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Reliability and Responsibility: A Theory of Endogenous Commitment[1]

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Abstract

A common assumption in Political Science literature is policy commitment: candidates maintain their electoral promises. We drop such assumption and we show that costless electoral campaign can be an effective way of transmitting information to voters. The result is robust to relevant equilibrium refinements. An unavoidable proportion of ambiguous politicians emerges.
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1 Introduction

It is commonplace to say that electoral promises cannot be taken at their face value. However, parties and candidates invest a considerable amount of effort and resources in producing electoral messages. Presumably, electoral campaign is believed to be a credible mean to attract voters' support. But if campaigns are a mere act of promising why should they influence citizens?

Intuitively, campaigns convey information useful to predict future policies and future policies should be predictable from present ones. Otherwise, the electoral process could not accomplish its very objective of selecting and retaining politicians according to electors' views. Electoral campaigns, to be meaningful, must alter electors' beliefs about the policies the elected officials will implement. A widely employed explanation is that politicians and elected officials seek reelection. Electoral promises affect voters' expectations about the policies that the elected officials will choose. They provide a benchmark linking promises, policies and reelection (retrospective voting), because a credible threat to reelection is imposed (see Barro (1973), Ferejohn (1986) and Austen-Smith and Banks (1989)).

Nevertheless, the disciplining role of electoral competition is only one face of the coin. I prove here that electoral promises provide also a solution to the informational asymmetries between candidates and politicians. The difficulty arises because campaigns are "cheap talk" changing electoral messages alone doe not alter agents' payoff.

Downs (1957) himself underlies intimate the relationship between preelection statements and post election behavior. He asserts that it is necessary for rational voting being meaningful.

> Now we try to prove that a party's ideology must be consistent with either (1) its actions in prior election periods, or (2) its statements in the preceding campaign (including its ideology), or (3) both... A party is **reliable** if its policy statements at the beginning of an election period-including those in its preëlection campaign-can be used to make accurate predictions of its behavior... A party is **responsible** if its policies in one period are consistent with its actions (or statements in the preceding period),... (pp. 103-105)... The absence of reliability means that voters cannot predict the behavior of parties from what the parties say they will do. The absence of responsibility means party behavior cannot be predicted by consistently projecting what parties have done previously....We conclude that reliability is a logical necessity in any rational election system, and that responsibility-though not logically necessary-is strongly implied by rationality as we define it. Of course this conclusion does not prove that reliability and responsibility actually exist in our model. We can demonstrate that they do-and therefore that our model is rational-only by showing that political parties are inexorably driven

> by their own motives to be both reliable and responsible...(pp. 105-107). In our model it is necessary for each party's ideology to bear a consistent relation to its actions.... Any other procedure makes rational voting nearly impossible... (p. 113).

However, most of the classical models of electoral competitions like the Hotelling-Downs one assume that politicians commit to their electoral engagements. The questions about the credibility of campaign promises are left unanswered.

Building on Downs' intuition, the paper provides an explanation based on both informational asymmetries and dynamic aspects, in our case career concerns. Each one of the argument alone is not able to provide a satisfactory solution. Under complete information, politicians cannot credibly commit to policies different from their favorite ones unless elections are infinitely repeated (Alesina (1988)). The result can be relaxed only allowing for indifference in voters' preferences (Aragonés et al (2005))[1]. With the prospect of a unique election, costless electoral campaign cannot be meaningful (Harrington (1992a)) unless one drops the assumption of full policy enforceability (Harrington (1992b))[2].

The paper that is closest to our approach is Harrington (1993). He presents a model of finitely (twice) repeated elections under bilateral asymmetric information. Elected officials can choose between two policies. Candidates' and voters' types are the policies they think to be the most beneficial to their income. The type space is finite and beliefs are not consistent with the common prior assumption. While voters' only care about their income, candidates' preferences are lexicographic: they first care about holding the office and then about the policy they implement. In this case, each politician prefers to carry out the policy she beliefs the most effective. The author proves that there exist equilibria in which each candidate truthfully announces and implements her favorite policy. Policy preferences play only a tie-braking role so there is no interaction between reelection concerns and policy preferences.

This paper presents a model in which candidates' care both about office and about the policy they implement if elected. Politicians' and voters' preferences are private information. Differently from Harrington (1993), the type space is continuous and beliefs are derived from a common prior. The distribution of agents' preferences is symmetric with respect to the origin. Candidates' compete for election by announcing a particular policy. The campaign announcement is costless. The winning candidate implements a policy and runs for reelection against a randomly chosen opponent. The analysis focus on symmetric and monotonic equilibria in which centrist politicians are elected with higher probabilities and implement more centrist policies. Monotonic equilibria permit to rule out very unlikely behaviors where extremists present themselves as centrist,

[1] The authors themselves admit that this approach does not seems a "compelling explanation because of how campaign promises can have effect since it rests on the existence of a nontrivial set of indifferent voters".

[2] Costly electoral campaign can be relevant (Banks (1990) and Callander and Wilkie (2005)).

while moderates make an extremist campaign, and have an intuitive appeal. Furthermore, I show that in all non-monotonic equilibrium electoral campaign is meaningful. I refine out-of-equilibrium beliefs with regard to totally unexpected policies, using a refinement first introduced by Bernheim and Severinov (2003)[3].

This refinement is called monotonic D1 Criterion. It adapts the D1 Criterion, proposed by Cho and Kreps (1987), to monotonic environments. I characterize the set of these equilibria. Innovating on Harrington (1993), I prove reelection pressures and policy motivations interact giving relevance to electoral promises. A necessary and sufficient condition for informative campaign is a sufficiently high candidates' policy concern. Candidates suffer the tension between pleasing their constituencies and seeking the reelection. The cost of ambiguity is to implement policies that are faraway from the candidate's favorite one. So extremists are less willing to fully pay it. But they do pay a price, even when they fully reveal their preferences. It is because they are forced to please the centrist electorate to enhance their election chances. Centrist candidates prefer to pool on the same electoral campaign to increase their election perspectives. Reliability as commitment to the electoral promises of a relevant part of the politicians emerges endogenously. In the same way responsibility appears, present policies can be useful proxies to predict future ones. But ambiguous (or dishonest behavior cannot be eliminated). There will be always politicians who act as pure office seekers[4]. I show that their share decreases as the degree of policy concern increases. The ambiguity of centrist politicians captures a feature that Harrington (1993) was not able to account for: the partial (but relevant) responsiveness of policies to electoral announcements found by empirical work (see Harrington (1992a) and (1992b)). The result also connects with the debate on the nature of political center. It is compatible with the vision of a political center lacking of a well-defined ideology and better defined by its opportunistic behavior, which is popular between the general public (see Hazan (1997)).

The structure of the paper is the following. In Section 2 I describe agents' behavior and I introduce the characteristics of electoral competition and the definition of an Electoral Equilibrium. In Section 3 I present some preliminary results that clarify our choices and I prove the impossibility of fully honest behavior. Section 4 introduces the equilibrium refinement that I analyze in Section 5. Finally, Section 6 draws the conclusion and suggests possible directions of future research. An Appendix contains the proofs that are not included in the main text.

2 The Model

Candidates' and voters' preferences are private knowledge. Candidates compete for office by making campaign announcements. The winner chooses a policy taking into account reelection perspectives. There are two elections. The policy

[3] See also Kartik (2005).

[4] The result is general, does not depend on the equilibrium refinement used.

space is $P = [-D, D]$, where $D > 0$. There are two **candidates**: R(ight) and L(eft). Let $P_R = [0, D]$ and $P_L = [-D, 0]$ be candidate R and candidate L policy space, respectively.
Candidates' policy intentions (their types) are assumed to be independent and symmetric random variables. Candidate R type, $\alpha_R \in [0, D]$ is drawn from the cdf $F(\cdot)$ with continuous density, $f(\cdot) = F'(\cdot)$, where $f(\alpha) > 0$ if and only if $\alpha \in [0, D]$. Candidate L's policy intentions have symmetric density.

In the campaign stage each candidate $j = L, R$ can send a message $m \in P_j$. Based on campaign announcements (m_R, m_L) each voter casts her vote for one of the candidates. We assume that there are n voters, where $n \in \mathbf{N}$ is odd and publicly known. Once in office, the winning candidate implements a policy from her policy space, simultaneously a challenger is selected from the original distribution. Each voter observes incumbent's policy choice and, casts a vote to confirm or to fire her. The "world ends" after this election. Challenger's type is drawn from the original distribution F_L[5].
A voter of type $\alpha \in [-D, D]$ has preferences represented by the following utility function, $V(x, \alpha) = -(\alpha - x)^2$, where x is the policy implemented by the elected politician.
At each election, a **median voter**, m_v is drawn, independently across time, from a symmetric distribution G on $[-D, D]$. G is assumed to have a continuous density, $g(\cdot) = G'(\cdot)$. The assumption is equivalent to have a known median voter in 0 with a symmetric unknown idiosyncratic bias (see Austen-Smith and Banks (2005)).
Let $y > 0$ candidates' private benefit derived from holding the office, let $k > 0$ be their **degree of policy implication** and let $\delta \in (0, 1)$ be their intertemporal factor discount. Let x be the policy implemented by the incumbent
Candidate α's utility from winning the election is, at each period

$$U(x, \alpha) = y - k(\alpha - x)^2.$$

A defeated candidate gets 0 utility[6].
Let π_i be the probability candidate R wins the election i, for $i = 1, 2$. When an incumbent is confirmed in the office, she will implement her favorite policy. From implementing policy $x_R \in P_R$, a candidate of type $\alpha \in [0, D]$ will then derive utility:

$$U_R(\pi_1, \pi_2, x_R, \alpha) = \pi_1 \left[y - k(\alpha - x_R)^2 + \pi_2 \delta y \right].$$

2.1 Voters' behavior

Let m_v be the median voter. Let μ be her beliefs about candidates' policy preferences. She votes for candidate R if and only if $E\left[(m_v - \alpha_R)^2 \mid \mu\right] <$

[5] From Harrington (1992a and b) it follows that any campaign stage before the last election would be irrelevant.

