ECONOMICS WORKING PAPERS SERIES

Economics Working Paper 34

Price Competition in Segmented Industries*

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April 1993

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Keywords: Firm Behavior, Oligopoly and other imperfect markets.

Journal of Economic Literature classification: D21, L13.

* This article is based on Chapter 2 of my doctoral dissertation. I wish to thank Timothy Bresnahan, Frank Wolak, Avner Greif and participants in the Industrial Economics Workshop at Stanford University. I am responsible of any remaining errors.

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Abstract

Repeated interaction between duopolists in a segmented industry is considered. The industry is fragmented into two separate segments. The duopolists compete in prices and segment choice, assuming that pricing strategies are completely flexible while segment choice is irreversible. Initially the two firms are located in different segments of the market, but they can choose to extend their operation to the other segment, operating in the whole industry. It is shown that there exists an equilibrium involving segment locations and collusion in prices. The equilibrium path is further restricted by the magnitude of the fixed cost of entering the other segment, and by refinements on the equilibrium concept. Finally, the implications of the irreversibility of the entry decision are analyzed.

1 Introduction

The purpose of this article is to analyze the extent of price competition in a model where firms can determine the degree of segmentation of an industry. The industry's demand is fragmented into separate markets. The firms can change the degree of multi-market contact among themselves by choosing to remain in a single segment or deciding to operate in all the segments of the market. The main result is as follows. An equilibrium exists where firms extend their operation across all the segments of the market and collude in prices.

In what follows *segmentation* [1] refers to any kind of separation in an otherwise homogeneous market. This separation can arise because of the following reasons:

- Buyers segmented into different types.
- Different product lines.
- Geographic separation.

It is assumed that initially two local monopolists operate in separate segments of the market. The firms have the choice to break this segmentation by entering into the rival's segment and competing in prices.

The model presented here is similar to models of price-competition when capacity is constrained by an initial pre-commitment to quantities, as in Kreps and Sheinkman (1983) extended to more general settings by Benoit and Krishna (1987) and Osborne and Pitchik (1987). Stigler (1968), Spence (1977) and Dixit (1979) investigate the possibility of collusion and competition along pricing and non-pricing strategies. Fershtman and Muller (1986) consider semi-collusive equilibria in an industry where firms cooperate in setting a short-run variable such as prices, but their bargaining power is affected by their choice on a sticky variable that can be changed only in the long-run. On the other hand, Davidsen and Deneckere (1990) consider also semi-collusive equilibria, but in a two-stage game where firms choose capacities at the initial stage and compete in prices afterwards, constrained by the capacities chosen at the initial stage.

The demand structure used in this article is similar to that in Bernheim and Whinston (1990), where the demand for the industry is segmented but products and consumers in each segment are homogeneous. Two assumptions are imposed on the demand structure: (1) the operation of additional firms in a segment of the market expands the demand in that segment and (2) prices are set uniformly across the segments of the market. This demand structure

[1] The concept of fragmented duopoly has been used by development economists to study backward agriculture. See, for instance, Basu and Bell (1991).

corresponds to industries where each firm has a base of consumers, gained by brand reputation or loyalty, plus a pool of consumers that may be disputed among the competitors. This pool of consumers may decide to participate in the market or not by considering if the market may be monopolized. In the present model, if all firms decide to operate in one of the segments of the market, the consumers expect this market to be more competitive than if only one firm operates in it. This justifies the assumption of demand expansion. On the other hand, price uniformity is common in many industries, for instance industries where firms are required by regulation to treat all consumers equally, if they operate in markets with identical characteristics.

In a model with homogenous products, price–setting oligopolists compete away all profits by mutual undercutting of prices until the breakeven level is reached. This is known as the *Bertrand paradox*. Capacity constraints, product differentiation and repeated interaction allow firms to reduce the aggressiveness in pricing strategies, therefore allowing them to keep positive profits despite of their competition in prices. The current paper formulates a multi–period game where in each period the firms compete in segment choice and prices. It discusses an additional motive for a reduction in price aggressiveness. The two crucial assumptions are the state of initial absence of market contact and the uniformity in prices. Suppose that a firm is considering whether to undercut the prevailing market price. This firm may be deterred from undercutting if this move hurts it across all segments of the market. The particular equilibrium obtained, characterized by segment expansion and collusion in prices, has also interesting implications for price regulation.

The paper is organized as follows: Section 2 states the definitions and assumptions of the model, while section 3 presents the finite one–period and two–period games, section 4 analyzes the infinite game and section 5 extends the main result for renegotiation–proof equilibria and studies the implications of irreversibility of segment choice.

2 Definitions and Assumptions

Consider two firms producing a perfectly substitutable good at no cost [2]. There is a market for this good divided into two identical segments. Originally, each firm operates as a local monopolist in one segment.

Competition is modeled as an infinite non–cooperative game, in which the same game is played at each period t. The single period game is as follows: The firms decide simultaneously the price that they want to set and the segments where they want to operate. If a firm has not yet entered the other segment, it decides whether it wants to enter in the next period or not, paying a fixed

[2] All the results hold if we consider instead constant marginal cost c, using the net price $p - c$.

cost of location F in the current period. Simultaneous with this decision, the firms choose a price to compete in the current period according to the state of industry segmentation. There are no capacity limitations.

The state variable X_i indicates if firm i has entered the other segment or not, according to

$$X_i^t = \begin{cases} 1 & \text{if firm } i \text{ has entered,} \\ 0 & \text{otherwise.} \end{cases}$$

The policy variable M_i^t represents the location decision as follows:

$$M_i^t = \begin{cases} 1 & \text{if firm } i \text{ decides to enter,} \\ 0 & \text{otherwise.} \end{cases}$$

The following assumption is used in the next three sections of the paper:

(A1) Entry in the other segment is irreversible.

Two identical demand functions represent consumer preferences in each segment: $\phi_l(p)$ if there is only one firm operating in the segment and $\phi_c(p)$ if there are two firms. These demand functions are assumed to have the following properties:

(A2) $\phi_l(p)$ and $\phi_c(p)$ are twice continuously differentiable and concave, $\phi_l(p)$ ¡ $\phi_c(p)$, and with $\bar{p}_l < \bar{p}_c$ such that $\phi_l\ (\bar{p}_l) = \phi_c\ (\bar{p}_c) = 0$.

