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Working Paper 2009-08
http://www.bus.lsu.edu/economics/papers/pap09_08.pdf

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Abstract

It is widely reported in the literature that it is not possible to test nonmarket good preference restrictions against revealed preference. While it is clearly impossible to affirm any particular preference restriction as being "true," it is possible to show that a preference restriction is not feasible. A revealed preference feasibility test for weak complementarity is presented here. With weak complementarity defined by the observable property of nonessentiality and the unobservable property of no-existence-value, the latter is the actual preference restriction. It is shown that no-existence-value is feasible if and only if an observable revealed preference condition of "single-preference" is satisfied. This strong revealed preference condition is nontrivial when there are two or more market goods in addition to the weak complement. With simple Samuelsonian revealed preference we can falsify single-preference and thereby reject weak complementarity. Stone-Geary numeric examples are included to demonstrate these and other results.

Keywords: Weak complementarity, Nonessentiality, No-existence-value, Testing preference restrictions, Feasibility condition, Single-preference, Samuelsonian revealed preference.

1 Introduction

A core methodological problem of nonmarket goods is that revealed preference from market demand does not by itself provide sufficient information for welfare analysis. There are several diverse methodological approaches for providing the necessary additional preference information. One approach involves imposing intuitively appealing preference assumptions, typically called "preference restrictions" or "maintained hypotheses." The principal example is weak complementarity as introduced by Mäler.[1] It is widely thought that preference restrictions in general, and weak complementarity in particular, cannot be tested against revealed preference. In this paper I provide a nontrivial observable feasibility condition for weak complementarity that permits such testing.

This model is based on the standard understanding of consumer behavior in the context of nonmarket goods. Let z represent a nonmarket good that as a variable can take on any values of the set Z. A nonmarket good may be discrete or continuous, and the values it takes on might be scalars, vectors, or even non-numerical attributes such as $Z = \{$Poor Fishery, Thriving Fishery$\}$.[2] Superscripts are used to distinguish individual elements of Z, as in $z^a, z^b \in Z$. Let $X = \Re^L_+$ be the commodity consumption set with typical element $x = (x_1, \ldots, x_L)$, and define $Y = X \times Z$ with typical element (x, z). The consumer is thought to have a complete and transitive preference relation on Y, designated by $\succsim_Y$, which is typically represented by a utility function U_Y so that $U_Y(x^a, z^a) \geq U_Y(x^b, z^b) \iff (x^a, z^a) \succsim_Y (x^b, z^b)$ for all possible pairs of (x, z) vectors $(x^a, z^a), (x^b, z^b) \in Y$.[3] For each fixed value of z, I assume that $\succsim_Y$ is continuous on X, and also strictly convex and strongly monotone on the interior of X but only require it to be simply convex and weakly monotone on the boundary.[4] With these assumptions the demand relation is single valued, i.e., a function.

[1]The weak complementarity concept was first fully developed in Mäler (1971) but the terminology was not introduced until later in Mäler (1974).

[2]Mäler's first example was fishery quality, although as a continuous variable.

[3]I use both preference notation and utility functions in this presentation, often in parallel. This redundancy has a purpose so that I ask the reader to bear with me. The work presented here is conceptually based on preference theory and is most directly conveyed with preference notation. Moreover, overreliance on utility function representation of preference can open the door to some erroneous conclusions, as we shall see. However most readers are probably much more familiar with utility function representation than with preference notation. I have therefore also included some utility function representation to help convey the development. Of course, utility functions are indispensable for fully defining specific preference relations, as in the my examples section, and also for some other purposes.

[4]For example with Cobb-Douglas defined on the entirety of X using the traditional exponential utility function, it is strictly convex and strongly monotone only on the interior of X. However, along the boundary is only simply convex and weakly monotone.

The consumer is able to choose some $x \in X$ subject to his budget constraint, but is not able to choose $z \in Z$. Instead, the nonmarket good is treated as a state variable, like prices or wealth, that define the choice context. With the standard model, the consumer optimizes his choice by solving the constrained optimization problem,

$$\begin{aligned} \max_{x} \quad & U_Y(x, z) \\ s.t. \quad & p \cdot x \leq w, \\ & x \in X, \end{aligned} \tag{1}$$

where $p \in \Re^L_{++}$ is the vector of market good prices and $w > 0$ is the individual's wealth.[5] The consumer's demand function is defined as the solution function to this problem. It can also be characterized in purely preference-theoretic terms,

$$\widehat{x}(p, z, w) = \{x \in X | \, p \cdot x \leq w, \text{ and } (x, z) \succsim_Y (\bar{x}, z) \text{ for all } \bar{x} \in X \text{ such that } p \cdot \bar{x} \leq w\}\,. \tag{2}$$

For each value of z the consumer chooses the preference maximizing affordable commodity bundle. Thus the nonmarket good parameterizes the choice problem and the demand function in the same way as prices and wealth.

We are able to observe the consumers economic behavior as represented by the demand function, but are not able to directly observe the preference relation $\succsim_Y$ or any representative utility function such as U_Y.

2 Available revealed preference information

Consumers reveal their commodity preference as they choose bundles of goods for various combinations of p, z and w, as indicated by program (1) and equation (2). Since they are not able to choose z values in the market, revealed preference is not available for differences in these values. Furthermore, commodity revealed preference is only available across those bundles that the consumer might actually obtain as indicated by the demand function. For each $z \in Z$, the obtainable set in X is $\widehat{X}_z = \{x \in X \,|\, x = \widehat{x}(p, z, w) \text{ for some } (p, w) \in \Re^{L+1}_{++}\}$.[6]

[5] Following Mas-Colell et al. (1995), I use wealth instead of income. The seminal Mäler (1971) uses "lump sum income" which is arguably more akin to wealth, a stock variable, than income, a flow variable.

[6] Here are three examples of obtainable sets: 1) With Cobb-Douglas or CES preference, the obtainable set is the strictly positive orthant $\Re^L_{++}$ so that at least of some of each commodity is always consumed. 2) With Leontief preference the obtainable set is a ray from the origin (not including the origin). 3) With quasilinear preference, the obtainable set will typically include some commodity vectors with no consumption of some goods, i.e., corner solutions. This distinction between X and the obtainable subset is also used by Richter (1971).

I assume that each $\widehat{X}_z$ is closed under weak monotone superiority. Then with $z \in Z$, $x^1 \in \widehat{X}_z$ and $x^2 \in X$ with $x^2_\ell \geq x^1_\ell$ for all $\ell = 1, \ldots, L$, we also have $x^2 \in \widehat{X}_z$.[7]

Let $\bar{z} \in Z$, $p^a \in \Re^L_{++}$, $w^a > 0$ and $x^a = \widehat{x}(p^a, \bar{z}, w^a)$. Then with the nonmarket good fixed at $\bar{z}$, simple Samuelsonian revealed preference allows us to say that x^a is revealed preferred to all $x^b \in X$ such that $x^b \neq x^a$ and $p^a \cdot x^b \leq w^a$. In particular we then know that $(x^a, \bar{z}) \succ_Y (x^b, \bar{z})$ and $U_Y(x^a, \bar{z}) > U_Y(x^b, \bar{z})$ for all such x^b. With this approach the maximal revealed preference information that we can possibly recover is complete knowledge of $\succsim_Y$ for each fixed value of the nonmarket good as restricted to the respective obtainable sets.[8] For each $z \in Z$, let $\succsim_z$ be the preference relation on $\widehat{X}_z$ such that for all $x^a, x^b \in \widehat{X}_z$, $x^a \succsim_z x^b \iff (x^a, z) \succsim_Y (x^b, z)$. From the properties of $\succsim_Y$ we know that each $\succsim_z$ is transitive, complete, continuous, strictly convex and strongly monotone. The set $\{\succsim_z \mid z \in Z\}$ represents all possible revealed preference information. Utility function representations of these z-specific preference relations can be useful. For all $z \in Z$, there must exist representative utility functions $u_z(x)$ such that $u_z(x^a) \geq u_z(x^b) \iff x^a \succsim_z x^b$ for all $x^a, x^b \in \widehat{X}_z$.[9]

We are now working with two levels of revealed preference information. The first is simple Samuelsonian revealed preference, such as with the $(x^a, \bar{z}) \succ_Y (x^b, \bar{z})$ example at the beginning of the preceding paragraph. At the second level, we have the individual $\succsim_z$ preference relations which extend simple revealed preference to a complete relation on each $\widehat{X}_z$.[10] The corresponding representative u_z utility functions might be obtained by solving the integrability problem for each $z \in Z$.[11] I am not assuming that all of each $\succsim_z$ preference relation is available to the analyst, such as in the form of their representative utility functions, but will later indicate some of their possible uses and misuses when they are available. Such

[7]This monotonic closure of $\widehat{X}_z$ may well follow from the continuity, convexity and monotonicity assumptions. All well known preference families with these properties have monotonically closed obtainable sets.

