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Optimal Government Spending Reversal in a Small Open Economy*

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Abstract

This paper reexamines optimal debt stabilization policy in a small open economy borrowing from abroad. We incorporate spending reversals as a policy option available to policy-makers for stabilizing public debt. Results show that spending reversals can be welfare-improving and that there exists an optimal degree of spending reversal if the debt elasticity of the country-specific risk premium is high. The tradeoff between smoothing the tax rate and stabilizing the sovereign interest rate in the discussion of optimal tax rate policy (Bi, 2010) does not arise. Spending reversals can lower both the tax rate volatility and that of the interest rate.

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1 Introduction

Expansionary fiscal policies and the resulting rapid surge of public debt in developed and emerging countries attributable to the global financial crisis of 2007–2009, and the recent sovereign debt crises in the Euro zone (Greece, Spain, Portugal, Ireland) have stimulated the discussion of an exit strategy from expansionary fiscal policy and an appropriate mode of stabilizing public debt.

Many studies have examined debt stabilization policies from the perspective of welfare (e.g., Schmitt-Grohé and Uribe (2007) and Kollmann (2008)). Reflecting the recent sovereign debt crises in the Euro zone, Bi (2010) analyses optimal debt stabilization policies in a small open economy borrowing from abroad. Bi (2010) specifically examines optimal tax policy as a policy tool for stabilizing public debt, and thereby demonstrates that because the sovereign interest rate depends on the level of public foreign debt, the government faces a tradeoff between smoothing the tax rate and stabilizing the sovereign interest rate.

Corsetti, Meier and Müller (2009) argue that most existing analytical studies assume that higher current deficits engender future tax increases, whereas government spending does not respond to the level of public debt.[1] They then discuss the effects of an increase in government spending followed by spending reversals.[2] They find that with spending reversals, a fiscal ex-

[1]See papers cited in Corsetti, Meier and Müller (2009) for empirical evidence showing that not only taxes but also fiscal spending is sensitive to the level of public debt.

[2]If the government allows fiscal spending to respond systematically to the level of public debt, then we observe a decline of government spending below the trend after a current episode of deficit spending. 'Spending reversal' is the designation given to the dynamics.

pansion can have a crowding-in effect of consumption through a decline of the long-term interest rate because of an expected future fall in the short-term interest rate. Their results suggest that it would be important to consider not just tax increases but also spending reversals as policy tools for offsetting fiscal measures when conducting policy evaluation.

We reconsider optimal debt stabilization policy while considering spending reversals as a policy option that is available to policy-makers for stabilizing public debt. Following Bi (2010), we use a real business cycle model of a small open economy in which the government and households can borrow from abroad.[3] Using the perturbation methods presented by Schmitt-Grohé and Uribe (2004), we measure welfare levels under different degrees of spending reversals. Our analysis shows that spending reversals can be welfare-improving. Spending reversals can improve economic welfare in the case in which the debt elasticity of the country-specific risk premium is high (i.e., the friction in foreign borrowing is large). In the case in which the debt elasticity of the country-specific risk premium is high, the positive effect of spending reversals on reducing the volatility of interest rates through limitation of the deviation of foreign debt can be considerable. In this case, the positive effect of spending reversals of reducing the volatility of interest rates can dominate the distortionary effects of spending reversals on the equilibrium path of the other economic variables. Our analysis also shows that the tradeoff between the volatility of tax rates and that of interest rates

[3]Although Bi (2010) assumes that households borrow from abroad at a constant rate (but face an adjustment cost for their bond holdings), we assume that households face a country-specific interest rate premium as does the government. Therefore, the risk premium depends on the debt level of households as well as the public debt level.

does not arise when the government chooses the optimal degree of spending reversals. This contrasts with the finding of Bi (2010) that there exists a tradeoff between the volatility of tax rates and that of interest rates when the government chooses the optimal tax rate policy.

