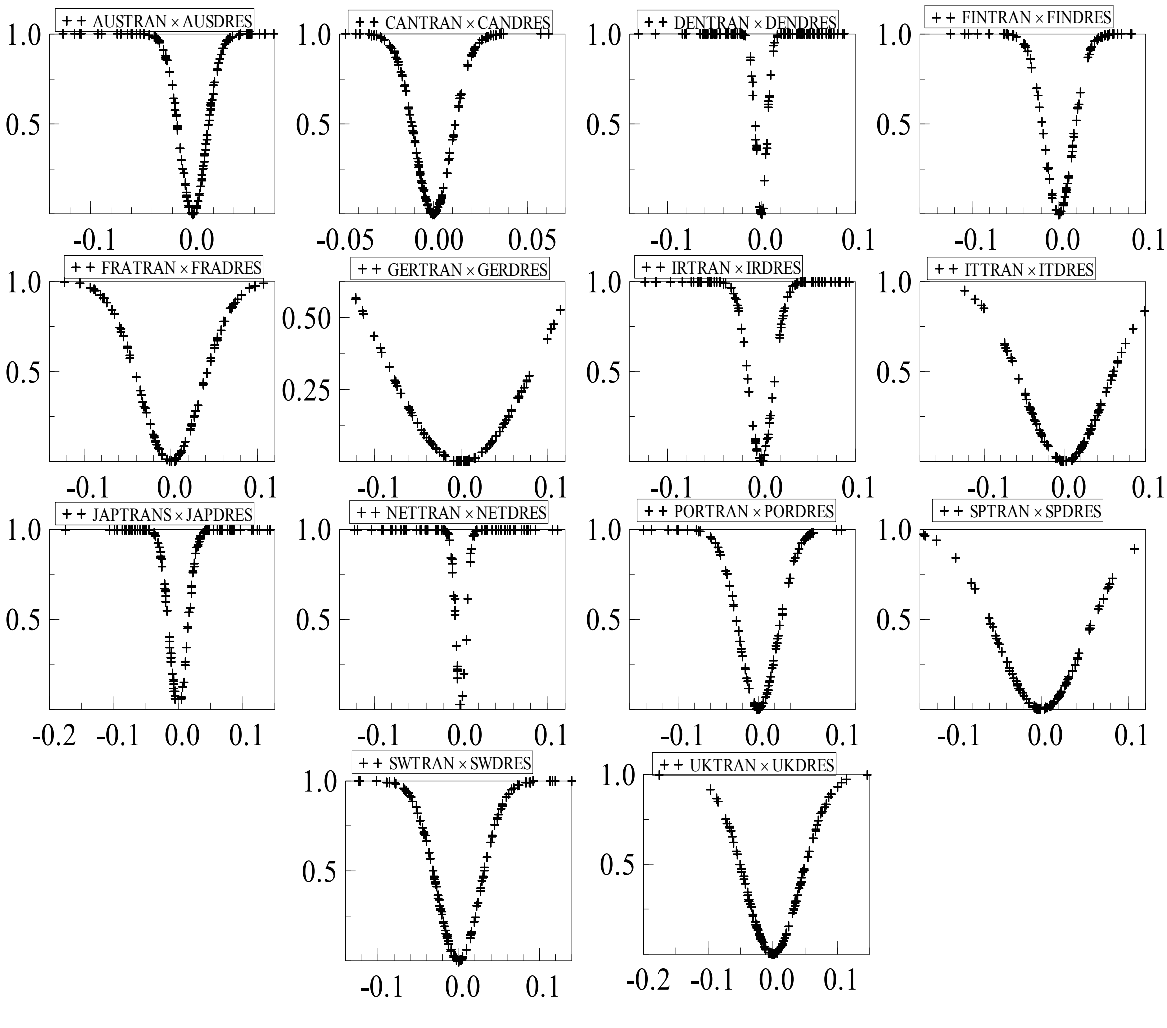
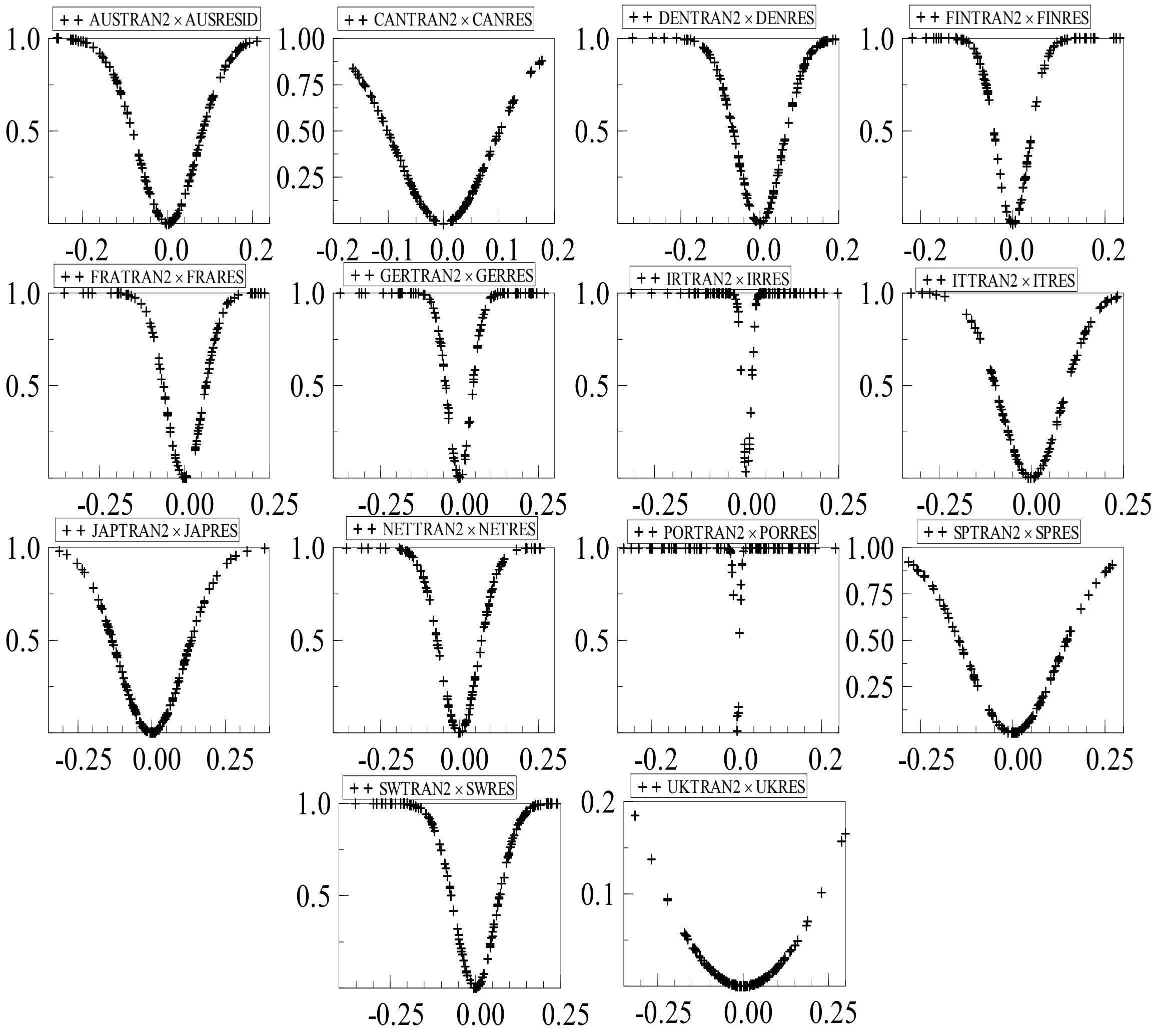


Figure 4: Transition function vs. residuals $\mathrm{F}-t_{NL}$ test



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