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STOCHASTIC SIMULATION AND DYNAMIC PROPERTIES
OF THE NEW VERSION OF THE ITALIAN MODEL

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Computations have been performed on the computer IBM/370, model 168, installed at CNUCE, Pisa.

1. INTRODUCTION

This paper describes the results of some stochastic simulation experiments performed on the most updated version of the Italian model.

Due to a change in the income accounts system, the model has been completely reestimated using the new quarterly data. It consists of 128 equations, 50 of which are stochastic. As regards to the structure of the model, the main differences with respect to the previous version [4] lie in the income sector: now the different components of income distribution are determined endogenously and disaggregated by sector, and affect directly private disposable income.

Stochastic simulation has been performed using the program described in [1]. The generation of pseudo-random numbers with multivariate normal distribution has been performed using the Box-Muller technique [3] and McCarthy algorithm [9].

2. STANDARD ERRORS OF THE REDUCED FORM EQUATIONS: STOCHASTIC AND ANALYTIC SIMULATION

In case of nonlinear models, the standard errors of the reduced form equations can be computed by means of stochastic simulation. In fact, they can be computed as sample standard deviations across the replicated stochastic simulation results.

However, to get great numerical accuracy in the results, the number of replications should be very high and hence the computation becomes quite expensive.

It could be, therefore, convenient to abandon the methodological correctness and use an alternative method which can more quickly produce sufficiently accurate results. Such a method, which could be called analytic simulation, is based on a linearization of the model in the neighborhood of the solution point at each period, obtained by computing the partial derivatives of each endogenous variable with respect to each structural disturbance.

Table 2.1 displays the reduced form standard errors in one-step simulation at 1976/4 for some of the main variables of the model computed, respectively, using 100, 1000 and 2000 replications of stochastic simulation and analytic simulation.

Table 2.1

Variable	One-step simulation at 1976/4. Reduced form standard errors			
	100 Repl.	1000 Repl.	2000 Repl.	Analyt. Simul.
TIBAL	187.11	183.16	181.71	182.02
IPIND	421.33	415.95	417.07	416.53
PI70	5.42	5.37	5.35	5.43
RLGF*	288.78	312.33	308.17	301.70
PC70	1.74	1.73	1.73	1.75

YLINT*	421.42	358.43	372.17	376.27
MG*	282.97	264.80	259.04	265.96
YG*	292.21	326.09	322.46	325.02
BALGS*	408.72	465.68	454.14	466.12
CF70*	147.74	138.18	137.40	137.79
IMI70*	44.37	40.26	40.92	39.76
IC70*	40.91	39.73	39.92	40.00
XCS70*	129.75	147.38	145.59	147.01
MCS70*	118.16	114.00	111.76	114.83
GDP70*	262.89	275.55	270.38	274.55
YDF*	469.92	450.89	460.62	451.07

From Table 2.1 one could get the strong impression that the stochastic simulation results converge to those of analytic simulation as the number of replications goes to infinity. This is of course impossible, due to the nonlinearity of the model, but clearly gives an idea of the great accuracy of the analytic simulation method, which requires one control solution and as many disturbed solutions as the number of stochastic equations (50, for this model).

The same experiment has been repeated for a dynamic simulation run of three years. Table 2.2 displays the results at the last quarter (1976/4).

It must be pointed out that the dynamic simulation experiments have been performed under the assumption of independence of the structural disturbances in different periods, in other words absence of serial correlation of any order.

It is clear from Table 2.2 that the nonlinearities of the model have a stronger effect in dynamic simulation. For several variables, in fact, there is a clear divergence between the reduced form standard errors computed by means of analytic simulation and those correctly computed (even if approximate) by means of stochastic simulation.

Table 2.2

Dynamic simulation 1974/1 - 1976/4. Results at 1976/4.

Variable	Reduced form standard errors			
	50 Repl.	200 Repl.	1000 Repl.	Anal.Sim.
TIBAL	214.77	229.47	232.34	279.28
IPIND	467.22	550.50	541.98	540.04
PI70	8.80	8.64	8.89	6.09
RLGF*	467.74	409.16	393.74	390.61
PC70	4.75	4.57	4.53	3.92
YLINT*	403.60	491.32	482.76	411.55
MG*	478.65	511.22	502.42	348.77
YG*	313.07	347.61	364.75	207.99
BALGS*	586.66	619.67	635.10	405.26
CF70*	258.05	258.23	247.38	301.44
IMI70*	73.00	87.31	81.76	84.72
IC70*	56.10	64.29	59.15	77.05
XCS70*	220.96	242.39	243.22	210.97
MCS70*	181.14	205.48	192.65	188.89

GDP70*	420.71	448.72	433.02	469.46
YDF*	543.91	598.95	575.83	523.46

3. THE DETERMINISTIC SIMULATION EIAS

It is a well known statement that, in nonlinear models, the deterministic simulation values "can be expected to diverge systematically from the corresponding" historical values [7,p.309]. In fact the nonlinear transformations of the random disturbances, when passing from the structural form to the reduced form, do not maintain zero means, so that the conditional expectation (given coefficients and values of the predetermined variables) of the solution error (computed minus observed value of each endogenous variable) will be generally non-zero.

It must be pointed out that, under the assumption of "exact" normal distribution of the structural disturbances, this conditional expectation could become infinite; in this case it is meaningless to assume an exactly normal distribution. The adopted pseudo-random numbers generators in practice truncate tails at plus or minus seven times the standard deviations.

To check the existence of a bias for some variables, the number of replications of stochastic simulation must be generally very high. A convenient stopping rule could be the following: increase the number of replications until the difference between the deterministic and the mean stochastic solution for the examined variable is greater than the standard deviation, computed across the replications, of the mean stochastic solution (which decreases and goes to zero as the number of trials increases) or, even better, greater than double the standard deviation.

Table 3.1 displays the results of this experiment for some variables in the one-step simulation at 1976/4. They were obtained after 5000 replications of stochastic simulation. The table includes some of the main variables of the model and some of the variables for which the estimated bias seems to be the largest. It is clear that the bias, even though known to exist for some variables, is small enough to be of no practical interest.

