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Abstract

Abstract In a common-values election with two candidates voters receive a signal about which candidate is superior. They can acquire information that improves the precision of the signal. Electors differ in their information acquisition costs. For large electorates a non negligible fraction of voters acquires information, but the quantity of informed voters and the quality of acquired information decline so fast that information aggregation fails to obtain.
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1 Introduction

There is considerable evidence that voters have little and uneven knowledge about policies and the backgrounds of elected governmental officials (see, for instance, Delli Carpini and Keeter, 1996). These facts are consistent with the rational ignorance hypothesis formulated by Schumpeter (1950) and Downs (1957): individual voters will choose to acquire little information, since each individual's vote has little impact on the outcome of a large election and information acquisition is costly. Determining the implications of this hypothesis has important implications about the quality of democratic deliberations.

A vast empirical literature attempts to assess the extent to which political judgments and deliberations would differ if voters were well informed (see Althaus 1998, 2003, Gilens 2001). According to Althaus (2003) "Knowledge does matter, and the way it is distributed in society can cause collective preferences to reflect disproportionately the opinions of some groups more than others. Sometimes collective preferences seem to represent something like the will of the people, but frequently they do not".

A first view suggests that the informational failure can be so severe that the vote would not be more likely to reflect the (informed) will of the electorate than a fair toss coin. Scholars have long feared that democracies cannot function if they are too large. Polybius (1992, book 6) in the second century B.C. argued that ochlocracy (mob-rule) is a natural evolution of democracy . In Madison's (in Hamilton 1788, 9 and 14) opinion the United States and even some states were too vast for direct democracy.

A second view suggests that aggregate opinion may be able to reflect the public interests even when most individuals are poorly informed. Condorcet (1786) argued that the larger is the population, the higher is the probability that a democracy will make the 'right' decision. According to this argument, in the process of preference aggregation, the more or less random opinion of poorly informed voters would cancel out (see Wittman 1989, 1995). This statement constitutes the so called Condorcet Jury Theorem.

The objective of this paper is to investigate how costly information acquisition in a large and heterogeneous electorate influences the quality of voting outcomes. A model where voters have to decide over two alternatives, A and B is introduced. Voters have common preferences but they do not know which one of the alternatives is better for them. They do not have free access to a reliable font of information, but they can acquire some information. Acquiring precise information is costly and voters may differ in their abilities of collecting and processing information, which reflects in different information acquisition costs. A voter who acquires information of quality x receives the correct signal with probability $\frac{1}{2}+x$ and faces a cost of $C(\alpha, x)$ where α is her type. C is strictly convex and increasing in x. Types with higher types faces increasingly higher costs.

The model incorporate the features of Martinelli's (2006, 2007). In Martinelli (2006) electors can acquire information of different quality but they all have the same cost function. Martinelli (2007) allows for heterogeneity in information acquisition costs but voters can buy information of one given quality. So Martinelli (2006, 2007) cannot account for uneven levels of information. Both works conclude that (at least partial) information aggregation in large election is always possible.

We focus on symmetric equilibriua and prove that an equilibrium with information acquisition exists if and only if the expected gains from reaching the right decision are equal at every state. As the number of electors grows the probability that any elector is decisive converges to zero. Only the electors with lower information acquisition costs acquire information. The fraction of informed electors and the expected quality of information they acquire decreases to zero. Asymptotically, the probability that the elections will reach the right decision converges to one half.

We investigate whether access to cheaper information can alleviate this informational failure. In this model the costs of information depend on two factors: the quality of information and the type of the agent. We prove that elections produce efficient results only if the marginal cost of acquiring information increases at a slower with respect to both the precision of information and the type of the agents, formally only if $C_{xx}(0,0) = C_{\alpha x}(0,0) = 0$. In this case an equilibrium with information acquisition exists for every parameter specification and elections perfectly aggregate information: the probability of reaching the right decision converges to one when the size of the population grows to infinity.

We reach different conclusions with respect to Martinelli (2006,2007) because our model jointly incorporates the feature he studies separately. In Martinelli (2006) voters acquire information of decreasing quality but every elector acquires the same quality of information (they have the same costs) so information aggregates, even if incompletely. In Martinelli (2007) a decreasing part of the electorate acquire information but the quality of information acquired is always the same so information aggregates. In our paper the two effects combine: a decreasing part of the electorate acquires information of decreasing quality as the number of voters grows. For this reason also the conditions for information aggregation are more demanding.

The introduction of heterogeneity in information acquisition costs allows to account for three empirically relevant facts:

(i) A small fraction of the electorate is informed.

(ii) The overall quality of information electors have is limited.

(iii) The distribution of information across electors is uneven.

Martinelli (2006) can account only for (ii) and Martinelli (2007) only for (i), none for (iii). The only paper which reflects (iii) is Oliveros (2006) who takes an orthogonal approach: voters have the same information acquisition costs but they differ in the losses they bear when the wrong decision is taken. He proves aggregation results similar to Martinelli (2006). In this model the existence of equilibria with information acquisition heavily relies on the introduction of "stubborn voters". A fraction of them always votes for alternative, A while the others vote for alternative B. In this way the probability that a voter is decisive is bounded away from zero.

The structure of the article is the following. Section 2 introduces the model. Section 3 characterizes equilibria with information acquisition. Section 4 tackles the existence of equilibria with information acquisition and its aggregation properties. Section 5 studies the aggregate costs of information acquired and the asymptotic efficiency of equilibria. Section 6 concludes.

Related Literature

The paper is related to the line of research about the Condorcet Jury Theorem. The first proofs were entirely statistical (see Berg 1993, Berend and Paroush 1998, Ladha 1992, 1993). They assumed that each individual privately observes a signal about the right candidate and then vote sincerely according to the signal. More recently the theorem has been proved under the assumption of strategic voting (see, e.g., Austen-Smith and Banks 1996, Feddersen and Pesendorfer 1996, 1997, 1999, and Myerson 1998). But all these papers assume that the information is freely available to voters. Interestingly Paroush (1998), in a non strategic setup, proved that elections can fail to aggregate information if the probability a voter receives the correct signal is not bounded away from one half. Yariv (2004) analyzes majority voting in common value two-option environments where voters have private information, the quality of which exogenous depends on the size of the electorate. She proves that information of low quality may lead to informational failures. In this paper we endogeneize the causes of the decreasing quality of information. In a recent work Mandler (2007) proved a similar negative result: if voters are uncertain of the quality of the initial signal elections can loose their ability to aggregate information.

The literature focusing on voting in committee has recently considered the issue of costly information acquisition. Persico (2004) and Mukkhopadhaya (2005) consider a setting in which committee members have identical and fixed costs of acquiring information. In this setup there is a maximum number of voters who can acquire information at equilibrium so that for large electorates there is no equilibrium with information acquisition. Oliveros (2007) presents a model based on Oliveros (2006) in which voters have the same information acquisition costs but they differ in the gains obtained from taking the right decisions. Voters can select whether to vote or abstain and the amount of information to acquire. He proves that there are equilibria where voters collect information of different qualities, there are informed voters that abstain, and information and abstention need not be inversely correlated for all voters.

2 The Model

There are $N = 2n + 1$ voters and two states of the world $\omega = a, b$. Electors have to select between two alternatives, A and B by majority voting. The prior probability of state a is q_a and the prior probability of state b is $q_b = 1 - q_a$. Before voting agents independently receive a signal $s \in \{s_a, s_b\}$. Before receiving the signal they can acquire information of quality $x \in \left[0, \frac{1}{2}\right]$. When a voter receives a signal of quality x the likelihood of receiving the signal s_ω conditional to ω is $p\left(s_\omega \mid \omega, x\right) = \frac{1}{2} + x$. Voters have different acquisition costs. One interpretation is literal: voters bear different costs of access to information. According to a different one voters have different ability in processing information or they have access to different fonts of information. So less skilled agents must invest more effort in order to extract the same amount of information. An elector of type α bears a cost $C\left(x, \alpha\right)$ to purchase information of quality x. Types are distributed in the interval $[0, 1]$, independently across the electorate. Let the types of each elector be distributed according to a continuous density function $f : [0, 1] \to \mathbb{R}_+$, with $f\left(0\right) \neq 0$.

The cost function is of class $C^2\left(\left[0, \frac{1}{2}\right] \times [0, 1]\right)$ and has the following properties:

NFL $C(0,\alpha)=0$ and $C(x,\alpha)>0$ for all $x>0$ and for all α.

CONV $C_x(x,\alpha)>0$, $C_{xx}(x,\alpha)>0$, for all $x\in\left(0,\frac{1}{2}\right)$,and for all $\alpha>0$. $C_x(0,0)=0$ and $C_{xx}(0,0)>0$.

SCR $C_\alpha(x,\alpha)>0$, $C_{x\alpha}(x,\alpha)>0$ for all $x>0$ and for all $\alpha>0$. $C_{\alpha x}(0,0)>0$.

Property NFL (no free lunch) states that acquiring a positive amount of information has a strictly positive cost, while acquiring no information entails no costs. Property CONV (convexity) states that the cost function is strictly increasing and strictly convex for all types. Type zero has zero marginal costs. Property SCR (single crossing) states that higher types α face increasingly higher costs.

We study the robustness of our results with respect to these assumption. We will analyze two cases where condition CONV and/or SCR are replaced by WCONV and WSCR.

WCONV $C_x(x,\alpha)>0$, $C_{xx}(x,\alpha)>0$, for all $x\in\left(0,\frac{1}{2}\right)$,and for all $\alpha>0$. $C_x(0,0)=0$, $C_{xx}(0,0)=0$. There exists $k\in\mathbb{N}$ such that $C_{x^{(k)}}(0,0)\neq 0$.

WSCR $C_\alpha(x,\alpha)>0$, $C_{x\alpha}(x,\alpha)>0$ for all $x>0$ and for all $\alpha>0$. $C_{x\alpha}(0,0)=0$. There exists $k\in\mathbb{N}$ such that $C_{x\alpha^{(k)}}(0,0)\neq 0$.

If WCONV holds the marginal cost of acquiring information of the lowest type is lower than with respect to when CONV holds. If WSCR holds the marginal cost of acquiring information of the lower types increases at a slower rate withe respect to when SCR holds.

