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Abstract

This paper characterizes optimal intercorporate guarantees, under the classical trade-off between bankruptcy costs and taxation. Conditional guarantees, allowing the guarantor - or Holding company - to maintain limited liability vis-à-vis the beneficiary - or Subsidiary - maximize joint value. They indeed achieve the highest tax savings net of default costs. We provide conditions ensuring that - at the optimum - guarantees increase total debt, which bears mostly on the Subsidiary. This difference in optimal leverage between Holding company and Subsidiary explains why optimal conditional guarantees (i) generate value independently of cash flow correlation (ii) are unilateral rather than mutual, at least for moderate default costs (iii) dominate the unconditional ones, that are embedded in mergers, at least when firms have high cash-flow correlation. We also endogenize the choice of the guarantor, showing that it has higher proportional bankruptcy costs, lower tax rates and bigger size.

Keywords: debt, taxes, bankruptcy costs, limited liability, capital structure, subsidiary, groups, mergers.

JEL classification numbers: G32, G34, L22

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1 Introduction

Corporations routinely guarantee the debt obligations of their subsidiaries[1]. Despite being so common, it is not clear why such guarantees exist: while providing gains to the Subsidiary (S), they generate a cost for the guaranteeing Holding company (H) so that net value creation is expected to be zero. At most one expects them to increase joint value to the extent that firm cash flows are less than perfectly correlated, because of diversification gains. Even in this case, it remains unclear whether any given firm should both provide and receive support, or specialize in either being a guarantor or a beneficiary. Another set of issues emerges if specialization obtains: which are the characteristics of the firm providing support relative to that receiving it? And how much debt should each one raise?

This paper analyzes intercorporate guarantees, that to our knowledge have not been systematically addressed before, simply assuming - as in Leland (2007) - that debt provides a tax shield but increases the likelihood of bankruptcy, absent any information or incentive problems. Such absence allows to deal with guarantees that are credible from the lenders' point of view, for instance because they are enforceable by law. Our main focus will be on conditional guarantees, that allow the supporting Holding company to retain limited liability with respect to the supported Subsidiary's lenders. Corporate limited liability is indeed the norm in major jurisdictions, according to the legal literature[2]. Also in practice, companies tend to terminate support to a struggling subsidiary when its needs are large with respect to group equity or when group profits turns negative ((Dewaelheyns and Van Hulle (2006); Gopalan et al. (2007); Emery and Cantor (2005)).[3]

Our main result is that such guarantees increase the joint value of H and S. Indeed, the Subsidiary is able to increase its debt financing and, as a consequence, tax savings net of default costs. Numerical simulations show that S's tax savings can be four times as large as in the base case of no guarantees, while its leverage can reach 99%, as in the seminal Modigliani and Miller analysis without bankruptcy costs (Miller and Modigliani (1963)). Thus, a guarantee has the effect of boosting tax avoidance, net of default costs. In more detail, we provide analytical conditions for S's debt to be higher than total debt of the two firms in the absence of any guarantee, and for H to be unlevered - so as to increase its ability of providing support.

[1] See Bodie and Merton (1992) for US firms and Chang and Hong (2000) for Korean chaebols. Deloof and Vershueren (2006) analyze contractual guarantees in Belgian groups, while Khanna and Palepu (2000) and Gopalan et al. (2007) observe that Indian group firms assist each other in times of financial distress. Standard & Poor's (2001) discusses how contractual or implicit guarantees affect credit ratings.

[2] See Hadden (1996) on Britain, France, Germany and the US, and Hill (1996) on Australia. This was already the case in US groups in the early twentieth century (Blumberg (1989)). The exception to this rule is the "piercing of the corporate veil" that requires to prove both the lack of separate existence of the subsidiary and parent company's conduct "akin to fraud".

[3] Herring and Carmassi (2009) discuss recent instances when financial intermediaries walked away from insolvent subsidiaries.

The guarantee described until now is unilateral, while a priori support could be mutual. We show that any given firm specializes in either providing or receiving support, at least for moderate proportional default costs. The intuition is that unilateral guarantees permit to save on both default costs and tax payments, thanks to both rescue and a different leverage in the Holding company and its Subsidiary. Mutual guarantees still save on default costs, but create a tension because each firm should at the same time increase its debt - since it is guaranteed - and decrease it - in its quality of guarantor. This tension results in lower total debt and tax savings than with unilateral guarantees. If default costs are moderate, tax incentives - and the asymmetric leverage which optimally exploits them thanks to unilateral guarantees - prevail.

The above conclusions on the properties of guarantees hold when Holding company and Subsidiaries differ in default costs, tax rates and size as well as when they do not. Our next set of results concerns the characterization of H and S when they do differ. The guarantor H should be the firm with higher bankruptcy costs (lower tax rates), because its optimal debt - and hence bankruptcy probability (foregone tax savings) - is lower. It should also be the bigger one, as it is likelier to have larger cash flows available for rescue, reducing S's expected default costs relative to the opposite arrangement.

So far we have been discussing guarantees between two separately incorporated activities. These, we will argue in section 6, are common in business groups and private equity funds and provide a rationale for their diffusion. In the real world we also observe conglomerate Mergers (M), in which activities' cash flows are pooled so that they become jointly liable vis-à-vis lenders. We think of this as a case of an unconditional guarantee being provided by each division to the other. Abstracting from any operational synergy, Leland (2007) shows that a purely financial Merger destroys value in a number of situations, when the loss of limited liability exceeds gains from higher optimal debt - for instance when the high correlation between activities' cash flows limits diversification opportunities. Below we show that conditional guarantees - of the HS type - are value-enhancing relative to the M case in such situations. This result highlights that, contrary to intuition, conditional guarantees work even when cash flow diversification is limited, to the extent that the Holding company is free to have lower debt and therefore higher cash-flows available for supporting its levered Subsidiary.

Last but not least, one may wonder whether these results have some real-world counterparts. Since our theoretical findings are broadly consistent with stylized observations on guarantees, leverage and firm scope, we argue that value creation due to intercorporate guarantees is able to explain the diffusion of Holding-Subsidiary structures.

The rest of the paper is organized as follows. Section 2 clarifies our contribution relative to previous theoretical literature. Section 3 lays out the model. Section 4 analyzes how value and optimal debt change due to guarantees, providing the main results summarized above. Section 5 presents numerical simulations so as to appreciate the magnitude of the effects on debt, value, tax savings and default costs. In section 6, we compare our findings with a number of related empirical observations. Section 7 concludes.

2 Previous literature

Our paper is closely related to the literature on non-synergistic firm combinations. Lewellen (1971) argues that merging imperfectly correlated Stand Alone (SA) activities has a coinsurance effect that, by reducing the risk of default, increases debt capacity, defined as the amount that lenders are willing to offer. He then conjectures that this always leads to greater optimal debt, greater tax savings, and value for the merged firms. We argue that an increase in leverage and tax savings obtains with an intercorporate conditional guarantee, as well. We further show that this generates value irrespective of coinsurance gains, and we provide analytical conditions ensuring greater optimal debt of HS relative to the non-guaranteed SA case.

Leland (2007) constructs a structural model that is able to determine optimal debt and the value of liabilities for Stand Alone firms and the Merger. He completes Lewellen's argument by splitting the change in value from a Merger into a pure limited-liability component and a leverage effect. The former is non-positive and independent of leverage: it is the loss in limited liability pointed out by Scott (1977) and Sarig (1985), that derives from the possibility of negative operational cash flows. The latter is the sum of tax and default cost changes associated with optimal leverage, and is positive only when there are coinsurance gains. As a consequence, Leland observes that Mergers may reduce value and provides conditions on Gaussian cash-flows distribution ensuring that activities either benefit or loose from a Merger. The case in which M is beneficial for any correlation lower than one is very special: the volatilities of the merging firms must be identical and moderate. When volatilities differ, or are identical but large, Mergers are undesirable at high correlations.

We embed the Merger case into a more general structural model, where cash flows have any (even non-Gaussian) distribution function and firm combinations encompass the HS case too. For this model we get analytical results on value enhancement with respect to both the Stand Alone and the Merger case. We first show that a conditional guarantee never destroys value. This is because the Holding company can commit to support its insolvent Subsidiary conditional on its own survival, thanks to corporate limited liability. Secondly, and consequently, we argue that a conditional guarantee is worth introducing when the unconditional guarantee deriving from a Merger is wasteful. Thirdly, we provide analytical results for larger optimal HS leverage - and larger value gains due to lower taxes net of default costs - as opposed to numerical computations in previous work.

The relevance of limited liability links our study also to previous work emphasizing this trait of business groups (Cestone and Fumagalli (2005) and Bianco and Nicodano (2006)). Both papers focus on incentive issues, instead of a tax-bankruptcy trade-off, and posit exogenous debt needs. In the first paper the benefit of groups stems from improved managerial effort that impacts on competition in the output market. The second paper characterizes the fraction of exogenous group debt to raise through a Subsidiary, when this stimulates risk shifting. Thus, neither paper uncovers the first order effects on debt, tax savings and ultimately firm value that lie at the heart of the current research.

Our reliance on tax benefits connects our paper to a large literature on tax avoidance and corporate finance (see the survey in Graham (2003)), which focusses on arbitrage in unequal tax rates. For instance, multinational groups raise more debt from subsidiaries in high-tax countries (Huizinga et al. (2008)). In our model, guarantees minimize the tax burden - net of default costs - even with equal tax rates across firms. Thus, we point out a powerful tax avoidance tool which, to our knowledge, has not been analyzed yet.

Last but not least, our paper owes to Merton (1977), who recognizes that the provision of a guarantee for all the debt of a company - with no corporate limited liability - is akin to the issue of a put option on that company assets, and prices it accordingly. We observe that a conditional guarantee is akin to an option on the Subsidiary's cash flows. We price it accordingly, taking into consideration its effects on optimal capital structure.

3 The model

We consider a no arbitrage environment with two dates $t = \{0, T\}$, in which every payoff is evaluated according to its expected discounted value, under the risk neutral measure. This basic set up is drawn from Leland (2007).

An entrepreneur owns two production units[4] Each activity i ($i = 1, 2$) generates a random operating cash flow X_i at time T. X_i is a continuous random variable, endowed with the first two moments ($X_i \in L^2$), that may take both negative and positive values.[5] We also assume that the joint density of the cash flows X_i - denoted as $f(x, y)$ - exists and is positive on the whole plane[6].The owner can "walk away" from negative cash flows thanks to personal limited liability. The unlevered firm value is $V_{0i} = (1 - \tau)\phi\mathbb{E}X_i^+$, where X_i^+ is the positive part of X_i, ϕ is the discount factor, $\phi \triangleq (1 + r_T)^{-1}$, and r_T is the riskless rate for the time span T.

At time zero the entrepreneur can lever up each firm by issuing a zero-coupon debt. Let the debt principal be $P_i \geq 0$. Assume it is due, with absolute priority, at T. The value at time 0 of such debt, D_{0i}, is cashed-in by the entrepreneur at issuance. We assume that there is an incentive to issue debt, as interest is a deductible expense. The promised interest payment is equal to:

$$P_i - D_{0i} \tag{1}$$

Taxable income is the operating one net of interests, $X_i - (P_i - D_{0i})$, when

[4] Please notice that, at $t > 0$, the entrepreneur, after having raised debt so as to maximize firm value, can also sell part of his own equity .

[5] Denote as F_i its distribution function; this means $0 < F_i(0) < 1$.

[6] This rules out the cases of linear correlation exactly equal to -1 and 1, which can be treated numerically.

positive. The zero-tax level of cash flow, X_i^Z, is therefore equal to:

$$X_i^Z = P_i - D_{0i} \tag{2}$$

A positive tax rate $\tau > 0$ applies when $X_i > X_i^Z$. Operating cash flows, net of tax payments, are $X_i^+ - \tau(X_i - X_i^Z)^+$, which means

$$X_i^n = \begin{cases} 0 & X_i < 0 \\ X_i & 0 < X_i < X_i^Z \\ X_i(1-\tau) + \tau X_i^Z & X_i > X_i^Z \end{cases} \tag{3}$$

However, issuing debt has costs as well. Similarly to Merton (1974), default occurs when net operating cash flow is smaller than the principal, namely $X_i^n < P_i$. The default triggering condition can be restated as $X_i < X_i^d$, where the default threshold X_i^d is defined as:

$$X_i^d = P_i + \frac{\tau}{1-\tau} D_{0i} = \frac{P_i - \tau X_i^Z}{1-\tau} \tag{4}$$

In the event of default, we assume that a fraction $\alpha > 0$ of operating cash flows is lost. Bondholders will receive a fraction $0 < 1 - \alpha < 1$ of operating cash flow, X_i, when this is positive. They must pay taxes when $X_i > X_i^Z$. There is then a trade-off between the dissipative default costs, αX_i, and the tax savings possibly generated by debt. As customary in the related literature, we are assuming that cash flows are exogenous, that the firm receives no tax refunds when they are negative[7] and that bankruptcy costs as proportional to cash-flows.

The entrepreneur chooses the non-negative face value of debt, P_i, in the two activities, given this tax-bankruptcy cost trade-off, so as to maximize the time-zero combined value of the two units. He can also allow one or both units to guarantee the lenders of the other activity.

