Intangibles Mismeasurement, Synergy, and Accounting Numbers: A Note

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Abstract:

For the last two decades, authors (e.g. Ohlson, 1995; Lev, 2000, 2001) have regularly pointed out the enforcement of limitations by traditional accounting frameworks on financial reporting informativeness. Consistent with this claim, it has been then argued that accounting finds one of its major limits in not allowing for direct recognition of synergy occurring amongst the firm intangible and tangible items (Casta, 1994; Casta & Lesage, 2001). Although the firm synergy phenomenon has been widely documented in the recent accounting literature (see for instance, Hand & Lev, 2004; Lev, 2001) research hitherto has failed to provide a clear approach to assess directly and account for such a henceforth fundamental corporate factor.

The objective of this paper is to raise and examine, but not address exhaustively, the specific issues induced by modelling the synergy occurring amongst the firm assets whilst pointing out the limits of traditional accounting valuation tools. Since financial accounting valuation methods are mostly based on the mathematical property of additivity, and consequently may occult the perspective of regarding the firm as an organized set of assets, we propose an alternative valuation approach based on non-additive measures issued from the Choquet’s (1953) and Sugeno’s (1997) framework. More precisely, we show how this integration technique with respect to a non-additive measure can be used to cope with either positive or negative synergy in a firm value-building process and then discuss its potential future implications for financial reporting.

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1. Introduction

For the last two decades, authors (e.g. Ohlson, 1995; Lev, 2000, 2001) have regularly pointed out the enforcement of limitations by traditional accounting frameworks on financial reporting informativeness. Consistent with this claim, one could argue that accounting finds one of its major limits in not allowing for direct recognition of synergy occurring amongst the firm intangible and tangible items (Casta, 1994). Although the firm synergy phenomenon has been widely documented in the recent accounting literature (see for instance, Hand & Lev, 2004; Lev, 2001) research hitherto has failed to provide a clear approach to assess directly and account for such a henceforth fundamental corporate factor.

Based on the mathematical property of additivity, the traditional financial methods used for firm valuation purpose raises therefore many concerns related to measurement, inexactness and uncertainty of accounting data. Because of its computational structure stemming from elementary arithmetic, the traditional accounting model does not appear anymore to be designed to deal with computing problems implied by firm assets’ synergy and more specifically with firm intangible assets valuation (Lev, 2000).

Indeed, while focusing on simple additive construction, the financial traditional methods leave behind the basic idea of firm assets’ synergy (or redundancy) directly related to the over-additivity (or under-additivity) principle that may be observed amongst the elements of an organised set such as firm assets (Casta & Bry, 1998). This synergy (respectively redundancy) effect may involve that the value of a set of assets be greater (respectively lower) than the sum of all the values of each independent asset. This hard avowal is commonly observed through the occurrence of goodwill in the traditional accounting framework. Subsequently, the strictly numerical fundamentals which underlie the classical accounting framework may encounter problems while attempting to cope with the inaccurate and/or uncertain nature surrounding the financial data environment as well as the ambiguity implied by fundamental valuation concepts. Epitomes of such issues documented by the literature are inexactness and subjectivity of accounting values and firm risk assessment (see March, 1987; Zebda, 1991; Bry and Casta, 1995; de Korvin et al., 1995; Casta and Lesage, 2001). As a result, the traditional accounting model may impede financial valuation process characterised by the key role assigned to human judgement, the use of qualitative information and the prominent function of subjective valuation (Scott, 1997).

Following these considerations, the objective of our paper is chiefly to raise and examine, but not address exhaustively, the specific issues induced by modelling the synergy occurring amongst the firm assets while pointing out the limits of traditional accounting valuation tools. After showing that financial accounting valuation methods are mostly based on the mathematical “additivity” property and consequently occult the perspective of regarding the firm as an organized set, we propose an alternative valuation approach based on non-additive measures issued from the Choquet’s (1953) and Sugeno’s (1977) framework. More precisely, we attempt to show how this integration technique with respect to a non additive measure can be used to handle either positive or negative synergy in a firm value-building process and discuss its potential future implications for financial reporting.