At state $\omega=a,b$, the utility of a voter of type α, who has invested in a level of signal precision x is, depending on the decision $d\in\{A.B\}$ taken is: $U(d,\omega)-C(x,\alpha)$ for $d=A,B$. We assume that A is the best alternative at state a and B is the right alternative at state b, which is $U(A,a)-U(B,a)=\Delta U_a>0$ and $U(B,b)-U(A,b)=\Delta U_b>0$.

For every voter of a given type a strategy specifies: (i) how much information she acquires and (ii) for which alternative she votes after receiving the signal.

Definition 1 *A strategy for voter i consists of a information acquisition strategy $x:[0,1]\rightarrow\left[0,\frac{1}{2}\right]$ and of a voting strategy $v:[0,1]\times\{s_a,s_b\}\rightarrow\{A,B\}$ such that x is measurable and $v(\cdot,s)$ is measurable for $s\in\{s_a,s_b\}$.*

A strategy of player i is denoted by (x_i, v_i), a strategy profile $(x_i, v_i)_{i=1,\ldots,2n+1}$ is denoted by (X, V) and $(X, V)_{-i}$ is the coalitional strategy of all voter but i. Given $(X, V)_{-i}$, we denote by

$$U(v \mid \omega) = \sum_{d \in \{A,B\}} U(d, \omega) \Pr\left(d \mid \omega, v, (X, V)_{-i}\right)$$

the expected utility from voting v at state ω, net of information acquisition costs, where $\Pr\left(d \mid \omega, v, (X, V)_{-i}\right)$ is the probability the outcome is d at state ω.

Given investment choice x and after receiving signal $s \in \{s_a, s_b\}$, the expected utility from voting v is

$$U\left(v \mid x, s, (X, V)_{-i}\right) = \sum_{\omega \in \{A,B\}} U(v \mid \omega)\, Pr(\omega \mid (x, s))$$

where $Pr(\omega \mid (x, s))$ is the likelihood of ω given investment x and signal s.

The expected utility from a player investing x and using a voting rule from using a strategy (x, v) when other agents play $(X, V)_{-i}$ is

$$U\left(x, v \mid (X, V)_{-i}\right) = \sum_{s \in \{s_a, s_b\}} U^i\left(v \mid x, s, (X, V)_{-i}\right) p(s)$$

where $p(s)$is the probability of receiving the signal s.

The equilibrium concept we employ is symmetric Bayesian equilibrium.

Definition 2 *A symmetric Bayesian equilibrium (SBE from now on) is given by a strategy* $(\hat{x}, \hat{v})$ *such that the profile* $\left(\hat{X}, \hat{V}\right) = (\hat{x}, \hat{v})_{i=1,\ldots 2n+1}$*satisfies:*

1. $U\left(v(\hat{x}(\alpha), s) \mid \hat{x}(\alpha), s, \left(\hat{X}, \hat{V}\right)_{-i}\right) \geq U\left(v \mid \hat{x}(\alpha), s, \left(\hat{X}, \hat{V}\right)_{-i}\right)$ *for all* $\alpha \in [0, 1]$, *for all* $v \in \{A, B\}$ *and for all* $s \in \{s_a, s_b\}$.
2. $U\left(\hat{x}(\alpha), \hat{v} \mid \left(\hat{X}, \hat{V}\right)_{-i}\right) - C(\hat{x}(\alpha), \alpha) \geq U\left(x, v \mid \left(\hat{X}, \hat{V}\right)_{-i}\right) - C(x, \alpha)$ *for all* $\alpha \in [0, 1]$ *for all voting rules* v.

An *SBE with information acquisition* is a *SBE* where a non-zero measure of types acquires information.

In a symmetric equilibrium all players agents employ the same strategy, voting strategies are optimal conditional to the signals received and information acquisition strategies are ex ante optimal.

Observe that SBE with no information acquisition always exist: voters do not acquire information and at least $n+2$ of them vote for the same alternative independently on their type and signal.

When no ambiguity is possible we omit any reference to $(X,V)_{-i}$. We now introduce some mathematical notation. Let $f,g : X \to \mathbb{R}$ where X is a metric space. Let $z \in X$. We write $f \approx g$ for $x \to z$ if $lim_{x\to z}\frac{f(x)}{g(x)} = 1$, $f = o(g)$ for $x \to z$ if $lim_{x\to z}\frac{f(x)}{g(x)} = 0$ and $f = O(g)$ for$x \to z$ if there exists $C > 0$ such that $|f(x)| \leq C|g(x)|$ in a neighborhood of z. Let $\{a_n\}_{n\in\mathbb{N}}$ and $\{b_n\}_{n\in\mathbb{N}}$ two sequences of real numbers. We write $a_n \approx b_n$ if $lim_{n\to\infty}\frac{a_n}{b_n} = 1$, $a_n = o(b_n)$ if $lim_{n\to\infty}\frac{a_n}{b_n} = 0$ and $a_n = O(b_n)$ if there exists $C > 0$ such that$|a_n| \leq C|b_n|$ for n large enough. With Φ we denote the standard normal distribution.

3 Characterization

The utility (net of information acquisition costs) that a voter derives from a voting strategy (v_a, v_b) is

$$\sum_{\omega\in\{a,b\}} p_\omega q_\omega \left[U(v_a \mid \omega)\, p(s_a \mid \omega) + U(v_b \mid \omega)\, p(s_b \mid \omega)\right] + U_{-i}$$

where $p_\omega = p\left(piv \mid \omega, (X,V)_{-i}\right)$ is the probability a player is pivotal at state ω, given other voters' strategies and $U_{-i} = U_{-i}\left((X,V)_{-i}\right)$ is a term that depends only on the strategies taken by agents other than i.

A voter who ignores the signal is always strictly better off by not investing in information. By playing $(0,A,A)$, her expected utility is

$$U(0,A,A) = p_a q_a U(A \mid a) + p_b q_b U(A \mid b) + U_{-i} =$$

$$= \frac{p_a q_a \Delta U_a + p_b q_b \Delta U_b}{2} + \frac{p_a q_a \Delta U_a - p_b q_b \Delta U_b}{2} + p_a q_a U(B \mid a) + p_b q_b U(A \mid b) + U_{-i}$$

By playing $(0,B,B)$, her expected utility is:

$$U(0,B,B) = p_a q_a U(B \mid a) + p_b q_b U(B \mid b) + U_{-i} =$$

$$= \frac{p_b q_b \Delta U_b + p_a q_a \Delta U_a}{2} + \frac{p_b q_b \Delta U_b - p_a q_a \Delta U_a}{2} + p_a q_a U(B \mid a) + p_b q_b U(A \mid b) + U_{-i}$$

Observe that $U(0,A,A) \geq U(0,B,B)$ if and only if $p_a q_a \Delta U_a \geq p_b q_b \Delta U_b$.

The benefit from acquiring x units of information and following the signal $U(x,A,B)$ is

$$U(x,A,B) = (p_a q_a \Delta U_a + p_b q_b \Delta U_b)\left(\frac{1}{2} + x\right) + p_a q_a U(B \mid a) + p_b q_b U(A \mid b) - C(\alpha, x) + U_{-i}$$

Let $\alpha = \alpha(p_a, p_b)$ such that

$$(p_a q_a \Delta U_a + p_b q \Delta U_b) - C_x(\alpha, 0) = 0 \tag{1}$$

if any exists and set $\alpha = \alpha(p_a, p_b) = 1$ otherwise. Type $\alpha(p_a, p_b)$ is the lowest type for whom is optimal not to acquire information. The function $\alpha(p_a, p_b)$ is differentiable in the interior of the set where (p_a, p_b) satisfies 1, with partial derivatives

$$\alpha_{p_a}(p_a, p_b) = \frac{q_a \Delta U_a}{C_{x\alpha}(\alpha(p_a, p_b), 0)}$$

$$\alpha_{p_b}(p_a, p_b) = \frac{q_b \Delta U_b}{C_{x\alpha}(\alpha(p_a, p_b), 0)}$$

and $lim_{(p_a,p_b)\to 0}\alpha(p_a, p_b) = 0$.[1] So, for p_a and p_b small, $\alpha(p_a, p_b) \in (0,1)$.

Thus, the optimal information investment for type α, $x = x(\alpha, p_a, p_b)$ solves

$$(p_a q_a \Delta U_a + p_b q_b \Delta U_b) = C_x(\alpha, x) \tag{2}$$

for $\alpha \leq \alpha(p_a, p_b)$. If $\alpha > \alpha(p_a, p_b)$, then $x(\alpha, p_a, p_b) = 0$. For $(p_a, p_b) \neq 0$ and $0 \leq \alpha < \alpha(p_a, p_b)$ from the

[1] If condition STR holds α is differentiable at $(0,0)$, too.

implicit function theorem we have

$$x_{\alpha}(\alpha, p_a, p_b) = -\frac{C_{x\alpha}(\alpha, x)}{C_{xx}(\alpha, x)}$$

$$x_{p_a}(\alpha, p_a, p_b) = \frac{q_a \Delta U_a}{C_{xx}(\alpha, x(\alpha, p_a, p_b))}$$

$$x_{p_b}(\alpha, p_a, p_b) = \frac{q_b \Delta U_b}{C_{xx}(\alpha, x(\alpha, p_a, p_b))}$$

and $lim_{(p_a,p_b)\to 0} x(\alpha, p_a, p_b) = 0$ for every α.[2]The function, $x(\alpha, p_a, p_b)$ is strictly increasing in p_ω for $\omega = a, b$ and strictly decreasing inα for $\alpha \leq \alpha(p_a, p_b)$, $x(\alpha, p_a, p_b)$.

For a voter of type α is optimal to follow the signal if and only if

$$U(x(\alpha, p_a, p_b), A, B) \geq \max\{U(0, A, A), U(0, B, B)\}$$

$U(x(\alpha, p_a, p_b), A, B) \geq \max\{U(0, A, A), U(0, B, B)\}$ or, equivalently if and only if

$$(p_a q_a \Delta U_a + p_b q \Delta U_b) x - C(\alpha, x) \geq \frac{|p_b q_b \Delta U_b - p_a q_a \Delta U_a|}{2}$$

for $x = x(\alpha, p_a, p_b)$.