Table 3.1

Variable	One-step simulation at 1976/4. 5000 Replications			
	Det.Sol.	Mean.Stoch.	Bias	St.Dev of Mean Stoch.
TIBAL	-2419.36	-2419.42	0.06	2.57
IPIND	15086.58	15080.98	5.60	5.93
PI70	247.10	247.00	0.10	0.08
RLGF*	14567.73	14563.34	4.38	4.37
PC70	214.60	214.60	0.01	0.02
YLINT*	22307.32	22307.20	0.12	5.24

MG*	9252.52	9248.02	4.50	3.79
YG*	9173.95	9176.51	-2.56	4.64
BALGS*	400.23	408.81	-8.58	6.67
CF70*	11212.23	11212.18	0.06	1.94
IMI70*	1378.88	1379.01	-0.13	0.57
IC70*	1699.73	1699.69	0.04	0.57
XCS70*	4368.20	4369.50	-1.31	2.10
MCS70*	3421.81	3422.80	-0.99	1.61
GDP70*	17887.42	17887.78	-0.35	3.89
YDF*	31992.52	31985.86	6.67	6.45
IDSSC	377.07	376.52	0.54	0.31
PMCS*	315.18	315.03	0.15	0.10
CASHF	4513.99	4516.19	-2.20	1.56
AMMC	3909.74	3911.82	-2.08	1.25
SI	484.88	497.49	-12.61	6.91
AMMP	3803.74	3805.82	-2.08	1.25

Table 3.2 displays the same information in the same quarter, but after 1000 dynamic simulation runs starting from 1974/1. The estimated bias could even be larger than before, but is still so small to be of no practical interest.

Table 3.2

Dynamic simulation 1974/1 - 1976/4. Results at 1976/4.

Variable	1000 Replications			
	Det.Sol.	Mean.Stoch.	Bias	St.Dev of Mean Stoch.
TIBAL	-2345.09	-2334.84	-10.25	7.35
IPIND	14448.14	14465.63	-17.49	17.14
PI70	256.74	256.57	0.16	0.28
RLGF*	14539.56	14567.85	-28.29	12.45
PC70	209.49	209.64	-0.15	0.14
YLINT*	21668.72	21661.62	7.10	15.27
MG*	8740.46	8746.65	-6.19	15.89
YG*	8699.50	8705.33	-5.83	11.53
BALGS*	428.58	426.05	2.53	20.08
CF70*	11250.11	11257.67	-7.56	7.82
IMI70*	1244.09	1247.59	-3.51	2.59
IC70*	1703.57	1705.11	-1.54	1.87
XCS70*	3962.37	3965.03	-2.66	7.69
MCS70*	3221.23	3225.28	-4.05	6.09
GDP70*	17589.10	17600.32	-11.22	13.69
YDF*	31448.11	31457.88	-9.77	18.21
IDF	2389.87	2384.73	5.13	3.80
IAPC	4116.56	4121.59	-5.03	3.36
PABNC	4396.22	4381.87	14.36	9.04
GDP*	38914.50	38953.43	-38.93	26.41
AIMI	30644.05	30667.37	-23.32	13.59
CFT	25415.66	25444.42	-28.76	16.04

4. HETEROSCHEDASTICITY OF THE REDUCED FORM

In the dynamic simulation of a dynamic econometric model [7], the reduced form errors are autocorrelated and heteroschedastic even if the structural disturbances are uncorrelated and homoschedastic.

Reduced form heteroschedasticity can also be generated by one-step simulation if the model is non-linear. This heteroschedasticity must be considered with great care when interpreting the simulation results, for example when considering the forecasting performances of a model. It must be recalled, in fact, that, when forecasting outside the sample period, the variance of the reduced form errors is a component of the variance of the forecast error [5].

Tables 4.1 and 4.2 display the computed values and the corresponding reduced form standard deviations for both one-step and dynamic simulation for a couple of variables. In the same tables we also present the coefficients of variation (ratio between the standard error and the mean value, if sufficiently far from zero, in percentage form).

On the right hand side of the table concerning the variable PYG70 we again display the reduced form standard errors computed, this time, using in the dynamic simulation the algorithm by McCarthy that takes account of serial correlation in the structural disturbances [9]. An even stronger effect of heteroschedasticity is present in this case.

Table 4.1
GDP70*

	One-Step			Dynamic		
	Comput. Value	Std. Dev.	Cf. Var.	Comput. Value	Std. Dev.	Cf. Var.
74/1	16452.7	303.	1.8	16452.7	303.	1.8
74/2	17017.0	276.	1.6	16652.4	338.	2.0
74/3	17422.8	262.	1.5	16797.1	333.	2.0
74/4	16734.2	299.	1.7	16449.3	378.	2.3
75/1	15927.1	263.	1.6	15821.5	378.	2.4
75/2	16346.0	281.	1.7	16260.4	385.	2.4
75/3	16635.6	247.	1.5	16595.6	388.	2.3
75/4	16876.4	257.	1.5	17000.5	385.	2.3
76/1	16677.9	263.	1.6	16948.3	399.	2.4
76/2	17135.1	285.	1.6	17052.8	412.	2.4
76/3	17285.3	266.	1.5	16971.1	417.	2.5
76/4	17887.4	263.	1.5	17589.1	442.	2.5

Table 4.2

PXG70

	One-Step			Dynamic			
	Comput. Value	Std. Dev.	Cf. Var.	Comput. Value	Std. Dev.	Cf. Var.	St.Dev. with Serial Corr.
74/1	166.8	2.85	1.7	166.8	2.85	1.7	2.85
74/2	171.4	2.94	1.7	181.4	4.36	2.4	3.99
74/3	185.4	3.23	1.7	192.5	5.93	3.1	5.38
74/4	199.1	3.57	1.8	203.2	7.44	3.6	6.68
75/1	202.6	3.63	1.8	206.3	8.38	4.0	7.15
75/2	202.8	3.48	1.7	207.4	8.96	4.3	8.05
75/3	204.4	3.92	1.9	213.5	9.58	4.5	9.41
75/4	213.3	3.66	1.7	220.5	9.59	4.3	10.2
76/1	224.2	4.22	1.9	236.8	10.5	4.4	11.2
76/2	244.7	4.35	1.8	267.8	12.6	4.7	12.9
76/3	252.0	4.56	1.8	273.3	12.7	4.6	13.7
76/4	271.2	5.04	1.9	288.8	13.9	4.8	15.8

5. REDUCED FORM VARIANCE DECOMPOSED BY SECTOR OF ORIGIN

The reduced form variance can be decomposed into the contributions induced by each stochastic equation.

