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Abstract

This paper explores the relationship between optimal leverage and credit risk under ownership links. It develops a structural model of a parent and a subsidiary, which issues debt in its own name under a guarantee by the parent. We find that zero leverage can be optimal for the guarantor, while leverage close to one can be optimal for the guaranteed company, as this optimally exploits the tax shield of debt while minimizing default costs. As far as credit risk is considered, their joint default probability is lower than that of stand alone units, despite their higher debt capacity. Higher group optimal leverage and lower default probability increase value with respect to conglomerate mergers and stand alone arrangements. Default probability, spreads and loss given default of the subsidiary are higher than for a stand alone with similar size and volatility.

We also study the situation when the subsidiary is constrained to a debt equal to the optimal stand alone level. Only in this case group credit risk depends on the ownership share.

Consistently with intuition, our unconstrained model rationalizes the capital structure typical of private equity; the constrained model instead is able to explain observed features of public business groups and more regulated environments.

Keywords: credit risk, default risk, structural models, optimal leverage, zero leverage, ownership structure, parent-subsidiary.

JEL classification numbers: G32, G33, G34

1 Introduction

Structural models of credit risk consider a company as a stand-alone unit. However, companies often own - at least partially - a subsidiary unit, which issues debt in its own name. These parent-subsidiary links characterize private equity arrangements and LBOs, as well joint ventures, project financing and traditional business groups. A considerable amount of empirical works investigates the leverage of these organizations, being them private or public, but no research model explains it. This paper takes a step in this direction.

The parent-subsidiary link may imply very different relationships depending on legal covenants, informal support agreements, ownership levels and shared names (Samson, 2001). At one extreme, the two firms can be treated like an integrated business. This is the case when either the subsidiary is fully owned, or when there is a legally binding guarantee issued by the parent. In such instances there is no difference between parent and subsidiary debt, because the parent is fully responsible for its subsidiary. At the other extreme the subsidiary is run independently, and the parent - which is totally unresponsible for its subsidiary's debt obligations -only receives dividends. These two cases respectively resemble the conglomerate merger and the stand-alone units which are analyzed in Leland (2007). We model the in-between case, where the parent provides support to bail out its insolvent subsidiary when it can afford to, but leverage in the two units is determined so as to maximize their joint value. The parent provides a guarantee, but still enjoys limited liability. As a result, we can analyze how the intermediate structure affects default probabilities, recovery rates, the associated spreads and ratings. We can thus contrast the credit quality of the two extreme situations with the intermediate one, that we label the group.

Observed parent-subsidiary credit links and the features of the corresponding internal capital markets, have been studied in a number of empirical papers. They all recognize that group membership affects the size, location and default risk of members' debt obligations (Emery and Cantor, 2005; Dewaelheyns and Van Hulle, 2006; Bianco and Nicodano, 2006). As a whole, cross subsidization seems to exist, but to be conditional on survival of the parent.

As for existence, Khanna and Palepu (2000) document it in Indian groups, which have distinguished features of LBOs, such as absence of agency costs. Khanna and Yafeh (2000) report the existence of liquidity smoothing. Gopalan, Nanda and Seru (2004) report the existence of asset transfers, both in terms of cash or subsidized loans or transfer pricing at off-market prices, which put in place conditional rescue of one group unit by the other.

As for conditioning, Boot et al. (1993) investigate why parent companies write comfort letters assuring subsidiaries' lenders that they would assist them in distress. These are seemingly useless documents, as they are legally unenforceable. However, precisely this feature allows parent companies to disregard them ex-post in states when supporting the insolvent subsidiary would undermine their own integrity. Thus Boot et al. (1993) point out, as a distinctive feature of groups, that the parent can choose whether to support its subsidiary or let it selectively default. Jensen (2007) too states that there are limits to the cross-subsidization among business units and the waste of free cash flow.

A paper which documents both cross subsidization and its conditional nature is Emery and Cantor (2005). The Authors, while recognizing that it is extremely difficult to compile a database that accounts for cross guarantees and support mechanisms, find that selective default occurs quite frequently, especially in ring fenced subsidiaries. Cross guarantees in these cases are not unconditionally effective or legally binding.

A few theoretical models build on our assumption that the parent is not legally responsible for its subsidiary debt obligations[1] (Cestone and Fumagalli, 2005; Bianco and Nicodano, 2006). They study how transfers from the parent impact on the spread charged to a subsidiary from its outside financiers, when either managerial effort or investment risk cannot be observed. Higher credit quality for the subsidiary is associated to either increased managerial effort or reduced risk shifting. The common feature of these models is that of incorporating the agency issues in group structures. Our model starts from the hypothesis that leverage in parent-subsidiary links can be better explained in a model without agency costs, such as a structural model. We will add a constraint proxying for agency costs at a later stage.

Well known models of credit risk - including the ones used in the financial industry - adopt a structural view at the multi firm level. However, with the exception of Leland's conglomerate, they do not explicitly incorporate any consideration of ownership structure, with associated selective default features. Giesecke (2004) allows for links between default boundaries, but does not model the firm relationships which justify them: as a consequence, the drivers and the strength of the link are not incorporated in his model. On the contrary, we model explicitly ownership, control and intra-group support mechanisms, which, according to empirical evidence, are expected to affect default and credit risk.

A preview of our main results is as follows. Debt financing - and the associated tax shield - is larger for group-affiliated than for stand alone firms. Despite this, the probability of joint default for group companies is very low when compared to stand alone units. This paradox is due to the optimal capital structure of the group, entailing a complete shift of the debt burden onto the subsidiary: the unlevered parent defaults only when its cash flow turns negative. The shift of debt onto the subsidiary optimally exploits the tax shield of debt while minimizing default costs, as the parent is able to rescue its subsidiary provided their cash flows are less than perfectly correlated. Despite such rescues, group affiliation dramatically worsens the credit quality of the subsidiary, with respect to its stand alone situation. Most of the time, the subsidiary default is selective, in the sense that the parent survives - a circumstance that is precluded in mergers (Sarig, 1985). In order to prevent a complete shift of the debt burden on the subsidiary, either external constraints on leverage or asymmetries between the parent and the subsidiary - such as greater size of the parent - are needed.

We therefore find that groups, or the existence of guarantees together with separate incorporation, rationalize the credit features observed in practice for private equity funds, LBOs and MBOs and, in particular, the relationship between debt and insolvency which

[1]This is the case in major jurisdictions, including the U.S., the U.K., Germany and France (Hadden, 1996; Blumberg, 1989).

uses to be "the least understood aspect of private equity" (Jensen , 2007).

Private equity indeed is characterized by very high leverage at the divisional level and by rare bankruptcies, even in the presence of leverage up to 95% or higher (Jensen, 1989, 2007). Kaplan (1989) documents the fact that MBOs and their leverage policies are driven by tax savings, Andrade and Kaplan (1998) show that, during the eighties, a number of LBOs and high levered transactions went bankrupt for financial, not economic reasons: firms in distress were those unable to repay overwhelming debt, not those with unsuccessful projects and poor value as going concerns. This is exactly what happens in our unconstrained model, where extreme subsidiary leverage turns out to be optimal and entails low group default probability.

At the same time, our uncostrained model, as well as the asymmetric versions of the unconstrained model, rationalize observed features of traditional business groups, whose leverage is lower and whose debt is split between parent and subsidiaries. Indeed, external constraints on leverage, meant to prevent agency costs or being a result of regulatory constraints, are expected to affect more public groups than private ones.

The paper is organized as follows. Section 2 presents the set up, and three organizational modes for two activities - stand alone, group and conglomerate. Section 3 analyzes these cases through a numerical example, so as to understand their properties. Section 4 compares optimal leverage, default probabilities, recovery rates and credit spreads across the three modes, for symmetric units, as the correlation between their cash flows varies. Section 5 extends the analysis to a subsidiary that is not allowed to raise more debt than a stand alone, for external or regulatory constraints. Section 6 examines the case where units differ in bankruptcy costs, size and volatility. The last section concludes.

2 The common set up

In this section we review Leland set up and his analysis of stand alone firms. We then extend it to the group case. We consider a no arbitrage environment with two dates $t = \{0, T\}$. There are two activities, and each activity i generates a random future operational (net) cash flow value X_i at time $t = T$. X_i is a continuous random variable. The riskfree interest rate over the time period T is r_T. No arbitrage implies that the value of the operational cash flow at $t = 0$ is its discounted expected value:

$$X_{0i} = (1 + r_T)^{-1}\mathrm{E}X_i \tag{1}$$

where $\mathrm{E}X_i$ is evaluated under the risk neutral measure. The owners can "walk away" from negative cash flows thanks to limited liability. Thus the (pre-tax) value of each activity with limited liability is

$$H_{0i} = (1 + r_T)^{-1}\mathrm{E}X_i^+ \tag{2}$$

where $X_i^+ = \max(X_i, 0)$, and the pre-tax value of limited liability is

$$L_{0i} = H_{0i} - X_{0i} \geq 0 \tag{3}$$

Now consider a tax rate on future cash flows equal to τ_i. The aftertax value of each unlevered activity is

$$V_{0i} = (1 - \tau_i) H_{0i} \tag{4}$$

and the present value of taxes paid (with no debt) is

$$T_{0i}(0) = \tau_i H_{0i} \tag{5}$$

Firms can issue zero-coupon bonds at time $t = 0$, due, with absolute priority, at $t = T$, with principal value P_i. They have an incentive to do so as interest on debt is a deductible expense. However, debt will also increase the probability of default, which is assumed to cost a fraction α_i of (positive) cash flows and to cause a loss proportional to the firm value[2].

Let us examine separately the effects of taxes and bankruptcy costs.

Let $D_{0i}(P_i)$ denote the value, at $t = 0$, of debt. The promised interest payment is

$$P_i - D_{0i}(P_i) \tag{6}$$

Taxable income is the operational one net of interest payment:

$$X_i - (P_i - D_{0i}(P_i)) \tag{7}$$

The zero-tax level of cash flow or tax shield, X_i^Z, is then

$$X_i^Z(P_i) = P_i - D_{0i}(P_i) \tag{8}$$

Hereafter the argument P_i of D_{0i} and X_i^Z is often suppressed.

We assume that no tax refunds are paid to the firm when $X_i < X_i^Z$. It follows that operational cash flows, net of tax payments, are

$$X_i^n = X_i^+ - \tau_i (X_i - X_i^Z)^+ = \begin{cases} 0 & X_i < 0 \\ X_i & 0 < X_i < X_i^Z \\ X_i(1-\tau) + \tau X_i^Z & X_i > X_i^Z \end{cases} \tag{9}$$

Similarly to Merton (1974), default occurs when net operational cash flow at T is smaller than the face value of the debt:

$$X_i^n < P_i \tag{10}$$

Having defined the default threshold X_i^d as

$$X_i^d(P_i) = P_i + \frac{\tau_i}{1 - \tau_i} D_{0i}(P_i) = \frac{P_i - \tau_i X_i^Z}{1 - \tau_i} \tag{11}$$

the default triggering condition (10) can be written in terms of the pre tax cash flows as $X_i < X_i^d$. Please notice that $X_i^Z < X_i^d$. In the event of default, we assume that bondholders will receive a fraction $(1 - \alpha_i)$ of operational cash flow, X_i, when this is positive. They will however pay taxes out of this fraction, whenever operational cash flows are greater than

[2] In our model indeed firm value and cash flow X_i coincide at maturity T

the tax shield. Debt holders pay D_{0i} at time 0 to the firm, namely to its initial owners or equity holders, face to their expected payoffs.

