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Collateral, Financial Arrangements and Pareto Optimality

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Collateral, Financial Arrangements and Pareto Optimality[1]

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[1] I am grateful to Paolo Siconolfi for his suggestions and help and to John Geanakoplos teaching while visiting Columbia for getting me interested in these topics.

Abstract

The existence of collateral requirements to guarantee repayment on issued securities reduces in general the efficiency of competitive equilibria. The general equilibrium analysis is presented in a world where reputation plays no role, and the lender always expects a future payment equal to the future market value of provided collateral. In this context I show that collateral requirements result in two distinct problems for efficiency.

I argue that two financial arrangements, tranching and financial pyramiding, arise in developed capital markets in response to the challenges posed by collateral requirements. If these arrangements are sufficiently developed, then the pareto efficiency of competitive equilibria is restored, even in the presence of collateral requirements.

JEL Classification: D5, E44
Keywords: Collateral, Pareto Optimality, Financial Arrangements, Tranching, Financial Pyramiding

1 Introduction

Do financial arrangements displayed by developed financial systems play an important role in achieving efficient allocations? This paper argues that, whenever credit is channeled through the exchange of securities, tranching and financial pyramiding are essential to move an economy toward Pareto efficiency.

A security is a promise to be paid, at some point in the future, a certain set of goods. The trade of a security entails two sides: a seller that borrows money today in exchange of a (possibly state contingent) payment in the future (the short side) and a buyer that does the opposite (the long side). Trading a security does two things: reshuffle one's endowment across contingencies and between the present and the future. The bundle of these activities is collected under the notion of credit markets.

In reality, most of borrowings - to finance present investment or consumption - is coupled with the provision of collateral. The collateral is nothing else that an easy method that the lender requires to commit the borrower to repay. Colleralized lending is important: central banks typically keep track and report its amount. The collateral may be held by either party involved in the transaction or by a third party entrusted by both sides. In developed financial systems, collateral is usually borrower held.

The introduction of collateral requirements in the credit market is a relatively recent feature of financial contracts. Contrary to what one may think, since collateral is a guarantee, credit in economies more uncertain than ours were not subject to collateral requirements. Cipolla (1976) reports how, at the very outset of financial development - in Italy during the centuries XII through XIV and in Northern Europe afterwards, lending was mostly related to trade and enforcement was reputation based. This situation did not substantially change since then and at least until the XIX century, most lending did not require any collateral. In facts, the first "modern" banks were almost exclusively involved in subscribing government bonds, bonds of quasi-public agencies (e.g. South Sea Company, East India Company) and funding the construction of major public infrastructure (mostly railways) . To summarize we can say that, before the industrial revolution, lending was trade oriented or, at most, directed to consumption smoothing, (production was in fact labor intensive). Reputation played a central role and even after the industrial revolution most lending was directed toward governments and quasi public institutions: their credibility was enough to waive collateral requirements. Collateralized credit arised at a later stage of financial development.

Collateralized credit started when the economic development and technological shifts increased the demand for investment. As more and more capitals needed to be built up, the financial structure began to stretch. Financial innovation became so the necessary condition for economic development. Collateralized credit was one of these innovations. An innovation responding to a new particular characteristic of economic development: the diffusion of private entrepreneurship made investment opportunities widespread. Reputation based lending within small groups was no longer enough to enforce the exchange of funds between parties that had no previous relationship. Collateral arose then as a practical enforcement mechanism able to move capital in largely decentralized market where anonymity is the rule more than the exception.

In order to study the role of collateral, I will work in a world in which reality is exacerbated because reputation plays no role. Naturally this is an unrealistic restriction that I adopt for clarity purpose. If someone gets a loan from someone else, the lender always expects a future payment equal to the future *market value* of the collateral. Therefore the lender always wants to secure his repayment through the provision of collateral by the borrower. The collateral typically (but not necessarily) is a physical durable good, like a house or a car. In more general term, anything entitling the owner to some share of positive wealth in the future is - potentially - collateral.