Let $\alpha'(p_a, p_b)$, satisfying

$$(p_a q_a \Delta U_a + p_b q \Delta U_b) x - C(\alpha, x) = \frac{|p_b q_b \Delta U_b - p_a q_a \Delta U_a|}{2} \quad (3)$$

for $x = x(\alpha, p_a, p_b)$ if any exists and let $\alpha'(p_a, p_b) = 1$ otherwise. Finally set

$$\alpha^*(p_a, p_b) = \min\{\alpha'(p_a, p_b), \alpha(p_a, p_b)\}$$

and observe that $\alpha^*(p_a, p_b) = \alpha(p_a, p_b)$ if an only if $p_b q_b \Delta U_b = p_a q_a \Delta U_a$, otherwise $\alpha^*(p_a, p_b) < \alpha(p_a, p_b)$. Given (p_a, p_b), every type $\alpha \leq \alpha^*(p_a, p_b)$ acquires the positive amount of information determined by Equation 2 and every type $\alpha > \alpha^*(p_a, p_b)$ does not acquire information. If $p_b q_b \Delta U_b = p_a q_a \Delta U_a$ the voters who do

[2]If condition CONV holds $x$is differentiable for$(p_a, p_b) = (0, 0)$, too.

not acquire information are indifferent among the two alternatives. Otherwise, they vote for alternative A if $p_b q_b \Delta U_b < p_a q_a \Delta U_a$ and for alternative B if $p_b q_b \Delta U_b > p_a q_a \Delta U_a$.

Let

$$\tilde{x}(p_a, p_b) = \int_0^{\alpha^*(p_a,p_b)} x(\alpha, p_a, p_b) f(\alpha) d\alpha$$

be the expected amount of information acquired by a voter of unknown type. Let $\lambda(\alpha) \in \{0,1\}$ be the probability a voter of type $\alpha > \alpha^*(p_a, p_b)$ votes for A and set $\tilde{\lambda}(p_a, p_b) = \int_{\alpha^*(p_a,p_b)}^1 \lambda(\alpha) f(\alpha) d\alpha = \lambda(p_a, p_b)(1 - F(\alpha^*(p_a, p_b)))$ for some $\lambda(p_a, p_b) \in [0,1]$. λ is the conditional probability a voter of unknown type votes for A, given that she does not acquire information. Finally set $\mu(p_a, p_b) = \lambda(p_a, p_b) - \frac{1}{2} \in \left[-\frac{1}{2}, \frac{1}{2}\right]$. The probability a voter votes for alternative A at state a is:

$$\frac{F(\alpha^*(p_a, p_b))}{2} + \tilde{x}(p_a, p_b) + \tilde{\lambda} = \frac{1}{2} + \tilde{x}(p_a, p_b) + \mu(p_a, p_b)(1 - F(\alpha^*(p_a, p_b)))$$

The probability a voter votes for alternative A at state b is:

$$\frac{F(\alpha^*(p_a, p_b))}{2} - \tilde{x}(p_a, p_b) + \tilde{\lambda} = \frac{1}{2} - \tilde{x}(p_a, p_b) + \mu(p_a, p_b)(1 - F(\alpha^*(p_a, p_b)))$$

The probability a voter is pivotal at state a is

$$P_{a\mu}(p_a, p_b) = \binom{2n}{n} \left\{ \frac{1}{4} - \left[\tilde{x}(p_a, p_b) + \mu(p_a, p_b)(1 - F(\alpha^*(p_a, p_b)))\right]^2 \right\}^n$$

The probability a voter is pivotal at state b is

$$P_{b\mu}(p_a, p_b) = \binom{2n}{n} \left\{ \frac{1}{4} - \left[\tilde{x}(p_a, p_b) - \mu(p_a, p_b)(1 - F(\alpha^*(p_a, p_b)))\right]^2 \right\}^n$$

Let $\tilde{\alpha}(p_a, p_b) \geq \alpha^*(p_a, p_b)$ be such that

$$\left(\mu(p_a, p_b) + \frac{1}{2}\right)(1 - F(\alpha^*(p_a, p_b))) = \int_{\alpha^*(p_a,p_b)}^{\tilde{\alpha}(p_a,p_b)} f(\alpha) d\alpha$$

Let $\alpha^* (p_a, p_b) < 1$, then $\mu + \frac{1}{2}$ coincides with the probability of a type votes for A, conditional of not acquiring information. If $\mu = 0$, $\tilde{\alpha}(p_a, p_b)$is the median of the conditional distribution.

From Stirling Formula:

$$\binom{2n}{n} \approx \frac{2^n}{\sqrt{\pi n}}$$

so that for $\omega = a, b$

$$P_\omega (p_a, p_b) = O\left(\frac{1}{\sqrt{\pi n}}\right)$$

for some $C > 0$.

If $C_x(0,0) \neq 0$ and for n large enough no voter of any type would acquire information. We can summarize these findings in the following Proposition.

Proposition 1 *A BSE with information acquisition equilibrium exists if and only if there are* $(p_a, p_b) \in [0,1]^2 \setminus \{(0,0)\}$ *and* μ *such that* $(P_{a\mu}(p_a, p_b), P_{b\mu}(p_a, p_b)) = (p_a, p_b)$. *Equilibrium strategies are given by* (x, v), *where:*

1. $(x, v)(\alpha) = (x(\alpha, p_a, p_b, A, B))$ *for* $\alpha \leq \alpha^*(p_a, p_b)$,
2. $(x, v)(\alpha) = (0, A, A)$ *if* $\alpha > \alpha^*(p_a, p_b)$ *and* $p_b q_b \Delta U_b - p_a q_a \Delta U_a > 0$,
3. $(x, v)(\alpha) = (0, B, B)$ *if* $\alpha > \alpha^*(p_a, p_b)$ *and* $p_b q_b \Delta U_b - p_a q_a \Delta U_a < 0$,
4. $(x, v)(\alpha) = (0, A, A)$ *if* $\alpha^*(p_a, p_b) < \alpha \leq \tilde{\alpha}(p_a, p_b)$ *and* $p_b q_b \Delta U_b - p_a q_a \Delta U_a = 0$,
5. $(x, v)(\alpha) = (0, B, B)$ *if* $\tilde{\alpha}(p_a, p_b) < \alpha \leq 1$ *and* $p_b q_b \Delta U_b - p_a q_a \Delta U_a = 0$.

Given any sequence of SBE and corresponding pivotal probabilities (p_{an}, p_{bn}): $lim_{n \to \infty} p_{\omega n} = 0$, *for* $\omega = a, b$.

4 Existence and informational failure(s)

For every n let (x_n, v_n) be a SBE strategy with $2n+1$ agents. Let $p_n = (p_{an}, p_{bn})$ be the corresponding pivotal probabilities. Set $\alpha_n = \alpha^*(p_n)$ and set $\tilde{x}_n = \tilde{x}(p_n)$. Finally let $\mu_n = \mu(p_n)$.

In every SBE with information acquisition and for large n, uninformed voters are indifferent between the two alternatives.

Lemma 1 *For n large, in every SBE with information acquisition* $p_{an}q_a\Delta U_a = p_{bn}q_b\Delta U_b$.

As the probability of being pivotal converges to 0 for large electorates the cutoff type, α_n and the quality of information acquired by every agent, $\tilde{x}_n$ converge to 0. In order to estimate how much information elections aggregate we need to estimate the speed of convergence to 0 of these quantities.

Proposition 2 *Let* $(x_{n_k}, v_{n_k})_{k\in\mathbb{N}}$*be a subsequence of SBE strategies. Let*

$$l^I = lim_{k\to\infty}\sqrt{n_k}\left(\tilde{x}_{n_k} + \mu_{n_k}\left(1 - F\left(\alpha_{n_k}\right)\right)\right)$$

$$l^{II} = lim_{k\to\infty}\sqrt{n_k}\left(\tilde{x}_{n_k} - \mu_{n_k}\left(1 - F\left(\alpha_{n_k}\right)\right)\right)$$

and assume they exist. Let $P_{\omega k}$*be the probability the right decision is taken at state* $\omega = a, b$*, at the corresponding SBE. Then*

$$P_{ak} \to \Phi\left(2\sqrt{2}l^I\right)$$

$$P_{bk} \to \Phi\left(2\sqrt{2}l^{II}\right)$$

In particular, the elections are asymptotically efficient along the subsequence if and only if $l^I = l^{II} = \infty$.

The proof is based on Lemma 1 and on the Berry-Esseen theorem (see Chow and Teicher 1997, p 322), which provides an estimate of the speed of converge to the standard normal distribution of the normalized sum of i.i.d random variables.

Theorem 1 *For n large an equilibrium with information acquisition exists if and only if* $q_a\Delta U_a = q_b\Delta U_b$. *As $n \to \infty$ the probability of taking the right decision converges to $\frac{1}{2}$ along every sequence of equilibria with information acquisition.*

The proof of Theorem 1 is broken in different lemmata.

Lemma 2 *If* $q_a\Delta U_a = q_b\Delta U_b$ *a SBE with information acquisition exists.*

Once the pivotal probabilities are known the amount of information acquired in equilibrium is determined. From Proposition 1 and Lemma 1 it follows that $p_a = p_b = p$ and $\mu = 0$. In this way we can reduce the dimension of the problem and work in the space of pivotal probabilities instead than in the infinite dimensional strategy space. Brower's fixed point theorem is employed in order to proof that $P_{a0}(p_a, p_b)$ has a fixed point.

The next result evaluates the speed of convergence of $\mu_n, \tilde{x}_n$ to 0.

Lemma 3 *For every sequence of equilibria with information acquisition,* $lim_{n\to\infty}\mu_n = 0$, $lim_{n\to\infty}\sqrt{n}\tilde{x}_n = 0$ *and* $lim_{n\to\infty}\sqrt{n}\left(\tilde{x}_n \pm \mu_n\left(1 - F\left(\alpha_n\right)\right)\right) = 0$.

In Martinelli (2006 and 2007) $lim_{n\to\infty}\sqrt{n}\left(\tilde{x}_n \pm \mu_n\left(1 - F\left(\alpha_n\right)\right)\right) > 0$. While in Martinelli (2007) an equilibrium with information acquisition exists for $\frac{q_a \Delta U_a}{q_b \Delta U_b}$ in a neighborhood of 1, in our setup an equilibrium with information acquisition exists if and only if $\frac{q_a \Delta U_a}{q_b \Delta U_b} = 1$. The reason is that the pivotal probabilities goes to zero so fast that the marginal benefit of acquiring information is surpassed by its marginal costs. In order to prove the result we show that $\tilde{x}_n$ goes to 0 at the same speed of p_{an}^2.