[6] The results of the paper can be generalized to the case in which candidates care also about the policy implemented by any incumbent.

$E\left[(m_v - \alpha_L)^2 \mid \mu\right]$ which is if and only if $m_v > e(\mu)$ where

$$e(\mu) = \frac{1}{2}\frac{E\left[\alpha_R^2 \mid \mu\right] - E\left[\alpha_L^2 \mid \mu\right]}{E\left[\alpha_R \mid \mu\right] - E\left[\alpha_L \mid \mu\right]}$$

is the **decisive median voter**.
Then R is elected with probability $\pi(\mu) = 1 - G(e(\mu)) = \frac{1}{2} + G(-e(\mu))$, because G is symmetric.
The next example presents the values of decisive median voters for two different specifications of the beliefs, that will be used along the paper.

Example 1

If voters believe that candidate R is of type $\alpha > 0$ and that candidate L is randomly drawn from her original distribution then the decisive median voter is

$$e\left(\alpha, f\left(\cdot\right)\right) = \frac{1}{2}\frac{\alpha^2 - \int_0^D \beta^2 f\left(\beta\right) d\beta}{\alpha + \int_0^D \beta f\left(\beta\right) d\beta}.$$

In this case we denote by $\pi\left(\left(\alpha, f\left(\cdot\right)\right)\right)$ R's probability of election.

Instead, if voters believe that candidate R's type belongs to interval (α_1, α_2) and it is drawn from the distribution F, while candidate L is randomly drawn from her original distribution then the decisive median voter is

$$e\left(\left[\alpha_1, \alpha_2\right], f\left(\cdot\right)\right) = \frac{1}{2}\frac{\int_{\alpha_1}^{\alpha_2} \beta^2 f\left(\beta\right) d\beta - \left(F(\alpha_2) - F(\alpha_1)\int_0^D \beta^2 f\left(\beta\right) d\beta\right.}{\int_{\alpha_1}^{\alpha_2} \beta f\left(\beta\right) d\beta + \left(F(\alpha_2) - F(\alpha_1)\int_0^D \beta f\left(\beta\right) d\beta\right.}.$$

We denote by $\pi\left(\left(\alpha_1, \alpha_2, f\left(\cdot\right)\right)\right)$ R's probability of election. Using elementary Analysis it can be shown that $e\left(\alpha_3, f\left(\cdot\right)\right) > e\left(\left[\alpha_1, \alpha_2\right], f\left(\cdot\right)\right) > e\left(\alpha_0, f\left(\cdot\right)\right)$ if $\alpha_3 > \alpha_2 > \alpha_1 > \alpha_0 > 0$. $e\left(\alpha, f\left(\cdot\right)\right)$ is strictly increasing in α. $e\left(\left[\alpha_1, \alpha_2\right], f\left(\cdot\right)\right)$ is strictly decreasing in α_1, α_2 (separately).
Finally, $\lim_{\alpha_1 \to \alpha_2^-} e\left(\left[\alpha_1, \alpha_2\right], f\left(\cdot\right)\right) = e\left(\alpha_2, f\left(\cdot\right)\right)$ and
$\lim_{\alpha_2 \to \alpha_1^+} e\left(\left[\alpha_1, \alpha_2\right], f\left(\cdot\right)\right) = e\left(\alpha_1, f\left(\cdot\right)\right)$.

2.2 The Electoral Equilibrium

A **campaign strategy** for candidate $j = R, L$ is a function $m_j : P_j \to P_j$. j's campaign strategy is a costless announcement of a policy by candidate j.
If elected, candidate j has to choose a policy P_j. A **policy strategy** for incumbent j is a function $s_j : P_j^2 \times P_k \to P_j$, $j \neq k$.

Remark 1 *Each incumbent is opposed to a randomly chosen challenger, then there is no loss of generality in considering policy strategies that are independent from the other candidate's campaign message. We consider policy strategies that depend only on incumbent's type and from the electoral message she sent,* *which is policy of the form $s_j : P_j^2 \to P_j$. With $s_R(\alpha, m)$ we denote the policy implemented a right wing incumbent of type α, who had sent an electoral message m. Analogous notation is used for candidate L.*

A **voting strategy** is a 4-uple $(r_{1j}, r_{2j})_{j=R,L}$ where $r_{1j} : P_R \times P_L \times P \to \{0, \frac{1}{2}, 1\}$, $r_{1R} + r_{2L} = 1$ and $r_{2j} : P_R \times P_L \times P_j \times P \to \{0, \frac{1}{2}, 1\}$. $r_{1j}(m_R, m_L, \gamma)$ represents the probability the median voter votes for candidate j at the first election, when she is of type γ and has observed electoral messages (m_R, m_L). If j results elected after campaign messages (m_R, m_L), $r_{2j}(m_R, m_L, s_j, \gamma)$ denote the probability that the median voter of type γ votes for her, when she implements policy s_j.

Remark 2 *Like in Remark 1, there is no loss of generality in considering second stage voting strategies independent of first stage loser's campaign, Then, we consider voting strategies that depend only on incumbent electoral message, on the policy she implemented and on median voter's type, which is $r_{2j} : P_j^2 \times P \to \{0, \frac{1}{2}, 1\}$, for $j = R, L$.*

A **belief** at the first election about candidates is a function μ_1 from the cartesian product of campaign messages $P_L \times P_R$ to the set of joint probability distributions on P^2. A belief at the second election is a function μ_2 from the cartesian product of campaign messages, first stage voting outcomes, and policy outcomes to the set of joint probability distributions on P^2.

Definition 1 *An* ***electoral equilibrium*** *consists of strategies (m_R, s_R), (r_{1R}, r_{2R}), (m_L, s_L), (r_{1L}, r_{2L}) and beliefs (μ_1, μ_2) such that*
(1) For all $\alpha \in P_R$, $m_R(\alpha)$ maximizes in m

$$\int_{-D}^{D}\int_{-D}^{0} r_{1R}(m, m_L(\beta), \gamma)\left[y - k(\alpha - s_R(\alpha, m)^2\right] f(\beta) g(\gamma) d\beta d\gamma +$$
$$\int_{-D}^{D}\int_{-D}^{0} r_{1R}(m, m_L(\beta), \gamma_1) r_{2R}(m, s_R(\alpha, m), \gamma_2) \delta y f(\beta) g(\gamma_1) g(\gamma_2) d\beta d\gamma_1 d\gamma_2.$$

(2) For all $(\alpha, m) \in [0, D] \times [0, D]$, $s_R(\alpha, m)$ maximizes in $s \in [0, D]$:

$$-k(\alpha - s)^2 + \int_{-D}^{D} r_{2R}(m_R(\alpha), s_j, \gamma)) \delta y g(\gamma) d\gamma.$$

Analogous requirement are imposed on candidate L's strategies
(3) For all $(m_R, m_L, \gamma) \in [0, D] \times [-D, 0] \times [-D, D]$:
$r_{1R}(m_R, m_L, \gamma) = 1 \, if \, E\left[(\gamma - s_R(\cdot))^2 \mid \mu_1(m_R, m_L)\right] < E\left[(\gamma - s_L(\cdot))^2 \mid \mu_1(m_R, m_L)\right]$.
$r_1(m_R, m_L, \gamma) = \frac{1}{2} \, if \, E\left[(\gamma - s_R(\cdot))^2 \mid \mu_1(m_R, m_L)\right] = E\left[(\gamma - s_L(\cdot))^2 \mid \mu_1(m_R, m_L)\right]$.

$r_1(m_R, m_L, \gamma) = 0$ *if* $E\left[(\gamma - s_R(\cdot))^2 \mid \mu_1(m_R, m_L)\right] > E\left[(\gamma - s_L(\cdot))^2 \mid \mu_1(m_R, m_L)\right]$.
Expectations are taken with respect to μ_1.
Analogous requirements are imposed on candidate L's first term probability of election.

(4) For all $(m, s, \gamma) \in [0, D] \times [0, D] \times [-D, D]$:
$r_{2R}(m, s, \gamma) = 1$ *if* $E\left[(\gamma - \alpha_R)^2 \mid \mu_2(m_R, s)\right] < E\left[(\gamma - \alpha_L)^2 \mid \mu_2(m_R, s)\right]$.
$r_{2R}(m, s, \gamma) = \frac{1}{2}$ *if* $E\left[(\gamma - \alpha_R)^2 \mid \mu_2(m_R, s)\right] = E\left[(\gamma - \alpha_L)^2 \mid \mu_2(m_R, s)\right]$.
$r_2R(m, s, \gamma) = 0$ *if* $E\left[(\gamma - \alpha_R)^2 \mid \mu_2(m_R, s)\right] > E\left[(\gamma - \alpha_L)^2 \mid \mu_2(m_R, s)\right]$.
Expectations are taken with respect to μ_2.
Analogous requirement are imposed on candidate L's second term probability of election

(5) Beliefs are computed using Bayes' rule whenever possible.

Conditions (1) and (2) say that each candidate's electoral and policy strategies are sequentially optimal given her opponent's strategies and voters' decision. Conditions (3) and (4) say that voters' decisions are optimal at each election, given their beliefs.

Definition 2 *An electoral equilibrium* $\{(m_j, s_j), (r_{1j}, r_{2j}), (\mu_1, \mu_2)\}_{j=R,L}$ *is* ***symmetric*** *if* $(m_R(\alpha), s_R((\alpha), m_R(\alpha))) = -(m_L(-\alpha), s_L((-\alpha), m_L(-\alpha)))$ *for all* $\alpha \in [0, D]$.