[8]This statement is a slight simplification. Any actual instance of simple Samuelsonian revealed preference involves a single point that is realized by the demand function and is therefore a member of the obtainable set. However the other points that are revealed to be inferior to the first point are not restricted to the obtainable set. For example, x^a as defined at the beginning of the paragraph is a member of $\widehat{X}_{\bar{z}}$, but the x^b points need not be.

[9]The existence of these utility functions follows from standard utility representation theory such as presented in Mas-Colell et al. (1995), section 3C, and does not imply that they are known to the analyst. The use of z as an index on $\succsim_z$ and $u_z(x)$ might suggest that Z is a countable set. However as I indicated previously, z may be a real valued continuous variable or even a real vector. In that context $\{\succsim_z\}$ would probably be obtained as a parametric continuum, so that $u_z(x)$ would be specified with a parametric representation defined on $z \in Z$.

[10]This distinction between the two levels of revealed preference is equivalent to the distinction that Varian (1988) makes in his second paragraph between "revealed preference theory" and "integrability theory."

[11]See Mas-Colell et al. (1995), section 3H.

an assumption would be equivalent to assuming that all of the z-specific Hicksian demand functions are available.[12]

There is another way of seeing that the set $\{\succsim_z \mid z \in Z\}$ includes all possible revealed preference information. By definition, Samuelsonian revealed preference information is only revealed through the demand function as with our earlier example. It follows that any preference information that is not needed for the construction of the demand function cannot be revealed from the demand function. Our demand function can be fully specified purely in terms of $\{\succsim_z \mid z \in Z\}$, without any of the additional preference information available in $\succsim_Y$,

$$\widehat{x}(p,z,w) = \{x \in \widehat{X}_z \,|\, p \cdot x \leq w, \text{ and } x \succsim_z \bar{x} \text{ for all } \bar{x} \in \widehat{X}_z \text{ such that } p \cdot \bar{x} \leq w\}.$$

Therefore none of the additional preference information available in $\succsim_Y$ can be recovered as revealed preference. The preference information content of the demand function and the set $\{\succsim_z \mid z \in Z\}$ are exactly the same.

This can also be seen with the representative utility functions $\{u_z \,|\, z \in Z\}$. For each $z \in Z$, let $x_z(p,w)$ be the solution function to the constrained optimization problem,

$$\begin{aligned} \max_x \quad & u_z(x) \\ s.t. \quad & p \cdot x \leq w, \\ & x \in \widehat{X}_z. \end{aligned}$$

Then for all $p \in \Re^L_{++}$, $z \in Z$ and $w > 0$, we have $\widehat{x}(p,z,w) = x_z(p,w)$. Thus the ordinal preference information of all the u_z representative utility functions is sufficient for constructing the $\widehat{x}$ demand function, and none of the additional information available in U_Y can be recovered from the demand function.

It is well understood that utility function representations are not unique so that we can apply monotonic transformations to the u_z functions and obtain different but equally valid representative utility functions. These monotonic transformations can vary with z.

[12] With Marshallian demand and the u_z utility functions we could easily construct the z-specific indirect utility functions, and from them obtain z-specific expenditure functions and finally the z-specific Hicksian demand functions. Even ordinary exact welfare values, such as for changes in commodity prices, cannot be obtained directly from Marshallian demand. Either Hicksian demand is required, or its equivalent such as with expenditure and indirect utility functions. Presupposing the availability of Hicksian demand is a substantial assumption. Supposing the accessibility of the u_z utility functions is an equivalent assumption. As this paper is not focused on specifying welfare measures, neither assumption is necessary. Furthermore, forgoing the latter assumption about the u_z functions underscores the development later in the paper showing that weak complementarity can be directly falsified with Marshallian demand through simple revealed preference.

For each $z \in Z$, let $g_z : \Re \rightarrow \Re$ be a strictly increasing function and define the utility function $u_z^g : \widehat{X}_z \rightarrow \Re$ by $u_z^g(x) = g_z(u_z(x))$. Then the set of utility functions $\{u_z^g \,|\, z \in Z\}$ includes exactly the same ordinal preference information as the original set so that there is no justification for choosing one over the other based on revealed preference.

3 Indeterminate welfare analysis and testing preference restrictions

Suppose that we wish to measure the change in welfare for a shift in the nonmarket good value from z^a to z^b with price and wealth respectively fixed at $\bar{p}$ and $\bar{w}$. For example, with the compensating variation welfare measure we are seeking the CV value such that,[13]

$$\left(\widehat{x}\left(\bar{p}, z^a, \bar{w}\right), z^a\right) \quad \sim_Y \quad \left(\widehat{x}\left(\bar{p}, z^b, \bar{w} - CV\right), z^b\right),$$

or in terms of the unknown representative utility function,

$$U_Y\left(\widehat{x}\left(\bar{p}, z^a, \bar{w}\right), z^a\right) \;=\; U_Y\left(\widehat{x}\left(\bar{p}, z^b, \bar{w} - CV\right), z^b\right). \tag{3}$$

This requires preference information across different z values. That is, for at least some pairs of elements from the preference domain, $(x^a, z^a), (x^b, z^b) \in Y$ with $z^a \neq z^b$, we need to know whether or not $(x^a, z^a) \succsim_Y (x^b, z^b)$. However, as we have seen this information is not available from revealed preference.

If the analyst has available a set of utility functions such as $\{u_z \,|\, z \in Z\}$ that represents the set of z-fixed preference relations $\{\succsim_z \,|\, z \in Z\}$, there may be some temptation to construct from these a utility function on Y, $U_0(x, z) = u_z(x)$ for $(x, z) \in Y$ and $x \in \widehat{X}_z$, and use this for welfare analysis involving changes in the nonmarket good.[14] However, there is no basis for concluding that $U_0(x, z)$ represents the consumer's original unobserved preference relation $\succsim_Y$.

Consider for example an alternative set of representative utility functions such as $\{u_z^g \,|\, z \in Z\}$ obtained from the original set with a set of z-specific transforms $\{g_z(u) \,|\, z \in Z\}$, as

[13] See Mas-Colell et al. (1995), section 3I, for the standard definition of this measure, i.e., without nonmarket goods. Bockstael and Kling (1988), Larson (1991), Bockstael and McConnell (1993), Herriges et al. (2004), Smith and Banzhaf (2004), and Bullock and Minot (2006) all use compensating variation measures for a change in the nonmarket good value that are equivalent to the one used here, while Ebert (1998) uses an equivalent variation ("EV") measure.

[14] Technically U_0 is not necessarily defined on the entirety of Y but rather on the obtainable subset $\widehat{Y} = \{(x, z) \in Y \,|\, x \in \widehat{X}_z \text{ for } z \in Z\}$. However this is sufficient for any welfare question involving changes in p, z and w.

discussed at the end of the preceding section. We can construct another utility function on Y with these, $U_g(x,z) = u_z^g(x)$ for all $(x,z) \in Y$ with $x \in \widehat{X}_z$. If any two of the g_z functions are not identical, $g_{z^a} \neq g_{z^b}$ for some $z^a, z^b \in Z$, then for any simple monotonic transformation $f : \Re \rightarrow \Re$ with $f'(u) > 0$ we would have $f(U(x,z)) \neq U_g(x,z)$ for at least some $(x,z) \in Y$, so that U_0 and U_g represent different preference relations on Y. With two sets of utility functions such as $\{u_z \,|\, z \in Z\}$ and $\{u_z^g \,|\, z \in Z\}$, since there is no reason for choosing one over the other based on revealed preference, we also have no basis for choosing between U_0 and U_g, and hence no basis for choosing either one as a representation of $\succsim_Y$.

