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Decisions on Investment Allocation in the Post-Keynesian Growth Model*

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Abstract

In this article the analysis developed by Feldman (1928) and Mahalanobis (1953) are incorporated to the Post-Keynesian Growth Model to consider the decisions of investment allocation on economic growth. By adopting this approach it is possible to study the interaction between distributive features and investment allocation which allows us to determine the rate of investment allocation according to the equilibrium decisions of investment and savings. Finally, an additional condition is added to the Post Keynesian Growth Model in order to fully characterise the equilibrium path in an extended version of this framework, where capital goods are also needed to produce capital goods.

Keywords: Post-Keynesian growth model, structural change, multi-sector models.

JEL Classification: E21, O11.

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1. Introduction

The Post-Keynesian growth model – PKGM hereafter – designates the growth models that were initially developed by Kaldor (1956) and Robinson (1956, 1962) and extended by Dutt (1984), Rowthorn (1982) as well as by Bhaduri and Marglin (1990). The PKGM passes through three principal phases that are labeled as 'generations'. Although Kaldor (1956) has built his seminal model on the notion of full capacity utilization, Dutt (1984) and Rowthorn (1982), working independently, have built what is known as the second generation of the PKGM by endogenizing the rate of capacity utilization in the lines of Steindl (1952). One of the main contributions of this generation is the possibility of disequilibrium and the presence of a stagnationist regime in which an increase in the profit share implies a reduction in capacity utilization. The key assumption behind this result is that the growth rate of investment is a function not only of the profit rate, as in Kaldor-Robinson but also of the rate of capacity utilization.

Bhaduri and Marglin (1990) have challenged this view by considering that the growth rate of investment is a direct function not of the profit rate but of the profit share. According to them the profit rate has already been implicitly considered in the equation of the growth rate of investment through its relation with the rate of capacity utilization[1]. Hence by substituting the profit rate by the profit share in the expression of the growth rate of investment avoids to consider twice the effects of the former. One of the properties of the third generation model, as it became known, is the possibility of a non-stagnationist regime in which eventual falls in consumption due to a lower real wage are overcompensated by an increase in investment led by a profit share expansion.

[1] This is given by $r = \pi.u$, where r is the profit rate, π is the profit share and u is the rate of capacity utilization.

Although these characteristics are shared by other models in the Post-Keynesian tradition there is a remarkable lack of theoretical cohesion between them and the PKGM – an argument highlighted by Pasinetti (2005, p. 839-40) to explain why the Keynesian School has somewhat failed as a successful alternative paradigm to mainstream economics. Of course some effort was made in order to establish connections among these approaches or even to build a general PKGM. Intending to build reconciliation between the Kaleckian effective demand and Sraffian normal prices Lavoie (2003), for instance, has built a bridge between the PKGM and the Sraffian model. He considers that "a large range of agreement has remained, in particular about a most crucial issue, the causal role played by effective demand in the theory of capital accumulation".

Araujo and Teixeira (2011) have built a multi-sector version of the PKGM by considering it as a particular model of the Pasinetti's model of structural change and economic growth. By adopting this approach it is possible to show that the structural economic dynamics is conditioned not only to patterns of evolving demand and diffusion of technological progress but also to the distributive features of the economy, which can give rise to different regimes of economic growth.

In the present paper we show that the cross-fertilization between the PKGM and models in the structural economic dynamic tradition may render new results to central issues of economic growth such as investment allocation and structural change. In this article, we intend to study how investment allocation may be incorporated in the PKGM by considering the two sector model of Feldman-Mahalanobis, hereafter F-M model. Feldman (1928) and Mahalanobis (1953) models, are generally used as benchmarks to study the effects of the investment allocation on economic growth[2]. In order to

[2] Dutt (1990, p. 120) considers that no discussion related to models with investment and consumption good sectors is complete without considering the contribution of Feldman-Mahalanobis.

introduce a normative criterion to these approaches, Bose (1968) and Weitzman (1971) established an optimum rate of investment allocation in a context of dynamic optimisation of consumption. However, these analyses did not take into account the composition of consumption demand. In order to mitigate the limitations of the F-M model in relation to the passive role of per capita consumption demand, Araujo and Teixeira (2002) have shown that the F-M model may be treated as a particular case of Pasinetti's model of structural change. In this case it was possible to establish the rate of investment allocation which guarantees that the economy is in its stable growth path.