Set:

$$\pi_{1R}(m_R, m_L(\cdot)) = \int_{-D}^{D} \int_{-D}^{0} r_{1R}(m_R, m_L(\beta), \gamma) f(\beta) g(\gamma) d\beta d\gamma.$$

$$\pi_{2R}(m_R, s) = \int_{-D}^{D} r_{2R}(m, s, \gamma) g(\gamma) d\gamma.$$

π_{1R} and π_{2R} are candidate R's probabilities of winning the first and the second election, respectively.
Define analogous quantities for candidate L.

Definition 3 *An electoral equilibrium* $\{(m_j, s_j), (r_{1j}, r_{2j}), (\mu_1, \mu_2)\}_{j=R,L}$ *is* ***monotonic*** *if:*

(R) $\pi_{1R}(m_R(\alpha), m_L(\cdot))$ *and* $\pi_{2R}(m_R(\alpha), s_R(\alpha))$ *are decreasing on* $[0, D]$*, and* $s_R(\alpha, m_R(\alpha))$ *is increasing on* $[0, D]$[7].

(L) $\pi_{1L}(m_R(\cdot), m_L(\alpha))$ *and* $\pi_{2L}(m_L(\alpha), s_L(\alpha))$ *are increasing on* $[-D, 0]$ *and* $s_L(\alpha, m_L(\alpha))$ *is decreasing on* $[-D, 0]$.

[7] Unless otherwise stated, decreasing and increasing will stay for weakly decreasing and weakly increasing, respectively.

In words, an electoral equilibrium is monotonic if centrist candidates have higher probabilities of being elected, and implement more centrist policies.
Let $\{(m_j, s_j), (r_{1j}, r_{2j}), (\mu_1, \mu_2)\}_{j=R,L}$ be an electoral equilibrium.
Let $x \in [0, D]$ and set $\Omega(x) = \{\alpha : m_R(\alpha) = x\}$. $\Omega(x)$ is called an **electoral pool.**
Let $x, z \in [0, D]$ and set $\Omega(x, z) = \{\alpha : s_R(\alpha, x) = z\}$. $\Omega(m_R(\alpha), s_R(\alpha, m_R(\alpha)))$ is called a **policy pool**.
Let $x, \alpha, \alpha' \in [0, D]$, $\alpha \neq \alpha'$. If $\Omega(m_R(\alpha)) = \{\alpha\}$ or $\Omega(m_R(\alpha), s_R(\alpha, m_R(\alpha))) = \{\alpha\}$ we say that α **separates in campaign** or **in policy**, respectively. Otherwise, we say that α **pool**. If $\{\alpha, \alpha'\} \subset \Omega(m_R(\alpha))$ or if $\{\alpha, \alpha'\} \subset \Omega(m_R(\alpha), s_R(\alpha, m_R(\alpha)))$ we say that α and α' **pool** (together) in campaign or in policy, respectively. If $\Omega(m_R(\alpha)) = \{\alpha\}$ and $\Omega(m_R(\alpha), s_R(\alpha, m_R(\alpha))) = \{\alpha\}$ for all $\alpha \in [0, D]$, then the equilibrium is **fully separating**. If only one of the two conditions holds, then the equilibrium will be said **separating in campaign** and **in policy**, respectively.
Analogous definitions hold for candidate L. All along the paper, we devote our attention to symmetric monotonic equilibria. Then, in the analysis, it suffices to consider only one of the two candidates. We will analyze R's strategies omitting the subscript R, when there is no risk of ambiguity. Furthermore we use $s(\alpha)$ for $s(m(\alpha), \alpha)$, $\pi_1(\alpha)$ for $\pi_1(m_R(\alpha), m_L(\cdot))$ and $\pi_2(\alpha)$ for $\pi_2(m(\alpha), s(\alpha))$.
The following Remark points out the connectivity properties of monotone equilibria.

Remark 3 *Let* $\{(m_j, s_j), (r_{1j}, r_{2j}), (\mu_1, \mu_2)\}_{j=R,L}$ *be a monotonic equilibrium.*
(1) For each $\alpha \in [0, D]$, $\Omega(m(\alpha), s(\alpha))$ *is connected, hence it is an interval.*
(2) If $s(\alpha) = s(\alpha')$ *then* $\pi_1(\alpha) = \pi_1(\alpha')$ *and* $\pi_2(\alpha) = \pi_2(\alpha')$.
There is no loss of generality in assuming that candidates having the same probability of election at the first stage use the same electoral campaign (we assume candidates only use pure strategies). Under this assumption, we can state (2) as.
(3) $s(\alpha) = s(\alpha') \Rightarrow m(\alpha) = m(\alpha')$.

Monotonicity has a very intuitive appeal and helps to get rid of unlikely equilibria, for example of situations in which centrists and moderates present different electoral platforms, but extremists pool with centrists.
We say that the electoral campaign is **meaningful** if there exist α, $\alpha' \in [0, D]$, such that $\pi_1(\alpha) \neq \pi_1(\alpha')$.

3 Preliminary results on monotonic equilibria

The first result provides an additional reason that makes monotonic equilibrium a reasonable choice in this environment. Any electoral equilibrium is, locally, monotonic. In all electoral pools, equilibrium policies are monotonic and second stage election probabilities are decreasing.

Lemma 1 *Let $x \in [0, D]$ be a campaign message. In all symmetric equilibria:*
(i) $s(\alpha, x)$, is increasing in α on $\Omega(x)$.
(ii) $\pi_2(x, s(\alpha, x))$ is decreasing in α on $\Omega(x)$[8]*.*
Symmetric claims hold for candidate L.

From Lemma 1, it follows:

Corollary 1 *In any non monotonic equilibrium the electoral campaign is meaningful.*

In any monotonic equilibrium, if the types (α_1, α_2) belong to the same policy pool, then there is a set of unused policies. This result will be frequently used. It implies that the policy function has a discontinuity, at the end of any policy pool.

Lemma 2 *Let $x \in [0, D]$ be a policy and let $\alpha_1 < \alpha_2$. If $s(\alpha) = x$ for all $\alpha \in (\alpha_1, \alpha_2)$, then there exists $h > 0$ such that policies in $(x, x + h)$ are not used or $s(\alpha) = x$ on $(\alpha_2, D]$.*

The next result shows that the threat of reelection is effective on the incumbent. In order not to decrease her chances of reelection, she will implement a policy which is more centrist than her favorite one.

Lemma 3 *In a monotonic equilibrium, if $s(\alpha)$ is separating on $[\alpha_1, \alpha_2)$ then $s(\alpha) < \alpha$ on $[\alpha_1, \alpha_2)$.*

It follows that in any monotonic equilibrium some candidates' types are pooling in order to increase the probability of winning the elections. This fact implies that a full separating equilibrium does not exist.

Proposition 4 *There exists no policy separating monotonic equilibrium. Hence there is no full separating equilibrium.*

Proof. Otherwise, from Lemma 3 $s(0) < 0$. Any full separating equilibrium is equivalent to a monotonic equilibrium so the second claim follows from the first one. ■

4 The MD1 refinement

In this section, we present an equilibrium refinement, introduced by Bernheim and Severinov (2003) and studied also in Kartik (2005) for one round signalling games in which costless and costly messages are present. Differently from Kartik (2005) in our model there are two senders and receiver's type is unknown. Furthermore, cheap talk and costly signalling are not simultaneous. We then

[8] Property (i) holds in any electoral equilibrium, either symmetric or asymmetric.

adapt the refinement to our framework. We apply it only to policies that are never used in equilibrium.
Before the Monotonic D1 criterion is defined, we introduce some notation. We will refer to the lower and highest probability of election, following a given policy x. For all $x \in [0, D]$ set

$$\begin{aligned} \pi_{lR}(x) &= \sup_{s_R(\alpha)>x} \pi_{2R}(\alpha) \text{ if } s_R(\alpha) > x \text{ for some } \alpha \in [0, D] \\ \pi_{lR}(x) &= \pi_{2R}(D, f(\cdot)) \text{ otherwise} \end{aligned}$$

and

$$\begin{aligned} \pi_{hR}(x) &= \inf_{s_R(\alpha)<x} \pi_{2R}(\alpha), \text{ if } s_R(\alpha) < x \text{ for some } \alpha \in [0, D] \\ \pi_{hR}(x) &= \pi(0, f(\cdot)) \text{ otherwise} \end{aligned}$$

Analogous bounds are symmetrically defined for candidate L.

Definition 4 *An electoral equilibrium satisfies the* ***monotonic D1 (MD1) criterion*** *if*
(1) It is monotonic
(2) Let $m^* = (m_R(\beta_R^*), m_L(\beta_L^*))$ *for some* $(\beta_R^*, \beta_L^*) \in P_R \times P_L$. *Let* $x \in [0, D]$ *with* $\mu(x \mid \beta_R, \beta_L) = 0$ *for all* $(\beta_R, \beta_L) \in P_R \times P_L$. *Assume that there exists a non-empty set of types* $\Omega \subset [0, D]$ *such that, for each* $\alpha \notin \Omega$, *there exists some* $\alpha' \in \Omega$ *such that for all* $\pi \in [\pi_{lR}(x), \pi_{hR}(x)]$:
$\pi_{1R}(\beta)\left(y - k(x-\alpha)^2 + \pi\delta y\right) \geq \pi_{1R}(\alpha)\left(y - k(s_R(\alpha) - \alpha)^2 + \pi_2(\alpha)\delta y\right) \Longrightarrow$
$\pi_{1R}(\beta)(y - k(x-\alpha')^2 + \pi\delta y) > \pi_{1R}(\alpha')\left(y - k(s_R(\alpha') - \alpha')^2 + \pi_2(\alpha')\delta y\right)$
then $\mu(\cdot, \cdot \mid m, x) = \mu_R(\cdot) f(\cdot)$ *and* $supp\mu_R(\cdot \mid m, x)) \subset \Omega$.
Analogous requirement is symmetrically imposed on candidate L.