The level of debt determines both the probability of default, PRD_i, and the (undiscounted) expected loss. The latter can be computed as the difference between the full repayment, P_i, and the expected recovery, $D_{0i}(1+r_T)$. The percentage expected loss is then

$$\frac{P_i - D_{0i}(1+r_T)}{P_i}$$

By taking the ratio of each company expected loss to the corresponding default probability, we get the loss given default, and therefore the recovery rate

$$R_i = 1 - \frac{P_i - D_{0i}(1+r_T)}{P_i \times PRD_i} \tag{12}$$

Last but not least, the endogenous spread y can be determined from the ratio between the face and present value of debt[3]:

$$y_i = (P_i/D_{0i})^{1/T} - 1 - r_T \tag{13}$$

We will assume that the leverage policy of the firm aims at maximizing ν_{0i}, the sum of equity and debt, which in turn pairs the after-tax asset value of the firm. The value of equity and debt is the expected present value of cash flows accruing to shareholders and lenders respectively, evaluated under the risk neutral measure. The latter value is included in the maximization since it is cashed in by shareholders at time 0.

Clearly, the cash flows accruing to debt and equity holders vary with parent subsidiary links, which we analyze below.

2.1 Stand alone companies

Let the two activities, $i = 1, 2$, be separately incorporated and independently managed, as in Leland (2007). Thus the face value of debt issued by firm i maximizes the value of firm i:

$$\nu_{0i}(P_i) = E_{0i} + D_{0i} \tag{14}$$

The payoff E_i to shareholders at time $t = T$ is operational cash flow less taxes and the repayment of principal, when the difference is positive:

$$E_i(P_i) = (X_i^n - P_i)^+ \tag{15}$$

By no arbitrage the value of equity is simply

$$E_{0i}(P_i) = (1+r_T)^{-1}\mathrm{E}(X_i^n - P_i)^+ \tag{16}$$

The payoff D_i to lenders at time $t = T$ will equal P_i when $X_i > X_i^d$ and the firm is solvent. Recalling that the government has priority for tax payments before lenders, the latter will

[3] This is the spread over r which makes the principal P_i the compound amount of D_{0i}, over the specified horizon T.

absorb a tax liability $\tau_i(X_i - X_i^Z)$ in default when $X_i^Z < X_i < X_i^d$. The payoff to lenders is therefore

$$D_i(P_i) = \begin{cases} (1-\alpha_i)X_i & 0 < X_i < X_i^Z \\ (1-\alpha_i)X_i - \tau_i(X_i - X_i^Z) & X_i^Z < X_i < X_i^d \\ P_i & X_i > X_i^d \end{cases}$$

In figure 1 we represent such payoff for a portfolio of stand alone companies. When cash flow is below (above) X_i^d for both units, there is joint default (survival). Otherwise there is selective default.

Insert here Figure 1

Debt present value $D_{0i}(P_i)$, the value of zero-coupon debt given the principal P_i, can be written as

$$D_{0i}(P_i) = (1+r_T)^{-1}\mathrm{E}\left[\begin{array}{c} (1-\alpha_i)X_i\ \mathbf{1}_{\{0<X_i<X_i^Z\}}+ \\ \left[(1-\alpha_i)X_i - \tau_i(X_i - X_i^Z)\right]\mathbf{1}_{\{X_i^Z<X_i<X_i^d\}}+ \\ +P_i\ \mathbf{1}_{\{X_i>X_i^d\}} \end{array}\right] \tag{17}$$

where $\mathbf{1}_{\{\bullet\}}$ is the usual indicator function. The value of debt is negatively affected by taxes and bankruptcy costs paid in default states, given P_i. This feeds back on both the spread in equation (13) and on equity value (16) primarily through its effect on net income (7).

Note that (17) is an implicit equation, since X_i^Z and X_i^d are themselves a function of D_{0i} through (8) and (11). Numerical methods are required for its solution. Since D_{0i} determines the thresholds and the latter enter the equity value, the solution approach for finding firm value ν_{0i} consists in finding a fixed point for D_{0i} and then determine X_i^Z, X_i^d and E_{0i}.

2.2 Groups

We now depart from Leland (2007) and analyze the case where the two activities are still separately incorporated, but one of the two - the parent company - transfers cash flows to the subsidiary in order to honour debt obligations when this allows the survival of both. This is consistent with the existence of comfort letters, which motivated Boot et al. (1993) model, and which is still in force, according to Standard & Poor's reports (Samson, 2001) and rating agencies evaluations. It is also consistent with the evidence in Dewaelheyns and Van Hulle (2006), who report that "private business groups support struggling subsidiaries [..]. However, once groups profitability turns negative, groups tend to terminate support to weak subsidiaries". In general, cash or asset transfers are the way in which our stylized model can incorporate those forms of support from the parent to the subsidiary which in reality take more complex forms, such as transfer pricing at off-market prices, support in restructuring or renegotiating the terms of debt, collateral provision.

Let us denote with X_h and X_s the pretax operational cash flows of the parent $(i = h)$ and of the subsidiary $(i = s)$, with $X_i^d, X_i^Z, i = h, s$ their thresholds. Please notice that the default thresholds $X_i^d,\ i = h, s$ and the tax shield X_i^Z will be related to debt principal and present value by (11) and (8), as in the stand alone case. Moreover, since, as we will see, the holding and subsidiary optimal debts will differ from the stand alone ones, also their thresholds will.

Following the legal literature, we assume that the parent company enjoys limited liability if the subsidiary defaults, being not responsible for the subsidiary's debt obligations. Therefore, equations from (1) to (5) still hold for both the holding and the subsidiary, with $i = h, s$ instead of $i = 1, 2$, but not for the group $(i = g)$, since limited liability is preserved.

The transfer takes place if the subsidiary is in default while the parent is not. The parent limited liability implies that there is no rescue if the operational cash flows of the subsidiary are negative, as the parent would otherwise bear an operational loss that it could have avoided. Put together, these restrictions mean that transfer occurs if

$$\begin{cases} 0 < X_s < X_s^d, \\ X_h > X_h^d \end{cases} \tag{18}$$

In addition, the parent intervenes only if she is not drag into default by rescue. The transfer honours the subsidiary debt obligations when the after-tax parent cash flow, net of debt repayment exceeds the corresponding difference for the subsidiary. Since the latter difference is negative, the sufficient condition for rescue is

$$X_h^n - P_h > P_s - X_s^n \tag{19}$$

Overall, a state-contingent transfer will occur if and only if (18) and (19) both hold.

In what follows, we denote the occurrence of these conditions as event A. When rescue occurs, the holding transfers exactly what the subsidiary is short of in order to repay debt, namely $P_s - X_s^n$. The amount of the transfer then is $(P_s - X_s^n)\mathbf{1}_{\{A\}}$.

The initial owner or shareholder is assumed to choose the face value of debt in the parent and in the subsidiary so as to maximize levered group value. If we include in the holding equity (E_{0h}) dividends from the subsidiary, the group value is:

$$\nu_{0g} = \nu_0(P_h, P_s; \omega) = E_{0h} + D_{0h} + (1 - \omega)E_{0s} + D_{0s} \tag{20}$$

where ω is the ownership share of the parent in the subsidiary: $\omega \in [0, 1]$. The maximization is subject to the state contingent payoffs which we now characterize. We first posit that the parent controls the subsidiary with an infinitesimal equity share[4]: $\omega = 0$. We later remove this simplifying assumption and study what happens when the parent receives dividends from its subsidiary. For the sake of simplicity, we write down the model assuming that the tax rate and default costs do not differ across the two units. Therefore, $\alpha_i = \alpha$ and $\tau_i = \tau$. This assumption is removed in the numerical analysis of asymmetric cases.

[4]It is well known that separation of ownership from control is a possibility in business groups. A parent may directly control a subsidiary with 50% of its voting equity, may indirectly control a second layer subsidiary with 25% and so on. This feature lies at the basis of several models of group behavior, such as Bebchuk, et al. (2000).

2.2.1 Control without dividends

When $\omega = 0$ and no dividends are received, the only difference in events relevant to payoffs with respect to the stand alone case is the existence of A. The corresponding area denoted as "joint survival with rescue" in Figure 2:

Insert here Figure 2

Selective default of the subsidiary (i.e., default of the subsidiary and survival of the parent) takes place when its own cash flow falls below its default threshold, while the parent cash flow exceeds its own default threshold but either is not used to support the subsidiary (since $X_s < 0$), or is not sufficient to support it ($X_s < 0$ and (19) not satisfied).[5] Figure 2 depicts also the combinations of cash flow realizations leading to either joint default ($X_i < X_i^d, i = h, s$), or selective default of the parent, or joint survival without rescue. Comparison with figure 1 allows to appreciate the potentially positive effect of group structure on the subsidiary credit quality, keeping the thresholds fixed: the area of its selective default shrinks as a consequence of the parent transfer. However, as stated above, the optimal thresholds are endogenously determined.

The cash-flow accruing to shareholders of the parent company is equal to the stand alone one, $(X_h^n - P_h)^+$, less the transfer amount. As a consequence, it depends on both principals P_h and P_s:

$$E_h(P_h, P_s) = (X_h^n - P_h)^+ - (P_s - X_s^n)\mathbf{1}_{\{A\}} \tag{21}$$

The equity value is

$$E_{0h}(P_h, P_s) = (1 + r_T)^{-1}\mathrm{E}\left[(X_h^n - P_h)^+ - (P_s - X_s^n)\mathbf{1}_{\{A\}}\right] \tag{22}$$

The payoff to subsidiary lenders is the same as in the stand alone case, in the states where no transfer takes place.[6] These states are formally characterized as

$$\begin{cases} X_s < X_s^d \\ X_h > X_h^d \\ X_h^n - P_h < P_s - X_s^n \end{cases}$$

and are denoted as event $B \cup C$. Events B or C in turn occur respectively when the subsidiary does not pay taxes ($X_s < X_s^Z$) or pays them ($X_s^Z < X_s < X_s^d$).

The payoff to lenders must instead be augmented by the transfer in the transfer area, denoted as event A. There, it would have been X_s^n. Including the transfer, it becomes $X_s^n + (P_s - X_s^n) = P_s$.

[5] The equation of the straight lines which bound the "joint survival with rescue" zone from below are known in closed form.

[6] We are assuming that there is no consolidation of assets in the event of default of the parent - which seems consistent with what happens in most real-world cases (Samson, 2001).

Since the subsidiary debt is the present expected value of these final payoffs, it becomes:

$$D_{0s}(P_s, P_h) =$$

$$(1 + r_T)^{-1} \mathrm{E} \left[\begin{array}{c} X_s(1-\alpha)\mathbf{1}_{\{B\}} + \\ + \left[X_s(1-\alpha) - \tau(X_s - X_s^Z)\right] \mathbf{1}_{\{C\}} + \\ + P_s \left[\mathbf{1}_{\{A\}} + \mathbf{1}_{\{X_s > X_s^d\}}\right] \end{array} \right] \tag{23}$$

The reader can notice that debt depends on the principals of both subsidiary and parent companies, since the transfer does. As in the stand alone case, a fixed point of the debt function determines its value and consequently those of the thresholds X_s^d and X_s^Z. Thus both E_{0h} and D_{0s}, as well as the corresponding thresholds, depend on principals P_h and P_s, which must be simultaneously chosen.

The payoffs to lenders of the parent do not change with respect to the stand alone case, as the transfer to the subsidiary occurs only after the service of the parent debt. Similarly, equity holders of the subsidiary are unaffected, as the transfer occurs for the sake of servicing debt. As a consequence equations (17) and (15) still hold for $i = h$ and $i = s$ respectively, and

$$D_{0h} = D_{0h}(P_h), E_{0s} = E_{0s}(P_s) \tag{24}$$

It should be noted that nothing prevents, so far, the switch of the labels "subsidiary" and "parent". In other words, rescue goes in one direction only - from one company to the other - but the two companies are otherwise symmetric. In the next section we instead allow only the parent to receive dividends.[7]

2.2.2 Control with dividend flows

We now consider the general case of non-zero dividend flows from the subsidiary to the parent, $0 < \omega \leq 1$. Dividends are another type of state- contingent transfer: they are not distributed when the subsidiary is in default and are proportional to its profit after interest and taxes otherwise. Thus, cash flows received by stakeholders do not change as long as the subsidiary defaults, namely when $X_s < X_s^d$, or, equivalently, $X_s^n < P_s$. In the opposite case, the parent cash flows include both operational earnings and dividends.

