## Ramón y Cajal: Mediation and <br> Meritocracy

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No.22, September 2006
www.carloalberto.org

# Ramón y Cajal: Mediation and Meritocracy ${ }^{1}$ 

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September $2006^{4}$

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#### Abstract

The Ramón y Cajal Program promotes the hiring of top researchers in Spanish R\&D centers and academic institutions. The centralized mechanism associated to the Program is analyzed. The paper models it as a two-sided matching market and studies if it provides the incentives to increase the quality of the researchers hired. We analyze the mechanism both under complete and incomplete information. The comparison of the theoretical findings with the available data points out that the mechanism provides poor incentives and does not prevent collusion between research departments and candidates in the hiring process. JEL Classification Numbers: C78, D78. Keywords: Matching Markets; Preagreements; Implementation.


## 1 Introduction

Spanish Research Policy has been constantly under scrutiny, fostering an intense debate that has been echoed by international scientific journals. Two issues were especially raised: "the lack of sufficient funding and the existence of social networks that regardless of the candidates' scientific merit, systematically award positions to one of their members." (Navarro and Rivero (2001), see also Camacho (2001)). Many researchers attacked the inbreeding system that a long-standing intellectual tradition with supporting empirical results (see, for instance, Eisenberg and Wells (2000) and Soler (2001)), links with poor scientific productivity. Many advocated for drastic solutions: "... first, every position should be advertised internationally; second, there should be no local members on appointment committees; and third, lecturers or full professors with low scientific productivity should not serve on committees that appoint professorships." (Soler (2001)). In addition, the Spanish academic world has been repeatedly accused of hostility towards researchers who had completed their scientific training abroad (see Ferrer (2000)).

After April 2000 elections, a reshaping of the whole Science and Technology policy domain took place. A R\&D committee was created to evaluate the situation. The findings substantially agreed with the mentioned critiques. The committee pointed out the lack of meritocracy in researchers' selection and the high incidence of endogamic practices as reasons for recruitment distortions. It also remarked that the labor market presented serious problems regarding careers opportunities and long-term perspectives.

An ambitious National R\&D Plan was expected to operate, but only at the end of 2000, because of pressures from Universities representatives, a new instrument was put on the agenda, the Ramón y Cajal Program. The Program was designed to overcome recruitment distortions. It was intended to provide subsidies to public research centers for hiring researchers. The instrument selected was a partially funded 5 years contract ${ }^{1}$. The objective of the Program was to identify the best quality researchers and promote their employment within the Spanish R\&D system. The Ramón y Cajal Program attempted to introduce a competitive and centralized selection of the applicants through an objective evaluation of their merits and potentialities realized by a governmental agency (see also Bosch (2001) and Sanz Menénedez and al. (2003)). The design of the assigning procedure involved a complex bargaining stage between the main actors involved: the Ministry of Technology, the direct responsible of the Program, the Ministry of Education, having regulatory powers on Universities, the Universities, legal and accounting advisers and aspirant researchers. Each one of them tried to put its particular objectives across. The Ministry was concerned about selecting the best researchers, the universities feared an attempt to weakening their independence and the researchers expressed the need of stabilization and of improvements of their working conditions (for a detailed account see Sanz Menéndez (2003)).

[^1]In this paper we analyze the mechanisms used to match researchers and research departments within the Program in its two main versions. The original matching procedure has been used, with no great adjustments for the first three years of the Program. After public expressions of concern about its performance, the procedure was redesigned for the fourth edition. Our aim is to provide insights on the ability of the mechanisms to carry out its objective. We explore if the different interests that influenced the design undermined the goal of incorporating the most promising researchers into the Spanish R\&D system. We compare the mechanism objectives with its performance, through an analysis that is both theoretical and empirical.

We present a bilateral matching model that describes the main features of the assignment mechanism. We derive predictions compatible with the data available. We explain why the first model provided poor incentives and needed a redesign. The new matching mechanism can solve part of the problems found in the previous one.

To provide an intuition for these findings, we need to describe the procedure used with minor variations in the first triennium. Initially, each research department was endowed with a quota of researchers, which is the maximum number of researchers they were allowed to hire. Any researcher who wanted to apply for Ramón y Cajal Contracts had to contact the department or the departments she would like to join and to ask for a preliminary acceptance. To get it, each applicant had to provide the department with a scientific curriculum and a detailed research proposal. The applicants who had been preliminary accepted by at least one department, were evaluated for a Ramón y Cajal contract, by a panel of national and international specialists in each field. They had to rank the applicants in each field up to fill the number of financed contracts. Once the lists of researchers entitled to financed contracts was published, the researchers had to choose which departments to join compatibly with the preacceptances and the ranking. The top ranked applicant in each research area decided which department to join, among the ones that had previously accepted her. The other applicants could choose among the departments that had accepted them and had not filled all positions with better ranked researchers. At this stage, the departments were passive. Each one had to hire all applicants preaccepted in the preliminary stage up to complete the number of financed positions assigned. If at the end of this round all acceptable applicants got a position the procedure ended. Otherwise, in a second round of the matching procedure, the departments left with unfilled positions were asked to consider the researchers unmatched after the first round. If a department and a researcher agreed they were matched together. This second round was informal. In the first edition of the Program, almost all matchings ended with the first assignment. Also, in most of this second round matchings the researcher finally decide to opt out without signing any contract.

The matching procedure resembled the Gale-Shapley algorithm with applicants proposing and departments assuming as preferences the official ranking on the preliminary accepted applicants. We prove that the original procedure was unable to reconcile a competitive and meritocratic selection of the appli-
cants with department's co-responsibility. The mechanism aimed at creating incentives for departments to hire top researchers. The coordination between departments and researchers prevented this objective. The procedure gave departments the possibility of vetoing any applicant through the preliminary acceptance phase. Therefore, departments and applicants had possibilities to collude. For example, any department could have decided to accept only the applicants who had applied only to this department. An applicant who did not get approval by at least one institution did not enter the selection. In conclusion the possibility to create a large set of collusive equilibria existed (Proposition 2). Institutions and applicants were able to jointly manipulate the mechanism and make the centralized selection procedure irrelevant to the outcome (Theorem 1).

In Spain most research departments' priority is either to settle researchers already working on short-term contracts or to hire researchers they already know. Therefore, academic merit was not departments' main concern, at the beginning of the Program. Given this preferences, once the applicants with financed contracts are known, the matching is independent of the ranking of financed researchers. Therefore, the mechanism does not create incentives in hiring top researchers if by "creating incentive" we mean inducing the department to internalize the meritocratic ranking provided by the Committee in each field. It does not produce changes in departments' choices following the ranking. This is because it was indifferent for the departments to hire the best or the second-best applicant. Even worse, there was a problem of constrained eligibility. The procedure only guaranteed that researchers hired were the best among those that applied because only they will be financed but no all applicants can be evaluated. In particular, an applicant alien to the system might have had problems to get the preacceptance she needs. Nevertheless, it was appealing for departments to take part in the Program because it reduced the financial load of new long-term contracts. It also favored researchers' objectives by improving their labor conditions and stabilization perspectives.

Because of such concerns about the performance of the mechanism a new procedure has been in place since the fourth edition (2004). The new procedure is simpler and removes the preliminary acceptance stage. Each researcher interested to enter the selection must send a unique scientific curriculum and research proposal to the Ministry. The applicants are ranked and the best are granted with a financed contract conditioned on final matching with a research department. Then departments and researchers have to settle agreements in a decentralized way. The contracts of the applicants who signed an agreement are confirmed, the rest are withdrawn. The new mechanism does not prevent any applicant from being evaluated by the Commission, and reduces considerably the application costs. We will see along the paper that it does not prevent collusion between centers and applicants, because assignments are decentralized. The main limits seem to be the possible incompatibilities between applicants and research centers (Proposition 4). If the academic world is already collusive (see Navarro and Rivero (2001)), it might deter outsiders' entry. Still, without any doubt it is an improvement with respect to the old matching mechanism,
because it prevents low ranked applicants to be financed through the Program.
The paper proceeds as follows: in Section 2 we present the formal model. In Section 3 we study the sequential game that models the mechanism used in the first edition by the Ramón y Cajal Program. We explore the strategic incentives of the players. We characterize the Subgame Perfect Nash Equilibrium (SPE from now on) outcomes that results to have particular stability properties. The outcome set only depends on agents' preferences and on the number of positions assigned to each department. It is not responsive to the order in which the Committee ranks the applicants (Theorem 1). In Section 3.6 we analyze the mechanism in use since the fourth edition of the Program. We show that, while the outcome is not independent of the ranking, it is still possible that research departments and applicants collude (Proposition 4).

In Section 4 we introduce informational incompleteness in the original model. We would like to know if the lack of information can change departments' attitude towards meritocracy. With this purpose in mind, we simplify the preferences: we assume that each department only distinguishes between acceptable and not acceptable applicants. This is a charitable assumption: it is equivalent to postulate that departments assume the ranking on the set of their acceptable researchers. Further, we assume that each department, given the information each researcher sends to it, is able to discover the relative ranking of its applicants. If uncertainty affects only the ranking then it does not produce any incentive effect (Corollary 3). If the uncertainty is on agents' preferences, two sources of instability emerge. One stems from the cut of poorly ranked researchers, the other one is manipulative. We see that both such instabilities cause inefficient outcomes in which some contracts are not assigned. Our data show that the number of contracts signed between researchers and departments in the second round was small. Therefore, either such uncertainty did not exist or the mechanism induces agents to share the information they need to prevent the inefficiencies.

In Section 5 the data available on the first call of the Program is analyzed. We show how uncertainty affected applicants. A small number of applicants applied to more than one research department, despite the costs of presenting different research proposals. This behavior can be explained in our model as an effect of incomplete information on other agents' preferences. Most applicants were well-informed about the market, but some "outsiders" did not know the possibility they had of entering it but they were confident in their possibilities of being assigned with a contract, once preaccepted by some department.

The study of sequential matching mechanism under perfect and complete information is not new in the literature even with few examples as Alcalde et al. (1998), Alcalde and Romero-Medina (2000) and (2005). Our model apparently shares features of the sequential mechanism described by Alcalde and RomeroMedina (2005). There, each applicant can apply to one department according to some priority order and the SPE outcome is the applicants' optimal stable matching, which is independent of any initial order. First, in our model, the preliminary acceptance phase gives departments larger strategic possibilities (that is, veto power or the possibility to exclude candidates). Then, each researcher
can apply to any number of departments, fact that could produce instabilities, if applications weren't costly (see Example 2). Therefore, the SPE outcomes set is larger. We deal with a more general problem in which the matching procedure is restricted by the scarcity of contracts with respect to departments' wishes. Our mechanism can be seen also as a generalization to the many-to many-case of the model analyzed by Sotomayor (2003). She presents a one-to-one matching model where departments can preliminary accept any number of aspirants. Then, each applicant has to select, in a given order, one department among the ones who have accepted her and are still unmatched. She proves that the stable set is implemented in Nash equilibrium.

Roth and Rothblum (1999) and Ehlers (2004) consider the incentives applicants have under incomplete information in misrepresenting their preferences when the Gale and Shapley mechanism is used and information is symmetric. They show that when information is symmetric they cay can at best have incentives to truncate their preferences list. For a game-theoretic analysis the two main references are Ordoñez de Haro and Romero-Medina (2004), who study a sequential hiring game similar to the one analyzed in Alcalde et al. (1998) and Pais (2005), who studies a particular set of equilibria of the Gale Shapley algorithm under incomplete and symmetric information.

## 2 The model

We consider a bilateral matching market with $k$ research departments (or departments) and $f \geq k$ applicant researchers. Let $D=\left\{d_{1}, \ldots, d_{k}\right\}$ be the set of the research departments and let $R=\left\{r_{1}, \ldots, r_{f}\right\}$ be the set of applicant researchers. Each $d \in D$ is endowed with a complete, strict, transitive preference relation, $P_{d}$ on $2^{R}$, where $2^{R}$ is the set of subsets of $R$, which is the set of all possible research groups. For every $S, S^{\prime} \in 2^{R}, S P_{d} S^{\prime}$ means that $d$ prefers to employ research group $S$ to researchers group $S^{\prime}$. If $S=\emptyset$ we say that $S^{\prime}$ is unacceptable to $d$, meaning that $d$ would prefer not to hire any new researcher rather than hire group $S^{\prime}$. Given $P_{d}$ and $S \in 2^{R}$, the choice set $C h_{d}(S)$ is the research group that $d$ would like to hire most when $S$ is available, formally $C h_{d}(S)=\sup _{P_{d}}\left\{S^{\prime}: S^{\prime} \subset S\right\}$. For all $r, r^{\prime} \in R$, we write $r P_{d} r^{\prime}$ instead of $\{r\} P_{d}\left\{r^{\prime}\right\}$. For each $d \in D$ d's quota, $q_{d}$ is the maximum number of researchers that $d$ is willing to employ, formally $q_{d}=\max \sharp\left\{S: S P_{d} \emptyset\right\}^{2}$. Set $q_{i}=q_{d_{i}}$ and $q=\left(q_{1}, \ldots, q_{k}\right)$. Each researcher $r$ has complete, strict and transitive preferences $P_{r}$ on $D \cup\{r\}$. For every $d, d^{\prime} \in D, d P_{r} d^{\prime}$ means that $r$ would prefer to be employed by $d$ rather than by $d^{\prime}$. Any department $d$ such that $r P_{r} d$ is said to be unacceptable to $r$, which means that $r$ would prefer to stay unemployed rather than join department $d$. If $d P_{r} r, d$ is said to be $a c$ ceptable to $r$. In the paper we make also use of a cardinal representation of $P_{r}, u_{r}: D \cup\{r\} \rightarrow \mathbf{R}$. Let $P=\left(P_{d_{1}}, \ldots, P_{d_{k}}, P_{r_{1}}, \ldots, P_{r_{f}}\right)$ denote a preference profile. The triple $M=(D, R, P)$ is called matching market.