The combined value is given by the sum of the two firms' equities and debts, determined as the present value of the future expected payoffs to equity and bond holders, respectively:

$$\sum_{i=1}^{2} \nu_{0i} \triangleq \sum_{i=1}^{2} E_{0i} + D_{0i} = \sum_{i=1}^{2} \phi \mathbb{E}[E_i + D_i] \tag{5}$$

where $E_i + D_i$ are the payoffs at time $T, \mathbb{E}$ is the risk-neutral expectation operator. Tedious algebra shows that, for each and every guarantee, the levered values ν_{0i} coincide with the value of the unlevered firm, V_{0i}, plus tax savings from interest deduction, TS_i, less the present value of default costs, DC_i[8]:

[7] In the real world, companies may carry forward some losses, in order to reduce the asymmetric nature of taxation - which however remains substantial.

[8] In the absence of guarantees, the proof is given in Leland (2007). In the presence of them, it can be given only once appropriate expressions for DCs are introduced (see later sections).

$$\nu_{0i} = V_{0i} + TS_i - DC_i \tag{6}$$

Tax savings TS_i are equal to the discounted differential tax burden of the unlevered and the levered firm, which are denoted as $T_i(0)$ and T_i respectively:

$$TS_i \triangleq T_i(0) - T_i = \tau_i \phi \left[\mathbb{E}X_i^+ - \mathbb{E}(X_i - X_i^Z)^+ \right] \tag{7}$$

Given these definitions, levered firm value can be written as

$$\nu_{0i} = V_{0i} + T_i(0) - T_i - DC_i = \phi \mathbb{E}X_i^+ - T_i - DC_i \tag{8}$$

Since both unlevered firm value and its tax burden are independent of leverage, the value maximization problem coincides with that of minimizing the tax burden plus default costs, through an appropriate choice of the principals $P_i \geq 0$, for any guarantee:

$$\min_{P_i} \sum_{i=1}^{2} (T_i + DC_i) \tag{9}$$

Two key points for understanding this problem - and how it differs across guarantees - are the following. First, for each firm, the tax burden has the same expression, namely $T_i = \mathbb{E}(X_i - X_i^Z)^+$, independently of the guarantees. However, it has not the same value, for given principal: T_i depends on the tax shield, $X^Z = P - D_0$, which in turn depends on the market value of debt, D_0. The latter is determined as the present value of its payoffs D_i, which are affected by the guarantees. As a consequence, for any given principal, T_i differs across guarantees because D_0 does. Second, default costs DC_i have a different expression across guarantees because the provision of support affects them. On top of that, costs differ across guarantees because, similarly to taxes, they depend on the thresholds X^d and X^Z. The latter depend on D_0, which has different expressions across guarantees. So, in spite of the common formalization of firm value, (9), its dependence on the principals is different across guarantees. Below we will outline such dependence by working out the debt holders' payoff and default costs over guarantee types.

3.1 No guarantees

When there are no guarantees, firms coincide with the Stand Alone activities in Leland (2007). In this case the cash flows to lenders at time T are equal to[9]:

$$D_i(P_i) = \begin{cases} (1-\alpha)X_i & 0 < X_i < X_i^Z \\ (1-\alpha)X_i - \tau(X_i - X_i^Z) & X_i^Z < X_i < X_i^d \\ P_i & X_i > X_i^d \end{cases} \tag{10}$$

[9] The payoff to equity holders is: $E_i(P_i) = (X_i^n - P_i)^+$. Its present discounted value E_{0i} is a call option with underlying X_i^n and exercise price P_i. Debt instead is a portfolio of plain vanilla puts and the present value of the principal.

When firm i is insolvent, lenders receive cash flows net of bankruptcy costs, $(1-\alpha)X_i$, if gross cash flows are positive but lower than the tax shield, i.e. $X_i < X_i^Z$. When cash flows exceed the tax shield but fall short the default threshold, that is $X_i^Z < X_i < X_i^d$, the government has priority for tax payments, and debtholders also bear a tax liability, $\tau(X_i - X_i^Z)$. Lenders receive reimbursement equal to P_i when the firm is solvent, i.e. $X_i > X_i^d$. These inflows make D_i defined implicitly, since the zero-tax and default thresholds X_i^Z and X_i^d depend on its present value. Fixed points arguments apply to both the theoretical and numerical characterization of the latter.

Without external support the firm defaults as soon as its gross cash flows fall short the default threshold, $X_i < X_i^d$, its default costs are equal to:

$$DC_i(P_i) = \alpha\phi\mathbb{E}\left[X_i \; \mathbf{1}_{\left\{0<X_i<X_i^d\right\}}\right] \tag{11}$$

where $1_{\{\bullet\}}$ is the indicator function.

Appendix A proves that the market value of debt increases less than proportionally with respect to its face value, and that both the tax shield and the default threshold also increase in the face value of debt, as intuition suggests. As a consequence, default costs increase in the face value of debt, because the set of default states gets larger. As for the tax burden, it is decreasing in both the tax shield and the face value of debt. The latter enlarges interest deductions and the associated tax shield, increasing default costs but alleviating the tax burden. In other words, the tax burden is a call option on X_i with strike X_i^Z. The call is decreasing in the principal, since the strike is increasing in it. Default costs are a barrier call option on X_i with zero strike and barriers equal to zero and X_i^d. The call is increasing in the principal, since the upper barrier is increasing in it.

Appendix A studies the ensuing trade-off and proves that, under the standard convexity assumptions on the objective of minimization, the Stand Alone firm is always levered. If we denote its optimal debt as P_i^*, we have:

Lemma 3.1 *If the tax burden and default costs $T_i + DC_i$ are convex in the principal P_i, the Stand Alone company is optimally levered: $P_i^* > 0, i = 1, 2$.*

3.2 Conditional guarantee

This section models a guarantee between a Holding company and its Subsidiary, studies its properties and assesses their effects on lenders' payoffs. For the sake of comparison, we posit that cash flows are equal to the Stand Alone case, i.e. that $X_H = X_1$; $X_S = X_2$.

We start from unilateral guarantees, postponing the case of mutual guarantees. The key feature of these guarantees is corporate limited liability, as we argued in the introduction. This implies that the Holding company provides support if two conditions hold. First, the cash flows of the defaulting company are non negative, else the guarantor would bear an operating loss that it can avoid by using its limited liability:

$$0 < X_S < X_S^d \tag{12}$$

Second, joint cash flows are sufficient to honour all debt obligations:

$$X_H^n - P_H > P_S - X_S^n \tag{13}$$

The previous condition can be written as $X_H > h(X_S)$, where

$$h(X_S) = \begin{cases} X_H^d + \frac{P_S}{1-\tau} - \frac{X_S}{1-\tau} & X_S < X_S^Z \\ X_H^d + X_S^d - X_S & X_S > X_S^Z \end{cases} \tag{14}$$

The transfer from H to S, associated with a conditional guarantee, is equal to:

$$(P_S - X_S^n)\mathbf{1}_{\{0<X_S<X_S^d, X_H>h(X_S)\}} \tag{15}$$

where the term in curly brackets represents the "rescue area", i.e. the cash-flow combinations which lead to rescue.

We now determine how the transfer changes both the payoffs to the lenders and the default costs with respect to the Stand Alone case.

The final payoffs to the Subsidiary lenders become:

$$D_S(P_H, P_S) = \begin{bmatrix} X_S(1-\alpha)+ \\ +\tau(X_S - X_S^Z)\mathbf{1}_{\{X_S>X_S^Z\}} \end{bmatrix} \mathbf{1}_{\{0<X_S<X_S^d, X_H<h(X_S)\}} + \tag{16}$$

$$+P_S\left[\mathbf{1}_{\{X_S>X_S^d\}} + \mathbf{1}_{\{0<X_S<X_S^d,\ X_H>h(X_S)\}}\right]$$

The first square bracket refers to the case when the Subsidiary defaults and the Holding company does not support it because its own cash flow is insufficient. In this situation, lenders have to pay taxes only if cash flows exceed the tax shield. The second square bracket takes into account that the Subsidiary is able to reimburse its debt either when it is solvent on its own or thanks to the Holding company transfer.For fixed principals, the payoff to the Holding company's lenders does not change with respect to the Stand Alone case, as it provides support to the Subsidiary only after the service of its own debt. Thus H's debt is unaffected: $D_H(P_H) = D_1(P_H)$. [10]

Moreover, H default costs are unaffected as it provides support only if solvent. On the contrary, the guarantee reduces the Subsidiary's default costs - for fixed principals. These are equal to:

$$DC_S(P_H, P_S) = \alpha\phi\mathbb{E}\left[X_s\mathbf{1}_{\ \{0<X_s<X_s^d,\ X_H<h(X_S)\}}\right] \tag{18}$$

[10] So is the value of H equity. The payoff to S shareholders instead becomes:

$$\left[(X_S^n - P_S)^+ - (P_H - X_H^n)1_{\{0<X_H<X_H^d, X_S>h(X_H)\}}\right] \tag{17}$$

For any debt level of H and S, denote with Γ the savings in (expected, discounted) default costs of the Subsidiary with respect to the no-guarantee case:

$$\Gamma(P_H, P_S) \triangleq DC_2 - DC_S = \alpha\phi\mathbb{E}\left[X_s\mathbf{1}_{\{0<X_s<X_s^d,\ X_H>h(X_S)\}}\right] \tag{19}$$

These savings are akin to an option on the Subsidiary's cash flows. It is non-negative and becomes strictly positive as soon as $P_S > 0$. Appendix B establishes the other properties of Γ. It is non-increasing in the Holding company debt. Indeed, for any joint cash flow distribution and any capital structure, reducing debt in the Holding company enlarges - or at least does not shrink - its ability to provide support, and consequently default costs of the Subsidiary. The effect associated with changes in Subsidiary debt is less obvious. Raising P_S contributes to such savings by increasing the value of S cash flows that are saved when H succeeds in rescuing it. On the other hand, raising P_S reduces savings by making it less likely that H cash flows will be sufficient to service S debt. When debt in the Subsidiary diverges, the second effect dominates and the marginal value of savings is negative.

Appendix B proves that - as a consequence of the properties of Γ - the optimal principal of the Subsidiary is positive, exactly as in the no-guarantee case.

Lemma 3.2 *If $T_H + DC_H + T_S + DC_S$ is convex in the principals P_H, P_S, the Subsidiary is optimally levered: $P_S^* > 0$.*

Remark 3.1 *From the lemma, which entails $X_S^d > 0$, together with $f(x, y) > 0$, it follows that the rescue area is non-empty and the probability associated to rescue is non-null.*

So far we have been addressing the case of only one firm supporting the other. It is however possible for the entrepreneur to establish a mutual, but still conditional, guarantee. This is composed of a guarantee from H to its S - as above - and by a guarantee from S to its H - which is triggered by the fact that $0 < X_H < X_H^d,\ X_S > h(X_H)$. Default costs of both firms are now reduced with respect to the Stand-Alone case. They coincide with those obtained with unilateral guarantee for the Subsidiary; for the Holding company they have a symmetric expression. Denote as Γ_m the overall savings in (expected, discounted) default costs which a mutual conditional guarantee provides with respect to no-guarantee at all, for given principals:

$$\Gamma_m(P_H, P_S) = \alpha\phi\mathbb{E}\left[X_S 1_{\left\{0<X_S<X_S^d, X_H>h(X_S)\right\}} + X_H 1_{\left\{0<X_H<X_H^d, X_S>h(X_H)\right\}}\right] \tag{20}$$

One can recognize that they can be split into the values of the corresponding unilateral savings (from H to S and viceversa). Name the two parts $\Gamma_{12} \triangleq DC_S - DC_2 = \Gamma$ and $\Gamma_{21} \triangleq DC_H - DC_1$.

The expression for S debt - $D_S(P_H, P_S)$ - is unchanged with respect to the unilateral guarantee case described above. Debt of the Holding company

- denoted as $D_H^{\Gamma_m}(P_H, P_S)$ - differs instead because now the Holding company receives a guarantee too.

$$D_H^{\Gamma_m}(P_H, P_S) = \left[\begin{array}{c} X_H(1-\alpha)+ \\ +\tau(X_H - X_H^Z)\mathbf{1}_{\{X_H > X_H^Z\}} \end{array} \right] 1_{\{0<X_H<X_H^d, X_S<h(X_H)\}} + \\ +P_S \left[\mathbf{1}_{\{0<X_H<X_H^d,\ X_S>h(X_H)\}} + \mathbf{1}_{\{X_H>X_H^d\}} \right] \tag{21}$$

3.3 Unconditional guarantees

The entrepreneur may also decide that both firms will transfer their cash flows unconditionally to each other, i.e. that they will be jointly liable vis-à-vis all lenders. We consider the Merger (M) case in Leland (2007) as the case of unconditional guarantees.

The Merger value obtains when substituting into equation (6) both the cash flow $X_M = X_1 + X_2$ and the debt $P_M \geqslant 0$. The expressions for both lenders' payoffs and default costs similarly obtain from equations (10) and (11). Thus, we can replicate the proof in Lemma 3.1 to demonstrate that the optimal Merger debt is always positive, $P_M^* > 0$, under the usual convexity assumption.

4 Main results: guarantees and debt

4.1 Conditional guarantees versus no guarantee

This section examines whether and how conditional guarantees create value with respect no guarantees, starting with unilateral ones.