The remainder of the paper is organized as follows. The following section underlines for analysis purposes the limitations of the traditional financial reporting and accounting models. While section 3 examines the financial and accounting valuation problems involved by the occurrence of firm synergy, section 4 discusses alternative financial methods based on Choquet’s (1953) computation framework as a potential remedy to classical accounting treatments. Finally, section 5 brings over some concluding remarks for the paper and draws out some potential future research avenues.
2. Financial reporting and accounting model

The strictly numerical approach which underlies the classical accounting representation is not easily compatible with the imprecise and/or uncertain nature of the data or with the ambiguity of concepts such as imprecision and subjectivity of the accounting valuations, poorly defined accounting categories, subjective nature of any risk evaluation method (see: March, 1987; Zebda, 1991; Casta, 1994; Bry and Casta, 1985; de Korvin et al., 1995; Casta and Lesage, 2001).

Moreover, because its computational structure stems from elementary arithmetic, the traditional accounting model is not designed to handle features linked to synergy. For these problematic issues, we propose extensions of this model which deal with the synergy affecting the data used in the elaboration of financial statements. However, this approach requires a thorough re-examination of the semantics of the accounting measurement of value and income of the firm.

Our discussion concerns the operating rules governing quantification used in accounting. These rules are based on a rigorous conception of "numericity" which relates back to a given state of mathematical technology linked to the concept of measurement used.

2.1 Measure theory and accounting

Generally speaking, Measure Theory, in the mathematical sense, relates to the problem of mapping the structure of a space corresponding to observed elements onto a space allowing numerical representation; the set $\mathbb{R}$ of real numbers for example. The concept of measurement used in accounting has been influenced by two schools of thought:

- the classic approach — the so-called measure theory — directly inspired by the physical sciences according to which measurement is limited to the process of attributing numerical values, thereby allowing the representation of properties described by the laws of physics and presupposing the existence of an additivity property;
- the modern approach — the so-called measurement theory — which has its origin in social sciences and which extends the measure theory to the evaluation of sensorial perceptions as well as to the quantification of psychological properties (Stevens, 1951, 1959).

The quantitative approach to the measurement of value and income is present in all the classic authors for whom it is a basic postulate of accounting. The introduction by Mattessich (1964), Sterling (1970) and Ijiri (1967, 1975) to Stevens’ work provoked a wide-ranging debate on the modern theory of measurement but did not affect the dominant model (see Vickrey, 1970). Following criticisms of the traditional accounting model whose calculation procedures were considered to be simple algebraic transformations of measurements (Abdel-Magid, 1979), a certain amount of work was carried out, within an axiomatic framework, with a view to integrating the qualitative approach. However, the restrictive nature of the hypotheses (complete and perfect markets) (see Tippett, 1978; Willet, 1987) means that their approach cannot be generally applied.

Efforts to integrate the qualitative dimension into the theory of accounting did not come to fruition. From then on, the idea of measurement which underlies financial accounting remained purely quantitative.
2.2 Calculation and role of the double-entry bookkeeping

In a given historical and economic context, financial accounting is a construction which is based on a certain number of principles generally accepted by accounting practice and by theory. An understanding of economic reality through the accounting model representing a firm is largely conditioned by the choice of these principles. The formal principle of double-entry occupies a specific place. By prescribing, since the Middle Ages, the recording of each accounting transaction from a dual point of view, it laid down an initial formal constraint which affected both the recording and the processing of the data in the accounts. Later, with the emergence of the balance sheet concept, the influence of this principle was extended to the structuring of financial statements.

There is a unique algebraic structure of double-entry accounting. On a formal level, the underlying algebraic structure has been explained by Ellerman (1986). Going beyond Ijiri's classic analysis in integrating both the mechanism of the movement of accounts and the balance sheet equation, Ellerman identifies a group of differences: a group constructed on a commutative and cancelling monoid, that of positive real numbers endowed with addition. He calls this algebraic structure the Pacioli group. The Pacioli group $P(M)$ of a cancelling monoid $M$ is constructed through a particular equivalence relationship between ordered couples of elements of $M$.