[^2]Definition $1 A$ matching on $(D, R)$ is a mapping $\mu: D \cup R \rightarrow 2^{R} \cup D$, such that, for every $d \in D$ and $r \in R$ :
(1) $\mu(d) \in 2^{R}$ and $\mu(r) \in D \cup\{r\}$.
(2) $\mu(r)=d$ if and only if $r \in \mu(d)$.

Let $\mu$ be a matching on $(D, R)$ and let $M=(D, R, P)$ be a matching market. With $\mathcal{M}(D, R)$ we denote the set of matchings on $(D, R)$. With abuse of notation, for all $S \subset D, \mu(S)$ denotes $\bigcup_{d \in S} \mu(d)$.

Definition 2 The matching $\mu$ is individually rational for $d \in D$ if $r P_{d} \emptyset$ for each $r \in \mu(d)$ or $\mu(d)=\emptyset ; \mu$ is individually rational for $r \in R$ if $\mu(r) P_{r} r$ or $\mu(r)=r$.

Definition 3 The matching $\mu$ is blocked by a pair $(d, r) \in D \times R$ if $d P_{r} \mu(r)$ and $r \in C h_{d}(\mu(d) \cup\{r\}\}$.

Definition $4 \mu$ is stable in market $(D, R, P)$ if (1) $\sharp \mu(d) \leq q_{d}$ for all $d \in D$ (2) $\mu$ is individually rational for all agents and (3) $\mu$ does not have any blocking pair.

In words a matching $\mu$ is stable if there are not a department $d$ and an applicant $r$ such that $r$ does not belong to $\mu(d)$ but $r$ would prefer to join $d$ rather than $\mu(r)$ and $d$ would choose to have $r$ in its research group if it was available together with $\mu(d)$.

Definition 5 Given a bilateral matching market $M=(D, R, P)$ the stable set of $M$, denoted by $\Gamma(M)$ is the set containing the matchings that are stable in market $M$.

In general the stable set may be empty. This is why the literature has focused on the restriction that researchers are seen as substitutes. More precisely, a department's preferences are substitutable if it wants to hire a researcher even when other researchers become unavailable.

Definition 6 Let $d \in D$. The preference relation $P_{d}$ is said to be substitutable if for each $S \in 2^{R}$ and for all $r, r^{\prime} \in S, r \neq r^{\prime}$, whenever $r \in C h_{d}(S)$ then $r \in C h_{d}\left(S-\left\{r^{\prime}\right\}\right)$.

If all departments have substitutable preferences we say that preferences are substitutable. Roth and Sotomayor (1990) have shown that, under this hypothesis, the deferred acceptance algorithm produce either the departmentoptimal or the researcher-optimal stable matching, depending on whether the departments or the students make the offer.

Substitutability is the most general condition, known in the literature, which assures the non emptiness of the stable set. There are other additional properties that hold in the case of one-to-one matchings or under responsiveness (see Roth and Sotomayor (1990)), but not under substitutability. In particular, in the
former cases the set of unmatched agents is the same in all stable matchings, a property that we need to prove Proposition 4. A general condition under which this property holds joins substitutability and separability with respect to departments' quotas (Martinez and al (2000)). Preferences are separable whenever only adding acceptable students makes any given set of worker a better one. Preferences are quota $q$-separable if the property holds only with respect to subsets having less than $q$ elements. Formally:

Definition 7 Let $d \in D$ and let $q>0$ be a natural number. The preference relation $P_{d}$ is said to be $q$-separable if for all $S \in 2^{R}$ :

$$
\begin{gathered}
\sharp S<q, r \notin S: r P_{d} \varnothing \Leftrightarrow S \cup\{r\} P_{d} S . \\
\sharp S>q \Rightarrow \varnothing P_{d} S .
\end{gathered}
$$

All along the paper, we assume that each department $d$, has substitutable and quota $q_{d}$-separable preferences ${ }^{3}$.

In our analysis we compare the preferences of the departments with a ranking $T$ over $R$. $T$, according to the Government's views, evaluates researchers' merits. If a department evaluates' research group according to $T$, we say that its preferences are meritocratic. More precisely a department has meritocratic preferences if she would prefer to hire researcher $r$ rather than $r^{\prime}$, if and only if $r$ is better ranked than $r^{\prime}$ according to $T$. And this, independently of the other components of the research group.

Definition 8 Let $T$ be a strict order on $R$. Let $P$ be a profile of strict preferences on $2^{R} \cup\{d\}$. Department d's preferences are T-meritocratic if, for all $r, r^{\prime} \in R \cup\{\varnothing\}$ and for all $S \in 2^{R}$ such that $\sharp S \leq q_{d}-1$

$$
S \cup\{r\} P_{d} S \cup\left\{r^{\prime}\right\} \Leftrightarrow r T r^{\prime}
$$

In other words, $P_{d}$ is meritocratic if they are responsive to $T$. In particular, $T$-meritocratic preferences are substitutable and quota $q$-separable.

Finally, let us recall the notion of subgame perfect implementation in a matching market framework. Let $D$ and $R$ be two disjoint and non-empty sets of departments and researchers, respectively.

Definition 9 Let $\Phi$ be a class of matching markets and let $F: \Phi \rightarrow \mathcal{M}(D, R)$. An extensive form mechanism $(D, R, \Gamma)$ implements $F$ in Subgame Perfect Equilibrium if (1) for each $(D, R, P) \in \Phi$ and for each $\mu \in F(D, R, P)$ there exists a SPE of the game $(D, R, \Gamma, P)$ yielding $\mu$ as outcome (2) each SPE outcome of $(D, R, \Gamma, P)$ belongs to $F(D, R, P)$.

Throughout the paper, we consider equilibria in pure strategies only.

[^3]
## 3 The mechanisms

In this section we introduce the two matching mechanisms that we analyze in the paper. The first one models the matching procedure which was used in the three first editions of the Program. We call it the "old mechanism". It needs a preliminary acceptance of the applicant from each of the departments she is applying to. The "new mechanism" has been used since the fourth edition. In this new procedure the applicants send their applications to the Ministry without preacceptance.

### 3.1 The old mechanism

In a preliminary stage each research department communicates to the Ministry its scientific projects in some areas and asks for some contracts to be financed. The Ministry assigns to each department a maximum number of contracts. The positions offered are linked to the specific projects presented by the departments. Let $n_{i}$ be the number of financed contracts that are assigned to department $d_{i}$. $N=\sum_{i=1}^{k} n_{i}$ is the maximum number of new contracts to financed. Set $n=\left(n_{1}, \ldots, n_{k}\right)$. We assume that the number of positions and projects to be financed at each department is given and public knowledge. In the same way we assume that the Commission's ranking criteria are known and they can be summarized by a strict order $T$, on $R$. Then the matching takes place as a five stage game.

Stage 1: Researchers' Preacceptance Applications. Each applicant asks to some departments the preacceptances for joining them. Joint with each application, the applicant must send a detailed research project related with one presented to the Ministry by the department itself. The application must specify also applicants' preferences on the participant institutions ${ }^{4}$. Then each application presents a cost for the applicant. We introduce it in her utility function as a linear term. Let $\psi_{d}^{r}$ be the cost that $r$ faces in applying to $d$. We assume $0<\psi_{d}^{r}$ for all $r \in R$, for all $d \in D$ and $\psi_{d}^{r}<u_{r}(d)$ whenever $u_{r}(r)<u_{r}(d)$. For each $r \in R$ let $D_{1}(r)$ be the set of departments $r$ applied to. For each $d \in D$ let $R_{1}(d)=\left\{r: d \in D_{1}(r)\right\}$ be the set of researchers applying to department $d$. Denote $m_{R}^{1}=\left\{D_{1}(r)\right\}_{r \in R}$ the action profile for researchers at stage 1. With abuse of notation $m_{R}^{1}$ denotes the applications received by departments which is $\left\{R_{1}(d)\right\}_{d \in D}$.
Stage 2: Departments Preacceptance Decision. Each department accepts or rejects the demands it receives. For each $d \in D$ let $R_{2}(d) \subset R_{1}(d)$ the set of researchers preaccepted by department $d$. For each $r \in R . \quad D_{2}(r) \subset D_{1}(r)$ be the set of departments that

[^4]has accepted $r$. Set $m_{D}^{2}=\left\{R_{2}(d)\right\}_{d \in D}$. Each department, $d$ which has accepted at least one researcher, communicates $R_{2}(d)$ to the ministry.
Stage MR: Ministry Ranks Preaccepted Researchers. The result of this process is a ranking on $\cup_{i=1}^{k} R_{2}\left(d_{i}\right)$, formally the restriction of $T$ to $\cup_{i=1}^{k} R_{2}\left(d_{i}\right)$. We denote the $i^{t h}$ ranked researcher by $r_{T}^{i}$ or $r^{i}$ when no ambiguity is possible.
Stage 3: First Assignment. The researchers who have been accepted by at least one department are assigned, until $N$ positions are filled. Priority is given to the best ranked applicants. Researcher $r^{1}$ is assigned to the department she chooses among the ones in $D_{2}\left(r^{1}\right)$. For $i \leq N, r^{i}$ is assigned to the department she chooses among the ones that have some free positions in $D_{2}\left(r^{i}\right)$, if any. The rest remain unmatched. Each researcher accepted by some department must choose a department if at least one among the ones that accepted her has some free positions. The result of such a process is a matching $\mu^{1}$. If at least one researcher is unmatched and at least one university has some unfilled positions, the procedure goes to the fourth stage. Otherwise the matching process ends and $\mu=\mu^{1}$ is the resulting matching. For $i=1, \ldots, k$ let $n_{i}^{\prime}=n_{i}-\sharp \mu^{1}\left(d_{i}\right)$, be the number of department $d_{i}$ 's unfilled positions.
Stage 4: Second Preacceptance Decision. For any department $d_{i}$ such that $n_{i}^{\prime}>0$ the ministry asks to it to submit a new set $R_{4}\left(d_{i}\right)$, of acceptable researchers among the $r^{i}(i \leq N)$ not matched under $\mu^{1}$. Let $T^{\prime}$ be the restriction of the ranking $T$ to such applicants. For each such $r^{i}$, let $D_{4}\left(r^{i}\right)$ be the set of departments that accepted $r^{i}$.
Stage 5: Second Assignment. All researchers $r_{i}$ such that $D_{4}\left(r_{i}\right) \neq \varnothing$ are assigned to the departments following the procedure used in Stage 3. Applicant $r_{T^{n}}^{1}$ is assigned to the department she chooses among the ones that have some free positions in $D_{4}\left(r_{T}^{1}\right)$, if any exists. At step $i$ if some empty position is left and if some applicant is not yet assigned to any department applicant $r_{T^{\prime}}^{i}$ is assigned to the department she chooses among the ones that have some free positions in $D_{4}\left(r_{T^{\prime}}\right)$. A second matching $\mu^{2}$ is concluded involving the not yet assigned researchers. The process ends at this point: the researchers matched to some department at the end of Stage 3 are assigned by such an institution, the other ones are assigned by $\mu^{2}$. For $i \leq N$ set $\mu\left(r^{i}\right)=\mu^{1}\left(r^{i}\right)$ if $\mu^{1}\left(r^{i}\right) \in D, \mu\left(r^{i}\right)=\mu^{2}\left(r^{i}\right)$ otherwise. Set $\mu(r)=r$ otherwise. At any point of the process each applicant can leave the game.