We do establish that such guarantees create value, absent any assumption on the correlation of the firm's cash flow, and we split such an increase into a limited liability, a rescue and a leverage component. We also show that, at the optimum, the Holding company is less levered than before offering the guarantee, the intuition being that this increases its chances of being able to provide support (theorem 4.3). Under convexity, we further show that the Holding company is unlevered (theorem 4.2) and that the Subsidiary's principal is greater than the sum of the debts of the original Stand Alone companies. In other words, we show that guarantees increase total optimal debt of the two companies.

In the light of the general expression (8), the combined value of the firms cum-guarantee is equal to:

$$\nu_{0HS}(P_H, P_S) = \phi\mathbb{E}X_H^+ - T_H - DC_H + \phi\mathbb{E}X_S^+ - T_S - DC_S$$

We call "value of the guarantee" the difference between this combined value and the sum of the two Stand Alone firms' values. Theorem 4.1 shows that it is positive:

Theorem 4.1 *Conditional guarantees are value increasing with respect to no guarantees (weakly , if $T_H + DC_H + T_S + DC_S$ is not convex in the principals).*

Proof. The value of the guarantee can be formalized as

$$\nu_{01}(P_H) + \nu_{02}(P_S) + \Gamma(P_H, P_S) - \nu_{01}(P_1) - \nu_{02}(P_2) \tag{22}$$

Suppose that the HS maintains the initial debt levels ($P_H = P_1^*, P_S = P_2^*$). Since $\nu_{01}(P_H) = \nu_{01}(P_1^*), \nu_{02}(P_S) = \nu_{02}(P_2^*)$, the value of the guarantee reduces to

$$\nu_{0HS}(P_1^*, P_2^*) - \nu_{01}(P_1^*) - \nu_{02}(P_2^*) = \Gamma(P_1^*, P_2^*)$$

where $\Gamma(P_1^*, P_2^*) > 0$ since $P_2^* > 0$ (≥ 0 if no convexity is required). For the suboptimal principals $P_H = P_1^*, P_S = P_2^*$ the value of the guarantee is then positive (non-negative). A fortiori it is positive (non-negative) when principals of the guarantor and guaranteed company are optimized, $P_H = P_H^*, P_s = P_S^*$. ■

We now split the optimal value of the guarantee into three components. The first term is the difference in the unlevered firm value, which is the "limited liability effect" defined in Leland (2007). The second one is the reduction in default costs due to rescue, when leverage - and hence the tax burden - is the same as in the Stand Alone case. The last term is the increase in value associated with the possibility of levering up more thanks to the guarantee.

$$\nu_{0HS}(P_H^*, P_S^*) - \nu_{01}(P_1^*) - \nu_{02}(P_2^*) =$$
$$= \underbrace{\nu_{0HS}(0,0) - \nu_{01}(0) - \nu_{02}(0)}_{\text{limited liability effect}} + \underbrace{\Gamma(P_1^*, P_2^*)}_{\text{rescue effect}} + \underbrace{\nu_{0HS}(P_H^*, P_S^*) - \Gamma(P_1^*, P_2^*)}_{\text{leverage effect}}$$

It is easy to show that the first term is zero, because there is no loss in limited liability when shifting from two Stand Alone firms to a Holding Subsidiary structure. The proof of the above theorem shows that the second and third term are positive (or non negative).

The next theorem studies how debt levels in Holding company and Subsidiary differ from their Stand Alone counterparts.

Theorem 4.2 *Under the convexity assumption, i) the Holding company is optimally unlevered ($P_H^* = 0$); ii) the Subsidiary principal - and, a fortiori, the HS one - is higher than in two Stand Alone companies ($P_S = P_S^* + P_H^* > P_1^* + P_2^*$) if and only if the ratio of default costs to the tax rate is bounded above by a constant Q.*

Proof. See Appendix B, which also provides the expression for Q. ■

Clearly, tax savings increase in total debt, which optimally coincides with Subsidiary's debt since setting P_H to zero reduces both H and S default costs. We saw that increasing P_S may reduce H ability to support S, thus increasing default costs. The Q condition ensures that marginal tax gains exceed marginal default costs at $P_H + P_S = P_1^* + P_2^*$.

Another version of this theorem, which does not require convexity, is proved in Appendix B too. In the first part it also claims that there is a shift of debt from the guarantor to the guaranteed party.[11] However, it need not entail zero leverage for the Holding company. The second part provides an alternative condition for higher optimal debt.

Theorem 4.3 *(i) the entrepreneur creates value by decreasing the Holding company debt below P_1^* and increasing the Subsidiary debt above P_2^*; (ii) provided that the value of the guarantee is decreasing in P_S at P_2^*, with $\partial\Gamma\left(P_1^*, P_2^*\right)/\partial P_S > \partial\Gamma\left(P_1^*, P_2^*\right)/\partial P_H$, then the overall optimal debt is greater than $P_1^* + P_2^*$.*

Proof. See Appendix B. ■

In our numerical experiments, which are built on the Leland (2007) BBB parametrization, we find results consistent with both theorems 4.2 and 4.3 for equally distributed cash flows ($X_1 = X_2$), while they are consistent only with theorem 4.3 for cash flows that differ in size ($X_1 = 5X_2$).

To anticipate on observational findings that we describe in section 6, Masulis et al. (2008) find that group affiliates and Holding companies are respectively more and less levered than comparable Stand Alone firms, in line with with both propositions.

4.2 On mutual conditional guarantees

This section clarifies the reason why we have so far focussed on a unilateral guarantee. In principle, each firm can grant support to the other, but this turns out to be dominated by unilateral support, at least for moderate default costs.

Theorem 4.4 first highlights that the value of the HS structure achieves local maxima when only one firm is levered and, therefore, the guarantee is unilateral. It then provides a condition ensuring that mutual guarantees are not optimal.

Theorem 4.4 *Unilateral guarantees maximize the combined value of H and S. There exists a default cost level α^* below which they are the only optimal guarantees.*

Proof. See Appendix B. ■

The intuition for this result is the following. Unilateral guarantees permit to save on tax payments net of default costs by concentrating debt in the beneficiary, which allows to increase overall leverage. With mutual guarantees each firm should instead both increase its debt - since it receives support - and decrease it - in its quality of guarantor. This tension results in lower total debt and tax savings. The theorem indicates that it is not profitable to give

[11] The intuition for the transfer of debt from H to S is straightforward. Without any guarantee, marginal tax savings equal marginal default costs at debt levels P_1^*, P_2^*. With the guarantee, the marginal cost of H debt is higher (because it reduces net cash flows available for rescue), while the marginal default cost associated with S debt is lower (because of the rescue by H). It follows that debt in H (S) must fall (increase) in order to re-establish equality with tax savings, thus $P_H < P_1^*$ and $P_S > P_2^*$.

up the increase in overall debt which unilateral guarantees permit, at least if default costs are moderate: tax incentives - and the asymmetric leverage which optimally exploits them - prevail.

The previous theorem does not shed light on the respective features of the firm providing or receiving support. This leads us to characterize the ideal Holding company in the following section.

4.3 Which firm provides support?

In the previous sections we assumed that a company, called the Holding company, was providing a guarantee. In this section we explicitly consider different characteristics for firms 1 and 2, and find conditions ensuring that it is optimal for firm 1 to be the guarantor. This is the case if

$$v_{12}(P_{1H}^*, P_{2S}^*) > v_{21}(P_{1S}^*, P_{2H}^*) \tag{23}$$

where the left-hand side (rhs) is defined as total HS value when 1 supports 2 (2 supports 1). Notice that P_{iH}^* (P_{iS}^*) is the optimal debt of firm i when it acts as Holding company (Subsidiary).

Observe that the inequality (23) can be written as:

$$TG_1 + DCG_1 < TG_2 + DCG_2 \tag{24}$$

where TG_i and DCG_i are the incremental tax burden and default costs when firm i shifts from providing to receiving support (and optimally levers up):

$$TG_i(P_{iH}^*, P_{iS}^*) \triangleq T_i(P_{iH}^*) - T_i(P_{iS}^*)$$

$$DCG_i(P_{jH}^*, P_{iH}^*, P_{iS}^*) \triangleq DC_i(P_{iH}^*) - DC_S(P_{jH}^*, P_{iS}^*),\ j \neq i$$

Firm 1 should provide support if and only if it has smaller incremental tax burden net of default costs, relative to firm 2. It is now easy to demonstrate that:

Proposition 4.1 *i) If $X_1 = X_2$ (in distribution) and tax rates as well as default costs are equal across the two firms, then each firm can either provide or receive support; (ii) under the convexity assumption, firm 1 supports firm 2 if - all others equal - it has a higher percentage default cost ($\alpha_1 > \alpha_2$) or a lower tax rate ($\tau_1 < \tau_2$); iii) firm 1 guarantees firm 2 if it has a larger size, in the following sense:*

$$\mathbb{E}\left[X_1 1_{\left\{0<X_1<X_1^d(P_{1S}^*), X_2<h(X_1)\right\}}\right] > \mathbb{E}\left[X_2 1_{\left\{0<X_2<X_2^d(P_{2H}^*), X_1<h(X_2)\right\}}\right] \tag{25}$$

Proof. See Appendix B. ■

Why do we relate the last part of the previous proposition to size? With Gaussian distributions higher mean cash flows (which we can think of as a proxy for size) map into higher expected positive cash flows.[12] With general distributions, too, we can interpret bigger size of firm 1 as $\mathbb{E}X_1^+ >> \mathbb{E}X_2^+$. Observe that:

a) a bigger Stand Alone levers up more than a smaller one, i.e. $P_1^* > P_2^*$; a fortiori, a bigger Subsidiary levers up more than its smaller Holding company: $P_{1S}^* > P_{2H}^*$,

b) as a consequence of different debts, the default thresholds differ too. In particular, the one of the smaller Holding is lower: $X_2^d(P_{2H}^*) < X_1^d(P_{1S}^*)$. Hence the set of states when the bigger Subsidiary defaults exceeds the set in which the smaller Holding defaults.

c) observations a) and b) together with bigger size imply

$$\mathbb{E}\left[X_1 1_{\{0<X_1<X_1^d(P_{1S}^*)\}}\right] > \mathbb{E}\left[X_2 1_{\{0<X_2<X_2^d(P_{2H}^*)\}}\right]$$

d) we also expect the set of states when a smaller Holding does not support its bigger Subsidiary, $X_2 < h(X_1)$, to be larger than the one over which the converse holds, $X_1 < h(X_2)$. This observation and c) make condition (25) more likely to be satisfied when $\mathbb{E}X_1^+ >> \mathbb{E}X_2^+$.

The theorem suggests that the Holding company is the firm with larger cash flows available for support. In the case of equal cash flow distributions, H is the one that would lever up less even as stand alone, because of lower tax rates or higher default costs. With unequal cash flow distribution but equal α and τ, it is the bigger firm.

Another remark is in order. It will be clear by now that the Holding company does not necessarily own its Subsidiary in our set up, as we sidestep intercorporate ownership for simplicity[13]. However, it is straightforward to consider an alternative organization in which an unlevered super-holding company owns 100% of both H and S. The super-holding company indirectly provides the guarantee to S by collecting cash flows and distributing them as in the direct case addressed so far. That is, this indirect case leads to the same optimal guarantor H described in Proposition 4.1 and the same capital structure of Theorem 4.2.

4.4 Unconditional guarantees

Leland (2007) points out that merging two firms, i.e. introducing an unconditional guarantee between two firms, may reduce their joint value. We now show that, in such situations, it is still possible to create value through a unilateral conditional guarantee, if correlation is high. This is the content of theorem 4.5 below.

[12] In the Gaussian case, we have:

$$\mathbb{E}X^+ = \int_0^{+\infty} xn(x)dx = \mu N(\mu/\sigma) + \sigma n(-\mu/\sigma)$$

where N and n are the distribution and density functions of a standard normal. This expectation is increasing in the mean - as one can check by computing $\partial \mathbb{E}X^+/\partial\mu = N(\mu/\sigma) > 0$.

[13] The case with intercorporate dividends is available from the authors upon request.

Theorem 4.5 *Unilateral conditional guarantees are value increasing (non decreasing, if convexity does not hold) with respect to a Merger if either (i) activities cash flows are equal and perfectly correlated, or (ii) they are Gaussian, with* $\rho^Q < \rho \leq 1$ *and (common) volatility* $\sigma > \sigma_L$*, where* $\sigma_L = \arg\min \nu^*(P_M)$ *or (iii) they are Gaussian, with* $\rho^R < \rho \leq 1$ *and distinct volatilities:* $\sigma_H \neq \sigma_S$*.*

Proof. See Appendix B. ρ^Q and ρ^R are characterized in Leland. ■

Leland (2007) shows that a Merger is value decreasing - at least for high volatility, i.e. $\sigma > \sigma_L$ - when diversification gains disappear, i.e. when cash flow correlation across two equal activities is perfect. Case (i) strikingly implies that - with perfect correlation and independently of the level of volatility - gains from the conditional guarantee obtain: the Holding company is still able to rescue S because its own debt is optimally lower than S's. Debt diversity - i.e., the fact that H and S have distinct principals - preserves the value of the guarantee when diversification opportunities vanish, independently of volatility. In cases (ii) and (iii), Leland (2007) shows that the loss in limited liability - due to the Merger - is large enough to make it less desirable than separation. A fortiori - due to theorem 4.1 - a Merger is less desirable than a HS structure.