The remainder of the paper is organized in three sections. In section 2, we present a real business cycle model of a small open economy in which the government and households can borrow from abroad. Section 3 explains the method of conducting policy evaluation, and presents an examination of how different degrees of spending reversals affect the economy's welfare level. Conclusions are presented in Section 4.

2 The Model

2.1 Households

The model examined in this paper resembles the model used by Schmitt-Grohé and Uribe (2003). Consider a small open economy in which there are many homogeneous households, each of which maximizes the following utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t), \tag{1}$$

where E_t denotes the mathematical expectations operator conditional on information available at time t, $\beta \in (0, 1)$ is the discount factor, c_t signifies consumption in period t, h_t represents labor effort, and U denotes a period utility function assumed to be strictly increasing in its first argument, strictly decreasing in its second argument, and strictly concave.

The households' flow budget constraint is

$$b_t = (1 + r_{t-1})b_{t-1} - (1 - \tau_t)[w_t h_t + (r_t^k - \delta)k_t] + c_t + (i_t - \delta k_t), \quad (2)$$

where b_t is the foreign debt position of the household, k_t stands for capital, i_t signifies investment, τ_t is the rate of income tax, w_t is the real wage, r_t^k is the rental rate of capital, and δ represents the depreciation rate of physical capital. The interest rate at which households can borrow from abroad, r_t, is given as

$$r_t = r + p(s_t), \quad (3)$$

where r is the world interest rate, and $p(\cdot)$ is a country-specific interest rate premium that is increasing in the aggregate level of foreign debt s_t, which is the sum of the foreign debt position of the household and that of the government as

$$s_t \equiv b_t + d_t, \quad (4)$$

where d_t stands for the foreign debt position of the government. The process of capital accumulation is given as

$$k_{t+1} = (1 - \delta)k_t + i_t - \Phi(k_{t+1} - k_t), \quad (5)$$

where $\Phi(\cdot)$ is the capital adjustment cost, which is assumed to be convex and which satisfies $\Phi(0) = \Phi'(0) = 0$.

The household takes as given the processes $\{r_t, w_t, r_t^k\}_{t=0}^{\infty}$ as well as b_{-1} and k_0, and chooses $\{c_t, h_t, i_t, k_{t+1}, b_t\}_{t=0}^{\infty}$ to maximize the utility function (1) subject to Eqs. (2), (5) and a no-Ponzi-game condition:

$$\lim_{j\to\infty} E_t \frac{s_{t+j}}{\prod_{z=1}^{j}(1+r_z)} \leq 0. \tag{6}$$

The optimality conditions associated with the households' maximization problem are given as

$$U_c(c_t, h_t) = \lambda_t, \tag{7}$$

$$U_h(c_t, h_t) = -\lambda_t(1-\tau_t)w_t, \tag{8}$$

$$\lambda_t = \beta(1+r_t)E_t\lambda_{t+1}, \tag{9}$$

and

$$\lambda_t[1+\Phi'(k_{t+1}-k_t)] = \beta E_t\lambda_{t+1}[(1-\tau_{t+1})r_{t+1}^k+\delta\tau_{t+1}+1-\delta+\Phi'(k_{t+2}-k_{t+1})], \tag{10}$$

where λ_t is the Lagrange multiplier on Eq. (2).

2.2 Firms

The production function is given as

$$y_t = A_t F(k_t, h_t), \tag{11}$$

where A_t is a stochastic productivity shock, and the function F is assumed to be increasing in both arguments and concave. The productivity is exogenously evolving according to the following process:

$$\ln A_t = \rho_A \ln A_{t-1} + \varepsilon_{A,t}, \quad \varepsilon_{A,t} \sim i.i.d.\mathcal{N}(0, \sigma_A^2). \tag{12}$$