In the case in which $[\pi_{lR}(x), \pi_{hR}(x)]$ is substituted by $[\pi(D, f(\cdot)), \pi(0, f(\cdot))]$ we would have an adaptation to our setup of the D1 criterion introduced by Cho and Kreps (1987). (2) extends the monotonicity requirements to out of equilibrium beliefs. If an elected official implements out of equilibrium policy x, she should expect of being reelected with probability between $\pi_{lR}(x)$ and $\pi_{hR}(x)$. The refinement assign positive probability only to those types who benefit most from this deviation. From now on, we will refer to MD1 equilibria only, if not otherwise stated.

5 Equilibrium characterization and existence

If $s(\cdot)$ is increasing then it has at most a countable set of discontinuity points and it is differentiable almost everywhere (see Royden (1988)). There is no loss of generality in assuming that the electoral campaign is monotonic increasing and that $m(\alpha) = \alpha$ when type α is separating, and that agents having the same probability of being elected at the first election make the same announcement.

Denote by $\alpha_1, \alpha_2, ...\alpha_k, ...$ where $0 \leq \alpha_1 < \alpha_2 < ... < \alpha_k < ... \leq D$ the discontinuity points of s.

Lemma 4 *Assume that, for some $i = 1, 2, ..., k, ...,$ the types in (α_i, α_{i+1}) belong the same policy pool. Then they belong to the same campaign pool.*

Proof. Let $0 \leq \alpha < \alpha' < \alpha''$. By contradiction, assume that some types in $(\alpha, \alpha') \subset (\alpha_i, \alpha_{i+1})$ send message m and that the types in $(\alpha', \alpha'') \subset (\alpha_i, \alpha_{i+1})$ send message m'. Let $\alpha' + \varepsilon$ imitate type $\alpha' - \varepsilon$. The gain in the probability of being elected at the first stage is bounded below by a strictly positive constant. The gain in second election probability is non negative. For $\varepsilon \to 0$ the loss in policy term goes to 0 by continuity, so the deviation would be profitable for ε small enough. ■

Observe that $\alpha = \arg\max_{\alpha'} \pi_1(\alpha')) \left[y - k\left(s(\alpha') - \alpha\right)^2 + \pi_2\left(\alpha'\right)\delta y\right]$

So, almost everywhere.

$$\pi_1'(\alpha)\left[y - k\left(s(\alpha) - \alpha\right)^2 + \pi_2\left(\alpha\right)\delta y\right] + \pi_1(\alpha))\left[-2ks'(\alpha))\left(s(\alpha) - \alpha\right) + \pi_2'\left(\alpha\right)\delta y\right] = 0 \tag{1}$$

In particular, if all agents in (α', α'') pool on the same campaign but use different policies $(\pi_1(\alpha)))' = 0$ so that

$$\left[-2ks'(\alpha))\left(s(\alpha) - \alpha\right) + \pi_2'\left(\alpha\right)\delta y\right] = 0 \text{ on } \left(\alpha', \alpha''\right) \tag{2}$$

Remark 5 *If π_1 and π_2 are C^1 and strictly decreasing both problems defined by the differential equations above and the terminal condition $S(D) = D$ have a unique solution such that $s(\alpha) < \alpha$ on $(D - \varepsilon, D)$, for arbitrary small $\varepsilon > 0$. The result follows from Lemma 5 in the Appendix (see also Kartik (2005)). Such solution is such that $s(\alpha) < \alpha$ on $(0, D)$. Furthermore $s(0) < 0$. Otherwise the graph of s should crosses the diagonal at some $\alpha^* > 0$. In this case $\lim_{\alpha \to \alpha^{*+}} s'(\alpha) = \infty$. This is impossible: if the graph cross the diagonal it must be from below because $s(\alpha) < \alpha$ on $(0, D)$.*

For all α, α' such that $m(\alpha) = m(\alpha')$, set

$$T(\alpha, \beta, x, \pi_2) = \pi_1(\beta)\left(y - k(x - \alpha)^2 + \pi_2 \delta y\right) - \pi_1(\alpha)\left(y - k(s(\alpha) - \alpha)^2 + \pi_2(\alpha)\delta y\right).$$

Remark 6 *Condition (2) of Definition 4 can be written in this case as Let $m^* = (m_R(\beta_R^*), m_L(\beta_L^*))$ and let $x \in [0, D]$ with $\mu(x \mid \beta_R, \beta_L) = 0$ for all $(\beta_R, \beta_L) \in P_R \times P_L$. If there exists a non-empty set of types $\Omega \subset [0, D]$ such that, for each $\alpha \notin \Omega$, if there exists some $\alpha' \in \Omega$ such that, for all $\pi \in [\pi_l(x), \pi_h(x)]$*

$$T(\alpha, \beta, x, \pi_2) \geq 0 \Longrightarrow T(\alpha', \beta, x, \pi_2) > 0,$$

then $\mu(\cdot, \cdot \mid m, x) = \mu_R(\cdot) f(\cdot)$, where $supp\mu_R(\cdot \mid m, x)) \subset \Omega$.

When there is no risk of ambiguity we omit the arguments β, x, π_2 and we write simply $T(\alpha)$ for $T(\alpha, \beta, x, \pi_2)$ and $T'(\alpha)$ for $\frac{\partial T}{\partial \alpha}(\alpha, \beta, x, \pi_2)$.

From Equation 1 it follows that

$$T'(\alpha) = 2k\left[\pi_1(\alpha)(\alpha - s(\alpha)) - \pi_1(\beta)(\alpha - x)\right]. \tag{3}$$

Any MD1 equilibrium is characterized by a cutoff type that divides pooling types from separating types.

Proposition 7 *Any symmetric MD1 is essentially equivalent*[9] *to an equilibrium in which, for all i, there exists $\alpha^* \in (0, D]$ such that*
(i) $s_R(\alpha) = 0$ on $[0, \alpha^*]$
(ii) If $\alpha^* < D$ then $s_R(\alpha)$ is separating on $(\alpha^*, D]$ and $s_R(D) = D$.

We can go further and characterize all MD1 equilibria. According to Proposition 7 they can belong to four categories:
(i) **Babbling:** equilibria in which all types pool in campaign and policy.
(ii) **Campaign irrelevant but policy significative** equilibria, in which all types send the same electoral message, but the more extremist types separate in policy.
(iii) **Weakly expressive campaign** equilibria in which centrists and extremists form different campaign pools but extremists separate in policy
(iv) **Expressive campaign** equilibria where centrists pool on the same electoral promise and on the same policy and extremists separate both in campaign and in policy.

The larger is the degree in which candidates cares about the policy they implement, the larger are the possibilities of relevant electoral campaign.

Theorem 8 *A symmetric MD1 equilibrium exists. There exist $k_0 < k_1 < k_2$ and there exists strictly decreasing functions $\alpha_1(k), \alpha_2(k), \alpha_3(k)$ with $\lim_{k\to\infty} \alpha_i(k) = 0$ for $i = 1, 2, 3$, such that*

(i) For $k \leq k_0$ all MD1 equilibria are fully pooling, which is $m(\alpha) = m(0)$ and $s(\alpha) = 0$ for all $\alpha \in [0, D]$. If $k > k_0$ such equilibria are not MD1.

(ii) For $k \geq k_0$ there exists an MD1 equilibria such that $m(\alpha) = m(0)$ for all $\alpha \in [0, D]$, $s(\alpha) = 0$ for all $\alpha \in [0, \alpha_1(k)]$, $s(\alpha)$ is separating on $(\alpha(k), D]$.

(iii) For $k \geq k_1$ there exists an MD1 equilibrium in which $m(\alpha) = m(0)$ for all $\alpha \in [0, \alpha_2(k))$ and $m(\alpha) = m(\alpha_2(k)) \neq m(0)$ for all $\alpha \in [\alpha_2(k), D]$, $s(\alpha) = 0$ for all $\alpha \in [0, \alpha_2(k)]$ and $s(\alpha)$ is separating on $[\alpha_2(k), D]$.

(iv) For $k \geq k_2$ there exists an MD1 equilibrium in which $m(\alpha) = m(0)$ for all $\alpha \in [0, \alpha_3(k))$ and $m(\alpha) = \alpha$ for all $\alpha \in [\alpha_3(k), D]$. $s(\alpha) = 0$ for all $\alpha \in [0, \alpha_2(k)]$ and $s(\alpha)$ is separating on $[\alpha_3(k), D]$.

Any symmetric MD1 equilibrium is essentially equivalent to one of the equilibria described above.

[9] Essentially equivalent means that it is equal, excepted, at most a zero measure set of types.

Observe that the expressive campaign equilibrium of point (iv) asymptotically converges to a fully separating equilibrium, in which electoral promises are maintained.
Theorem 8 relies on the possibilities of threatening the reelection perspectives of the incumbent, if she shirks. It requires beliefs to be correlated outside of the equilibrium path.
In the real world electoral disappointment does have an effect on electors. The model we presented does not capture this aspect because the idiosyncratic shocks defining median voter exact position is independent across periods and uncorrelated to actions. Electoral disappointment can be introduced as a shift of voters distribution, correlated with the degree of electoral fulfillment. To make things simple as possible assume that median voter distribution is shifted to left in the case of an R incumbent, or to the right in the case of an L incumbent of a fix factor $x > 0$, if the elected officer deviates from the expected policy(ies)[10]. The reader can easily verify that the claim of Theorem 8 holds even if we impose voters' beliefs about the two candidates to be independent. More precisely, if $x \geq D - \frac{D^2 - \int_0^D \beta f(\beta) d\beta}{D + \int_0^D \beta f(\beta) d\beta}$ the k_0, k_1, k_2 found in the proof of the result would stay the same. Otherwise their value would be larger as it would harder to induce extremists not to pool in campaign. We conjecture that a similar result can be obtained also through a shock which is continuously dependent from the distance between expected policy and implemented one.

