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Ex Post Portfolio Performance with Predictable Skewness and Kurtosis*

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Abstract

This paper examines the ex-post performance of optimal portfolios with predictable returns, when the investor horizon ranges from one month to ten years. Due to the investor's ability to anticipate shifts from bull to bear markets, predictability involves the risk premium, volatility and correlations, and may extend to third and fourth moments. We analyze three different equity portfolios datasets, each covering more than eight indexes, including the commonly used US Industry and International Book-to-Market portfolios. Allowing for regimes improves portfolio performance for at least a subset of investment horizons in all datasets. Despite large non-normalities in both the Industry and the BM dataset, gains from predicting higher order moments obtain only in the latter - where third rather than fourth moments matter.

The equally weighted strategy usually leads to lower ex-post performance measures than optimizing ones, despite simple econometrics and power utility preferences underlying optimal strategies.

Key words: Stock Market Regimes, Return Predictability, Skew and Kurtosis, Equity Diversification

JEL code: G11, F37, C22, C51.

1. **Introduction**

Risk-adjusted profits of portfolio managers derive from their ability in forecasting returns out-of-sample. Recently, Bossaerts and Hillion (1999), Ang and Bekaert (2007) and Goyal and Welch (2008) cast doubts on prevailing linear methods for predicting out-of-sample, which are reinforced by the inability of optimizing strategies in obtaining out-of-sample gains relative to a naive equally-weighted strategy (DeMiguel, Garlappi

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and Uppal, 2009). Avramov and Chordia (2006) do find large out-of-sample gains[1]. However they consider individual stocks as opposed to diversified equity portfolios, which favour optimal strategies over simpler ones.

Importantly, all these papers restrict attention to mean-variance preferences in computing out-of-sample welfare gains, thus overlooking the fact that investors appear to care about both asymmetries and tails of the wealth distribution - as indicated by the asset pricing literature (Harvey and Siddique, 2000; Dittmar, 2002; Kumar, 2005; Guidolin and Timmermann, 2008a). This leaves open the possibility of out-of-sample welfare gains deriving from the impact of predictable higher order moments of stock returns on optimal portfolio composition.

Moreover, there is mounting evidence that non-linear models in general and Markov-switching models in particular provide superior fit. Ang and Chen (2002) report that regime switching models replicate the asymmetries in correlations observed in US stock returns better than GARCH-M. Lettau and Van Nieuwerburgh (2008) suggest that the presence of changing steady state means in the dividend-price ratio may explain why it proves so difficult to predict stock returns out of sample with such ratio. Guidolin and Nicodano (2008) find that, both in-sample and out-of-sample, regime switching models with time-varying covariance matrix fare as well as or better than multivariate ARCH models in an international dataset of size indexes.

This paper provides extensive evidence of out-of-sample performance of optimal portfolio strategies in three datasets that are commonly used by both academics and the industry, at monthly frequencies. We analyze ten US industry portfolios (IND) and eleven Book-to-Market (BM) international portfolios, along with eight international and emerging (IE) market stock indexes. In most cases we find out-of-sample gains relative to the equally-weighted strategies for investors who have one period horizons, as those studied by DeMiguel et al.(2009).

Several papers already indicate that predicting higher order moments changes the composition of optimal portfolios (e.g. Ang and Bekaert (2002), Guidolin and Timmermann (2008a), Guidolin and Nicodano (2008), Jondeau and Rockinger (2009)), because investors overweight equity indexes that increase positive wealth skewness and reduce excess wealth kurtosis relative to mean-variance portfolios. These papers - with the exception of Ang and Bekaert (2002) and Guidolin and Timmermann (2008a) - use however weekly data, which amplify the importance of higher order moments relative the commonly used monthly returns. Moreover, each focusses on one set of equity indexes only. Importantly, they offer little discussion of out-of-sample gains and - when they do (as in Jondeau and Rockinger (2009)) - they do not explore the effects on predictability on longer run portfolio performance.[2] A second contribution of our paper consists in analyzing how the investor horizon, which ranges from 1 month to 10 years, affects ex-post gains.