The firm's profit is given as

$$\pi_t = A_t F(k_t, h_t) - r_t^k k_t - w_t h_t. \tag{13}$$

The optimality conditions associated with the firm's maximization problem are given as

$$r_t^k = A_t F_k(k_t, h_t), \tag{14}$$

and

$$w_t = A_t F_h(k_t, h_t). \tag{15}$$

2.3 Government

The government collects income tax and borrows from abroad to finance the government spending. The government's budget constraint is given as

$$d_t = (1 + r_{t-1}) d_{t-1} + g_t - \tau_t [w_t h_t + (r_t^k - \delta) k_t], \tag{16}$$

where g_t denotes government spending. Government spending and the tax policy are respectively assumed to follow the following rules:

$$\hat{g}_t = \rho_{gg}\hat{g}_{t-1} + \rho_{gd}(d_{y,t-1} - d_y) + \varepsilon_{g,t}, \quad \varepsilon_{g,t} \sim i.i.d.\mathcal{N}(0, \sigma_g^2), \tag{17}$$

and

$$\tau_t = \tau + \theta(d_{y,t-1} - d_y). \tag{18}$$

Therein, $\hat{g}_t$ denotes the percentage deviation of government spending from its steady state level (i.e., $\hat{g}_t \equiv \ln g_t - \ln g$), and g and τ denote the steady state levels of g_t and τ_t, respectively. Also, $d_{y,t}$ is the ratio of public debt to output (i.e., $d_{y,t} \equiv d_t/y_t$), and d_y denotes its steady-state level. $\rho_{gd}(\leq 0)$ and $\theta(\geq 0)$ are parameters respectively representing the sensitivities of government spending and the tax rate to the level of public debt. This paper specifically examines ρ_{gd} because the parameter directly governs the amplitude of the government spending reversals. As ρ_{gd} becomes smaller, government spending becomes more sensitive to the public debt and consequently displays a stronger reversal.

2.4 Equilibrium

By combining Eqs. (2) and (16) and using (4), we obtain the economy's current account ca_t as

$$ca_t \equiv -s_t + s_{t-1} = tb_t - r_{t-1}s_{t-1}, \tag{19}$$

where tb_t is the economy's trade balance:

$$tb_t \equiv y_t - c_t - i_t - g_t. \tag{20}$$

2.5 Functional forms

Following Schmitt-Grohé and Uribe (2003), we adopt the following standard forms for the utility function, production function, capital adjustment cost, and risk premium:

$$U(c,h) = \frac{[c^{\omega}(1-h)^{1-\omega}]^{1-\gamma} - 1}{1-\gamma}, \tag{21}$$

$$F(k,h) = k^{\alpha}h^{1-\alpha}, \tag{22}$$

$$\Phi(x) = \frac{\phi}{2}x^2, \quad \phi > 0, \tag{23}$$

and

$$p(s) = \psi(e^{s-\bar{s}} - 1), \tag{24}$$

where $\bar{s}$ is the steady state level of the economy's aggregate foreign debt position.

2.6 Calibration

Parameter values are presented in Table 1. Following Schmitt-Grohé and Uribe (2003), we set the inverse of intertemporal elasticity of substitution γ, the preference parameters for consumption ω, the share of capital in output α, the adjustment cost parameter for capital ϕ, the rate of depreciation of

Table 1: Calibration.

Parameters	Value	
γ	2	Inverse of intertemporal elasticity of substitution
ω	0.22	Relative preference for consumption
α	0.32	Share of capital in output
ϕ	0.084	Capital adjustment cost parameter
δ	0.1	Depreciation rate of capital
β	0.96	Discount factor
h	0.2	Steady-state value of labor
τ	0.25	Steady-state value of the income tax rate
g/y	0.2	Steady-state value of the government spending to output ratio
ρ_A	0.62	Persistence: productivity shock
ρ_{gg}	0.62	Persistence: government spending shock
σ_A	0.02	Standard deviation: productivity shock
σ_g	0.02	Standard deviation: government spending shock
θ	0.5	Debt elasticity of tax rate

capital δ, the discount factor β, and the steady state value of labor h to 2, 0.22, 0.32, 0.084, 0.1, 0.96 and 0.2, respectively. Following Kollmann (2008), the steady-state value of the income tax rate τ and that of the government spending to output ratio $\frac{g}{y}$ are respectively set to 0.25 and 0.2. For the degree of persistence of the productivity shock and the government spending shock, we follow Bi (2010) and set ρ_A and ρ_{gg} to 0.62 with standard deviation $\sigma_A = \sigma_g = 0.02$.