If we exclude double taxation of subsidiary income, the cash flows of the parent become:

$$X_h^n + \omega(X_s^n - P_s)^+ \tag{25}$$

By absolute priority these cash flows, when positive, first repay debt, up to its face value P_h, then equity.

The payoff to parent lenders, which by a fixed point argument determines its debt current value, D_{0h}, is then equal to:

$$D_h(P_s, P_h) = \left\{ \begin{array}{ll} 0 & X_h^n + \omega(X_s^n - P_s)^+ < 0 \\ (1-\alpha)\left[X_h^n + \omega(X_s^n - P_s)^+\right] & 0 < X_h^n + \omega(X_s^n - P_s)^+ < P_h \\ P_h & X_h^n + \omega(X_s^n - P_s)^+ > P_h \end{array} \right. \tag{26}$$

[7] While nothing logically prevents the subsidiary from receiving dividends, cross-holdings are often prohibited in the real world.

In the first case, the parent cash flows are negative despite dividends. Thus, lenders get zero. In the second case, cash flows are positive but the service of debt is only partial. In the last case, cash flows gross of dividends exceed the face value of debt which is fully reimbursed.

It follows from the previous expression that X_h^d, the parent default threshold with infinitesimal ownership, remains the default threshold under the new ownership structure only until dividends are not received. When the subsidiary pays out dividends, the default threshold is the level of operational cash flows, net of taxes but gross of dividends, that equals P_h. It is the level of X_h such that

$$X_h^n + \omega\left(X_s^n - P_s\right) = P_h \tag{27}$$

This new default threshold then depends on the subsidiary cash flow X_s. It can be shown to be greater than the parent tax shield, and therefore to entail tax payments, as long as $X_s < X_s^{\circ}$. The latter threshold is a known function of the default thresholds with infinitesimal ownership, the parent tax shield and the ownership share ω. As a whole, the new holding default threshold is depicted as a dotted line in figure 3 below.

In Figure 3 we visualize also the corresponding default and non default events.

Insert here Figure 3

The payoff to parent equity holders, E_h, is similarly affected by the shift of the default threshold, with respect to the infinitesimal ownership case, due to dividends. It amounts to:

$$X_h^n + \omega(X_s^n - P_s)^+ \tag{28}$$

when $X_h^n + \omega(X_s^n - P_s)^+ > P_h$.

Dividends from the subsidiary may therefore rescue the parent whenever they cover the parent operational losses. As long as no dividends are paid ($X_s < X_s^d$), the occurrence of joint and selective default does not differ across figures 2 and 3[8]. The subsidiary dividends are able to rescue the parent from default when they are "large enough", namely if $X_h < X_h^d$ and $\omega\left(X_s^n - P_s\right) > -\left(X_h^n - P_h\right)$. This happens in the zone which we label "joint survival with dividend rescue". The boundaries of this zone are again a known (but unreported here) function of the levels X_i^Z,X_i^d, and of the principals P_s, P_h.

The equity and debt value in the parent obtain by discounting the expectation of cash flows to shareholders and lenders, respectively. The problem is complicated by the fact that now they both depend also on the face value of the subsidiary debt[9], i.e.

[8] The optimal threshold levels (X_i^{Z*}, X_i^{d*}, the rescue/no rescue lines) will differ, since firm values - the parent debt in particular - do.

[9] As a matter of fact parent equity holders receive no dividends when the subsidiary is in default, an occurrence that depend on the face value of the subsidiary debt. When the subsidiary pays out dividends, parent equity holders have right to them once any operational loss is offset and parent lenders are reimbursed, an occurrence which depends on the parent face value of debt.

$$D_{0h} = D_{0h}(P_h, P_s) = (1+r_T)^{-1}E\left[\begin{array}{c}(1-\alpha)\left[X_h^n + \omega(X_s^n - P_s)^+\right]\mathbf{1}\left\{_{0<X_h^n+\omega(X_s^n-P_s)^+<P_h}\right\} + \\ +P_h\mathbf{1}_{\left\{X_h^n+\omega(X_s^n-P_s)^+>P_h\right\}}\end{array}\right] \tag{29}$$

$$E_{0h} = E_{0h}(P_h, P_s) = (1 + r_T)^{-1}E\left[\left[X_h^n + \omega(X_s^n - P_s)^+\ \right]\mathbf{1}_{\left\{X_h^n+\omega(X_s^n-P_s)^+>P_h\right\}}\right] \tag{30}$$

The value of subsidiary debt is unaffected by dividend payment, and remains equal to $D_{0s} = D_{0s}(P_s, P_h)$. The value of its equity, $(1-\omega)E_{0s} = (1-\omega)E_{0s}(P_s)$, is also unchanged. They can be represented respectively as in (23) and as a fraction $(1-\omega)$ of (15).

Given the above payoffs parent and subsidiary principals are chosen so as to maximize group value[10]

$$(P_h^*(\omega), P_s^*(\omega)) = \arg\max v_{0g}(P_{h,}P_s; \omega) \tag{31}$$

obtained by substituting in (20) the expressions in section 2.2.1 or 2.2.2.

Once the optimal principals are determined, the current values of both debt, $D_{0h}^*(\omega)$, $D_{0s}^*(\omega)$, and equity, $E_{0h}^*(\omega)$, $E_{0s}^*(\omega)$, can be computed. The optimal tax shields $X_i^{Z*}(\omega)$ and default thresholds $X_i^{d*}(\omega)$ follow.

The probabilities corresponding to the different events in figures 2 and 3 can be computed once the optimal tax shields, default thresholds and principal values are known. In what follows we will denote the selective default probabilities as $PRDS_i,\ i = h, s$, the joint default probability as $PRDJ$ and the marginal default probabilities $(PRDS_i + PRDJ)$ as $PRD_i,\ i = h, s$. We will also be interested in the rescue probability, PRR, and in the joint survival (with or without rescue), $PRND$. Finally, we will obtain recovery rates and spreads over Treasury.

2.3 Conglomerates

The conglomerate merger case - introduced in Leland (2007) - obtains when the two activities $X_i,\ i = 1, 2$, are incorporated as one company. It may also obtain, despite separate incorporation, when a parent company deliberately becomes legally responsible for its subsidiary debt obligations by issuing a legally binding guarantee.

The merger cash flow X_m is the sum of the cash flows of the original activities:

$$X_m = X_1 + X_2 \tag{32}$$

[10] There is no closed formula for the group value, even with infinitesimal ownership. As evident in the text, at least the current value of debt and the thresholds (tax shield and default) of each name depend on the principals. We study the maximization problem numerically, in a base case - whose parameter values are drawn from Leland (2007) - and under some alternative parameter combinations. In the base case without dividends group value is concave in the subsidiary principal, for given holding principal, and decreasing in the latter, for given subsidiary principal. In the alternative cases too we explored the monotonicity properties of the group value in order to present the global maximum and not a local one.

Equations from (1) to (5) hold for the whole conglomerate ($i = m$), since one firm can drag the other into default. The unique choice variable is the face value of debt, P_m, which maximizes[11]

$$\nu_{0m} = \nu_0(P_m) = E_0(P_m) + D_0(P_m) \tag{33}$$

where $E_0(P_m)$ and $D_0(P_m)$ are computed according to (15) and (17) with $i = m$. Debt is again a fixed point, while X_m^Z and X_m^d are defined as in (8) and (11). After having selected the optimal debt value for the merger, $P_m^* = \arg\max \nu_{0m}$, one obtains via (15) and (17) the current value of optimized debt and equity, $E_0(P_m^*)$ and $D_0(P_m^*)$. The optimal leverage ratio follows, together with the optimal tax shield and default threshold, X_m^{Z*} and X_m^{d*}.

In the conglomerate case the probability of selective default is zero. Thus the probability of joint default $PRDJ$ coincides with the default probability of the merger, PRD_m. The recovery rate and the spread are determined using (12) and (13) in the common set up.

3 Credit risk: a base case

We will numerically study the credit risk implications of the organizational structures outlined above assuming that - for each company - annual cash flows are Normal i.i.d.

We start from a base case, whose parameters are borrowed from Leland (2007), in which companies have identically - although not independently - distributed cash flows. We will refer to them as being symmetric. The parameters which characterize the symmetric case are calibrated to those of firms that - as stand alone - issue BBB-rated unsecured debt. In particular, the debt maturity is assumed to be five years, consistent with investment grade evidence. Given an annual riskless interest rate of 5%, expected operational cash flow for each activity, $Mu = 127.6$, is chosen such that its present value is $X_0 = 100$. Operational cash flow at the end of 5 years has a standard deviation (Std) of 49.2. Given that annual cash flows are independent in time, this is consistent with an annual standard deviation of 22.0 ($= 49.2/\sqrt{5}$). Henceforth we express volatility σ as an annual percent of initial activity value X_0, e.g. $\sigma = 22\%$. The tax rate $\tau = 20\%$ and the default cost parameter $\alpha = 23\%$ are chosen so as to generate optimal leverage and recovery rates consistent with the BBB choice (see Leland, 2007).

Insert here Table 1

When the correlation coefficient between the units cash flows is equal to 0.2, as in Leland (2007) we obtain the optimal capital structure and credit risk indicators of table 2. The first column reports values for a stand alone. The second and the third refer to parent and subsidiary respectively. The fourth refers to the overall group, while the last column to a conglomerate.

Insert here Table 2

[11] In the conglomerate case - as well as in the stand alone - firm value is not a monotonic function of the principal value of debt. However, in this case too we will present the global maximum and not a local one.

Let us analyze the stand alone versus group situation (columns one to four) first and compare it with its cum dividend counterpart. The stand alone model and the group versus conglomerate comparison (columns four and five) will follow.

3.1 Group versus stand alone

The first, important feature of Table 2 is that the overall group debt is on the subsidiary's shoulders. The best way to solve the bankruptcy costs versus tax savings trade off is to raise capital via the subsidiary, given the possibility of supporting it when insolvent. The subsidiary can indeed be saved from bankruptcy, so as to avoid the proportional loss of value inherent in default. It can also be left alone when transferring money to it would deplete the overall group value without avoiding bankruptcy costs.[12]

The resulting face value of debt for the subsidiary is higher than that of two stand alone companies (219 versus 114.4). This characteristic of the optimal solution is consistent with the empirical evidence in Dewaelheyns and Van Hulle (2006), who notice that the "decreased potential costs of financial distress allow group members to ex ante take on more debt, thus realizing more tax gains". It is a fortiori consistent with the very high leverage observed in project financing, LBOs and private equity, which are closer to our assumption of no agency costs.[13]

Such a high debt produces a considerable increase in the no tax threshold of the subsidiary (102.32 versus 14.98), and an associated increase in its asset value with respect to that of one stand alone (116.71 versus 81.23). The overall group value ν_{0g}^{*} (165.91) is higher than twice that of a stand alone (162.46), even though the parent value - which does not raise debt and stands ready to rescue its subsidiary - falls with respect to the stand alone situation (from 81.23 to 49.2). Thus parent-subsidiary links that preserve limited liability while allowing for state-contingent support create value for financiers. This explains the pervasiveness of parent-subsidiary arrangements, not only as business groups, but also as private equity funds and LBOs.

Leverage impacts on the marginal default probabilities. These go from $PRD_1 = PRD_2 = 11.2\%$ for each stand alone to $PRD_h = 0.34\%$ for the parent and $PRD_s = 46.54\%$ for its subsidiary. The latter is more likely to default than a stand alone, in spite of rescue (which takes place 52.35% of the times), because of its extreme leverage. However, in the absence of rescue, its default probability would be even larger - actually close to one (99.3%). This is a potential explanation for the coexistence of highly leveraged transactions and a comparatively fairly low number of defaults in private equity and LBOs (Jensen, 2007).