With all the possible sets of g-transforms we have a vast variety of different feasible preference relations on Y that will yield wildly divergent welfare measure values.[15] For example, Herriges et al. (2004) include an empirical example with two substantially positive CV estimates where one is more than three times the size of the other. However the problem is more serious than this example indicates. Some preference relations on Y represented by U^g-type utility functions will yield very positive CV values and others will yield very negative values. Consequently it is not possible to know from revealed preference alone even whether the change from z^a to z^b is beneficial or detrimental.[16]

There are several diverse methodological approaches for providing sufficient additional preference information so that we can obtain a well-defined unique welfare measure value, such as for compensating variation. These include various combinations of survey methods, indirect measures and "reasonable" assumptions.[17] Implicity the goal of each of these methodologies is to isolate one of the many feasible preference relations on Y, and to identify it as being the most reasonable based on the rationale of the given methodology. One class of methodologies involves simply imposing one or more intuitively appealing preference assumptions. These are typically called "preference restrictions" or "maintained hypotheses." The assumption or set of assumptions is usually sufficiently strong so that the analyst is able to isolate a unique feasible preference relation on Y without additional nonmarket data such as survey information. Weak complementarity is the most prevalent preference restriction in the literature. It will be examined in the next section.

A preference relation on Y that is identified with a given preference restriction is equally consistent with any revealed preference information as all the other diverse feasible preference relations on Y. Therefore, there can be no test using revealed preference for verifying that the

[15] Feasibility requires consistency with all possible revealed preference, $\{\succsim_z \,|\, z \in Z\}$. Ebert (1998, 2001), Herriges et al. (2004) and von Haefen (2007) also provide clear statements of the diversity of feasible preference relations characterized in terms of utility function transformations like my set of g_z transforms.

[16] This is illustrated in my section of numeric examples.

[17] See Champ et al. (2003) for a practical survey of these techniques.

identified preference relation is the "true" relation $\succsim_Y$. This is well known in the literature. Unfortunately this relatively narrow result is usually asserted only implicity in the context of a much more general statement such that the preference restriction cannot be tested at all against revealed preference.[18]

These general statements are fallacious. The inability to affirm that $\succsim_Y$ satisfies a particular preference restriction does not imply an inability to deny it. Thus we must distinguish between two types of tests. The test discussed in the preceding paragraph seeks to affirm that the "true" relation $\succsim_Y$ satisfies the preference restriction, given that there exists a feasible preference relation on Y that is consistent with the given preference restriction. We have seen that this type of test is not possible. The general statements of testing impossibility implicity assume the existence of such a feasible and consistent preference relation. However this implicit assumption need not be true. It is possible that none of the feasible preference relations on Y are consistent with the given preference restriction, and this can be tested. The set of feasible preference relations on Y is only limited by the set of possible revealed preference, $\{\succsim_z \mid z \in Z\}$. If all of the feasible preference relations on Y are inconsistent with the given preference restriction, then that preference restriction must be inconsistent with revealed preference in the form of $\{\succsim_z \mid z \in Z\}$ and is itself not feasible. It follows that we can sometimes test the preference restriction against revealed preference and conclude that the restriction is not feasible.

Thus while it is not possible to affirm preference restrictions with revealed preference, it is possible to deny preference restrictions with revealed preference. I demonstrate this by showing that the feasibility of weak complementarity requires a strong condition in revealed preference.

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[18]For example Ebert (2001, p. 374) states that "one is unable to reject" preference restrictions. Several others provide this kind of general impossibility statement for the specific case of weak complementarity. For instance Bockstael and McConnell (1993, p. 1254, footnote 9) state that "[w]e can never test the hypothesis of weak complementarity; we can only judge whether this link between the public and private good is reasonable" (where the public good is a nonmarket good). von Haefen (2007, p. 16) characterizes weak complementarity as an "intuitive but untestable restriction." In the context of a specific application, Herriges et al. (2004, p. 63) do provide a concise characterization of the actual narrow result, describing the choice between two alternative welfare measures as "the choice between non-testable preference restrictions." If an valid exact welfare measure is properly obtained from a preference restriction, then that preference restriction must be affiliated with a feasible preference relation on Y and hence cannot be tested. However the overall theme of Herriges et al. (2004) emphasizes the general non-testability of preference restrictions.

4 Preference properties of weak complementarity

Weak complementarity is the most common preference restriction in the literature and has been called "the foundation of the theory of welfare measurement of environmental quality changes."[19] It involves a relationship between the nonmarket good and one of the market goods, the "weak complement."[20] Mäler's first example was the relationship between the quality of a public fishery and sport fishing (nonmarket and market good respectively).[21] A similar example from Bockstael and McConnell (1993) involves wildlife populations in a sanctuary and tourist trips to the sanctuary. In some presentations the nonmarket good is a quality of the market good, while others simply require that they be "consumed" together. Without loss of generality, I shall assume that the nonmarket good is associated with the first market good, x_1. Given this relationship, weak complementarity requires two preference properties, one which may be observed from revealed preference and another imposed as a preference assumption. This second property is thus the actual "preference restriction" or "maintained hypothesis."

The first property is nonessentiality. There are two versions of it in the literature. Willig (1978) originally defined nonessentiality as requiring that "any bundle including good 1 can be matched in the preference ordering by some other bundle which excludes good 1."[22] Formally, for any $x^a \in X$ and $z \in Z$ there must exist some $x^b = (x_1^b, \ldots, x_L^b) \in X$ with $x_1^b = 0$ such that $(x^a, z) \sim_Y (x^b, z)$. Most of the weak complementarity literature requires a slightly stronger property whereby the preference matching is restricted to those commodity vectors that can be obtained via the demand function.[23] Then for any $z \in Z$ and any $x^a \in \widehat{X}_z$, there must exist some $x^b \in \widehat{X}_z$ with $x_1^b = 0$ such that $(x^a, z) \sim_Y (x^b, z)$, or in terms of utility, $U_Y(x^a, z) = U_Y(x^b, z)$. The distinction between the two types of nonessentiality is equivalent to the distinction between infinite and finite choke prices for good one. Henceforth I shall use the term nonessentiality to refer to the stronger property.

[19] Bockstael and Kling (1988, p. 63).

[20] It is possible that there are more than one weak complement, as with Bockstael and Kling (1988). The work presented here can be easily generalized to accommodate that.

[21] Mäler (1971).

[22] The word "other" is not actually operational in the definition so that the two bundles can be the same. von Haefen (2007) provides an equivalent characterization of nonessentiality.

[23] The weak complementarity literature actually tends to be rather vague on nonessentiality. It is sometimes not explicitly considered while clearly still an implicit requirement such as with Larson (1991), Herriges et al. (2004), and Bullock and Minot (2006). Others, such as Smith and Banzhaf (2004) initially state it in terms Willig's original characterization but in their analysis clearly require the stronger version adopted here. My statement of the stronger version is equivalent with the characterizations provided by Bockstael and McConnell (1993), and Palmquist (2005).

No-existence-value is the second required preference property associated with weak complementarity.[24] This property tells us that the consumer does not care about the value of z when the consumption bundle is fixed with $x_1 = 0$. Thus the nonmarket good does not have any stand-alone existence value, but rather has only use value in conjunction with the consumption of the weak complement, good one.[25] Beginning with Mäler (1971), this property is typically formally defined by the partial differential equation,

$$\frac{\partial U_Y}{\partial z}(0, x_2, x_3, \ldots, x_L, z) = 0.$$

For the purposes of this paper it is more convenient to use a preference characterization of no-existence-value: for any $x \in X$ with $x_1 = 0$ and any $z^a, z^b \in Z$ we have $(x, z^a) \sim_Y (x, z^b)$, or in terms of utility, $U_Y(x, z^a) = U_Y(x, z^b)$.[26]

One of these two properties, nonessentiality, can be restated purely in terms of the set of z-fixed preference relations, $\{\succsim_z \mid z \in Z\}$: for any $z \in Z$ and any $x^a \in \widehat{X}_z$, there must exist some $x^b \in \widehat{X}_z$ with $x_1^b = 0$ such that $x^a \sim_z x^b$, or in terms of utility, $u_z(x^a) = u_z(x^b)$. Therefore nonessentiality can be verified against revealed preference and is not a "preference restriction." However the definition of no-existence-value requires preference comparison across different z values and consequently cannot be verified with revealed preference. It is a "preference restriction" or "maintained hypothesis." I next present an observable revealed preference condition that is necessary and sufficient for the feasibility of no-existence-value.