The MD1 refinement applies only to zero probabilities policies. It is strong enough to shrink dramatically the set of possible equilibria. The key, as for Universal Divinity, is that we ask the support of the distribution to be minimal. If the function T has a a unique maximizer, α, then to such maximizer must be given probability one. Like in Banks (1990), this leads to equilibria characterized by a unique cutoff type.

The claim of Proposition 4 relies on the boundedness of the type space. Allowing for an unbounded type space can lead to full separation in sender-receiver games with both costly messages and cheap talk (see Kartik (2005)). It is not the case here. We would obtain full separation in policy, but total pooling in campaign. The reason is that, asymptotically, candidates utilities is null, so it is the effect of career concerns. Very extremist candidates would be incomparably better off by maximizing their first election probability. Full separation could probably be obtained in the case in which candidates care also about challengers' implemented policy.

6 Conclusions

The paper has presented a model of electoral competition under incomplete information in which candidates care about both office and the policy. It introduces incomplete information and the dynamic aspects of a double election and it proves that electoral campaign is able to convey relevant information to

[10] Excluding the case of a totally out of equilibrium policy, which defines the MD1 criterion.

voters even when campaigning is not costly. The work opens a possibility for endogenous commitment. The result is driven by both candidates' career concern and the threat of failed reelection. The impossibility for candidates to sustain policies that are too faraway from their ideal ones shapes then policy they carry out in the first term. Extending Harrington (1993), I find that not only reelection pressure but also policy motivation can give relevance to electoral promises. Despite of it centrists' electoral opportunism cannot be eliminated. It can be only be reduced if candidates' degree of policy implication is high enough. This is consistent with the empirical literature which estimates that only a part (even if relevant) of policies are responsive to electoral compromises.

The investigation can be extended in different directions. On the one hand toward the study of more complex models of competition. In our model the " world ends" after the second election. So just before there is no place for meaningful electoral competition before the last election. Allowing for repeated interactions should make it relevant. A suitable and realistic model would be the one of an overlapping generation of politicians that can stay in the office for a fixed number of terms. The threat to reelection imposed on the incumbent would be reinforced, and so the degree of commitment.

On the other hand, a partially unexplored field is the nature itself of electoral campaign. It is usually modeled as a one-shot policy announcements (either costly or cheap). Despite of it, in the real world, electoral campaigns are complex and longer interactions between electors and politicians. Voters are continuously exposed to announcements. Politicians invest many resources in pools to discover electors' intentions and tastes. The empirical literature considered these aspects as an important part of the process of information transmission ((see for instance Alvarez (1998)). Parties try both to send reliable messages and to get information about electors. There is little theoretical investigation about such phenomena, but the models of repeated cheap talk (see Krishna and Morgan (2004)) could provide useful tools to deal with the topic.

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8 Appendix

8.1 Section 3

Proof of Lemma 1 Let $0 \leq \alpha < \alpha'$. Set $t = s(\alpha, x)$, $t' = s(\alpha', x)$, $\pi = \pi_2(x, s(\alpha, x))$ and $\pi' = \pi_2'(x, s(\alpha', x))$.
(i) The proof of the claim is by contradiction. Assume that $t' < t$. From incentive compatibility it follows:

$$\begin{aligned} -k(t-\alpha)^2 + \pi\delta y &\geq -k(t'-\alpha)^2 + \pi'\delta y. \\ -k(t-\alpha')^2 + \pi'\delta y &\geq -k(t-\alpha')^2 + \pi\delta y. \end{aligned}$$

Which is:

$$\begin{aligned} (\pi - \pi')\,\delta y + k\left[(t'-\alpha)^2 - (t-\alpha)^2\right] &\geq 0. \\ (\pi' - \pi)\,\delta y + k\left[(t-\alpha')^2 - (t'-\alpha')'^2\right] &\geq 0. \end{aligned}$$

Summing up the two inequalities:

$$(t'-\alpha)^2 - (t-\alpha)^2 + (t-\alpha')^2 - (t'-\alpha') \geq 0.$$

Simplifying:

$$(\alpha - \alpha')\,(t - t') \geq 0$$

that yields a contradiction because $\alpha < \alpha'$.
(ii) The proof of the claim is by contradiction. Assume that $\pi > \pi'$. From (i) and from the definition of monotonic equilibrium it follows that it cannot be the case that α and α' belong to different policy pools or that α and α' belong the same policy pool or that α' pools with some other type while α separate. It must be the case that α pools and α' separates. From Remark 3, the pool α belongs to is an interval (α_1, α_2) (or $[\alpha_1, \alpha_2]$, or $(\alpha_1, \alpha_2]$, or $[\alpha_1, \alpha_2)$). In such a case the decisive median voter for α is:

$$e(\alpha) = \frac{1}{2}\frac{\int_{\alpha_1 0}^{\alpha_2} \beta^2 f(\beta)\,d\beta - (F(\alpha_2) - F(\alpha_1))\int_0^D \beta^2 f(\beta)\,d\beta}{\int_{\alpha_1}^{\alpha_2} \beta f(\beta)\,d\beta + (F(\alpha_2) - F(\alpha_1))\int_0^D \beta f(\beta)\,d\beta}$$

while the decisive median voter for $\alpha' > \alpha$ is:

$$e(\alpha') = \frac{1}{2}\frac{(\alpha')^2 - \int_0^D \beta^2 f(\beta)\,d\beta}{\alpha' + \int_0^D \beta f(\beta)\,d\beta} > e(\alpha)$$

which yields a contradiction.

Proof of Lemma 2 From Remark 3, part (3) $m(\alpha)$ constant on (α_1, α_2). There is no loss of generality in assuming that (α_1, α_2) is the interior of the corresponding policy pool. From monotonicity $s(\alpha_2) \geq x$.
Consider first the case $s(\alpha_2) > x$. Then policies in $(x, s(\alpha_2))$ are not used

in equilibrium.

Now let $s(\alpha_2) = x$ and set $\widehat{x} = \lim_{\alpha \searrow \alpha_2} s(\alpha) = \inf_{\alpha > \alpha_2} s(\alpha)$. Observe that $s(\alpha) > x$ for $\alpha > \alpha_2$. By contradiction, suppose that $\widehat{x} = x$. Set $\pi_{1\varepsilon} = \pi_1(\alpha_2 + \varepsilon)$, $\pi_1 = \pi_{10}$, $\pi_{2\varepsilon} = \pi_2(\alpha_2 + \varepsilon)$, $\pi_2 = \pi_{20}$. It must be the case that $\pi_{2\varepsilon} < \pi_2$ and $0 < \pi_{1\varepsilon} \leq \pi_1$ for all $\varepsilon > 0$. The difference $\pi_2 - \pi_{2\varepsilon}$ is bounded below by some positive constant c. Furthermore, $(s(\alpha_2 + \varepsilon) - (\alpha_2 + \varepsilon))^2 > (x - \alpha_2 - \varepsilon)^2$ for all $\varepsilon > 0$ Otherwise, $\alpha_2 + \varepsilon$ could profitably deviate by mimicking α_2.

For all $0 < \varepsilon < \varepsilon^*$ set

$L(\varepsilon) = \pi_1 \left(y - k(x - \alpha_2 - \varepsilon)^2 + \pi_2 \delta y\right) -$

$\pi_{1\varepsilon} \left\{y - k\left[s(\alpha_2 + \varepsilon) - (\alpha_2 + \varepsilon)\right]^2 + \pi_{2\varepsilon}\delta y\right\}$

$L(\varepsilon)$ is the net loss or the net gain to type $\alpha_2 + \varepsilon$ from imitating type α_2. As it is an equilibrium $L(\varepsilon) \leq 0$ for all $\varepsilon > 0$.

$L(\varepsilon) \geq \pi_1 k \left[(s(\alpha_2 + \varepsilon) - (\alpha_2 + \varepsilon))^2 - (x - \alpha_2 - \varepsilon)^2\right] + \pi_{1\varepsilon^*}(\pi_2 - \pi_{2\varepsilon})\delta y \geq$

$\pi_1 k \left[(s(\alpha_2 + \varepsilon) - (\alpha_2 + \varepsilon))^2 - (x - \alpha_2 - \varepsilon)^2\right] + \pi_{1\varepsilon^*} c \delta y.$

$\inf_{\varepsilon > 0} \pi_1 k \left[(s(\alpha_2 + \varepsilon) - (\alpha_2 + \varepsilon))^2 - (x - \alpha_2 - \varepsilon)^2\right] = 0$, then for ε small enough $L(\varepsilon) > 0$, a contradiction.

Proof of Lemma 3 By contradiction, suppose that $s(\widehat{\alpha}) = \widehat{\alpha}$ for some $\widehat{\alpha} \in (\alpha_1, \alpha_2)$. Let $\varepsilon \geq 0$ and set $\pi_{1\varepsilon} = \pi_1(\widehat{\alpha} + \varepsilon,)$, $\pi_1 = \pi_{10}$, $\pi_{2\varepsilon} = \pi_{2R}(\widehat{\alpha} + \varepsilon)$ $\pi_2 = \pi_{20}$. As s is strictly increasing $\pi_{2\varepsilon} < \pi_2$ and $\pi_{1\varepsilon} \leq \pi_1$for all $\varepsilon > 0$. Let $L(\varepsilon)$ be the net loss or the net gain to type $\widehat{\alpha} + \varepsilon$ from imitating type $\widehat{\alpha}$. At equilibrium $L(\varepsilon) \leq 0$ for all $\varepsilon > 0$.