Assumption (A2) implies that the revenue functions are concave and single–peaked. It also implies that demand will be expanded when two firms are operating in the same segment instead of one. This expansion in demand is needed to rank the outcomes of the model. With demand expansion, the firms will find it profitable to extend operation to all segments of the market, if a collusive outcome is sustainable. All the results would hold (*weakly*) without this assumption, since the firms would be indifferent in a long–run equilibrium between staying as local monopolists or operating in both markets and colluding in prices.

Let Π_l and Π_c denote the monopoly profits for ϕ_l and ϕ_c, [3] with associated monopoly prices p_l ¡ p_c. Define Π_{cl} as the monopoly profits obtained when there is a firm located in both segments, while the other firm is operating only in its own segment, that is $\Pi_{cl} = \max_p p(\phi_c + \phi_l)$ with associated price p_{cl}. Additionally, define the price $p_R = \min\{p; p(\phi_l + \phi_c) = \Pi_l\}$, which is the minimum price at which a firm is indifferent between operating as a local monopolist in its own market or operating in both segments. Furthermore, Π_{Rc} and Π_{Rl} are defined as $p_R\phi_c(p_R)$ and $p_R\phi_l(p_R)$. Obviously, $\Pi_{Rc} + \Pi_{Rl} = \Pi_l$. Figure 1 illustrates these definitions.

If one firm sets a lower price than its rival, it gains all the market for its product since there are no capacity limitations. If the firms set the same price, than their share of the market is assumed to be one half. Prices are uniform

[3]The argument of the demand function is dropped when it can be inferred from context.

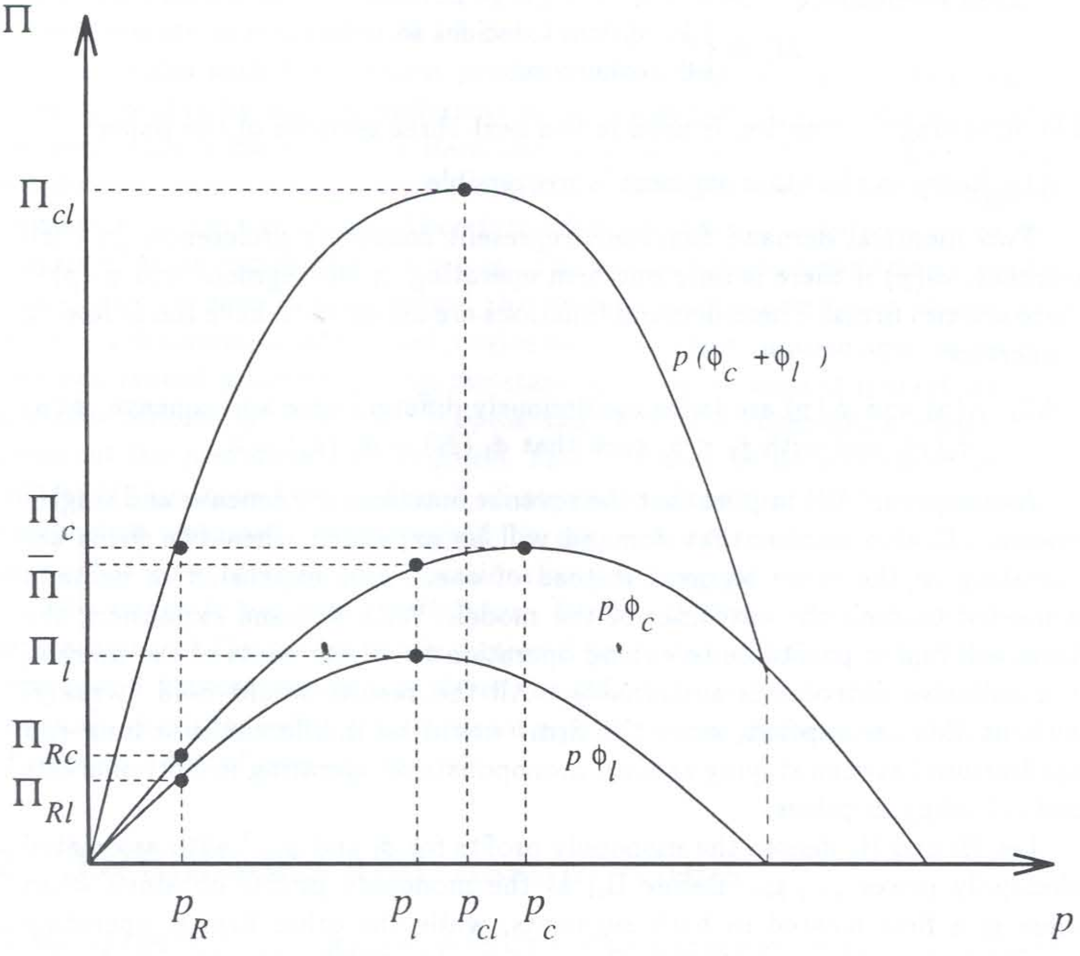

Figure 1: Revenue functions

across segments of the market. This can be summarized by the following assumption over the profit functions:

(A3) The profit function for firm i given that it sets p_i and firm j sets p_j, $i, j = 1, 2$, $i \neq j$ is

$$\Pi(p_i, p_j) = \begin{cases} L_i(p_i) = p_i\big((X_i + X_j)\phi_c + (1 - X_j)\phi_l\big) & \text{if } p_i < p_j \\ C_i(p_i) = p_i\big(\frac{1}{2}(X_i + X_j)\phi_c + (1 - X_j)\phi_l\big) & \text{if } p_i = p_j \\ U_i(p_i) = p_i(1 - X_j)\phi_l & \text{if } p_i > p_j \end{cases}$$

Figure 2 illustrates these functions for different values of the state variables. The payoff functions present a discontinuity at $p_i = p_j$, but are continuous outside this region. According to the value of the state variables, we can have four types of one–period games, which are denoted as $g(X_1, X_2)$. Finally, the sequence of decisions on the infinite game is specified for later use:

(A4) Infinite payoff streams are discounted with a common factor δ to the beginning of period 1, and profits are obtained at the end of each period.

To analyze possible equilibria in the infinite game, first simple one–period and two–period games are discussed.