From Lemma 3and Proposition 2 we have:

Corollary 1 *Along every sequence of SBE with information acquisition the probability of taking the right decision converges to* $\frac{1}{2}$ *when the size of the population converges to infinity.*

Lemma 3 is then used to prove the last part of Theorem 1.

Lemma 4 *If* $q_a \Delta U_a \neq q_b \Delta U_b$ *a SBE with information acquisition does not exist for* n *large.*

Intuitively, replacing CONV by WCONV reduces the marginal costs of acquiring information for the lowest types so they should acquire information of higher quality. Replacing SCR by WSCR reduces the growth of marginal costs when the type increases so the cut-off type should be higher.

Nonetheless The same negative results hold when 3. is replaced by 3' but 2 holds and when 2 is replaced by 2' but 3 holds. The proof is similar to the one of Theorem 1

Theorem 2 *Assume C satisfies NFL, CONV, WSCR or it satisfies properties NFL, WCONV, SCR. For* n *large an equilibrium with information acquisition exists if and only if* $q_a \Delta U_a = q_b \Delta U_b$*. As* $n \to \infty$ *the probability of taking the right decision converges to* $\frac{1}{2}$.

In order for the election to aggregate information it is needed that both CONV and SCR are replaced by WCONV and WSCR respectively. It turns out that, in this case the Condorcet Jury Theorem holds.

Theorem 3 *Assume C satisfies NFL, WCONV and WSCR. If $q_a > 0$ and if n is large an equilibrium with information acquisition exists and the probability of taking the right decision converges to 1 as $n \to \infty$.*

5 The aggregate cost of information and voters' welfare

The expected aggregate cost of information is

$$(2n+1) \int_{0}^{\alpha(p_a,p_b)} C(\alpha, x(\alpha(p_a, p_b))) f(\alpha) \, d\alpha$$

Proposition 3 *Along a sequence of equilibria with information acquisition the aggregate cost of information converges to zero as the number of voters goes to infinite.*

The expected utility for a voter in a sequence of equilibria with information acquisition is asymptotically equivalent to

$$U_n = q_a \left(1 - \Phi\left(J_n^I\right)\right) U(A \mid a) + q_a \Phi\left(J_n^I\right) U(B \mid a) +$$

$$q_b \left(1 - \Phi\left(J_n^{II}\right)\right) U(A \mid b) + q_b \Phi\left(J_n^{II}\right) U(B \mid b) - \int_{0}^{\alpha_n} C(\alpha, x(\alpha(p_{an}, p_{bn}))) f(\alpha) \, d\alpha$$

where

$$J_n^I = \sqrt{n}\left(\tilde{x}_n + \mu_n (1 - F(\alpha_n))\right)$$

$$J_n^{II} = \sqrt{n}\left(\tilde{x}_n - \mu_n (1 - F(\alpha_n))\right)$$

The expected utility of the best uninformative equilibrium is

$$U_0 = q_a U(A \mid a) + q_b U(A \mid b)$$

When hold, the expected utility of a voter converges to

$$U_\infty = q_a U(A \mid a) + q_b U(B \mid b)$$

which is the maximum possible value of utility that can be reached then the equilibria are asymptotically efficient.

In all other cases let $q_a \Delta U_a = q_b \Delta U_b = r$, otherwise SBE with information acquisition do not exist. The expected utility of every voter converges to U_0 that coincides with the utility of every uninformed equilibrium. Assume that 1,2,3. Let $\tilde{\alpha}_n = \frac{1}{\sqrt{n}}$. Let $\beta_n$be the median of the conditional distribution $F(\alpha \mid \alpha \geq \tilde{\alpha}_n)$. Set $x(\alpha) = x\left(\alpha, \frac{1}{\sqrt{n}}\right)$for every $\alpha \leq \tilde{\alpha}_n$and set $x(\alpha) = 0$ otherwise. Let types $\alpha \leq \tilde{\alpha}_n$voting according to the signal, types with $\tilde{\alpha}_n < \alpha \leq \beta_n$voting for A and types in $\beta_n < \alpha \leq 1$ voting for B. If it is the case

$$\sqrt{n}\tilde{x}_n \approx Ce^{-4\left(\sqrt{n}\tilde{x}_n\right)^2}$$

for some $C > 0$, so that $lim_{n\to\infty}\sqrt{n}\tilde{x}_n = l$, where l satisfies $l = Ce^{-4l^2}$.
Furthermore, $(2n+1)\int_0^{\alpha(p_a,p_b)} C(\alpha, x(\alpha(p_a,p_b)))\, f(\alpha)\, d\alpha \to 0$. So the sequence of equilibria are not asymptotically efficient.
Choosing appropriately $\tilde{\alpha}_n = n^{-\beta}$, $\beta > 0$ the same result can be proved also when 1,2',3 or 1,2,3' hold.

Proposition 4 *If either NFL, CONV and SCR or NFL, CONV and WSCR or NFL, WCONV and SCR hold SBE with information acquisition are asymptotically inefficient. If NFL, WCONV and WSCR hold they are asymptotically efficient.*

6 Conclusions

When voters can acquire information of different qualities and have different information acquisition results large election fail to aggregate information, in general. This is consistent with the most pessimistic view of the rational ignorance hypothesis. Information aggregation is possible only under quite restrictive assumptions.

There are aspects not reflected here could have important implications. First of all, in our model information acquisition is independent among voters. It is not clear the impact of communication or correlation among different sources of information as it would introduce new strategic considerations.[3] Furthermore, the information and its cost are exogenously provided. Competition among information providers might reduce its costs and improve election efficiency. Also the possibility of abstention might affect our results. If less informed voters abstain the probability an informed voter is decisive increases and so the incentive to acquire information (see also Feddersen and Pesendorfer (1999)).

Appendix: Proofs

Proof of Lemma 1. By contradiction, assume there exist a sequence of equilibria with information acquisition such that $p_{an}q_a\Delta U_a > p_{bn}q_b\Delta U_b$ for infinitely many n. With no loss of generality we assume $p_{an}q_a\Delta U_a > p_{bn}q_b\Delta U_b$ for every n. So the highest type who acquire information, $\alpha = \alpha^*(p_{an}, p_{bn})$ and $x = x(\alpha(p_{an}, p_{bn}), (p_{an}, p_{bn}))$ solve the system of equation.

$$\begin{cases} C_x(\alpha, x) = p_{an}q_a\Delta U_a + p_{bn}q\Delta U_b \\ (p_{an}q_a\Delta U_a + p_{bn}q\Delta U_b)\, x - C(\alpha, x) = \frac{p_{an}q_a\Delta U_a - p_{bn}q_b\Delta U_b}{2} \end{cases}$$

Set $y = p_{an}q_a\Delta U_a + p_{bn}q\Delta U_b$ and set $z = \frac{p_{an}q_a\Delta U_a - p_{bn}q_b\Delta U_b}{2}$.

Consider the system

$$\begin{cases} C_x(\alpha, x) = y \\ yx - C(\alpha, x) = z \end{cases} \tag{4}$$

[3]Gerardi and Yariv (2007) study a model of pre-voting communication communication, without information acquisition.

Let $\bar{\alpha}$ satisfying

$$C_x(\bar{\alpha}, 0) = y.$$

Such an $\bar{\alpha}$ exists for y small enough. The equation

$$C_x(\alpha, x) = y \tag{5}$$

has a solution x if and only if $\alpha \leq \bar{\alpha}$. Such solution, $x^I(\alpha)$ is unique and it is a continuous and strictly decreasing function of α.

The equation

$$yx - C(\alpha, x) = z \tag{6}$$

has a solution if and only if if and only if

$$yx^I(0) - C\left(0, x^I(0)\right) \leq z$$

If this condition is met, for every α the solution is unique. We denote it by $x^{II}(\alpha)$, which is continuous. Observe that the graphs of x^I and x^{II} never intersect in their interior, because the derivative of $x^{II}(\alpha)$ explodes and changes of sign when it intersects $x^I(\alpha)$.

If

$$yx^I(0) - C\left(0, x^I(0)\right) < z$$

then $x^{II}(0) > x^I(0)$. In this case the only possible solution of of system4 is $\alpha = \bar{\alpha}$ and $x = 0$. At the corresponding SBE we have $\alpha(p_{an}, p_{bn}) = \alpha^*(p_{an}, p_{bn})$, in contradiction with $p_{an}q_a\Delta U_a > p_{bn}q_b\Delta U_b$.

If

$$yx^I(0) - C\left(0, x^I(0)\right) = z$$

the only possible solution of system 4 is $\alpha = 0$ and $x = x^I(0)$. At the corresponding SBE we have $\alpha(p_{an}, p_{bn}) = 0$. If it was the case only a zero measure set of voters would acquire information and with probability one all types would vote for alternative A, so the probability of being pivotal would be null, a contradiction.

Proof of Proposition 2. Without loss of generality assume that the sequences themselves converge. Given equilibrium strategies, let the event of a given voter voting for A in state a corresponds to a Bernoulli trial with probability of success $\frac{1}{2} + \tilde{x}_n + \mu_n (1 - F(\alpha_n))$. For $i = 1, ..., 2n+1$, set

$$V_i^n = \begin{cases} \frac{1}{2} + \tilde{x}_n + \mu_n (1 - F(\alpha_n)) \text{ if voter } i \text{ votes for} A \\ \frac{1}{2} - [\tilde{x}_n + \mu_n (1 - F(\alpha_n))] \text{ if voter } i \text{ votes for } B \end{cases}$$

The V_i^n are i.i.d. Furthermore,$E(V_i^n) = 0$, $E\left((V_i^n)^2\right) = \frac{1}{4} - [\tilde{x}_n + \mu_n (1 - F(\alpha_n))]^2$ and $E\left(|V_i^n|^3\right) = \frac{1}{4} - 2[\tilde{x}_n + \mu_n (1 - F(\alpha_n))]^4$.