### 3.2 The extensive form of the game with complete information

We assume that $n_{i} \leq q_{i}$ for all $i$, which is consistent with the fact than departments cannot be forced to hire any researcher against their will. Let ( $D, R, P$ ) be a matching market and let $n=\left(n_{1}, \ldots, n_{k}\right)$. For each $d \in D$ let $P_{d}^{n}$ be preferences that coincide with $P_{d}$ except for the fact that all sets of cardinality larger than $n_{d}$ are not acceptable. Observe that if $P_{d}$ are substitutable and $q_{d}$-quota separable then $P_{d}^{n}$ are substitutable and $n_{d}$-quota separable. For each $r \in R$ let $P_{r}^{n}=P_{r}$. Finally, set $P^{n}=\left(P_{d_{1}}^{n}, \ldots, P_{d_{k}}^{n}, P_{r_{1}}^{n}, \ldots, P_{r_{f}}^{n}\right) . P^{n}$ is called the effective preference profile.

For $i=1, \ldots, 5, z_{i}$ we denote a node of the game belonging to stage $i$. $Z_{i}$ denotes the set of stage $i$ nodes. We assume that at each node $z$, all agents $z$ belongs to, have complete information of what happened before. Each node of the second stage $z_{2}$, is characterized by the applications strategies leading to it. Let $R\left(z_{2}\right)$ be the set of researchers who sent at least one application and let $D\left(z_{2}\right)$ be the set of departments that received at least one application. $P^{n}\left(z_{2}\right)$ denotes the profile of preferences in which departments' preferences coincide with the ones of $P^{n}$ and researcher $r$ 's preferences $P_{r}^{n}\left(z_{2}\right)$ ranks departments in the same order as $P^{n}$, but each department she did not applied to is not acceptable. The matchings belonging to $\Gamma\left(D\left(z_{2}\right), R\left(z_{2}\right), P^{n}\left(z_{2}\right)\right)$ is called $z_{2}$-stable. Let $\mu_{z_{2}}^{R}$ be the researchers' optimal stable matching for $\left(D\left(z_{2}\right), R\left(z_{2}\right), P^{n}\left(z_{2}\right)\right)$.

Definition 10 The reduced game is the extensive form game that ends with the first assignment.

For each $z_{4}$ let $D\left(z_{4}\right)$ be the set of departments $d$ that filled less than $n_{d}$ positions. Let $R\left(z_{4}\right)$ be the set of idoneous researchers who did not sign for any department. If $D\left(z_{4}\right)=\varnothing$ or if $R\left(z_{4}\right)=\varnothing$ the first matching is the definitive one. Let $\mu_{z_{4}}^{1}$ be the first matching at $z_{4}$. Define the profile of preferences $P^{n}\left(z_{4}\right)$ as follows: for each $d \in \mathrm{~d}\left(z_{4}\right)$ for all $S, S^{\prime} \subset R\left(z_{4}\right), S P_{d}^{n}\left(z_{4}\right) S^{\prime}$ if and only if $\left(\mu_{z_{4}}^{1}(d) \cup S\right) P_{d}^{n}\left(\mu_{z_{4}}^{1}(d) \cup S^{\prime}\right)$, researchers' preferences coincide with the original one. If $P_{d}^{n}\left(z_{2}\right)$ is substitutable and $n_{d}$-quota substitutable and $z_{4}$ follows $z_{2}$ then $P_{d}^{n}\left(z_{4}\right)$ is substitutable and $\left(n_{d}-\sharp \mu_{z_{4}}^{1}(d)\right)$-quota substitutable.

The outcome matching is be the union of a first matching and of a residual matching $\mu=\mu_{z_{4}}^{1} \cup \mu_{z_{5}}^{2}$ where $z_{4}$ is a node of the fourth stage and $z_{5}$ is a terminal node of the fifth stage following $z_{4}$.

Definition 11 The full game is the game that ends with the second assignment.

### 3.3 Analysis

We show that the assignment procedure does not prevent collusion between the two sides of the market. There exists a set of SPE outcomes which depends only on the total number of contracts assigned and on agents' preferences and not on the ministerial ranking. Such an outcome set is "stable" compatibly with the
limit imposed by the number of contracts assigned to each department. In such equilibria the departments do not compete for the best applicants according to the ranking, they compete for their favorite researchers. The research departments can exclude any researcher by not accepting her at the second stage. If the ranking used by the Committee and the preferences of the departments differ substantially any researcher could be excluded in the matching because no department gave her the preacceptance. In this case she would never appear in the ranking compiled after the second stage. We show there are equilibrium outcomes that can result only from contracts signed at the end of the fifth stage and build on "redundant acceptances". We suggest (Example 1) that such equilibria could hurt some departments. It might explain why a low number of fifth stage contracts were signed in the first edition of the Program. It could be the result of a departments' attempt of reducing the hurting matchings. This is the reason why we consider the reduced game that yields sharper predictions. All SPE outcomes are stable, considering the departments' preferences as shaped by quota. Furthermore applicants' equilibrium strategies reduce to a unique application, which resembles the data of the first application.

The result does not longer hold if one assumes zero costs of application. In such a case unstable SPE equilibria could occur because of multiple applications and incoherent behavior of the departments. Unstable equilibria build on departments that in subgames starting at the second stage use a different acceptance policy even when they receive the same set of applications. We first note that any preaccepted applicant has as dominant strategy, at each node she owns at the third and at the fourth stage to accept the best offer it holds. This must be her SPE strategy.

Proposition 1 Let $r \in R$. At the third stage of the reduced and of the full game she has a unique dominant strategy: to join her favorite department among the ones that have preaccepted her and have vacant positions. It holds both for the reduced and for the full game. In the fifth stage of the full game she has a unique dominant strategy: to join her favorite available department.

Proof. The proof is trivial for the reduced game and for the fifth stage of the full game. For the third stage of the full game it suffices to observe that $r$ is called to play at most once between the third and the fifth stage.

### 3.4 The lack of meritocratic incentives

First we show that the procedure is not immune to agents' manipulation. We prove that the set of SPE includes a "stable set", which is independent of the ranking elaborated by the Ministry. It follows that the mechanism does not provide enough incentives for hiring the best applicants. Further it might foster the collusion among departments in order to end the game after the first assignment. The SPE outcome sets of the full and of the reduced game contain the "stable set" $\Gamma\left(D, R, P^{n}\right)$, which is independent of any ranking $T$ of the applicants. Once the number of financed contracts is known, it depends only on agents' preferences. The result does not build on application costs.

Proposition 2 Let $n$ be given, let $P$ be a profile of preferences and let $\mu \in$ $\Gamma\left(D, R, P^{n}\right)$. Then there exists a SPE of full game yielding $\mu$ as outcome, in which all matchings are concluded after the first assignment. Such strategies are SPE of the reduced game that yield $\mu$ as outcome, too.

Proof. Let $\mu \in \Gamma\left(D, R, P^{n}\right)$. Consider the following strategy profile: Stage 1: Each $r$ applies $\mu(r)$. Stage 2: At each $z_{2} \in Z_{2}$, department $d$ accepts the applicants $r \in \mu_{z_{2}}^{R}(d)$. Stage 3: Each researcher plays her dominant strategy. Stage 4: At each $z_{4} \in Z_{4}$, department $d$ accepts the applicants $r \in \mu_{z_{4}}^{R}(d)$. Stage 5: Each researcher plays her dominant strategy. Given such a profile of strategies, at each $z_{2} \in Z_{2}$ no researcher is accepted by more than one department. The stability of $\mu_{z_{2}}^{R}$ in $\left(D\left(z_{2}\right), R\left(z_{2}\right), P^{n}\left(z_{2}\right)\right)$ and of $\mu_{z_{4}}^{R}$ in $\left(D\left(z_{4}\right), R\left(z_{4}\right), P^{n}\left(z_{4}\right)\right)$, for all $z_{2}$ and for all $z_{4}$ imply subgame perfection. The same strategy profile, "cut" after the third stage constitutes a SPE of the reduced game and both yield $\mu$ as outcome.

Remark 1 The strategy profile used in the proof of Proposition 2 is consistent with the empirical evidence found on the first edition of the Program (see Section 5). Most applicants presented applications to few departments. In addition, most institutions preaccepted only the researchers that they would have hired. Furthermore, almost all matchings were settled at the end of the third stage. In the contracts closed at the fifth stage typically the applicant postponed the time of join the department and finally opted out. It seems that agents deliberately chose not to use stage 4 and 5 .

### 3.5 The reduced game

The equilibrium constructed in the proof of Proposition 2 ends with the first matching. Not all equilibria conclude with a first definitive matching. More important, there are equilibrium outcomes that cannot be result of strategies ending with the first matching if costs are strictly positive. They build on "redundant acceptances". In these equilibria some department $d$ gives a redundant preacceptance to an applicant $r$ that it is not going to hire. Researcher $r$ will finally join a department $d^{\prime}$ which accepts her as residual. Any applicant that $d^{\prime}$ prefers to $r$ has been excluded from the ranking due the acceptance policies of the other departments, and they do not enter the process due to application costs. Department $d^{\prime}$ would be strictly better off, if no researcher had been redundantly accepted. Therefore, if a fifth stage matching is not redundant it hurts some departments. The following example formalizes the intuition and shows that not all SPE end with the first assignment.

## Example 1

Let $k=2$ and let $f=3$. Let $n_{1}=n_{2}=1$ and $N=2$. Let $0<\psi_{d}^{r}<u_{r}(d)$ for all $d$ and for all $r$. Let $T: r_{1}, r_{2}, r_{3}, P_{d_{1}}=r_{1}, r_{2} P_{d_{2}}=r_{3}, r_{2}, P_{r_{1}}=d_{1}$, $P_{r_{2}}=d_{2}, d_{1}, P_{r_{3}}=d_{2}$. Consider the following strategy profile: First Stage: $D_{1}\left(r_{1}\right)=\left\{d_{1}\right\}, D_{1}\left(r_{2}\right)=\left\{d_{1}\right\}, D_{1}\left(r_{3}\right)=\varnothing$. Second Stage: let $d_{1}$ accepting $r_{1}$ and/or $r_{2}$ whenever at least one of them applies. Let $d_{2}$ not accepting anyone else. Let $d_{2}$ always accepting $r_{3}$, whenever she applies Let $d_{2}$ accepting $r_{2}$ whenever $r_{3}$ has not applied to any university at the first stage. Let $d_{2}$ rejecting $r_{2}$ when $r_{2}$ and $r_{3}$ apply. Let $d_{3}$ not accepting any applicant different from $r_{3}$ and $r_{4}$. Third Stage: let researchers play their dominant strategies. Fourth Stage: for each $z_{4}$ let department $d$ accepting only the applicants in $\mu_{z_{4}}^{R}(d)$ (see Proposition 2). Fifth Stage: let researchers play their dominant strategies. The described strategies yield the following matching, $\mu$ as outcome.

$$
\begin{array}{lll}
d_{1} & d_{2} & \varnothing \\
r_{1} & r_{2} & r_{3}
\end{array}
$$

They constitute a $S P E$. It is easily checked that $d_{1}, r_{1}, d_{2}$ and $r_{2}$ behave optimally in any subgame. If $r_{3}$ deviates, it is sufficient to consider the case in which she applies to $d_{2}$. By applying to $d_{2}$ the three researchers would be preaccepted, $r_{3}$ would not get any position because there are only two positions available and two better researched have been preaccepted.
Now we prove that there exits no SPE yielding $\mu$, where all the agreements end at the first assignment. Buy contradiction assume that there exists one. Then, it must be the case that $r_{4}$ did not apply to any department, otherwise she would pay positive costs. Further $r_{3}$ applied only to $d_{3}$. Otherwise consider the following deviation for $r_{3}, D_{1}\left(r_{3}\right)=\left\{d_{3}\right\}$. In the subgame induced by such deviation $d_{3}$ accepts $r_{3}$, because it is the unique acceptable application it receives. Then such a deviation would be profitable for $r_{3}$ because she would save the costs of multiple applications.

In Example 1, the redundant acceptance of $r_{3}$ by $d_{2}$ forces $r_{4}$ to be excluded by the process. This hurts $d_{3}$, which would prefer to hire $r_{4}$ rather than $d_{3}$ but it cannot, because $r_{4}$ entry is prevented by $r_{3}$ redundant acceptance.

An assignation can end at the fifth stage only if some applicants have been accepted by a department, that fills all its positions with better ranked researchers at the third stage. The redundant acceptances either hurt some departments or the process could have been concluded at the third stage. Why should a department preaccept an applicant whom it is not going to hire? Departments could deliberately chose to collude and to play the reduced game only. It would prevent unwanted assignations and it would be an equilibrium strategy, according to Proposition 2. This is consistent with the fact that most of contracts signed in the first edition of the Program (16 out of 802) were concluded at the fifth stage. Therefore, we study the reduced game whose equilibrium set is smaller and more appealing, at least for the departments. The reduced game has no equilibrium outcomes outside the stable set. So they do not depend on
the ranking used by the Ministry, but only on the number of financed contracts. The introduction of the ranking alone cannot induce departments to compete for the best researchers.