5 Single-preference and no-existence-value feasibility

"Single-preference" is a potential property of the set of z-fixed preferences $\{\succsim_z \mid z \in Z\}$ in relationship to some subset of X. With single-preference all of the individual $\succsim_z$ relations agree with each other on the subset so that there is a single preference relation on that limited preference domain. Based on this intuitive understanding we would have single-preference on some subset $\widetilde{X} \subseteq X$ if $x^1 \succsim_{z^a} x^2 \Leftrightarrow x^1 \succsim_{z^b} x^2$ for all $z^a, z^b \in Z$ and all $x^1, x^2 \in \widetilde{X}$.

However, this characterization implicity requires that both $\succsim_{z^a}$ and $\succsim_{z^b}$ be complete on $\widetilde{X}$ so that $\widetilde{X} \subseteq \widehat{X}_{z^a}$ and $\widetilde{X} \subseteq \widehat{X}_{z^b}$. A somewhat more complicated statement of single-preference is required when some $\succsim_z$ might not be complete on $\widetilde{X}$. In this context the key

[24]Beginning with Mäler (1974), weak complementarity is defined most often in the literature as the property that I call no-existence-value. However the need for both properties is often not clear in these presentations. My two-property definition of weak complementarity follows Palmquist (2005) and von Haefen (2007), and facilitates a more clear understanding of the distinct roles of both properties.

[25]See Herriges et al. (2004) for a more precise understanding of use and existence value.

[26]Willig (1978) also uses this utility equality characterization of no-existence-value.

intuition of single-preference is that for any $z^a, z^b \in Z$, the two preference relations $\succsim_{z^a}$ and $\succsim_{z^b}$ do not disagree with each other on $\widetilde{X}$, which is equivalent to requiring them to agree with each other on the intersection $\widetilde{X} \bigcap \widehat{X}_{z^a} \bigcap \widehat{X}_{z^b}$:

Definition. For a given a set of z-fixed preference relations $\{\succsim_z \mid z \in Z\}$ defined respectively on the obtainable sets $\widehat{X}_z$, we have single-preference on some subset of commodity space $\widetilde{X} \subseteq X$ when $x^1 \succsim_{z^a} x^2 \Leftrightarrow x^1 \succsim_{z^b} x^2$ for all $z^a, z^b \in Z$ and all $x^1, x^2 \in \widetilde{X} \bigcap \widehat{X}_{z^a} \bigcap \widehat{X}_{z^b}$.

In the context of no-existence-value we are concerned with the subset of X where the supposed weak complement, good one, is zero, $X_1^0 = \{x \in X \mid x_1 = 0\}$. With $L = 2$ the set X_1^0 is the vertical axis, and with $L = 3$ it is a quarter-plane taking on all nonnegative combinations of the other two goods. In general it is the nonnegative portion of a hyperplane with dimension $L - 1$.

We have this important relationship between no-existence-value feasibility and single-preference on X_1^0:

Theorem 1. *No-existence-value is feasible if and only if the set of possible revealed preference* $\{\succsim_z \mid z \in Z\}$ *has single-preference on* X_1^0.

Thus without single-preference on X_1^0, the nonmarket good must have existence value. Single-preference on X_1^0 can also be characterized in terms of indifference:

Theorem 2. *The set of possible revealed preference* $\{\succsim_z \mid z \in Z\}$ *has single-preference on* X_1^0 *if and only if* $x^1 \sim_{z^a} x^2 \Leftrightarrow x^1 \sim_{z^b} x^2$ *for all* $z^a, z^b \in Z$ *and all* $x^1, x^2 \in X_1^0 \bigcap \widehat{X}_{z^a} \bigcap \widehat{X}_{z^b}$.

As I will demonstrate, this second theorem allows us to recognize the absence or presence of single-preference on X_1^0 respectively by whether or not indifference curves cross for different z values. The proofs of both theorems are provided in the appendix.

The observable property of nonessentiality also has a role here. It directly follows from nonessentiality that for each $z \in Z$ the set $\widehat{X}_z \bigcap X_1^0$ is not empty. Moreover with each $\succsim_z$ continuous, strictly convex and strongly monotone, with nonessentiality we will typically find that the interior of X_1^0, in the $L-1$ dimensional hyperplane defined by $x_1 = 0$, is included in each obtainable set $\widehat{X}_z$. For example with $L = 2$ we would find all of the standard vertical axis, except for the origin, included in each obtainable set. With $L = 3$ the interior of X_1^0 is the open quarter-plane with $x_1 = 0$ and all strictly positive combinations of the other two goods. Thus with nonessentiality, the presence or absence of single-preference on X_1^0 may be readily observed from revealed preference, allowing us to easily test the feasibility of no-existence-value.

When there are only two market goods, single-preference on X_1^0 is always satisfied so that no-existence-value is always feasible and the testing implications of Theorem 1 are trivial.[27] However with $L \geq 3$ single-preference on X_1^0 is not automatic. Then Theorem 1 provides us with a nontrivial revealed preference test of no-existence-value. Lack of single-preference is depicted in Figure 1 for $L = 3$. From Theorem 2, without single-preference there must be two points $x^1, x^2 \in X_1^0$ such that for some $z^a, z^b \in Z$, we have $x^1, x^2 \in \widehat{X}_{z^a} \bigcap \widehat{X}_{z^b}$ and $x^1 \sim_{z^a} x^2$ but not $x^1 \sim_{z^b} x^2$. This is all depicted in Figure 1 as x^1 and x^2 are on the same z^a-indifference set in X_1^0 represented by I^a but not on the same z^b-indifference set. The z^b-indifference set that includes x^1 is represented by I^b. It is depicted as passing above x^2 so that with monotonicity x^2 must be on a lower z^b-indifference set and hence $x^1 \succ_{z^b} x^2$.

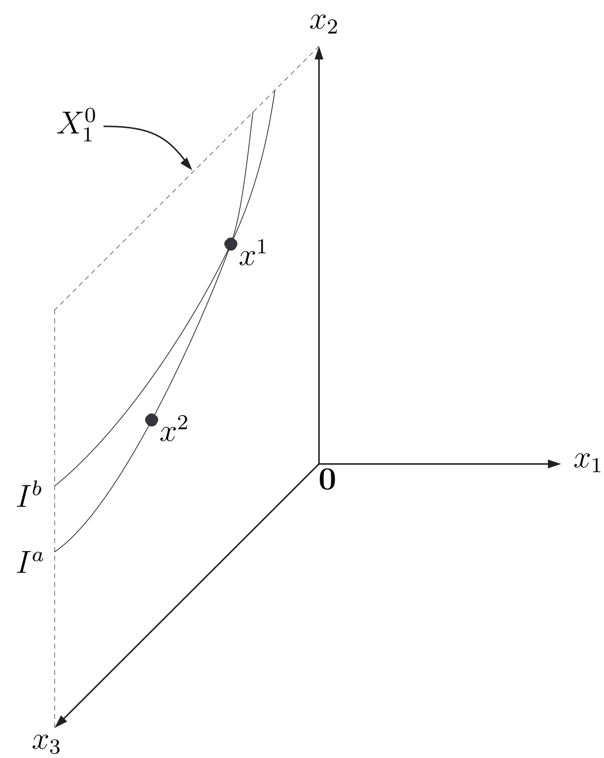

Figure 1: Without single-preference on X_1^0, no-existence-value is not feasible

We can also see in Figure 1 that no-existence-value is not feasible. From $\succsim_{z^a}$ we have $x^1 \sim_{z^a} x^2$ and hence $(x^1, z^a) \sim_Y (x^2, z^a)$. Similarly, from $\succsim_{z^b}$ we have $x^1 \nsim_{z^b} x^2$ and hence $(x^1, z^b) \nsim_Y (x^2, z^b)$.[28] If no-existence-value were feasible then we could also have

[27]With $L = 2$, X_1^0 is the vertical axis so that with monotonicity all preference relations on any subset of X_1^0 are fully defined by the increasing value of good two. Therefore all of the individual $\succsim_z$ relations agree with each other on X_1^0, giving us single-preference on X_1^0. Then from Theorem 1, no-existence-value is always feasible when $L = 2$.