5 Numerical analysis

We now turn turn to simulations, for four reasons. First, they will allow us to appreciate the magnitude of the effects of conditional guarantees on debt, value, tax burdens, default costs and leverage, i.e. the ratio between the market value of debt and the market value of the firm. The second reason for looking at simulations is to assess the robustness of our results when we relax the assumption of convexity, which is necessary and sufficient in the analytical results we provided for general cash flow distributions. We will see below that capital structure and value rankings align with our predictions, even if - in this example - convexity does not hold in the whole domain. The third motivation is that we can get some insight on the case, which we cannot treat analytically, of different risk across firms. Finally, we will also assess whether asymmetry in bankruptcy costs, volatility or size makes guarantees more valuable with respect to the case of equal firms. Throughout, we impose the assumption of Gaussian cash flows so as to compare our results with those of Leland.

5.1 Equal Firms

The base-case parameters also coincide with Leland's. The horizon T is 5 years, the per-annum interest rate - which, under no arbitrage, determines also the cash flow return - is 5%. When firms are equal, as in Table 1, operating cash flow for each activity has expected present value $X_0 = 100$, annual volatility $\sigma = 22$ and expected final gross returns $\mu = 127.6$. Annual cash flows are independently distributed over time. The correlation coefficient between the units' cash flows ρ is set equal to 0.2. The tax rate, $\tau = 20\%$, and the default cost parameter,

$\alpha = 23\%$, generate optimal leverage and recovery rates consistent with a BBB firm issuing unsecured debt. In Table 1 the first column refers to a Stand Alone firm. The second, third and fourth columns refer to a Holding company, a Subsidiary and an "average" of H and S respectively, while the last column to half of a conglomerate. All figures must be interpreted as percentages of the present value of cash flow.

The optimal face value of debt, which is equal to 57.1 in a Stand Alone firm, reaches 220 in the Subsidiary, raising its leverage from 51.81 to 99.9. The Holding company is optimally unlevered, thus debt almost doubles in the average HS, reaching 110. This increases its expected tax savings by more than expected default costs (from 2.32 to 7.31 and from 0.89 to 4.07 respectively). As a result, the average firm value (83.29) exceeds the one of a firm without guarantees (81.47), resulting in a value of the guarantee equal to 3.64. In contrast, the value of half a Merger is only slightly larger than that of a Stand Alone (81.57), with a value of the unconditional guarantee of 0.2. However, the Merger entails both lower tax savings (2.18) and lower default costs (0.61).

The previous effects on leverage and value are not limited to the case of low cash flow correlation underlying Table 1. In Figure 1 (bottom right panel) we see that the face value of debt in HS grows much larger than without guarantees as the correlation coefficient ρ increases from -0.8 to 0.8. Tax savings increase almost linearly in the correlation coefficient, while default cost increase at a decreasing rate. Indeed, high correlation means that the unlevered Holding has huge cash flows available for rescue precisely when its highly levered Subsidiary would also incur into important default costs because of its own huge cash flows. Thus, HS value - and the value of the guarantee - is also increasing in correlation. In Mergers, the advantages of diversification increase as correlation falls. But even with correlation as low as -0.8, the value of HS exceeds that of Mergers.

5.2 Different Firms

Numerical results so far refer to the base-case of identical activities, while we now allow them to differ in either proportional bankruptcy costs (α), cash flow volatility (σ), or size (μ). The results described below refer to cash-flow correlation equal to 0.2, but hold as well for the unreported range $[-0.8, +0.8]$.

Table 2 refers to activities with proportional default costs respectively equal to 23%, as in the base case, and 75%. With larger bankruptcy costs, the optimal value of a Stand Alone firm drops from 81.47 (see the second column) to 80.83 (first column) as its face value of debt reduces from 57.1 to 33. As a consequence, the average Stand Alone firm in the third column has debt equal to 45.05, with lower tax savings of 1.79 and lower default costs of 0.68. Its value drops to 81.15. In the case of HS, by contrast, we know by Proposition 4.1 that the activity with larger bankruptcy costs is the Holding company, which is optimally unlevered even when bankruptcy costs are eas low as 23%. Unsurprisingly, it is still unlevered when it has higher default costs. Hence both the optimal capital structure and HS value do not change. The Merger case is intermediate, with debt, tax savings, default costs and value falling relative to the symmetric case (respectively to 46.5, 1.69, 0.17 and 81.24). The value of the Merger stays higher than in the average Stand Alone. Let us now assess how asymmetric default costs affect the value of the guarantees. The value of the conditional guarantees grows in DC diversity, from 3.64 in Table 1 to 4.29 in Table 2, while the unconditional one falls from 0.20 to 0.17 - possibly because of lack of debt diversity.

Table 3 concerns the case of different risk. We design it so as to keep total firm volatility constant for firm combinations, when the 0.2 correlation coefficient is accounted for. To do so, we consider one firm having 15% volatility while the other has 27.52%. The average Stand Alone firm raises more debt than the firm in the base case (60.5 versus 57.10), its value slightly increases (81.70 instead of 81.47) as well as its tax savings (2.47 instead of 2.32) relative to default costs (0.9 versus 0.89). Figures for the Merger case do not change, as volatility parameters are set so as to keep total M volatility constant. What changes, though, is the value differential relative to the Stand Alone case: the value of the unconditional guarantee drops to -0.26, because loss of limited liability is now larger due to the different cash flow volatility of units.

On the contrary, the value of the conditional guarantee climbs to 4.32. This occurs despite a fall in debt and tax savings relative to the base case (from 110 to 107.5; and from 7.31 to 6.87) coupled with a reduction in default costs (from 4.07 to 3.15). Intuitively, the conditional guarantee is an option with two separate underlyings, which benefits from their different volatilities even when the overall volatility is kept constant.

Finally, Table 4 describes the case of different size: the expected cash flow of one firm is five times the other, keeping the total present value equal to 200. In the SA case, the larger (lower) firm has higher (lower) debt, tax savings, default costs and value. As a result, the average Stand Alone almost coincides with the base case one.

As for HS, we know from Proposition 4.1 that the larger firm must be the guarantor. For the first time we see that such a large Holding company raises debt as well, so as to reduce its own tax burden, without compromising the provision of support to its Subsidiary, that remains 100% levered. With respect to the base case, the HS has lower debt, lower tax savings and lower default costs The asymmetric Merger also has lower debt, but this translates into higher tax savings (because the unreported value of debt goes down more than the face value of debt, increasing the tax shield) and higher default costs. In all the organizations, value does not increase relative to the equally distributed case (SA stays constant at 162.94; HS falls from 166.58 to 166.13 and M from 163.14 to 162.98).

What is the impact of size asymmetry on guarantees? Differently from the previous two cases, the value of the conditional guarantee falls from 3.44 to 3.19. Unreported simulations show that this obtain when the value difference exceeds a certain threshold. For instance, when H is 4/3 and S is 2/3, the value of the guarantee is higher than in the base case.[14] The value of the unconditional

[14] In this paper the size of HS components is exogenous. By endogenising it, we would never

guarantee does not become negative as in the case of asymmetric volatility, but falls from 0.2 to 0.04.

The following proposition summarizes our main numerical findings:

Conclusion 1 *Consider BBB calibrated companies and cash flow correlation* $\rho \in$ [-0.8, 0.8]. *Then (i) Subsidiary leverage may reach up to 100%; (ii) conditional guarantees are value increasing but generate much higher tax savings and default costs than both unconditional and no guarantees; (iii) when cash flow volatility differs across firms, the Holding company company is the safer one; (iv) the Holding company is also levered when the size difference exceeds a certain threshold; (v) the value of the conditional guarantee may increase in bankruptcy costs, risk and size asymmetries between activities.*

6 Intercorporate guarantees in practice

The current section relates our model to stylized empirical facts. After challenging our main assumption and listing some evidence that our model helps rationalizing, we explain why it does not capture all relevant aspects of the empirical spectrum.

As for our main assumption, empirical studies point out the existence of conditional guarantees in both listed and unlisted business groups (Khanna and Palepu (2000); Gopalan et al. (2007), Deloof and Vershueren (2006)). Rating agencies are also well aware of this and take intra-group guarantees - as well as the opposite, namely ring-fencing - into consideration when evaluating credit risk (Standard & Poor's(2001)). For instance, a study by Emery and Cantor (2005) analyzes the ensuing occurrence of either selective (S only) or multiple (both H and S) defaults in a sample of 670 non-financial U.S. HS structures, including 1741 rated firms.

As for our results, we now describe some observations that witness the diffusion of Holding-Subsidiary structures and their higher debt and tax savings with respect to competing organizations.

Holding-Subsidiary structures embedding conditional guarantees are indeed pervasive. For instance, they are the norm in both emerging markets (Khanna and Yafeh (2007)) and continental European countries (De Jong et al. (2009), Barca and Becht, 2001). They are also present in innovative industries in the US and the UK (Allen (1998); Sahlman (1990); Mathews and Robinson (2008)), as well as in the private equity groups.[15] Risk taking in the banking industry appears to be related with the presence of intercompany guarantees as well (Dell'Ariccia and Marquez (2010)), which can be imposed by the Federal Reserve (Herring and Carmassi (2009)).

see the case of Table 4.

[15] Conditional guarantees are implicit in private equity. Partners need to periodically raise funds in the market because of the limited temporal commitments of financiers, and this is possible only if their reputation concerning participation in restructurings is good. Moreover, the managers of LBO targets receive bonuses only when they repay their debt obligations. See Jensen (2007).

As far as debt financing is concerned, group affiliates do appear to rely on it more than comparable Stand Alone firms (Masulis et al. (2008), Deloof and Vershueren (2006)), consistent with our analytical results. This is especially true in the case of leveraged buy-outs, a situation where our assumption of no agency costs applies reasonably well. In such a stylized world, our results point to zero debt in the Holding company and a highly levered Subsidiary. In the real world, the private equity fund is unlevered while portfolio firms display extraordinary debt levels (Kaplan (1989)). In Table 1 the tax burden of debt drops from 17.70% of operating cash-flow of a non-guaranteed firm (that we may consider as a Stand Alone public company) to 5.39% for a guaranteed one (the LBO portfolio firm). Kaplan (1989) reports that, in firms taken private, the tax burden dropped from 20% to 4.8% in the third year.

We did not find data comparing HS's tax savings to those of competing organizations. However, we see that 22% of cross-country variation in Subsidiary leverage of US multinationals' subsidiaries is explained by cross-country variation in tax rates (Desai et al. (2004)). Moreover, we indirectly understand the relevance of tax savings and guarantees in HS since several countries (including Australia, China, Germany, Italy, the Netherlands, UK and the US) adopt rules limiting interest deductions if the debt-equity ratio or interest expenses exceed certain thresholds. These rules are called "thin capitalisation" as large interest deductions go together with high leverage - as in our simulations. For instance, Her Majesty Revenue and Customs (INTM541010) explains that "thin capitalisation commonly arises where a company is funded by another company in the same group. It can also arise where funding is provided to a company by a third party, typically a bank, but with guarantees or other forms of comfort provided to the lender by another group company or companies (typically the overseas Holding company). The effect of funding a UK company or companies with excessive intra-group or Parentally- guaranteed debt is, potentially, excessive interest deductions. It is the possibility of excessive deductions for interest which the UK legislation on thin capitalisation seeks to counteract."

In practice, we also observe Stand Alone organizations that in our set up should not exist, and Mergers that should not exist if correlation between activities' cash flows is high enough. We discuss below some of our simplifying assumptions, such as credible guarantees, no frictions and no regulation, that could be relaxed so as to reflect such empirical regularities.

Our model posits credibility of the conditional guarantee, which can be associated for instance to enforceability in court. In practice, alternative jurisdictions ensure different degrees of lenders' protection associated with the same guarantee (Herring and Carmassi (2009)). Moreover, within a given jurisdiction, the parties may write alternative contracts that make the ensuing guarantee more or less binding. Comfort letters, that are legally unenforceable promises of rescue sent by the Holding company to Subsidiary's lenders, are also common (Boot et al. (1993)) and derive their credibility from the guarantor's reputation. Herring and Carmassi (2009) cite cases of financial institutions providing additional funds to troubled SIV, despite the absence of legal obligations, so as to protect their reputation. When we embed these less-than-fully credible guarantees in numerical simulations of our model, the Merger may become an optimal arrangement also for higher cash flow correlations, because the unconditional guarantee is more reliable than the conditional one. This reconciles our theoretical conclusions with empirical evidence, in which Holding-Subsidiary structures coexist with Mergers, both under different legislations and under the same one.

Financial frictions associated with firm combinations may also explain the coexistence of HS and Stand Alone firms. For instance, previous models highlight that internal capital markets, which are present inside both Mergers and HS, may distort allocations (see Inderst and Mueller (2003), Faure-Grimaud and Inderst (2005), Rajan et al. (2000)); and that shareholders' heterogeneity may lead to minority shareholders expropriation in groups (Almeida and Wolfenzon (2006)). Non-financial frictions and diseconomies of scale may similarly explain discrepancies between our frictionless model and reality, if they generate a cost of HS which offsets the financial synergy we uncover.

Last but not least, regulation has been targeting complex organizations in certain countries - for instance in the US since the New Deal (see Morck (2005)). Regulation is absent in our frictionless model.