Theorem 1 For each n, the reduced game implements $\Gamma\left(D, R, P^{n}\right)$ in SPE. In all SPE each applicant applies to at most one department.

The result extends straightforwardly to the equilibria of the full game in which all contacts are signed at the end of the third stage. From Theorem 1 and from Proposition 2 it follows that

Corollary 1 For each $n$ the set of SPE outcomes in which contacts are signed at the end of the third stage is $\Gamma\left(D, R, P^{n}\right)$.

To proof Theorem 1 we need some preliminary Lemmas. The SPE allocations are "ex-post" stable.

Lemma 1 Let $z_{2}$ be second stage node where at most $N$ researchers $\left(D\left(z_{2}\right) \leq\right.$ $N$ ) have applied. The game beginning at $z_{2}$ implements the set of $z_{2}$-stable matchings, $\Gamma\left(D\left(z_{2}\right), R\left(z_{2}\right), P^{n}\left(z_{2}\right)\right)$ in SPE.

Proof. Without loss of generality assume $D\left(z_{2}\right)=D$ and $R\left(z_{2}\right)=R$. We first prove each matching $\mu$, that is stable in $\left(D, R, P^{n}\left(z_{2}\right)\right)$ is an equilibrium outcome. Consider the following strategies. For each $d \in D$ let $R_{2}(d)=\mu(d)$. Let each $r \in R$ playing her dominant strategy at each node of the game. No researcher can profitably deviate. If some $d$ deviates let $\mu^{\prime}(d)$ be the outcome matching, keeping fixed other agents' strategies. Let $\left.r \in \mu^{\prime}(d)\right) \backslash \mu(d)$. Then $d P_{r}\left(z_{2}\right) \mu(r)$ as no applicant has been accepted by more than one department different from $d$. The matching $\mu$ is stable in $\left(D, R, P\left(z_{2}\right)\right)$ so $r \notin C h_{d}(\mu(d) \cup$ $\{r\})$. In particular, $\mu(d) P_{d}\left(z_{2}\right) \mu^{\prime}(d)$. The matching $\mu$ is stable in $\left(D, R, P\left(z_{2}\right)\right)$, so it cannot be the case that $d$ can profitably deviate simply by not accepting some applicants in $\mu(d)$. Then, no deviation is profitable for $d$.
We complete the prove by showing that each SPE outcome matching of $z_{2}$ is stable in $\left(D, R, P^{n}\left(z_{2}\right)\right)$. Let $d \in D$ and assume that $d P_{r}\left(z_{2}\right) \mu(r)$ for some $r \in R$. Consider the following deviation for $d: R_{2}(d)=C h_{d}((\mu(d) \cup\{r\})$, where, as $d$ preferences we consider $P_{d}\left(z_{2}\right)$. It must be the case that such deviation produces a matching $\mu^{\prime}$ such that $\mu^{\prime}(d)=C h_{d}((\mu(d) \cup\{r\})$ because the applicants decide sequentially according to ranking $T$, and at any SPE each one selects her best available offer at her turn. Then if $r \in C h_{d}(\mu(d) \cup\{r\})$ the deviation would be profitable for $d$. Then $\mu$ is stable in $\left(D, R, P\left(z_{2}\right)\right)$.

The next result shows that, in any SPE in which researchers apply to one department or none, the outcome is stable under the effective preferences, $P^{n}$.

Lemma 2 All SPE equilibria outcomes of the reduced game in which each matched researcher applies to exactly one department are stable in $\left(D, R, P^{n}\right)$.

Proof. Let $\mu$ be an SPE outcome in which each researcher applies to exactly one department. By contradiction, assume that $(d, r)$ blocks $\mu$ in $\left(D, R, P^{n}\right)$. Consider the following deviation for $r$ : she applies only to $d$ and she conforms to SPE strategies afterwards. In the subgame induced by such a deviation $d$ receives the applications of the agents in $\mu(d) \cup\{r\}$. Given subgame perfection, it must be the case that the deviation matches $r$ with $d$, yielding a contradiction.

The main result follows.
Proof of Theorem 1. >From Lemma 2, it suffices to show that, at equilibrium, all researchers apply to no more than one department. At equilibrium no unmatched agent researcher to any department because of the costs. So, at equilibrium no more than $N$ researchers apply. To complete the proof, let $m^{*}$ be an SPE yielding a matching $\mu^{*}$ as outcome. Let $\left\{D_{1}^{*}(r)\right\}_{r \in R}$ be the equilibrium application profile. Let $r^{*} \in R$ and let $d^{*} \in D$ such that $\mu^{*}(r)=d^{*}$. Let $z_{2}$ be the node in which each $r \neq r^{*}$ has applied to $D_{1}^{*}(r)$ and $r^{*}$ has applied to only $d^{*}$. The matching $\mu^{*}$ is stable in $\left(D\left(z_{2}\right), R\left(z_{2}\right), P^{n}\left(z_{2}\right)\right)$ and $d^{*}$ is $r$ 's unique stable partner in $\left(D\left(z_{2}\right), R\left(z_{2}\right), P^{n}\left(z_{2}\right)\right)$. By Lemma 1 if $D_{1}^{*}\left(r^{*}\right) \supsetneqq\left\{d^{*}\right\}$ the deviation $D_{1}\left(r^{*}\right)=\left\{d^{*}\right\}$ would be profitable to $r^{*}$, because it would produce the same matching as $m^{*}$ with a lower number of application so with lower costs. Then it must be the case that $D_{1}^{*}(r)=\left\{\mu^{*}(r)\right\}$ for all $r \in R$.

As it was to expect, the system performs correctly if the preferences of the departments coincide with the ranking criteria. When preferences are $T$ meritocratic there exists a unique stable matching in $\left(D, R, P^{n}\right), \mu^{T}$. Matching $\mu^{T}$ matches the best researcher to her favorite department, the second ranked researcher to her favorite department among the ones that have empty positions and so on.

Proposition 3 Let all departments' preferences be $T$-meritocratic then the reduced game implements $\mu^{T}$ in SPE.

Proof. If researchers play their SPE strategy the best each department can do at every $z_{2} \in Z_{2}$ is to accept all applicants regardless of other departments' moves. So, in any SPE of $z_{2}$ it must be as well as playing such a strategy. Then, the SPE outcome of $z_{2}$ must be the unique stable matching of $\left(D\left(z_{2}\right), R\left(z_{2}\right), P\left(z_{2}\right)\right)$. Let then $\mu^{*}$ be an SPE outcome of the game. Without loss of generality let $T=r_{1}, \ldots, r_{N}, \ldots$ By contradiction, assume that there exists $r_{i}$ such that $\mu^{*}\left(r_{i}\right) \neq \mu^{T}\left(r_{i}\right)$. Let $r_{i}$ be the best ranked of such applicants which means $\mu^{*}\left(r_{j}\right) \neq \mu^{T}\left(r_{j}\right)$ for $j<i$ and $\mu^{*}\left(r_{i}\right) \neq \mu^{T}\left(r_{i}\right)$. If $d P_{r} \mu^{T}\left(r_{i}\right)$ then $\mu^{T}(d) \subset\left\{r_{1}, \ldots, r_{i-1}\right\}$ and $\sharp \mu^{*}(d)=n_{d}$. Then $\mu^{T}(d)=\mu^{*}(d)$ by hypothesis. Therefore, $r_{i}$ can join any department $d \in\left\{d: \sharp \mu^{T}(d)<n_{d}\right\}$ by applying to $d$. But $r_{i}$ 's most preferred department in this set is exactly $\mu^{T}\left(r_{i}\right)$.

Remark 2 The result is independent of any assumption on costs.
When there are no application costs and preferences are not $T$ meritocratic the mechanism might produce unstable matchings. An unstable matching may result at equilibrium if some agents apply to more than one department.

## Example 2

Let $k=f=3$ and let $\psi_{d}^{r}=0$ for all $r$ and for all $d$. Let $n_{1}=n_{2}=n_{3}=1$. Let $P_{r_{1}}=d_{1}, d_{2}, d_{3}, P_{r_{2}}=d_{2}, d_{1}, d_{3}, P_{r_{3}}=d_{2}, d_{3}$. Let $P_{d_{1}}=r_{2}, r_{1}, r_{3}, P_{d_{2}}=$ $r_{1}, r_{3}, r_{2}, P_{d_{3}}=r_{3}, r_{1}, r_{2}$. Consider the following matching, $\mu$.

$$
\begin{array}{lll}
d_{1} & d_{2} & d_{3} \\
r_{1} & r_{2} & r_{3}
\end{array}
$$

It is blocked by $\left(d_{2}, r_{3}\right)$ in $\left(D, R, P^{n}\right)$, but it is an SPE outcome of the reduced game. Consider the following strategies. Researcher $r_{1}$ and $r_{2}$ : apply to $\left\{d_{1}, d_{2}\right\}$ at the first stage and use the strict dominant strategies at the third stage. Researcher $r_{3}$ : applies to $d_{3}$ at the first stage and uses the strict dominant strategies at the third stage. At any $z_{2}$ of the second stage $d$ accepts only the applicants in $\mu_{z_{2}}^{R}(d)$. It is easy to verify that such strategies constitute an SPE: $r_{1}$ and $r_{2}$ have no profitable deviation as $\mu$ assign them to their first choice. If $r_{3}$ deviates she is matched to $r_{3}$ whenever she includes it in her application, otherwise she ends unmatched. The stability of $\mu_{z_{2}}^{R}$ assures that no department can profitably deviate from the described strategies at $z_{2}$. So $\mu$ is a SPE outcome of the reduced game. It can be easily seen that it is a SPE outcome of the full game with no costs.

When the costs are positive and the information is complete no researcher applies to more than one department, at equilibrium (Theorem 1). Multiple applications create a coordination problem to departments. Even if a candidate gets more than one preacceptance she can join only one department. On the other side the example relies on an "incoherent" behavior on departments' side. In the subgames in which $r_{3}$ applies to $d_{2}$ while the other researchers behave as in equilibrium $d_{1}$ accepts only $r_{2}$ even if it receives the same applications as on the equilibrium path (by $r_{1}$ and $r_{2}$ ). A convenient behavioral assumption on departments' strategies would extend the stability result to the case of zero costs. Matching $\mu$ can be sustained as an equilibrium of the full game without costs, too.

It is not clear which kind of SPE outcomes can be sustained without costs: it is larger than the set of stable matchings, but it is strictly contained in the set of individually rational matchings ${ }^{5}$. The next example proves that there exists individually rational matchings that are not SPE outcomes.

## Example 3

Let $k=f=2$. Let $n_{1}=n_{2}=1$ and let $N=2$. Let $P_{r_{1}}=d_{1}, d_{2}$, $P_{r_{2}}=d_{2}, d_{1}, P_{d_{1}}=r_{1}, r_{2}$ and $P_{d_{2}}=r_{2}, r_{1}$. The following matching is IR (but not stable)

$$
\begin{array}{ll}
d_{1} & d_{2} \\
r_{2} & r_{1}
\end{array}
$$

[^5]It cannot be sustained by any SPE of the reduced game, whatever is $T$. It cannot be sustained by single application strategies because is not stable (Lemma 2). It cannot be sustained by multiple application strategies: each department who receives more than one application would be better strictly better off by accepting its favorite researcher only.

### 3.6 The New Mechanism

Concerns were publicly expressed on the effectivity of the old mechanism in selecting the best researchers. At the same time, the fears of the universities of losing their autonomy were lessening, so it was possible to reshape the assignment procedure. The main innovation is the elimination of the preacceptances. Under the new mechanism nobody is prevented from being ranked, if she chooses to participate. Only one proposal is necessary to enter the selection. This reduces the application cost to the participants. On the other hand the matching stage is completely decentralized. In this way the universities preserved their independence in hiring new personnel.

Stage 1: Candidates' application. Applicants simultaneously send their scientific CV and a research proposal to the Ministry. Let $R_{1}$ be the set of agents who apply.
Stage MR: The Ministry ranks all applicants The first $N=$ $\sum n_{i}$ ranked researchers have the right to see their contract (eventually) financed through the Program. We call them idoneous. The other students are definitively out of the Program.
Stage 2: Assignment. In a decentralized way the departments and the idoneous applicants sign contracts. Each department $d_{i}$ cannot sign more than $n_{i}$ contracts with idoneous researchers. A matching $\mu$ is agreed.

We assume the decentralized matching takes place as in the following way. Once the set of idoneous researchers is known, each department reveals the applicants it is willing to hire. Then, sequentially each researcher decides which department to join among the ones that have admitted her and are left with vacancies ${ }^{6}$. The mechanism is an extension of Sotomayor (2003) to the many-toone case and can be formally described as follows. Let $N^{\prime} \leq N$ be the number of idoneous researchers. Let $r_{i_{1}}, \ldots, r_{i_{N^{\prime}}}$ the order in which researchers can choose which department to sign for. This order agrees with the restriction of $T$ to the best $N^{\prime}$ applicants.