Araujo and Teixeira (2002) have introduced a normative criterion to define the rate of investment allocation but it is important to note that their result is only normative and it remains the question of what will be the rate of investment allocation in a positive economy. Here we answer this question by showing that the PKGM may be treated as an aggregated version of the F-M model. This fact is not a novelty since both models are vertically integrated.

By following this approach it is possible to determine the rate of investment allocation compatible with the equilibrium in the credit market given by the PKGM growth model. Then it is possible to compare this rate with the normative one obtained from the F-M model as found by Araujo and Teixeira (2002). These results points to the importance of the credit market in determining the existing conditions for capital accumulation. If the decisions on investment allocation were distorted as a consequence of wrong expectations of savers and investors then less capital may be accumulated than what is necessary to endow the economy with the required capital goods to keep the economy in equilibrium.

This paper is structured as follows: in the next section we present a brief overview of the PKGM. In section 3 we show that the PKGM may be disaggregated

into a two sector model in the lines of the F-M model by using the device of vertical integration. Furthermore the rate of investment allocation is also derived and it is compared with the one warranted rate of investment allocation obtained from the Pasinetti's model. Section 4 searches to extends this results to a more disaggregated economy. Section 5 summarizes the results.

2. The Post-Keynesian Growth Model: A Brief Overview

An important characteristic of the PKGM is the existence of independent investment and savings functions that depends on income distribution. The saving propensities, for instance, are particular to each class may it be workers or capitalists. Unlike the Neoclassical model, the PKGM considers that neither savings nor technological progress is the variable that drives the growth process. The rationale is that investment is determined essentially by the availability of credit in the financial sector as well as the 'animal spirits'. Once investment is made effective demand determines output which in turns determines savings.

The main assumptions behind the PKGM are noted: the economy is closed and produces only one good that can be both a consumption as well as a capital good. Technology is characterized by fixed coefficients. Likewise, there are constant returns to scale. There is no government, and the monetary side is ignored. All firms are equal in the sense that they wield no differences in market power. In such an economy, the value of net aggregate output is equal to the sum of the wages and profits, namely:

$$pX = wN + rpK \qquad (1)$$

where p is the price level, X is the level of real output, w is the nominal wage rate, N is the level of labour employment, r is the rate of profit and K is the stock of capital. Expression (1) may be rewritten as:

$$p = w\frac{N}{X} + rp\frac{K}{X} \tag{1'}$$

Now define $l = \frac{N}{X}$ as the labour per unit of output, $v = \frac{K}{X_{fe}}$ as the capital-output ratio and $u = \frac{X}{X_{fe}}$ as the rate of capacity utilization, where X_{fe} stands for the full employment output. By using this notation $\frac{K}{X} = \frac{u}{v}$ and assuming that v is constant and normalized to one we can rewrite expression (1)' as:

$$p = wl + rpu^{-1} \tag{1''}$$

Let us assume that prices are given by a mark-up rule over wage according to:

$$p = (1+\tau)wl \tag{2}$$

Where τ is the mark-up rate. By substituting expression (2) into (1)'', simple algebraic manipulation allows us to obtain the following relationship between the profit share, the rate of profit and the rate of capacity utilization:

$$r = \pi u \tag{3}$$

Implicit in this result is the fact that the profit share is given by: $\pi = \frac{\tau}{(1+\tau)}$. Expression (3) gives us the profit rate from the supply side of the model. In order to find the profit rate from the demand side let us consider separately the contribution of some authors in order to emphasize the evolution of the model.