The recovery rate falls from 48.1% for the stand alone to 31.2% for the subsidiary. Indeed, the parent is more likely to be unable to support its subsidiary when the latter

[12] Since a symmetric manoeuvre is not possible, the subsidiary is leveraged while the parent is not. However all debt would be borne by the parent in a group arrangement where rescue transfers are possible from the subsidiary to the parent only. Indeed, in the case under analysis there is no difference between the companies - other than the direction of the rescue possibility and the label.

[13] They are also close to our rescue committment in spite of potential ex post inefficiency, since, as Jensen (1989) says "[...] an LBO partnership that tries to profit at the expense of its creditors or walks away from a deal gone sour will not be able to raise funds for future investments."

losses are larger - leaving these low recovery cases to lenders. The decline in recovery when the default probability increases - or, equivalently, the increase in loss given default, its complement to one - is an important feature to capture, since empirical evidence supports it (see for instance Altman and Fanjul, 2004). As a consequence of high default probability and low recovery, the credit spread dramatically increases for the subsidiary with respect to the stand alone (8.4% versus 1.26% over five years).

The previous figures are better understood if we consider not only the marginal default probabilities, but also the selective, rescue and joint default probabilities. Under the group organization, selective default of the subsidiary occurs in $PRDS_s = 46.2\%$ of the possible cases, while rescue occurs in 52.35% of the occurrences. These are the two most likely scenarios: either the subsidiary defaults, because of the leverage ratio it has been charged together with inability of the parent to rescue her, or it is indeed rescued. Joint default, given that the parent is unlevered, is very rare (0.34%) compared to joint default of two stand alone firms (2%). These observations imply that the risk of firm portfolios will be affected by the incidence of subsidiaries (or parents) in the portfolio, as well as by the type of parent subsidiary link.

When the parent not only exerts control, but also receives 100% of the subsidiary dividends ($\omega = 1$), the whole debt should be again borne by the subsidiary. The overall capital structure and credit risk implications of the model are invariant, as apparent from Table 3. Therefore the amount of expected dividends and the associated increase in the parent company equity value are negligible (0.037 when $\rho = 0.2$), given that the subsidiary is highly leveraged.

Insert here Table 3

The lesson we draw is that when agency costs can be neglected the internal capital market, and more specifically state-contingent transfers targeted to rescue, determine the optimal capital structure. Non targeted ones, like dividends in our model, are not crucial as they do not affect the trade-off between the tax-shield and bankruptcy costs.

3.2 Conglomerate versus stand alone and group

It is well known that divisions of a conglomerate diversify away some risk provided that their operational cash flows are less than perfectly correlated (Lewellen, 1971). This justifies the findings that the conglomerate raises more debt than the two stand alone firms (Leland, 2007). Because of risk sharing, the higher tax advantages induced by higher debt are not completely offset by higher expected default costs, and such a leverage policy creates a merge value ν_m^* greater than twice the optimal value of two stand alone firms: merging is profitable. These results can be visualized by comparing the first with the last column of Table 2: for the conglomerate, the overall debt (117.4) and company value (163.15) are greater than for two stand alone companies (114.4 and 162.46 respectively).

A conglomerate is more levered than two stand alone units because its debt is issued against a diversified portfolio of assets. Its default probability is higher (6.5% instead of 2%, the joint default probability of two stand alone units) since one activity can drag the other, profitable one into default (Sarig, 1985). For the same reason, this is accompanied by higher recovery (56.5 instead of 48.1%). The increase in recovery outweighs that in default probability, thus reducing the spread (0.6% instead of 1.26%)

We now turn to the comparison between a conglomerate and a group. Our model shows that group debt capacity is greater than for conglomerates (219 versus 117.4 in terms of face value). The group is able to implement state-dependent rescue, as opposed to the state independent one inherent in cash flow pooling of mergers[14]**.** As a consequence, in spite of higher face value of debt and endogenous spread and default probability, the value of a group, $\nu_{0g}^{*} = 165.91$, exceeds that of a conglomerate, $\nu_{0m}^{*} = 163.15$. The specificities of the group - namely limited liability and separate incorporation - allow to tailor capital structure so as to increase the no tax profit level (to 102.32 from 14 for the merger) and halve bankruptcy costs. Indeed, while the expected tax payments decrease from 35.62 for the merger to 25.38 for the group, the corresponding default costs raise only from 1.24 to 7.98. The proportional nature of default costs can be responsible for such a result: however, it is consistent with empirical evidence and it will be confirmed also in section 5 below, where the subsidiary leverage is constrained and the wedge default costs - tax savings cannot be fully exploited. There is indeed increasing evidence of the role of taxes in determining the capital structure for public firms (see Graham, 2003). The prevalence of tax savings over default costs has also been empirically detected for bank financed, non listed, small or medium sized firms (see f.i. Bartholday and Mateus, 2005).

The probability of joint default falls to 0.34% from 6.5% even if the optimal leverage ratio is greater for groups than for conglomerates: 70% for the former and 54.8% for the latter.

It will be evident from the next section, namely from the fact that the group arrangement creates value with respect to the merger for any correlation level, that its comparative advantage does not come from diversification only. It comes from the fact that it exploits diversification better than a merger: in the latter an insolvent unit can drag the other into default, so that rescue is symmetric. In a group the Sarig effect cannot occur and only profitable rescues occur.[15]

Parent-subsidiary relationships in groups are value enhancing with respect to mergers, even tough they considerably deplete the credit worthiness of subsidiaries, have lower recovery, greater default probabilities and far higher spreads than mergers.

[14] It is also able to exploit the asymmetry of taxation, namely the different elasticity of the tax shield and of the default threshold with respect to leverage. Tax asymmetry is studied in a companion paper, Luciano and Nicodano (2007).

[15] The transfer can turn out to be ex post inefficient: this drawback is overcome in an infinite horizon version of the current model. Indeed, it can be shown that, at least in the numerical case analyzed here, shareholders' cash flows are always greater when they pay back loans and outside financiers continue partecipating than in the opposite case, when they fail to rescue and outside financing is truncated. As an alternative, one can envisage a full commitment versus partial committment to rescue. The group case reported here would correspond to full commitment, the stand alone one to no committment. Partial commitment would produce groups unable to maximize value with respect to alternative arrangements. Such a commitment role - which was pointed out to us by H. Leland - is studied in a revised version of the companion paper Luciano and Nicodano (2007).

4 Credit risk as correlation changes

It is evident that correlation should play a role in the results obtained so far. Exploring Leland's model, we get the perhaps unsurprising result that the merger should raise as much debt as two stand alone units when correlation equals 1. This is because the distinctive characteristic of conglomerates is diversification.

For the same reason, one may expect that the optimal face values of debt in groups will converge to the stand alone level as correlation among cash flow increases, since the transfers from the parent to the subsidiary will become less likely. This intuition is incorrect: debt in the parent continues to be zero, because this still allows to eliminate the parent bankruptcy costs. The tax shelter differential between raising debt in the parent and raising it in the subsidiary is evidently not strong enough to move debt from the subsidiary to the parent - i.e. from the company which can be rescued to the other one - since the same face amount of debt in the initially unlevered parent deserves less interests than in the highly levered subsidiary. When correlation increases, support from the holding decreases, recovery tends to increase and this allows for further debt to be issued by the subsidiary. Figure 4, top left corner, reports the optimal leverage ratio for the three types of parent/subsidiary links. As correlation increases, the effect of diversification vanishes and the optimal leverage of a conglomerate converges to that of a stand alone as discussed above. On the contrary, the one of groups falls from over 80% to less than 70%, a figure which remains 30% higher than leverage of conglomerates and stand alone firms. As a consequence, the value differential with respect to the stand alone situation is increasing for groups, but decreasing for the merger, since in the latter case the lack of diversification when correlation increases is not counterbalanced by the ability to raise the tax shield. The behavior of ν_0^* is shown in Figure 4, top right corner.

The bottom plots of Figure 4 represent the recovery rate (left) and credit spread (right): apart from the fact that the inequalities across organizational forms hold throughout, we notice that the spread sensitivity to correlation are higher in the group case. Cross subsidization makes the merger recovery higher than under no support (stand alone) and under conditional rescue (group), since the latter is likely to be ineffective in front of large losses. Merger spread too benefits from cross subsidization, while the group spread boosts up because of high leverage and conditional rescue only.

Insert here Figure 4

We can summarize the results from Figure 4 as follows:

Proposition 1 *Assume positive bankruptcy costs, fiscal deductibility of interest and the ability of the parent company to commit to state contingent transfers to its subsidiary for BBB companies. Then the leverage of a group, as well as the resulting firm value, exceeds the one of conglomerates and of the corresponding stand alone companies. The holding is unlevered. The recovery rate of the group, which coincides with the subsidiary one, is lower, while the spread over Treasuries is larger than in other parent-subsidiary links.*

The leverage results are reflected in default probabilities. In the top panel of Figure 5 we contrast the marginal default probability of a stand alone and a subsidiary. At a correlation equal to -0.8, the default probabilities are equal, despite the much higher debt in the subsidiary. This is clearly due to the high probability of support by the parent. For comparison, we also add to the picture the default probability of a stand alone with the optimal subsidiary leverage: this is close to 100% for all correlation coefficients, as the amazing leverage of the subsidiary becomes unsustainable when no rescue through an internal capital market takes place.

In the bottom panel of Figure 5, we add to the merger and group joint probabilities of default the chances for two stand alone companies to default together, evaluated at their own optimal capital structure. They would default more often than the group, even if the latter is more leveraged, given that they cannot support each other. They would however default less than the merger, since the latter is more levered and subsidization is not state contingent. Figure 5 then confirms the beneficial effects of groups in terms of value and default probabilities, at the expense of the subsidiary safety.

Insert here Figure 5

Figure 5 is especially important for understanding how parent/subsidiary links may affect the default probability of firm portfolios, and therefore bank stability. Two stand alone firms would default less than the merger, since they would be far less leveraged and none of them would drag the other into default. At the same time, as correlation between activity cash flows increases, the joint default probability of two stand alone firms dramatically departs from the joint default probability of the group. Thus, a portfolio of group-affiliated parent and subsidiary couples appears to be more resilient than a portfolio of stand-alone companies, which already improves over a merger.

Another way to assess the impact of different parent-subsidiary links onto their portfolio behavior consists in computing default correlation, which is the correlation between the default indicators[16]. Figure 6 presents the default correlation of stand alone companies and their group, as a function of their asset correlation: forming a group out of two stand alone companies eliminates negative default correlation, and smooths out the effect of cash flow correlation in the portfolio. A portfolio of stand alone companies trades off a mild negative default correlation - close to -10% - when their activities are negatively correlated for a destabilizing high positive correlation - close to 55% - in the opposite case. When the independently managed firms turn into a group, their default correlation stays close to 5% for all levels of asset correlation: stability is achieved since joint default occurrence is very low, but not very sensitive to asset correlation. At the same time negative default correlation is lost.

Insert here Figure 6

[16] It is computed as follows:

$$\frac{PRDJ - PRD_i PRD_j}{\sqrt{PRD_i\left(1 - PRD_i\right) PRD_j\left(1 - PRD_j\right)}}$$

5 The case of a constrained subsidiary

We now optimize leverage in the holding, imposing a subsidiary debt level equal to the stand alone one (57.2), in order to mimic agency costs or regulatory constraints on leverage. Agency costs are indeed a well known reason for firms to maintain leverage ratios lower than the optimal ones: in their presence capital markets reduce the size of the issue or demand strict covenants. As for regulatory covenants, there are at least two reasons for observing them. First, in some subsidiaries, shareholders may not have any stake in the holding. This situation for instance occurs when only subsidiaries are listed on public exchanges. Several jurisdictions impose to subsidiary managers to act in the interest of the subsidiary shareholders, rather than implement what is optimal for the group (Hadden, 1996). In this circumstance, a leverage close to 100% and a value of equity close to zero in the subsidiary can easily be considered as a violation of the rule. Second, regulation against thin capitalization, such as the Italian one, can prevent high leverage in the subsidiary. We examine both the case of infinitesimal ownership and the finite one.

