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Abstract

A price reveal auction is a Dutch auction in which the current price of the item on sale remains hidden. Bidders can privately observe the price only by paying a fee, and every time a bidder does so, the price falls by a predetermined amount. We show that in equilibrium, no rational bidder should enter into such an auction. Contrary to this prediction, data about actual price reveal auctions run on the Internet show that bidders do enter and that the mechanism is profitable for the seller.

JEL Classification: D44, C72.

Keywords: price reveal auctions, pay-per-bid auctions.

1 Introduction

This paper provides the first analysis of a new and peculiar online selling mechanism, the so-called price reveal auctions. A price reveal auction is a Dutch auction (i.e., descending price) in which the current price of the item

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on sale is not publicly observable. Each bidder can privately observe this price by paying a fee c. The bidder is then given a limited amount of time (say, 30 seconds) to decide if he wants to buy the good at the current price. If the bidder buys the good, the auction ends. Otherwise, the price is decreased by a fixed amount $\Delta < c$ and the auction keeps going.[1] In other words, in a price reveal auction the price is hidden and falls by Δ every time a bidder observes it. Therefore, and contrary to standard Dutch auction procedures, the price does not fall exogenously at a predetermined speed but rather endogenously in response to bidders' behavior.

A price reveal auction is an example of the more general category of pay-per-bid auctions. These mechanisms enrich traditional auction formats with some more "exotic" elements and have recently gained some traction on the Internet. The reason is that items are usually sold for extremely low prices (often less than 5% of the market value). Nevertheless, these auction formats turn out to be profitable for the seller, as the revenues that he collects through the bidding fees more than compensate for the low selling price.

Lowest unique bid auctions (LUBAs) and penny auctions are certainly the most prominent examples of pay-per-bid auctions. In a LUBA bidders place secret bids, and the winner is the one who submits the lowest unique offer, i.e., the lowest offer that is not matched by any other bid. Theoretical and/or empirical analysis of LUBAs appear in Eichberger and Vinogradov (2008), Gallice (2009), Östling *et al.* (2009), Rapoport *et al.* (2009) and Houba *et al.* (2010). In a penny auction each bid increases the current price by a fixed amount (a penny) and restarts a public countdown; the winner is the bidder who holds the winning bid at the moment the countdown expires.

[1] Notice that the decision to observe the price remains private so that no bidder can infer the current price from the number of times the price has been observed.

Various aspects of penny auctions are analyzed in Augenblick (2009), Byers *et al.* (2010), Hinnosaar (2010) and Platt *et al.* (2010).

The attention that pay-per-bid auctions have raised should not come as a surprise, as there are many interesting aspects that characterize these mechanisms. First, these are games in which bidders face complex strategic situations whose equilibria are non trivial. Second, it is relatively easy to retrieve field and/or experimental data about bidders' actual behavior; this allows researchers to test theoretical predictions and identify other empirical regularities. Finally, the optimal design of these mechanisms is still unclear.[2]

In this paper we investigate these issues in the context of price reveal auctions. We show that this auction format has a very clear cut prediction. In fact, according to the perfect Bayesian equilibrium of the game, no bidder should submit any bid; rational bidders realize that no player has any incentive to submit the first few bids given that the private costs of observing the price outweigh the benefits. As such, the price does not fall, the item remains unsold and the auctioneer makes zero profits. Contrary to this prediction (and thus contrary to the hypothesis of rational behavior of the bidders), data about actual price reveal auctions show that entry occurs and that the mechanism is moderately profitable. We discuss the introduction of two rules that could further stimulate entry and increase the profitability of this auction format.

2 The model

We model the sale of a single indivisible item through a price reveal auction. Let $N = \{1, ..., n\}$ be a set of risk-neutral potential buyers. The auctioneer (a

[2]This is an important issue, as pay-per-bid mechanisms seem particularly suitable for charitable purposes.

non-strategic player) sets the starting price at $p_0 = v_r$ where v_r is the retail price of the good. The starting price remains always visible while the current price p_t is not publicly observable. At any period $t \in \{1, ..., T\}$ each bidder $i \in N$ plays $a_{i,t} \in \{\phi, obs \times \{nb, b\}\}$. Action $a_{i,t} = \phi$ indicates that player i remains inactive. Action $a_{i,t} = (obs, nb)$ indicates that agent i observes the current price p_t but decides not to buy the good. Action $a_{i,t} = (obs, b)$ indicates that agent i observes the price p_t and decides to buy the good. A bidder is charged the fee $c > 0$ whenever he observes p_t, i.e., whenever $a_{i,t} \neq \phi$. Every time that a bidder plays $a_{i,t} \neq \phi$, the price decreases to $p_{t+1} = p_t - \Delta$ with $\Delta \in (0, c)$.[3] Otherwise, $p_{t+1} = p_t$. The auction ends at $t_e \in \{1, ..., T\}$ where $t_e = T$ if $a_{i,t} \neq (obs, b)$ for any i and any t while $t_e = \hat{t}$ as soon as a bidder plays $a_{i,\hat{t}} = (obs, b)$.