3 Single Period Competition

3.1 One–period game

The state variables X_i, X_j determine which kind of game it is being played, according to the state of segmentation:

$g(0,0)$: This is a degenerate case where the firms do not interact, since they are operating as local monopolists in their own segments. The equilibrium is $p_1 = p_2 = p_l$ with $\pi_1 = \pi_2 = \pi_l$.

$g(1,1)$: In this case both firms have started operating in the other segment. This is a classical Bertrand game with no capacity constraints, so the equilibrium implies $p_1 = p_2 = 0$ and $\pi_1 = \pi_2 = 0$.

$g(0,1)$: This case is equivalent to $g(1,0)$ with firm i and j reversed. We will discuss this second case.

$g(1,0)$: There is an equilibrium in mixed strategies.

To establish the result for $g(1,0)$ the reaction functions are introduced. Notice that firm 1 remains a local monopolist in its own market. When is it

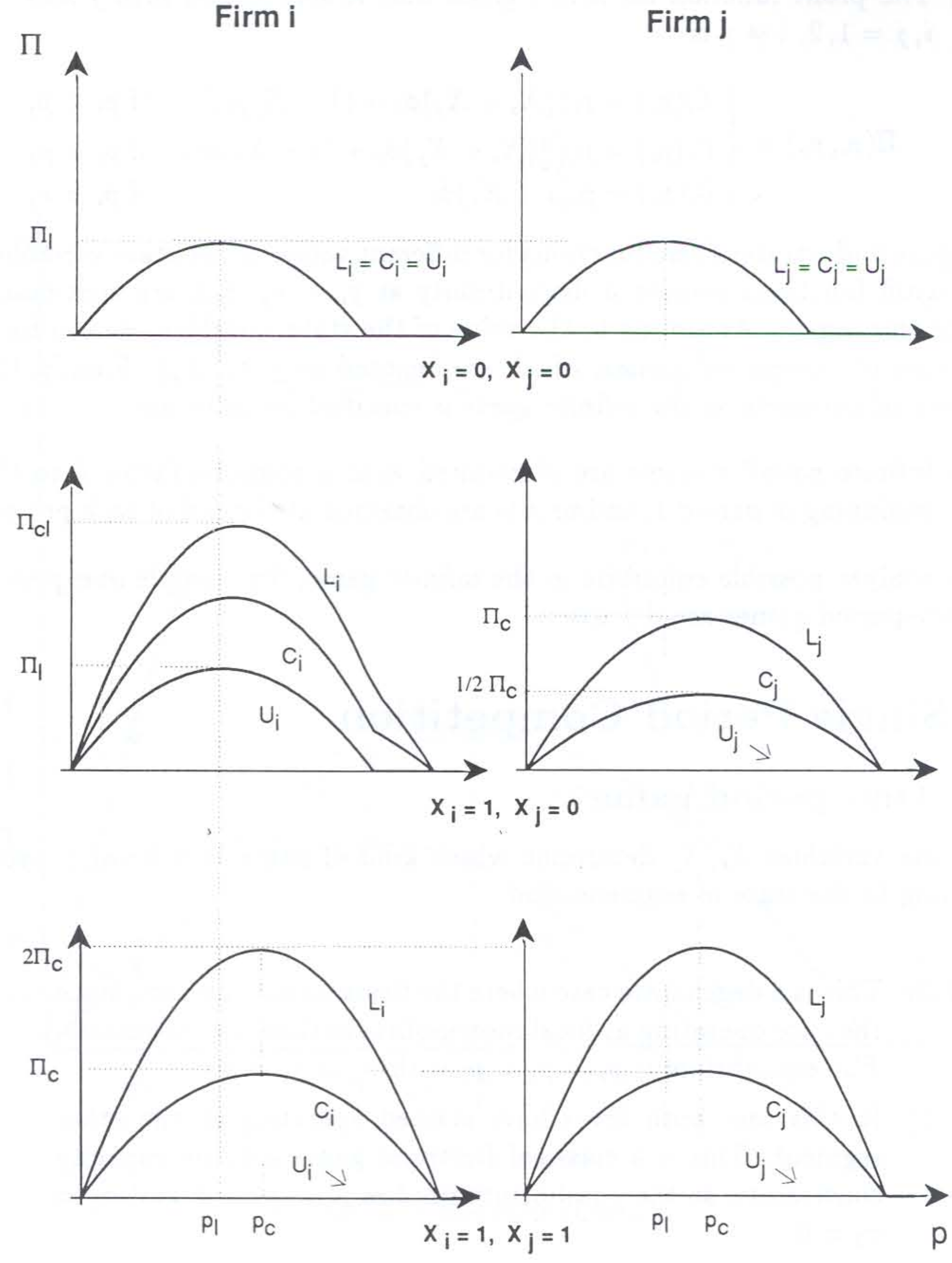

Figure 2: Revenue functions according to industry segmentation

profitable for firm 1 to undercut the price of firm 2? Only when the profits that it gets from operating exclusively in its own market are less than the profits that it gets from operating in both markets. Formally, if the price of firm 2 is such that $p_2 > p_R$, then undercutting is profitable for firm 1, but if $p_2 > p_{cl}$ then it sets p_{cl} since this price maximizes profits for aggregate demand in both segments. Firm 2 always finds it profitable to undercut firm 1. Hence, the reaction functions can be written as:

$$R_1(p_2) = \begin{cases} p_l & \text{if } 0 \leq p_2 \leq p_R \\ \\ p_2 - \epsilon & \text{if } p_R \leq p_2 \leq p_{cl} \\ \\ p_{cl} & \text{if } p_2 \geq p_{cl} \end{cases}$$

$$R_2(p_1) = p_1 - \epsilon$$

where ϵ is positive and close to 0.

The absence of an equilibrium in pure strategies can be determined by inspecting the reaction functions. Firm 1 can always set a price equal to p_l for a minimum profit of Π_l, but this will trigger mutual undercutting until prices are under p_R. This in turn will cause firm 1 to set its price to p_l starting the cycle again[4].

Despite the discontinuity of the payoff functions, an equilibrium in mixed strategies can be constructed[5]. A rationale for the use of mixed strategies, that is a randomization over a range of prices, can be found in a model by Varian (1980). Varian's model consists of firms that have sales, where prices are randomly chosen to take advantage of informational asymmetries among consumers.