[28]The symbol "$\nsim$" indicates "not $\sim$" or "not indifferent to."

$(x^1, z^a) \sim_Y (x^1, z^b)$ and $(x^2, z^a) \sim_Y (x^2, z^b)$ without fear of contradiction. However with these two and $(x^1, z^a) \sim_Y (x^2, z^a)$, we get $(x^1, z^b) \sim_Y (x^2, z^b)$ from transitivity which directly contradicts $\succsim_{z^b}$. Thus the basic preference properties implied by lack of single-preference on X_1^0 precludes the possibility of no-existence-value.

Absence of single-preference can be also observed with Marshallian demand and simple Samuelsonian revealed preference. Without single-preference on X_1^0 (and with $L \geq 3$) there must be intersections of indifference sets for different z-values such as depicted at point x^1 in Figure 1.[29] With continuity of preference such intersections cannot exist in isolation. Instead we will have an infinite mesh of such intersections which can be imagined with Figure 1 by adding z^a-indifference curves parallel to I^a and z^b-indifference curves parallel to I^b. These intersections or crossings allow us we to observe from simple Samuelsonian revealed preference that no-existence-value is not feasible.

Another indifference set intersection is depicted in Figure 2, again with $L = 3$ but with a different axes alignment than presented in Figure 1. Here we again have a $\succsim_{z^a}$ indifference curve I^a, and a $\succsim_{z^b}$ indifference curve I^b, both in X_1^0 that this time cross at the point x^0. Almost any such crossing can be detected with simple Samuelsonian revealed preference.[30] In Figure 2 we have points $x^a \in I^a$ and $x^b \in I^b$ on the tangents of these indifference curves with two budget lines so that $x^a = \widehat{x}(p^a, z^a, w^a)$ and $x^b = \widehat{x}(p^b, z^b, w^b)$ for some price and wealth combinations $(p^a, w^a), (p^b, w^b) \in \Re_{++}^{L+1}$.[31] It is clear that x^b is in the budget set when x^a is chosen and x^a is in the budget set when x^b is chosen ($p^a \cdot x^b < w^a$ and $p^b \cdot x^a < w^b$) so that x^a is z^a-revealed preferred to x^b, and x^b is z^b-revealed preferred to x^a. Thus from simple Samuelsonian revealed preference we know that $x^a \succ_{z^a} x^b$ and $x^b \succ_{z^b} x^a$, which violates single-preference on X_1^0. Then from Theorem 1 no-existence-value is impossible.

With just the two observed demand points $x^a = \widehat{x}(p^a, z^a, w^a)$ and $x^b = \widehat{x}(p^b, z^b, w^b)$ in Figure 2 we can also contradict no-existence-value directly without reference to Theorem 1. With no-existence-value we would have $(x^a, z^a) \sim_Y (x^a, z^b)$ and $(x^b, z^a) \sim_Y (x^b, z^b)$. Simple revealed preference from $x^a = \widehat{x}(p^a, z^a, w^a)$ and $p^a \cdot x^b \leq w^a$ gives us $(x^a, z^a) \succ_Y (x^b, z^a)$ so that with transitivity we would have $(x^a, z^b) \succ_Y (x^b, z^b)$. However this conflicts with the simple revealed preference observation that $(x^b, z^b) \succ_Y (x^a, z^b)$ [from $x^b = \widehat{x}(p^b, z^b, w^b)$ and

[29] Without any such intersections, for any two $z^a, z^b \in Z$, the partition of $X_1^0 \bigcap \widehat{X}_{z^a} \bigcap \widehat{X}_{z^b}$ into indifference sets would the same for both $\succsim_{z^a}$ and $\succsim_{z^b}$. Then from Theorem 2 we would have single-preference on X_1^0.

[30] A possible exception is when the crossing occurs at a tangent that is common to both curves. However such tangent crossings are individually isolated and are locally surrounded by crossings that are not tangent, such as the one depicted in Figure 2.

[31] The two budget lines depicted in Figure 2 are the intersections of X_1^0 with the budget planes respectively defined by $p^a \cdot x = w^a$ and $p^b \cdot x = w^b$. Obtaining x^a and x^b from the demand function requires that the price of good one be greater than or equal to the choke price. This is demonstrated in the next section.

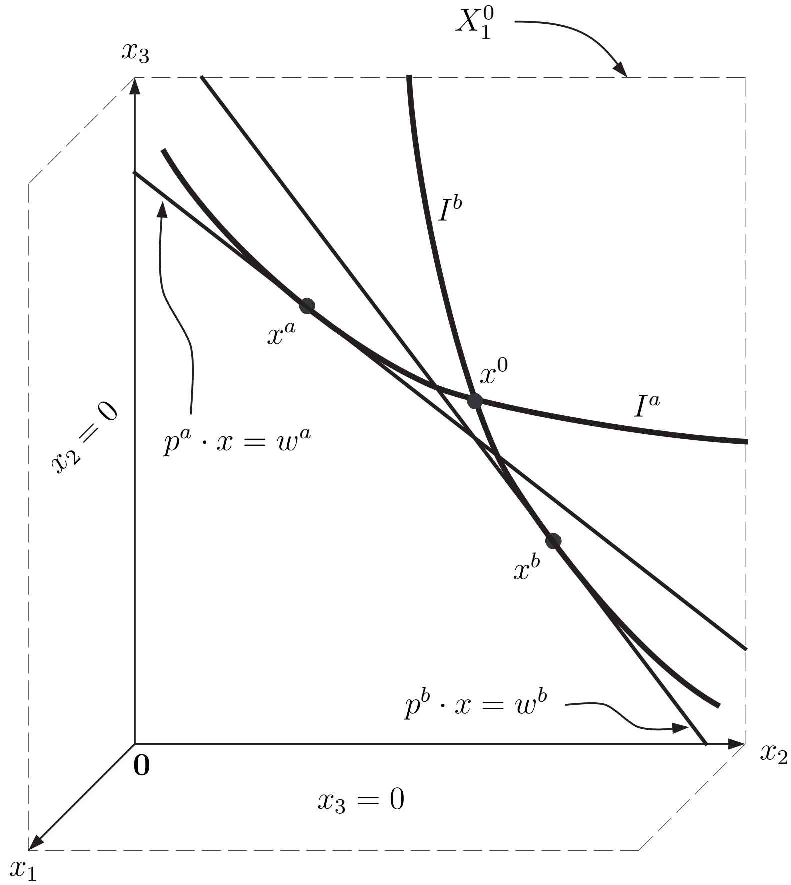

Figure 2: Falsifying no-existence-value with revealed preference

$p^b \cdot x^a \leq w^b$]. Therefore with simple Samuelsonian revealed preference we can directly falsify no-existence-value and thus demonstrate by direct observation that weak complementarity is not feasible.