5.1 Infinitesimal ownership

Insert here Table 4

Table 4 shows that the optimal debt in the parent company has a face value which is closer to the subsidiary one, and - as a consequence - the difference in the value of debt and equity is lower than in the unconstrained case. Group value still exceeds that of stand alone firms, but the differential shrinks, as expected default costs for the parent are now positive.

Let us focus for the moment on the case $\rho = 0.2$. The relative leverage of parent and subsidiaries (50% and 53%) is now closer to the one observed in Belgian and Italian groups, where the former tends to exceed the latter (Bianco and Nicodano, 2006; Dewaelheyns and Van Hulle, 2007). Similarly, the implied optimal group leverage (51%) is close to the stand alone level of Table 2 - and closer to that part of evidence which shows that the group leverage tends to be smaller than stand alone leverage (Deloof and Verschueren, 2001; Dewaelheyns and Van Hulle, 2007). In spite of lower leverage, the group has greater value than two stand alone companies (162.79 instead of 162.46), so that its existence is justified. The comparison between the unconstrained and constrained case gives a stylized picture of the differential between more and less regulated parent-subsidiary links. According to our model, highly levered arrangements such as the ones observed in private equity can indeed be explained by weaker regulation, while the milder ones in public groups are consistent with the latter being subject to greater external monitoring and regulation.

The Table also presents the recovery, marginal default probabilities and spreads for the group affiliated and for the corresponding stand alone units. All the endogenous credit evaluations for the subsidiary differ from those of a stand alone, despite their common face value of debt. In particular, its default probability is much smaller, since the holding can support it. Consequently the spread it deserves is also smaller, even if its recovery continues to unfavorably compare to that of the stand alone (58 bp, instead of 126 for the stand alone). The evaluations for the holding too are better than those of a stand alone: the face value of its debt is slightly less than the subsidiary, i.e. the stand alone, one; since the holding provides support only when this does not endanger her lenders, despite an almost unchanged recovery, it deserves a lower spread than a stand alone (112 bp). The probability of joint default for two stand alone firms still exceeds that for the group (2% versus 1.81%).

Despite the similar debt burden, the parent recovery rate is much higher than that of a subsidiary, as it does not depend on support (46% versus 24%). The joint default probability reaches 1.81%, versus 0.34% in the unconstrained case. Conversely, the occurrence of selective defaults in the subsidiary reduces to 9.6%, down from 46% in the unconstrained case. Finally, the probability of selective default of the parent (7.8%) now by far exceeds that of the subsidiary (1.8%) - which has similar leverage but receives support from its parent.

Moving out of the $\rho = 0.2$ correlation case, we observe that the holding - and therefore the group - debt decreases as diversification opportunities vanish, contrary to what happens in the unconstrained case. In fact, debt in the subsidiary cannot increase in order to counteract the rising recovery rate associated with reduced support - as was happening in the unconstrained case. In addition, the spread of the holding decreases with correlation, since rescue opportunities are reduced.

The following proposition summarizes these results:

Proposition 2 *Consider a subsidiary with face value of debt equal to that of a BBB stand alone company. Assume positive bankruptcy costs, fiscal deductibility of interests, the ability of the holding company to commit to state contingent transfers to its subsidiary and control with infinitesimal ownership. Then the group leverage exceeds the one of the corresponding BBB stand alone companies only for low correlation. The firm value is always greater for groups than for stand alone firms. The parent optimal leverage is positive and close to the subsidiary and stand alone one; the default probability of both the subsidiary and the parent, their recovery rate and spreads are lower than the stand alone ones.*

Also for this case we computed the historical default probabilities and the implicit rating assignment.

Default correlations too are affected by constraints. Figure 7 below compares the default correlation of two stand alone firms with the one in their constrained group: even though debt principal is unchanged for the subsidiary and is not far from the stand alone one for the holding, default correlation deteriorates considerably. It turns from negative to positive or - when positive - increases.

Insert here Figure 7

Introducing a group structure while maintaining fixed the debt face value of the subsidiary - and optimizing the holding one - has a positive impact on the credit worthiness of both firms: both firms' spreads lower. However, in contrast with the unconstrained case, the portfolio effect is not so favorable: face to an almost unchanged joint default probability, the correlation worsens.

5.2 Finite ownership

Insert here Table 5

Table 5 compares the optimized features of holdings and constrained subsidiaries under infinitesimal ownership with the finite ownership ones, when $\omega = .5$ or 1, and correlation is mild ($\rho = .2$). Ownership share, which was almost irrelevant in the unconstrained case, turns out to affect the optimal policies. The holding gets more levered as ownership increases, since it can count on a greater dividend payoff to pay debt back (from 0 to 39). Greater leverage increases the overall group value, which departs more from the two stand alone firms' level.

The holding default probability and spread decrease, in spite of greater leverage, since it gets more dividends, which can be used to repay loans. Overall, therefore, the greater debt burden on the holding does not impact negatively on its credit quality, since it is counterbalanced by greater dividends.

As for the subsidiary, even though the face value of debt is constant, its present value slightly decreases. The holding uses cash flows to cover its loans, instead of rescuing: as a result, the tax shield increases, the default threshold lowers, as well as default probability, spread goes slightly up. In spite of constant face value of debt, the subsidiary's credit worthiness is therefore weakly damaged by the greater aggressiveness in leveraging the holding: spread indeed suffers.

6 Asymmetric companies

In this section we consider the leverage and credit risk outcomes for non identically distributed activities. In particular, we will in turn analyze the cases of lower default costs (Table 6), higher volatility (Table 7) and smaller size (Table 8) for the subsidiary. These cases have been shown to be value enhancing with respect to the opposite ones. That is, the expected value of the group would be lower if the subsidiary were less volatile, costlier and larger than its holding company, at their optimal unconstrained capital structure (see Luciano and Nicodano, 2007). We maintain the $\rho = 0.2$ assumption.

Insert here Table 6

Higher default costs in the holding do not change the type of optimal capital structure, in the sense that the whole debt burden is still borne by the subsidiary only. Setting to zero the holding company leverage is a fortiori optimal with larger default costs. These will never be incurred in, and hence cannot affect credit quality or spreads or value. Even if costs are as high as 75%, exactly the same face value of debt obtains as in the symmetric case above. Comparison across Table 6 and Table 3 reveals that both stand alone entities' and conglomerates' values suffer from the increase in bankruptcy costs of one unit. This indicates that the capital structure flexibility allowed by group structure can be especially valuable when there are asymmetric bankruptcy costs across activities.

Insert here Table 7

A riskier subsidiary faces a reduced probability to both independently survive and to be rescued by the holding, for given leverage. As a consequence, the subsidiary turns out to have a slightly lower leverage. Consider the case in which risk in the subsidiary is twice as large as in the holding, since volatility is equal to 44% and 22% for the subsidiary and the holding respectively. The optimal subsidiary leverage is 97.3% instead of 100% in the base case. Its default probability increases (48.7% instead of 46.5%); the corresponding recovery and spread are 20.2% and 10.9%, which are respectively smaller and higher than in the symmetric case (31.2% and 8.4%)[17].

When risk doubles, the stand alone spread jumps to 6.2% from 1.26%. In conglomerates diversification opportunities help, but the spread still more than doubles (from 0.6% to 2%). In groups it increases without doubling (from 8.4% to 10.9%) Thus, the credit quality of a group appears to be less sensitive to highly volatile cash flows in one of its units; its capital structure flexibility can help maintaining credit quality in situations of asymmetric risk.

Insert here Table 8

Size asymmetry makes it profitable to shift some debt onto the holding. Let us explore the case in which the holding is five times as large as its subsidiary in the sense that the mean of the final operational cash flow (Mu) is such a multiple. Percentage volatility is kept fixed and equal for the two units. The holding leverage ratio rises from zero to 51%, while that of the subsidiary is unchanged relative to the symmetric case. The holding cash flow is comparatively large enough to be able to rescue its subsidiary despite its positive debt commitment, which reduces its tax burden: the rescue probability is 66%. The credit quality of the holding drops, as its default probability increases to 4.5% from 0.34% in the symmetric case. However, its selective default probability is still zero: it defaults when also the subsidiary does. Since the holding is leveraged, the impact - in terms of portfolio default correlation - of creating a group out of two stand alone firms becomes relevant for lenders. The group default correlation stays close to 30% for all levels of asset correlation.

Overall, we may conclude that the insights obtained in the symmetric case are robust to parametric changes.

7 Summary and concluding remarks

Our model provides optimal leverage policies and credit risk measurement in a situation of interdependence between firms which, as far as we know, was not modelled explicitly in previous studies. It contributes to the literature on credit risk by showing how state-contingent support by a parent company modifies optimal capital structure and the associated default probability in affiliated companies.

In the absence of regulatory constraints and with symmetric firms, optimal capital structure entails a highly-leveraged subsidiary, with a face value of debt that can be almost four times the stand-alone one and the one of a conglomerate division. By contrast, dividends

[17] Also in the group case, we find that as default probability grows for higher volatility the recovery rate falls. This is a stylized fact in the literature on credit risk (Altman et al., 2004).

from the subsidiary to the parent company hardly affect optimal leverage and credit quality, because they leave the tax-bankruptcy cost trade-off unchanged. In such cases, our model predicts zero optimal leverage for supporting companies. We are thus suggesting one potential explanation both for the presence of a large proportion of zero debt firms (Strebulaev and Yang, 2006) and for the presence of high leverage holding companies in fairly unregulated structures such as going private arrangements, private equity funds and project financing, for which there is strong empirical evidence. At the same time, since the group default probability in our model is extremely low, we are resolving the conflict between highly leveraged transactions and the infrequent occurrence of default (Jensen, 1989, Jensen, 2007, Andrade and Kaplan, 1998, Kaplan, 1989).

The implied optimal capital structure of group firms may however be considered at odds with the less extreme leverage of many non-stand alone companies. By introducing constraints and asymmetric companies, our model is able to capture also such situations, proper of public ownership arrangements, such as most traditional business groups and more regulated environments.

Our theory offers insights into how parent-subsidiary links affect default probabilities of the affiliated units. It explains why the prediction of default frequency conditional on firm debt improves when the credit standing of the other affiliated units is taken into account (Dewaelheyns and Van Hulle, 2006). More than that, it rationalizes selective defaults and the discriminating assignments of ratings by most agencies when a parent subsidiary link is in place (Emery and Cantor, 2005). It also identifies some characteristics that should be related to selective defaults. These are the correlation between operating cash-flows, the size of the affiliate relative to the group and the relative risk of its operations.

Last but not least, our paper can provide a basis for studying the default correlation properties of firm portfolios - and therefore the stability of banks. Our model predicts that such properties crucially depend on the type of parent-subsidiary link, on the existence of regulatory constraints on leverage and on the incidence of subsidiaries in the portfolio, because of size.