Firstly, I analyze what problems, in terms of equilibrium efficiency, arise because of collateral requirements on borrowing. Then I will focus on the role of financial arrangements - particularly tranching and pyramiding - have in easing the collateralization constraint and optimizing the use of available collateral. This will help to study the problems arising with collateralized lending, complementing some previous works[1] and highlighting the necessary conditions to restore the pareto optimality of market allocations.

1.1 Endogenous Asset Structure

The introduction of collateral, and the related possibility of default, can result in the creation of many different securities, starting from an elementary asset structure. The first objective in the present work is to analyze how active financial contracts - defined by the couple actual payoffs and collateral level - are endogenously selected in equilibrium. In general - I argue - the specific financial contracts that will be traded depend on the interaction between two factors: the underlying characteristics of the goods/assets that can be used as collateral and the distribution of individual endowments. The properties of the available collateral are crucial to the determination of the particular credit contract that will be exchanged and thus to determine when and why markets may fail to complete.

To focus the matter one may think about a firm borrowing on the market. The firm must then provide some collateral. Since the amount of collateral determines the actual payoffs of this security, the specific value of collateral across contingencies affects the kind of security that can be created. For instance, consider an economy where the underlying formal promise is a bond paying the same in every contingency: if the agent is forced to pool all his collateral together, he may be unable to default and thus offer securities with state contingent payments. This may constitute a problem in many instances where the firm is looking for risk sharing contracts. It is immediate to observe that two factors shape the ability of an economic agent to create a security and sell it: firstly, what part of his future wealth one can use as collateral and, secondly, how linked together his wealth is. Given the outlined factors, I show that tranching and pyramiding address these problems.

1.2 Financial Arrangements

Once the financial contracts arising in equilibrium are identified, it is natural to turn to the question of pareto optimality of the equilibrium in collateralized economies. I show that the

[1] Geanakoplos and Zame (2002), Geanakoplos (2002).

market equilibrium is pareto efficient if two conditions hold: firstly, a sufficiently large share of one's future endowment may be pledged as collateral; secondly, individual endowment can be "offered" in sufficiently small subsets to back one's short positions. In particular - it may be shown -, if aggregate endowment in the economy is derived only through production and is sufficiently "separable"[2], the equilibrium asset structure is always complete and the equilibrium allocation is pareto optimal. I argued that two important financial arrangements - tranching and financial pyramiding - are important to restore the pareto optimality of these equilibria. They arise in the attempt to push the economy toward Pareto optimal allocations.

These financial arrangements are common in financial markets (e.g. credit insurance provided by investment banks, the market for mortgage pools and derivatives) and are widely used by financial intermediaries in their attempt to sell risk insurance to economic agents. I now turn to describe them in greater details and highlight the specific problems they address, before introducing the model of collateralized economies (KE from now on).

1.2.1 Tranching

By tranching I denote the possibility that the same amount of wealth (physical ofr financial) is used as collateral to back more than one short position (promise to pay in the future). In reality this is admitted at a variety of levels that ranges from the derivatives market to the possibility of starting more than one mortgage on the same house. Tranching is an important arrangement. In particular, one may recall from the discussion above that the "separability" of future endowment affects dramatically individual's ability to sell financial contracts and, thus, borrow.

In principle, if the entire future endowment of one individual were linked together, she may be forced to choose between one financial contract and the other but being unable to achieve any combination of them. Recall the example of our firm: it is possible to identify the value of a company but not of one of its asset separatedly. Then one can write a financial contract using the whole company as collateral but not parts of it. This forces the company to choose among financial contracts without being able to combine them. Tranching, allowing different financial contracts to be backed by the same collateral, is the financial arrangement that prevents this problem. Observe that in Arrow - Debreu economies every agent can borrow using any subset of his individual future endowment. Preventing this possibility pushes the economy away from pareto optimality.

1.2.2 Pyramiding

By financial pyramiding, or simply pyramiding, I define the chain of promises that arises when securities/financial contracts can be used as collateral. Think about the following case: all borrowing is collateralized in the economy but agent 1 holds a (physically collateralized) promise to be paid by agent 0. Agent 1 could then issue a promise to pay agent 2, backed by the promise he holds from agent 0. Agent 2 could then do the same: issue a promise to pay agent 3 in the future, backed now by the promise of 1. It is only the collateral on

[2] This will be made precise later on.

the first security - the one issued from 0 and sold to 1 - to be required to be some physical asset, like a house or a factory plant.