In the following experiments, the decomposition is performed in relation to each sector of the model. For this purpose, instead of introducing the generated pseudo-random disturbances into all the stochastic equations simultaneously, they have been only introduced into the stochastic equations of each sector one at a time, while no disturbances have been inserted into the others [2].

The effect of one sector on the reduced form standard error of each variable depends on the historical unexplained variation of the sector (i.e. the structural variance) and the linkages between the sector and the variable under investigation.

The model has been divided into five sectors:

- a)- Demand (D)
- b)- Prices and Wages (PW)
- c)- Production, Employment and Income Distribution (PED)
- d)- Foreign Trade (FT)
- e)- Government (G)

In the Tables from 5.1 to 5.5 ((a)=one-step, (b)=dynamic, (c)=dynamic with autocorrelated disturbances), after 200 replications, the experimental results are displayed for the most important variables of each sector in the following way:

col. 1 - Endogenous variable name (+ sign before the name refers to structural stochastic equation).

col. 2 - Structural standard error for the structural stochastic equations that do not require normalization in the simulation phase.

col. 3 - Reduced form standard error computed with

pseudo-random disturbances introduced into all the stochastic equations of the model.

col. 4, 5, 6, 7, and 8 present the reduced form standard errors computed when shocking the stochastic equations of one sector only and in the same order as above:

- 4- Demand (D)
- 5- Prices and Wages (PW)
- 6- Production, Employment and Income Distribution (PED)
- 7- Foreign Trade (FT)
- 8- Government (G)

Under these last five values there are the reduced form variances induced by the corresponding sector, expressed as % of the total reduced form variance.

The results refer to the quarter 1976/4; in the case of dynamic simulation the starting simulation quarter was 1974/1.

DEMAND at 1976/4

Tab. 5.1.a. One-step simulation

	STR.FORM STD.ERR.	RED.FORM STD.ERR.	D	PW	PED	FT	G
GDP70*	-	274.55	104.94 14%	50.73 3%	50.98 3%	195.05 50%	15.39 0%
+CF70*	95.51	137.79	99.31 51%	43.04 9%	55.39 16%	4.48 0%	16.13 1%
+IMI70*	36.14	39.76	38.18 92%	2.37 0%	18.12 20%	16.48 17%	0.61 0%
IC70*	-	40.00	37.93 89%	9.47 5%	1.10 0%	4.42 1%	0.33 0%
XCS70*	-	147.01	2.45 0%	5.93 0%	12.01 0%	147.71 100%	0.87 0%
MCS70*	-	114.83	18.65 2%	3.54 0%	9.13 0%	119.58 108%	1.93 0%
YDF*	-	451.07	22.04 0%	5.37 0%	458.99 103%	29.59 0%	118.56 6%

Tab. 5.1.b. Dynamic simulation from 1974/1

GDP70*	-	442.00	110.43 6%	179.59 16%	107.54 5%	301.52 46%	35.37 0%
+CF70*	95.51	246.70	103.27 17%	159.45 41%	129.17 27%	37.88 2%	39.67 2%
+IMI70*	36.14	85.14	40.60 22%	22.32 6%	27.53 10%	61.31 51%	4.65 0%
IC70*	-	58.33	47.09 65%	19.90 11%	7.57 1%	18.00 9%	3.12 0%
XCS70*	-	253.48	10.90 0%	13.77 0%	43.24 2%	246.51 94%	2.72 0%
MCS70*	-	194.95	27.15 1%	23.06 1%	38.09 3%	197.38 102%	10.72 0%
YDF*	-	581.66	34.82 0%	196.14 11%	529.69 82%	76.63 1%	132.87 5%

Tab. 5.1.c. Dynam. simul. from 1974/1, autocorr. disturb.

GDP70*	-	523.33	112.78 4%	301.75 33%	156.64 8%	215.29 16%	57.09 1%
+CF70*	95.51	426.97	103.74 5%	271.26 40%	194.90 20%	24.71 0%	64.45 2%
+IMI70*	36.14	82.21	37.97 21%	33.18 16%	40.74 24%	47.57 33%	7.65 0%
IC70*	-	75.59	49.66 43%	26.18 11%	11.15 2%	12.77 2%	4.81 0%
XCS70*	-	254.41	13.54 0%	9.19 0%	63.11 6%	231.95 83%	3.08 0%
MCS70*	-	196.88	27.69 1%	30.74 2%	60.78 9%	220.64 125%	17.49 0%
YDF*	-	578.15	29.73 0%	245.91 18%	566.63 96%	87.16 2%	124.02 4%

PRICES AND WAGES at 1976/4

Tab. 5.2.a One-step simulation

	STR.FORM STD.ERR.	RED.FORM STD.ERR.	D	PW	PED	FT	G
PGDP*	-	2.06	0.14 0%	1.68 66%	0.06 0%	1.26 37%	0.21 1%
+PI70	-	5.43	0.02 0%	4.40 65%	0.22 0%	2.81 26%	0.0 0%
PC70	-	1.75	0.08 0%	1.79 104%	0.05 0%	0.08 0%	0.27 2%
WA	-	0.0	0.0 0%	0.0 0%	0.0 0%	0.0 0%	0.0 0%
ULC	-	1.55	0.12 0%	0.06 0%	1.35 75%	0.50 10%	0.02 0%

Tab. 5.2.b Dynamic simulation from 1974/1

PGDP*	-	4.70	0.18 0%	3.62 59%	0.59 1%	2.66 32%	0.57 1%
+PI70	-	9.01	0.11 0%	8.10 80%	1.08 1%	3.10 11%	0.10 0%
PC70	-	4.18	0.12 0%	4.01 92%	0.34 0%	0.74 3%	0.62 2%
WA	-	33.55	1.67 0%	32.36 93%	2.55 0%	5.36 2%	7.04 4%
ULC	-	2.65	0.19 0%	1.39 27%	1.86 49%	1.06 16%	0.24 0%

Tab. 5.2.c Dynam. simul. from 1974/1, autocorr. disturb.