3 Welfare

3.1 Welfare benefit measure

We use the perturbation methods presented by Schmitt-Grohé and Uribe (2004) that compute second-order accurate solutions to measure the levels of unconditional life time utility. By comparing the measured welfare levels under different degrees of spending reversals with the welfare level under no spending reversals, we conduct policy evaluations of spending reversals.

We define unconditional lifetime utility in the case of no spending reversals (i.e. $\rho_{gd} = 0$) as

$$E\,V_0^b \equiv E\,\sum_{t=0}^{\infty} \beta^t U(c_t^b, h_t^b),$$

where E denotes the unconditional expectations operator. We next define the unconditional life time utility with different degrees of spending reversals as alternative policy regimes:

$$E\,V_0^a \equiv E\,\sum_{t=0}^{\infty} \beta^t U(c_t^a, h_t^a).$$

We then define λ as the welfare benefit of adopting policy regime a rather than policy regime b. Formally, λ is defined as

$$E\,V_0^a = E\,\sum_{t=0}^{\infty} \beta^t U((1+\lambda)c_t^b, h_t^b).$$

In other words, λ is the fraction of regime b's consumption process that compensates a household to be as well off under regime b as under regime a. For the particular form of the utility function (21), λ can be expressed as

$$\lambda = \left\{ \frac{(1-\gamma)E\,V_0^a + (1-\beta)^{-1}}{(1-\gamma)E\,V_0^b + (1-\beta)^{-1}} \right\}^{\frac{1}{\omega(1-\gamma)}} - 1. \qquad (25)$$

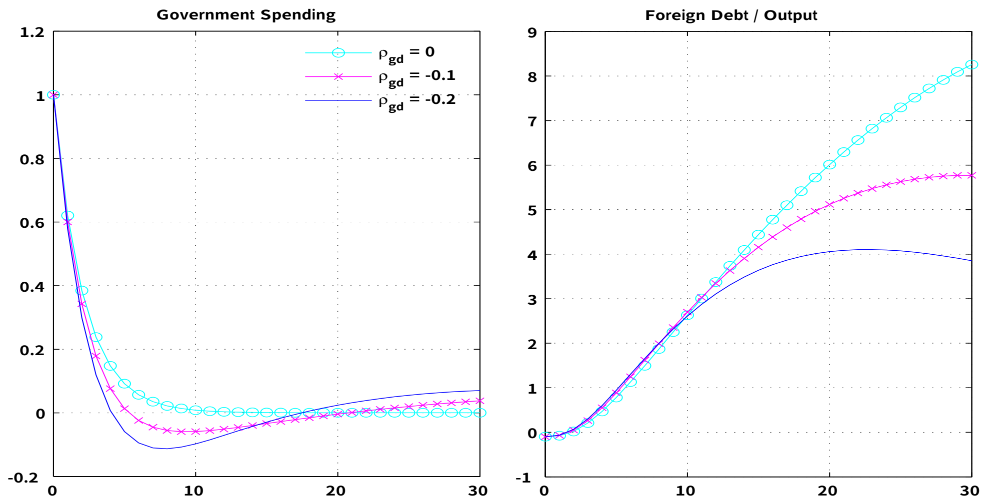

Figure 1: Impulse responses to government spending shock.

3.2 Effect of spending reversals on economic welfare

As a starting point, we first examine the effects of spending reversals on an economy in which the country-specific interest rate premium ψ is set as 0.001.[4] Before we analyze the effect of spending reversals on economic welfare, we examine the effects of spending reversals on the debt-to-output ratio.