Bidders have a valuation v_i for the good on sale. We assume that each v_i is independently and identically distributed on the interval $[0, v_r]$ according to the cumulative distribution function F which is strictly increasing and continuously differentiable with density f. In line with the standard independent private value assumption, each bidder knows v_i and knows that every $v_{j \neq i}$ is drawn from F. Finally, let $\eta_{i,t} \in \mathbb{N}_0$ be the number of times bidder i observes the price (and thus pays the fee c) up to period t included. Payoffs, therefore, take the following form:

$$u_{i,t_e} = \begin{cases} v_i - p_{t_e} - c\eta_{i,t_e} & \text{if } i \text{ buys the good} \\ -c\eta_{i,t_e} & \text{otherwise} \end{cases} \qquad \text{for } i \in N$$

A price reveal auction is thus an extensive game with imperfect information, as bidders do not know p_t and do not observe rivals' valuations and actions.

[3]If $\Delta > c$ then a bidder could drive p_t down to zero by playing $\frac{v_r}{\Delta}$ times action $a_{i,t} = (obs, nb)$. This strategy would cost $\frac{c}{\Delta} v_r < v_r$ and would thus ensure positive profits as far as $\frac{c}{\Delta} v_r < v_i$ where v_i is bidder i's valuation.

As a solution concept, we apply the notion of perfect Bayesian equilibrium. Before properly defining such an equilibrium (Proposition 1), we first discuss some of its characteristics and introduce some additional notations.

First, notice that bidders accumulate sunk costs c every time they observe the price. Therefore, a player would ideally observe p_t only once, discover a price that he likes and buy the item. More precisely, the bidder should observe the price when he believes p_t has reached what he thinks to be its optimal level $b^*(v_i)$. Let $\mu_{i,t}(\cdot)$ indicate the beliefs bidder i holds at time t. A bidder should then play $a_{i,t} = (obs, b)$ when $\mu_{i,t}(p_t = b^*(v_i)) \geq \tilde{\mu}_i$ where $\tilde{\mu}_i$ is some threshold to be defined later. On the contrary, if $\mu_{i,t}(p_t = b^*(v_i)) < \tilde{\mu}_i$ then the agent should play $a_{i,t} = \phi$.

As for $b^*(v_i)$, this value optimally solves the trade-off between the surplus that the agent may realize and the risk of being preempted by the rivals. To explicitly define $b^*(v_i)$, we analyze the bidding behavior that would emerge in a standard Dutch auction characterized by the same parameters v_r, n, F, and c. In a Dutch auction p_t falls exogenously and is always visible so that a player does not have to form beliefs about the current price; thus, his optimal strategy is to buy the product when $p_t = b^*(v_i)$. Exploiting a fundamental result of auction theory (see Klemperer, 1999 and Krishna, 2002), we know that such a Dutch auction is strategically equivalent to a first price sealed bid auction with entry fee c.

The latter situation has been carefully investigated by Menezes and Monteiro (2000), who show that in the symmetric Bayesian equilibrium, bidders enter the auction only if their valuation is above a cutoff value ρ. This value is implicitly defined by the condition $\rho F(\rho)^{n-1} - c = 0$ which imposes indifference in terms of the entry of a player for which $v_i = \rho$. Menezes and Monteiro

(2000) also show that a bidder with $v_i \geq \rho$ bids according to:

$$b^*(v_i) = \frac{1}{F(v_i)^{n-1}} \int_{\rho}^{v_i} (n-1)xF(x)^{n-2}f(x)dx \tag{1}$$

Integrating the above expression by parts, we can reformulate the optimal bidding strategy as:[4]

$$b^*(v_i) = v_i - \rho\frac{F(\rho)^{n-1}}{F(v_i)^{n-1}} - \frac{1}{F(v_i)^{n-1}} \int_{\rho}^{v_i} F(x)^{n-1}dx \tag{2}$$

which clearly shows that $b^*(v_i) < v_i$ for any $v_i > \rho$ and any $n \geq 2$. We can now properly define the equilibrium of a price reveal auction:

Proposition 1 *In the unique Bayesian perfect equilibrium of a price reveal auction, bidders play* $\left(a^*_{i,t}\right)_{t=1}^{T}$*, where* $a^*_{i,t}$ *is such that:*

$$a^*_{i,t} = \begin{cases} (obs, b) & \text{if } \mu_i(p_t \leq b^*(v_i)) \geq \tilde{\mu}_i \\ \phi & \text{otherwise} \end{cases}$$

where $b^*(v_i)$ *is as in (2),* $\tilde{\mu}_i = \frac{c}{v_i - b^*(v_i)}$ *and every bidder holds beliefs* $\mu_{i,t}(p_t \leq b^*(v_i)) = 0$ *at any* t*. Therefore, on the equilibrium path, all bidders play* $(\phi)_{t=1}^{T}$*.*