2.3 The measurement of value in accounting: the balance sheet equation

The accounting model for the measurement of value and income is structured by the double-entry principle through what is known as the balance sheet equation. It gives this model a strong internal coherence, in particular with regard to the elaboration of financial statements. In fact the balance sheet equation expresses an identity in terms of assets and liabilities:

$$\text{Assets}(T) \equiv \text{NetEquities}(T) + \text{Debts}(T)$$

Since this is a description of a tautological nature of the company's value, this relationship is, by nature, verifiable at any time. As a result, this balance sheet equation involves that financial reporting is classically based on the additive property which tends to become unsuitable while firm synergy occurs amongst its assets as we shall see in the following section.
3. Synergy modelling and financial valuation

The determination of the value of a set of assets results from a subjective aggregation of viewpoints concerning characteristics which are objective in nature. As we have seen, the usual methods of financial valuation are based on additive measure concepts (as sums or integrals). They cannot, by definition, express the relationships of reinforcement or synergy which exist between the elements of an organised set such as assets. In particular, this can be obviously observed while intangible and tangible assets are associated efficiently. Subsequently, such an association can generate an abnormal value which is what accountants call goodwill.

3.1 Unsuitability of the classic measurement concept

First, methods of evaluating assets presuppose, for the sake of convenience, that the value \( V \) of a set of assets is equal to the sum of the values of its components, that is:
\[
V_i = \sum_{i=1}^{I} f(x_i)
\]

The additivity property, based on the hypothesis of the interchangeability of the monetary value of the different elements, seems intuitively justified. However, this method of calculation proves particularly irrelevant in the case of the structured and finalised set of assets which makes up a patrimony. Indeed, the optimal combination of assets (for example: brands, distribution networks, production capacities, etc.) is a question of know-how on the part of managers and appears as a major characteristic in the creation of intangible assets. This is why an element of a set may be of variable importance depending on the position it occupies in the structure; moreover, its interaction with the other elements may be at the origin of value creation such as:
\[
V = \sum_{i=1}^{I} f(x_i)
\]

Secondly, the determination of value is a subjective process which requires viewpoints on different objective characteristics to be incorporated. In order to model the behaviour of the decision-maker when faced to these multiple criteria, the properties of the aggregation operators must be made clear. Indeed, there exists a whole range of operators which reflect the way in which each of the elements can intervene in the aggregated result such as: average operators, weighted-average operators, symmetrical sums, t-norms and t-conorms, mean operators, ordered weighted averaging (OWA).

Depending on the desired semantics, the following properties may be required (Grabisch et al., 1995): continuity, increase (in the widest sense of the term) in relation to each argument, commutativity, associativity, and the possibility of weighing up the elements and of expressing the way the various points of view balance each other out, or complement each other. However, these operators of simple aggregation do not allow to fully express the modalities of the decision-maker's behaviour (tolerance, intolerance, preferential independence) or to model the interaction between criteria (dependence, redundancy, synergy) which is characteristic of the structuring effect.

As an example, let us consider a firm using three independent and identifiable assets (whether tangible or intangible), namely A, B and C. The underlying structure of the firm is depicted by exhibit 1:
In this setting, it is implicitly assumed that assets A, B and C do not exhibit any interrelationships with each other. As a consequence, this particular setting hypothesizes that the sum of each asset fair value is equal to the fair value of the sum of all assets.

Taking deliberately an opposite point of view, let us consider now a structured set X made of three assets (i.e. A, B and C). We can then graphically represent the set P(X) of the subsets of X in order to analyze the potential interrelationships (synergy, redundancy, complementarity and so on) which might occur amongst these three assets (see exhibit 2).

At each node of the lattice, it is then possible to take into account the asset interrelationships (i.e. dependency, redundancy, synergy) of two or three components, through the measure $\mu$. In order to alleviate the problem of financial information irrelevance and to control for the weaknesses of intangible financial reporting (see Lev and Zarowin, 1999), it is crucial to raise the computational process impact (Lev, 2000). As shown in exhibit 1 and 2, we assume that intangibles recognition issues come from the computational structure underlying traditional financial reporting and accounting framework. Translating this value-added
creation resulting from non-identifiable intangible assets use then appears to be necessary. In this respect, paraphrasing Lev (2000), we propose to use new maths for new economy. The approach proposed in this study is simply based on non-additive measures and more specifically non-additive integrals.

4. Non-additive measures and integrals

The concept of non-additive integrals is in direct continuity with non-additive measures and extends the integrals to measures which are not necessarily additive. It characterises integrals of real functions in relation to a given non-additive measure. (Denneberg, 1994).