Kaldor (1956) and Robinson (1956, 1962) have built models on the notion of full employment and full capacity utilization that contemplate both the supply and demand sides to determine the growth rate of a closed economy. There are some differences between the approaches developed by these authors; however, the core of their models may be described as follows. It is assumed that workers do not save and the economy operates at full capacity[3]. The growth rate of investment, g_I, is assumed to be given by:

$$g_I = g_o + \alpha r \tag{4}$$

where $\alpha > 0$ measures the influence of the investment to the interest rate, r, and g_o stands for the growth rate of autonomous investment. The positive effect of the rate of profit on investment decisions relies on the relation between actual and expected profits.

In order to take into account the possibility of disequilibrium, Dutt (1984) and Rowthorn (1982), by working independently, have built what is known as the second generation of the Post-Keynesian growth model by endogenizing the rate of capacity utilization in the lines of Steindl (1952). One of the main contributions of this second generation is the possibility of a stagnationist regime in which an increase in the profit share implies a reduction in the capacity utilization. The key assumption behind this result is that the growth rate of investment is a function not only of the profit rate, as in Kaldor-Robinson but also of the rate of capacity utilization [Steindl (1952)]:

$$g_I = g_o + \alpha r + \beta u \tag{5}$$

where $\beta > 0$ measures the sensibility of the growth rate of investment to the capacity utilization and captures the accelerator effect: a high rate of capacity utilization induces

[3] Robinson (1956, 1962) refers to a ‘normal’ rate of capacity utilization to express that degree of utilization of productive capacity that producers consider as ideally suited to fulfill demand requirements.

firms to expand capacity in order to meet anticipated demand while low utilization induces firms to contract investment.

Bhaduri and Marglin (1990) have challenged this view by considering that the growth rate of investment is a function of the rate of capacity utilization and of the profit share. According to them the rate of profit has already been implicitly considered in the equation of the growth rate of investment through the rate capacity utilization and due to the following macroeconomic relation $r = \pi.u$. Hence by substituting the rate of profit by the profit share in the expression of the growth rate of investment avoids to consider twice the effects of the former on the growth rate of investment. One of the properties of this third generation model, as it became known is the possibility of a non-stagnationist regime. In Bhaduri and Marglin (1990) the investment function now reacts positively to profits and capacity utilization, given that the profit-share is used as a measure of profitability[4]. Therefore:

$$g_I = h(\pi, u) \tag{6}$$

with partial derivatives $h_\pi(\pi, u) > 0$ and $h_u(\pi, u) > 0$.

According to Bhaduri and Marglin (1990, p. 380), influences of existing capacity on investment cannot be captured satisfactorily by simply introducing a term for capacity utilization. The investment function should also consider profit share and capacity utilization as independent and separate variables in the lines of expression (6). Following Blecker (2002, p. 137) let us assume, for the sake of convenience only, a linear version of the investment function:

[4] Bhaduri and Marglin (1990) do not linearize the investment function but some authors such as Blecker (2002) adopted a linearized version to obtain closed form solutions for the endogenous variables.

$$g_I = g_o + \alpha\pi + \beta u \qquad (6)'$$

The growth rate of savings, g_S, is given by the Cambridge equation in all generations:

$$g_S = sr \qquad (7)$$

where s is the saving propensity, with $0 \le s \le 1$. Note that equation (7) does not establish the rate of profit as in the Kaldor-Pasinetti process – where the natural growth rate is given – and determines the rate of profit once the propensity to save is exogenous [See Araujo (1992-93)].