8 References

Altman, E., and G. Fanjul (2004), Defaults and returns in the high yield bond market: the year 2003 in review and market outlook, NYU-STERN S CDM 04 01

Andrade, G., and N. Kaplan (1998), How costly is Financial (not Economic) Distress? Evidence from Highly Leveraged Transactions that Became Distressed, *Journal of Finance*, 53, 5, 1443-1493

Bartholdy, J. and C. Mateus, (2005), Debt and Taxes: Evidence from bank-financed small and medium-sized firms, SSRN Working Paper

Bebchuk, L., R.Kraakman, G.Triantis (2000), Stock Pyramids, Cross-Ownerships and Dual Class Equity: the Mechanisms and Agency Costs of Separating Control from Cash Flow Rights, in *Concentrated Corporate Ownership*, R.Morck (ed.), 445-460

Bianco, M. and G. Nicodano, (2006), Business Groups and Debt, *European Economic Review*

Blumberg, P. I., (1989), *The Law of Corporate Groups*, Boston: Little, Brown and Company

Boot, A.W.A., S.I. Greenbaum and A.V. Thakor, (1993), Reputation and Discretion in Financial Contracting, *The American Economic Review*, 83(5), 1165-1183

Cestone G., and C. Fumagalli, (2005), The Strategic Impact of Resource Flexibility in Business Groups, *Rand Journal of Economics*, 36, 193-214

Deloof M., and I. Verschueren, (2001), How Does Intragroup Financing Affect Leverage? Belgian Evidence, *Journal of Accounting Auditing and Finance*, 83-108

Dewaelheyns N. and C. Van Hulle, (2007), Internal Capital Markets and Capital Structure: Bank versus Internal Debt, Katholieke Universiteit Leuven, Working Paper 0703

Dewaelheyns N. and C. Van Hulle, (2006), Corporate Failure Prediction Modeling: Distorted by Business Groups' Internal Capital Markets?, *Journal of Business Finance and Accounting*, 33(5)(6), 909-931

Emery, K.M., and R. Cantor, (2005), Default Correlation Among Non-Financial Corporate Affiliates, *The Journal of Fixed Income*, 87-96

Giesecke, K., (2004), Correlated Default with Incomplete information, *Journal of Banking and Finance*, 28, 1521-1545

Gopalan, R., V. Nanda and A. Seru, (2004), Bankruptcy and Business Groups, ASU Working Paper

Graham, J., (2003), Taxes and Corporate Finance: A Review, *Review of Financial Studies*, 16, 1074-1128.

Hadden, T., (1996), Liabilities in Corporate Groups, in *I Gruppi di Società, Proceedings of the International Conference on Business Groups, Venice*, Rossi Guido et al. (eds.), 1361-1388

Jensen, M., (1989), Eclipse of the Public Corporation, *Harvard Business Review,* revised 1997.

Jensen, M., (2007), The Economic Case For Private Equity, Harvard NOM Research Paper No. 07-02

Kaplan, S. (1989), Management Buyouts: Evidence on Taxes as a Source of Value, *Journal of Finance*, 44, 3, 611-632

Khanna, T., and K. Palepu, (2000), Is Group Membership Profitable in Emerging Markets? An Analysis of Diversified Indian Business Groups, *Journal of Finance*, 55, 867-891

Khanna, T., and Y.Yafeh, (2000), Business Group and Risk Sharing around the World, Harvard Business School Working Paper

Leland, H., (2007), Purely financial sinergies and the optimal scope of the firm: implications for mergers, spin offs, and structured finance, *Journal of Finance*, 62, 765-807

Lewellen, W., (1971), A Pure Financial Rationale for the Conglomerate Mergers, *Journal of Finance*, 26, 521-537

Luciano, E., and G. Nicodano, (2007), Firm Scope, Debt, and Value: A Structural Model of Business Groups, RICAFE Working Paper

Merton, R.C., (1974), On the pricing of corporate debt: the risk structure of interest rates, *Journal of Finance*, 449-70

Samson, B., (2001), Corporate Ratings Criteria, Standard & Poor's

Sarig, O., (1985), On mergers, divestments, and options: a note, *Journal of Financial and Quantitative Analysis*, 385-89

Strebulaev, I. A., and B.Yang, (2006), The Mystery of Zero-Leverage Firms, GSB Stanford University Working Paper

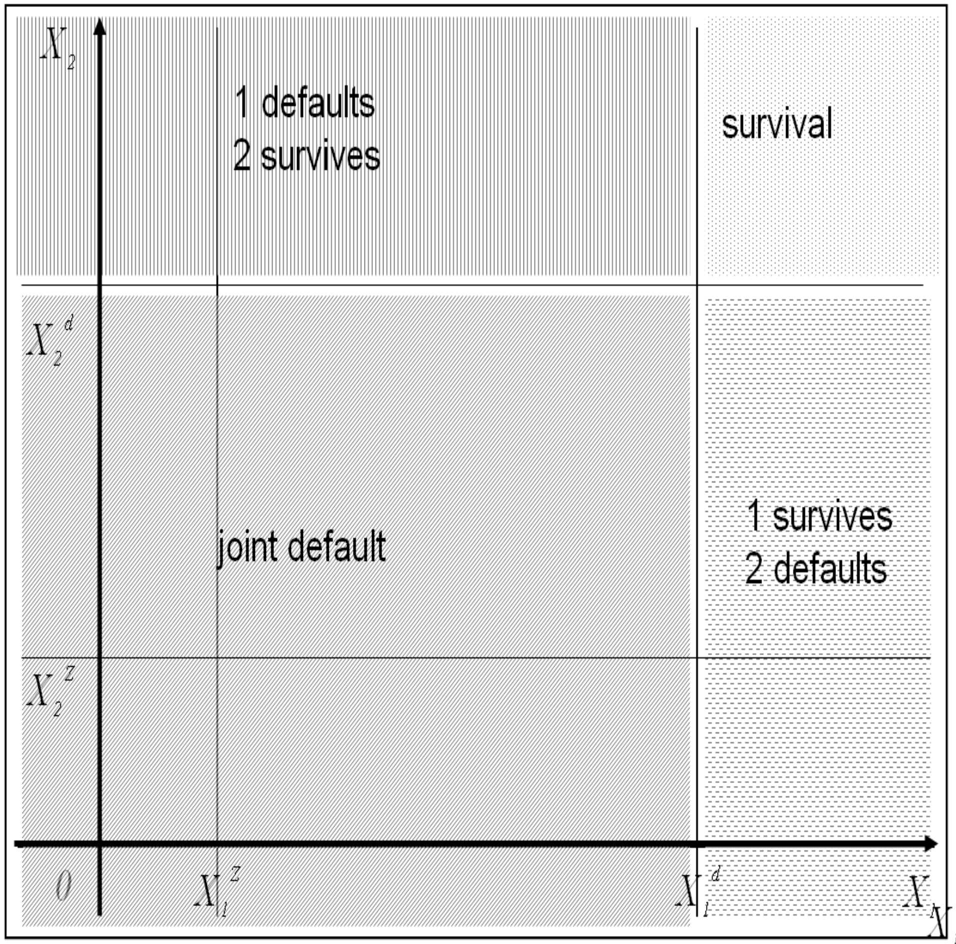

Figure 1: This figure displays combinations of cash flows such that either joint default or selective defaults or survival obtain. The cash flows of the stand alone firms 1 and 2 are on the horizontal and the vertical axis, respectively.

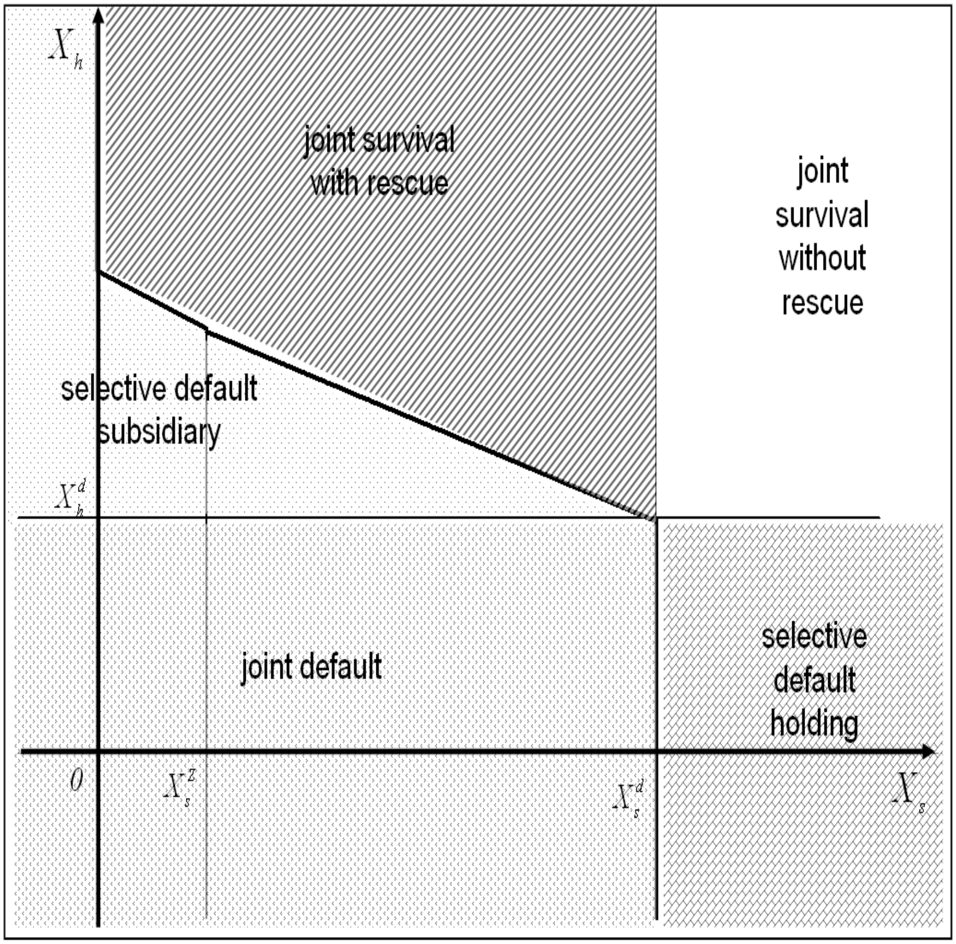

Figure 2: This figure shows the cash flows combinations ensuring the rescue of an insolvent subsidiary, as well as the areas of joint or selective defaults with infinitesimal ownership. It represents the cash flow of the subsidiary on the horizontal axis and of the holding on the vertical axis.

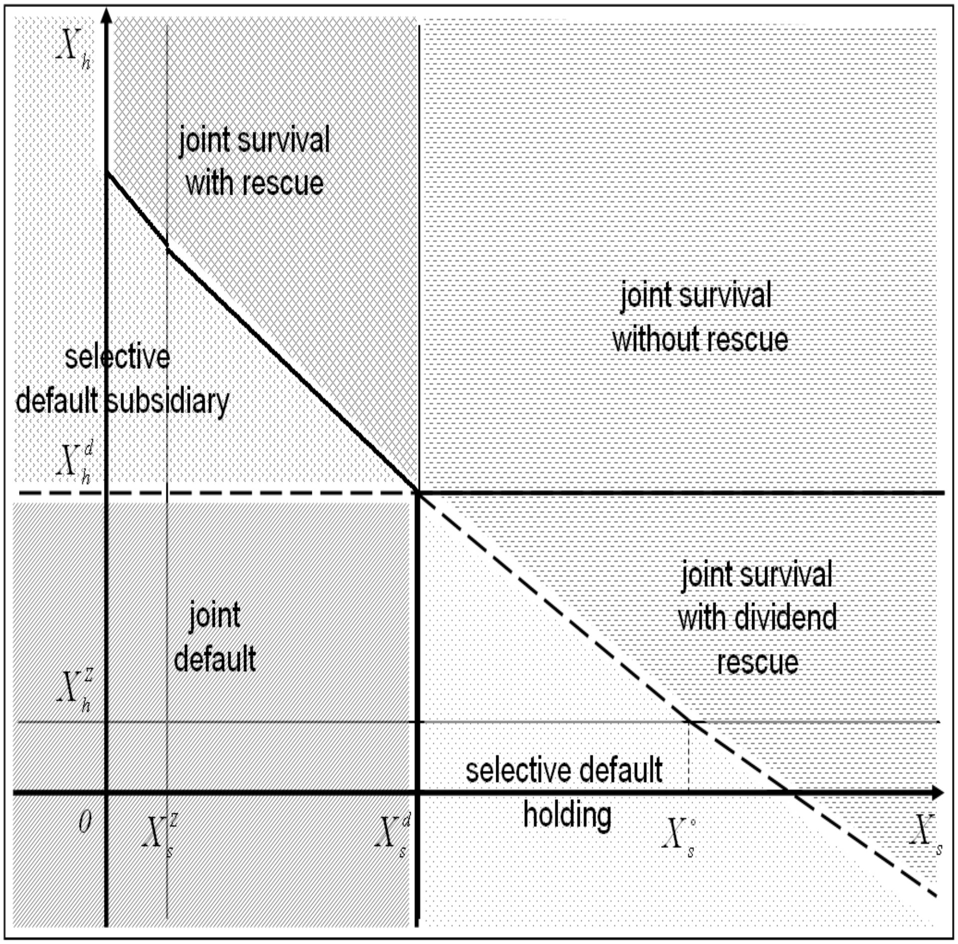

Figure 3: Cash flows of the subsidiary and of the parent, when ownership is non infinitesimal, are on the horizontal and the vertical axis, respectively. This figure shows when there is a subsidiary rescue, as well as the areas of joint or selective defaults. It can be seen that - if the thresholds remain fixed - the area of selective holding default shrinks, with respect to Figure 2, thanks to dividends.