Stage 2.1 Each department $d$ selects a subset $R(d) \subset\left\{r_{i_{1}}, \ldots, r_{i_{N^{\prime}}}\right\}$.
Let $D(r)$ be the set of departments that have accepted $r$.

[^6]Stage 2.1.1 $r_{i_{1}}$ chooses a mate $x_{i_{1}}$ in $D\left(r_{i_{1}}\right) \cup\left\{r_{i_{1}}\right\}$. Set $\mu^{1}\left(r_{i_{1}}\right)=$ $x_{i_{1}} \mu^{1}\left(x_{i_{1}}\right)=\left\{r_{i_{1}}\right\}$ if $x_{i_{1}} \in D$, set $\mu^{1}(r)=r$ for all $r \neq r_{i_{1}}$ and set $\mu^{1}(d)=\varnothing$ for all $d \neq x_{i_{1}}$.
Stage 2.1.t $\left(2 \leq t \leq N^{\prime}\right) r_{i_{t}}$ chooses a mate $d_{i_{t}}$ in the set $D\left(r_{i_{t}}\right) \backslash\left\{d: \sharp \mu^{1}(d) \geq q_{d}\right\} \cup$ $\left\{r_{i_{t}}\right\}$. Set $\mu_{t}\left(r_{i_{i}}\right)=d_{i_{t}}, \mu_{t}\left(d_{i_{t}}\right)=\mu_{t-1}\left(d_{i_{i}}\right) \cup\left\{r_{i_{t}}\right\}$, and $\mu_{t}(x)=$ $\mu_{t-1}(x)$ for all $x \neq r_{i_{1}}, x_{i_{t}}$.
Set $\mu=\mu_{N^{\prime}}$.
Let $z$ be an initial node of the second stage and let $R(z)$ be the set of idoneous researchers at $z$. Let $\left(D, R(z), P^{n}(z)\right)$ be the matching markets in which researchers' preferences are the original ones and let each department $d$ 's preferences, $P_{d}^{n}(z)$ to coincide with $P^{n}$ on $2^{R} \backslash\{\varnothing\}$ and to rank as not acceptable all subsets containing agents in $R \backslash R(z)$. We call a matching belonging to $\Gamma\left(D, R(z), P^{n}(z)\right), z$-stable. The next result characterizes the SPE outcomes of the subgames beginning at the second stage, extending the result of Sotomayor (2003), to the many-to-one case.

Corollary 2 Let $z_{2}$ be an initial node of the second stage. Then the game starting at $z_{2}$ implements the $z_{2}$-stable set, $\Gamma\left(D, R(z), P^{n}\left(z_{2}\right)\right)$ in SPE stable matching.

Proof. Observe that the subgame beginning at $z_{2}$ is a subgame of the full game, that begins at the fourth stage with the empty set and agents choosing in the order $r_{i_{1}}, \ldots, r_{i_{N^{\prime}}}$. Then the claim follows from Lemma 1.

The main result about the new mechanism reveals the two faces of the coin. The new system is positively responsive to the ranking. A good researcher can enter and get the right to a financed contract. However, assignments are completely decentralized: it is impossible to guarantee her with a position according to her preferences. In the worst of the cases, this possibility could deter her entry. The mechanism does not reward better ranked researchers: there is no difference in being ranked first or $N^{t h 7}$.

Proposition 4 Let $T=r_{1}, \ldots, r_{N}, \ldots r_{f}$. Let $\nu \in \Gamma\left(D, R, P^{n}\right)$.
(i) In any SPE at most $\sharp \nu(D)$ are employed.
(ii) Let $\sharp \nu(D) \geq N$ and let $\mu$ be a matching in which exactly $N$ researchers are employed. Let $r_{j^{*}}$ be the worst ranked employed researcher. Let $j^{*}>N^{8}$. The matching $\mu$ is an SPE outcome of the new mechanism if and only if $\mu$ is stable in the market in which all non employed researchers are excluded $\left(D, \mu(D), P_{D}, P_{\mu(D)}\right)$ and for all $j<j^{*}$ such that $\mu\left(r_{j}\right)=r_{j}, r_{j}$ is single in $\left.\operatorname{market}\left(D, \mu(D) \backslash\left\{r_{j^{*}}\right\}\right) \cup\left\{r_{j}\right\}, P_{D}, P_{\left(\mu(D) \backslash\left\{r_{j^{*}}\right\}\right) \cup\left\{r_{j}\right\}}\right)$.
(iii) Let $\mu$ be a matching such that $\sharp \mu(D)<N$. Then $\mu$ is an SPE outcome of the new mechanism if and only if $\mu$ is stable for $\left(D, \mu(D), P_{D}, P_{\mu(D)}\right)$ and for all $j$ such that $\mu\left(r_{j}\right)=r_{j}, r_{j}$ is single in market $\left(D, \mu(D) \cup\left\{r_{j}\right\}, P_{D}, P_{\left(\mu(D) \cup\left\{r_{j}\right\}\right.}\right)$.

[^7](iv) Let participation costs to be strictly positive for each agent. At any SPE the same set of $N$ agents is matched. The set of SPE outcomes coincides with the set of matchings described in (ii).
(v) Let $\sharp \nu(D) \leq N$ and let participation costs to be strictly positive for each agent. The set of SPE outcomes of the new mechanism is $\Gamma\left(D, R, P^{n}\right)$.

Proof. See Appendix A.
According to part (i), if the stable matchings of $\Gamma\left(D, R, P^{n}\right)$ employ strictly less than $N$ researchers than some financed contracts will be wasted. Consider a full employment equilibrium in which a poorly ranked researcher signs a financed contract and some better researcher did not apply for the grant. According to part (ii), it is because had the better researcher decided to participate in the selection she would not have got any acceptable position (and a contract would have been wasted). Part (iv) and (v) analyze the case of strictly positive application costs: in this case any researcher who will not get any position does not apply. If costs are null then some researcher might apply even if she knows that she is not going to get any position. If she ranked between the best $N$ aspirants then some contracts will be wasted. The new mechanism does not completely prevent collusion, because of the decentralized assignment. Anyway it removes several collusive equilibria of the old mechanism as is proved in the next example.

## Example 4

Assume Let $k=3$ and let $f=4$. Let $n_{1}=n_{2}=n_{3}=1$ and $N=3 . T$ : $r_{1}, r_{2}, r_{3}, r_{4} P_{d_{1}}=r_{2} P_{d_{2}}=r_{1}, r_{3} P_{d_{3}}=r_{4}, r_{3} P_{r_{1}}=d_{2} P_{r_{2}}=d_{1} P_{r_{3}}=d_{2}, d_{3}$ $P_{r_{4}}=d_{3}$ The unique stable matching of $(D, R, P)=\left(D, R, P^{n}\right)$ is $\mu$ :

| $d_{1}$ | $d_{2}$ | $d_{3}$ | $\varnothing$ |
| :--- | :--- | :--- | :--- |
| $r_{2}$ | $r_{1}$ | $r_{4}$ | $r_{3}$ |

By Theorem $1 \mu$ is the unique equilibrium of the reduced game. Only $r_{1}, r_{2}, r_{4}$ are employed. By Proposition $4 \mu$ cannot be sustained as a SPE outcome of the new mechanism. Actually, the unique SPE outcome of the new mechanism is:

$$
\begin{array}{cccc}
d_{1} & d_{2} & d_{3} & \varnothing \\
r_{2} & r_{1} & r_{3} & r_{4}
\end{array}
$$

Through the new mechanism we recover some efficiency, with respect to $T$. This was one of the designers' concerns. The new mechanism may act exactly by preventing the formation of inefficient stable markets, preventing the entry of poor researchers. Anyway, the mechanism cannot carry out the task if preferences are too endogamic: researchers are excluded by the market itself (from (ii) and (iii)), whatever is their position in $T$.

## 4 More general information structure

In this section we relax the assumption of perfect and complete information and we allow for a more general structure. Let $S$ be the (finite) set of state of the world. A state of the world, $s$ is characterized by a ranking $T(s)$ and a profile of preferences, $U(s)=\left(U_{x}(\cdot \mid s)\right)_{x \in R \cup D}$. In addition each agent, $x \in R \cup D$ has a fixed prior distribution $\pi^{x}$ on $S$. An allocation is a function $M$ from $S$ to the set of possible matchings.

The agents play one of the games described in the previous section. A path in the game is a pair $(s, h)$, where $s \in S$ and $h$ is an history of the actions taken by the players. To any terminal path is associated an outcome matching $\mu=\mu(h, s)$.

An information set for a player $x$ is a set, $I^{x}$ of histories identifying those paths player $x$ is not able to distinguish. $\Im^{x}$ denotes $x$ 's collection of information set. Let $I^{x}(s)$ to denote $x$ 's information set at the beginning of the game. Behavioral strategies for $x \in R \cup D, \sigma^{x}$, specify the actions taken by agents at each information set. Let $\sigma=\left(\sigma^{x}\right)_{x \in R \cup D}$ be a profile of strategies.

A profile of (behavioral) strategies, $\sigma^{*}$ is sequentially rational if, for all $x \in R \cup D$, for all $s \in S$ and for every information set $I^{x} \in \Im^{x}$

$$
\left.\mathbf{E}_{\pi^{x}}\left[U_{x}\left(\mu\left(\sigma^{*}, s\right)\right)\right) \mid I^{x}\right] \geq \mathbf{E}_{\pi^{x}}\left[U_{x}\left(\mu\left(\sigma^{-x *}, \sigma^{x}, s\right)\right) \mid I^{x}\right] \text { for all } \sigma^{x} .
$$

where $\left.\mathbf{E}_{\pi^{x}}[\cdot, s)\right]$ denotes the expectation operator with respect to to $\pi^{x}$. Beliefs are determined using Bayes' Rule whenever it is possible. It is however necessary to model beliefs outside the equilibrium path, which is if $I^{x}$ has probability 0 under $\left(\sigma^{*}, \pi\right)$, then an appropriate conditional distribution has to be assigned. We use Perfect Bayesian Equilibrium (PBE) and Sequential Equilibrium (SE) as equilibrium concepts. Observe that in the second case the information at any information set must be consistent with initial information through Bayesian updating.

### 4.1 The reduced old mechanism

Let $\mathcal{T}$ be the set of all possible rankings on $R$. Agent $x$ 's information determines a probability distribution $\rho^{x}$ on $\mathcal{T}$, where $\rho^{x}(T)=\sum_{T=T(s)} \pi^{x}\left(s \mid I^{x}(s)\right)$. Each department $d$, once it receives the applications from the researchers in some subset $R_{1}$ of $R$, is able to determine the relative ranking among the members of $R_{1}$. Its information set at $(s, h)$ can be then identified by $I^{d}=\left(R_{1}, T_{\mid R_{1}}\right)$ where $T_{\mid R_{1}}$ is the restriction of $T$ to $R_{1}$. Department $d$ 's beliefs are a probability distribution $\pi^{d}\left(\cdot \mid I^{d}\right)$. We assume that the induced probability distribution on $\mathcal{T} \rho^{x}(\cdot \mid$ $\left.I^{d}\right)$ is consistent with $\rho^{x}$ for all $I^{d}$. Let $\mathcal{T}^{d}\left(I^{d}\right)=\left\{T: T \mid R_{1}(d)=T\left(I^{d}\right)_{\mid R_{1}(d)}\right\}$, the set of rankings consistent with $d$ 's information at $I^{d}$ 's. This induces a differential information structure. Departments are better informed than applicants on the final ranking. Each department knows only the ranking of its applicants but does not have detailed information about other participants' positions on
ranking. Pooling of all departments' information would reveal the full ranking on the applicants.

Remark 3 At the final stage of the game, all relevant information has already been disclosed. Then, like in Proposition 1, each researcher has a strictly dominant strategy: to accept the best offer she holds whenever she is called to choose. The result holds at any equilibrium consistent with sequential rationality.

If $\mu$ is the outcome function, and $(h, s)$ is a path, let $R_{1}^{d}(h, s)$ be the set of researchers applying to $d$ along that path. For all $(h, s),\left(h^{\prime}, s^{\prime}\right)$ belonging to the same information set $I^{d}$, we have $R_{1}^{d}(h, s)=R_{1}^{d}\left(h^{\prime}, s^{\prime}\right)=R_{1}^{d}\left(I^{d}\right) . \quad R_{2}^{d}(h, s)=$ $R_{2}^{d}\left(I^{d}\right) \subset R_{1}^{d}\left(I^{d}\right)$ denotes the set of researchers accepted by department $d$ at $I^{d}$. Let $T^{d}\left(I^{d}\right)$ be the a profile of preferences for $d$, responsive to the following order on individuals.

$$
\begin{aligned}
& r T^{d}\left(I^{d}\right) d \Leftrightarrow r \in R_{2}^{d}(h, s) \text { for some }(h, s) \in I^{d} . \\
& r T^{d}\left(I^{d}\right) r^{\prime} \Leftrightarrow r T(s) r^{\prime} .
\end{aligned}
$$

Such preferences are meritocratic at $I^{d}$.
Let $\mathcal{M}\left(I^{d}\right)$ be the matching market in which researchers have their original preferences as in state $s$ and each department $d$ has preferences $T^{d}\left(I^{d}\right)$. Let $\mu^{T}\left(I^{d}\right)$ be the unique stable matching in $\mathcal{M}\left(I^{d}\right)$. Then the following result holds.