Let W^n be the normalized sum of the V_i^n.

$$W^n = \frac{\sum_{i=1}^{2n+1} V_i^n}{\sqrt{(2n+1) E\left((V_i^n)^2\right)}}$$

Let F_n be the p.d.f. of W^n. The alternative A wins if and only if it gets at least $n+1$ votes which is if and only if

$$\sum_{i=1}^{2n+1} V_i^n > -\frac{1}{2} - (2n+1) [\tilde{x}_n + \mu_n (1 - F(\alpha_n))]$$

The probability of reaching the right alternative at state a is $1 - F_n(J_n)$ where

$$J_n = \frac{-\frac{1}{2} - (2n+1) [\tilde{x}_n + \mu_n (1 - F(\alpha_n))]}{\sqrt{\left\{\frac{1}{4} - [\tilde{x}_n + \mu_n (1 - F(\alpha_n))]^2\right\} (2n+1)}}$$

$$J_n \approx \frac{-\frac{1}{2} - 2\sqrt{n} \left\{\sqrt{n} [\tilde{x}_n + \mu_n (1 - F(\alpha_n))]\right\}}{\sqrt{\frac{n}{2} - 2\left\{\sqrt{n} [\tilde{x}_n + \mu_n (1 - F(\alpha_n))]\right\}^2}} \approx -2\sqrt{2}\left\{\sqrt{n} [\tilde{x}_n + \mu_n (1 - F(\alpha_n))]\right\}$$

From the Berry-Esseen Theorem (see Chow and Teicher 1997, p 322).

$$|F_n(J_n) - \Phi(J_n)| = O\left(\frac{1}{\sqrt{n}}\right)$$

So

$$lim_{n\to\infty}P_{an} = 1 - \Phi\left(-2\sqrt{2}l^{I}\right) = \Phi\left(2\sqrt{2}l^{I}\right)$$

whether l^I is finite or infinite.

The proof of the case $\omega = b$ is similar.

Proof of Lemma 2. It must be the case that $p_a = p_b = p$ (see Lemma 1). Furthermore with the notation of Proposition 1, $\mu = 0$. Set $r = q_a \Delta U_a$. Let $\hat{\alpha}(p)$ satisfying $2rp = C_x(\alpha, 0)$ if any such α exists and $\hat{\alpha}(p) = 1$ otherwise. The function $\hat{\alpha}(p)$ is continuous. Let the function $x(\alpha, p)$ be defined on $[0, \hat{\alpha}(p)]$ satisfying $2rp = C_x(\alpha, x)$ for every $\alpha \in [0, \hat{\alpha}(p)]$. set

$$T(p) = \binom{2n}{n}\left[\frac{1}{4} - \left(\int_{0}^{\hat{\alpha}(p)} x(\alpha, p) f(\alpha) d\alpha\right)^{2}\right]^{n}$$

The function $T : [0, 1] \to [0, 1]$ is well defined and continuous so it has a fixed point. Let p^* be a fixed point of T. Define $\tilde{\alpha}$ as follows:

$$F(\tilde{\alpha}) - F(\hat{\alpha}(p^*)) = \int_{\hat{\alpha}(p)}^{\tilde{\alpha}} f(\alpha) d\alpha = \frac{1 - F(\hat{\alpha}(p^*))}{2}.$$

Type $\tilde{\alpha}$ is the median type, conditional on the types who do not acquiring information. Consider the strategy (x, v), where $(x, v)(\alpha) = (x(\alpha, p^*), A, B)$ for $\alpha \leq \hat{\alpha}(p^*)$, $(x, v)(\alpha) = (0, A, A)$ for $\alpha(p^*) \leq \alpha < \tilde{\alpha}$ and $(x, v)(\alpha) = (0, A, A)$ for $\tilde{\alpha} < \alpha \leq 1$. It is easily seen that $(x, v)_i = (x, v)$ for $i = 1, ...2n + 1$ is a SBE. We next prove that there is information acquisition. By contradiction assume that there is no information acquisition then $\alpha(p^*) = 0$. It follows that $p^* = 0$, but $T(0) = \binom{2n}{n}\frac{1}{4^n} \neq 0$, a contradiction.

Proof of Lemma 3. (i) From Proposition 1, at any equilibrium $p_a q_a \Delta U_a = p_b q_b \Delta U_b$. Let $p \in [0, 1]$ and r be such that $pr = p_a q_a \Delta U_a = p_b q_b \Delta U_b$. Let Let $\hat{\alpha}(p)$ satisfying $2rp = C_x(\alpha, 0)$ if any such α exists and $\hat{\alpha}(p) = 1$ otherwise. The function$\hat{\alpha}(p)$ is continuous. Let the function $x(\alpha, p)$ be defined on $[0, \hat{\alpha}(p)]$ satisfying $2rp = C_x(\alpha, x)$ for very $\alpha \in [0, \hat{\alpha}(p)]$. Define $\tilde{x}(p) = \int_0^{\hat{\alpha}(p)} x(\alpha, p) f(\alpha) d\alpha$. From Proposition 1 and 1 for n large enough there exists $\mu \in \left[-\frac{1}{2}, \frac{1}{2}\right]$ satisfying

$$q_a \Delta U_a \binom{2n}{n} \left\{ \frac{1}{4} - \left[\tilde{x}\left(p_a, p_b\right) + \mu \left(1 - F\left(\hat{\alpha}\left(p_a, p_b\right)\right)\right)\right]^2 \right\}^n = C_x\left(\hat{\alpha}\left(p_a, p_b\right), 0\right)$$

$$q_b \Delta U_b \binom{2n}{n} \left\{ \frac{1}{4} - \left[\tilde{x}\left(p_a, p_b\right) - \mu \left(1 - F\left(\hat{\alpha}\left(p_a, p_b\right)\right)\right)\right]^2 \right\}^n = C_x\left(\hat{\alpha}\left(p_a, p_b\right), 0\right)$$

$$p_a = \binom{2n}{n} \left\{ \frac{1}{4} - \left[\tilde{x}\left(p_a, p_b\right) + \mu \left(1 - F\left(\hat{\alpha}\left(p_a, p_b\right)\right)\right)\right]^2 \right\}^n$$

$$p_b = p_a \frac{q_a \Delta U_a}{q_b \Delta U_b}$$

Set $p = p_a$ set $x(\alpha, p) = x\left(\alpha, p, \frac{q_a \Delta U_a}{q_b \Delta U_b} p\right)$ and set $\alpha(p) = \alpha\left(p, \frac{q_a \Delta U_a}{q_b \Delta U_b} p\right)$. Then

$$2 p q_a \Delta U_a = C_x\left(\alpha, x\left(\alpha, p\right)\right)$$

and

$$2 p q_a \Delta U_a = C_x\left(\alpha\left(p\right), 0\right)$$

$$p \Delta = C_x\left(\alpha, 0\right)$$

It follows:

$$\alpha_p\left(p\right) = \frac{2 p q_a \Delta U_a}{C_{x\alpha}\left(\alpha\left(p\right), 0\right)}$$

and for $p \to 0$

$$\alpha\left(p\right) = \frac{2 q_a \Delta U_a}{C_{x\alpha}\left(0, 0\right)} p + o\left(p\right)$$

Similarly

$$x_\alpha\left(\alpha, p\right) = -\frac{C_{x\alpha}\left(\alpha, x\left(\alpha, p\right)\right)}{C_{xx}\left(\alpha, x\left(\alpha, p\right)\right)}$$

and

$$x_p\left(\alpha, p\right) = \frac{2 q_a \Delta U_a}{C_{xx}\left(\alpha, x\left(\alpha, p\right)\right)}$$

Observe that the second partial derivatives of C are continuous so that C_{xx} , $C_{\alpha x}$ are bounded away from zero in a neighborhood of $(0,0)$ and $\alpha(p) = O(p)$ uniformly in a neighborhood of 0. For $p \to 0$ and $\alpha < \alpha(p)$, for some $\gamma \in (\alpha, \alpha(p))$

$$x(\alpha,p) = -x_\alpha(\gamma,p)(\alpha(p)-\alpha) + x_p(\gamma,p)\,p = \frac{C_{x\alpha}(\gamma,p)}{C_{xx}(\gamma,p)}(\alpha(p)-\alpha) + \frac{2q_a\Delta U_a}{C_{x\alpha}(\gamma,p)}p$$

Then.

$$x(\alpha,p) = \frac{C_{x\alpha}(0,0)}{C_{xx}(0,0)}(\alpha(p)-\alpha) + \frac{2q_a\Delta U_a}{C_{x\alpha}(0,0)}p + o(p) + o(\alpha)$$

uniformly in $\alpha < \alpha(p)$. It follows that (see Olver 1974)

$$\widetilde{x}(p) = \int_0^{\alpha(p)} x(p,\alpha) f(\alpha)\,d\alpha \approx \frac{C_{x\alpha}(0,0)}{C_{xx}(0,0)}\int_0^{\alpha(p)} (\alpha(p)-\alpha) f(\alpha)\,d\alpha + \frac{2q_a\Delta U_a}{C_{x\alpha}(0,0)}p\int_0^{\alpha(p)} f(\alpha)\,d\alpha$$

$$\approx \frac{C_{x\alpha}(0,0)}{C_{xx}(0,0)}f(0)\frac{\alpha^2(p)}{2} + \frac{2q_a\Delta U_a}{C_{x\alpha}(0,0)}F(\alpha(p)) \approx \frac{C_{x\alpha}(0,0)}{C_{xx}(0,0)}f(0)\frac{\alpha^2(p)}{2} + \frac{2q_a\Delta U_a}{C_{x\alpha}(0,0)}f(0)\,p\alpha(p)$$

$$\approx \left(\frac{C_{x\alpha}(0,0)}{2C_{xx}(0,0)} + 1\right)\left(\frac{2q_a\Delta U_a}{C_{x\alpha}(0,0)}\right)^2 f(0)\,p^2$$

For $p \to 0$.

$$p^2 \approx C\widetilde{x}(p) \tag{7}$$

where $C = \left[\left(\frac{C_{x\alpha}(0,0)}{2C_{xx}(0,0)} + 1\right)\left(\frac{\Delta}{C_{x\alpha}(0,0)}\right)^2 f(0)\right]^{-1}$

At the SBE

$$p_{an} = \binom{2n}{n}\left\{\frac{1}{4} - \left[\tilde{x}_n + \mu_n(1-F(\alpha_n))\right]^2\right\}^n$$

and

$$p_{bn} = \binom{2n}{n}\left\{\frac{1}{4} - \left[\tilde{x}_n - \mu_n(1-F(\alpha_n))\right]^2\right\}^n$$

Assume $lim_{n\to\infty}\mu_n = M$ exists and is non negative.[4] Observe that $M = lim_{n\to\infty}\tilde{x}(p)+\mu(1-F(\alpha(p))) \geq 0$ and that $-M = lim_{n\to\infty}\tilde{x}(p)-\mu(1-F(\alpha(p)))$.