One of the major innovations of developed financial systems is to allow the use of securities - i.e. purely financial contract - as collateral. The rationale for it should be, by now, straight forward: this is how financial markets loosen the collateralization constraints, allow agents to use larger shares of their future endowments and maximize potential borrowing. In conclusion: pyramiding is the financial arrangement that allows to borrow against the *financial* part of individual wealth. In this way it economizes on the use of collateral and, avoiding the selling of long positions, may substitute missing financial markets. I turn now to the formal analysis.

2 Collateralized Economy (KE)

2.1 Two Periods Setup[3]

We want to study how the asset structure arises endogenously in KEs and how we evaluate their equilibrium allocations in terms of pareto efficiency. Assume an economy lasting for two periods. In the second period, S contingencies can realize. We will refer to the first period by $s = 0$ and to a contingency in the second period by $s \in \{1, ..., S\}$. The basic features of the economy are described herebelow.

1. Commodities: L consumption goods in each $s \in \{0, 1, ..., S\}$ so that the total consumption bundle of agent h is $c^h \in R^{(S+1)L}$. I define c_{sl} and p_{sl} as, respectively, consumption and spot market price of commodity l in contingency s;

2. Asset Structure: lending is channeled through the exchange of securities which must be guaranteed by the provision of collateral. Security j is defined by a vector of deliveries D^j and a level of collateral j such that:

$$D_j = \begin{bmatrix} D_j^1 \\ ... \\ D_j^S \end{bmatrix}; D_j \in \Re^S; j \in \Re^{SL};$$
$$D_j^s = \left\{p_s^T \cdot j_s\right\}; \forall s$$
$$j \in \left\{k/10^{12} : k \in N^{SL}\right\}$$

   Moreover, for clarity, it is convenient to use different letters to denote short and long positions since short positions must be backed by collateral. I define φ_j^h as the number of units of security j issued by borrower h and θ_j^h as the number of units of security j purchased by lender h.

[3] This setup is related to Geanakoplos [2002], and Genakoplos and Zame [2002].

3. Individuals: there is a continuum of agents $h \in [0, y]$ in th economy, $y > 1$ maximizing:

$$U\left(c^h\right) \text{ s.t.}$$
$$B^h\left(p, q; e^h\right) = \begin{cases} s = 0 : \sum_l p_{0l}\left(c^h_{0l} - e^h_{0l}\right) \leq -\sum_j q_j\left(\theta^h_j - \varphi^h_j\right) \\ s \in \{1, ..., S\} : \sum_l p_{sl}\left(c^h_{sl} - e^h_{sl}\right) \leq \sum_j D_j\left(\theta^h_j - \varphi^h_j\right) \end{cases} \tag{1}$$
$$U(.) \text{ is weakly monotonic, i.e. } x >> y \Rightarrow U(x) > U(y)$$
$$x, y \in \Re^{(S+1)L}$$

where q_j is the price of security j. *Individual* short positions are subject to the two following additional constraints:

- **Collateralization Constraints:**

$$\sum_{\varphi^h_j} D_j \varphi^h_j \leqslant \sum_l p_{sl}\left(i^h_{sl}(j) + d^h_{sl}(j)\right) + \underbrace{\sum_{\theta^h_j \in C} p_{sl} \cdot j}_{\text{pyramiding allowed}} \tag{2}$$

where

$$\sum_{\varphi^h_j \in C} d^h_{sl}(j) = d^h$$
$$\sum_{\varphi^h_j \in C} i^h_{sl}(j) = i^h$$
$$d^h + i^h = S(e^h_1) \leq e^h_1 \tag{3}$$

where d^h represents the share of individual's future endowment that can be freely split to back different security while i^h represents the share of individual's future endowment that can be employed as collateral to back securities but can not be used to back more than one security at the same time. Finally, $S(e^h)$ measures the total share of an individual endowment that can pledged as collateral and so it tracks the tightness of the collateralization constraint. In addition to future endowment, I allow for the possibility that individuals also use the payments of the security they purchased (their long position), $\sum_{\theta^h_j \in C} p_{sl} \cdot j$, as collateral (financial pyramiding). The securities that an individual can employ as collateral are described by the follwoing set:

$$C = \left\{ \theta^h_j \middle| \text{security purchased by } h, \text{ pledgeable as collateral} \right\} \subseteq \left\{ \theta^h_j \middle| \forall j \right\}$$