Lemma 3 Let $I^{d}$ be an information set belonging to the second stage and let researchers playing conforming to Proposition 1. Then for any $(h, s)$ belonging to $I^{d}$, the outcome of $(h, s)$ is $\mu^{T}\left(I^{d}\right)$ at any SE and at any PBE.

Proof. Let $r_{1} T \ldots T r_{N^{\prime}}$, let $N^{\prime} \leq N$ be the applicants who receive a financed contract on path $(h, s)$. Set $d_{1}=\max _{P^{r_{1}}(s)}\left\{d: r_{j} \in R_{2}^{d}(h, s)\right\}$ and, for all $j \leq$ $N^{\prime}-1$ set
$d_{j+1}=\max _{P^{r_{j+1}(s)}}\left\{d: r_{j} \in R_{2}^{d}(h, s),\left|\left\{0 \leq k \leq j-1: d=d_{j-k}\right\}\right|<n^{d}\right\}$, with the convention $\max _{P^{r}(s)} \varnothing=r$. Let $d_{j}$ be $r_{j}$ 's partner at the end of $h$ at state $s$. Executing the deferred acceptance algorithm with departments applying, where each department $d$ is substituted by $n_{d}$ replicas in the order $\left(d_{1}, \ldots, d_{N^{\prime}}\right)$, one obtains the same matching. The first claim follows by taking into account that the outcome of such procedure is the departments' optimal stable matching in $\mathcal{M}(h, s)$ for any order of application. The second claim follows by executing the deferred acceptance algorithm with applicants applying the order defined by the ranking.

Then, departments' strategy is equivalent to "revealing" a profile of preferences from a restricted set. If the application stage does not convey more information than the initial one to the departments and if preferences are public knowledge the conclusions of previous section applies. Analogous result holds in the case of complete information.

Corollary 3 Assume that the information is complete on agents' preferences and the informational structure is the same than in Section 3.1
(i) the full game weakly implements the stable set in PBE and in SE
(ii) the reduced game fully implements the stable set in PBE and in SE.

Information asymmetries may have a different impact on the analysis if and only if it affects agents' preferences and not only the ranking.

In order to make the formal analysis simpler we restrict our attention to preference profiles such that the departments can only differentiate between acceptable and not acceptable applicants. They only care in filling their positions with acceptable researchers. They assume the official ranking on the set of their acceptable researchers, but it does not mean that their preferences are meritocratic.

Let $u_{A}^{d}(s)$ be $d$ 's utility from an acceptable researcher at $s \in S$ and let $u_{N A}^{d}(s)$ be $d$ 's utility from an unacceptable researcher at $s \in S$. Let $u^{d}(\varnothing, s)$ be the utility of not hiring any researcher at state $s \in S$. With no loss of generality we assume $u^{d}(\varnothing, s)=0$. Let $u_{N A}^{d}(s)<0<u_{A}^{d}(s)$. Let $A^{d}=A^{d}(s)$ be the set of acceptable researchers for department $d$ at $s \in S$. Set

$$
\begin{aligned}
u^{d}\left(R^{\prime}, s\right) & =\left|A^{d}(s) \cap R^{\prime}\right| u_{A}^{d}(s)+\left|R^{\prime} \backslash A(d, s)\right| u_{N A}^{d}(s), R^{\prime} \neq \varnothing,\left|R^{\prime}\right| \leq q_{d} . \\
u^{d}\left(R^{\prime}, s\right) & <u^{d}(\varnothing, s) \text { if }\left|R^{\prime}\right|>q_{d} .
\end{aligned}
$$

We will say that a department strategy is essentially dominant if it is dominant in the game where the researchers play according to the strategy in proposition 1. In Appendix B we prove that it is essentially dominant, for all departments, to accept all acceptable researchers.

Proposition 5 Accepting all acceptable applicants is the unique essentially dominant departments' strategy. Furthermore it is the unique departments' strategy which resists to the iterated elimination of weakly dominated strategies.

### 4.2 Core stability and efficiency

Now we consider the issue of core stability.
Let $\mathcal{M}=(D, R, P)=\left(D, R, P^{D}, P^{R}\right)$ be a matching market where $P^{D}$ is of the same form as in 1 . Let $n=\left(n^{d}\right)_{d \in D}$ an assignment of financed contracts.

Definition $12 A$ matching $\mu$ on $\mathcal{M}$ is $n-$ stable in $\mathcal{M}$ if it is individually rational and if there exists no $d \in D$ and no $\widehat{R} \subset A^{d}$ such that $n^{d} \geq|\widehat{R}|>|\mu(d)|$ and $d P^{r} \mu(r)$ for all $r \in \widehat{R} \backslash \mu(d)$.

The definition adapts to our setup the notion of stability. A matching is stable with respect to a given assignation of contracts if it satisfies two conditions. It must be individually rational and no department can increase the number of hired acceptable researchers with applicants who prefer to join it rather than their current employer.

Incomplete information mitigates the clear-cut results obtained under the hypothesis of full information. In contrast with Theorem 1, a researcher could apply to some department even if she is not sure to join it. Consider the case of an applicant who beliefs that, if she got at least one preacceptance, she would be ranked in a good position. Assume that she would like to join some department, but she does not know with certainty if she is acceptable to it. If she cared enough for the profession (i.e. if application costs are low enough) she would apply. It is similar to the case in which an applicant applies to some department despite the risk of ending unemployed. She is not sure of passing the cut but the conditional probability of joining the department, having passed the cut is high enough.

On departments' side, it might happen that, for some realizations, a department has accepted more applicants than the ones it finally hires. Even if the strategy is ex-post redundant it can be strictly optimal at departments' information set. This behavior might cause an idoneous applicant not to be employed. Actually, any unstable equilibrium matching results in not assigned financed positions.

There are two causes of such instabilities. The first one we could call riskinduced. This instability is because of applicants who would like to join a department which otherwise would not fill all its vacancies are too low ranked with respect to the other idoneous applicants. It comes from departments because of lack of information and risk aversion over accepted applicants. The other form of instability is called manipulative. It derives from the rejection of a good ranked applicant by some department, in order not to loose lower ranked researchers. This form of instability needs more precise information: rejecting a well ranked applicant in favor of a lower ranked one reduced the probability of a matching with an acceptable researcher. It can emerge in a structure in which departments' preferences are strict. From Proposition 5 such kinds of strategies are (weakly) dominated.

Proposition 6 Only risk-induced or manipulative instabilities can emerge at equilibrium. In both cases at least one preaccepted and idoneous researcher is unemployed. If departments play their essentially dominant strategy only meritocratic instabilities can emerge at equilibrium otherwise. So an outcome matching is stable iff it employs $N$ researchers.

Looking at the data instabilities seems not likely to occur. All contracts in budget (and even more) have gone assigned. An explanation is that ex-ante information is precise. It is also likely that the mechanism itself induces agents, mainly departments, to share the information they have on applicants and on the ranking. The argument is that the lack of information may induce redundant acceptances and instabilities that harm departments by reducing the number of the researchers assigned to some departments. Voluntary disclosure of the information can prevent such effects. Given the dimension and the structure of the market, communication costs are lower than the ones in which departments would incur in losing financed positions.

## 5 Data and results

In this section we study the empirical evidence that supports the theoretical findings on the mechanism associated to the first call of the Ramón y Cajal Program. The data analyzed have been provided by the Dirección General de Investigación of the Ministerio de Ciencia y Tecnología. We have data on researcher's applications and information provided by the 151 research institutions that participated in the Program. These institutions requested 2064 contracts. Most institutions have more than one research department among the 24 scientific areas in with the applicants where divided. Once the demand of contracts of an institution was decided, research departments played as independent agents. The Program was designed to provide funding for a maximum of 800 contracts. 2807 researchers applied to at least one research center and got the preacceptances that allow then to be evaluated. Each researcher could present at most two independent research projects. 122 researchers decided to present two research projects. Therefore, 2939 different research projects were presented. These projects received 3974 preacceptances.

A total of 24 Committees created by the "Agencia Nacional de Evaluación y Prospectiva (ANEP)" (see Siune (1999)) evaluated the applicants. Overall, 341 experts took part in the evaluation. If a contract was granted to a researcher, she could join the research departments that had preaccepted her. If more than one research institution had preaccepted her, she would have to rank the institutions in her application. Details on the number of preacceptances obtained by the researchers are presented in Table 1.

Table 1: preacceptances

| Number of preacceptances | 1 | 2 | 3 | 4 | 5 | More than 5 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Applicants | 2229 | 486 | 124 | 45 | 24 | 14 |
| (percentage) | $76.3 \%$ | $16.6 \%$ | $4.2 \%$ | $1.5 \%$ | $0.8 \%$ | $0.4 \%$ |
| Granted | 562 | 150 | 31 | 11 | 12 | 8 |
| (percentage) | $72.6 \%$ | $19.4 \%$ | $4.0 \%$ | $1.4 \%$ | $1.6 \%$ | $1.0 \%$ |

Finally, 802 applicants were selected and 782 contracts signed. The first call of the Ramón y Cajal Program served the purpose of stabilizing investigators who had arrived recently to the Spanish system of R\&D. The Program has also incorporated 315 new researchers, of whom 104 are national of countries other than Spain (more details about the first call of the Ramón y Cajal Program can be found at Sanz Menéndez L. et al. (2003)).

We analyze several of the insights from our theoretical findings using the data of the 2939 individual projects that were evaluated by the different Committees. First, we analyze if Spanish researchers tended to (ask and) get preacceptances from the research center where they achieved their Ph.D. This is evidence of endogamic behavior: as general rule universities tend not to hire their own graduate students. 998 researchers were preaccepted by the department where they achieved their Ph.D. Out of the 2229 applicants with only one preacceptance, 616 were preaccepted by the research department where they achieved their

Ph.D. Also, 158 research departments gave preacceptances only to their own graduates. Therefore, we can infer that endogamy was present in the preacceptance phase of the Ramón y Cajal Program. On Table 2 we analyze the consequences of this behavior and we present a model where we control for the effects of endogamy in the different research areas.

Table 2: The Model of endogamy


On Table 2, we present two probit regressions over a variable that has value one if a research project has requested and achieved a preacceptance in a research center were the researcher achieved her Ph.D. Notice that in the data set there is a group of research centers that cannot be endogamic by definition. This is because they cannot provide Ph.D.s and therefore cannot preaccept and, eventually, hire their own Ph.D. graduates. We consider a variable that collects the percentage of public funding of the institutions preferred by the applicant for her project among those that preaccepted her. Public research centers as
public universities can obtain private funds through research contracts with the private sector. We can infer that a researcher increases the chances to ask for and get a preacceptance from the center where she achieved her Ph.D. as the share of public funding of the institution increases. Finally, the chances to incur in endogamy increase with the number of preacceptance. This means that the research centers are viewed as entry points into the research career for their graduates. Therefore, if a graduate considers to apply to more than one center she includes the center where she graduates. It is worth to notice that 261 of the finally selected applicants, have been preaccepted by the research centers where they obtained their Ph.D. and 198 of them have only the preacceptances of these centers. We can conclude that in the first call of the Ramón y Cajal Program the Spanish research institutions have a tendency to preaccept their former Ph.D. Students. In addition, this is more likely as the share of private funding decreases. In our second model we can see how area dummies control, among other things, for the probability to incur in endogamic behavior. We use Economics as reference area because is the one that contributes less to increase the probability of endogamy.

We have considered the problem of endogamy and proved that the mechanism is not able to prevent it. Another variable can give us more insights about our theoretical results. Although we have no direct information from the applicants' curricula we can model the scores given by the different evaluating committees to the projects presented by the applicants. The model is presented in Table 3. The applicants with more preacceptances and US residents get better scores; notice that this also applies to Spaniards resident in the US for doctoral or postdoctoral training. To have achieved her Ph.D. in Spain or to present more than one research project influences negatively the score achieved by the applicant. The difference between the scores induced by these variables goes from 2 to 8 points over a hundred.