4.1 The concept of non-additive measure

For a finite, non-empty set \( X \), composed of \( n \) elements, a non-additive measure is a mapping \( \mu \), defined over the set \( P(X) \) of the subsets of \( X \), with values in \((0,1]\), such that:

\[
\begin{align*}
(1) & \quad \mu(\emptyset) = 0 \\
(2) & \quad \mu(X) = 1 \\
(3) & \quad \forall A \subseteq B, \quad \mu(A) \leq \mu(B)
\end{align*}
\]

There is no additivity axiom. As a result, for two disconnected sets \( E \) and \( F \), a non-additive measure can, depending on the modeling requirement, behave in the following manner:

- additive: \( \mu(E \cup F) = \mu(E) + \mu(F) \)
- over-additive: \( \mu(E \cup F) \geq \mu(E) + \mu(F) \)
- under-additive: \( \mu(E \cup F) \leq \mu(E) + \mu(F) \)

The definition of a non-additive measure requires the measures of all subsets of \( X \) to be specified, that is to say \( 2^n \) coefficients to be calculated.

4.2 Reassessment of the integral concept and the “additivity” principle

The redefinition of the concept of measurement implies calling into question the definition of the integral in relation to a non-additive measure. Choquet integrals (Choquet, 1953), used as an operator of non-additive integration, enable us to model the synergy relation which often underlies financial valuation. We present the concepts of non-additive measure and Choquet integrals (see Appendix) and we then suggest various learning techniques which allow the implementation of a financial valuation model which includes the synergy relation (Casta and Bry, 1998; Casta and Lesage, 2001).

Choquet integral involves the sum and the usual product as operators. It reduces to Lebesgue integral when \( \mu \) is Lebesgue measure, and therefore extends it to possibly non additive measures. As a result of monotonicity, it is increasing with respect to the measure and to the integrand. Hence, Choquet integral can be used as an aggregation operator.
4.3 Principal applications of Choquet's integrals

Choquet’s integrals found an especially suitable field of application in economic theory to be made on subjects such as non-additive probabilities, expected utility without additivity (Schmeidler, 1989), and the paradoxes relating to behaviour in the presence of risk (Wakker, 1990). More recently, they have been used as aggregation operators for the modelling of multicriteria choice, particularly in the case of problems of subjective evaluation and classification (Grabisch and Nicolas, 1994; Grabisch et al., 1995). With regard to the latter applications, non-additive integrals exhibit the properties usually required from an aggregation operator whilst providing a very general framework for formalization.

The Choquet’s integral approach means that the defects of classical operators can be compensated for (Grabisch et al., 1995). Including most other operators as particular cases, Choquet’s framework can provide detailed modelling of such features as:

- The redundancy through the specification of the weights on the criteria, but also on the groups of criteria. Taking into account the structuring effect makes possible to take interaction and the interdependency of criteria into account; $\mu$ is under-additive when the elements are redundant or mutually inhibiting; $\mu$ is additive for the independent elements; $\mu$ is over-additive when expressing synergy and reinforcement.
- The compensatory effect: all degrees of compensation can be expressed by a continuous change from minimum to maximum.
- The semantic underlying the aggregation operators.

4.4 Non-additive measure learning method

Modelling through Choquet’s integral presupposes the construction of a measure which is relevant to the semantic of the problem. Since the measure is not a priori decomposable, it becomes necessary to define the value of $2^n$ coefficients $\mu(A)$ where $A \in P(X)$. Casta & Bry (1998) suggest an indirect econometric method for estimating the coefficients. Moreover, in cases where the structure of the interaction can be defined approximately, it is possible to reduce the combinatorial part of the problem by restricting the analysis of the synergy to the interior of the useful subsets.

4.5 Numerical illustration

Now, as an example of this technique, let us consider an entity using three identifiable assets (whether tangible or intangible), namely A, B and C whose fair value are respectively 1000, 2000 et 500. If the computational process is strictly speaking additive, the sum $V$ of all the assets’ fair values is necessarily equal to the value of the set (i.e. in the example, $V = 3,500$) (see Exhibit 3).
In an additive property setting, the implicit hypothesis \( \mu(A) + \mu(B) + \mu(C) = 1 \) is made. Thus, in a non-additive computational process, additional information on the measure \( \mu \) which allows modelling the different interrelationships type (dependency, redundancy, synergy and so on) is required.