Assume first that $M=0$. For $n\to\infty$

$$p_{an} \approx \frac{e^{-4\left\{\sqrt{n}[\tilde{x}_n+\mu_n(1-F(\alpha_n))]\right\}^2}}{\sqrt{\pi n}}$$

and

$$p_{bn} \approx \frac{e^{-4\left\{\sqrt{n}[\tilde{x}_n-\mu_n(1-F(\alpha_n))]\right\}^2}}{\sqrt{\pi n}}$$

From 7 it follows that, for $n\to\infty$:

$$\left(\sqrt{n}\tilde{x}_n\right)e^{8\left\{\sqrt{n}[\tilde{x}_n+\mu_n(1-F(\alpha_n))]\right\}^n} \approx \frac{1}{\pi C\sqrt{n}}$$

$$\left(\sqrt{n}\tilde{x}_n\right)e^{8\left\{\sqrt{n}[\tilde{x}_n-\mu_n(1-F(\alpha_n))]\right\}^2} \approx \left(\frac{q_b\Delta U_b}{q_a\Delta U_a}\right)^2\frac{1}{\pi C\sqrt{n}}$$

Combining the two equivalence we obtain the claim.

Now let $0 < M < \frac{1}{2}$. Set $\delta = \sqrt{\frac{1}{4}-M^2}$ and set $y_n = [\tilde{x}_n-\mu_n(1-F(\alpha_n))]^2 - M^2 = o(1)$ and set $z_n = \sqrt{n}\,[\tilde{x}_n-\mu_n(1-F(\alpha_n))]^2 - M^2 = o(1)$. We have

$$p_{an} \approx \frac{(2\delta)^{2n}}{\sqrt{\pi n}}e^{-n\frac{y_n}{\delta^2}}$$

$$p_{bn} \approx \frac{(2\delta)^{2n}}{\sqrt{\pi n}}e^{-n\frac{z_n}{\delta^2}}$$

Furthermore, from Lemma 1:

$$lim_{n\to\infty}e^{-n\frac{y_n-z_n}{\delta^2}} = \frac{q_b\Delta U_b}{q_a\Delta U_a}$$

so

$$lim_{n\to\infty}-n\frac{y_n-z_n}{\delta^2} = ln\frac{q_b\Delta U_b}{q_a\Delta U_a}$$

[4] There is no loss of generality: the argument can be used along every convergent subsequence.

Then, 7 imply that

$$\widetilde{x}(p) \approx O\left(\frac{(2\delta)^{4n}}{\pi n}\right)$$

But then, as $n \to \infty$

$$n(y_n - z_n) = -4n\mu_n(1 - F(\alpha_n))\widetilde{x}_n \to 0$$

because $\delta < \frac{1}{2}$. A contradiction.

The case $M = \frac{1}{2}$ is similar. In order to conclude and obtain a contradiction one has to observe that now $\widetilde{x}_n \approx o\left(\frac{(2\delta)^{4n}}{\pi n}\right)$ for every $\delta > 0$. So it must be that $M = 0$.

Proof of Lemma 4. If an equilibrium with information acquisition exists there are $\mu \in \left(-\frac{1}{2}.\frac{1}{2}\right)$ and (p_a, p_b)such that:

$$2\Delta U_a q_a \binom{2n}{n}\left[\frac{1}{4} - \left(\widetilde{x}(p_a, p_b) + \mu(1 - F(\alpha(p_a, p_b)))\right)^2\right]^n = C_x(\alpha(p_a, p_b), 0) \tag{8}$$

$$2\Delta U_b q_b \binom{2n}{n}\left[\frac{1}{4} - \left(\widetilde{x}(p_a, p_b) - \mu(1 - F(\alpha(p_a, p_b)))\right)^2\right]^n = C_x(\alpha(p_a, p_b), 0) \tag{9}$$

Assume an equilibrium with information exists for infinitely many n. Let $\{n_k\}_{k\in\mathbb{N}}$ be a subsequence such that a SBE with information acquisition exists when there are $2n_k + 1$voters. With the notation of Lemma 3 we have.

$$lim_{k\to\infty}\frac{p_{an_k}}{p_{bn_k}} = lim_{n\to\infty}\frac{\binom{2n}{n}\left\{\frac{1}{4} - \left[\tilde{x}_n - +\mu_n(1 - F(\alpha_n))\right]^2\right\}^n}{\binom{2n}{n}\left\{\frac{1}{4} - \left[\tilde{x}_n - \mu_n(1 - F(\alpha_n))\right]^2\right\}^n} = lim_{n\to\infty}\frac{e^{-4\left[\sqrt{n}(\tilde{x}_n + \mu_n(1-F(\alpha_n)))\right]^2}}{e^{-4\left[\sqrt{n}(\tilde{x}_n - \mu_n(1-F(\alpha_n)))\right]^2}} = 1$$

but according to Lemma 1. This is possible only if $q_a\Delta U_a = q_b\Delta U_b$.

Proof of Corollary 1. The claim follows from Lemma 3 and Proposition 2.

Proof of Proposition 2. Lemmas 1, 2 hold also when CONV is replaced by WCONV and when SCR is replaced by WSCR. Consider first the case where SCR is replaced by WSCR The proof of the other results need only minor changes. Here we sketch them. Let k be the minimal integer such that $D = C_{x\alpha^{(k)}}(0, 0) \neq 0$.

We have $D > 0$. Using a k^{th} order Taylor's approximation, as $n \to \infty$

$$\alpha\left(p_n\right) \approx \left(D^{-1} 2 p_{an} q_a \Delta U_a\right)^{\frac{1}{k}}.$$

Working as in the proof of Lemma 3, we get

$$\widetilde{x}_n = C\left(p_{an}\right)^{1+\frac{1}{k}}$$

as $n \to \infty$, from which the claims of Lemma 3, Lemma 4 and Lemma 1 follow exactly as above.

Now consider the case where CONV is replaced by WCONV. At equilibrium $p_{an} = p_{bn} = p_n$, let $\alpha\left(p_a\right) = \alpha\left(p_a, \frac{q_a \Delta U_a}{q_b \Delta U_b} p_a\right)$. Let k be the lowest integer such that $C_{x^{(k)}}\left(0,0\right) \neq 0$

For $p_a \to 0$ (see the proof of Lemma 3):

$$\alpha\left(p_a\right) \approx \frac{2 p_a q_a \Delta U_a}{C_{x\alpha}\left(0,0\right)}$$

We have:

$$4 p_a q_a \Delta U_a = 2 C_x\left(\alpha, x\right) = 2 C_{x\alpha}\left(0,0\right)\alpha + C_{x^{(k)}}\left(0,0\right) x^{k-1} + o\left(\alpha\right) + o\left(x^2\right)$$

uniformly in $\alpha \leq \alpha\left(p_a\right)$, and in $x \leq x\left(0, p_a\right)$.

So, for $\alpha \leq \bar{\alpha}\left(p_a\right) = min\left\{\alpha\left(p\right), \frac{2 p_a q_a \Delta U_a}{C_{x\alpha}(0,0)}\right\}$

$$x\left(\alpha, p\right) \approx \left(C_{x^{(k)}}\left(0,0\right)\right)^{-\frac{1}{k-1}} \left(p_a q_a \Delta U_a - 2 C_{x\alpha}\left(0,0\right)\alpha\right)^{\frac{1}{k-1}}$$

uniformly in α.

Observe that $\bar{\alpha}\left(p_a\right) - \alpha\left(p_a\right) = O\left(p_a^2\right)$. Integrating (see also Segala 1999 and Olver 1974), for $p_a \to 0$

$$\tilde{x}\left(p_a, \frac{q_a \Delta U_a}{q_b \Delta U_b} p_a\right) \approx \left(C_{x^{(k)}}\left(0,0\right)\right)^{-\frac{1}{k-1}} \int_0^{\bar{\alpha}(p_a)} \left(p_a q_a \Delta U_a - 2 C_{x\alpha}\left(0,0\right)\alpha\right) f\left(\alpha\right) d\alpha$$

and

$$\tilde{x}(p_a) \approx \frac{(k-1)(4\alpha(p_a) q_a \Delta U_a)^{\frac{k}{k-1}} f(0)}{k C_{x\alpha}(0,0)\left(C_{x^{(k)}}(0,0)\right)^{\frac{1}{k-1}}}$$

Then

$$\tilde{x}(p_a) \approx \frac{(k-1)\left(8q_a^2(\Delta U_a)^2\right)^{\frac{k}{k-1}} f(0)}{k\left(C_{x\alpha}(0,0)\right)^{\frac{k+1}{k-1}}\left(C_{x^{(k)}}(0,0)\right)^{\frac{1}{k-1}}}(p_a)^{\frac{k}{k-1}}$$

from which the claims of Lemma 3, Lemma 4 and Lemma 1 follow exactly as above.

Proof of Theorem 3. Also in this case Proposition 1 and Lemma 1 hold. First assume $C_{xxx}(0,0) \neq 0$ and $C_{x\alpha\alpha}(0,0) \neq 0$.

At every SBE $p_{an} = \frac{q_b \Delta U_b}{q_a \Delta U_a} p_{bn}$. Set $p_{nb} = p_n$. Set $\alpha(p) = \alpha\left(\frac{q_b \Delta U_b}{q_a \Delta U_a} p, p\right)$, set $x(\alpha, p) = x\left(\alpha, \left(\frac{q_b \Delta U_b}{q_a \Delta U_a} p, p\right)\right)$ and set $\tilde{x}(p) = \tilde{x}\frac{q_b \Delta U_b}{q_a \Delta U_a} p, p$. Observe that $C_{xxx}(0,0) > 0$ and $C_{x\alpha\alpha}(0,0) > 0$.