6 Numeric Examples

The results presented above are illustrated in this section using z-specific preference relations that can be represented by Stone-Geary utility functions of the form

$$u_z(x) = (x_1 + \phi)^{\alpha_1(z)} \prod_{\ell=2}^{L} (x_\ell)^{\alpha_\ell(z)},$$

where $\phi > 0$, and for all $z \in Z$ and all $\ell = 1, \ldots, L$, $\alpha_\ell(z) > 0$ and $\sum_{\ell=1}^{L} \alpha_\ell(z) = 1$.[32] Nonessentiality is satisfied with the $\phi > 0$ restriction. For each $z \in Z$, the choke price of the

[32]Larson (1991) and Palmquist (2005) also provide examples based on Stone-Geary preference.

weak complement (good one) is

$$p_1^c(z) \quad = \quad \frac{w\alpha_1(z)}{\phi\,(1-\alpha_1(z))}.$$

Each of the L component demand functions includes two cases, one for interior solutions when the price of good one is less than this choke price and one for corner solutions in X_1^0. For good one this becomes

$$\widehat{x}_1(p,z,w) \quad = \quad \begin{cases} (w+\phi p_1)\,\dfrac{\alpha_1(z)}{p_1} - \phi & \text{if} \quad p_1 < p_1^c(z) \\ 0 & \text{if} \quad p_1 \geq p_1^c(z) \end{cases}, \tag{4}$$

and for the other goods, $j = 2, \ldots, L$, we have

$$\widehat{x}_j(p,z,w) \quad = \quad \begin{cases} (w+\phi p_1)\,\dfrac{\alpha_j(z)}{p_j} & \text{if} \quad p_1 < p_1^c(z) \\ \dfrac{w\alpha_j(z)}{p_j\,(1-\alpha_1(z))} & \text{if} \quad p_1 \geq p_1^c(z) \end{cases}. \tag{5}$$

From these demand functions we can see that the obtainable set is the same for all $z \in Z$, $\widehat{X}_z = \{x \in X \,|\, x_1 \geq 0 \text{ and } x_\ell > 0 \text{ for } \ell = 2, \ldots, L\}$. Therefore each obtainable set $\widehat{X}_z$ includes the entire interior of X_1^0.

I will work with $L = 3$ and up to three z values, $Z = \{z^a, z^b, z^c\}$. In this context I am able to adopt a simplified notation so that the set of possible revealed preference is $\{\succsim_a, \succsim_b, \succsim_c\}$ with the individual relations respectively represented by

$$\begin{aligned} u_a(x) &= (x_1+12)^{1/4}\,(x_2)^{1/4}\,(x_3)^{1/2}, \\ u_b(x) &= (x_1+12)^{1/2}\,(x_2)^{1/6}\,(x_3)^{1/3}, \\ u_c(x) &= (x_1+12)^{1/4}\,(x_2)^{1/2}\,(x_3)^{1/4}. \end{aligned}$$

These three utility functions clearly represent three different preference relations on $\widehat{X}_z$. With these I first demonstrate the wide variability of possible welfare measure values when we only have revealed preference to work with. I then provide applications of Theorem 1, first to preclude the possibility of weak complementarity, and then to guarantee the existence of a feasible weak complementarity solution. I close this section with a demonstration of how weak complementarity may be falsified from simple Samuelsonian revealed preference.

A compensating variation welfare measure can be constructed with any utility function defined on Y that is consistent with the set of possible revealed preference. For each $\theta > 0$, the utility function $u_\theta(x) = (u_b(x))^\theta$ represents $\succsim_b$ on $\widehat{X}_z$. We can construct a parameterized family of feasible utility functions on Y by combining these with u_a and u_c,

$$U_\theta(x,z) \quad = \quad \begin{cases} (x_1+12)^{1/4}\,(x_2)^{1/4}\,(x_3)^{1/2} & \text{if} \quad z = z^a \\ \left((x_1+12)^{1/2}\,(x_2)^{1/6}\,(x_3)^{1/3}\right)^\theta & \text{if} \quad z = z^b \\ (x_1+12)^{1/4}\,(x_2)^{1/2}\,(x_3)^{1/4} & \text{if} \quad z = z^c \end{cases}. \tag{6}$$

With these representations of preference on Y we can see that z^b is valued more with higher values of θ. With each of these utility functions we can use equation (3) to solve for CV values. With $\bar{p} = (1, 1, 1)$, $\bar{w} = 52$ and a change from z^a to z^b, $\theta = 0.7$ gives us $CV = -173.8$ while $\theta = 1.4$ gives us $CV = 38.5$.[33] Both of these CV values are large in the context of our wealth value. Thus with only revealed preference, we cannot even determine whether the change in z represents a major improvement or a major reduction in welfare.

This last result illustrates the rather dramatic need for some methodology such as applying weak complementarity as a preference restriction. However, any implementation of weak complementarity requires that no-existence-value be feasible. Since $L = 3$, this feasibility is not a trivial issue and may be tested via Theorem 1.

Theorem 1 is only concerned with preference on X_1^0, i.e., where $x_1 = 0$. In this limited preference domain,

$$\begin{aligned} u_a(0, x_2, x_3) &= (12)^{1/4} (x_2)^{1/4} (x_3)^{1/2}, \\ u_b(0, x_2, x_3) &= (12)^{1/2} (x_2)^{1/6} (x_3)^{1/3}, \\ u_c(0, x_2, x_3) &= (12)^{1/4} (x_2)^{1/2} (x_3)^{1/4}. \end{aligned}$$

We can then apply these monotonic transformations,

$$\begin{aligned} h_a(u) &= \left(u \Big/ (12)^{1/4}\right)^{4/3}, \\ h_b(u) &= \left(u \Big/ (12)^{1/2}\right)^{2}, \\ h_c(u) &= \left(u \Big/ (12)^{1/4}\right)^{4/3}, \end{aligned}$$

to obtain preference relations on X_1^0 respectively defined by

$$\begin{aligned} u_a^h(x_2, x_3) &= h_a\left(u_a(0, x_2, x_3)\right) = (x_2)^{1/3} (x_3)^{2/3}, \\ u_b^h(x_2, x_3) &= h_b\left(u_b(0, x_2, x_3)\right) = (x_2)^{1/3} (x_3)^{2/3}, \\ u_c^h(x_2, x_3) &= h_c\left(u_c(0, x_2, x_3)\right) = (x_2)^{2/3} (x_3)^{1/3}. \end{aligned}$$

From this we can see that the preference set $\{\succsim_a, \succsim_b, \succsim_c\}$ does not have single-preference on X_1^0. Even though $\succsim_a$ and $\succsim_b$ are identical on X_1^0, $\succsim_c$ is different so that single-preference is not satisfied. It then follows from Theorem 1 that no-existence-value is not feasible, and therefore weak complementarity cannot be applied.

[33] Initial demand $\widehat{x}(\bar{p}, z^a, \bar{w})$ is an interior solution with equations (4) and (5). For both values of θ, the demand after compensation $\widehat{x}(\bar{p}, z^b, \bar{w} - CV)$ is also an interior solution. Values of $\theta \geq 1.44$ will yield corner solutions such that $x_1 = 0$. Taking limits across both interior and corner solutions, as $\theta \to 0$ we get $CV \to -\infty$, and as $\theta \to \infty$ we get $CV \to \bar{w}$.

Suppose now that the set of permissible nonmarket good values is instead $\widetilde{Z} = \{z^a, z^b\}$ with the set of possible revealed preference $\{\succsim_a, \succsim_b\}$. We have already seen that these two preference relations are the same on X_1^0 so that single-preference is satisfied, and from Theorem 1 it follows that no-existence-value is feasible. Since nonessentiality is also satisfied, we should be able to specify a weak complementarity preference relation on Y that is consistent with $\{\succsim_a, \succsim_b\}$. A utility function on Y representing weak complementarity preference can be obtained by applying h_a and h_b respectively to u_a and u_b,

$$U_{wc}(x, z) = \begin{cases} \left(\dfrac{x_1 + 12}{12}\right)^{1/3} (x_2)^{1/3} (x_3)^{2/3} & \text{if} \quad z = z^a \\ \dfrac{x_1 + 12}{12} (x_2)^{1/3} (x_3)^{2/3} & \text{if} \quad z = z^b \end{cases} .$$

This preference relation on $X \times \widetilde{Z}$ is unique in being consistent with $\{\succsim_a, \succsim_b\}$ while also satisfying no-existence-value.[34] It is not represented by any member of the U_θ family of utility functions presented in equation (6).[35] Again with fixed $\bar{p} = (1, 1, 1)$ and $\bar{w} = 52$, and a change from z^a to z^b we can use equation (3) with U_{wc} to obtain a unique weak complementarity CV value, $CV = 13.6$.