PGDP*	-	5.71	0.21 0%	5.58 95%	0.74 1%	2.65 21%	0.84 2%
+PI70	-	7.53	0.09 0%	6.55 75%	1.26 2%	2.75 13%	0.16 0%
PC70	-	6.27	0.14 0%	6.14 95%	0.43 0%	0.69 1%	0.90 2%
WA	-	41.83	1.32 0%	39.66 89%	3.18 0%	5.36 1%	8.31 3%
ULC	-	2.86	0.17 0%	1.47 26%	1.81 40%	0.85 8%	0.29 1%

PROD. EMPL. AND INCOME DISTRIB. at 1976/4

Tab. 5.3.a One-step simulation

	STR.FORM STD.ERR.	RED.FORM STD.ERR.	D	PW	PED	FT	G
+VIM70*	54.50	165.01	36.88 4%	19.87 1%	63.79 14%	127.14 59%	5.00 0%
+VC70*	13.25	25.80	18.17 49%	4.45 2%	13.25 26%	2.93 1%	0.19 0%
VT70*	-	129.58	62.05 22%	30.69 5%	54.88 17%	78.64 36%	17.91 1%
LDNA	-	87.27	5.53 0%	1.42 0%	86.78 98%	4.35 0%	0.44 0%
YLINT*	-	376.27	21.91 0%	9.32 0%	357.73 90%	25.96 0%	5.15 0%
+RLGF*	296.90	301.70	22.84 0%	10.81 0%	285.86 89%	65.26 4%	2.66 0%
+IPIND	284.50	416.53	58.08 1%	29.80 0%	288.38 47%	234.64 31%	8.68 0%
UTLP	-	2.42	0.36 2%	0.18 0%	1.58 42%	1.44 35%	0.05 0%

Tab. 5.3.b Dynamic simulation from 1974/1

+VIM70*	54.50	252.61	37.60 2%	62.51 6%	99.23 15%	198.63 61%	10.68 0%
+VC70*	13.25	34.23	22.64 43%	10.30 9%	12.97 14%	11.58 11%	1.65 0%
VT70*	-	206.26	64.33 9%	107.04 26%	78.88 14%	120.60 34%	27.90 1%
LDNA	-	165.24	15.05 0%	21.49 1%	160.89 94%	32.39 3%	5.23 0%
YLINT*	-	457.47	33.68 0%	160.02 12%	403.12 77%	63.23 1%	39.99 0%
+RLGF*	296.90	421.10	36.75 0%	88.50 4%	366.61 75%	115.41 7%	19.67 0%
+IPIND	284.50	541.46	74.00 1%	105.01 3%	280.51 26%	421.84 60%	22.34 0%
UTLP	-	2.99	0.46 2%	0.56 3%	1.53 26%	2.34 61%	0.12 0%

Tab. 5.3.c Dynam. simul. from 1974/1, autocorr. disturb.

+VIM70*	54.50	231.75	39.19 2%	104.31 20%	100.13 18%	149.25 41%	17.04 0%
+VC70*	13.25	38.75	23.73 37%	14.06 13%	14.31 13%	7.86 4%	2.65 0%
VT70*	-	300.04	64.91 4%	180.84 36%	101.05 11%	103.01 11%	36.73 1%
LDNA	-	196.57	17.44 0%	36.11 3%	197.74 101%	22.62 1%	7.92 0%
YLINT*	-	423.10	31.95 0%	177.82 17%	378.60 80%	47.75 1%	44.58 1%
+RLGF*	296.90	474.68	37.76 0%	97.80 4%	450.45 90%	97.07 4%	22.28 0%
+IPIND	284.50	476.50	68.67 2%	178.81 14%	309.56 42%	376.20 62%	37.04 0%
UTLP	-	2.72	0.46 2%	0.93 11%	1.72 39%	2.20 65%	0.19 0%

FOREIGN TRADE SECTOR at 1976/4

Tab. 5.4.a One-step simulation

	STR.FORM STD.ERR.	PED.FORM STD.ERR.	D	PW	PED	FT	G
+XG*	-	325.02	5.88 0%	3.02 0%	26.06 0%	328.84 102%	0.88 0%
+MG*	-	265.96	50.12 3%	8.79 0%	24.59 0%	279.17 110%	5.78 0%
BALGS*	-	466.12	65.14 1%	11.43 0%	55.21 1%	477.33 104%	7.04 0%
+PYG70	-	4.78	0.0 0%	0.0 0%	0.0 0%	4.78 100%	0.0 0%
+PXCS*	1.45	3.69	0.02 0%	0.41 1%	0.01 0%	3.68 99%	0.06 0%
+PMCS*	-	6.80	0.0 0%	0.0 0%	0.0 0%	6.80 100%	0.0 0%

Tab. 5.4.b Dynamic simulation from 1974/1

+XG*	-	369.40	19.08 0%	35.52 0%	73.91 4%	362.98 96%	9.86 0%
+MG*	-	501.99	72.80 2%	68.20 1%	102.88 4%	510.12 103%	30.28 0%
BALGS*	-	603.34	102.24 2%	88.40 2%	171.24 8%	622.76 106%	41.45 0%
+PYG70	-	13.92	0.28 0%	0.87 0%	2.25 2%	13.49 93%	0.18 0%
+PXCS*	1.45	10.08	0.20 0%	1.35 1%	1.62 2%	9.61 90%	0.22 0%
+PMCS*	-	6.70	0.0 0%	0.0 0%	0.0 0%	6.70 100%	0.0 0%

Tab. 5.4.c Dynam. simul. from 1974/1, autocorr. disturb.