Proof. First notice that, at any t, action $a_{i,t} = (obs, nb)$ is strictly dominated for any possible beliefs player i holds about p_t. More precisely, $a_{i,t} = (obs, nb)$ is dominated by $a_{i,t} = (obs, b)$ whenever $\mu_{i,t}(p_t \leq b^*(v_i)) \geq \tilde{\mu}_i$ and by $a_{i,t} = \phi$ whenever $\mu_{i,t}(p_t \leq b^*(v_i)) < \tilde{\mu}_i$. Therefore only actions $a_{i,t} = (obs, b)$ and $a_{i,t} = \phi$ can be part of the equilibrium. To solve for the beliefs threshold $\tilde{\mu}_i$, consider the condition:

$$\tilde{\mu}_i(v_i - b^*(v_i) - c) - (1 - \tilde{\mu}_i)c = 0$$

[4] Let $g(x) = x$ and $h(x) = F(x)^{n-1}$. Then $b^*(v_i) = \frac{1}{h(v_i)} \int_{\rho}^{v_i} g(x)h'(x)dx$. Integration by parts leads to $b^*(v_i) = \frac{1}{h(v_i)} \left(g(v_i)h(v_i) - g(\rho)h(\rho) - \int_{\rho}^{v_i} g'(x)h(x)dx\right)$.

which imposes indifference between the expected payoffs associated with action $a_{i,t} = (obs, b)$ and action $a_{i,t} = (\phi)$. Solving for $\tilde{\mu}_i$, one gets $\tilde{\mu}_i = \frac{c}{v_i - b^*(v_i)}$ which defines the equilibrium. Now, given that $b^*(v_i) < v_i \leq v_r$ for every i and that $\Delta < c$, on the equilibrium path all bidders play $a_{i,1} = \phi$. Therefore, it is common knowledge that $p_2 = v_r$. This implies $\mu_{i,2}(p_2 \leq b^*(v_i)) = 0 < \tilde{\mu}_i$ for every i so that $a_{i,2} = \phi$ and $p_3 = v_r$. The same reasoning iterates as sequential rationality implies $p_t = v_r$ and $\mu_{i,t}(p_t \leq b^*(v_i)) = 0 < \tilde{\mu}_i$ for any i and any t. It follows that, on the equilibrium path, all bidders play $(\phi)_{t=1}^T$. ■

Proposition 1 states that in equilibrium, no bidder ever observes p_t. Therefore, the good remains unsold, $u_{i,t_e} = 0$ for every i with $t_e = T$ and the auction raises zero profits for the seller.[5] Notice that this result holds for any distribution F as well as for any n. As such, it also holds in cases in which players do not know the number of opponents and/or individual valuations are highly dispersed.

Example 1 *Consider a price reveal auction with* $v_r = 100$, $c = 0.8$, $\Delta = 0.4$, *and* $i \in \{1, 2, 3\}$. *Let* F *be uniform on* $[0, 100]$ *with* $v_1 = 10$, $v_2 = 50$, *and* $v_3 = 95$. *Then,* $\rho = \sqrt[3]{8000} = 20$. *As such agent* 1 *does not enter the auction while* $b^*(v_2) = 26.4$ *and* $b^*(v_3) = 59.42$. *But given that the condition* $p_t \leq b^*(v_2)$ *(resp.* $p_t \leq b^*(v_3)$*) requires* p_t *to be observed* 184 *(resp.* 102*) times and that with rational players this event has zero probability to occur, then* $\left(a_{2,t}^*\right)_{t=1}^T = \left(a_{3,t}^*\right)_{t=1}^T = (\phi)_{t=1}^T$. *Therefore, none of the three players submits any bid.*

Thus, the current design of price reveal auctions is flawed as the mechanism does not trigger the initial entry of the bidders. Clearly, not much would change

[5] Payoffs for the seller s with valuation $v_s \in [0, v_r]$ are given by $u_{s,t_e} = p_{t_e} - v_s + c\sum_{i \in N} \eta_{i,t_e}$ if a bidder buys the good and by $u_{s,t_e} = c\sum_{i \in N} \eta_{i,t_e}$ otherwise.

if the auctioneer set the initial price p_0 at a level $\tilde{v} < v_r$.[6] In what follows, we will briefly discuss two alternative rules that could stimulate entry and thus improve the profitability of the mechanism.