Previously we used Theorem 1 with the set of possible revealed preference $\{\succsim_a, \succsim_b, \succsim_c\}$ to reject the feasibility of no-existence-value. In such a situation we can also use simple Samuelsonian revealed preference to reject no-existence-value and with it weak complementarity. Let $p^b = (8, 7, 8)$, $w^b = 84$, $p^c = (3, 5, 3)$, $w^c = 45$, $x^b = \widehat{x}(p^b, z^b, w^b)$ and $x^c = \widehat{x}(p^c, z^c, w^c)$. Then from equations (4) and (5) we get $x^b = (0, 4, 7)$ and $x^c = (0, 6, 5)$ so that both commodity vectors are in X_1^0. We also have $p^b \cdot x^c = 82 \leq w^b$ and $p^c \cdot x^b = 41 \leq w^c$, so that x^b is z^b-revealed preferred to x^c and x^c is z^c-revealed preferred to x^b. This is a revealed preference violation of single-preference on X_1^0, so that with Theorem 1 we are able to reject no-existence-value based on observed (or estimated) Marshallian demand. We can also reject no-existence-value directly, without reference to the theorem, based on the reasoning developed at the end of the previous section.

[34] No-existence-value defines a unique relation on $X \times \widetilde{Z}$ by affiliating indifference sets in X across different z values based on their common subsets in X_0^1. These common subsets would not exist without single-preference on X_0^1.

[35] Our weak complementarity utility function cannot be obtained as a simple monotonic transformation of any of the parameterized family of utility functions presented in equation (6) after eliminating z^c. That is, there is no increasing real valued function $f(u)$ such that $U_{wc}(x, z) = f(U_\theta(x, z))$ for some $\theta > 0$ and for all $x \in X$ and all $z \in \widetilde{Z}$. Therefore none of the U_θ utility functions represent a weak complementarity compliant preference relation even after dropping z^c from Z.

7 Conclusions

It is well understood that there are multiple feasible preference relations on Y, and hence it is not possible to determine the "true" one based on revealed preference. Preference restrictions such as weak complementarity are one way of dealing with this problem. They are imposed as assumptions with the goal of creating sufficient additional preference structure so that a unique welfare measure value may be identified, such as with compensating variation. If the preference restriction is feasible, it follows that the restriction cannot tested against revealed preference to see if it describes the "true" $\succsim_Y$.

All of this is well understood in the literature except that it has not been recognized that a given preference restriction may not be feasible in the first place. Omitting this possibility from consideration has led to two types of erroneous claims. The first vastly inflates testing impossibility such that it is simply not possible to test preference restrictions against revealed preference. Usually these statements simply ignore the possibility that the restriction may not be feasible.[36] However some explicitly reject the possibility of nontrivial observable feasibility conditions.[37] The second kind of erroneous claim immediately follows from the first. If we cannot reject a given preference restriction then it is technically universally applicable so that the only practical limitation on its application is the analyst's intuition as to what makes sense.[38]

This paper has focused on the example of weak complementarity to show that there can be observable nontrivial feasibility conditions for preference restrictions. With weak complementarity defined by the observable property of nonessentiality and the unobservable property of no-existence-value, the latter is the actual preference restriction. From Theorem

[36] The statements quoted in note 18 are mostly of this type.

[37] For example Ebert (1998, p. 242) states that preference restrictions are untestable in general because "there are no observable implications of the properties imposed." Similarly, Bockstael and McConnell (1993, p. 1250) state that weak complementarity "has implications for the structure of preferences which are not observable (and hence not testable)." In the same vein Herriges et al. (2004, p. 56) state that "apparent violations of [weak complementarity] are conditional on an *assumed* functional form for preferences, but unfortunately these preferences are observationally indistinguishable from alternative functional forms satisfying the [weak complementarity] assumption" (italics in original).

[38] For example Larson (1991, p. 98) claims to have demonstrated "a method for recovering weakly complementary preferences when integrating back from *any* Marshallian demands to recover the quasi-expenditure function" (italics in original). Even if his methodology can be applied to any Marshallian demand function, it cannot yield weakly complementary preference if the single-preference condition is not satisfied. Later in this paper Larson concludes that weak complementarity "can be imposed if judged appropriate" (p. 107). Some of the statements quoted in note 18 also refer to the role of analyst intuition and judgement as the only real constraint on application.

1 we know that single-preference on X_1^0 is necessary and sufficient for the feasibility of no-existence-value. This observable feasibility condition is not trivial when there are two or more market goods that are not prospective weak complements to the nonmarket good. Thus when $L \geq 3$, the absence of single-preference is a testable condition that allows us to reject no-existence-value and with it weak complementarity.[39] In particular, I have shown that both single-preference and no-existence-value can be tested with simple Samuelsonian revealed preference.

Single-preference on X_0^1 is a strong condition which increases in strength with L. If we arbitrarily chose two preference relations that are complete on $X = \Re^3_+$ $(L = 3)$ and satisfy nonessentiality, it is highly unlikely that they will agree with each other on X_0^1. This chance occurrence is equivalent to arbitrarily choosing two preference relations on $X = \Re^2_+$ and finding they are exactly the same on the entirety of X. Single-preference requires this exact matching not just between two preference relations, but across all the preference relations $\succsim_z$ for $z \in Z$. With ever larger values of L, single-preference on X_0^1 requires this matching on ever higher dimensional commodity spaces, and is hence even more unlikely to occur by chance. Thus single-preference on X_0^1 places a strong restriction on the set of possible revealed preference $\{\succsim_z \mid z \in Z\}$. For the vast majority of potential $\{\succsim_z \mid z \in Z\}$ sets, single-preference is not satisfied and therefore weak complementarity is not feasible.[40]

The presence of an observable nontrivial feasibility condition does not appear to be a consequence of some unique characteristic of weak complementarity in relationship to other preference restrictions. This is an example of what may well be a prevalent property. For example, preference restrictions similar to weak complementarity, such as those presented towards the end of Smith and Banzhaf (2004), also have feasibility conditions requiring single-preference on a specified set. It may be that other preference restrictions have other kinds feasibility conditions that do not involve single-preference.

[39]All of the models in the papers previously cited in notes 18, 37 and 38 allow for more than two market goods.

[40]Most of the literature is specifically concerned with an explicitly continuous nonmarket good, which is only an open possibility in this paper. The argument presented here is still valid in that context. Suppose that Z is the set of positive real numbers and $\succsim_Y$ has continuous preference over $X \times Z$. Thus when $|z^a - z^b| < \varepsilon$ for some small $\varepsilon > 0$ we would expect $\succsim_{z^a}$ and $\succsim_{z^b}$ to be very similar on X, including X_0^1. However "very similar" is categorically different from being the same. Single-preference would still be special.

Appendix: Proofs of Theorems

Theorem 1

Proof. ($\Rightarrow$): For this part I need to establish that feasibility of no-existence-value implies single-preference on X_1^0. Feasibility of no-existence-value requires that it is possible to have a rational preference relation on Y indicated by $\succsim_\alpha$ that satisfies no-existence-value and is consistent with all $\succsim_z$ for $z \in Z$. Let $z^a, z^b \in Z$. We then have single-preference on X_1^0 if it follows that $x^1 \succsim_{z^a} x^2 \Leftrightarrow x^1 \succsim_{z^b} x^2$ for all $x^1, x^2 \in X_1^0 \bigcap \widehat{X}_{z^a} \bigcap \widehat{X}_{z^b}$. Let $x^1, x^2 \in X_1^0 \bigcap \widehat{X}_{z^a} \bigcap \widehat{X}_{z^b}$ such that $x^1 \succsim_{z^a} x^2$ Then from the feasibility of $\succsim_\alpha$ we have $(x^1, z^a) \succsim_\alpha (x^2, z^a)$. From satisfaction of no-existence-value we also have $(x^1, z^a) \sim_\alpha (x^1, z^b)$ and $(x^2, z^a) \sim_\alpha (x^2, z^b)$. Then by transitivity we get $(x^1, z^b) \succsim_\alpha (x^2, z^b)$ and hence $x^1 \succsim_{z^b} x^2$. I have shown that $x^1 \succsim_{z^a} x^2 \Rightarrow x^1 \succsim_{z^b} x^2$. Since z^a, z^b are general members of Z we also have $x^1 \succsim_{z^b} x^2 \Rightarrow x^1 \succsim_{z^a} x^2$, and hence $x^1 \succsim_{z^a} x^2 \Leftrightarrow x^1 \succsim_{z^b} x^2$ so that $\{\succsim_z \mid z \in Z\}$ has single-preference on X_1^0.