- **Separability Constraint:** within $S(e^h)$ the share of future endowment that can be used as collateral, there is some part that can be employed to back promised only in "fixed" portions

$$0 < i^h \leq S(e^h) \tag{4}$$

Whenever $0 < i^h$, I will say that collateralizable endowment is not fully separable.

2.2 Competitive Equilibrium in KE:

The equilibrium in a KE is defined as the n-tuple ($c^h \in \Re^{(S+1)L}$, $\left(\theta^h, \varphi^h\right) \in \Re^{10^{12}}$, $\forall h$; $p \in \Re^{(S+1)L}$, $q \in \Re^{10^{12}}$) satisfying:

- Agents Optimize:

$$U\left(c_0^h, c_1^h\right) \geq U\left(c_0, c_1\right), \ \forall \left(c_0, c_1\right) \in B^h\left(p, q; e^h\right) \tag{5}$$

- Commodity Market Clears in $s \in \{0, 1, ..., S\}$:

$$\int_0^y c_{s,l}^h dh = \int_0^y e_{s,l}^h dh, \forall l \tag{6}$$

- Asset Markets Clear:

$$\int_0^y \left(\theta_j^h - \varphi_j^h\right) dh = 0, \ \forall j \tag{7}$$

The setup is quite general and departs from the standard Arrow-Debreu framework in only one feature: collateral must be provided anytime an agent goes short in some security.

Each agent decides which contracts he will trade depending on his endowment and the level of collateral each contract requires. All the contracts will be priced but only some will be actively traded in equilibrium. This is to say that the asset structure will be endogenous. This allows us to draw important conclusions as we will discuss in the following section. Finally, note that each collateral level defines a different financial contract and each financial contract defines a market that must clear in equilibrium.

2.3 Assessing Pareto Efficiency of KE Equilibrium

In setups like this one, as in Geanakoplos (2002) and Geanakoplos and Zame (2002), the market selects which financial contracts will be actively traded in the CE, among all the possible ones. These contracts are the ones that suit best the characteristics of individual endowments. In this section I want to push this observation to its non immediate implications. This will highlight the two factors crucial for the competitive equilibrium in KEs to replicate the Pareto efficient Arrow-Debreu allocation. This may eventually allow us to provide an explanation of why certain financial arrangements (tranching and pyramiding) arise.

To this purpose, I exploit the proposed general setting and try to analyze when and why it may diverge from the Arrow-Debreu allocation implied by complete markets. The first feature one has to look at to address the question of economic efficiency uses the following definition:

Definition 1 *Future endowment is* ***fully collateralizable*** *iff* $S(e_1^h) = e_1^h$, $\forall h$, *and financial pyramiding is admitted, i.e. each individual can put up his entire future physical endowment and all his long positions as collateral in order to back the securities he issues. In this case constraint (2) becomes:*

$$S(e^h) + \sum_{\theta_j^h} (p_{sl} \cdot j) = e_1^h + \underbrace{\sum_{\theta_j^h \in C} (p_{sl} \cdot j)}_{\textit{pyramiding allowed}}; \forall h$$

The fact that future endowment is fully collateralizable is of immediate intuitive importance. If no part of one's future endowment can be used to collateralize his short positions, his ability to borrow from the future and trasfer wealth across contingencies is substantially invalidated. If, on the other hand, agents can employ their entire future wealth, both physical and financial, as collateral they trade the financial contracts of their preference, i.e. the contracts that best fit their needs. Therefore, full collateralization makes the completion of the asset structure more likely. The potential problem of lacking fully collateralizable future endowment is illustrated through the following example, which illustrates a case with less than perfect collateralization of physical endowment:

Example 1 *There is an economy with* $S = 2$, $L = 1$ *and many agents with different endowments. Agent* h *is endowed by* $e^h = (0, 1, 2)$. *One can think about the real life case of an unemployed looking for a job that can be high or low paying. It would be very hard for this individual to borrow against his future income, him being unemployed. At the same time, if he has a strictly concave utility, he would like to smooth his consumption across contingencies and between the present and the future. In order to do so he needs to borrow today using future endowment as collateral. If, for instance, he can not use his endowment at* $s = 2$ *as collateral (h may find difficult to go to a bank and say that he will have an investment banker salary next year!), he can not borrow using the contingency in which he is richer. Therefore, his ability to smooth is substantially reduced.*

The second example that follows illustrates the limitations to risk smoothing that individuals face if they can not use as collateral their long positions, i.e. if financial pyramiding does not take place. This is an important fact since, in both complete and incomplete markets economies, it is always assumed that each individual can use his assets payoffs to balance his budget constraint. Not allowing to do so can result in important departures from the standard analysis as the following example shows:

Example 2 *Consider an economy with* $S = 2$, $L = 1$. *Agent* h *is endowed with* $e^h = (2, 1, 0)$. *Assuming* h *has strictly concave utility, he would like to save for the future and split is consumption equally across contingency at the given market prices. Assume that, given the fundamentals in the economy, only two financial contracts arise in equilibrium and they have the following delivery vectors:*

$$D = \begin{array}{cc} a_1 & a_2 \\ \left[\begin{array}{cc} 1 & 1/2 \\ 1/2 & 1 \end{array}\right] \end{array} \quad \begin{array}{c} s=1 \\ s=2 \end{array}$$

Clearly the span of these assets is R^2 and so markets are complete. In order for h to achieve $c^h = (1,1,1)$, he must be able to purchase the portfolio: $a_1 = \left(-\frac{2}{3}\right)$ and $a_2 = \left(\frac{4}{3}\right)$. This portfolio can not be acquired if h is not allowed to guarantee the payment in $s = 2$ of asset 1 with asset 2. Pyramiding is therefore crucial to fully collateralize individual endowment and so achieve his optimal consumption.

Fully collateralizable wealth does not solve all the problems that collateral requirements impose on the economy. The discussion above highlighted a second factor that must be considered in addressing the issue of pareto optimality in th economy's allocation: the indivisibility of individual future endowment. To this purpose I define:

Definition 2 *Future endowment is* ***fully separable*** *if any element of the power set of individual endowment can be put up as collateral, i.e. if constraint (4) becomes:*

$$0 = i^h < S(e^h)$$

The issue of "indivisibility" in future endowment is quite important in reality. Future endowment can not always be used as collateral in the desired subset. If each individual could use any subset of his future endowment as collateral, it would be very easy for individuals to issue Arrow securities. The lack of separability in future endowments may explain why, in the real world, these securites are rarely traded.

To build an intuition, one may think about the case when individual future endowment is a house. A house is a good taking strictly positive, though different, values in all contingencies. When this is employed as collateral, the agent determines the financial contract he trades by the share of the house that he uses as collateral. In particular, since the actual payment is equal to collateral worth in all contingencies, the individual may be selling positive future payments even when he may not want to. This forced overcollateralization easily results in the trade of (subjectively) suboptimal contracts and "waste" of collateral. Pareto suboptimality of the equilibrium is just consequential. The following example illustrates the argument:

Example 3 *Consider an economy with $S = 2, L = 1$ where each agent can put his entire future endowment as collateral and the exogenously given asset pays off in the numeraire good: $A = [1]$, $\forall s$. There are three individuals g, h and i, with identical probability assessment (i.e. $\Pr(s = 1) = 1/2$), strictly concave utility and endowment, respectively, $e^g = (3,0,3)$, $e^h = (3,3,0)$, $e^i = (0,3,6)$. The endowment of individual i is partially indivisible. It is in fact partitioned into the following two subsets: $\alpha = (0,1,6)$ and $\beta = (0,2,0)$. This resembles the real life situation in which an individual future wealth is twofold (e.g. a house and a capital gain). i can thus trade only two financial contracts, i.e. only two level of collateral j coupled with A, as he wants to smooth his consumption across time and contingencies. To this purpose he tries to reallocate the income available in $s = 1,2$ toward $s = 0$. In equilibrium he wishes to sell only securities delivering $D_{j=\alpha} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ backed by the α subset of his endowment. Since his future endowment can not be separated, he can sell only one unit of the asset $a_{j=\alpha}$. g or h can trade any financial contract satisfying their* *budget constraint. The equilibrium of the economy with the abovementioned non separable endowment is* $x^g = (9/4, 9/4, 39/20)$, $x^h = (123/52, 123/52, 41/20)$, $x^i = (18/13, 18/13, 5)$, $q_{[1,1]} = 14/13$, $q_{[1,0]} = 1/2$, $q_{[0,1]} = 15/26$; $U^g = 1.55$, $U^h = 1.65$, $U^i = 1.30$. *Notice that this is different from the Arrow Debreu equilibrium* $x^g_{AD} = (9/4, 9/4, 27/8)$, $x^h_{AD} = (2, 2, 3)$, $x^i_{AD} = (7/4, 7/4, 21/8)$, $q^{AD}_{[1,1]} = 5/6$, $q^{AD}_{[1,0]} = 1/2$, $q^{AD}_{[0,1]} = 1/3$; $U^g = 1.82$, $U^h = 1.59$, $U^i = 1.32$. *The KE equilibrium is not a pareto optimal allocation. In fact individual* i *would be willing to sell asset* $a_{[0,1]}$ *at a price strictly smaller than* $q_{[0,1]} = 15/26$ *and individuals* g *and* h *would gladly oblige. But this is exactly what they are not allowed to do. Notice finally that the KE equilibrium is not pareto dominated by the allocation in Arrow Debreu.*

If economic agents can put *any* subset of their future endowment as collateral, then, using perfectly anticipated payments, they can trade as many different financial contracts as necessary to complete capital markets.

We can now turn to the question of efficiency in KEs. These economies are such that short sales constraints, i.e. borrowing constraints, exist. Borrowing constraints are enough to invalidate the pareto optimality of the market allocation. In our setup, since the collateral requirement can not be waived, $h's$ ability to borrow is limited by how he can use his future endowment and portfolio as collateral. This makes limited participation to the asset market endogenous in our setup. It is probably interesting to observe that, if everyone's entire future endowment is fully collateralizable, completely separable and pyramiding is allowed, the equilibrium allocation is no different from the one obtainable in the corresponding Arrow Debreu economy. This is summarized in the following theorem:

Theorem 1 *The KE equilibrium allocation is equivalent to the one in the Debreu Economy defined as follows:*

Lemma 1 *1.* $h \in [0, y]$ *maximizes*

$$U\left(c^h\right) \ \ s.t.$$

$$B^h\left(p; e^h\right) = \left\{\sum_l \sum_s p_{sl}\left(c^h_{sl} - e^h_{sl}\right) = 0\right\}$$

$$U(.) \text{ is weakly monotonic, i.e. } x >> y \Rightarrow U(x) > U(y)$$

$$x, y \in R^{(S+1)L}$$

2. Commodity Market Clears:

$$\int_0^y c^h_{s,l} dh = \int_0^y e^h_{s,l} dh, \forall l, s$$

if e^h_s, $h \in [0, y]$ *and* $s \in \{1, ..., S\}$, *is fully collateralizable, fully separable and financial pyramising is admitted..*