The main results of the PKGM are summarized in the table 1 according to its respective generation:

	Kaldor-Robinson	Neo-Kaleckian	Bhaduri-Marglin
Rate of capacity utilization	$u^* = 1$	$u^* = \frac{g_o}{\pi(s-\alpha)-\beta}$	$u^* = \frac{g_o + \alpha\pi}{s\pi - \beta}$
Profit Rate	$r^* = \frac{g_o}{s-\alpha}$	$r^* = \frac{\pi g_o}{\pi(s-\alpha)-\beta}$	$r^* = \frac{\pi(g_o + \alpha\pi)}{s\pi - \beta}$
Growth rate	$g^* = \frac{sg_o}{s-\alpha}$	$g^* = \frac{s\pi g_o}{\pi(s-\alpha)-\beta}$	$g^* = \frac{s\pi(g_o + \alpha\pi)}{s\pi - \beta}$

Table 1

The main properties of the PKGM are well known and may be summarized as: (i) both in the first and in the second generation we have a wage-led regime in which an increase in the profit share yields smaller profit and growth rates. This result is a stagnationist regime of economic growth. In the third generation, possibilities arise of

an exhilarationist regime in which the growth is profit-led, that is, an increase in the profit share yields higher profit and growth rates and, (ii) the prevalence of the wage or the profit-led growth regime depends on the magnitude of the parameters of the model.

3. The Model

A procedure to prove our results is to perform an initial disaggregation of the PKGM into two sectors in the lines of the Feldman's two sector growth model. According to Bhaduri and Marglin (1990, p.377) in the PKGM "we can think of the representative firm as vertically integrated using directly and indirectly a constant amount of labour per unit of final output." Araujo and Teixeira (2002) and Halevi (1996) also have shown that F-M model may be seen as a particular case of Pasinetti (1981) by using the device of vertical integration. Then by focusing on the degree of aggregation it is possible to say that the main difference between the PKGM and F-M model is that while the former is aggregated in one sector the latter is aggregated in two sectors. But the device adopted to build these models is the same, namely vertical integration.

This view is also supported by other authors such as Lavoie (1997) and Scazzieri (1990) for whom the concept of vertical integration has been extensively but implicitly used in macroeconomic analysis[5]. From this standpoint let us consider a two-sector version of the PKGM with special focus on the supply side. We know from the Feldman's model that the investment sector grows at:

[5] This view is confirmed by Steedman (1992, p. 136) for whom "Kaleckian writings frequently appeal to vertically integrated representations of the economy."

$$\frac{I}{K} = \frac{X_{k_1}}{K} \tag{8}$$

where X_{k_1} is the production of capital goods, which is described by Leontief production functions and the limiting factor of production is the stock of capital goods. Hence:

$$X_{k_1} = \min\left[\frac{K_{k_1}}{v_{k_1}}, \frac{L_{k_1}}{\upsilon_{k_1}}\right] \Rightarrow X_{k_1} = \frac{K_{k_1}}{v_{k_1}} \tag{9}$$

where K_{k_1} refers to the stock of investment goods and v_{k_1} stands for the capital-output ratio in the capital goods sector while L_{k_1} and υ_{k_1} are the quantity of employed working force and the labour coefficients respectively. By substituting (9) into (8) we obtain:

$$\frac{I}{K} = \frac{K_{k_1}}{v_{k_1} K} \tag{10}$$

For the sake of convenience only, it is assumed that there is no depreciation of capital goods, the investment goods cannot be imported and the production of capital goods does not depend on the production of consumption goods sector. Now it is possible to establish the growth rate of investment. The change in investment is given by:

$$\dot{X}_{k_1} = \dot{K}_{k_1} / v_{k_1} \tag{11}$$

But the variation in stock of capital in sector k_1 depends only on the proportion of the total output of this sector that is allocated to itself. We assume that a proportion λ of the current production of the investment sector is allocated to itself while the remaining, $1 - \lambda$, is allocated to sector 1 ($1 \geq \lambda \geq 0$). Hence:

$$\dot{K}_{K_1} = \lambda X_{k_1} \tag{12}$$

Substituting (12) into (11) leads to the growth rate of the investment sector:

$$\frac{\dot{X}_{k_1}}{X_{k_1}} = \frac{\lambda}{v_{k_1}} \tag{13}$$

Let us assume that the production in the consumption sector is also described by Leontief production function with the limiting factor of production the stock of capital goods.