The reader may finally object that conditional guarantees go hand-in-hand with intercorporate ownership, while in this version of our model the Holding company does not hold any shares in the Subsidiary. Unreported simulations indicate that the fully-credible conditional guarantees are insensitive to intercorporate ownership, because the Subsidiary is hardly ever able to distribute dividends due to its debt obligations. When we numerically assess the case of less credible guarantees, the Subsidiary becomes able to pay a dividend thanks to its lower debt service. In turn the Holding company may lever up, the more so the higher are its dividend receipts. This makes the value of the conditional guarantee sensitive to ownership only when it is not fully credible.

7 Conclusions and extensions

This paper provides new insights on intercorporate guarantees and firm scope. Up to our knowledge, it models for the first time the provision of these guarantees, the associated optimal leverage and their impact on tax savings net of default costs. Given the correspondence between conditional guarantees and HS, our model offers a rationale for the diffusion of Holding-Subsidiary structures without relying on previous insights relating to internal capital markets and expropriation of minority shareholders. It also explains their observed reliance on debt and their high tax gains, which appear to be of concern to tax authorities around the world.

Importantly, our model is just a first step towards a better understanding of intercorporate guarantees in Holding companies and their subsidiaries, as it relies on a simple static setting with two activities and no agency problems. Developments relying on more general settings are postponed to further work.

The first obvious extension would be to allow for more than two firms. The logic of our characterization of the Holding company suggests that the features of the third (or N-th) firm - namely its relative default costs, tax rate and size - will determine whether it will act as a guarantor to or a beneficiary of the initial H and S. Debt seniority will then emerge endogenously among the members of the enlarged pool.

The comparative welfare properties of HS also appear to deserve further attention, as they may be socially wasteful despite being value maximizing. Unreported simulations show that the joint default probability of H and S with conditional guarantees is very low. Still, they indicate that HS appear to incur into much higher bankruptcy costs than competing organization when they do go bankrupt, despite the provision of a guarantee to lenders and the absence of moral hazard. This perspective may also shed light on the recent crisis, triggered by institutions that were interconnected and had large leverage.

8 Appendix A

8.1 The optimization problem without guarantees

This Appendix studies the maximization of firm value with respect to non-negative debt levels, $P_i \geq 0$, with $i = 1, 2$, through its equivalent problem, namely the minimization of the tax burden plus default costs. The optimal debt results from trading off these effects, as known from seminal results by Kim (1978), among others. We first establish some properties of the market value of debt:

Lemma 8.1 *Debt is increasing less than proportionally in the face value of debt:*

$$0 \leq dD_{0i}(P_i)/dP_i < 1 \qquad \textit{with} \qquad \lim_{P_i \to 0+} \frac{dD_{0i}(P_i)}{dP_i} > 0$$

Proof. Observe first that, as default costs and taxes approach zero, we have:

$$0 \leq dD_{0i}(P_i)/dP_i = (1 - F_i(P_i))\phi \leq \phi < 1$$

In particular, when the face value of debt tends to zero, we have:

$$\lim_{P_i \to 0+} \frac{dD_{0i}(P_i)}{dP_i} = (1 - F_i(0))\,\phi > 0$$

since the probability that X_i is positive is positive ($F_i(0) < 1$).

For positive default cost and tax rates, when closed form expressions for $D_{0i}(P_i)$ do not obtain, we need to prove both that $0 \leq dD_{0i}(P_i)/dP_i < 1$ and that $\lim_{P_i \to 0+} \frac{dD_{0i}(P_i)}{dP_i} > 0$.

The non-negativity of $dD_{0i}(P_i)/dP_i$ is proved in Regis (dissertation). In order to demonstrate that $dD_{0i}(P_i)/dP_i < 1$ we use the fact that risky debt D_{0i}

can be written as the difference between the corresponding riskless debt, $P_i\phi$, and lenders' discounted expected loss.

The first part of the proof proceeds by contradiction. Thus assume instead $dD_{0i}(P_i)/dP_i \geq 1$. Observe that the derivative of riskless debt with respect to the face value of debt (ϕ) is smaller than one. In order for the risky debt to have a derivative not smaller than one, the discounted expected loss should have a derivative smaller than zero, i.e. it should decrease in the face value of debt. This contradicts the minimal requirement that both default probability and expected default costs increase in the face value of debt.

As for the limit when the principal tends to zero, let $\lim_{P_i\to 0+} \frac{dD_{0i}(P_i)}{dP_i} \leq 0$. This implies that the discounted expected loss has a derivative, when $P_i \to 0+$, positive and not smaller than ϕ. This in turn implies that lenders' expected loss has a derivative greater than one with respect to debt , which is absurd. ■

This Lemma implies that both the tax shield and the default threshold are increasing in the face value of debt:

$$0 < \frac{dX_i^Z}{dP_i} = 1 - \frac{dD_{0i}(P_i)}{dP_i} < 1, \tag{26}$$

$$\frac{dX_i^d}{dP_i} = 1 + \frac{\tau}{1-\tau}\frac{dD_{0i}(P_i)}{dP_i} \geq 1 \tag{27}$$

Now we show that lemma 3.1 holds.

Proof. As remarked by Leland (2007), the entrepreneur's problem in the Stand Alone case is

$$\min_{P_i \geq 0} [T_i(P_i) + DC_i(P_i)] \tag{28}$$

The Kuhn-Tucker (KT) conditions for this problem are:

$$\begin{cases} \frac{dT_i(P_i^*)}{dP_i} + \frac{dDC_i(P_i^*)}{dP_i} \geq 0 \\ P_i^* \geq 0 \\ \left[\frac{dT_i(P_i^*)}{dP_i} + \frac{dDC_i(P_i^*)}{dP_i}\right] P_i^* = 0 \end{cases} \tag{29}$$

Conditions (29) are necessary and sufficient if $T_i + DC_i$ is convex in $P_i \geq 0$. The derivative of tax burdens and default costs, appearing on the lhs of (29), is equal to:

$$\frac{dT_i(P_i)}{dP_i} + \frac{dDC_i(P_i)}{dP_i} = -\tau(1 - F_i(X_i^Z))\frac{dX_i^Z}{dP_i}\phi + \alpha X_i^d f_i(X_i^d)\frac{dX_i^d}{dP_i}\phi \tag{30}$$

where f_i is the density of X_i. If $\tau = 0$, a minimum exists, with $P_i^* = X_i^{d*} = X_i^{Z*} = 0$. If $\tau > 0$, then a minimum at $P_i = 0$ cannot exist, since the first condition in (29) is violated. The optimum is interior, and (30) is set to zero. ■

9 Appendix B - Optimal guarantees

9.1 Proofs of lemma 3.2, theorems 4.2 and 4.3

We first prove a lemma which characterizes savings in default costs Γ.

Lemma 9.1 Γ *a) is non increasing in P_H and has a null derivative if and only if $P_S = 0$; b) has a null derivative with respect to P_S at $P_S = 0$; c) is decreasing in P_S when the latter diverges.*

Proof. Part (a) requires $\frac{\partial \Gamma}{\partial P_H} \leq 0$, that is:

$$\frac{\partial \Gamma}{\partial P_H} = -\alpha\phi \times \frac{dX_1^d}{dP_1} \times$$

$$\times \left[\begin{array}{c} \int_0^{X_S^Z} xf\left(x, X_H^d + \frac{P_S}{1-\tau} - \frac{x}{1-\tau}\right) dx + \\ + \int_{X_S^Z}^{X_S^d} xf(x, X_H^d + X_S^d - x) dx \end{array} \right] \leq 0 \qquad (31)$$

which is true by (27).

Equality in (31) holds if and only if the third term is zero, that is $X_S^d = X_S^Z = 0$, which in turn happens if and only if $P_S = 0$.

As concerns part (b), we compute:

$$\frac{\partial \Gamma}{\partial P_S} = \alpha\phi \times \left\{ -\frac{1}{1-\tau} \int_0^{X_S^Z} xf\left(x, X_H^d + \frac{P_S}{1-\tau} - \frac{x}{1-\tau}\right) dx + \right. \qquad (32)$$

$$\left. -\frac{dX_2^d}{dP_2} \int_{X_S^Z}^{X_S^d} xf(x, X_H^d + X_S^d - x) dx + \frac{dX_2^d}{dP_2} \int_{X_H^d}^{+\infty} X_S^d f(X_S^d, y) dy \right\} =$$

$$= \frac{\alpha}{(1-\tau)(1+r_T)} \times \left\{ -\int_0^{X_S^Z} xf\left(x, X_H^d + \frac{P_S}{1-\tau} - \frac{x}{1-\tau}\right) dx + \right.$$

$$\left. + \left(1 - \tau + \tau \frac{dD_{02}(P_2)}{dP_2}\right) \left[\begin{array}{c} -\int_{X_S^Z}^{X_S^d} xf(x, X_H^d + X_S^d - x) dx + \\ + \int_{X_H^d}^{+\infty} X_S^d f(X_S^d, y) dy \end{array} \right] \right\}$$

When $P_S = 0$, then $X_S^d = X_S^Z = 0$, all the integrals vanish and the previous derivative is null.

As concerns part (c), namely

$$\lim_{P_S \to +\infty} \frac{\partial \Gamma(P_H, P_S)}{\partial P_S} < 0,$$

consider that, when $P_S \to +\infty$, definition (4) implies that

$$\lim_{P_S \to +\infty} X_S^d = +\infty$$

For fixed y, $\lim_{x \to +\infty} xf(x, y) = 0$ - since f is a density - implies that, for any sequence x_n which goes to $+\infty$, then $x_n f(x_n, y)$ converges to zero. We suppose that the function $f_n(y)$ satisfies the dominated convergence property. This allows us to exchange integration and limit:

$$\lim_{n \to +\infty} \int_{X_H^d}^{+\infty} x_n f(x_n, y) dy = \int_{X_H^d}^{+\infty} \lim_{n \to +\infty} x_n f(x_n, y) dy = 0.$$

As a consequence,

$$\lim_{X_S^d \to +\infty} \int_{X_H^d}^{+\infty} X_S^d f(X_S^d, y) dy = 0$$

Together with (26) this entails

$$\lim_{P_S \to +\infty} \frac{\partial \Gamma}{\partial P_S} =$$

$$= \lim_{P_S \to +\infty} \frac{\alpha}{(1-\tau)(1+r_T)} \times \left\{ -\int_0^{X_S^Z} x f\left(x, X_H^d + \frac{P_S}{1-\tau} - \frac{x}{1-\tau}\right) dx - \right.$$

$$\left. - \left(1 - \tau + \tau \frac{dD_{02}(P_2)}{dP_2}\right) \int_{X_S^Z}^{X_S^d} x f(x, X_H^d + X_S^d - x) dx \right\} < 0$$

and proves part (c). ■

We are now ready to prove lemma 3.2.

Proof. Let us examine the KT conditions for a minimum of the total tax burden and default costs of the Holding and Subsidiary, namely

$$T_{HS} + DC_{HS} \triangleq T_H + DC_H + T_S + DC_S$$

with respect to non-negative Subsidiary debt. Recall that such conditions are necessary and sufficient, under the convexity assumption of the lemma.

$$\left\{ \begin{array}{c} \frac{\partial T_{HS}(P_H^*, P_S^*)}{\partial P_S} + \frac{\partial DC_{HS}(P_H^*, P_S^*)}{\partial P_S} = \frac{dT_2(P_S^*)}{dP_S} + \frac{dDC_2(P_S^*)}{dP_S} - \frac{\partial \Gamma(P_H^*, P_S^*)}{\partial P_S} \geq 0 \\ P_S^* \geq 0 \\ \left[\frac{dT_2(P_S^*)}{dP_S} + \frac{dDC_2(P_S^*)}{dP_S} - \frac{\partial \Gamma(P_H^*, P_S^*)}{\partial P_S}\right] P_S^* = 0 \end{array} \right. \tag{33}$$

Let us examine whether the KT conditions are satisfied at $P_S = 0$. As a consequence of part (b) of the previous lemma, if $P_S^* = 0$ we have

$$\frac{dT_2(0)}{dP_S} + \frac{dDC_2(0)}{dP_S} - \frac{\partial \Gamma(P_H^*, 0)}{\partial P_S} = \tag{34}$$

$$= \frac{dT_2(0)}{dP_S} + \frac{dDC_2(0)}{dP_S} = -\tau(1 - F_2(0)) \left[1 - \frac{dD_{02}}{dP_2} |_{P_2=0}\right] \phi$$

where the last term follows from (30), since $X_S^d = X_S^Z = 0$ when $P_S = 0$. Such derivative is negative, since $F_2(0) < 1$ and

$$\lim_{P_2 \to 0+} \frac{dD_{02}}{dP_2} < 1 \tag{35}$$

by lemma 8.1. The KT conditions are then violated when the Subsidiary is unlevered. This concludes the proof. ■

We are now ready for the proof of theorem 4.2

Proof. Consider part i). Under the convexity assumption, the KT conditions for a minimum of $T_{HS}+DC_{HS}$, with respect to non-negative debt for both the Holding and the Subsidiary, under the constraint

$$P_H^*+P_S^*\geq P_1^*+P_2^*:=K \tag{36}$$

are necessary and sufficient. They are equal to:

$$\begin{cases} \frac{\partial T_{HS}(P_H^*,P_S^*)}{\partial P_H}+\frac{\partial DC_{HS}(P_H^*,P_S^*)}{\partial P_H}= & \\ =\frac{dT_1(P_H^*)}{dP_H}+\frac{dDC_1(P_H^*)}{dP_H}-\frac{\partial\Gamma(P_H^*,P_S^*)}{\partial P_H}=\mu_1+\mu_3 & (i) \\ P_H^*\geq 0 & (ii) \\ \mu_1 P_H^*=0 & (iii) \\ \frac{\partial T_{HS}(P_H^*,P_S^*)}{\partial P_S}+\frac{\partial DC_{HS}(P_H^*,P_S^*)}{\partial P_S}= & \\ =\frac{dT_2(P_S^*)}{dP_S}+\frac{dDC_2(P_S^*)}{dP_S}-\frac{\partial\Gamma(P_H^*,P_S^*)}{\partial P_S}=\mu_2+\mu_3 & (iv) \\ P_S^*\geq 0 & (v) \\ \mu_2 P_S^*=0 & (vi) \\ P_H^*+P_S^*\geq K & (vii) \\ \mu_3(P_H^*+P_S^*-K)=0 & (viii) \\ \mu_1\geq 0,\mu_2\geq 0,\mu_3\geq 0 & (ix) \end{cases} \tag{37}$$

We temporarily ignore constraints (vii) and $(viii)$, and set $\mu_3=0$ in $(i),(iv)$ and (ix). We want to demonstrate that there exists a point $(0,P_S^*)$ which solves them. All the conditions but *(iv)* are easy to discuss.