$L(\varepsilon) = \pi_1 \left(y - k\varepsilon^2 + \pi_2 \delta y\right) - \pi_{1\varepsilon} \left\{y - k\left[s(\widehat{\alpha} + \varepsilon) - (\widehat{\alpha} + \varepsilon)\right]^2 + \pi_{2\varepsilon}\delta y\right\}$

$L(\varepsilon) \geq \pi_1 \left(y - k\varepsilon^2 + \pi_2 \delta y\right) - \pi_{1\varepsilon}\left\{y + \pi_{2\varepsilon}\delta y\right\} \geq \pi_{1\varepsilon^*}\left[-k\varepsilon^2 + (\pi_2 - \pi_{2\varepsilon})\delta y\right]$, for some fixed $\varepsilon^* > 0$. $\pi_{1\varepsilon^*} > 0$ and $\pi_{2\varepsilon} = \pi(\widehat{\alpha} + \varepsilon, f(\cdot))$.

Set $B(\varepsilon) = \pi_{1\varepsilon^*}\left[-k\varepsilon^2 + (\pi_2 - \pi_{2\varepsilon})\delta y\right]$, then:

$\frac{dB(\varepsilon)}{d\varepsilon} = -\pi_{1\varepsilon^*}\left(2k\varepsilon + \frac{d\pi_{2\varepsilon}}{d\varepsilon}\right)$ and

$\frac{dB(\varepsilon)}{d\varepsilon}_{|\varepsilon=0} = -\pi_{1\varepsilon^*}\left(\frac{d\pi(\widehat{\alpha}+\varepsilon, f(\cdot))}{d\varepsilon}_{|\varepsilon=0}\right) > 0$ from Example 1.

As $B(0) = 0$, then $B(\varepsilon) > 0$ for ε small enough. But then type $\widehat{\alpha} + \varepsilon$ could profitably mimic type $\widehat{\alpha}$, for ε small enough, yielding a contradiction.

By contradiction, assume that $s(\alpha) > \alpha$. Consider type $\alpha' = s(\alpha) > \alpha$. By monotonicity $s(\alpha') > s(\alpha)$ because agents in $[\alpha_1, \alpha_2)$ separate. But then α' could profitably imitate α: the probabilities of election at both stages increase and she would not pay policy costs.

8.2 Section 5

Lemma 5 *Let f be a strictly negative C^1 functions defined on $[0, D] \times B$ where B is a real interval such that $[0, D] \subsetneq B \subset (-\infty, D]$. Then there exists a solution, defined on $[0, D]$, to the following ordinary differential equation problem:*

$$\begin{cases} y'(x)(y-x) = f(x, y(x)) \\ y(D) = D \\ y(x) \leq x \end{cases}$$

Furthermore, if there exists $\delta > 0$ *such that* $f_y(x,y) \geq 0$ *for every* $(x,y) \in \{(x,y) \in B : ||(x,y) - (D,D)|| < \delta,\ y < x,\}$*, then the solution is unique.*

Proof. The problem does not satisfy the local Lipschitz conditions in a neighborhood of D. The existence part of the Proof is by approximation. Let y_ε be the solution of the following Cauchy problem:

$$\begin{cases} y'(x)(y(x)-x) = f(x, y(x)) \\ y(D) = D - \varepsilon \end{cases}$$

Here the local existence and uniqueness theorem applies. In order to prove that $y_\varepsilon(x)$ can be extended to the interval $[0, D]$ it suffices to show that there exists no $x^* \in [0, D)$, such that $\lim_{x \to x^{*+}} y'_\varepsilon(x) = \infty$. In this case the extension theorem applies. First observe that if y_ε is defined and C^1in the interval $(x^*, D]$. From $y_\varepsilon(D) = D - \varepsilon$ and $y'_\varepsilon(x)(y_\varepsilon(x) - x) < 0$ it follows that $y'_\varepsilon(x) > 0$ and $y_\varepsilon(x) < x$ on $(x^*, D]$. If $\lim_{x \to x^{*+}} y'_\varepsilon(x) = \infty$, then $\lim_{x \to x^{*+}} y_\varepsilon(x) = x^*$. It follows that, for $\delta > 0$ small enough, $y'_\varepsilon(x) > 2$ on $(x^*, x^* + \delta]$. Let $0 < \delta' < \delta$. By the intermediate value theorem $y_\varepsilon(x^* + \delta) - (x^* + \delta) = y_\varepsilon(x^* + \delta') - (x^* + \delta') + y'_\varepsilon(x^* + \delta'')\left(\delta - \delta'\right)$ for some $\delta' < \delta'' < \delta$ but then $y_\varepsilon(x^* + \delta) - (x^* + \delta) > y_\varepsilon(x^* + \delta') - (x^* + \delta') + 2\left(\delta - \delta'\right)$. Let $\delta' \to 0$. From the previous observations it follows that the RHS converges to 2δ while the LHS is independent of $\delta' < \delta$. Then $y_\varepsilon(x^* + \delta) - (x^* + \delta) > 2\delta > 0$, which yields a contradiction. $y_\varepsilon(x)$ is C^1 with respect to ε on $[0, D)$ (Pontryagin (1962), ch. 23). $y_\varepsilon(D) \to D$ for $\varepsilon \to 0$. By contradiction, assume that, for some $x \in [0, D)$, $y_\varepsilon(x)$ is not converging for $\varepsilon \to 0$. In particular, for some $0 < \delta < D$, the Ascoli-Arzelá Theorem does not apply in $[0, D - \delta]$. The family $\{y_\varepsilon\}_{\varepsilon > 0}$ is uniformly bounded in $[0, D - \delta]$ (because $y_\varepsilon(x) \leq x$ on $[0, D - \delta]$). It must be the case that $\{y_\varepsilon\}_{\varepsilon > 0}$ it is not uniformly continuous then $\sup_{\varepsilon > 0} y'_\varepsilon = \infty$. As above, it follows that $y_\varepsilon(x) > x$ for some ε and some $x \in [0, D - \delta]$, a contradiction. So y_ε converges uniformly to some y in each interval $[0, D - \delta]$. Each y_ε satisfies $y'(x)(y(x) - x) = f(x, y(x))$, and y'_ε converges uniformly to some continuous z. Then $y' = z$. The local existence and uniqueness theorem implies that y is independent of the choice of δ. The function y is defined and differentiable on $[0, D)$ and satisfies $y'(x)(y(x) - x) = f(x, y)$ because each y_ε satisfies it. $y(x) < x$ on $[0, D)$ otherwise $y'(x) \to \infty$ for $x \to x^*$, some x^* against the uniform convergence of y'_ε. The existence part is proved by setting $y(D) = \lim_{x \to D} y(x) = 0$.

Now we prove uniqueness. Let f such that that, for some $\delta > 0$, $f_y(x,y) \geq 0$ for every $(x,y) \in \{(x,y) \in B : ||(x,y) - (D,D)|| < \delta,\ y < x,\}$. By contradiction, assume that y_1 and y_2 two different solutions of the problem. The local existence and uniqueness theorem implies that the graphs of the function cross only at (D, D). There is no loss of generality then in assuming that $y_1(x) < y_2(x)$ on

$[0, D)$. Then, for some δ small enough $y_1'(x) > y_2'(x)$ for all $x \in [D-\delta, D)$. For x next to D, we have $y_2'(x)(y_2(x)-x) \geq y_1'(x)(y_1(x)-x) = f(x, y_1(x)) \geq f(x, y_2(x))$ with at least one strict inequality. This yields a contradiction because y_2 solves the ODE problem. ■

From Lemma 2 follows that if some types $(\alpha', \alpha'') \subset (\alpha_i, \alpha_{i+1})$ are in the same policy pool then (α_i, α_{i+1}) is included in the same policy pool. So if (α_i, α) with $\alpha \leq \alpha_{i+1}$ are separating then types in (α_i, α_{i+1}) are all separating.

Lemma 6 *In a symmetric MD1 equilibrium*
(i) $s(0) = 0$
(ii) If types in (α_i, α_{i+1}) *are in the same policy pool then agents in* $(\alpha_{i+1}, \alpha_{i+2})$ *separate.*
(iii) If types in (α_i, α_{1+1}) *separate then* $\alpha_{i+1} = D$
(iv) $s(D) = D$