+XG*	-	321.69	23.72 0%	57.91 3%	105.66 10%	285.80 78%	14.73 0%
+MG*	-	516.67	74.44 2%	106.06 4%	163.82 10%	578.66 125%	49.98 0%
BALGS*	-	580.90	108.40 3%	127.35 4%	280.18 23%	635.81 119%	66.86 1%
+PYG70	-	15.78	0.31 0%	0.93 0%	2.78 3%	14.65 86%	0.22 0%
+PXCS*	1.45	11.03	0.21 0%	1.97 3%	1.99 3%	10.27 86%	0.31 0%
+PMCS*	-	7.17	0.0 0%	0.0 0%	0.0 0%	7.17 100%	0.0 0%

GOVERNMENT SECTOR at 1976/4

Tab. 5.5.a One-step simulation

	STR.FORM STD.ERR.	RED.FORM STD.ERR.	D	PW	PED	FT	G
IDP*	-	97.92	0.0 0%	0.0 0%	0.0 0%	0.0 0%	97.92 100%
IIP*	-	101.94	5.04 0%	0.88 0%	2.47 0%	28.06 7%	100.29 96%
SIC*	-	94.12	0.0 0%	0.0 0%	0.0 0%	0.0 0%	94.12 100%
TIBAL	-	182.02	5.55 0%	41.59 5%	2.80 0%	28.53 2%	167.20 84%
+INT	43.22	45.01	0.25 0%	1.59 0%	0.13 0%	1.32 0%	44.86 99%

Tab. 5.5.b Dynamic simulation from 1974/1

IDP*	-	97.40	8.04 0%	5.15 0%	11.61 1%	27.58 8%	96.89 98%
IIP*	-	111.67	18.09 2%	17.08 2%	28.90 6%	52.35 21%	96.13 74%
SIC*	-	139.24	7.76 0%	36.29 6%	94.92 46%	16.01 1%	86.53 38%
TIBAL	-	228.64	30.65 1%	49.35 4%	132.71 33%	67.72 8%	162.90 50%
+INT	43.22	66.89	9.47 2%	7.99 1%	34.31 26%	14.77 4%	51.34 58%

Tab. 5.5.c Dynam. simul. from 1974/1, autocorr. disturb.

IDP*	-	101.33	8.30 0%	8.32 0%	15.16 2%	33.96 11%	92.33 83%
IIP*	-	111.29	19.82 3%	11.89 1%	44.20 15%	60.18 29%	88.67 63%
SIC*	-	115.50	9.02 0%	36.38 9%	81.19 49%	13.45 1%	87.12 56%
TIBAL	-	295.12	32.56 1%	52.35 3%	145.26 24%	66.96 5%	163.06 30%
+INT	43.22	128.32	11.44 0%	14.48 1%	51.71 16%	12.59 0%	67.90 27%

6. STOCHASTIC SIMULATION AND SPECIFICATION ANALYSIS

In nonlinear dynamic models the use of simulation experiments is the easiest way to obtaining information about 1) the size of multipliers of exogenous variables 2) the performance of each endogenous variables 3) the existence of correct transmission effects among equations.

We can assume that the use of stochastic simulation improves substantially our knowledge on the second and the third point. The results may be particularly interesting if utilized, as in our case, in the phase of model building.

Single variable analysis cannot be but descriptive: the comparison between the standard error of the reduced form of the model and the standard error of the estimated equations throws some light on the effects of simultaneity (see tables in section (5.)), while the behavior of reduced form standard errors through

time gives some information on the degree of heteroschedasticity.

With regards to the latter problem, in Table 6.1 the ratios between the standard error and the mean value of the stochastic simulation results for the more representative variables of each sector of the model are presented.

Table 6.1
Ratio between standard error and mean value
Pearson's coefficient of variation
(Dynamic simulation 1974/1 - 1976/4)

	CF70*	IMI70*	XCS70*	MCS70*	YDF*	PC70	IPIND
1974/4	2.15	4.32	4.06	5.42	2.71	1.00	3.43
1975/4	2.14	5.80	5.90	6.01	1.98	1.67	3.82
1976/4	2.20	6.84	6.39	6.07	1.85	2.00	3.75

In connection with a transmission effects analysis, from the first simulations performed on the new version of a model it is easy to find that the results obtained are not completely satisfactory. If we want to investigate the contribution to the error given by each sector in the case of deterministic simulation, a simple way is to compare the results of different simulations obtained exogenizing one sector at a time. The main problem with such a procedure is that the results obtained are not directly comparable since they refer to different reduced forms of the same model. This might explain the inconsistencies we find when carrying out such experiments.

Examples of such inconsistencies are given in Table 6.2, where we present the results of five different simulation experiments. Whereas the simulation errors obtained for the government sector appear to be significantly large, the exogenization of this sector does not reduce the simulation errors on GDP70*.

Table 6.2
Deterministic simulations 1974/1 - 1976/4
Percentage errors calculated on average yearly
values for GDP70*

	Total model	D	PW	PED	FT	G
1974	-1.64	-0.46	-0.71	-1.21	-1.20	-0.64
1975	0.91	0.22	2.76	-1.34	0.57	2.77
1976	-0.28	0.28	1.60	-0.77	-0.73	1.55

Actually, with stochastic simulation it is possible to obtain information on the same problem in a more consistent way. Let us examine some of the possible interpretations of the more significant results obtained from the experiments performed in section (5.).

The price-wages sector has a small impact effect on the rest of the model. The effect however increases through time, and at the end of the simulation period it becomes important on GDP70*.

owing to the fact that prices and wages gradually affect income distribution and then consumption and investment (see Tables 5.1.b, 5.3.b). Moreover, looking at the influence on this sector of the rest of the model (Table 5.2.b) two facts emerge. First, the main contribution to standard errors stems from the shocks given to the sector itself, reflecting the working of the indexation mechanism which is present in the determination of wages. Second, comparing the standard error decomposition for wholesale prices and unit labor cost respectively, we notice that the latter is affected by variables of the production-income distribution sector much more significantly than the former. This reflects the incomplete translation of labor costs onto prices which characterizes the price determination mechanism. Thus the results appear to be consistent with the hypotheses underlying the specification of the model.