Figure 1 presents the effects of the government spending shock to the government spending and the economy's (aggregate) foreign debt-to-output ratio for different values of ρ_{gd} ($\rho_{gd} = 0$, -0.1, and -0.2). The smaller the value of ρ_{gd}, the further the government spending falls below zero (after the shock) and the more limited is the expansion of the foreign debt-to-output ratio.

We next examine the effect of spending reversals on the economy's welfare

[4]Following Schmitt-Grohé and Uribe (2003), we set ψ to 0.001 as our benchmark case.

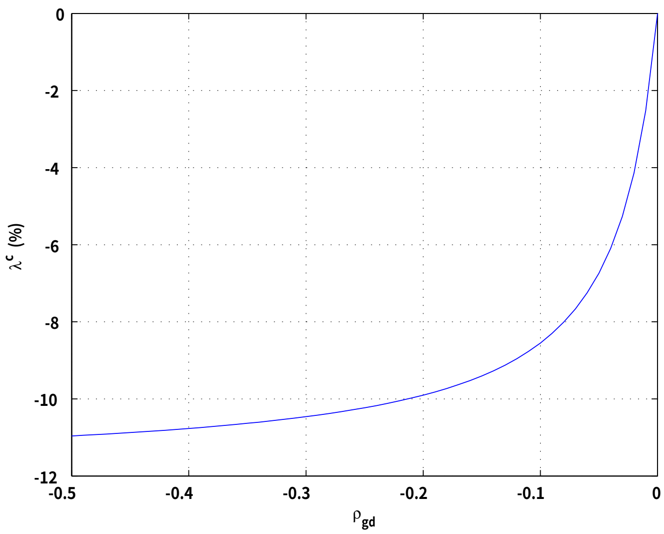

Figure 2: Welfare benefit ($\psi = 0.001$).

in the benchmark case. Figure 2 presents the welfare benefit of spending reversals ($\lambda^c(\%)$) corresponding to the various values of ρ_{gd} in the benchmark case. Figure 2 shows that the economy's welfare level deteriorates with the smaller ρ_{gd}. In the benchmark case in which the friction in foreign borrowing is small, the positive effect of spending reversals of reducing the volatility of interest rates through limitation of the deviation of foreign debt would be limited. Therefore, the negative effect of spending reversals of distorting the equilibrium path of economic variables through increase of the volatility of government spending dominates the positive effect of reducing the volatility of interest rates in this case.

We now analyze the case in which spending reversals can be welfare-improving. This is the case in which the friction in foreign borrowing is greater than the benchmark case. Figure 3 portrays the welfare benefit of spending reversals ($\lambda^c(\%)$) corresponding to the various values of ρ_{gd} in the

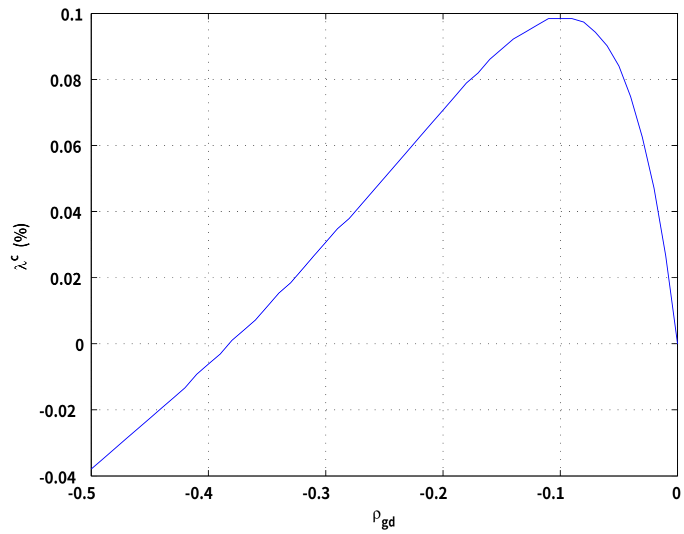

Figure 3: Welfare benefit ($\psi = 0.003$).

case in which ψ is 0.003. In Figure 3, the economy's welfare curve displays a hump shape. It is apparent that some range of $\rho_{gd}\,(-0.38 < \rho_{gd} < 0)$ improves welfare levels in contrast to the benchmark case. The rationale for our analysis results are explainable as follows. In the case in which friction in foreign borrowing is large (compared to the benchmark case), the positive effect of spending reversals of reducing the volatility of interest rates through limiting the deviation of foreign debt can be significant. Although spending reversals increase the volatility of government spending and distort the equilibrium path of the other economic variables, the positive effect of spending reversals attributable to the reduction of the volatility of interest rates can dominate the negative effect of causing distortions in this case.