Table 3: The Model of scores

|  | Model 1 | Std.Err. | Model 2 | Std.Err. | Endogamy | Std.Err. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| More than one research project | -4.557 | 1.439 | -4.583 | 1.392 | -6.036 | 2.155 |
| US residence | 8.725 | 1.740 | 8.222 | 1.682 | 2.009 | 3.066 |
| Spanish Ph.D. | -2.920 | 1.025 | -3.307 | 1.027 | - | - |
| Constant | 64.38 |  | - | - | - | - |
| Physic and Space Sciences |  |  | 74.584 | 1.452 | 75.577 | 1.791 |
| Earth Sciences |  |  | 63.505 | 1.821 | 60.348 | 2.590 |
| Materials Science and Technology |  |  | 66.826 | 1.800 | 65.378 | 2.724 |
| Chemistry |  |  | 66.654 | 1.481 | 65.271 | 1.464 |
| Chemical technology |  |  | 64.792 | 2.392 | 62.801 | 4.051 |
| Plant and animal biology. Ecology |  |  | 67.158 | 1.661 | 65.294 | 2.125 |
| Agriculture |  |  | 66.424 | 1.786 | 65.660 | 3.314 |
| Livestock and fishing |  |  | 63.157 | 2.231 | 62.012 | 4.120 |
| Food Science and technology |  |  | 61.354 | 2.117 | 61.419 | 3.124 |
| Molecular and cellular Biology and genetics |  |  | 63.992 | 1.262 | 62.091 | 1.302 |
| Physiology and Pharmacology |  |  | 66.596 | 1.834 | 66.072 | 2.368 |
| Medicine |  |  | 61.880 | 1.793 | 57.844 | 2.814 |
| Mechanical, Ship and Aeronautical Engineering |  |  | 60.877 | 4.628 | 38.000 | 10.458 |
| Electrical and Electronic Engineering and Robotics |  |  | 65.899 | 4.300 | 65.006 | 7.403 |
| Civil Engineering and architecture |  |  | 53.952 | 4.229 | 42.888 | 6.037 |
| Mathematics |  |  | 70.912 | 2.515 | 74.051 | 3.958 |
| Computer Sciences |  |  | 51.557 | 3.482 | 34.000 | 12.808 |
| Information \& Communication Technologies |  |  | 62.921 | 3.087 | 64.000 | 7.394 |
| Economics |  |  | 70.781 | 2.711 | 40.000 | 12.808 |
| Law |  |  | 56.206 | 4.934 | 49.200 | 6.846 |
| Social Sciences |  |  | 30.512 | 2.794 | 20.000 | 7.394 |
| Psychology and Education Sciences |  |  | 52.724 | 3.650 | 52.703 | 5.732 |
| Philology and Philosophy |  |  | 61.294 | 2.268 | 66.481 | 3.486 |
| History and Art |  |  | 62.964 | 1.959 | 58.774 | 2.536 |


|  | Model 1 | Prb. $>$ F. | Model2 | Prb. $>$ F. | Endogamy | Prb. $>$ F. |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: |
| Number of observations | 2906 |  | 2906 |  | 986 |  |
| F tests for joint significance of slopes | 14.78 | 0.000 | 15.51 | 0.000 | 4.20 | 0.015 |
| $\chi^{2}$ tests for equality of Area dummies |  |  | 294.63 | 0.000 | 1352.76 | 0.000 |

The most relevant characteristic of the scores is their variation among the different research areas. This variation cannot be attributed to the fact that each area has a different evaluating committee. The evaluation criteria and the coordination standards prevent them to diverge in their valuation. The criteria were based on evaluation of academic contributions (publications, patents, etc. and using standard citations index for the different areas, up to 60 points),
potential of the applicant (up to 20 points) and merits including postdoctoral studies in international research centers (up to 20 points). Another natural source of these results is the different quality of the applicants. The score was based on academic achievements, therefore it is independent of the ratio applicant/position in the different areas and on the total number of applicants. The contracts were granted only to applicants that were excellent according to the evaluation criteria. There were 1006 applicants qualified as excellent and in all areas there were more excellent applicants than contracts available. In fact, the number of contracts had to be increased from 800 to 802 because of unbreakable ties.

Then, if there were excellent applicants in all areas and the average points are significantly different, it can be due only to different degrees of difficulty to get a preacceptance in the different areas. This possibility of the research centers to include or not applicants during the selection process is highlighted by our theoretical results and, from Table 3, there is evidence that it has been used. We compare Model 2 in Table 3 with the Endogamy Model. In the latter, we include only the researchers that applied to the centers where they had obtained their Ph.D. We can see how the scores are generically smaller and very different among some areas. We do not know if the research departments have succeeded in prevent some agents from taking part to the selection but it is clear that many departments where willing to preaccept low qualified local applicants.

Finally, we want to test the results of our theoretical model on how information conditions the behavior of the applicants. We use an ordered probit to analyze the relation between the number of preacceptances and the characteristics of the researchers, that we assume to be related with their information about the Spanish institutions. We claim that the information of a resident in Spain is more precise than the one of an outsider and therefore the former will seek preacceptances in fewer institutions than the latter, given that this is costly and time-consuming. In this sense we claim that those researchers whose residence is in Spain or has been previously in Spain shall have better information than a researcher without this background.

In Table 4 we present the main results of the empirical part. A Spanish residence decreases the number of preacceptances an applicant well seek and get. If the applicant has achieved her Ph.D. in US she will seek more preacceptances. Notice that once an applicant has been preaccepted by a center she likes, her only source of uncertainty arises from the selection process. In fact, of the 782 contracts that were finally signed, 732 were signed with the applicant first choice and the $96.16 \%$ of them in the first matching round. Therefore, the reason to increase the number of preacceptances for the same project, as it happened in $23.7 \%$ of the cases, lies in the uncertainty about the possibility of getting to sign the contract once granted. This uncertainty can only be caused by lack of information about the demand of the centers. This can be seen in Table 4, the positive and negative coefficients of those who obtain their Ph.D. in the US and those who have Spanish residence can be understood as actions of persons with different level of information on the agents involved in the process. The results
confirm the insights provided by the model with incomplete information.
Table 4: The Model of number of total preacceptances

|  | $\text { Model } 1(1)$ | Std.Err. | $\text { Model } 2(1)$ | Std.Err. |
| :---: | :---: | :---: | :---: | :---: |
| Spanish residence | -0.279 | 0.063 | -0.203 | 0.065 |
| US Ph.D. | 0.448 | 0.135 | 0.378 | 0.140 |
| More than one research project | 1.582 | 0.076 | 1.724 | 0.078 |
| Apply to the research institution where she achieve her Ph.D. | 0.510 | 0.050 | 0.504 | 0.051 |
| Physic and Space Sciences |  |  | 1.952 | 0.454 |
| Area Earth Sciences |  |  | 1.000 | 0.463 |
| Materials Science and Technology |  |  | 1.316 | 0.459 |
| Chemistry |  |  | 1.062 | 0.455 |
| Chemical technology |  |  | 0.618 | 0.488 |
| Plant and animal biology. Ecology |  |  | 1.301 | 0.458 |
| Agriculture |  |  | 1.213 | 0.461 |
| Livestock and fishing |  |  | 0.887 | 0.474 |
| Food Science and technology |  |  | 1.279 | 0.465 |
| Molecular and cellular Biology and genetics |  |  | 1.218 | 0.452 |
| Physiology and Pharmacology |  |  | 1.309 | 0.460 |
| Medicine |  |  | 0.790 | 0.463 |
| Mechanical, Ship and Aeronautical Engineering |  |  | 1.165 | 0.551 |
| Electrical and Electronic Engineering and Robotics |  |  | 1.545 | 0.524 |
| Civil Engineering and architecture |  |  | - | - |
| Mathematics |  |  | 2.216 | 0.470 |
| Computer Sciences |  |  | 1.222 | 0.506 |
| Information \& Communication Technologies |  |  | 0.867 | 0.513 |
| Economics |  |  | 0.998 | 0.494 |
| Law |  |  | 0.963 | 0.578 |
| Social Sciences |  |  | 1.422 | 0.483 |
| Psychology and Education Sciences |  |  | 1.241 | 0.504 |
| Philology and Philosophy |  |  | 1.638 | 0.468 |
| History and Art |  |  | 0.947 | 0.468 |


|  | Model 1 | Model 2 |  |
| :--- | :--- | :---: | :---: |
| Log Likelihood | -2388.59 | -2283.01 |  |
| Number of observations | 2914 | 2914 |  |
| LR tests for joint significance of slopes | 544.33 | 0.000 | 555.10 |
| LR tests for joint significance of Area Dummy |  | 206.54 | 0.000 |

(1) Maximum Likelihood estimates for ordered probit estimates.

If we consider the results on Table 2 and the data on Table 1 we can infer that this can be because most applicants present only one preacceptance. This preacceptance suits to the institution they are working in and it is given by the
institution regardless of the quality of the applicant. Nevertheless, on Table 1 we have shown how $76.3 \%$ of researchers present only one research project and receive only one preacceptance. Therefore, we can show that the uncertainty was higher for those out of the system but there where few of then in the allocation process. Most of the applicants who achieved a research contract on the first call of the Program were Spaniards. In fact, $75 \%$ of the 698 Spaniards awarded with contracts were living in Spain at the time of the application. From the 104 foreigners 36 of then ( $5 \%$ of the total contracts) were living in Spain.

## 6 Conclusions

Successful mechanism design should take into account agent's motives in order to provide them with the right incentives to perform its goals. The Ramón y Cajal Program was created to improve Spanish scientific researchers' base by promoting the recruiting of top-researchers while preserving the autonomy of research institutions. The model we have presented provides insights on the way these two objectives have interacted in the design of the assigning mechanisms.

As a result we got a design failure. The limits to recruit Spanish scientists came from the lack of a merit criterion caused mainly by endogamous behavior. The objective of the mechanism was to induce departments to accept high quality researchers.

So the meritocratic concerns resulted in the selection of applicants, which had been taken far from concerned departments, and in giving better applicants priority. On the other hand, to preserve researcher centers' independence a preliminary acceptance phase was introduced. The interacting of these two features produced large possibility of collusion between applicants and research centers. Large information asymmetries seem able to prevent such collusion.

The analysis of the data confirms the theoretical findings. Most of applicants have fewer preacceptances and were preaccepted by the research centers where they obtained their Ph.D. On the informational side, insiders seem likely to present fewer applications than non-Spanish resident, which confirms the results in Section 4. Further, the quality of the applicant has little correlation with the number of preacceptances achieved.

We can attribute the redesign of the mechanism to the failures that it presented. In the redesign, the preacceptance stage has been removed. Therefore, all applicants are considered by the evaluation committees. Also, the matching has been decentralized and any priority has been removed.

The new design of the mechanism is an improvement. However, it does not guarantee that departments are competing for the best researchers. It only can assure the over all quality of the applicants selected.

An interesting point for future research, both theoretical and empirical, is the evolution of the agents' behavior through the different calls of the Program. The data on the other calls could also help in clarifying the role of the applicants that were granted a contract but decided to abandon the Program. Of the selected applicants 732 signed a contracts with the institution they preferred
and only 16 used the second round of the mechanism and signed contract with a research center that initially did not preaccept then. This supports the idea of a reasonably complete information environment, but it precludes testing this hypothesis.

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## 8 Appendix A

We prove Proposition 4 under a more general assumption on the decentralized mechanism beginning in the second stage.

Condition 2 Let $z$ be an initial node of the second stage. Let $R^{\prime}=R\left(z_{2}\right)$ be the set of idoneous researchers. Any SPE outcome of the decentralized matching starting at $z$ is stable in $\left(D, R(z), P^{n}(z), P_{R^{\prime}}\right)$ students.

We now prove Proposition 4 in a more general form, changing the claim to take into account Condition 2

Proposition 7 Assume Condition 2 holds. Let $T=r_{1}, \ldots, r_{N}, \ldots, r_{f}$ and let $\nu$ be stable in $\left(D, R, P^{n}\right)$.
(i) At any SPE of the new mechanism no more than $\sharp \nu(D)$ researchers are employed.
(ii) $\sharp \nu(D)>N$ and let $\mu$ be a matching in which exactly $N$ researchers are employed. Let $r_{j^{*}}$ be the worst ranked employed researcher. Let $j^{*}>N$. Then $\mu$ is an SPE outcome of the new mechanism if and only if $\mu$ is a SPE outcome at $z_{\mu}$ where $z_{\mu}$ is the subgame induced by each department $d$ accepting only agents in $\mu(d)$ and if for all $j<j^{*}$ such that $\mu\left(r_{j}\right)=r_{j}, r_{j}$ is single in the stable set of $\left(D, R\left(z_{\mu}\right) \backslash\left\{r_{j^{*}}\right\}\right) \cup\left\{r_{j}\right\}, P_{D}^{n}\left(z_{\mu}, P_{\left(\mu(D) \backslash\left\{r_{j^{*}}\right\}\right) \cup\left\{r_{j}\right\}}^{n}\right)$.
(iii) Let $\mu$ be a matching in which $N^{\prime}<N$ researchers are employed. Then $\mu$ is a SPE outcome at $z_{\mu}$ where $z_{\mu}$ is the subgame induced by each department $d$ accepting only agents in $\mu(d)$ for $\left(D, R\left(z_{\mu}\right) \cup\left\{r_{j}\right\}, P_{D}^{n}\left(z_{\mu}, P_{\left(\mu(D) \cup\left\{r_{j}\right\}\right.}^{n}\right)\right.$.
(iv)If $\sharp \nu(D)>N$ and each researcher has positive application costs, exactly $N$ researchers are employed at equilibrium. The set of SPE coincides with the set of matchings described in (ii)
(iv)Let $\sharp \nu(D) \leq N$ and assume that each researcher has positive application costs. Then the set of SPE at $(D, R, P)$ is the set of SPE outcomes of the decentralized matching process characterized starting at $R_{1}(z)=\nu(D)$.