Unlike the price-wage sector, the foreign trade sector has an impact on the rest of the model much larger than the short-run linkages would imply. This may be attributed to the misspecification of the foreign trade equations, as is indicated by the size of their unexplained variance.

Moreover if we look at the way in which such errors are distributed through time on some variables, for instance on domestic demand components (private consumption and investment in Tables 5.1.a and 5.1.b), the effects we notice are difficult to interpret and seem to indicate the existence of misspecification problems in the linkages between domestic demand and foreign trade. In other words the differential impact of foreign trade on consumption and investment does not seem to reflect the hypotheses we had in mind when specifying the model.

The picture looks different for the government sector. Here a strong unexplained variance (single equation misspecification) is accompanied by a correct transmission mechanism to the other sectors. As a matter of fact, if we look carefully at the results presented in the tables, we observe that the relative size of the contribution given by the government sector to the standard error of the key variables is in line with our a priori ideas.

The few remarks made so far show that from this kind of results on stochastic simulation it is possible to get useful insights into the model, and they seem to encourage further investigation in this direction.

7. SPECTRAL ANALYSIS OF STOCHASTIC SIMULATION RESULTS

In order to perform the spectral analysis, a stochastic simulation of the model was carried out 120 periods into the future from 1976/4, keeping the exogenous variables equal to the mean of the last four observed values. The reason for doing this was to ensure second order (variance) stationarity, that is to eliminate the problem of heteroschedasticity induced by trending exogenous variables in the nonlinear model used.

It is worth mentioning that by keeping the exogenous variables fixed at their last observed value, the model did not converge

after a few periods (this is also due to the presence of seasonal dummy variables). Further, keeping the exogenous variables in a given quarter fixed at the last observed value in that quarter had the effect of amplifying the seasonal component in the power spectra.

The first 20 simulated data were excluded in the analysis in order to remove the effects of transients.

In order to have first order (mean) stationarity, the detrending procedure used included the following steps: first, the dynamic deterministic solutions were subtracted from each stochastic simulation run and, second, since some trend was still found to be present, a third order polynomial trend was subtracted.

Power spectra were computed for the detrended and shortened (100 data) series using the Parzen window with truncation points $M=24$ and $M=40$.

The results reported in Tables 7.1 and 7.2 correspond to the means over 25 replications for the variable GDP70*; in fact there was very little difference between these means and those calculated over 5 and 10 replications thus suggesting high stability in the power spectrum estimates.

The above experiment was performed for the hypotheses of uncorrelated and autocorrelated errors. In both cases the errors were generated by the McCarthy's technique [9].

In the case of uncorrelated errors, the following observations may be made:

- The spectra of the variables in the demand and production sectors have peaks at the frequencies corresponding to the 6 or 10 year cycle and the annual (4 quarters) frequency; for example GDP70* has cycles of 6 years and one year, whereas VC70* has cycles of 10 years and one year.
- The variables in the prices and wages sectors have peaks in the spectra only to the annual component.
- Most of the foreign trade variables have a 10 year cyclical component whereas a few contain instead a 6 year cycle.
- No general conclusion may be drawn for the variables in the public sector; some, like IIP*, have only an annual cycle, others, like TIBAI, have cycles of one year and 6 years whereas a few, like SIC*, have cycles of 6 years or of 10 years as in the case of INT.

When we consider autocorrelated errors, the peak corresponding to the annual cycle is very greatly reduced in most cases and in some (foreign and public sectors) are no longer present. Peaks at the 10 year frequency on the other hand are very much increased where they were already present, or have been introduced in those cases where previously no 10 year cycle existed (prices and wages sector).

Very few variables appear to have a 6 year cyclical component. GDP70* has a peak only at the 10 year frequency.

We may hence conclude that the model generates cycles which have period longer than the standard business cycles, and that the presence of the annual cyclical behaviour in the uncorrelated error case is due mainly to the model specification; some equations, in fact, include four-quarter differences.

Table 7.1
Stochastic simulation spectrum of GDP70* after 25 replications
Serially uncorrelated disturbances

Period	Power Spectr.	Std. Dev.	
0.00	1783.	246.	
48.00	2123.	241.	*****
24.00	2317.	192.	*****
16.00	1815.	123.	*****
12.00	1274.	86.	*****
9.60	1006.	63.	*****
8.00	849.	56.	*****
6.86	688.	41.	*****
6.00	567.	44.	*****
5.33	536.	57.	*****
4.80	711.	63.	*****
4.36	1252.	114.	*****
4.00	1562.	174.	*****
3.69	1082.	120.	*****
3.43	505.	37.	*****
3.20	354.	23.	*****
3.00	351.	22.	*****
2.82	328.	20.	*****
2.67	298.	17.	*****
2.53	284.	21.	*****
2.40	298.	24.	*****
2.29	310.	19.	*****
2.18	315.	18.	*****
2.09	326.	25.	*****
2.00	333.	32.	*****

Table 7.2
Stochastic simulation spectrum of GDP70* after 25 replications
Serially correlated disturbances

Period	Power Spectr.	Std. Dev.	
0.00	1509.	400.	
80.00	2677.	572.	*****
40.00	4135.	634.	*****
20.00	2920.	195.	*****
16.00	2405.	234.	*****
13.33	2236.	236.	*****
11.43	1822.	229.	*****
10.00	1045.	115.	*****
8.00	165.	18.	**
7.27	98.	11.	*
6.67	142.	17.	**
5.71	258.	33.	***
5.00	503.	91.	*****
4.71	630.	120.	*****
4.44	515.	76.	*****
4.21	411.	46.	*****

4.00	415.	55.	*****
3.64	259.	25.	***
3.48	226.	26.	**
3.33	199.	22.	**
3.08	89.	9.	*

8. SPECTRAL ANALYSIS USING ANALYTIC SIMULATION

The computation of the power spectrum of the variable GDP70* was performed also using the analytic simulation algorithm proposed by Howrey and Klein [8].