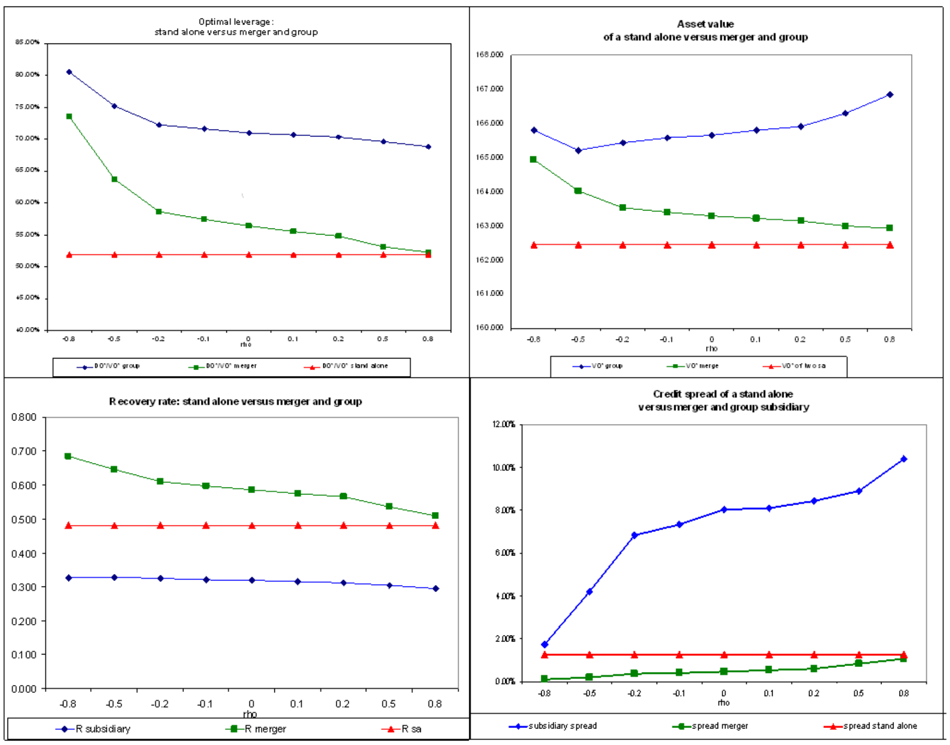

Figure 4: This figure depicts optimal leverage, total asset value, recovery rate and credit spreads for the three types of parent-subsidiary links, as correlation between activity cash flows increases.

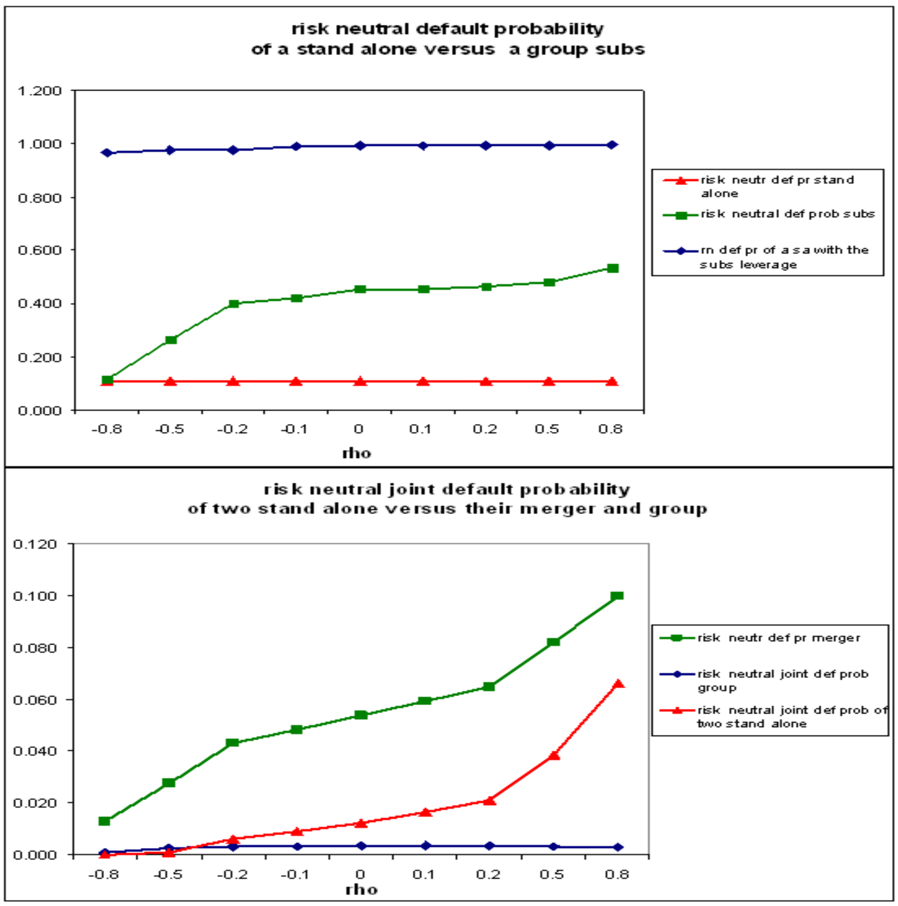

Figure 5: This figure depicts the risk-neutral default probability for the three types of parent-subsidiary links as correlation between activity cash-flows increases.

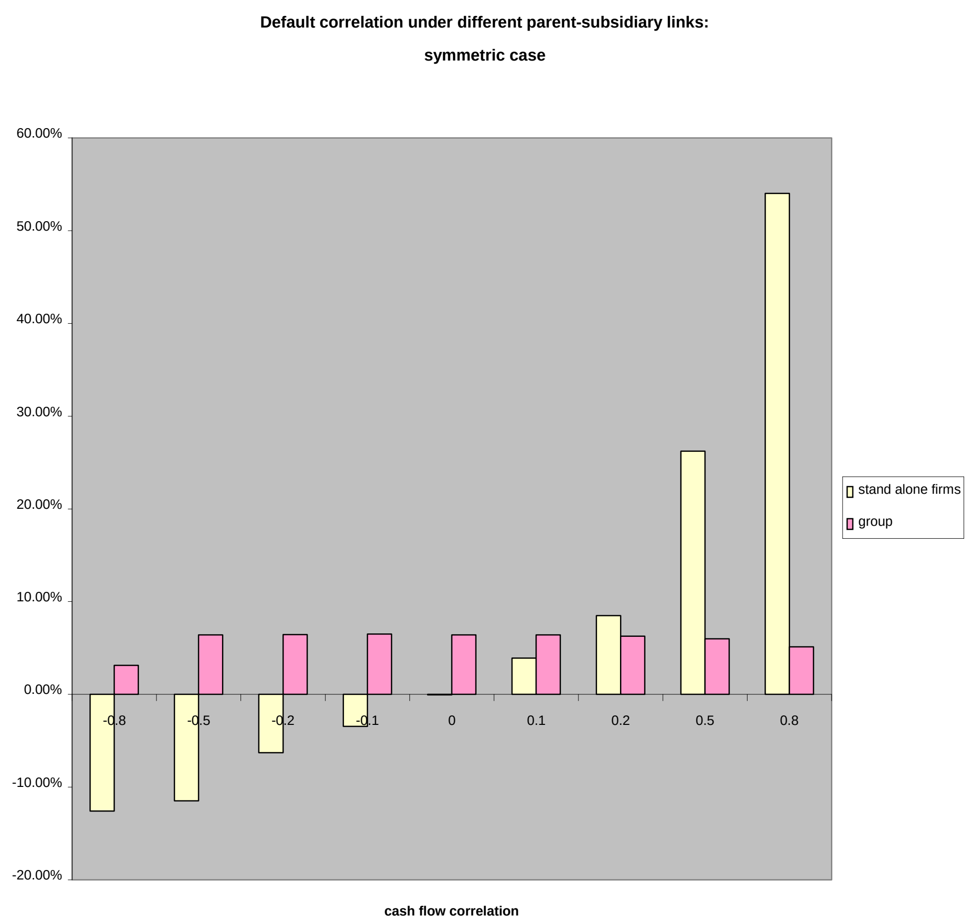

Figure 6: The figure presents the default correlation, i.e. the correlation between the five-year default events, for symmetric, unconstrained firms under different parent-subsdiary links: separately incorporated versus group structure.

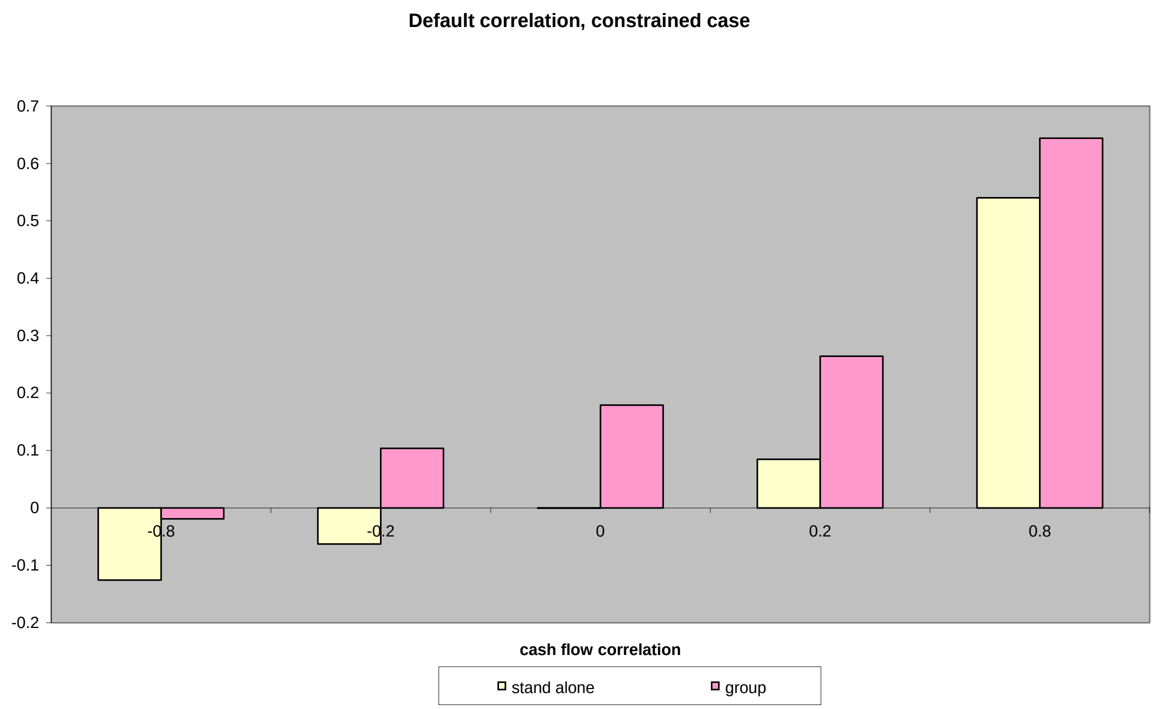

Figure 7: The figure presents the default correlation, i.e. the correlation between the five-year default events, for symmetric, constrained firms under different parent-subsdiary links: separately incorporated versus group structure.