Proof. *In order to prove the equivalence between the two equilibria allocations we only need to show that the individual maximization problem is equivalent in the two economies* *and that the asset markets always clear in KE. Consider the budget constraint in the KE equilibrium:*

$$B^h\left(p,q;e^h\right)=\begin{cases} s=0:\sum\limits_l p_{0l}\left(c_{0l}^h-e_{0l}^h\right)\le-\sum\limits_j q_j\left(\theta_j^h-\varphi_j^h\right)\\ s\in\{1,...,S\}:\sum\limits_l p_{sl}\left(c_{sl}^h-e_{sl}^h\right)\le\sum\limits_j D_j\left(\theta_j^h-\varphi_j^h\right)\end{cases}$$

where the only assets traded are $j_s=\begin{bmatrix}0\\...\\1\\...\\0\end{bmatrix};\forall s$, *entitling the buyer to a payment of the numeraire good in contingency* s. *These assets are just Arrow securities and clearly complete the markets. Since* $\left\{e^h\right\}_{h\in[0,y]}$ *is fully collateralizable and separable and financial pyramiding is allowed, everyone can trade any of the Arrrow-like assets compatible with his budget constraint. Finally, since the individual problem in a KE is reduced to the one in an asset economy with Arrow securities, the KE is equivalent to the Arrow economy. By the isomorphism between Arrow and Debreu economies I conclude the proof.* ■

Therefore I have the desired result:

Corollary 1 *If future endowment,* e_1^h, *is separable, fully collateralizable and financial pyramiding is admitted* $\forall h$, *then the KE competitive equilibrium is pareto efficient.*

2.3.1 Financial Arrangements, Pareto Optimality and Lack of Pareto Rankability

It is clear by now why the conditions of full collateralization and separability are necessary to ensure that the pareto optimality of the equilibrium. Why financial markets develop tranching of collateral: it allows to separate individual future endowment. As we have seen, the separability of endowment is one of the two crucial conditions on which pareto efficiency is based.

It is worthy to recall here the role of financial pyramiding to enhance the other condition necessary for pareto efficiency: full collateralization of future endowment. In fact, the future wealth of one individual consists, in general, of physical endowment (e.g. labor and capital income) and financial assets. Pyramiding, allowing individuals to borrow against the future value of their financial assets, helps the individual to mobilize a higher fraction of his wealth. This is indeed essential for the attainment of pareto efficiency. The formal argument illustrating this intuition will require the introduction of a multiperiod framework. This will be the purpose of the following sections and applications.

It is now worthwhile to summarize the two straight forward results of our analysis:

Remark 1 *In any KE where the collateralization constraint is binding for at least one* $h\in[0,y]$ *the equilibrium is pareto suboptimal.*

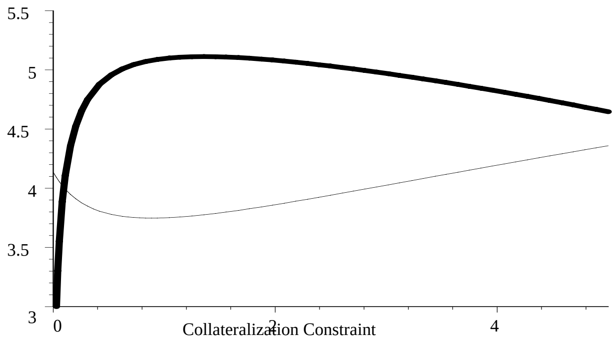
Obviously one could write an economy with endowments and preferences such that the collateralization constraint does not bind. Such economy would already be in a Arrow Debreu equilibrium, before the relaxation of the constraint. Thus the definition of KE (and the discussion above) would be irrelevant. But whenever the constraint binds, its pareto suboptimality would result in equilibrium. The reason is obvious: the binding constraint does not allow at least one borrower to choose the tangency point of his indifference curve and pareto optimality is thus prevented. Example 4 provides an illustration.

The second result is:

Remark 2 *The equilibrium of a KE is, in general, not pareto rankable with respect to the Arrow Debreu allocation.*