Dasgupta and Maskin (1986) discuss the existence of equilibria in games with discontinuous pay–offs. They find that for certain types of discontinuities there exist equilibria for these kind of games. The discontinuity in the current game is of the *right* kind, and an equilibrium exists according to the following theorem:

Theorem 1 *(Dasgupta and Maskin) Let the set of strategies for each player $A_i \subseteq R^1$ be a closed interval and let the payoff function $\Pi_i : A \rightarrow R^1$, $(i = 1,2)$ be continuous except on a subset of $A_1 \times A_2$ of dimension 1. Suppose $\Pi_1 + \Pi_2$ is continuous and $\Pi_i(p_i, p_j)$ is bounded and $\liminf_{a_i \xrightarrow{-} \bar{a}_i} \Pi_i(a_i, a_j) \geq \Pi_i(\bar{a}_i, a_j)$ (left lower semi–continuous in a_i at $\bar{a}_i$). Then the game has a mixed strategy equilibrium.*

[4]This argument is similar to the classical example by Edgeworth (1897) for capacity constrained price–setting firms.

[5]The procedure used here is similar to the one used in Osborne and Pitchick (1986).

This is a version of Theorem 5 in Dasgupta and Maskin (1986), where the assumptions are stronger than needed. The game $g(1,0)$ has continuous payoffs except for the diagonal $D = \{(p_1, p_2); p_1 = p_2\}$. If firm i lowers its price from a position where $p_1 = p_2$, it discontinuously increases its profit. Hence, the profit function $\Pi_i(p_i, p_j)$ is everywhere lower semi-continuous in a_i. Furthermore, $\Pi_1 + \Pi_2 = p\phi_l + p\phi_c$ is continuous everywhere.

Before characterizing a mixed strategy equilibrium, notice that firm 1 should get an equilibrium profit of Π_l, since this is what it gets in its own market and firm 2 can compete away all the profits in the other market. Firm 2 should get at least Π_{rc}, since firm 1 will not call prices below p_R. What drives this result is that firm 1 is not very aggressive in the market of firm 2 because given that it has to set prices uniformly in both markets, it is hurt in the other market by lowering its price too much.

Let $[\underline{p}_1, \bar{p}_1]$ and $[\underline{p}_2, \bar{p}_2]$ be the support for the equilibrium distributions of firm 1 and firm 2 respectively. The following facts are trivial but helpful to characterize a mixed strategy equilibrium:

(A) The minimum price that any of the firms wants to call is p_R. Firm 1 will always choose p_l for prices below p_R, and firm 2 does not have incentives to undercut below p_R (but may undercut below p_l).

(B) Firm 1 will not call a price higher than p_{cl}, since this price maximizes profits in both markets, while firm 2 will not call a price higher than p_c.

(C) $\bar{p}_2 \leq \bar{p}_1$, since for $\bar{p}_2 > \bar{p}_1$ firm 2 gets $U_2(p_2) = 0$. This holds for the interval $\bar{p}_1 \leq p_2 \leq \bar{p}_2$, so that this interval can be eliminated from the support of prices of firm 2.

Lemma 1 *Suppose $\bar{p}_1 \geq \bar{p}_2$ and firm 2 sets $\bar{p}_2$ with probability 0. Then*

1. $\bar{p}_1 = p_l$ *and* $\Pi^* = \Pi_l$.

2. $\underline{p}_1 = \underline{p}_2 = \underline{p}_R$.

Proof: 1. If firm 1 sets $\bar{p}_1$ and firm 2 sets $\bar{p}_2$ with probability 0 or $\bar{p}_1 > \bar{p}_2$, then firm 1 gets $\Pi_1 = U_1(\bar{p}_1) = p_1\phi_l(p_1)$. This is the equilibrium profit of firm 1, since firm 1 has to be indifferent about which price to call on the support of its equilibrium strategy. This is maximized at p_l for a profit of Π_l.

2. Suppose that $\underline{p}_i < \underline{p}_j$. If $\underline{p}_j$ is below p_c, then firm j can raise its profit by setting $\underline{p}_j < p < p_c$. So by fact (B) for this to be an equilibrium we must have $\underline{p}_2 = p_c$ or $\underline{p}_1 = p_{cl}$. But $\underline{p}_i < \bar{p}_1 = p_l$, for a contradiction. Therefore $\underline{p}_1 = \underline{p}_2$. If $\underline{p}_1 = \underline{p}_2 > p_R$ then firm 1 can call p such that $p_R < p < p_1$ for a profit greater than Π_l, which contradicts 1.

QED

Lemma 2 *If $\underline{p}_1$ and $\underline{p}_2$ are not named with positive probability then the equilibrium profit of firm 2 is Π_{Rc}.*

Proof: By fact (A) we know that $\Pi^* \geq \Pi_{Rc}$. By Lemma 1, $p_1 = p_2 = p_R$. If p_1 and p_2 are not named with positive probability then $\Pi_2(p_R) = L_2(p_R) = \Pi_{Rc}$. This has to be firm's 2 equilibrium profit, since firm 2 is indifferent about which price to call on the support of its prices.

QED

An equilibrium will be constructed now with coincident support for both firms equal to $[p_R, p_l]$ and consistent with the conditions and results of Lemma 1 and 2. The profits that firm i obtains if it calls a price p_i and firm 2 randomizes its prices using some distribution F_j is:

$$\Pi_i^* = U_i(F_j - \alpha_j) + C_i\alpha_j + L_i(1 - F_j),$$

where $\alpha_j(p_i)$ is any jump of F_j. Simplifying, and assuming that $\alpha = 0$,

$$F_j = \frac{L_i - \Pi_i^*}{L_i - U_i}.$$

Under $g(1,0)$ we have:

$$F_1(p) = \frac{p\phi_c(p) - \Pi_1^*}{p\phi_c(p)} \quad (1)$$

$$F_2(p) = 1 + \frac{p\phi_l(p) - \Pi_2^*}{p\phi_c(p)} \quad (2)$$

Notice that $F_1(p)$ is always strictly less than 1 and concave, so that it will have a jump at the maximum price of its range. Taking the minimum price of the range, p_R, we can determine Π_1^* and Π_2^*:

$$F_1(p_R) = \frac{\Pi_{Rc} - \Pi_2^*}{\Pi_{Rc}} = 0$$

$$F_2(p_R) = \frac{\Pi_{Rc} + \Pi_{Rl} - \Pi_1^*}{\Pi_{Rc}} = 0$$

implies $\Pi_1^* = \Pi_l$ and $\Pi_2^* = \Pi_{Rc}$.