Consider all the conditions except *(iv)* first. Having $\mu_3=0$, condition (i) becomes

$$\frac{dT_1(P_H^*)}{dP_H}+\frac{dDC_1(P_H^*)}{dP_H}-\frac{\partial\Gamma(P_H^*,P_S^*)}{\partial P_H}=\mu_1 \tag{38}$$

The left-hand side is positive at $P_H=P_1^*$, since the first two derivatives are null and $-\partial\Gamma/\partial P_H>0$ (by part (a) of lemma 9.1 and lemma 3.2, which rules out $P_S^*=0$). The first two terms on the left hand side are negative, if $P_H^*<P_1^*$, given convexity of T_i+DC_i for a Stand Alone. We also know that the third term $(-\partial\Gamma/\partial P_H)$ is still positive if $P_H^*<P_1^*$. When $P_H^*\to 0$, the left-hand side of (38) cannot be negative, since this would contradict the convexity assumption on the objective function. Thus $P_H^*=0$ and conditions (i,ii,iii) are satisfied by letting μ_1 equal to the (non-negative) difference between $\frac{dT_1(P_H^*)}{dP_H}+\frac{dDC_1(P_H^*)}{dP_H}$ and $\frac{\partial\Gamma(P_H^*,P_S^*)}{\partial P_H}$.

If later we choose $P_S^*>0$, also conditions (v,vi) are satisfied, provided that we select $\mu_2=0$. Given that we chose $\mu_1\geq 0,\mu_2=\mu_3=0$, condition (ix) holds. Let us turn to condition (iv), which has to provide us with a choice $P_S^*>0$. In view of the other conditions, (iv) becomes

$$\frac{dT_2(P_S^*)}{dP_S}+\frac{dDC_2(P_S^*)}{dP_S}-\frac{\partial\Gamma(P_H^*,P_S^*)}{\partial P_S}=0 \tag{39}$$

Consider its left-hand side as a function of P_S, denoting it with $\zeta(P_S)$ We know from the limit behavior of Γ (part (b) of lemma 9.1)) and from convexity of the

Stand Alone taxes and default costs $(T_2 + DC_2)$, that ζ has a negative limit when the Subsidiary debt tends to zero, and a positive limit (even non finite) when P_S diverges. It follows that there exists a positive debt level which satisfies condition (39). This proves part i) of the theorem, since all the KT conditions are satisfied.

Let us turn to part ii). We want to demonstrate that there exists a point $(0, P_S^*)$, with $P_S^* > K$, which solves conditions (i) to (ix). As above, we start by considering all the conditions but *(iv)*, which requires some caution.

We are interested in a solution for which the constraint *(vii)* is not binding, implying $\mu_3 = 0$ in condition *(viii)*. As above, we choose $P_H^* = 0$ and let μ_1 be equal to the (non-negative) difference between $\frac{dT_1(P_H^*)}{dP_H} + \frac{dDC_1(P_H^*)}{dP_H}$ and $\frac{\partial\Gamma(P_H^*, P_S^*)}{\partial P_H}$. Thus P_H^* and $\mu_1 \geq 0$ satisfy conditions (i, ii, iii). If later we also choose $P_S^* > K$, conditions (v, vi, vii) are satisfied as well, provided that we select $\mu_2 = 0$. Given that we chose $\mu_1 \geq 0, \mu_2 = \mu_3 = 0$, both conditions $(viii, ix)$ hold.

Let us turn to condition (iv), which has to provide us with a choice $P_S^* > K$. In view of the other conditions, (iv) becomes again (39). We are going to show that, under the conditions posited sub (ii), $\zeta(P_1^* + P_2^*) < 0$, which implies $P_S^* > P_1^* + P_2^*$. We have:

$$\zeta(P_1^* + P_2^*) = \phi \left\{ -\tau(1 - F_2(X_S^{Z**}))\frac{dX_S^{Z**}}{dP_S} + \alpha X_S^{d**} f_2(X_S^{d**})\frac{dX_S^{d**}}{dP_S} + \right.$$

$$+\frac{\alpha}{(1-\tau)} \int_0^{X_S^{Z**}} x f\left(x, \frac{P_1^* + P_2^*}{1-\tau} - \frac{x}{1-\tau}\right) dx +$$

$$\left. -\alpha \frac{dX_S^{d**}}{dP_S} \times \left[-\int_{X_S^{Z**}}^{X_S^{d**}} x f(x, X_S^{d**} - x) dx + \int_0^{+\infty} X_S^{d**} f(X_S^{d**}, y) dy \right] \right\}$$

where X_S^{d**} and X_S^{Z**} are the default and tax shield thresholds corresponding to $P_S = P_1^* + P_2^*$. Omitting ϕ, we can write the condition $\zeta < 0$ as

$$-\tau(1 - F_2(X_S^{Z**}))\frac{dX_S^{Z**}}{dP_S} + \alpha \frac{dX_S^{d**}}{dP_S} X_S^{d**} \int_{-\infty}^0 f(X_S^{d**}, y) dy +$$

$$+\frac{\alpha}{(1-\tau)} \int_0^{X_S^{Z**}} x f(x, h(x)) dx + \alpha \frac{dX_S^{d**}}{dP_S} \int_{X_S^{Z**}}^{X_S^{d**}} x f(x, h(x)) dx < 0$$

or, recognizing that both $1/(1-\tau)$ and $\frac{dX_S^{d**}}{dP_S}$ are $\partial h / \partial P_S$,

$$-\tau(1 - F_2(X_S^{Z**}))\frac{dX_S^{Z**}}{dP_S} + \alpha \frac{dX_S^{d**}}{dP_S} X_S^{d**} \int_{-\infty}^0 f(X_S^{d**}, y) dy +$$

$$+\alpha \frac{\partial h}{\partial P_S} \int_0^{X_S^{d**}} x f(x, h(x)) dx < 0$$

The last formulation can be written as

$$\alpha/\tau < \frac{\left(1-F_2(X_S^{Z**})\right)\frac{dX_S^{Z**}}{dP_S}}{X_S^{d**}\frac{dX_S^{d**}}{dP_S}\int_{-\infty}^{0} f(X_S^{d**},y)dy + \frac{\partial h}{\partial P_S}\int_0^{X_S^{d**}} xf(x,h(x))dx}$$

$$= \frac{\Pr(X_S > X_S^{Z**})\frac{dX_2^{Z**}}{dP_S}}{X_S^{d**}\frac{dX_S^{d**}}{dP_S}\Pr\left(X_S = X_S^{d**}, X_H < 0\right) + \frac{\partial h}{\partial P_S}\int\limits_0^{X_S^{d**}} xf(x,h(x)dx} \equiv Q$$

which is the condition in the theorem. This proves part ii). ■

We then consider theorem 4.3.

Proof. i) In order to increase value, we need to decrease the following function:

$$T_{HS}(P_H, P_S) + DC_{HS}(P_H, P_S) =$$
$$= T_1(P_H) + T_2(P_S) + DC_1(P_H) + DC_2(P_S) - \Gamma(P_H, P_S)$$

Since the derivative of both $T_1 + DC_1$ and $T_2 + DC_2$ with respect to their own arguments is null at the optimum leverage, P_1^*, P_2^*, then the impact of a local variation depends on the sign of the derivatives of Γ. We know from lemma 9.1 that decreasing the Holding debt increases it at any positive leverage of the Subsidiary, and therefore reduces $T_{HS} + DC_{HS}$, as needed. Given this, one can reduce the Holding debt and increase the Subsidiary one so that $T_{HS} + DC_{HS}$ decreases, as follows. We have:

$$d\left(T_{HS}\left(P_1^*, P_2^*\right) + DC_{HS}\left(P_1^*, P_2^*\right)\right) = -\frac{\partial\Gamma\left(P_1^*, P_2^*\right)}{\partial P_H}dP_H - \frac{\partial\Gamma\left(P_1^*, P_2^*\right)}{\partial P_S}dP_S \tag{40}$$

and the differential is negative if and only if

$$+\frac{\partial\Gamma\left(P_1^*, P_2^*\right)}{\partial P_H}dP_H + \frac{\partial\Gamma\left(P_1^*, P_2^*\right)}{\partial P_S}dP_S > 0$$

Consider the expression of $\frac{\partial\Gamma(P_1^*,P_2^*)}{\partial P_S}$, neglect $\alpha\phi$ and recognize that such derivative has a negative and a positive part. Define them as follows:

$$\Gamma_S^n \triangleq -\frac{1}{1-\tau}\int_0^{X_S^Z} xf\left(x, h(x)\right)dx +$$
$$-\frac{dX_2^d}{dP_2}\int_{X_S^Z}^{X_S^d} xf(x,h(x))dx$$
$$\Gamma_S^p \triangleq \frac{dX_2^d}{dP_2}\int_{X_H^d}^{+\infty} X_S^d f(X_S^d, y)dy$$

Then the differential (40) is negative **if** there exists a couple $dP_H < 0, dP_S > 0$ such that

$$\frac{\partial \Gamma\left(P_1^*, P_2^*\right)}{\partial P_H} dP_H + \Gamma_S^n dP_S = 0$$

For such couple $\Gamma_S^p dP_S > 0$. It suffices to take

$$dP_H < 0, dP_S = -\frac{\partial \Gamma\left(P_1^*, P_2^*\right)}{\partial P_H} \frac{1}{\Gamma_S^n} dP_H$$

since the last differential is positive.

ii) provided that $\frac{\partial \Gamma\left(P_1^*, P_2^*\right)}{\partial P_S} < 0$ and $dP_H < 0$,in order to demonstrate the assert we need to show that there exists a variation in P_S, starting from P_2^*, of the type $dP_S = -dP_H + \varepsilon, \varepsilon > 0, dP_H < 0$, such that the differential (40) is negative. This is value increasing and the corresponding principals represent a locally preferred optimal debt. The differential is negative if and only if:

$$\varepsilon < -\frac{\frac{\partial \Gamma\left(P_1^*, P_2^*\right)}{\partial P_H} - \frac{\partial \Gamma\left(P_1^*, P_2^*\right)}{\partial P_S}}{\frac{\partial \Gamma\left(P_1^*, P_2^*\right)}{\partial P_S}} dP_H$$

The right hand side of the last expression is positive, as required, as soon as $\partial \Gamma\left(P_1^*, P_2^*\right) / \partial P_H - \partial \Gamma\left(P_1^*, P_2^*\right) / \partial P_S < 0$. ■

9.2 Proof of theorem 4.4

Recall that $\Gamma^m = \Gamma_{12} - \Gamma_{21}$. Observe that, as known from the unilateral case, the derivative of the first component with respect to the guarantor's debt is non-positive. It is null iff the guaranteed party's debt is null, i.e. $P_S = 0$. Similarly, the derivative of the second part with respect to the guarantor debt is non-positive:

$$\frac{\partial \Gamma_{21}}{\partial P_S} = -\alpha\phi \times \frac{dX_2^d}{dP_2} \times$$

$$\times \left[\begin{array}{c} \int_0^{X_H^Z} yf\left(h(y), y\right) dy + \\ + \int_{X_H^Z}^{X_H^d} yf(h(y), y)\ dy \end{array} \right] \leq 0 \tag{41}$$

It is null iff guaranteed party's debt is null, i.e $P_H = 0$. The derivative of the first component with respect to the guaranteed party's debt can have any sign, since $\frac{\partial \Gamma_{12}}{\partial P_S} = \frac{\partial \Gamma}{\partial P_H}$.

However, we know from Appendix B that it is null when the guaranteed party's debt P_S is high enough (since its limit for $P_S \to \infty$ is negative) and when it is null, or $P_S = 0$. Similarly, the derivative of the second component with respect to the guaranteed party's debt can have any sign

$$\frac{\partial \Gamma_{21}}{\partial P_H} = \alpha\phi \times \left\{ -\frac{1}{1-\tau} \int_0^{X_H^Z} yf\left(h(y), y\right) dy + \right. \tag{42}$$

$$-\frac{dX_1^d}{dP_1}\int_{X_H^Z}^{X_H^d} yf(h(y),y)dy+\frac{dX_1^d}{dP_1}\int_{X_S^d}^{+\infty} X_H^d f(x,X_H^d)dx\Bigg\}$$

Due to the symmetry with respect to the first, also this derivative is null when the guaranteed party's debt P_H is high enough (since its limit for $P_H \to \infty$ is negative) and when it is null, i.e. $P_H = 0$.