For $p \to 0$ (see:

$$\alpha(p) \approx \left(\frac{2pq_b \Delta U_a}{C_{x\alpha\alpha}(0,0)}\right)^{\frac{1}{2}}.$$

We have:

$$4pq_b \Delta U_b \Delta = 2C_x(\alpha, x) = C_{x\alpha\alpha}(0,0)\alpha^2 + C_{xxx}(0,0)x^2 + 2C_{xx\alpha}(0,0)\alpha x + o\left(\|(\alpha, x)\|^2\right)$$

So, for $\alpha \leq \bar{\alpha}(p) = min\left\{\alpha(p), \left(\frac{2pq_b \Delta U_b}{C_{x\alpha\alpha}(0,0)}\right)^{\frac{1}{2}}\right\}$

$$x(\alpha, p) = \frac{-C_{xx\alpha}(0,0)\alpha + \sqrt{\left[C_{xx\alpha}^2(0,0) - C_{x\alpha\alpha}(0,0) C_{xxx}(0,0)\right]\alpha^2 + 4pq_b \Delta U_b}}{C_{xxx}(0,0)} + o(\alpha) + o(\sqrt{p})$$

uniformly in $\alpha \leq \bar{\alpha}(p)$ (see also Segala 1999), because x is non negative.

Observe that $\bar{\alpha}(p) - \alpha(p) = O(p)$. Then Integrating, for $p \to 0$

$$\tilde{x}(p) \approx \int_0^{\bar{\alpha}(p)} \frac{-C_{xx\alpha}(0,0)\alpha + \sqrt{\left[C_{xx\alpha}^2(0,0) - C_{x\alpha\alpha}(0,0) C_{xxx}(0,0)\right]\alpha^2 + 4pq_b \Delta U_b}}{C_{xxx}(0,0)} f(\alpha) d\alpha$$

$$\tilde{x}(p) \approx \frac{-C_{xx\alpha}(0,0)\alpha^2(p) f(0)}{2C_{xxx}(0,0)} +$$

$$\left[\frac{\alpha f\left(0\right)\sqrt{\left[C_{xx\alpha}^{2}\left(0,0\right)-C_{x\alpha\alpha}\left(0,0\right)C_{xxx}\left(0,0\right)\right]\alpha^{2}+4\frac{\alpha f(0)\sqrt{[C_{xx\alpha}^{2}(0,0)-C_{x\alpha\alpha}(0,0)C_{xxx}(0,0)]\alpha^{2}+4pq_{b}\Delta U_{b}}}{2C_{xxx}(0,0)}q_{b}\Delta U_{b}}}{2C_{xxx}\left(0,0\right)}\right]_{\alpha=0}^{\alpha=\alpha(p_{a})}+$$

$$\left[\frac{f\left(0\right)2pq_{b}\Delta U_{b}log\left(2\sqrt{C_{x\alpha\alpha}\left(0,0\right)}\alpha+2\sqrt{\left[C_{xx\alpha}^{2}\left(0,0\right)-C_{x\alpha\alpha}\left(0,0\right)C_{xxx}\left(0,0\right)\right]\alpha^{2}+4pq_{b}\Delta U_{b}}\right)}{\left(C_{xxx}\left(0,0\right)\right)^{\frac{3}{2}}}\right]_{\alpha=0}^{\alpha=\alpha(p_{a})}$$

Simplifying and eliminating infinitesimal of higher order:

$$\tilde{x}\left(p\right)\approx Cplog\left(p\right)$$

for some constant $C<0$.

Observe that if $q_b\Delta U_b = q_a\Delta U_a$ then Lemma 2 holds.

As $n\to\infty$

$$p_{an}\approx\frac{1}{\sqrt{\pi n}}e^{-4\left(\sqrt{n}\tilde{x}_{n}\right)^{2}}$$

So

$$\sqrt{n}\tilde{x}_{n}\approx\frac{C}{\sqrt{\pi}}e^{-4\left(\sqrt{n}\tilde{x}_{n}\right)^{2}}\left[-\frac{1}{2}log\left(\pi n\right)-4\left(\sqrt{n}\tilde{x}_{n}\right)^{2}\right]$$

And

$$lim_{n\to\infty}\sqrt{n}\tilde{x}_{n}=\infty$$

We can conclude with Proposition 2.

Now assume $q_b\Delta U_b < q_a\Delta U_a$. For every $\alpha,\gamma\in[0,1]$ define $x\left(\alpha,\gamma\right)$ as a solution of

$$C_{x}\left(\alpha,x\left(\alpha,\gamma\right)\right)=C_{x}\left(\gamma,0\right)$$

and define

$$\widetilde{x}\left(\gamma\right)=\int_{0}^{\gamma}x\left(\alpha,\gamma\right)f\left(\alpha\right)d\alpha$$

A SBE exists if and only if there exists $(\gamma,\mu)\in[0,1]\times\left[-\frac{1}{2},\frac{1}{2}\right]$ satisfying:

$$2\Delta U_a q_a \binom{2n}{n} 2^{-2n}\left\{1-4\left[\widetilde{x}(\gamma)+\mu(1-F(\gamma))\right]^2\right\}^n = C_x(\gamma,0) \tag{10}$$

$$2\Delta U_b q_b \binom{2n}{n} 2^{-2n}\left\{1-4\left[\widetilde{x}(\gamma)-\mu(1-F(\gamma))\right]^2\right\}^n = C_x(\gamma,0) \tag{11}$$

The SBE has information acquisition if and only if $-\frac{1}{2}<\mu<\frac{1}{2}$ so that $\gamma>0$. Let $\gamma_n^I(\mu)$ the solution of equation 10 and let $\gamma_n^{II}(\mu)$ the solution of equation 11. The function $\gamma_n^I(\mu)$ has a maximum γ_n^I which satisfies

$$2\Delta U_a q_a 2^{-2n}\binom{2n}{n} = C_x\left(\gamma^I,0\right)$$

and it is reached for $\mu_n=\mu_n^I$ where

$$\mu_n^I=\frac{-\widetilde{x}\left(\gamma_n^I\right)}{\left(1-F\left(\gamma_n^I\right)\right)}$$

The function $\gamma_n^{II}(\mu)$ has a maximum γ_n^{II} which satisfies

$$2\Delta U_b q_b 2^{-2n}\binom{2n}{n} = C_x\left(\gamma_n^{II},0\right)$$

and it is reached for $\mu_n=\mu_n^I$ where

$$\mu_n^I=\frac{\widetilde{x}\left(\gamma_n^{II}\right)}{\left(1-F\left(\gamma_n^{II}\right)\right)}$$

Observe that $-\frac{1}{2}<\mu_n^I<0<\mu_n^{II}<\frac{1}{2}$ and $0<\gamma_n^I<\gamma_n^{II}$. All sequences converge to 0 as $n\to\infty$ because

$$2^{-2n}\binom{2n}{n}\approx\frac{1}{\sqrt{\pi n}}$$

Furthermore $\gamma_n^{II}\left(\mu_n^I\right)<\gamma_n^I$ so that in order to prove that a SBE with information acquisition exists for n large enough it suffices to prove that $\gamma^{II}\geq\gamma^I\left(\mu^{II}\right)$ for n large enough . The left hand side of equation 10 is decreasing in γ for $\mu>0$. So this is equivalent to check that

$$\left\{1-4\left[\widetilde{x}\left(\gamma_n^{II}\right)+\mu_n^{II}\left(1-F\left(\gamma_n^{II}\right)\right)\right]^2\right\}^n\leq\frac{\Delta U_b q_b}{\Delta U_a q_a} \tag{12}$$

for n large enough. For $n \to \infty$:

$$\left\{1-4\left[\widetilde{x}\left(\gamma_n^{II}\right)+\mu_n^{II}\left(1-F\left(\gamma_n^{II}\right)\right)\right]^2\right\}^n \approx e^{-2\left\{\sqrt{n}\left[\widetilde{x}\left(\gamma_n^{II}\right)+\mu_n^{II}\left(1-F\left(\gamma_n^{II}\right)\right)\right]\right\}}$$

Set

$$p_n = 2^{-2n}\binom{2n}{n} \approx \frac{1}{\sqrt{\pi n}}$$

Observe that

$$\gamma_n^{II} = \alpha\left(p_n, p_n\right)$$

and

$$\tilde{x}\left(\gamma_n^{II}\right) = \tilde{x}\left(p_n, p_n\right)$$

So, for some $C < 0$

$$\tilde{x}\left(\gamma_n^{II}\right) \approx C p_n log\left(p_n\right) \approx \frac{C}{\sqrt{\pi n}} log\left(\frac{1}{\sqrt{\pi n}}\right)$$

so

$$lim_{n\to\infty}\sqrt{n}\tilde{x}\left(\gamma_n^{II}\right) = \infty$$

and the left hand side of inequality 12 converges to 0. Then a SBEequilibrium with information exists for n large enough.

We have

$$p_{an} = \binom{2n}{n}\left\{\frac{1}{4}-\left[\tilde{x}_n+\mu_n\left(1-F\left(\alpha_n\right)\right)\right]^2\right\}^n$$

and

$$p_{bn} = \binom{2n}{n}\left\{\frac{1}{4}-\left[\tilde{x}_n-\mu_n\left(1-F\left(\alpha_n\right)\right)\right]^2\right\}^n$$

So

$$p_{an} \approx \frac{1}{\sqrt{\pi n}} e^{-4\left\{\sqrt{n}\left[\tilde{x}_n+\mu_n\left(1-F\left(\alpha_n\right)\right)\right]\right\}^2}$$

$$p_{bn} \approx \frac{1}{\sqrt{\pi n}} e^{-4\left\{\sqrt{n}\left[\tilde{x}_n-\mu_n\left(1-F\left(\alpha_n\right)\right)\right]\right\}^2}$$

Without loss of generality assume $\mu_n \leq 0$ for infinitely many n.[5]

$>$From

$$\tilde{x}\left(p_{bn}\right) \approx C p_{bn} log\left(p_{bn}\right)$$

we have

$$\sqrt{n}\tilde{x}\left(p_{bn}\right) \approx C\left\{-4\left\{\sqrt{n}\left[\tilde{x}_n - \mu_n\left(1 - F\left(\alpha_n\right)\right)\right]\right\}^2 - \frac{1}{2}log\left(\pi n\right)\right\}\frac{1}{\sqrt{\pi}}Ce^{-4\left\{\sqrt{n}\left[\tilde{x}_n + \mu_n\left(1 - F\left(\alpha_n\right)\right)\right]\right\}^2}$$

So

$$lim_{n\to\infty}\sqrt{n}\left[\tilde{x}_n + \mu_n\left(1 - F\left(\alpha_n\right)\right)\right] = \infty$$

$>$From $\frac{p_{an}}{p_{bn}} = \frac{q_a \Delta U_a}{q_b \Delta U_b}$ it follows that also

$$lim_{n\to\infty}\sqrt{n}\left[\tilde{x}_n - \mu_n\left(1 - F\left(\alpha_n\right)\right)\right] = \infty$$

We can conclude by Proposition 2.