Proof. (i) The proof is trivial.
(ii) Let $\mu$ and $j^{*}$ as in the claim and set $R^{\prime}=\mu(D)$. Let $j<j^{*}$ such that $\mu$ is an SPE outcome of the new mechanism if and only by participating to the selection, no such $r_{j}$ can get a position. By deciding to participate $j<j^{*}$
would result idoneous. Let $z(j)$ be the subgame induced by such a deviation. $R^{\prime}(j)=\left(R^{\prime} \backslash\left\{r_{j^{*}}\right\}\right) \cup\left\{r_{j}\right\}$. The matching $\mu$ is a SPE if and only if $r_{j}$ is single as result of such a deviation. From Condition 2 and Martinez and al (2000) follows (ii) part of the claim.
(iii) is proven as (ii).
(v) follows from (ii) once observed that with positive application costs, at equilibrium, no unmatched agent applies.
(iv) follows from (ii) once observed that with positive application costs, at equilibrium, no unmatched agent applies.

## 9 Appendix B

Let $(h, s)$ be a path in the game and let $\sigma^{-d}$ be a profile of strategies for players other than $d$.

Let $R^{d}\left(h, s, \sigma^{-d}\right)=A(d, s) \cap R_{1}^{d}\left(h, s, \sigma^{-d}\right)$ be the set of acceptable application received by $d$.

Let $R^{-d}\left(s, \sigma^{-d}\right)=\widehat{R^{d}}\left(h, s, \sigma^{-d}\right) \cup\left[\bigcup_{d^{\prime} \neq d} R_{2}^{d^{\prime}}\left(h, s, \sigma^{-d}\right)\right]=\left\{r_{1}, \ldots, r_{l}\right\}$, where $r_{1} T \ldots . T r_{l} . R^{-d}\left(s, \sigma^{-d}\right)$ is the set of researchers that would be preaccepted at state $s$ if $d$ accepted only its acceptable applicants.

Consider the following algorithm.
Let $d_{1}=\max _{P^{r_{1}}}\left\{d: r_{1} \in R^{-d}\left(h, s, \sigma^{-d}\right)\right\}$ and let
$d_{j+1}=\max _{P^{r_{j+1}(s)}}\left\{d: r_{j} \in R_{2}(d, h, s),\left|\left\{0 \leq k \leq j-1: d=d_{j-k}\right\}\right|<n^{d}\right\}$.
Let $\mu\left(h, s, \sigma^{-d}\right)\left(r_{j}\right)=d_{j}$ and let $\mu\left(h, s, \sigma^{-d}\right)(r)=r$ for all other $r$.
Let $\sigma\left(h, s, \sigma^{-d}\right)(d)=\mu\left(h, s, \sigma^{-d}\right)(d)$ for all $(h, s)$ and set $R_{2}^{d}\left(I^{d}, \sigma^{-d}\right)(d)=$ $\bigcup_{\pi^{d}\left(h, s \mid I^{d}, \sigma^{-d}\right)(d)>0} \sigma\left(h, s, \sigma^{-d}\right)(d)$.

We call $\sigma\left(I^{d}, \sigma^{-d}\right)(d)$ the secure strategy against $\sigma^{-d}$ at $I^{d}$. The secure strategy against $\sigma^{-d}$ amounts in accepting acceptable researchers that with positive probability will join $d$ at the following stage. Playing secure strategy assure the best ranked ones.

Then it is the best each department can do if it only cares to fill all its position, whatever information it owns. Formally:

Lemma 4 Let $d \in D$. Let $\sigma^{-d}$ be a profile of strategies for players other than d, in which researchers play according to the strategies described in Proposition 1. Using the secure strategy profile at $I^{d}$ against is $\sigma^{-d}$ sequentially rational for $d$ at $I^{d}$. And the secure strategy maximizes the number of researchers can get at each state.

Proof. Let $\pi^{d}\left(h, s \mid I^{d}, \sigma^{-d}\right)(d)>0$. Accepting any $R_{2} \supset \sigma_{2}^{d}\left(h, s, \sigma^{-d}\right)(d)$ matches $d$ with all the agents in $\mu\left(h, s, \sigma^{-d}\right)(d)=\sigma\left(h, s, \sigma^{-d}\right)(d)$. So we can consider subsets of $R_{1}\left(d, I^{d}\right)$ such that $\sigma\left(I^{d}, \sigma^{-d}\right) \backslash R_{2} \neq \varnothing$. In particular, for
some $(h, s)$ such that $\pi^{d}\left(h, s \mid I^{d}, \sigma^{-d}\right)(d)>0 \sigma\left(h, s, \sigma^{-d}\right)(d) \backslash R_{2} \neq \varnothing$. It suffices to show that for any such $(h, s)$ accepting such $R_{2}$ does not increase the cardinality of $\mu(h, s)(d)$ where $\mu(h, s)$ denotes the outcome function at $(h, s)$, fixed $\sigma^{-d}$. We can assume also that $R_{2} \backslash \sigma\left(I^{d}, \sigma^{-d}\right) \neq \varnothing$ and that $R_{2}$ contains only acceptable researchers. Let $\mu_{0}$ be the outcome matching after $(h, s)$ when $d$ plays her secure strategy and let $\mu$ be the outcome matching. after $(h, s)$ when $d$ accepts the researchers in $R_{2} . \mu_{0}(d)=\sigma\left(h, s, \sigma^{-d}\right)(d)$. As the outcome for $d$ after $(h, s)$ is the same if d accepts $R_{2}$ and if it accepts $\mu(d)$ we can assume that $\mu(d)=R_{2}(d)$. Let $\mu_{0}(d) \backslash R_{2}=\left\{\widehat{r_{1}}, \ldots, \widehat{r_{s}}\right\}, \widehat{r_{1}} T \ldots T \widehat{r_{s}}$ and let $\mu^{1}(d) \backslash \mu_{0}(d)=\left\{\overline{r_{1}}, \ldots, \overline{r_{p}}\right\} \overline{r_{1}} T \ldots T \overline{r_{p}}$. Observe that it must be the case that $\widehat{r_{1}} T \overline{r_{1}}$, by construction of $\sigma\left(I^{d}, \sigma^{-d}\right)$. Consider the following algorithm. Let $\mu^{0}$ be the outcome matching when $d$ accepts the researchers in $\mu_{0}(d) \cup R_{2}$. For $1 \leq j \leq s: \operatorname{Step} \mathbf{j}: R^{j}=\mu^{j-1}(d) \cup R_{2} \backslash\left\{\widehat{r_{s-j+1}}\right\}$. Let $\mu^{j}$ be the outcome matching when $d$ accepts the researchers in $R^{j}$. We have $\mu^{s}=\mu$. Accepting $\mu_{0}(d) \cup R_{2}$ matches $d$ with $\mu_{0}(d)$. We now prove that $\left|\mu^{j+1}(d)\right| \leq\left|\mu^{j}(d)\right|$ for all $0 \leq j \leq s$. First of all observe that $\mu^{j-1}\left(\widehat{r_{s-j+1}}\right)=d$. Rejecting $\widehat{r_{s-j+1}}$ at step $j$ does not affect the choices of applicants ranked above $\widehat{r_{s-j+1}}$. This rejection can create at most one cycle of rejections interesting only lower ranked applicants. Then $d$ is matched at most with one applicant different than $\widehat{r_{s-j+1}}$ at each step. So $\left|\mu^{j+1}(d)\right| \leq\left|\mu^{j}(d)\right|$
and $|\mu(d)|=\left|\mu^{s}(d)\right| \leq\left|\mu^{0}(d)\right|$.
In general not only supersets of the secure strategies are best responses with respect to a given strategy. From the proof of Lemma 4 follows that are someway robust to changes in the information setup. Playing different strategies means to believe in the possibility of a matching with low ranked applicants. But lower ranked applicants have in general lower probability of passing the "cut".

Proof of Proposition 5. It follows from Lemma 4 and Remark 3.
Proof of Proposition 6. Let $\sigma^{*}$ be an equilibrium and let $\mu^{*}=\mu^{*}(s, T)$ be the equilibrium outcome matching at state $(s, T)$. Let $d \in D$ and let $\widehat{R} \subset A(d, s)$ such that $n^{d} \geq|\widehat{R}|>\left|\mu^{*}(d)\right|$ and $d P^{r}(s) \mu^{*}(r)$ for all $r \in \widehat{R} \backslash \mu^{*}(d)$.. At equilibrium $\left|\mu^{*}(d)\right|$ is the maximum number of researchers $d$ can get at $(s, T)$ against $\sigma^{*-d}$ (from Lemma 4). Let $r \in \widehat{R} \backslash \mu^{*}(d)$ such that $\left|\left\{r^{\prime} \in \bigcup_{d^{\prime}} A_{2}\left(d^{\prime}, s, I^{* d^{\prime}}\right): r^{\prime} T r\right\}\right|<$
$N$. Consider first the case in which $r$ applied to $d$. By contradiction, suppose $r^{\prime} T r$ for all $r^{\prime} \in \mu^{*}(d)$. Consider the following deviation for $d$ : accepts $\mu^{*}(d) \cup\{r\}$. It would match $d$ with $\mu^{*}(d) \cup\{r\}$ at $(T, s)$. But it contradicts Lemma 4:it would follows that $d$ could obtain at least $\left|\mu^{*}(d)\right|+1$ researchers by playing its secure strategy against $\sigma^{*-d}$. Now consider the case in which $r$ did not applied to $d$. Consider the following deviation for $r$ : applies to the same departments as in $\sigma^{* d}$ and to $d$. It changes the information set of no department but $d$. Starting from such path $d$ can get at least $\left|\mu^{*}(d)\right|$ researchers at $(T, s)$. As costs are small it must be the case that $d$ is not matched with $r$ as result of such deviation. If. it was not the case applying also to $d$ would increase $r$ 's probability of better matchings. Then $d$ can get at most $\left|\mu^{*}(d)\right|$. By contra-
diction, assume that $r^{\prime} T r$ for all $r^{\prime} \in \mu^{*}(d)$. Accepting only $\mu^{*}(d) \cup\{r\}$ would procure $\left|\mu^{*}(d)\right|+1$ researchers which, via Lemma 4 , constitutes a contradiction. The last part of the claim follows by observing that $d$ has a vacancy at $(s, T)$.


[^0]:    ${ }^{1}$ We thank Jordi Massó, César Alonso, Elena Iñarra, the partecipants of the II Workshop on Social Choice and Welfare Economics, Malaga, June $2^{n d}-4^{t h} 2005$ and of the Conference LGS4, June $22^{\text {nd }}-24^{\text {th }}$, of seminars at Universidad Carlos III and University of Basque Country for very helpful comments and suggestions. Romero-Medina acknowledges financial support from DGESIC,
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    ${ }^{4}$ © 2006 by Matteo Triossi and Antonio Romero-Medina Corchon. Any opinions expressed here are those of the authors and not those of the Collegio Carlo Alberto.

[^1]:    ${ }^{1}$ Each Ramón y Cajal contract is financed in decreasing terms during the five years. The first year the entire burden is on the Ministry, the last year it is on the department.

[^2]:    ${ }^{2}$ For any set $S, \sharp S$ denotes the cardinality of $S$.

[^3]:    ${ }^{3}$ Although, all results until Proposition 4 hold under substitutability only, unless stated otherwise.

[^4]:    ${ }^{4}$ The order provided has only informative purpose. It does not compel agents to conform to their revealed preferences. It seems that the organization used it to arrange the residual matchings.

[^5]:    ${ }^{5}$ The point has been made by Jordi Massó.

[^6]:    ${ }^{6}$ The main result of this section holds under a more general assumption on the decentralized assignment process (see the Appendix).

[^7]:    ${ }^{7}$ Part (i), (ii) and (iii) of the claim hold also if costs are zero.
    ${ }^{8}$ It is possible because better ranked researchers could have decided not to enter.