Let us now suppose that there is a synergy occurring between assets A and B. As a result, the following inequality will be observed:

\[
\mu(A,B) \geq \mu(A) + \mu(B)
\]

with for instance, \( \mu(A,B) = 0.75 ; \mu(A) = 0.25 \text{ et } \mu(B) = 0.40 \).

Following this, it would be more relevant to sort the different assets A, B and C by a fair value ascending order as shown in exhibit 4 in order to apply to this set the Choquet’s integral system (see Appendix) in a discrete case setting.

Turning back to our example, the set value will be

\[
V^+ = 3500 + (1000-500) \times 0.75 + (2000 - 1000) \times 0.40 = 4275.
\]

**Exhibit 3. Classical representation of the sum of the assets’ fair values**

**Exhibit 4. Representation of the entity’s value-ascending ordered assets**
5. Conclusion

Consistent with authors' (e.g. Ohlson, 1995; Lev, 2000, 2001) previous claims stating that traditional accounting frameworks enforces on financial reporting major limitations to financial reporting informativeness, we argue that the lack of firm assets synergy recognition by the accounting model may be chiefly responsible of this weakness.

Indeed, based on the mathematical property of additivity, the traditional financial methods used for firm valuation purpose raises therefore many concerns related to measurement, inexactness and uncertainty of accounting data. Because of its computational structure stemming from elementary arithmetic, the traditional accounting model does not appear anymore to be designed to deal with computing problems implied by firm assets’ synergy and more specifically with firm intangible assets valuation (Lev, 2000). Although the firm synergy phenomenon has been widely documented in the recent accounting literature (see for instance, Hand & Lev, 2004; Lev, 2001) research hitherto has failed to provide a clear approach to assess directly and account for such a henceforth fundamental corporate factor.

After discussing the possibilities offered by Choquets' (1953) and Sugeno’s (1977) integration framework, we attempt to shed light on the potential application fields in financial accounting for this non-additive system. We argue that this framework enables the effects of micro-structure, synergy and redundancy, which are opaque in classical linear accounting models, to be further analyzed and incorporated into accounting valuation methods. Although we show that these techniques allow to limit the purely combinatory effects which appear at the learning stage of the methodology, we then underline that this sophistication appears to be more costly in terms of computational complexity.

Finally, through a simple example, we attempt to shed light on how this integration technique with respect to a non additive measure can be used to handle either positive or negative synergy in a firm value-building process and discuss its potential future implications for financial reporting.
6. Appendix: Some developments on Choquet’s integral

Choquet’s (1953) integral of a measurable function \( f: \mathcal{X} \rightarrow [0,1] \) relative to a measure \( \mu \) is defined as:

\[
C(f) = \int \mu \left( \{ x \mid f(x) > y \} \right) dy
\]

**Exhibit 5. Choquet integral representation**

\[
f(x)
\]

\[
y + dy
\]

\[
y
\]

\[
\{ x / f(x) > y \}
\]

\[
X
\]

For example, in the case of a finite set \( X = \{ x_1, x_2, \ldots, x_n \} \) with:

\[
0 \leq f(x_1) \leq \ldots \leq f(x_n) \leq 1
\]

and \( A_i = \{ x_i, \ldots, x_n \} \), we have:

\[
C(f) = \sum_{i=1}^{n} (f(x_i) - f(x_{i-1})) \mu(A_i)
\]

Moreover, \( 1(A=B) \) being the "indicator function" which takes value 1 if \( A=B \) and 0 otherwise, we can write:

\[
C(f) = \int \left( \sum_{A \in \mathcal{P}(X)} \mu(A) \cdot 1(A = \{ x \mid f(x) > y \}) \right) dy
\]

\[
C(f) = \sum_{A \in \mathcal{P}(X)} \mu(A) \cdot (\int 1(A = \{ x \mid f(x) > y \}) dy)
\]

If we denote \( g_A(f) \) as the value of the expression \( \int 1(A = \{ x \mid f(x) > y \}) dy \), Choquet’s integral may be expressed in the following manner:

\[
C(f) = \sum_{A \in \mathcal{P}(X)} \mu(A) \cdot g_A(f)
\]
7. References