The maximum of the range for firm 2, $\bar{p}_2$, can be determined directly from substitution into (2):

$$F_2(\bar{p}_2) = 1 + \frac{\bar{p}_2\phi_l(\bar{p}_2) - \Pi_l}{\bar{p}_2\phi_c(\bar{p}_2)} = 1,$$

which implies $\bar{p}_2 = p_l$, and taking the same support for both firms $\bar{p}_1 = p_l$. Substituting into F_1, we have

$$F_1(\bar{p}_1) = 1 - \alpha_1(\bar{p}_1) = \frac{\bar{p}_1\phi_c(\bar{p}_1) - \Pi_{Rc}}{\bar{p}_1\phi_c(\bar{p}_1)}$$

which implies that $\alpha(p_l) = \Pi_{Rc}/\bar{\Pi}$, where $\bar{\Pi} = p_l\phi_c(p_l)$.

The following proposition summarizes the results in this section:

Proposition 1 *Under assumptions* (A1-A3) *the following equilibria arise according to the state of the game:*

1. *$g(0,0)$: This is a degenerate case with no interaction among the firms. Given that both markets are identical, we have $p_1 = p_2 = p_l$ with equilibrium profits $\Pi_1^* = \Pi_2^* = \Pi_l$.*

2. *$g(1,1)$: The equilibrium is in pure strategies with $p_1 = p_2 = 0$ and $\Pi_1^* = \Pi_2^* = 0$.*

3. *$g(1,0)$: There is a unique equilibrium in mixed strategies, with distribution functions:*

$$F_1(p) = \begin{cases} 0 & p < p_R \\ 1 - \frac{\Pi_{Rc}}{p\phi_c} & p_R \leq p < p_l \\ 1 & p \geq p_l \end{cases}$$

$$F_2(p) = \begin{cases} 0 & p < p_R \\ 1 + \frac{p\phi_l - \Pi_l}{p\phi_c} & p_R \leq p < p_l \\ 1 & p \geq p_l \end{cases}$$

and $\Pi_1^* = \Pi_l, \Pi_2^* = \Pi_{Rc}$.

Given the interaction of the decision on location (M_I^t) and the state in the following period (X_I^{t+1}) it is necessary to analyze the two-period game starting from any of the possible states.

3.2 The two-period game

In this section the two-period game is analyzed. It consists of a decision on entering the other segment in the first period and fixing a price for the second period[6]. The following proposition states the results for this game:

Proposition 2 *Under* (A1-A4)*:*

1. *Starting from $(X_i^1, X_j^1) = (1,1)$ there is a unique equilibrium consisting of $(p_1, p_2) = (0,0)$ and $(\Pi_1, \Pi_2) = (0,0)$*

2. *Starting from $(X_i^1, X_j^1) = (1,0)$ there is a sub-game perfect equilibrium consisting of firm j staying out and payoffs (Π_l, Π_{Rc}) at each period.*

[6] Superscripts denote time.

Table 1: Starting state $g(1,0)$: Pay-off matrix

	Firm j ($X_j = 0$) $M_j = 0$	$M_j = 1$
Firm 1 ($X_i = 1$)	$(1+\delta)\Pi_l$, $(1+\delta)\Pi_{Rc}$	Π_l, $\Pi_{Rc} - F$

3. Starting from $(X_i^1, X_j^1) = (0,0)$ *there is a sub-game perfect equilibrium consisting of both firms staying as local monopolists with payoff* (Π_l, Π_l) *each period.*

Proof: First, consider the case where both firms have entered their rival's segment. This corresponds to the standard finite repeated Bertrand game, that has a unique equilibrium consisting of price equal marginal cost at each period. Second consider the case where in period 1 the game is $g(0,1)$. The decision of firm 1 over the policy variable M_1^1 is irrelevant because of the irreversibility of the segment decision. Firm 2 faces the decision to enter the other segment and play $g(1,1)$, that implies $\Pi_2^2 = 0$, or keep playing $g_2^2(1,0)$, that implies $\Pi_2^2 = \Pi_{Rc}$. Therefore it is optimal for firm 2 to stay out of the other segment of the market. This is a sub-game perfect equilibrium, since the outcome of period 2 is the one–period Nash–equilibrium. Table 1 presents this game in matrix form, where the pay-offs are the sum of the two periods considering the fixed cost F of entering the other segment.

Third, consider the case where both firms start as local monopolists in their own markets. In this case both firms will stay as local monopolists, since this is a dominant strategy, as can be noticed by inspecting the game in matrix form given in Table 2.

QED

Proposition 2 illustrates the importance of segmentation and price uniformity in reducing price aggressiveness. Given that one of the firms has a market that it monopolizes, it is not in its interest to fight its rival in the other segment of the market. Knowing this, and under the threat of a price war if it deviates, the firm located in a single segment will not enter its rival segment, and an asymmetric state will be maintained. The one–period and two–period games are useful for determining the sub–game perfect reversion in the infinite game, which is analyzed next.

Table 2: Starting state $g(0,0)$: Pay-off matrix

Firm j ($X_j = 0$)

		$M_j = 0$	$M_j = 1$
Firm i ($X_i = 0$)	$M_i = 0$	$(1+\delta)\Pi_l$, $(1+\delta)\Pi_l$	$\Pi_l + \delta\Pi_{Rc}$, $(1+\delta)\Pi_l - F$
	$M_i = 1$	$(1+\delta)\Pi_l - F$, $\Pi_l + \delta\Pi_{Rc}$	$\Pi_l - F$, $\Pi_l - F$

4 Infinite game

In this section repeated interaction among the firms is considered. A model of repeated competition is useful to examine the degree of cooperation that firms can sustain implicitly. I adopt two concepts of strategic equilibrium: sub-game perfect and renegotiation-proof equilibrium. First consider a simple trigger strategy equilibrium, requiring that all continuation equilibria are sub-game perfect:

Proposition 3 *Under* A1-A4 *for all $\delta \geq 1/2$ and t, there exists $p^* = p_1^t = p_2^t$ in the range $[p_m, p_c]$ with $p_m \phi_c(p_m) = \frac{1-\delta}{\delta} F + \Pi_{Rc}$, such that the following is a sub-game perfect equilibrium:*

Firm i sets $M_i = 1$ and

$$p_i = \begin{cases} p^* & \text{if } X_j^{t+1} = 1 \text{ and } (p_j^{t-1}, p_j^{t-2}, \ldots) = (p^*, p^*, \ldots) \\ 0 & \text{if } X_j^{t+1} = 1 \text{ and } p_j^t - 1 \neq p^* \\ p \in [p_R, p_l] \text{ chosen according to } F_1 & \text{if } X_j^{t+1} = 0 \end{cases}$$

Proof: Suppose that $M_j^1 = 1$. Firm j will have no incentives to deviate from p^* if:

$$\frac{\delta}{1-\delta}\Pi^* \geq 2\delta\Pi^*, \tag{3}$$

where $\Pi^* = p^*\phi_c(p^*)$. Equation (3) holds only if $\delta \geq 1/2$. Firm j also could choose to stay out, avoiding the fixed cost F in the current period and getting Π_{Rc} forever, but would have no incentives to do so if: $\Pi_l - F + \frac{\delta}{1-\delta}\Pi^* \geq$

$\frac{\delta}{1-\delta}\Pi_{Rc} + \Pi_l$, that holds if $\frac{\delta}{1-\delta}(\Pi_c - \Pi_{Rc}) \geq F$. This also sets up the range for supportable collusion profits. The higher the fixed cost is, the higher the lower limit of this range and the smaller the range is.

QED

Figure 3 illustrates the additional restriction on the set of sustainable collusive profits implied by the fixed cost needed to extend operation to the other segment. The minimum sustainable collusive profit (Π_m) depends on the discount rate (δ) and on the fixed cost (F). In Figure 3 sustainable profits as a function of F are shown for the two extreme values of δ, $\delta = 1/2$ and $\delta = 1$. For each value of δ in the interval $[1/2, 1]$ the minimum sustainable profit is a linear function of the fixed cost F.

As a corollary to proposition 3, suppose that $\delta \leq 1/2$ so that collusion is not sustainable if both firms have entered. An asymmetric equilibrium is still possible as follows: Firm i enters and sets p^* above p_l. If firm 2 enters it knows that all profits will be competed away, so it will stay out.

5 Extensions

In this section proposition 3 is extended to allow for more credible punishment paths (renegotiation–proof equilibria, Farrell and Maskin (1983)) and next the assumption of irreversibility of entry is dropped.

5.1 Renegotiation–proof Equilibria

We now restrict the set of equilibrium profit paths by further refining the equilibrium concept. Sub-game perfect equilibria imply suboptimal punishments, since the harsh punishment schemes involved in the continuation plays are Pareto dominated. Renegotiation proof equilibria instead, require that no continuation payoff be dominated by another continuation payoff. In other words, it specifies that each firm begins to play cooperatively and does so as long as the other firm plays cooperatively. If firm j undercuts firm i price, than firm i sets the Nash–equilibrium price until firm j cooperates again, in which case it returns to cooperation. This type of punishment scheme is not always feasible. Define k as the number of periods where a firm will be punishing its rival for a deviation from the collusive price. Take the minimum k such that

$$\frac{\delta}{1-\delta}\Pi^* \geq 2\delta\Pi^* + \sum_{t=k+2}^{\infty} \delta^t \Pi^* \tag{4}$$

hence

$$k(\delta) = min\{k \quad ; \quad \sum_{t=1}^{k+1} \delta \geq 2\delta\} \tag{5}$$

If $\delta = 1/2$ then only for $k = \infty$ will (4) be satisfied (with equality). As δ increases the value of k decreases. Proposition 3 now is modified:

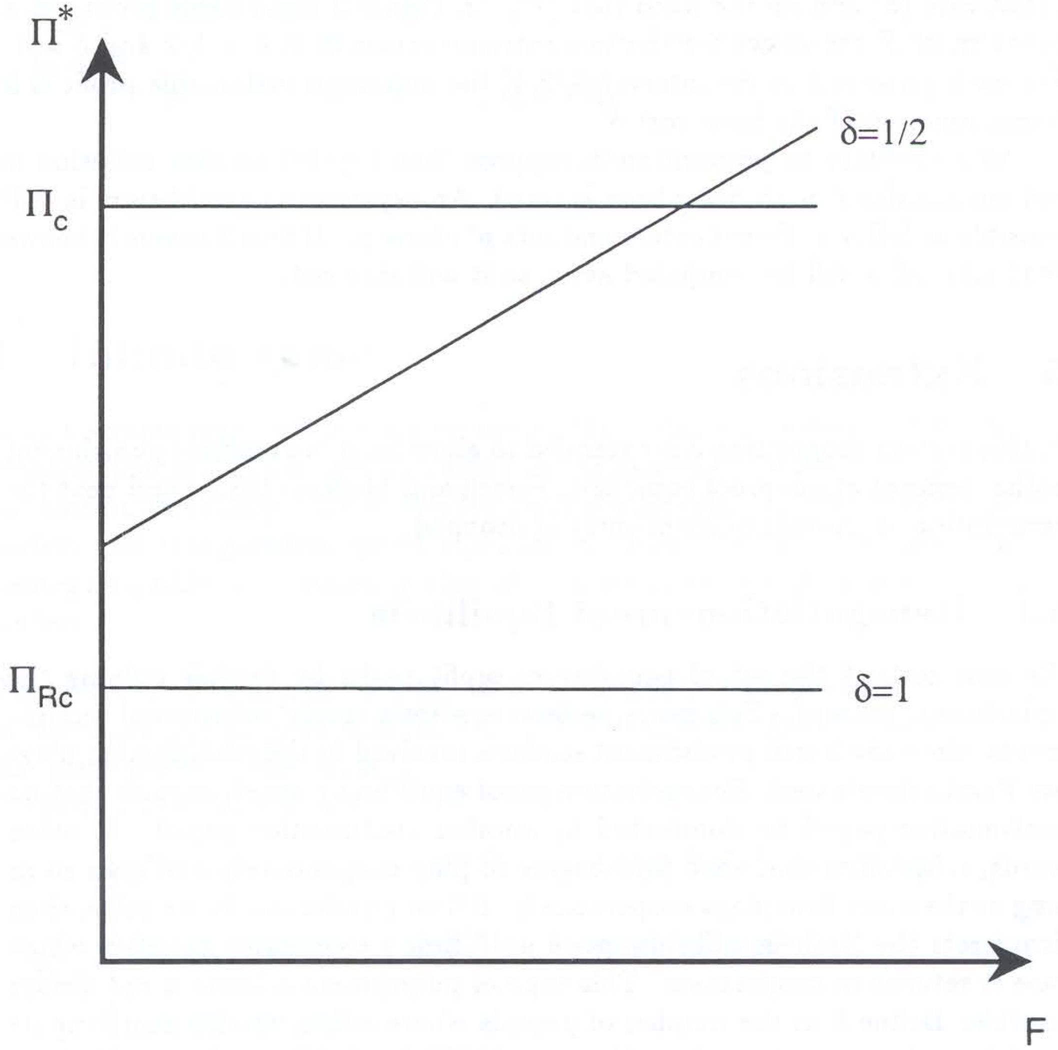

Figure 3: Sustainable collusive profits

Proposition 4 *Under* (A1-A4) *for all* $\delta \geq 1/2$ *and* t *there exists*

1. $k(\delta)$ *satisfying* (5)

2. $p^* = p_1^t = p_2^t$ *in the range* $[p_m, p_c]$ *with* $p_m \phi_c(p_m) = \frac{1-\delta}{\delta}F + \Pi_{Rc}$.

such that the following forms a renegotiation–proof equilibrium:

$$M_i = 1$$

$$p_i = \begin{cases} p^* & \text{if } X_j^{t+1} = 1 \text{ and } p_j^{t-1} = p^* \\ 0 & \text{if } X_j^{t+1} = 1 \text{ and } p_j^t - 1 \neq p^* \\ p \in [p_R, p_l] \text{ chosen} & \\ \text{according to } F_1 & \text{if } X_j^{t+1} = 0 \end{cases}$$

Proof: We have to show that no continuation path is dominated by another continuation path, and that no firm has incentives to deviate from the specified equilibrium path. Considering firm i (by symmetry this argument is valid for both firms), there are three possible continuation paths:

1. Always cooperate:

$$(\frac{\delta}{1-\delta}\Pi^*, \frac{\delta}{1-\delta}\Pi^*)$$

2. Firm i is being punished. It gets $\Pi = 0$ for k periods, setting $p = p_c$ at the k-th period to signal the return to cooperation:

$$(\sum_{t=k+1}^{\infty} \delta^t \Pi^*, \delta^* \Pi^* + \sum_{t=k+1}^{\infty} \delta^t \Pi^*)$$

3. Firm j is being punished with the same scheme as 2.:

$$(\delta^* \Pi^* + \sum_{t=k+1}^{\infty} \delta^t \Pi^*, \sum_{t=k+1}^{\infty} \delta^t \Pi^*)$$

Since k was chosen according to (5) no continuation payoff is dominating. To complete the proof, it is necessary to show that there is no incentive to deviate from the specified continuation paths. First, we now that by our choice of k there is no incentive to deviate from continuation path 1. Path 2. specifies that in a punishment phase the punished firm has to set p_c at period $k+1$. This firm will have no incentives to deviate from this path if the continuation payoff ($\sum_{t=k+1}^{\infty} \delta^t \Pi^*$) is bigger than what it gets from deviating ($2\delta^{k+1}\Pi^*$) plus what it gets after a second punishing phase ($\sum_{t=2k+1}^{\infty} \delta^t \Pi^*$), that is,

$$\sum_{t=k+1}^{\infty} \delta^t \Pi^*) \geq 2\delta^{k+1}\Pi^* + \sum_{t=2k+1}^{\infty} \delta^t \Pi^* \tag{6}$$

Similarly, the third restriction specifies that no firm has incentives to deviate from returning to cooperation after it has been signalled by the other firm:

$$\delta^k\Pi^* + \sum_{t=k+1}^{\infty} \delta^t\Pi^*) \geq 2\delta^{k+1}\Pi^* + \sum_{t=2k+1}^{\infty} \delta^t\Pi^* \qquad (7)$$

Restrictions (6) and (7) are also satisfied by $k(\delta)$.

5.2 Relaxing the Irreversibility of Entry

In this section (A1) is relaxed so that entry into the other segment of the market is reversible. It is assumed that if $M_i^t = 0$ and $X_i^{t-1} = 1$ then in order to exit from the opposite segment of the market an exit cost E has to be paid. The transition equation is now simply:

(A1′) $X_i^t = M_i^{t-1}$.

The results for the one–period game (Proposition 1) are still valid, since they only take into account the current state of the industry. Proposition 2 in modified to take into account the possibility of exit:

Proposition 5 *Under* (A1′),(A2-A4)

1. *Starting from* $(X_i^1, X_j^1) = (1,1)$ *there are two sub–game perfect equilibria in pure strategies:*
   (a) $(M_i^1, M_j^1) = (1,0)$ *with* $(p_i^1, p_j^1) = (0,0)$ *and* (p_i^2, p_j^2) *set according to* $F_i(p) = 1 - \frac{\Pi_{Rc}}{p\phi_c}$ *and* $F_j(p) = 1 + \frac{p\phi_l - \Pi_l}{p\phi_c}$ *over the range* $[p_R, p_l]$ *for a total profit of* $(\Pi_i, \Pi_j) = (\delta\Pi_l, \delta\Pi_{Rc} - E)$.
   (b) $(M_i^1, M_j^1) = (0,1)$ *with* i *and* j *reversed from 1.1.*
2. *Starting from* $(X_i^1, X_j^1) = (1,1)$ *there is a unique sub–game perfect equilibrium in pure strategies consisting of* $(p_i^1, p_j^1) = (p_i^2, p_j^2)$ *set according to* $F_i(p) = 1 - \frac{\Pi_{Rc}}{p\phi_c}$ *and* $F_j(p) = 1 + \frac{p\phi_l - \Pi_l}{p\phi_c}$ *over the range* $[p_R, p_l]$ *for a total profit of* $(\Pi_i, \Pi_j) = \big((1+\delta)\Pi_l, \Pi_{Rc} + \delta\Pi_l\big)$.
3. *Starting from* $(X_i^1, X_j^1) = (0,0)$ *there is a sub–game perfect equilibrium consisting of both firms staying as local monopolists with payoff* (Π_l, Π_l) *each period.*