Given these properties, let us proceed to the proof of theorem 4.4.

Proof. We demonstrate separately that i) unilateral guarantees permit to maximize the combined value of P and S; ii) there exists a default cost level α^* below which they are the only optimal guarantees.

Recall that maximizing the value is equivalent to minimizing the tax burden and default costs of the corresponding Stand Alone firms, net of Γ^m:

$$\min_{P_H \geq 0, P_S \geq 0}\left[\begin{array}{c} T_1(P_H)+DC_1(P_H)+T_2(P_S)+DC_2(P_S)+ \\ -\Gamma_{12}(P_H,P_S)-\Gamma_{21}(P_H,P_S)\end{array}\right] \tag{43}$$

When one of the principals is null, the conditions for optimality of the previous function collapse into the ones for optimality with Γ_{12} only in place (if $P_H = 0$), or Γ_{21} only in place (if $P_S = 0$).

For case i), we prove that there is an optimum characterized by a unilateral guarantee from 1 to 2 - $\Gamma_{12}(0, P_S^*)$ - where P_S^* is exactly the principal which maximized the conditional one-way guarantee. The existence of an optimum in $\Gamma_{21}(P_H^*, 0)$ can be demonstrated in a perfectly symmetrical way.

The derivatives of the function 43wrt P_H, P_S are:

$$\begin{cases} \frac{dT_1(P_H)}{dP_H}+\frac{dDC_1(P_H)}{dP_H}-\frac{\partial\Gamma_{12}(P_H,P_S)}{\partial P_H}-\frac{\partial\Gamma_{21}(P_H,P_S)}{\partial P_H}, \\ \frac{dT_2(P_S)}{dP_S}+\frac{dDC_2(P_S)}{dP_S}-\frac{\partial\Gamma_{12}(P_H,P_S)}{\partial P_S}-\frac{\partial\Gamma_{21}(P_H,P_S)}{\partial P_S}, \end{cases} \tag{44}$$

For $P_H^* = 0, P_S > 0$ we have:

$$\frac{\partial\Gamma_{21}}{\partial P_S}=\frac{\partial\Gamma_{21}}{\partial P_H}=0$$

In this case, we know from Appendix B that (44) admit a positive solution $P_S^* > K$. As a consequence, the group value is maximized by $\Gamma_{12}(0, P_S^*)$, as needed.

We can now proceed to the proof of part ii). We want to provide conditions under which the system of KT conditions for maximality of a mutual guarantee has no solutions other than the corner ones (P_H or $P_S = 0$). Otherwise said, we want to provide conditions under which it admits no solutions $P_H > 0$, $P_S > 0$ (44).

Recall that:

$$\frac{dT_1}{dP_H}+\frac{dDC_1}{dP_H}-\frac{\partial\Gamma_{12}}{\partial P_H}-\frac{\partial\Gamma_{21}}{\partial P_H}=\frac{dT_1}{dP_H}+\frac{dDC_1}{dP_H}-\alpha\phi\times$$

$$\left\{\begin{array}{c} -\frac{dX_1^d}{dP_1}\int_0^{X_S^Z} xf\left(x,h(x)\right)dx - \frac{dX_1^d}{dP_1}\int_{X_S^Z}^{X_S^d} xf(x,h(x))\ dx+ \\ -\frac{1}{1-\tau}\int_0^{X_H^Z} yf\left(h(y),y\right)dy - \frac{dX_1^d}{dP_1}\int_{X_H^Z}^{X_H^d} yf(h(y),y)dy+ \\ +\frac{dX_1^d}{dP_1}\int_{X_S^d}^{+\infty} X_H^d f(x,X_H^d)dx \end{array}\right\}$$

$$\frac{dT_2}{dP_S}+\frac{dDC_2}{dP_S}-\frac{\partial\Gamma_{12}}{\partial P_S}-\frac{\partial\Gamma_{21}}{\partial P_S}=\frac{dT_2}{dP_S}+\frac{dDC_2}{dP_S}-\alpha\phi\times$$

$$\left\{\begin{array}{c} -\frac{1}{1-\tau}\int_0^{X_S^Z} xf\left(x,h(x)\right)dx - \frac{dX_2^d}{dP_2}\int_{X_S^Z}^{X_S^d} xf(x,h(x))dx+ \\ \frac{dX_2^d}{dP_2}\int_{X_H^d}^{+\infty} X_S^d f(X_S^d,y)dy - \frac{dX_2^d}{dP_2}\int_0^{X_H^Z} yf\left(h(y),y\right)dy+ \\ -\frac{dX_2^d}{dP_2}\int_{X_H^Z}^{X_H^d} yf(h(y),y)\ dy \end{array}\right\}$$

The two derivatives are null when:

$$\left\{\begin{array}{c} -\frac{dX_1^d}{dP_1}\int_0^{X_S^Z} xf\left(x,h(x)\right)dx - \frac{dX_1^d}{dP_1}\int_{X_S^Z}^{X_S^d} xf(x,h(x))\ dx+ \\ -\frac{1}{1-\tau}\int_0^{X_H^Z} yf\left(h(y),y\right)dy - \frac{dX_1^d}{dP_1}\int_{X_H^Z}^{X_H^d} yf(h(y),y)dy+ \\ +\frac{dX_1^d}{dP_1}\int_{X_S^d}^{+\infty} X_H^d f(x,X_H^d)dx = \left[\frac{dT_1(P_H)}{dP_H}+\frac{dDC_1(P_H)}{dP_H}\right]\frac{1}{\alpha\phi} \\ -\frac{1}{1-\tau}\int_0^{X_S^Z} xf\left(x,h(x)\right)dx - \frac{dX_2^d}{dP_2}\int_{X_S^Z}^{X_S^d} xf(x,h(x))dx \\ +\frac{dX_2^d}{dP_2}\int_{X_H^d}^{+\infty} X_S^d f(X_S^d,y)dy - \frac{dX_2^d}{dP_2}\int_0^{X_H^Z} yf\left(h(y),y\right)dy+ \\ -\frac{dX_2^d}{dP_2}\int_{X_H^Z}^{X_H^d} yf(h(y),y)\ dy = \left[\frac{dT_2(P_S)}{dP_S}+\frac{dDC_2(P_S)}{dP_S}\right]\frac{1}{\alpha\phi} \end{array}\right.$$

Consider the behavior of the lhs and rhs when $\alpha\to 0+$. On the lhs, all the default thresholds' derivatives ($\frac{dX_1^d}{dP_1},\frac{dX_2^d}{dP_2}$) are positive and bounded; the integrals are positive and bounded too, since both the tax shields and the default thresholds keep bounded: $X_S^Z,\ X_H^Z, X_S^d,\ X_H^d<\infty$. In particular, the last integral, which extends over an infinite range, is bounded, since

$$\frac{dX_1^d}{dP_1}X_H^d\int_{X_S^d}^{+\infty} f(x,X_H^d)dx=\frac{dX_1^d}{dP_1}X_H^d\Pr f(X_1>X_S^d,X_2=X_H^d)<\infty$$

The rhs diverges when $\alpha\to 0+$, since - from Appendix A - both for $i=H$ and $i=S$

$$\lim_{\alpha\to 0+}\left[\frac{dT_i(P_i)}{dP_i}+\frac{dDC_i(P_i)}{dP_i}\right]=-\tau(1-F_i(X_i^Z))\frac{dX_i^Z}{dP_i}\phi<\infty$$

It follows that there exists an α^* below which the rhs and the lhs cannot be equal. Below α^* there is no maximum with bilateral instead of unilateral guarantee. ■

9.3 Proof of proposition 4.1

Proof. i) With equally distributed cash flows and equal parameters ($\alpha_i=\alpha,\tau_i=\tau$), we have $P_{1S}^*=P_{2S}^*, P_{1H}^*=P_{2H}^*$,

$$TG_1(P_{1H}^*,P_{1S}^*)+DCG_1(P_{2H}^*,P_{1H}^*,P_{1S}^*)=TG_2(P_{2H}^*,P_{2S}^*)+DCG_2(P_{1H}^*,P_{2H}^*,P_{2S}^*)$$

and the value difference is null. Indifference between firm 1 being a beneficiary or a guarantor follows.

ii) recall that, under the convexity assumption, the guarantor is unlevered: $P^*_{iH} = 0, i = 1, 2$. Its default threshold and tax shield are null: $X^Z_{iH} = X^d_{iH} = 0$. On the tax side, this implies that[16] $T_i(P^*_{iH}) = T_i(0)$.

On the default-cost side, the unlevered guarantor has no expected default costs: $DC_i(P^*_{iH}) = DC_i(0) = 0$. This means that the default cost savings - modulo a sign change - are the ones incurred by the firm as a beneficiary: $DCG_i(0, 0, P^*_{iS}) = -DC_S(0, P^*_{iS})$.

Putting the tax and cost effects together, the value inequality $\nu_{12} > \nu_{21}$ becomes

$$TG_1(0, P^*_{1S}) - DC_S(0, P^*_{1S}) < TG_2(0, P^*_{2S}) - DC_S(0, P^*_{2S}) \tag{45}$$

We provide conditions under which this inequality is satisfied when debt of the Subsidiary does not optimize the first guarantee, but optimizes the second. To do this, we assume now that $P_{2S} = P^*_{1S}$ (instead of $P_{2S} = P^*_{2S}$). A fortiori, it will be satisfied when the Subsidiary debt in the first guarantee is optimized ($P_{2S} = P^*_{2S}$).

In order to examine the case of different default costs we need to assume $X_1 = X_2$ and $\tau_i = \tau$. If $P_{2S} = P^*_{1S}$, together with $X_1 = X_2$ and $\tau_i = \tau$, then $T_1(P^*_{1S}) = T_2(P^*_{1S})$. Inequality $\nu_{12} > \nu_{21}$ becomes

$$\alpha_2 \phi \mathbb{E}\left[X_2 1_{\left\{0 < X_2 < X_2^d(P_{2S}), X_1 > h(X_2)\right\}}\right] < \alpha_1 \phi \mathbb{E}\left[X_1 1_{\left\{0 < X_1 < X_1^d(P^*_{1S}), X_2 > h(X_1)\right\}}\right]$$

Notice that - since $X_1 = X_2$ and $P_{2S} = P^*_{1S}$ - the expectations are equal. The previous inequality reduces to

$$(\alpha_2 - \alpha_1)\, \phi \mathbb{E}\left[X_2 1_{\left\{0 < X_2 < X_2^d(P^*_{1S}), X_1 > h(X_2)\right\}}\right] < 0$$

which is true if and only if $\alpha_1 > \alpha_2$. The condition is necessary and sufficient when $P_{2S} = P^*_{1S}$, it becomes sufficient when P_{2S} is optimized in the first guarantee.

In order to examine the case of different tax rates we need to assume $X_1 = X_2$ and $\alpha_i = \alpha$. A reasoning similar to the previous one leads first to $DC_1(P^*_{1S}) = DC_2(P_{2S})$ and then to $\nu_{12} > \nu_{21}$being satisfied if and only if $\tau_1 < \tau_2$, when $P_{2S} = P^*_{1S}$. The condition is necessary and sufficient when $P_{2S} = P^*_{1S}$, it becomes sufficient when P_{2S} is optimized in the first guarantee.

iii) In order to examine the impact of size, we provide conditions under which firm 1 guarantees firm 2 even though the principals are the optimal ones for a guarantee from 2 in favour of 1. Formally, we provide conditions for

$$v_{12}(P^*_{1S}, P^*_{2H}) > v_{21}(P^*_{1S}, P^*_{2H}) \tag{46}$$

[16] The differential tax burdens between being a G and a B coincide with tax savings, as defined in section 3, computed at the optimal beneficiary's debt:

$$T_i(0) - T_i(P^*_{iS}) = TS_i(P^*_{iS})$$

Indeed, if this equality holds, a fortiori it will hold when the first guarantee is optimized, i.e. when it is computed at P_{1H}^{*}, P_{2S}^{*}, since

$$v_{12}(P_{1H}^{*}, P_{2S}^{*}) \geq v_{12}(P_{1S}^{*}, P_{2H}^{*})$$

If the principals are (P_{1S}^{*}, P_{2H}^{*}), then the difference in tax burden for each firm, under different guarantees, is null (recall indeed that taxes do not differ in expression across guarantees, they differ only when the principals do):

$$TG_i = \begin{cases} T_1(P_{1S}^{*}) - T_1(P_{1S}^{*}) = 0 \\ T_2(P_{2H}^{*}) - T_2(P_{2H}^{*}) = 0 \end{cases}$$

Inequality (46) reduces to:

$$DCG_1 > DCG_2$$

i.e.

$$\mathbb{E}\left[X_1 1_{\{0<X_1<X_1^d(P_{1S}^{*}),X_2<h(X_1)\}}\right] > \mathbb{E}\left[X_2 1_{\{0<X_2<X_2^d(P_{2H}^{*}),X_1<h(X_2)\}}\right] \quad (47)$$

On the lhs you have the default costs which firm 1 (when levered as a Subsidiary) incurs when firm 2 is unable to rescue her, since $X_2 < h(X_1)$. On the rhs the same quantity for firm 2, evaluated at P_{2H}^{*}. ■

9.4 Proof of theorem 4.5

Proof. Let us add to and subtract from the value differential $\nu_{0HS}(P_H, P_S) -$

$\nu_{0M}(P_M)$ the value of two Stand Alone firms with cash flows $X_H = X_1, X_S = X_2$:

$$\begin{aligned} & \nu_{0HS}(P_H, P_S) - \nu_{0M}(P_M) && (48) \\ = \quad & [\nu_{0HS}(P_H, P_S) - \nu_{01}(P_1) - \nu_{02}(P_2)] - [\nu_{0M}(P_M) - \nu_{01}(P_1) - \nu_{02}(P_2)] && (49) \end{aligned}$$

We know that the content of the first square brackets is always positive (non-negative without convexity) at the optimum. The content of the second one is

$$\begin{gathered} \nu_{0M}(P_M) - \nu_{01}(P_1) - \nu_{02}(P_2) = \\ \phi\left[-\mathbb{E}(X_M)^{+} + \mathbb{E}X_1^{+} + \mathbb{E}X_2^{+}\right] + \\ +\tau\phi[\mathbb{E}(X_M - X_M^Z)^{+} - \mathbb{E}(X_1 - X_1^Z)^{+} - \mathbb{E}(X_2 - X_2^Z)^{+}] + \\ +\alpha\phi\left[\mathbb{E}(X_M 1_{\{0<X_M<X_M^d\}}) - \mathbb{E}\left(X_1 \mathbf{1}_{\{0<X_1<X_1^d\}}\right) - \mathbb{E}\left(X_2 \mathbf{1}_{\{0<X_2<X_2^d\}}\right)\right] \end{gathered} \quad (50)$$

It cannot be signed without additional assumptions. We are going to show that - at the optimum - it is null under *(i)* and non-positive under *(ii)* and *(iii)*.