Proof. For $i = 1, ...,$. Set $\underline{s_i} = \lim_{\alpha \nearrow \alpha_i} s(\alpha)$ and set $\overline{s_i} = \lim_{\alpha \searrow \alpha_i} s(\alpha)$. Set $\underline{s_0} = 0$, $\overline{s_0} = s(0)$ $\underline{s_D} = \lim_{\alpha \nearrow D} s(\alpha)$ and set $\overline{s_D} = s(D) \leq D$. By definition $\underline{s_i} < \overline{s_i}$ for $i = 1, 2....$ and $\underline{s_0} \leq \overline{s_0}$, $\underline{s_D} \leq \overline{s_D}$. From equilibrium monotonicity it follows that for all $i = 1, 2...$, $x \in (\underline{s_i}, \overline{s_i})$ $T_1(\alpha, \beta, x, \pi) < 0$ if $\alpha > \alpha_i$ and $T_1(\alpha, \beta, x, \pi) > 0$ for $\alpha < \alpha_i$. Then from Condition (2) $\mu(\alpha_i \mid m(\beta), x) = 1$ because T increases as α approaches α_i. For $x \in (\underline{s_0}, \overline{s_0})$ and $\alpha > 0$, $T_1(\alpha, \beta, x, \pi) < 0$ so $\mu(\alpha_i \mid m(\beta), x) = 1$. Finally, for all $x \in (\underline{s_D}, \overline{s_D}) \cup (\overline{s_D}, D)$, $T_1(\alpha, \beta, x, \pi) > 0$ then for all $x \in (\underline{s_D}, \overline{s_D}) \cup (\overline{s_D}, D)$, $\mu(D \mid m(\beta), x) = 1$.
(i) By contradiction, suppose that $s(0) > 0$. From the result above, for ε small enough $\mu(0 \mid m(0), \varepsilon) = 1$. Then ε can profitably deviate by sending $(m(0), \varepsilon)$.
(ii) Let $s^* = s(\alpha)$ for all $\alpha \in (\alpha_i, \alpha_{i+1})$. If agents in $(\alpha_{i+1}, \alpha_{i+2})$ are pooling then, for ε small enough $\alpha_{i+1} + \varepsilon$ can profitably deviate by implementing policy $\overline{s_i} - \delta$, with δ small enough $\mu(\alpha_{i+1} \mid m(\alpha_{i+1} + \varepsilon), \overline{s_i} - \delta) = 1$. It is because, from the continuity of x on $(\alpha_{i+1}, \alpha_{i+2})$, the gain in second election probability is bounded below by a positive constant, while loss in policy term is of order δ^2.
(iii) If types in (α_i, α_{i+1}) are separating and types in $(\alpha_{i+1}, \alpha_{i+2})$ are pooling then type $\alpha_{i+1} + \varepsilon$ can profitably deviate by sending $(m(\alpha_{i+1} - \varepsilon), \overline{s_i} - \delta)$. $\mu(\alpha_{i+1} \mid (m(\alpha_{i+1} - \varepsilon), \overline{s_i} - \delta)) = 1$. For δ small enough, the loss in policy term is compensated by the gain in election probability. If types in (α, α_{i+1}) are pooling then, for ε small enough $\alpha_{i+1} + \varepsilon$ can profitably deviate by implementing policy $\overline{s_i} - \delta$, with δ small enough. $\mu(\alpha_{i+1} \mid m(\alpha_{i+1} + \varepsilon), \overline{s_i} - \delta) = 1$ The loss in policy term is of order δ^2, the gain in second election probability is bounded below by a positive constant.
(iv) If $s(D) < D$, for , $\mu(D \mid m(D), D) = 1$, because for all $x \in (s(D), D)$ the function T has a maximum between x and D. Then D can profitably deviate by implementing policy $D - \varepsilon$, with $\varepsilon < D - s(D)$. ■

Proof of Proposition 7 It suffices to show that $\alpha^* > 0$. By contradiction, assume that $\alpha^* = 0$. In this the equilibrium would have a monotonic electoral equilibrium with separating policies contradicting Proposition 4.

Proof of Theorem 8 We will always assume that, whenever beliefs are not imposed by Bayesian, consistency or by the MD1 refinement, if a candidate

announces a policy and implements a different one, then the median voter will not confirm her. This is consistent as we allow beliefs to be correlated. Let us consider the different possibilities.
(a) The first case is $\alpha^* = D$ so that the equilibrium is equivalent to an equilibrium in which all types are pooling together at 0 a and at both stages they are elected with probability $\frac{1}{2}$, and after the first election all pool on policy 0. The payoff for type α is $\frac{1}{2}\left[\left(1+\frac{\delta}{2}\right)y - k\alpha^2\right]$. This is an MD1 equilibrium if and only if $\frac{1}{2}\left[\left(1+\frac{\delta}{2}\right)y - kD^2\right] \geq \frac{1}{2}y$ and $\frac{\delta}{2}y - kD^2 \geq \delta y \pi_2(D)$, otherwise type D could profitably separate by implementing policy D (at the campaign and at the policy stage respectively [11]) which is as far as:

$$k \leq \min\left\{\frac{y\delta}{2D^2}, y\delta\left(\frac{1}{2}-\pi_2(D)\right)\right\} = y\delta\left(\frac{1}{2}-\pi_2(D)\right) = k_0$$

where $\pi_2(D)$ is the probability a candidate is elected at the second stage if perceived as type D and the other candidate is selected from F_L, which is with probability $\left[1 - G\left(\frac{1}{2}\frac{D^2-\int_0^D \beta^2 f(\beta)d\beta}{D+\int_0^D \beta f(\beta)d\beta}\right)\right] < \frac{1}{2}$, because of the symmetry of G. For $k < k_0$ a (continuous of) pooling equilibrium exists but it does not satisfies the MD1 criterion.
(b) The second case is that $\alpha^* < D$, and all types pool at the first stage. In such a case all types are elected with probability $\frac{1}{2}$ at the first election. At the second stage type $\alpha \in [0, \alpha^*]$ is elected with probability:

$$\pi_2([0,\alpha^*]) = [1 - G(e([0,\alpha^*], f(\cdot)))]$$

where:

$$e([0,\alpha^*], f(\cdot)) = \frac{1}{2}\frac{\int_0^{\alpha^*}\beta^2 f(\beta)\,d\beta - F(\alpha^*)\int_0^D \beta^2 f(\beta)\,d\beta}{\int_0^{\alpha^*}\beta f(\beta)\,d\beta + F(\alpha^*)\int_0^D \beta f(\beta)\,d\beta}.$$

$e([0,\alpha^*], f(\cdot))$ is the decisive voter when the types in $[0,\alpha^*]$ are pooling and matched to a challenger selected from the original distribution. Elementary analysis shows that $-D < e([0,\alpha^*], f(\cdot)) < D$ for $\alpha^* > 0$ and $e([0,\alpha], f(\cdot))$ it is strictly increasing. Furthermore, $\lim_{\alpha^*\to 0^+} e([0,\alpha^*], f(\cdot)) = -\frac{1}{2}\frac{\int_0^D \beta^2 f(\beta)d\beta}{\int_0^D \beta f(\beta)d\beta} \in [-D, 0]$. $\lim_{\alpha^*\to D^-} e([0,\alpha^*], f(\cdot)) = 0$. So $\pi_2(\alpha)$ is strictly decreasing and differentiable in α.
A type $\alpha \in (\alpha^*, D]$ is elected at the second stage with probability:

$$\pi_2(\alpha) = 1 - G(e(\alpha, f(\cdot)))$$

where:

$$e(\alpha, f(\cdot)) = \frac{1}{2}\frac{\alpha^2 - \int_0^D \beta^2 f(\beta)\,d\beta}{\alpha + \int_0^D \beta f(\beta)\,d\beta}.$$

[11] See the proof of Proposition 7 above.

We have $-D < e([0,\alpha^*], f(\cdot)) < e(\alpha, f(\cdot)) < D$. So $\pi_2(\alpha) < \pi_2([0,\alpha^*])$ for $\alpha > \alpha^*$. $e(\alpha, f(\cdot))$ is strictly increasing on $(\alpha^*, D]$. $\lim_{\alpha^* \to D^-} e(\alpha, f(\cdot)) = \frac{1}{2}\frac{D^2 - \int_0^D \beta^2 f(\beta) d\beta}{D + \int_0^D \beta f(\beta) d\beta} \in (0, D)$. So $\pi_2(\alpha)$ is strictly decreasing and continuously differentiable in α^*.
If $s(\alpha)$ is separating on $(\alpha^*, D]$, it must satisfy:

$$2ks'(\alpha))(s(\alpha) - \alpha) = \pi_2'(\alpha)\,\delta y$$

with the final condition $s(D) = D$. Furthermore, $\alpha^* = \alpha_1(k) > 0$ must be indifferent between separating and pooling, then:

$$\left(\frac{1}{2} + \delta\pi_2([0,\alpha^*])\right) y - k\alpha^{*2} = \left(\frac{1}{2} + \delta\pi_2(\alpha^*)\right) y - k\,(s(\alpha^*) - \alpha^*)^2 .$$

Set $H(\alpha) = \left(\frac{1}{2} + \delta\pi_2([0,\alpha])\right) y - k\alpha^2 - \left(\frac{1}{2} + \pi_2(\alpha)\,\delta\right) y + k\,(s(\alpha) - \alpha)^2$. $s(0) < 0$. So $H(0) > 0$. $H(D) = [\pi_2([0,D]) - \pi_2(D)]\,\delta y - kD^2 = \left[\frac{1}{2} - \pi_2(D)\right]\delta y - kD^2 \leq 0$ if $k \geq k_0$. $H'(\alpha) < 0 = \frac{d\pi_2([0,\alpha])}{d\alpha}\delta y - 2ks(\alpha) < 0$[12]. It is easily seen that $s(\alpha_1(k)) > 0$ because if $s(\alpha) = 0$ then $H(\alpha) > 0$.
Through implicit differentiation

$$\frac{dH(\alpha_1(k))}{dk} = H_\alpha(\alpha_1(k))\frac{d\alpha_1(k)}{dk} + H_k(\alpha_1(k)) = 0$$

so

$$\frac{d\alpha_1(k)}{dk} = \frac{-H_k(\alpha_1(k))}{H_\alpha(\alpha_1(k))} = -\frac{s^2(\alpha_1(k)) - 2\alpha_1(k)s(\alpha_1(k))}{\frac{d\pi_2([0,\alpha_1(k)])}{d\alpha}\delta y - 2ks(\alpha_1(k))} < 0.$$

Then $\alpha_1(k)$ is strictly decreasing in k.
From $H(\alpha_1(k)) = 0$ follows $\alpha_1(k) \to 0$ as $k \to \infty$.
(c) In the third case there are two campaign pools $[0,\alpha^*)$ and$(\alpha^*, D]$, with the second separating in policies. $[0,\alpha^*)$ types' election probabilities are

$$\overline{\pi}_1([0,\alpha^*]) = \frac{1}{2}F(\alpha^*) + (1 - F(\alpha^*))\,[1 - G(e([0,\alpha^*), (\alpha^*, D]))] .$$

and $\pi_2([0,\alpha^*))$, respectively, where $(e([0,\alpha^*), (\alpha^*, D]))$ is the decisive median voter of pool $[0,\alpha^*)$ against pool $(\alpha^*, D]$: her location is:

$$\frac{1}{2}\frac{(1 - F(\alpha^*))\int_0^{\alpha^*} \beta^2 f(\beta)\,d\beta - F(\alpha^*)\int_{\alpha^*}^D \beta^2 f(\beta)\,d\beta}{(1 - F(\alpha^*))\int_0^{\alpha^*} \beta^2 f(\beta)\,d\beta + F(\alpha^*)\int_{\alpha^*}^D \beta^2 f(\beta)\,d\beta}$$

which is, simplifying:

$$\frac{1}{2}\frac{\int_0^{\alpha^*} \beta^2 f(\beta)\,d\beta - F(\alpha^*)\int_0^D \beta^2 f(\beta)\,d\beta}{\int_0^{\alpha^*} \beta^2 f(\beta)\,d\beta + F(\alpha^*)\left[\int_{\alpha^*}^D \beta^2 f(\beta)\,d\beta - \int_0^{\alpha^*} \beta^2 f(\beta)\,d\beta\right]}.$$

[12] Because s solves the differential equation.