The first experiments, performed directly on the variable GDP70*, did not show anything except a strong trend component and the seasonal cycle (4 periods).

The computation was then repeated on the series of the first differences (the resulting spectrum is displayed in Table 8.1), dropping in this way the trend component, and on the series of the fourth differences, dropping in this way also the seasonal component (Table 8.2).

Analytic simulation was performed on 120 quarters outside the sample period. The cross spectrum matrix of the structural disturbances was computed under the hypothesis of absence of serial correlation (equal to the covariance matrix for each frequency).

The difference with respect to the spectrum computed by means of stochastic simulation is quite evident. Tables 8.1 and 8.2 do not suggest the existence of cycles except the seasonal cycle. The peak corresponding to a period of 12 quarters in Table 8.2 is, in fact, not well defined.

Therefore, from these experiments, it is impossible to derive any indication about the business cycle.

Table 8.1

Analytic simulation spectrum of $[GDP70*(t)-GDP70*(t-1)]$

Period	Power	
192.00	.946D-04	*
128.00	.333D-04	*
96.00	.277D-04	*
64.00	.522D-04	*
48.00	.474D-04	*
38.40	.429D-04	*
32.00	.717D-04	*
24.00	.957D-04	*
20.21	.112D-03	*
16.00	.132D-03	*
14.22	.140D-03	*
13.24	.152D-03	*
12.00	.173D-03	*
10.97	.191D-03	*
9.85	.189D-03	*
8.00	.239D-03	*

6.00	.229D-03	*
5.05	.351D-03	**
4.68	.447D-03	**
4.52	.478D-03	**
4.17	.124D-02	*****
4.04	.411D-02	*****
4.02	.109D-01	*****
4.00	.131D-01	*****
3.92	.819D-02	*****
3.88	.235D-02	*****
3.80	.569D-03	***
3.52	.334D-03	**
3.00	.455D-03	**
2.00	.643D-03	***

Table 8.2

Analytic simulation spectrum of $[GDP70*(t)-GDP70*(t-4)]$

Period	Power	
192.00	.157D-02	*****
128.00	.501D-03	*****
96.00	.554D-03	*****
76.80	.891D-03	*****
64.00	.710D-03	*****
54.86	.593D-03	*****
48.00	.753D-03	*****
32.00	.962D-03	*****
27.43	.120D-02	*****
24.00	.137D-02	*****
22.59	.143D-02	*****
20.21	.152D-02	*****
19.20	.156D-02	*****
17.45	.165D-02	*****
16.00	.175D-02	*****
14.22	.183D-02	*****
13.24	.185D-02	*****
12.00	.191D-02	*****
10.67	.189D-02	*****
8.53	.180D-02	*****
8.35	.174D-02	*****
8.17	.169D-02	*****
8.00	.161D-02	*****
7.53	.141D-02	*****
6.86	.111D-02	*****
6.30	.824D-03	*****
6.00	.696D-03	*****
5.65	.564D-03	*****
5.19	.408D-03	*****
4.80	.269D-03	*****
4.41	.110D-03	***
4.09	.191D-04	*
4.00	.781D-05	*

9. STOCHASTIC SIMULATION OF NONLINEAR MODELS WITH ADDITIVE RESIDUALS

From an operational point of view, when performing stochastic simulation, it is necessary to have:

- 1) the estimated residuals;
- 2) the correct specification of the Fortran code for each equation.

The pseudo-random disturbances, in fact, must be inserted into each stochastic equation in the proper specification. This can be automatically accomplished, without any modification of the code, only if the structural residuals are additive, otherwise it is necessary to know the estimated (from regression) residuals and the equation structure before and after normalization (if any).

Considering all the residuals as additive would be a strong tool towards stochastic simulation of the whole Link System. Unfortunately, due to the nonlinearities, the results so obtained would be incorrect. The question, now, is how incorrect they are and an approximate answer can be derived from this experiment performed on the Italian model.

Supposing the lack of the estimated regression residuals and of information about the normalized equations, the experiment has been performed first regenerating the "structural" residuals by means of residual check, as if they were additive for all the stochastic equations, then using these new residuals for the generation of the pseudo-random disturbances.

The reduced form standard deviations at 1976/4 in dynamic simulation (initial quarter 1974/1) are displayed in Table 9.1, together with the reduced form standard deviations computed using the regression residuals, as in section (2.). The table includes some of the main variables of the model and some of the variables for which the difference in the results obtained in the two different ways seems to be the largest.

The results seem to be not very encouraging, as they are too different in the two experiments.

Table 9.1

Dynamic simulation 1974/1 - 1976/4. Results at 1976/4.

Variable	Reduced form standard errors 200 Replications	
	Regression Residuals	Additive Residuals
TIBAL	229.47	319.00
IPIND	550.50	450.40
PI70	8.64	7.80
RLGF*	409.16	393.00
PC70	4.57	3.94
YLINT*	491.32	449.00
MG*	511.22	329.40
YG*	347.61	165.90

BALGS*	619.67	400.00
CF70*	258.23	237.70
IMI70*	87.31	67.44
IC70*	64.29	61.19
XCS70*	242.39	144.40
MCS70*	205.48	130.60
GDP70*	448.72	331.90
YDF*	598.95	547.00
IAPC	102.00	275.00
IAPVT	129.80	236.70

10. STOCHASTIC SIMULATION WITHIN THE LINK ENVIRONMENT

An experiment has been undertaken to take into account the possible effects of a stochastic simulation of the whole Link System on the Italian model. The simplified assumptions to perform this experiment are discussed in detail in [10].