Proof: The game corresponding to case 1 is summarized in matrix form in Table 3. If firm i chooses $M_i^1 = 0$ then firm j chooses $M_j^1 = 1$ since $\delta\Pi_l > \delta\Pi_l - E$. Similarly, if firm j chooses $M_j^1 = 1$ then firm i chooses $M_i^1 = 0$ since $\delta\Pi_l - E > \delta\Pi_{Rc} - E$. Therefore $(M_i^1 = 0, M_j^1 = 1)$ constitutes an equilibrium in pure strategies. By a symmetric reasoning, it can be established that $(M_i^1 =$

Table 3: Starting state $g(1,1)$: Pay-off matrix for reversible entry

Firm j ($X_j = 0$)

		$M_j = 0$	$M_j = 1$
Firm i ($X_i = 0$)	$M_i = 0$	$\delta\Pi_l - E$, $\delta\Pi_l - E$	$\delta\Pi_{Rc} - E$, $\delta\Pi_l$
	$M_i = 1$	$\delta\Pi_l$, $\delta\Pi_{Rc} - E$	0, 0

Table 4: Starting state $g(1,0)$: Pay-off matrix for reversible entry

Firm j ($X_j = 0$)

		$M_j = 0$	$M_j = 1$
Firm i ($X_i = 0$)	$M_i = 0$	$(1+\delta)\Pi_l - E$, $\Pi_{Rc} + \delta\Pi_l$	$\Pi_l + \delta\Pi_{Rc}$, $\Pi_{Rc} + \delta\Pi_l - F$
	$M_i = 1$	$(1+\delta)\Pi_l$, $(1+\delta)\Pi_{Rc}$	Π_l, $\Pi_{Rc} - F$

$1, M_j^1 = 0)$ constitutes another equilibrium. The game corresponding to point 2 is summarized in Table 4. From the inspection of this table, it can be checked that row 1 and column 1 are weakly dominating, hence $(M_i^1, M_j^1) = (0,0)$ is an equilibrium in pure strategies. Finally point 3 is equivalent to point 3 in proposition 2.

QED

The next step is to consider the infinite game when the assumption of of irreversible segment choice is dropped. Considering proposition 5 we can analyze if it is possible to support collusive outcomes.

Proposition 6 *Under* (A1′), (A2-A4) *a collusive outcome cannot be sustained.*

Proof The long-term payoff of collusion is given again by $\frac{\delta}{1-\delta}\Pi^*$ where $\Pi^* \in (\Pi_l, \Pi_c]$. The optimal deviation consists of undercutting the rival's price in the current period and exiting from the segment, for a pay-off of $2\Pi_c - E$. According to proposition 5, the sub-game perfect reversion specifies that firm i will remain in both segments while firm j will operate only in its segment, for a payoff of $\frac{\delta}{1-\delta}$. Therefore collusion is possible if there exists δ such that

$$\frac{1}{1-\delta} \geq 2\Pi_c - E + \frac{\delta}{1-\delta}\Pi_{Rc} \tag{8}$$

In the Appendix it is shown that there is no positive value of δ that satisfies this equation.

QED

6 Conclusions

In a model of segment choice and price competition it is shown that there is a robust equilibrium where firms enter all segments of the market and collude in prices. The availability of a flexible strategic variable allows the firms to collude implicitly on that variable while competing on a long-run inflexible variable. The existence of different segments in the market and price uniformity reduces the aggressiveness of pricing strategies.

The policy implications of this paper for case industries are the following: first, it helps in understanding the persistence in the use of non-price variables in industries where there is a flexible short-run strategy available. Second, it suggests that the use of non-price strategic variables as a mean to evade price regulation, may survive de-regulation sometimes.

Appendix

A Verification of Proposition 1

We have to show that expected profits are respectively $\Pi_1^* = \Pi_l$ and $\Pi_2^* = \Pi_{Rc}$, and that *almost all* [7] prices in the support for F_1 and F_2 yield the expected equilibrium profits. Expected profits for firm 1 are:

$$\begin{aligned} E(\Pi_1, F_2) &= \int_{p_R}^{p_l} [U_1 F_2 + L_1(1 - F_2)] dF_1 \\ &\quad + Pr(p_1 = p_l)[U_1(p_l)F_2(p_l) + L_1(p_l)(1 - F_2(p_l))] \\ &= \Pi_l(1 - \frac{\Pi_{Rc}}{\bar{\Pi}}) + \Pi_l \frac{\Pi_{Rc}}{\bar{\Pi}} \\ &= \Pi_l, \end{aligned}$$

and also,

$$\begin{aligned} E(\Pi_2, F_1) &= \int_{p_R}^{p_l} [U_2 F_1 + L_2(1 - F_1)] dF_2 \\ &= \Pi_{Rc}(\frac{\Pi_{Rl} - \Pi_l}{\Pi_{Rc}}) \\ &= \Pi_{Rc}. \end{aligned}$$

Furthermore, any price of the support of F_1 yields $\Pi_1^* = \Pi_l$ since F_1 was defined by (2) and *almost any* price of the support of F_2 yields Π_2^* since F_2 was defined according to (2) (if firm 2 chooses p_l and and firm 1 randomizes according to F_1 then firm 2 gets less than Π_{Rc}, but p_l has measure 0 on the support of F_2).

B Verification of proposition 6

The inequality (8) can be written as

$$\delta \leq \frac{\Pi_c - E}{E + \Pi_{Rc} - 2\Pi_c} \tag{9}$$

First notice that $E + \Pi_{Rc} - 2\Pi_c < E - \Pi_c$ since $\Pi_{Rc} < \Pi_c$. This implies that if $\Pi_c - E > 0$ then the denominator of (9) is negative and there is no δ satisfying the inequality. On the other hand, if $\Pi_c - E < 0$ then $E + \Pi_{Rc} - 2\Pi_c < 0$, and $E - \Pi_c > 0$, but this implies that $E - \Pi_c < \Pi_{Rc} - \Pi_c$, or $E < \Pi_{Rc}$, but this is a contradiction.

[7] Except possibly for a set of measure 0.

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