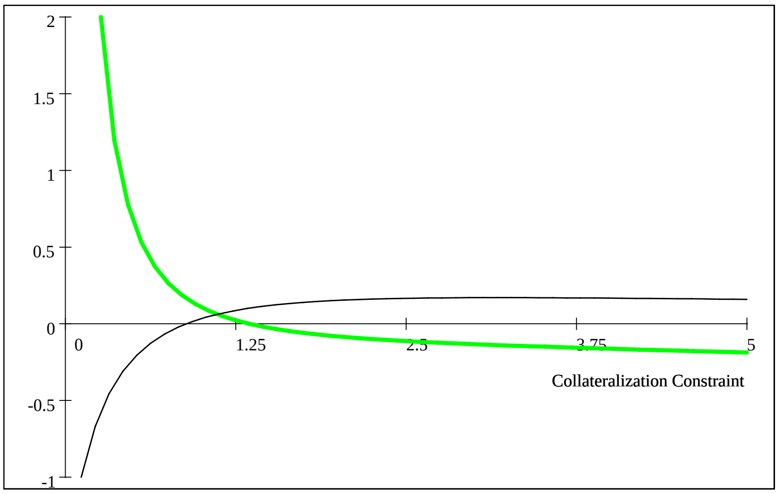
This simple observation is illustrated through the following example:

Example 4 *Assume an Economy with* $S = 1$ *(no uncertainty), one commodity today* (x_0) *and two commodities tomorrow* (x_1, x_2)*.* $U^i = \ln\left[x_0^i x_1^i x_2^i\right], \ i \in \{\alpha, \beta\}, \ e^\alpha = (0, 10, 5),$ $e^\beta = (10, 0, 5)$ *.The collateralization constraint binds such that no agent can put more that* j *units of good* 1 *as collateral. No units of* 2 *are ever available as collateral. There is one asset* $A = [1]$ *whose price is* q*. Moreover we define* $p = p_2/p_1$*. The pareto optimal Arrow Debreu allocation is* $p = q = 1$*;* $x^\alpha = x^\beta = (5, 5, 5)$ *. The KE economy displays a suboptimal equilibrium for* $j < 5$*. In particular individual utilities can be represented as follows when the collateralization constraint is relaxed to* $j = 5$*:*



U^β *in Thick Line*

Finally, the discussion above allows to discuss what happens to the equilibrium allocation when the collateralization constraint is relaxed. Unfortuntely very little can be established: some agents may benefit while others lose as well as pareto improved equilibria may be obtained. Any effect on individual welfare goes through the change in equilibrium prices that the constraint relaxation determines. For illustrative purposes we use the previous example. We graph below the change in indirect utility (including the change in equilibrium prices) when the constraint is relaxed. One may observe that for j *slightly bigger than* 1 *the economy (locally) generates pareto superior equilibria when the constraint is relaxed.*



Equilibrium Utility Gain (Loss) - U^β in Thick Line

3 Concluding Remarks

This paper focuses on the problems arising from the introduction of collateral requirements in security markets. Collateral requirements have been a major step in financial development and provide a rationale for endogenous borrowing constraints. The presented abstract economy helps to conceptualize the problems related to the existence of collateral requirements and how *sufficiently* developed tranching and pyramiding - financial arrangements available in actual financial markets - may restore Pareto optimality.

The general idea is that financial arrangements arise to loosen the constraints on borrowing that collateral imposes. This is in line with the basic intuition of Arrow (1964): as financial markets arise to add or replace missing markets so the specific financial arrangements respond to the constraints imposed on these markets. At an intuitive level, one is tempted to build an analogy between economies with incomplete markets and KEs. In fact borrowing limits, implied by collateral requirements, constraint agents' ability to create securities and reduce the asset span.

The discussion has so far abstracted from the presence of asymmetric information even though collateral requirements are usually justified by some pre-existing informational friction. In fact, if everyone were able to observe what everyone else is doing, collateral would not be necessary. It would suffice to observe whether each borrower respects his budget constraints. The provision of collateral ensures the purchaser of a security (the lender) that payments will be made, reducing the amount of information he needs to collect in order to forecast the actual payments. This is the reason why collateral requirements have provided a major step in financial development. The only factor the lender must check is the value of the collateral at the time when payments are scheduled. He does not need to observe whether the entire portfolio of the borrower is compatible with his budget constraint.

Therefore collateral acts as a device to economize on the information necessary in markets characterized by anonimity. As this study argues, the informational efficiency of collateral is not costless though. Collateral requirements constrain individual choices so that market equilibria may fail to be efficient. The point I have made here is that financial arrangements (tranching and pyramiding) arise to correct these inefficiencies.

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