$$X_1 = \min\left[\frac{K_1}{v_1}, \frac{L_1}{\upsilon_1}\right] \Rightarrow X_1 = \frac{K_1}{\upsilon_1} \tag{14}$$

where K_1 refers to the stock of investment goods and v_1 stands for the capital-output ratio in the consumption goods sector while L_1 and υ_1 are the quantity of employed working force and the labour coefficients respectively. Adopting the same procedure in relation to the consumption sector and considering that $\dot{K}_1 = (1-\lambda)X_{k_1}$,we establish its growth rate:

$$\frac{\dot{X}_1}{X_1} = \frac{(1-\lambda)X_{k_1}}{v_1 X_1} \tag{15}$$

Taking limits of both sides of expression (15) when t tends to infinity and applying the L'Hôpital rule lead us to conclude that the growth rate of consumption depends on the growth rate of investment and, in the long run, the former converges to the later, which will be the growth rate of the economy as a whole.

$$\lim_{t\to\infty}\frac{\dot{X}_1}{X_1} = \frac{\lambda}{v_{k_1}} \tag{16}$$

Besides the composition of capital goods in this economy will be given by:

$$\frac{K_{k_1}}{K_1} = \frac{\lambda}{1-\lambda} \quad (17)$$

The results in the third line of table 1 yield the investment in equilibrium normalized by the stock of capital. Table 2 shows this outcome:

	Kaldor-Robinson	Neo-Kaleckian	Bhaduri-Marglin
Growth rate	$\left(\frac{I}{K}\right)^* = \frac{sg_o}{s-\alpha}$	$\left(\frac{I}{K}\right)^* = \frac{s\pi g_o}{\pi(s-\alpha)-\beta}$	$\left(\frac{I}{K}\right)^* = \frac{s\pi(g_o+\alpha\pi)}{s\pi-\beta}$

Table 2

By equalizing these results with expression (10) we obtain for each case the following share for the stock of capital goods of sectors k_1 and 1 in total stock of capital. This is shown is table 3:

Kaldor-Robinson	Neo-Kaleckian	Bhaduri-Marglin
$\left(\frac{K_{k_1}}{K}\right)^* = \frac{v_{k_1} sg_o}{s-\alpha}$	$\left(\frac{K_{k_1}}{K}\right)^* = \frac{v_{k_1} s\pi g_o}{\pi(s-\alpha)-\beta}$	$\left(\frac{K_{k_1}}{K}\right)^* = \frac{v_{k_1} s\pi(g_o+\alpha\pi)}{s\pi-\beta}$
$\left(\frac{K_1}{K}\right)^* = \frac{s-\alpha-v_{k_1} sg_o}{s-\alpha}$	$\left(\frac{K_1}{K}\right)^* = \frac{(s-\alpha)\pi-\beta-v_{k_1} s\pi g_o}{\pi(s-\alpha)-\beta}$	$\left(\frac{K_1}{K}\right)^* = \frac{s\pi-\beta-v_{k_1} s\pi(g_o+\alpha\pi)}{s\pi-\beta}$
$\left(\frac{K_{k_1}}{K_1}\right)^* = \frac{v_{k_1} sg_o}{s-\alpha-v_{k_1} sg_o}$	$\left(\frac{K_{k_1}}{K_1}\right)^* = \frac{v_{k_1} s\pi g_o}{\pi(s-\alpha)-\beta-v_{k_1} s\pi g_o}$	$\left(\frac{K_{k_1}}{K_1}\right)^* = \frac{v_{k_1} s\pi(g_o+\alpha\pi)}{s\pi-\beta-v_{k_1} s\pi(g_o+\alpha\pi)}$

Table 3

By equalizing these results to (17) we obtain:

Kaldor-Robinson	Neo-Kaleckian	Bhaduri-Marglin
$\lambda^* = \dfrac{v_{k_1} s g_o}{s-\alpha}$	$\lambda^* = \dfrac{v_{k_1} s \pi g_o}{\pi(s-\alpha)-\beta}$	$\lambda^* = \dfrac{v_{k_1} s \pi (g_o + \alpha\pi)}{s\pi - \beta}$