Roughly speaking, the import prices of the four categories (more exactly their reciprocals) and the index of the total world trade were assumed to be affected by an additive random disturbance. These five disturbance terms were supposed to be independent from each other and from the structural disturbances of the model. To simplify the computations involved, the reduced form standard errors were assumed to be in the same ratio to the total values as the reduced form standard errors of the same variables were in the Wharton model of the U.S. economy. Furthermore, in dynamic simulation they were supposed to be heteroschedastic, with a time pattern computed by analogy with the corresponding variables of the Italian model. Tables 10.1 and 10.2 display, for 1976/4, respectively for one-step and dynamic simulation (initial quarter 1974/1), the reduced form standard deviations for some of the main variables.

Some theoretical problems arise in the stochastic simulation of the LINK system, regarding the mixing of different types of models. It is particularly notable that different models exogenize different variables. By exogenizing different variables different stochastic simulation properties will appear. In the context of this experiment it is particularly noticeable that there is different treatment of the oil export price in different models. The model of the developing regions uses an exogenous export price for oil, whereas the model of the U.S. economy has a behavioral equation for the oil export price with a rather large reduced form variance on the oil price. The large shocks on oil price used in this experiment are a result of assuming that the reduced form variance on oil prices for other countries are analogous to those of the U.S. In particular the reduced form variance on the reciprocal of the developing regions oil price (in dollars 1970=1.) was assumed to be 0.00089. If instead we used zero as the reduced form variance on OPEC oil export price the variance on Italy's oil import price should be much smaller.

The presence of zero reduced form variance on OPEC's oil

export price does not mean that there is no uncertainty associated with this price, since although exogenous it may be thought of as stochastic. It is clear that one must be careful in the interpretation of results of stochastic simulation of different models.

Table 10.1

Variable	One-step simulation at 1976/4. Reduced form standard errors 200 Replications	
	Only Italian Model	Within the LINK System
TIBAL	183.16	184.00
IPIND	415.95	450.10
PI70	5.37	9.63
RLGF*	312.33	315.74
PC70	1.73	1.75
YLINT*	358.43	412.20
MG*	264.80	458.45
YG*	326.09	337.40
BALGS*	465.68	624.30
CF70*	138.18	142.20
IMI70*	40.26	41.70
IC70*	39.73	39.90
XCS70*	147.38	153.50
MCS70*	114.00	126.09
GDP70*	275.55	288.39
YDF*	450.89	485.90
PIC*	3.93	5.50
PMCS*	6.70	11.64
SERP*	63.87	92.84
MGS*	304.00	543.66
DAZI70*	14.52	21.04

Table 10.2

Variable	Dynamic simulation 1974/1 - 1976/4. Results at 1976/4. Reduced form standard errors 200 Replications	
	Only Italian Model	Within the LINK System
TIBAL	229.47	253.41
IPIND	550.50	587.70
PI70	8.64	17.10
RLGF*	409.16	431.87
PC70	4.57	5.11
YLINT*	491.32	493.00
MG*	511.22	753.50
YG*	347.61	432.60
BALGS*	619.67	963.00

CF70*	258.23	263.88
IMI70*	87.31	89.01
IC70*	64.29	65.30
XCS70*	242.39	282.32
MCS70*	205.48	217.89
GDP70*	448.72	473.46
YDF*	598.95	621.46
PIMI*	3.32	5.12
PMCS*	6.43	17.33
BAL*	601.005	954.37
DAZI70*	22.28	32.00

Some conclusions can be drawn from this experiment.

- 1) In both, one-step and dynamic simulation, the Link effect on the reduced form standard errors seems to be rather small for most of the variables.
- 2) The Link effect is very strong on some price variables and on most of the variables of the foreign sector, for which the effect is direct.
- 3) The foreign trade sector is not yet complete in this model; it is expected to have, after completion, an even larger effect on the rest of the model.
- 4) Practically all the simplifications introduced to perform these last experiments, as discussed in [10] (independence of feedbacks, absence of serial correlation, shocks on the total world trade instead of directly Italian exports, etc.), lead to a reduction of the effects, in terms of variance; the complete Link effect, therefore, would theoretically be even larger.
- 5) The conclusion is encouraging towards continuation of experiments of this kind.
- 6) Further study in stochastic simulation of Link and, if possible, a complete stochastic simulation of the whole Link System would be desirable.

GLOSSARY

BALGS*	GOODS AND SERVICES BALANCE
CF70*	PRIV. CONSUMPTION EXPENDIT.
GDP70*	GDP AT MARKET PRICES
IC70*	INVESTM. IN CONSTRUCTION
IDP*	INCOME AND WEALTH TAXES (S.A.)
IIP*	INDIRECT TAXES (S.A.)
IMI70*	INVEST. : PLANT, MACHIN., TRANSP.
INT	INTERESTS ON PUBL. DEBT
IPIND	INDUSTRIAL PRODUCTION
LDNA	NON AGRIC. LABOR EMPL.
MCS70*	TOT. IMPORTS-COMM., SERVICES
MG*	VALUE TOT. MEFCH. IMPORTS FOB
PI70	WHOLESALE PR. INDEX NON AGRIC. PRODUCT
PMCS*	IMPL. PR. DEFLATOR FOR MCS
PXCS*	IMPL. PR. DEFLATOR FOR XCS
PXG70	UNIT VALUE INDEX COMM. EXP.

RLGF*	NON LABOR INCOME
SIC*	SOC. SECURITY CONTRIB. (S.A.)
TIBAL	BALANCE ON GOV. CURR. ACCOUNT
ULC	UNIT LABOR COST MANUF.
UTLP	RATE CAPAC. UTILIZ.-MANUF.
VC70*	GR. PROD. MARKET PR. CONSTRUCT.
VIM70*	GR. PROD. MARKET PRICES MANUF.
VT70*	GR. PROD. MARKET PR.: MARKET SERV.
WA	GR. HOURLY EARN. MANUF. (S.A.)
XCS70*	TOT. EXPORTS-COMM., SERVICES
XG*	VALUE TOT. MERCH. EXPORTS FOB
YDF*	PRIVATE DISPOSABLE INCOME
YLINT*	LABOR INCOME TOTAL

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