The first suggestion is to make the starting price p_0 random. More precisely, the rule should specify that the seller sets p_0 according to some distribution G defined on $[kv_r, v_r]$ with $k \in (0, 1)$. Bidders are not informed about p_0 but they know v_r, k, and G. The second suggestion is to modify the law of motion of p_t so as to make it partly exogenous. In particular, the rule should state that p_t falls by Δ not only every time a bidder observes the price but also every s periods where s is distributed on $\{s_{\min}, ..., s_{\max}\}$ according to some distribution H. In this case, bidders do not know s but they know $s_{\min}$, $s_{\max}$, and H. Both rules would make the game appealing also for rational players, as the event $p_t < b^*(v_i)$ would now have positive probability to occur.

3 Some data

In this section we present some summary statistics about actual price reveal auctions that have been organized by the website bidster.com.[7] This website first introduced price reveal auctions on the 29th of December 2009. On average a new price reveal auction is offered every 2-3 days and auctions do not expire (i.e., $T = \infty$). Therefore an auction closes only when a bidder buys the product. At the moment of writing, 70 auctions have closed and, in every

[6] In fact, if there exists at least a bidder i for which $v_i > \tilde{v} + c$, then this agent immediately buys the good and the auction ends at $t = 1$. On the contrary, if $v_i \leq \tilde{v} + c$ for every i, the situation is analogous to the one described in Proposition 1 so $u_{s,t_e} = 0$.

[7] Bidster.com is the European market leader for pay-per-bid auctions. Other websites offering price reveal auctions include dealwonders.com, bidmoo.com, bidenvelope.com, and many others.

auction, the relationship $\Delta = \frac{1}{2}c$ holds.[8]

Variables	average	min	max	st. dev.	sum
Retail price v_r	540.04	59	$1,700$	406.39	$37,803$
Selling price p_{t_e}	365.62	6	1240.25	294.10	$25,593$
Discount $(1 - p_{t_e}/v_r)$	0.36	0.07	0.97	0.17	
Bidding fee c	1.25	0.5	2	0.76	
Price decrease Δ	0.625	0.25	1	0.38	
# of times p_t has been observed	444.36	4	$3,595$	528.99	$31,105$
Estimated profits[9]	174.42	4	983	161.82	$12,209$
Estimated profits (% wrt v_r)	35.8	6.8	97	17.3	

Table 1. Some summary statistics of the data. All monetary values are in €.

By eyeballing Table 1, it is immediately evident that the data are hardly consistent with what the equilibrium analysis suggests. Proposition 1 indicates in fact that in equilibrium, no bidder ever observes the price. This implies that a price reveal auction should raise zero profits. The data show instead that, on average, the current price p_t has been observed 444.36 times in every auction. This entails average profits that amount to 35.8% of the retail price v_r. Profits are thus much lower than those that LUBAs or penny auctions raise but still significant.[10]

Notice that equilibrium analysis also indicates that the number of times bidders observe the price p_t remains at zero no matter the value of the item v_r

[8] The dataset is available upon request. Typical items include popular game consoles, mobile phones, and digital cameras.

[9] Estimated profits are computed according to the formula $p_{t_e} - v_s + c \sum_{i \in N} \eta_{i,t_e}$ and under the assumption $v_s = v_r$. The total number of times the price has been observed is given by $\sum_{i \in N} \eta_{i,t_e} = \frac{v_r - p_{t_e}}{\Delta}$.

[10] Gallice (2009) reports an estimated 441% for LUBAs while Byers *et al.* (2010) find a value of 86% in the case of penny auctions.

and the bidding fee c. In other words, $\#_\psi$ is independent of $v_{r,\psi}$ and c where we use the subscript $\psi \in \{1, ..., 70\}$ to index the individual auction. We test these relationships by estimating the equation $\#_\psi = \alpha + \beta_1 v_{r,\psi} + \beta_2 c_\psi + \epsilon_\psi$ using OLS. The estimated equation (with standard errors in parenthesis) is given by:

$$\# = \underset{(104.16)}{379.21} + \underset{(0.10)}{0.77} v_r - \underset{(54.65)}{280.61} c$$

which indicates that both coefficients β_1 and β_2 are significantly different from zero at the 5% level.

While these results could be easily rationalized by assuming the existence of some boundedly rational agents, they are clearly inconsistent with the assumption of perfect rationality of all the bidders.

4 Conclusions

The success that pay-per-bid mechanisms are experiencing on the Internet raises a number of interesting questions and it also provides a clear opportunity to observe people's behavior in the field. In this paper we have analyzed the most recent example of these mechanisms, the so-called price reveal auctions. We proved that if agents were fully rational, a price reveal auction should attract zero bids and thus lead to zero profits for the seller. Contrary to this prediction, data about actual price reveal auctions show that players submit enough bids to make the mechanism profitable. We interpret this inconsistency as evidence of bidders' limited rationality, and we claim that this "naïveté" is the only possible source of profits for the sellers.

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