Table 1: Base Case Parameters

Variables	Symbols	Values
Annual Riskfree Rate	r	5.00%
Time Period/Debt Maturity (yrs)	T	5.00
T-period Riskfree Rate	$r_T = (1+r)^T - 1$	27.63%
Capitalization Factor	$Z = (1+r_T)/r_T$	4.62
Unlevered Firm Variables		
Expected Future Operational Cash Flow at T	Mu	127.63
Expected Operational Cash Flow Value (PV)	$X_0 = Mu/(1+r)^T$	100.00
Cash Flow Volatility at T	Std	49.19
Annualized Operational Cash Flow Volatility	$\sigma = Std/T^{0.5}$	22.00
Tax Rate	τ	20%
Default costs	α	23%
Value of Unlevered Firm w/Limited Liability	V_0	80.05
Value of Limited Liability	L_0	0.057

Table 2: Credit risk in the base case, different organizational forms, $\rho = 0.2, \omega = 0$

Variables	Symbols	Values				
		Stand Alone	Holding	Subsidiary	Group	Conglomerate
Optimal Face Value of Debt	P^*	57.20	0	219	219	117. 4
Default Threshold	X^{d*}	67.75	0	248.169	-	69.87
No Tax Profit Level	X^{Z*}	14.98	0	102.32	-	14.00
Value of Optimal Debt	D_0^*	42.22	0	116.68	116.68	44.70
Value of Optimal Equity	E_0^*	39.01	49.2	0.037	49.237	36.88
Optimal Levered Firm Value	$\nu_0^* = D_0^* + E_0^*$	81.23	49.2	116.71	165. 91	163.15
Optimal Leverage Ratio	D_0^*/ν_0^*	52%	0	99,9%	70.33%	55%
Expected Bankruptcy Costs	DC_0^*	0.90	0	7.98	7.98	1.24
Expected Tax Payments	T_0^*	17.62	19.95	5.42	25.37	25.62
Annual Yield Spread of Debt (%)	y	1.26%	//	8.4%	-	0.6%
Value of Optimal Leveraging	$\nu_0^* - V_0$	1.18	- 30.60	36.91	6. 31	3.15
Recovery Rate	R	48.1%	//	31.2%	-	56.5%
Default Probability	PRD_i	0.112	0.0034	0.4654		
Selective Default Probability	$PRDS_i$		0	0.462		
Joint Default Probability	$PRDJ$				0.0034	0.065
Rescue Probability	PRR				0.5235	
Non Default Probability	$PRND$	0.888			0.5292	0.935

Table 3: Credit risk in the base case, different organizational forms, $\rho = 0.2, \omega = 1$

Variables	Symbols	Values			
		Stand Alone	Holding	Subsidiary	Group
Optimal Face Value of Debt	P^*	57.20	0	219	219
Default Threshold	X^{d*}	67.75	0	248.169	-
No Tax Profit Level	X^{Z*}	14.98	0	102.32	-
Value of Optimal Debt	D_0^*	42.22	0	116.68	116.68
Value of Optimal Equity	E_0^*	39.01	49.2	0.037	49.237
Optimal Levered Firm Value	$\nu_0^* = D_0^* + E_0^*$	81.23	49.2	116.71	165. 91
Optimal Leverage Ratio	D_0^*/ν_0^*	52%	0	99,9%	70.33%
Annual Yield Spread of Debt (%)	y	1.3%	//	8.4%	8.4%
Value of Optimal Leveraging	$\nu_0^* - V_0$	1.43	-30.60	36.91	6. 31
Recovery Rate	R	0.481	//	0.0533	0.0533
Default Probability	PRD_i	0.112	0.034	0.4654	
Selective Default Probability	$PRDS_i$		0	0.462	
Joint Default Probability	$PRDJ$				0.034
Rescue Probability	PRR				0.5235
Non Default Probability	$PRND$	0.888			0.057

Table 4: Credit risk in the constrained case, different values of rho

Variables	Symbols	ρ			
		-0.8	0	0.2	0.8
Face Value of Subsidiary Debt	P_{0s}^*	57.2	57.2	57.2	57.2
Optimal Face Value of Parent Debt	P_{0h}^*	57	56	54	51
Optimal Value of Subsidiary Debt	D_{0s}^*	44.47	43.82	43.59	42.78
Optimal Value of Parent Debt	D_{0h}^*	42.09	41.44	40.12	38.11
Optimal Levered Group Value	ν_{0g}^*	162.95	162.83	162.79	162.68
Optimal Value of Two Levered Stand Alone	$\nu_{01}^* + \nu_{02}^*$	162.46	162.46	162.46	162.46
Optimal Group Leverage Ratio	D_{0g}^*/ν_{0g}^*	53.1%	52.4%	51.4%	49.7%
Optimal Stand Alone Leverage Ratio	D_{01}^*/ν_{01}^*	52%	52%	52%	52%
Subsidiary's Recovery Rate	R_s	-	17.6%	23.8%	37.2%
Holding's Recovery Rate	R_h	46.7%	46.8%	46.1%	44.6%
Stand Alone's Recovery Rate	R_{sa}	48.1%	48.1%	48.1%	48.1%
Subsidiary's Default Prob	PRD_s	0.0031	0.0272	0.0362	0.0727
Holding's Default Prob	PRD_h	0.1084	0.1048	0.0963	0.0838
Stand Alone's Default Probability	PRD_{sa}	0.112	0.112	0.112	0.112
Joint Default Probability	$PRDJ$	0	0.0118	0.0181	0.0524
Annual Yield Spread (Subs)	y_s	0.2%	0.5%	0.6%	1.0%
Annual Yield Spread (Holding)	y_h	1.3%	1.2%	1.1%	1.0%
Annual Yield Spread (Stand Alone)	y_{sa}	1.3%	1.3%	1.3%	1.3%

Table 5: Credit risk in the constrained case, $\rho = 0.2$, different values of ω

Variables	Symbols	ω		
		0	0.5	1
Face Value of Subsidiary Debt	P_{0s}	57	57	57
Optimal Face Value of Parent Debt	P^*_{0h}	54	69	79
Value of Subsidiary Debt	D^*_{0s}	43.59	43.15	42.94
Optimal Value of Parent Debt	D^*_{0h}	40.12	51.52	59.04
Optimal Levered Group Value	ν^*_{0g}	162.79	162.94	163.08
Optimal Group Leverage Ratio	D^*_{0g}/ν^*_{0g}	51.4%	58.1%	62.5%
Subsidiary's Recovery Rate	R_s	23.8%	-	-
Holding's Recovery Rate	R_h	46.1%	-	-
Subsidiary's Default Prob	PRD_s	0.0362	0.0077	0.0077
Holding's Default Prob	PRD_h	0.0963	0.0034	0.0034
Joint Default Probability	$PRDJ$	0.0181	0.0008	0.0008
Annual Yield Spread (Subs)	y_s	0.6%	0.7%	0.8%
Annual Yield Spread (Holding)	y_h	1.1%	1.0%	1.0%
Dividends paid to holding	ωE_{0s}	0	19.5	39

Table 6: Credit risk with different αs, different parent-subsidiary link, $\alpha_s = 0.23,\ \alpha_h = 0.75, \rho = 0.2$

Variables	Symbols	Values				
		Stand Alone (α=0.75)	Holding	Subsidiary	Group	Conglomerate
Optimal Face Value of Debt	P^*	33	0	219	219	93
Default Threshold	X^{d*}	39.247	0	248.17	-	110.86
No Tax Profit Level	X^{Z*}	8.01	0	102.32	-	21.58
Value of Optimal Debt	D_0^*	24.99	0	116.68	116.68	71.42
Value of Optimal Equity	E_0^*	55.84	49.2	0.037	49.237	91.05
Optimal Levered Firm Value	$\nu_0^* = D_0^* + E_0^*$	80. 83	49.2	116.71	165. 91	162.47
Optimal Leverage Ratio	D_0^*/ν_0^*	30.9 2%	0	99,9%	70.33%	44%
Annual Yield Spread of Debt (%)	y	0.7%	//	8.4%	-	0.4%
Value of Optimal Leveraging	$\nu_0^* - V_0$	1.43	-30.60	36.91	6.31	2. 37
Recovery Rate	R	6.2%	//	5.3%		
Default Probability	PRD_i	0.036	0.0034	0.4654		
Selective Default Probability	$PRDS_i$		0	0.462		
Joint Default Probability	$PRDJ$				0.0034	0.029
Rescue Probability	PRR				0.5235	
Non Default Probability	$PRND$	0.964			0.5293	0.971

Table 7: Credit risk with different volatilities, different parent-subsidiary links, $\rho = 0.2, \sigma_s = 44\%, \sigma_h = 22\%$

Variables	Symbols	Values				
		Stand Alone(σ =44%)	Holding	Subsidiary	Group	Conglomerate
Optimal Face Value of Debt	P^*	83	0	223	223	118
Default Threshold	X^{d*}	95.19	0	248.169	-	69.50
No Tax Profit Level	X^{Z*}	34.25	0	102.32	-	17.01
Value of Optimal Debt	D_0^*	48.75	0	106.83	106.83	83.97
Value of Optimal Equity	E_0^*	36.10	60.29	3.01	63.30	79.28
Optimal Levered Firm Value	$\nu_0^* = D_0^* + E_0^*$	84.84	60.29	109.84	170.13	163.26
Optimal Leverage Ratio	D_0^*/ν_0^*	57.46%	0	97.3%	62.8%	51.4%
Annual Yield Spread of Debt (%)	y	6.2%	//	10.9%	-	2%
Value of Optimal Leveraging	$\nu_0^* - V_0$	4.79	-19. 51	30.04	10. 53	3. 16
Recovery Rate	R	1.64%	//	20.2%	-	negative
Default Probability	PRD_i	0.2548	0.0034	0.4871		
Selective Default Probability	$PRDS_i$		0.0001	0.4839		
Joint Default Probability	$PRDJ$				0.0033	0.0635
Rescue Probability	PRR				0.4019	
Non Default Probability	$PRND$	0.7452			0.5074	0.9365

Table 8: Credit risk with different size, different parent-subsidiary links, $\rho = 0.2$

Variables	Symbols	Values					
		Stand Alone(1/3)	Stand Alone(5/3)	Holding	Subsidiary	Group	Conglomerate
Optimal Face Value of Debt	P^*	19	95	63.33	121.33	184.66	115
Default Threshold	X^{d*}	22.512	112.56	75.39	138.90	-	136.62
No Tax Profit Level	X^{Z*}	4.951	24.77	15.08	51.07	-	28.53
Value of Optimal Debt	D_0^*	14.05	70.24	48.25	70.26	118.51	86.47
Value of Optimal Equity	E_0^*	13.11	65.54	47.15	0	47.15	76.51
Optimal Levered Firm Value	$\nu_0^* = D_0^* + E_0^*$	27.16	135.78	95.40	70.26	165.66	162.98
Optimal Leverage Ratio	D_0^*/ν_0^*	51.73%	51.73%	50.58%	100%	71.54%	53.06%
Annual Yield Spread of Debt (%)	y	1.2%	1.2%	0.6%	6.5%	-	0.9%
Value of Optimal Leveraging	$\nu_0^* - V_0$	0.48	2.36	15.6	-9.54	6.06	2.88
Recovery Rate	R	/	85.2%	38.4%	20.96%	-	32.1%
Default Probability	PRD_i	0.0163	0.3797	0.0454	0.3304		
Selective Default Probability	$PRDS_i$			0.0000	0.2849		
Joint Default Probability	$PRDJ$					0.0455	0.0597
Rescue Probability	PRR					0.6656	
Non Default Probability	$PRND$	0.9837	0.6203	0.9546	0.7696	0.6656	0.9403