($\Leftarrow$): For this part I need to establish that single-preference on X_1^0 implies feasibility of no-existence-value. No-existence-value is feasible if the requirement that $(x, z^a) \sim_Y (x, z^b)$ for all $x \in X_1^0$ and all $z^a, z^b \in Z$ is not contradicted by the set of possible revealed preference $\{\succsim_z \mid z \in Z\}$. Therefore we only need to consider preference interaction on the limited preference domain $\widehat{Y}_1^0 = \{(x, z) \in Y \mid x \in \widehat{X}_z \bigcap X_1^0 \text{ for } z \in Z\}$. Let $\succsim_\beta$ be a preference relation on Y such that for any $(x^a, z^a), (x^b, z^b) \in \widehat{Y}_1^0$ we have $(x^a, z^a) \succsim_\beta (x^b, z^b)$ if and only if $x^a \succsim_z x^b$ for at least some $z \in Z$.

I need to show that $\succsim_\beta$ is well defined on $\widehat{Y}_1^0$ in the sense that it is not possible to have both $(x^a, z^a) \succsim_\beta (x^b, z^b)$ and $(x^b, z^b) \succ_\beta (x^a, z^a)$ for some $(x^a, z^a), (x^b, z^b) \in \widehat{Y}_1^0$. This could occur only if there existed some $z^c, z^d \in Z$ such that $x^a \succsim_{z^c} x^b$ and $x^b \succ_{z^d} x^a$. However this would violate single-preference on X_1^0. Thus with single-preference on X_1^0, $\succsim_\beta$ is well defined on $\widehat{Y}_1^0$. It follows from this and the definition of $\succsim_\beta$ that for any $z \in Z$ and $x^a, x^b \in \widehat{X}_z \bigcap X_1^0$, $(x^a, z) \succsim_\beta (x^b, z)$ if and only if $x^a \succsim_z x^b$. Therefore $\succsim_\beta$ is consistent with all $\{\succsim_z \mid z \in Z\}$ on X_1^0.

For any $(x, z^a), (x, z^b) \in \widehat{Y}_1^0$ we have $x \sim_{z^a} x$ ($\sim_{z^a}$ is reflexive) so that $(x, z^a) \sim_\beta (x, z^b)$ and no-existence-value is satisfied. Therefore with single-preference on X_1^0, it is possible to have a no-existence-value preference relation that is consistent with the set of possible revealed preference, $\{\succsim_z \mid z \in Z\}$. Thus no-existence-value can not be contradicted by revealed preference and is hence feasible. $\square$

Theorem 2

Proof. ($\Rightarrow$): For this part I need to establish that with single-preference on X_1^0, then $x^1 \sim_{z^a} x^2 \Leftrightarrow x^1 \sim_{z^b} x^2$ for all $z^a, z^b \in Z$ and all $x^1, x^2 \in X_1^0 \bigcap \widehat{X}_{z^a} \bigcap \widehat{X}_{z^b}$. This implication is always true for single-preference irrespective of the commodity space subset $\widetilde{X} \subseteq X$.

With single-preference on $\widetilde{X}$, and any $z^a, z^b \in Z$ and $x^1, x^2 \in \widetilde{X} \bigcap \widehat{X}_{z^a} \bigcap \widehat{X}_{z^b}$, we have $x^1 \sim_{z^a} x^2 \Leftrightarrow [x^1 \succsim_{z^a} x^2 \text{ and } x^2 \succsim_{z^a} x^1] \Leftrightarrow [x^1 \succsim_{z^b} x^2 \text{ and } x^2 \succsim_{z^b} x^1] \Leftrightarrow x^1 \sim_{z^b} x^2$. The first and last equivalencies are from the definition of indifference, and the middle one comes from single-preference.

We have obtained the $x^1 \sim_{z^a} x^2 \Leftrightarrow x^1 \sim_{z^b} x^2$ indifference equivalency statement from general single-preference but the reverse implication does not hold; the indifference equivalency statement may be true when general single-preference does not hold. The next step is to show that the reverse implication does hold for the specific case of single-preference on X_1^0.

($\Leftarrow$): For this part I need to establish that whenever $x^1 \sim_{z^a} x^2 \Leftrightarrow x^1 \sim_{z^b} x^2$ for all $z^a, z^b \in Z$ and all $x^1, x^2 \in X_1^0 \bigcap \widehat{X}_{z^a} \bigcap \widehat{X}_{z^b}$, then we also have single-preference on X_1^0.

For the sake of clarity I need to change the commodity basket indexing of the indifference equivalency statement. Assume that $x^j \sim_{z^a} x^k \Leftrightarrow x^j \sim_{z^b} x^k$ for all $z^a, z^b \in Z$ and all $x^j, x^k \in X_1^0 \bigcap \widehat{X}_{z^a} \bigcap \widehat{X}_{z^b}$. Let $z^a, z^b \in Z$ and $x^1, x^2 \in X_1^0 \bigcap \widehat{X}_{z^a} \bigcap \widehat{X}_{z^b}$ such that $x^1 \succsim_{z^a} x^2$. We want to show that then $x^1 \succsim_{z^b} x^2$. Define $x^3 \in X$ such that the amount of each commodity is equal to the largest quantity of that commodity in x^1 or x^2, $x_\ell^3 = \max\{x_\ell^1, x_\ell^2\}$ for $\ell = 1, \ldots, L$. Then $x^3 \in X_1^0$ and x^3 is weakly monotonically superior to both x^1 and x^2 in that $x_\ell^3 \geq x_\ell^i$ for $i = 1, 2$ and all $\ell = 1, \ldots, L$. I assumed in the paper that all obtainable sets $\widehat{X}_z$ are closed under weak monotone superiority so that $x^3 \in \widehat{X}_{z^a}$. From monotonic preference we then have $x^3 \succsim_{z^a} x^1$ and the z^a-preference chain, $x^3 \succsim_{z^a} x^1 \succsim_{z^a} x^2$. From continuity of preference on X with $z = z^a$, there must be some x^4 in between x^2 and x^3 such that $x^1 \sim_{z^a} x^4$. More formally, there must exist some $\alpha \in [0, 1]$ such that with $x^4 = \alpha x^2 + (1 - \alpha) x^3$ we have $x^1 \sim_{z^a} x^4$.[41] Then from the indifference equivalency statement we also have $x^1 \sim_{z^b} x^4$.[42] The new point is weakly monotonically superior to x^2 so that $x^4 \succsim_{z^b} x^2$. From transitivity we have $x^1 \succsim_{z^b} x^2$.

I have shown that the indifference equivalency statement gives us the implication $x^1 \succsim_{z^a} x^2 \Rightarrow x^1 \succsim_{z^b} x^2$ for all $z^a, z^b \in Z$ and all $x^1, x^2 \in X_1^0 \bigcap \widehat{X}_{z^a} \bigcap \widehat{X}_{z^b}$. Since z^a and z^b are general

[41] Since x^3 is weakly monotonically superior to x^2, any point of the form $\alpha x^2 + (1 - \alpha) x^3$ ($\alpha \in [0, 1]$) is also weakly monotonically superior to x^2 and hence an element of $\widehat{X}_{z^a}$ under the monotonic closure assumption.

[42] From $x^2 \in \widehat{X}_{z^b}$ and the closure of $\widehat{X}_{z^b}$ under weak monotonic superiority we have $x^4 \in \widehat{X}_{z^b}$.

members of Z, we also have $x^1 \succsim_{z^b} x^2 \Rightarrow x^1 \succsim_{z^a} x^2$, and hence $x^1 \succsim_{z^a} x^2 \Leftrightarrow x^1 \succsim_{z^b} x^2$ so that $\{\succsim_z \mid z \in Z\}$ has single-preference on X_1^0. $\square$

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