To explain our analysis results more clearly, we present standard deviations of government spending, interest rate, and the tax rate corresponding to each case of different values of ρ_{gd} ($\rho_{gd} = 0$, -0.1, and -0.2) in Table 2.

Table 2 shows that the smaller ρ_{gd} increases the volatility of the government spending and reduces the volatility of the interest rate and that of the tax rate. Bi (2010) showed that the optimal tax rate policy is accompanied by a tradeoff between smoothing the sovereign interest rate and smoothing the tax rate: an aggressive tax adjustment can stabilize foreign debt and lower the volatility of the sovereign interest rate through the country premium, although it increases the volatility of the tax rate itself. As argued by Bi (2010), the tax rate affects the households' decisions related to consumption and the labor supply directly, whereas the sovereign interest rate affects the interest payment on the government debt and future tax rates.[5] Therefore, the government faces a tradeoff between smoothing the tax rate and the sovereign interest rate. In our case, however, as shown in Table 2, no tradeoff exists between the volatility of the tax rate and that of the interest rate. The optimal spending reversal policy is not accompanied by a tradeoff between smoothing the tax rate and the interest rate. In our case, instead, a tradeoff exists between the volatility of the government spending and both the volatility of the tax rate and that of the interest rate. In other words, a tradeoff exists between the volatility of the government spending and that of the foreign debt because the tax rate and the interest rate depend on the foreign debt.

[5]Because a country-specific interest rate premium is assumed to depend on the level of the households' debt as well as the public debt level in our model, the interest rate affects households' decisions on consumption and the labor supply directly as well as the tax rate in our case.

Table 2: Standard deviations.

	$\rho_{gd} = 0$	$\rho_{gd} = -0.1$	$\rho_{gd} = -0.2$
g_t	2.55	3.44	4.48
r_t	0.19	0.14	0.12
τ_t	6.58	4.77	3.83

Note) Standard deviations are reported in percentage points.

4 Conclusion

Regarding spending reversals as a policy option available to policymakers for stabilizing public debt, we have reexamined optimal debt stabilization policy in a small open economy with borrowing from abroad. Using the perturbation methods developed by Schmitt-Grohé and Uribe (2004), we measured unconditional life-time utility levels under different degrees of spending reversals. By comparing them with the welfare level under no spending reversals, we conducted policy evaluations of spending reversals.

Our analysis results show that spending reversals can improve the economic welfare in the case in which the debt elasticity of the country-specific risk premium is high (i.e., the friction in foreign borrowing is large). The intuition underlying our analysis results is straightforward. Spending reversals have two opposite effects on welfare levels. On one hand, spending reversals increase the volatility of the government spending and have distortionary effects on the equilibrium path of economic variables. On the other hand, spending reversals can curtail the foreign debt deviation and reduce the volatility of the country-specific risk premium. The imposition of spending reversals therefore can lower the volatilities of interest rates and tax rates and improve welfare. If the degree of spending reversals is too strict, however, the distortionary effect through increasing the volatility of the government spending would dominate the beneficial effect through withholding of foreign borrowing. Therefore, a hump-shaped relation might exist between spending reversals and welfare levels.

As argued above, our analysis shows that the imposition of spending reversals reduces both the volatility of interest rates and that of tax rates, which contrasts with that of Bi (2010), finding that a tradeoff exists between the volatility of tax rates and that of interest rates when the government chooses the optimal tax rate policy. When the government chooses the optimal degree of spending reversals, then the tradeoff between the volatility of tax rates and that of interest rates does not arise.