Now we consider the most general case. Let $k_1$be the lowest k such that $C_{x\alpha^{(k)}}\left(0,0\right) \neq 0$ and let $k_2$be the lowest k such that $C_{x^{(k)}}\left(0,0\right) \neq 0$. Then, for every positive constants C_1, C_2, C_3.

$$C_x\left(\alpha, x\right) = o\left(C_1 x^2 + 2C_2\alpha x + C_3\alpha^2\right)$$

for $\alpha \leq \alpha\left(p\right)$ and $x \leq x\left(0,p\right)$ for $p \to 0$.

Let C_1, C_2, C_3 be such that

$$\frac{-C_1\alpha + \sqrt{\left(C_2^2 - C_1C_3\right)\alpha^2 + 4pq_b\Delta U_b}}{C_1}$$

is well defined and non negative for all $\alpha \leq \alpha\left(p\right)$.

Then

$$\frac{-C_1\alpha + \sqrt{\left(C_2^2 - C_1C_3\right)\alpha^2 + 4pq_b\Delta U_b}}{C_1} = O\left(x\left(\alpha, p\right)\right)$$

[5] Otherwise the same procedure applies to p_{an}.

for $\alpha \leq \alpha(p)$ and $x \leq x(0,p)$ for $p \to 0$.

Integrating like in the first part of the proof one obtains that

$$|p(logp)| = O(\tilde{x}(p))$$

from which follows the claim.

Proof of proposition 4. Assume 1,2,3 hold then $q_a \Delta U_a = q_b \Delta U_b$ and $p_{an} = p_{bn} = p_n$. Furthermore, $D(\tilde{x}_n) \approx E(p_n)^2 \approx C(\alpha_n)^2$ and for some $C, D, E > 0$ (see the proofs of Lemma 1 in the appendix). Then

$$(2n+1)\int_0^{\alpha_n} C(\alpha, x(\alpha, p_n)) f(\alpha) d\alpha \approx \left(\frac{2n+1}{2}\right) \int_0^{\alpha_n} C_{xx}(0,0)(x(\alpha,p_n))^2 +$$

$$+C_{\alpha\alpha}(0,0)\alpha^2 + 2C_{x\alpha}(0,0)\alpha(x(\alpha,p_n)) f(\alpha) d\alpha$$

which is asymptotically equivalent (see the proof of Lemma 1)

$$\left(\frac{2n+1}{2}\right) \int_0^{\alpha_n} C_{xx}(0,0) \left(\frac{C_{x\alpha}(0,0)}{C_{xx}(0,0)}(\alpha_n - \alpha) + \frac{2q_a \Delta U_a}{C_{x\alpha}(0,0)} p\right)^2 +$$

$$+C_{\alpha\alpha}(0,0)\alpha^2 + 2C_{x\alpha}(0,0)\alpha \left(\frac{C_{x\alpha}(0,0)}{C_{xx}(0,0)}(\alpha_n - \alpha) + \frac{2q_a \Delta U_a}{C_{x\alpha}(0,0)} p_n\right) f(\alpha) d\alpha$$

Then, developing the integral one obtains

$$(2n+1)\int_0^{\alpha_n} C(\alpha, x(\alpha, p_n)) f(\alpha) d\alpha \approx nF\alpha_n^3$$

for some $F > 0$ so that

$$(2n+1)\int_0^{\alpha_n} C(\alpha, x(\alpha, p_n)) f(\alpha) d\alpha \to 0$$

as $n \to \infty$.

The cases where 1, 2,3' or 1, 2', 3 hold are proved proved similarly.

Now assume that $C_{xx}(0,0) = C_{x\alpha}(0,0) = 0$ and that $q_a \Delta U_a = q_b \Delta U_b$. We have $p_{an} = p_{bn} = p_n$. We have:

$\sqrt{n}\tilde{x}_n \to \infty$, $p_n \approx \frac{1}{\sqrt{\pi n}} e^{-4\left(\sqrt{n}\tilde{x}_n\right)^2}$ and , $\alpha_n \approx D\sqrt{p_n}$ for some $D > 0$ (see the proofs of Lemma 3).

$$(2n+1)\int_0^{\alpha_n} C\left(\alpha, x\left(\alpha, p_n\right)\right) f\left(\alpha\right) d\alpha$$

is asymptotically equivalent to

$$\frac{(2n+1)}{6}\int_0^{\alpha_n} C_{\alpha\alpha}\left(0,0\right) 3\alpha^2 + C_{\alpha\alpha\alpha}\left(0,0\right)\alpha^3 +$$

$$+3C_{\alpha\alpha x}\left(0,0\right)\alpha^2 x\left(\alpha,p_n\right) + 3C_{\alpha x x}\left(0,0\right)\alpha\left(x\left(\alpha,p_n\right)\right)^2 + 2C_{xxx}\left(0,0\right)\left(x\left(\alpha,p_n\right)\right)^3 f\left(\alpha\right) d\alpha$$

Let $\varepsilon > 0$. For n large enough the aggregate cost is strictly less than

$$n\int_0^{\alpha_n} \varepsilon\alpha f\left(\alpha\right) d\alpha$$

which is equivalent to

$$\frac{1}{2}\varepsilon n f\left(0\right)\left(\alpha_n\right)^2 \approx \frac{\varepsilon n}{2\pi n} e^{-2\left(\sqrt{n}\tilde{x}_n\right)^2}$$

which converges to zero. So does the aggregate cost. The proof of the case $q_a\Delta U_a > q_b\Delta U_a$ is identical.

References

[1] Althaus, Scott L., Information Effects in Collective Preferences, American Political Science Review 92 (1998), 545-558.

[2] Althaus, Scott. L. Collective Preferences in Democratic Politics: Opinion Surveys and the Will of the People. New York: Cambridge University Press, 2003.

[3] Austen-Smith, David and Banks, Jeffrey S., Information aggregation, rationality, and the Condorcet jury theorem. American Political Science Review, 90 (1996), 34–45.

[4] Berend, Daniel, Paroush Jacob, When is Condorcet's jury theorem valid? Social Choice and Welfare 15 (1998), 481-88.

[5] Berg, Sven, Condorcet's Jury Theorem, Dependency among Jurors. Social Choice and Welfare 10 (1993), 87-95

[6] Chow, Yuan Shih, Teicher Henry, Probability Theory: Independence, Interchangeability, Martingales. Springer-Verlag, 1997

[7] Condorcet, Marquis de, Essai sur l'application de l'analyse à la probabilité des décisions rendues à la probabilité des voix. Paris: De l'imprimerie royale, 1785.

[8] Delli Carpini, Michael X., Keeter, Scott, What Americans Know About Politics and Why It Matters. New Haven: Yale University Press, 1996.

[9] Downs, Anthony, An Economic Theory of Democracy, New York: Harper-Collins Publishers, 1957.

[10] Feddersen, Timothy J., Pesendorfer, Wolfgang, The Swing Voter' s Curse, American Economic Review, 86 (1996), 408-424.

[11] Feddersen, Timothy J., Pesendorfer, Wolfgang , Voting Behavior and Information Aggregation in Elections with Private Information, Econometrica, 65 (1997), 1029-1058.

[12] Feddersen, Timothy J., Pesendorfer, Wolfgang, Abstention in Elections with Asymmetric Information and Diverse Preferences, American Political Science Review, 93 (1999), 381- 398.

[13] Gerardi, Dino, Yariv, Leeat, Deliberative Voting, Journal of Economic Theory 134 (2007), 317-338.

[14] Gilens Martin, Political Ignorance and Collective Policy Preference. American Political Science Review 95 (2001), 379-396.

[15] Hamilton, Alexander, Madison James, Jay John. The Federalist: A Commentary on the Constitution of the United States . Modern Library; New Ed edition (1788), 2000.

[16] Ladha, Krish, The Condorcet Jury Theorem, Free Speech, and Correlated Votes, American Journal of Political Science 36 (1992), 617-34.

[17] Ladha, Krish, Condorcet's Jury Theorem in the Light of de Finetti's Theorem: Majority Rule with Correlated Votes. Social Choice and Welfare 10 (1993), 69-86.

[18] Mandler, Michael, The Fragility of Information Aggregation in Large Elections, Mimeo (2007).

[19] Martinelli, Cesar, Would Rational Voters Acquire Costly Information? Journal of Economic Theory 129 (2006), 225-251.

[20] Martinelli, Cesar, Rational Ignorance and Voting Behavior, International Journal of Game Theory 35 (2007), 315-335.

[21] Mukhopadhaya, Kaushik, Jury Size and the Free Rider Problem, Journal of Law, Economics and Organization 19 (2003), 24-44.

[22] Myerson, Roger B. Extended Poisson Games and the Condorcet Jury Theorem, Games and Economic Behavior 25 (1998), 111-131.

[23] Oliveros, Santiago, Who abstains in equilibrium? Mimeo, (2007).

[24] Oliveros, Santiago, Endogenous information in committees and aggregation of information in large elections, Mimeo (2006).

[25] Persico, Nicola, Committee design with endogenous information, Review of Economic Studies 71 (2004), 165-191.

[26] Polybius, Histories, Loeb Classical Library, 1992.

[27] Schumpeter, Joseph A., Capitalism, Socialism, and Democracy, third edition, New York: HarperCollins Publishers, 1950.

[28] Paroush, Jacob, Stay away from fair coins: A Condorcet jury theorem. Soc Choice Welfare 15 (1998): 15- 20.

[29] Wittman, Donald, Why Democracies Produce Efficient Results, Journal of Political Economy 97 (1989), 1395-1424.

[30] Wittman, Donald, The Myth of Democratic Failure: Why Political Institutions are Efficient, University of Chicago Press, 1995.

[31] Yariv, Leeat. When majority rule yields majority ruin. Mimeo (2004).

[32] Young, H. P. Condorcet's theory of voting. American Political Science Review 82 (1988), 1231—1244.