Table 4

The procedure adopted here ensures that the economic system will be endowed with the capital goods required to fulfil the requirements expressed by the equalization of savings and investment decisions in the PKGM. In order to proceed to capital accumulation it is necessary to build the background in terms of the expansion of the production of the capital sector to meet the demand requirements. But we know from the normative version of the F-M model that the rate of investment allocation is given by:

$$\lambda^* = v_{k_1}(n+\theta) \tag{18}$$

where n is the growth rate of population and θ is the growth rate of demand. Expression (18) is a normative criterion to the F-M model and may be seen as a warranted rate of investment allocation because if it is not fulfilled then the economic system will not have the productive capacity to produce the capital goods necessary to meet the demand requirements expressed by the growth rate of per capita demand. By equalizing expression (18) to the expressions in the table 4 we conclude that the saving rate that must be adopted by capitalists in order to ensure meet the warranted rate of investment allocation is given by:

Kaldor-Robinson	Neo-Kaleckian	Bhaduri-Marglin

$s^* = \frac{n+\theta-g_o}{\alpha(n+\theta)}$	$s^* = \frac{(n+\theta)(\pi\alpha+\beta)}{\pi(g_o+n+\theta)}$	$s^* = \frac{(n+\theta)\beta}{n+\theta-g_o-\alpha\pi}$

Table 5

Of course that in a capitalist economic there is no guarantee that these saving rates will be adopted by capitalists and equilibrium will occur only by a fluke as in the Harrod-Domar model. The situation is even worse if we consider a multi-sector economy in the lines of the Pasinettian model. This can be accomplished since F-M model were shown to be a particular case of the Pasinetti's model and then it is possible to assign to each sector a warranted rate of investment allocation.

From the perspective presented in this section, the limitations of the F-M model in relation to the passive role of per capita consumption demand are diminished. In the present case, the composition of investment will reflect, on the input side, the same order of priorities in which production of consumption goods is organised according to the consumer's preferences.

4. Towards a more disaggregated Economy

The analysis of the previous section may be extended to an arbitrary number of sectors. As shown by Araujo and Teixeira (2002) the Feldman's model is built under the notion of vertical integration and may be seen as a particular case of the Pasinetti's model of structural change and economic growth. Hence it is possible to consider the analysis of investment allocation in a multi-sector economy in each every sector is subject to a particular rate of growth of demand and technological progress. In this case the sectoral rate of rate of investment allocation is given by:

$$\lambda_i^* = v_{k_i}\left(n + \theta_i\right) \quad (18)'$$

Where θ_i is the growth rate of demand for the consumption good *i* and v_{k_i} is the capital-output ratio for the *i*-th sector. As shown by Araujo and Teixeira (2011) it is also possible to consider a multi-sector version of the PKGM and in this vein to consider sector expressions for the investment and savings according to the rationale to the generations of this model. According to them it is possible because the PKGM is also build on the notion of vertical integration. In this case the analysis of the previous sections may be extended to a multi-sector economy and each sector and the actual rate of investment allocation for each sector will be given by the following table according to each generation:

Kaldor-Robinson	Neo-Kaleckian	Bhaduri-Marglin
$\lambda_i^* = \dfrac{v_{k_i} s g_o^i}{s - \alpha_i}$	$\lambda_i^* = \dfrac{v_{k_i} s \pi_i g_o^i}{\pi_i (s - \alpha_i) - \beta_i}$	$\lambda_i^* = \dfrac{v_{k_i} s \pi_i (g_o^i + \alpha_i \pi_i)}{s\pi_i - \beta_i}$

Table 6

Now $\alpha_i > 0$ measures the influence of the investment to the interest rate in the *i*-th sector, π_i stands for the profit share in *i*-th sector and g_o^i stands for the growth rate of autonomous investment. By adopting the approach of the previous section, it is possible to understand now that each sector should have its own growth rate compatible with the correct allocation of capital goods according to the evolution of preferences. Hence by particularizing a saving rate for each sector we obtain:

Kaldor-Robinson	Neo-Kaleckian	Bhaduri-Marglin

$s_i^* = \dfrac{n+\theta_i - g_o^i}{\alpha_i(n+\theta_i)}$	$s_i^* = \dfrac{(n+\theta_i)(\pi_i\alpha_i+\beta_i)}{\pi_i(g_o^i+n+\theta_i)}$	$s_i^* = \dfrac{(n+\theta_i)\beta_i}{n+\theta_i - g_o^i - \alpha_i\pi_i}$

Table 7

These results show that the fulfilment of the capital accumulation conditions in each sector requires the existence of particular saving rates for each sector. Besides, Pasinetti (1981) shows that in fact each sector has to be a particular rate of profit in order to fulfil the demand requirements. He has called this profit rate as natural ones and has showed that for each sector the natural rate of profit is given by:

$$r_i^* = g + \theta_i \tag{19}$$

Note that if $r_i < g + \theta_i$ then capitalists in the *i*-th sector will not have the necessary amount of resources to invest in such sector in order to meet the expansion of demand. If $r_i > g + \theta_i$ then capitalist will overinvest in the *i*-th sector leading to excess of productive capacity. Araujo and Teixeira (2011) have shown that the multi-sectoral version of the PKGM also entails the derivation of the profit rate, which is in fact an actual profit rate.

	Kaldor-Robinson	Neo-Kaleckian	Bhaduri-Marglin
Profit Rate	$r_i^* = \dfrac{g_o^i}{s_i - \alpha_i}$	$r_i^* = \dfrac{\pi_i g_o^i}{\pi_i(s_i - \alpha_i) - \beta_i}$	$r_i^* = \dfrac{\pi_i(g_o^i + \alpha_i\pi_i)}{s_i\pi_i - \beta_i}$

Table 8

By equalizing the natural profit rate with the actual profit rate it is also possible to obtain the saving rate for each sector that fulfils the capital accumulation condition, namely:

	Kaldor-Robinson	Neo-Kaleckian	Bhaduri-Marglin
Saving rates	$s_i = \frac{g_o^i}{n+\theta_i} + \alpha_i$	$s_i = \frac{\beta_i}{\pi_i} + \frac{g_o^i}{n+\theta_i} + \alpha_i$	$s_i = \frac{\beta_i}{\pi_i} + \frac{g_o^i + \alpha_i \pi_i}{n+\theta_i}$

Table 9

It is important to note that the results of the table above are different from the one obtained to guarantee the equalization of the actual rate of investment allocation with the natural rate of investment allocation. Hence in general it is not possible to establish sectoral saving rates compatible with two different goals: endow vertically integrated sectors with the right composition of capital goods and give capitalists the warranted rate of profit.

5. Concluding Remarks

In this article, it was shown that by treating the PKGM as a particular case of the F-M model of investment allocation it is possible to obtain a new result concerning a central question on the theory economic development. The standpoint of the analysis is the concept of vertical integration which allows us to establish a correspondence between the two approaches. Then it was possible to study how the demand side, portrayed by the decisions of savings and investment may affect the decisions of investment allocation. The influence of these factors on the investment allocation between capital and consumption goods sectors were analysed in order to establish the rate of investment allocation subject to the equilibrium in the credit market. This rate is determined by taking into account the structure of consumer preferences. This fact shows that the structural economic dynamics is conditioned not only to patterns of

evolution of demand and diffusion of technological progress but also on the distributive features of the economy that can give rise to different regimes of economic growth.

It was also shown that when dealing with the most general version of the PKGM, where capital goods are considered, there is an additional expression in the system of equations that characterize the economic system to be verified. So we were able to formalise some important descriptive ideas contained in Halevi's paper, and therefore to proceed to a more technical discussion of these matters.

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