Observe that $G\left(e\left([0,\alpha^*),(\alpha^*,D]\right)\right) \leq \frac{1}{2}$, because $e\left([0,\alpha^*),(\alpha^*,D]\right) \leq 0$. $(\alpha^*,D]$'s election probabilities are:

$$\overline{\pi}_1\left((\alpha^*,D]\right) = \left(F(\alpha^*)\right)\left(1 - G\left(e\left((\alpha^*,D],[0,\alpha^*)\right)\right)\right) + \frac{1}{2}\left(1 - F(\alpha^*)\right)$$

and $\pi_2(\alpha)$, respectively, where:

$$e\left((\alpha^*,D],[0,\alpha^*)\right) = \frac{1}{2}\frac{F(\alpha^*)\int_{\alpha^*}^{D}\beta^2 f(\beta)\,d\beta - (1-F(\alpha^*))\int_0^{\alpha^*}\beta^2 f(\beta)\,d\beta}{F(\alpha^*)\int_{\alpha^*}^{D}\beta^2 f(\beta)\,d\beta + (1-F(\alpha^*))\int_0^{\alpha^*}\beta^2 f(\beta)\,d\beta}$$

From the symmetry of the distribution G, $G\left(e\left([0,\alpha^*),(\alpha^*,D]\right)\right) = 1 - G\left(e\left((\alpha^*,D],[0,\alpha^*)\right)\right) = 1 - G \leq \frac{1}{2}$. so:

$$\overline{\pi}_1\left((\alpha^*,D]\right) = \left(F(\alpha^*)\right)\left(G\left(e\left([0,\alpha^*),(\alpha^*,D]\right)\right)\right) + \frac{1}{2}\left(1 - F(\alpha^*)\right)$$

and $\overline{\pi}_1\left([0,\alpha^*]\right) = \overline{\pi}_1\left((\alpha^*,D]\right) + \frac{1}{2} - G\left(e\left((\alpha^*,D],[0,\alpha^*)\right)\right) \geq \overline{\pi}_1\left((\alpha^*,D]\right)$.
As above, on $(\alpha^*,D]$ s must satisfy:

$$2ks'(\alpha))\left(s(\alpha)-\alpha\right) = \pi_2'(\alpha)\,\delta y$$

and α^* must satisfy:
$\overline{\pi}_1[0,\alpha^*]\left[1 + \pi_2\left([0,\alpha^*]\right)\delta y - k\alpha^{*2}\right] =$
$\overline{\pi}_1\left((\alpha^*,D]\right)\left[1 + \pi_2(\alpha^*)\,\delta y - k\left(s(\alpha^*)-\alpha^*\right)^2\right]$
Set $H(\alpha,k) = \left\{\left(\overline{\pi}_1\left([0,\alpha^*]\right) + \delta\pi_2\left([0,\alpha^*]\right)\right)y - k\alpha^{*2}\right\} -$
$-\left\{\left(\overline{\pi}_1\left((\alpha^*,D]\right) + \pi_2(\alpha^*)\,\delta\right)y - k\left(s(\alpha^*)-\alpha^*\right)^2\right\}$
As above, it can be shown, that a unique solution to $H(\alpha_2(k),k) = 0$ exists if and only if $H(D) > 0$ which is if and only if $k \geq k_1^* > k_0$ where $H(D,k_1^*) = 0$. $\alpha_2(k)$ is strictly decreasing and $\alpha_2(k) \to 0$ as $k \to \infty$.
It must be checked that type D does not want to imitate type $\alpha_2(k)$ in the campaign and then implement D, which is
$\overline{\pi}_1[0,\alpha_2(k)]\,y \leq \overline{\pi}_1\left((\alpha_2(k),D]\right)\left[(1+\pi_2(D))\,\delta y\right]$, or, equivalently $\frac{1}{2} - G - \overline{\pi}_1(\alpha_2(k),D)\,\pi_2(D)\,\delta \leq 0$ but $\overline{\pi}_1(\alpha_2(k),D) = -F(\alpha_2(k))\left(\frac{1}{2}-G\right) + \frac{1}{2}$
Then the condition is $\left(\frac{1}{2}-G\right)\left(1 + F(\alpha_2(k))\,\pi_2(D)\,\delta\right) \leq \frac{\pi_2(D)\delta}{2}$.
Consider the function $R(\alpha) = \left(\frac{1}{2} - G\left(e\left([0,\alpha),(\alpha,D]\right)\right)\right)\left(1 + F(\alpha)\,\pi_2(D)\,\delta\right) - \frac{\pi_2(D)\delta}{2}$. $R(0) = -\frac{\pi_2(D)\delta}{2} < 0$, $R' > 0$. As $\alpha_2(k) \searrow 0$ as $k \to \infty$, there exists a unique $k^* > 0$ such that this kind of equilibrium exists only for $k \geq k^*$. Set $k_1 = \max\{k^*, k_1^*\}$.
(d) The last possible case is that agents in $[0,\alpha^*]$ pool in campaign and policy, and agents in $(\alpha,D]$ separate in campaign and in message. Then the probability of first stage election of types in in $[0,\alpha^*]$ is:

$$\pi_1([0,\alpha^*]) = \frac{1}{2}F(\alpha^*) + \int_{\alpha^*}^{D}\left[1 - G\left(e\left([0,\alpha^*],\beta\right)\right)\right]f(\beta)\,d\beta$$

where, for all $\alpha > \alpha^*$:

$$e\left(\left[0,\alpha^*\right],\alpha\right) = \frac{1}{2}\frac{\int_0^{\alpha^*}\beta^2 f\left(\beta\right)d\beta - F(\alpha^*)\alpha^2}{\int_0^{\alpha^*}\beta^2 f\left(\beta\right)d\beta + F(\alpha^*)\alpha}.$$

$e\left(\left[0,\alpha^*\right],\alpha\right) < 0$ as $\alpha > \alpha^*$. So $G\left(e\left(\left[0,\alpha^*\right],\alpha\right)\right) < \frac{1}{2}$.
If $\alpha^* < \alpha < D$ then is separating in campaign D is elected with probability:

$$\pi_1(\alpha) = F(\alpha^*)\left(1 - G\left(e\left(\alpha,\left[0,\alpha^*\right]\right)\right)\right) + \frac{1}{2}\int_{\alpha^*}^{D}\left[1 - G\left(\frac{\beta+\alpha}{2}\right)\right] f\left(\beta\right)d\beta$$

where

$$e\left(\alpha,\left[0,\alpha^*\right]\right) = \frac{1}{2}\frac{F(\alpha^*)\alpha^2 - \int_0^{\alpha^*}\beta^2 f\left(\beta\right)d\beta}{\int_0^{\alpha^*}\beta^2 f\left(\beta\right)d\beta + F(\alpha^*)\alpha}.$$

So $G\left(e\left(\alpha,\left[0,\alpha^*\right]\right)\right) = \frac{1}{2} + G\left(e\left(\left[0,\alpha^*\right],\right)\right)$
We can write

$$\pi_1(\left[0,\alpha^*\right]) = \frac{1}{2}F(\alpha^*) + \int_{\alpha^*}^{D}\left[\frac{3}{2} - G\left(\beta, e\left(\left[0,\alpha^*\right]\right)\right)\right] f\left(\beta\right)d\beta$$

which is:

$$\frac{3}{2} - F(\alpha^*) - \int_{\alpha^*}^{D}\left[G\left(\beta, e\left(\left[0,\alpha^*\right]\right)\right)\right] f\left(\beta\right)d\beta.$$

Now s must satisfy the differential equation:
$\pi_1'(\alpha)\left[y - k\left(s(\alpha)-\alpha\right)^2 + \pi_2\left(\alpha\right)\delta y\right] +$
$\pi_1(\alpha))\left[-2ks'(\alpha)\right)\left(s(\alpha)-\alpha\right) + \pi_2'\left(\alpha\right)\delta y] = 0$
And $\alpha^* = \alpha_3\left(k\right)$ has to satisfy $\pi_1\left(\left[0,\alpha^*\right]\right)\left[1 + \pi_2\left(\left[0,\alpha^*\right]\right)\delta y - k\alpha^{*2}\right] =$
$\pi_1\left(\alpha^*\right)\left[1 + \pi_2\left(\alpha^*\right)\delta y - k\left(s(\alpha^*) - \alpha^*\right)^2\right]$
Exactly as above one can prove the existence and uniqueness of $\alpha_3\left(k\right)$ and of k_2 such that the strategies are an equilibrium iff $k \geq k_2$.