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Appendix A Equilibrium conditions (Not for publication)

$$k_{t+1} = (1-\delta)k_t + i_t - \frac{\phi(k_{t+1}-k_t)}{2}$$

$$[c_t^{\omega}(1-h_t)^{1-\omega}]^{-\gamma}\omega c_t^{\omega-1}(1-h_t)^{1-\omega} = \lambda_t$$

$$[c_t^{\omega}(1-h_t)^{1-\omega}]^{-\gamma}(1-\omega)c_t^{\omega}(1-h_t)^{-\omega} = \lambda_t(1-\tau_t)w_t$$

$$\lambda_t = \beta(1+r_t)E_t\lambda_{t+1}$$

$$\lambda_t[1+\phi(k_{t+1}-k_t)] = \beta E_t\lambda_{t+1}[(1-\tau_{t+1})r_{t+1}^k+\delta\tau_{t+1}+1-\delta+\phi(k_{t+2}-k_{t+1})]$$

$$y_t = A_t k_t^{\alpha} h_t^{1-\alpha}$$

$$r_t^k = A_t\alpha k_t^{\alpha-1}h_t^{1-\alpha}$$

$$w_t = A_t(1-\alpha)k_t^{\alpha}h_t^{-\alpha}$$

$$d_t = (1+r_{t-1})d_{t-1} + g_t - \tau_t(w_t h_t + r_t^k k_t) + \delta\tau_t k_t$$

$$\hat{g}_t = \rho_{gg}\hat{g}_{t-1} + \rho_{gd}(d_{y,t-1} - d_y) + \varepsilon_{g,t}$$

$$\tau_t = \tau + \theta(d_{y,t-1} - d_y)$$

$$d_{y,t} = \frac{d_t}{y_t}$$

$$r_t = r + \psi(e^{s-\bar{s}} - 1)$$

$$tb_t = y_t - c_t - i_t - g_t$$

$$s_t = (1 + r_{t-1})s_{t-1} - tb_t$$

$$ca_t = tb_t - r_{t-1}s_{t-1}$$

$$tb_{y,t} = \frac{tb_t}{y_t}$$

$$ca_{y,t} = \frac{ca_t}{y_t}$$

$$s_t = b_t + d_t$$

$$s_{y,t} = \frac{s_t}{y_t}$$

$$b_{y,t} = \frac{b_t}{y_t}$$

Appendix B Steady state (Not for publication)

$$h = 0.2$$

$$\tau = 0.25$$

$$k = h\left(\frac{\beta^{-1} - \delta\tau - 1 + \delta}{\alpha(1-\tau)}\right)^{\frac{1}{\alpha-1}}$$

$$c = \frac{\omega}{1-\omega}(1-h)(1-\tau)(1-\alpha)\left(\frac{h}{k}\right)^{-\alpha}$$

$$y = k^{\alpha}h^{1-\alpha}$$

$$r^k = \alpha k^{\alpha-1}h^{1-\alpha}$$

$$w = (1-\alpha)k^{\alpha}h^{-\alpha}$$

$$i = \delta k$$

$$\lambda = [c^{\omega}(1-h)^{1-\omega}]^{-\gamma} \omega c^{\omega-1}(1-h)^{1-\omega}$$

$$r = \frac{1-\beta}{\beta}$$

$$\frac{g}{y} = 0.2$$

$$tb = y - c - i - g$$

$$\bar{s} = \frac{tb}{r}$$

$$ca = tb - r\,\bar{s}$$

$$d = \frac{-g + \tau(wh + r^k k) - \delta\tau k}{r}$$

$$b = \bar{s} - d$$

$$d_y = \frac{d}{y}$$

$$tb_y = \frac{tb}{y}$$

$$ca_y = \frac{ca}{y}$$

$$s_y = \frac{\bar{s}}{y}$$

$$b_y = \frac{b}{y}$$