(i) Consider the case of equally distributed cash flows for the two merged activities, and denote their common value as X. Notice that (a) the corresponding Stand Alone firms have the same optimal debt and cash-flow thresholds,

namely $P_i = P^*, X_i^d = X^d, X_i^Z = X^Z, i = 1,2$ (b) the volatility of $2X$ is twice the one of X, if $\rho = 1$. Since in this case the Merger can be thought of as a Stand Alone with double cash flow and double volatility, homogeneity of degree one of optimal debt applies, as in Leland (2007), and we have $P_M^* = 2P^*, X_M^d = 2X^d, X_M^Z = 2X^Z$. It follows that the unlevered value differential is null:

$$\phi\left[-\mathbb{E}(X_M)^+ + \mathbb{E}X_1^+ + \mathbb{E}X_2^+\right] = \phi\left[\mathbb{E}(2X)^+ - 2\mathbb{E}X^+\right] = 0.$$

The differential tax burden is null:

$$\tau\phi[\mathbb{E}(X_M - X_M^Z)^+ - \mathbb{E}(X_1 - X_1^Z)^+ - \mathbb{E}(X_2 - X_2^Z)^+] =$$

$$= \tau\phi[\mathbb{E}(2X - (P_M - D_M))^+ - 2\mathbb{E}(X - (\frac{P_M}{2} - \frac{D_M}{2}))^+] = 0 \tag{51}$$

and differential default costs are null too:

$$\alpha\phi\left[\mathbb{E}(X_M 1_{\{0<X_M<X_M^d\}}) - \mathbb{E}\left(X_1\mathbf{1}_{\{0<X_1<X_1^d\}}\right) - \mathbb{E}\left(X_2\mathbf{1}_{\{0<X_2<X_2^d\}}\right)\right] \tag{52}$$

$$= \alpha\phi[\mathbb{E}(2X\mathbf{1}_{\{0<2X<X_M^d\}}) - 2\mathbb{E}X\mathbf{1}_{\{0<X<\frac{X_M^d}{2}\}}] = 0$$

As a result, the value of M and two Stand Alone firms coincide. Summing up,

in this case we have

$$\begin{aligned} \nu_{0HS}(P_H^*, P_S^*) - \nu_{01}(P_1^*) - \nu_{02}(P_2^*) &> 0, \\ \nu_{0M}(P_M^*) - \nu_{01}(P_1^*) - \nu_{02}(P_2^*) &= 0 \end{aligned}$$

Thus the value differential is positive.

(ii) Let the cash flows be Gaussian with the same volatility (even with different means), and let their volatility satisfy condition *(ii)*. Then Proposition 2 in Leland (2007) indicates that there exists a correlation coefficient ρ^Q such that the second-square-bracket content in (48) is negative (Merger is undesirable), when evaluated at P_M^*, provided that correlation is greater than ρ^Q and less than perfect. For perfect correlation it is null.

(iii) If cash flows are Gaussian with different volatilities, then we know from Proposition 4 in Leland (2007) that there exists a correlation coefficient ρ^R such that the second-square-bracket content in (48) is negative, when evaluated at P_M^*, provided that correlation is greater than ρ^R and less than perfect. For perfect correlation it is null.

In cases *ii)* and *iii)* it follows that the value differential is positive (non-negative). ■

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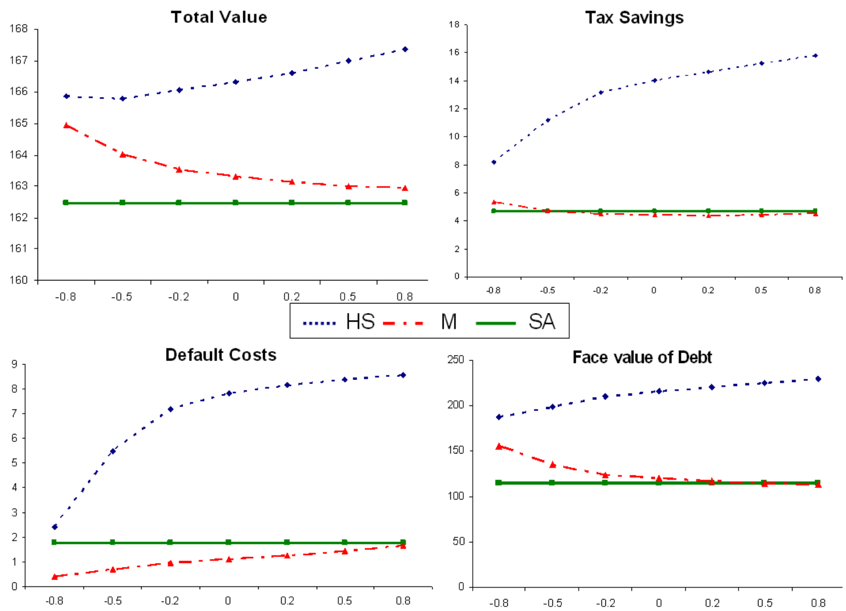

FIGURE 1. The upper left panel displays the the value of an HS (stars), a Merger (big and small dots) and two Stand Alone firms (dotted) as the correlation coefficient between the activities cash flows varies between -0.8 and +0.8. Similarly, the upper right panel displays the value of tax savings, the lower left panel the default costs and the last one the face value of debt.

Table 1: Optimal Capital Structure, Tax Savings and Guarantees - Base Case

		SA	H	S	1/2 HS	1/2 M
Face Value of Debt	P^*	57.10	0	220	110	58.5
Default Threshold	X^{d*}	67.65	0	249.27	-	69.64
Tax Shield	X^{Z*}	14.89	0	102.93	-	13.94
Levered Firm Value	$\nu_0^* = D_0^* + E_0^*$	81.47	49.46	117.13	83. 29	81.57
Leverage Ratio (%)	D_0^*/ν_0^*	51.81	0	99.9	70.26	54.62
Tax Savings of Leverage	TS_0^*	2.32	0	14.62	7.31	2.18
Expected Default Costs	DC_0^*	0.89	0	8.13	4.07	0.61
Value of the Guarantee	$\nu_{0i}^* - \nu_{01}^* - \nu_{02}^*, i =$HS,M				3.64	0.2

This table reports the optimal values of organizations under the base case parameters. The SA, H and S columns respectively refer to a Stand Alone unit, a Holding company and its Subsidiary. In the column 1/2 HS all the entries are the arithmetic mean of the figures of H and S, with the exception of the leverage ratio, which is $\frac{D_{0S}^*+D_{0H}^*}{\nu_{0HS}^*}$. The column 1/2 M reports half the figures of a Merger.

Table 2: Asymmetric default costs: capital structure and value

		SA	SA	1/2 SA	H	S	1/2 HS	1/2 M
Default Costs (%)	α	**75**	**23**		**75**	**23**		**49**
Face Value of Debt	P^*	33	57.10	45.05	0	220	110	46.5
Default Threshold	X^{d*}	39.247	67.65	-	0	249.27	-	55.42
Tax Shield	X^{Z*}	8.01	14.89	-	0	102.93	-	10.79
Levered Firm Value	$\nu_0^* = D_0^* + E_0^*$	80.83	81.47	81.15	49.46	117.13	83.29	81.24
Leverage Ratio (%)	D_0^*/ν_0^*	30.92	51.81	41.37	0	99.9	70.26	43.96
Tax Savings of Leverage	TS_0^*	1.25	2.32	1.79	0	14.62	7.31	1.69
Expected Default Costs	DC_0^*	0.46	0.89	0.68	0	8.13	4.07	0.46
Value of the guarantee	$\nu_{0i}^* - \nu_{01}^* - \nu_{02}^*$						4.29	0.17

This table reports optimal values for organizations whose units differ in default costs α.Other parameters are set to the base case. The SA, H and S columns respectively refer to a Stand Alone unit, a Holding company and its Subsidiary, taking into consideration that the optimal Holding company has higher default costs. In the column 1/2 HS entries are the arithmetic mean of the figures of H and S, with the exception of the leverage ratio, which is $\frac{D_{0S}^*+D_{0H}^*}{\nu_{0HS}^*}$. The column 1/2 M reports half the figures of a Merger. The DC of the Merger are set to the arithmetic mean of 23% and 75%.

Table 3: Asymmetric volatilities

		SA	SA	1/2 SA	H	S	1/2 HS	1/2 M
Volatility (%)	$\boldsymbol{\sigma}$	**15**	**27.5181**		**15**	**27.5181**		**32.63**
Face Value of Debt	P^*	61	60	60.5	0	215	107.5	58.5
Default Threshold	X^{d*}	72.70	67.65	-	0	243.60	-	69.64
Tax Shield	X^{Z*}	14.19	17.75	-	0	100.60	-	13.94
Levered Firm Value	$\nu_0^*= D_0^*+E_0^*$	81.69	81.71	81.70	52.87	114.84	83.86	81.57
Leverage Ratio (%)	D_0^*/ν_0^*	57.31	51.71	54.50	0	97.36	68.21	54.62
Tax Savings of Leverage	TS_0^*	2.22	2.71	2.47	0	13.74	6.87	2.18
Expected Default Costs	DC_0^*	0.53	1.27	0.90	0	6.29	3.15	0.61
Value of the Guarantee	$\nu_{0i}^* - \nu_{01}^* - \nu_{02}^*$						4.32	-0.26

This table reports optimal values for organizations whose units differ in cash flow volatility σ.Other parameters are set to the base case. The SA, H and S columns respectively refer to a Stand Alone unit, a Holding company and its Subsidiary, taking into consideration that the optimal Holding company has lower volatility. In the column 1/2 HS all the entries are the arithmetic mean of the figures of H and S, with the exception of the leverage ratio, which is simply $\frac{D_{0S}^*+D_{0H}^*}{\nu_{0HS}^*}$. The column 1/2 M reports half the figures of a Merger. The volatility of the Merger is, by definition, the volatility of the sum of its cash flows.

Table 4: Asymmetric size

		SA	SA	1/2SA	H	S	1/2 HS	1/2 M
Cash Flow Size		**1/3**	**5/3**	**1**	**5/3**	**1/3**	**1**	**1**
Face Value of Debt	P^*	19	95	57	68	121	94.5	57.5
Default Threshold	X^{d*}	22. 50	112. 54	67.52	72.08	134.06	-	67. 81
Tax Shield	X^{Z*}	4.98	24.85	14.92	16.31	52.25	-	13. 765
Levered Firm Value	$\nu_0^*= D_0^*+E_0^*$	27.16	135.78	81.47	97.38	68.75	83. 06	81.49
Leverage Ratio (%)	D_0^*/ν_0^*	51.73	51.73	51.73	53.08	100	71.95	53.06
Tax Savings of Leverage	TS_0^*	0.77	3.85	2.31	2.54	6.23	4.39	2.23
Expected Default Costs	DC_0^*	0.30	1.48	0.89	0.47	2.26	1.36	0.75
Value of the Guarantee	$\nu_{0i}^* - \nu_{01}^* - \nu_{02}^*$						3.19	0.04

This table reports optimal values for organization whose units differ in the mean level of cash flows. The smaller unit has size $\mu_i= 1/3\mu$ while the larger has $\mu_i= 5/3\mu$, where μ =127.63. As a consequence, the size of HS and M is 2. Other parameters are set to the base case. The SA, H and S columns respectively refer to a Stand Alone unit, a Holding company and its Subsidiary, taking into consideration that the optimal Holding company has bigger size. In the column 1/2 HS all the entries are the arithmetic mean of the figures of H and S, with the exception of the leverage ratio, which is simply $\frac{D_{0S}^*+D_{0H}^*}{\nu_{0HS}^*}$. The column 1/2 M reports half the figures of a Merger.