The prevailing linear forecasting methods - such as those in DeMiguel et al. (2009) - describe stock returns as randomly fluctuating around *one* mean return with *one given* volatility. Our portfolio strategies are instead based on models for returns that allow stock markets to persistently remain in either a bear or a bull regime. If the US stock market is in a bear regime, future returns of a given equity portfolio will be

[1] Allowing for a small amount of aversion to ambiguity about returns also leads to an *out-of-sample* increase in Sharpe ratio. See Garlappi et al., 2007.

[2] Exceptions are Fugazza *et al* (2009) and Diris *et al* (2008), who however focus on linear forecasting models.

expected to fluctuate around a given mean return with a given volatility - unless the stock market moves to a bull regime, which may happen with a positive probability that is updated on the basis of upcoming information. In that case, future returns fluctuate around a higher mean return with a lower volatility - unless the stock market returns to the bear regime. In our analysis, this representation fits the return data better than the gaussian i.i.d. representation according to standard statistical tests for both US Industry and International BM data.[3]

A state dependent return representation also has a number of advantages from the point of view of portfolio management. Given that returns are assumed to be normal conditional on a given regime, assets are characterized by the familiar expected return - variance statistics in each regime. It is therefore immediate to identify a defensive industry as one having a relatively high return in the bear state, compared to other industries. It also allows to generalize the concept to higher order moments. A truly defensive industry also contributes to increase the skewness of wealth, i.e. has a relatively low variance in the bear regime, and to reduce wealth kurtosis by displaying relatively low variance in highly volatile bear markets. North American stocks and Energy appear to be truly defensive portfolios in our data sets.

There are other well known econometric advantages from using such regime-switching representation. First, the data identify the number of stock market regimes, without the econometrician having to impose them exogenously. Second, it is possible to estimate higher order moments more precisely with a limited amount of observations, because they are a function of the transition probabilities plus conditional means and covariance terms (Timmermann, 2000). Thus, measuring the skewness requires the estimation of fewer parameters than a traditional representation. Third, it is possible to nest other simpler forecasting models as special cases of a general Markov-switching process.

Last but not least, methods that account for systematic skewness and kurtosis in a regime-switching setting are often cumbersome and/or do not allow for consideration of several securities[4]: this may prevent their use by investors. Our paper uses a tractable approach developed by Guidolin and Timmermann (2008b) which is convenient to implement in the presence of non-normalities and large asset menus.

Another related literature deals with predicting and timing volatility on daily data (see Fleming *et al.*, 2001, and references therein), assuming constant expected returns given the short horizon under scrutiny. Here we investigate whether there are economic gains from predicting and timing up to the fourth moment, all of which are likely to vary over a monthly - or longer - horizon.

The rest of the paper proceeds as follows. We describe the optimal asset allocation problem in Section 2. Section 2.1. contains a description of the portfolio strategies under analysis, whereas the remaining ones provide technical details on the solution method as well as on the specification test concerning the return generating process. Section 3 describes our data set and the distribution of returns conditional on two states

[3]This confirms previous specification tests performed by Ang and Chen (2009) and Guidolin and Nicodano (2009), who also extend the comparison to non-linear models such as GARCH-M and EGARCH-VAR. Earlier studies uncover non-normal features in retuns of equity portfolios (Longin and Solnik, 2001) as well as found evidence of regimes (e.g., Ang and Bekaert (2002), Turner, Startz and Nelson (1989)).

[4]Numerical techniques such as quadrature methods (Ang and Bekaert (2001), Lynch (2001)) or Monte Carlo simulations (Barberis (2000)) may not be very precise when the return distributions are not Gaussian (Keim and Stambaugh (1986), Fama and French (1988) and Pesaran and Timmermann,1995) as is strongly suggested by empirical research. By contrast, Monte Carlo methods rely on discretization of the state space and use grid that are computationally expensive.

of the market. Section 4 reports results concerning the composition of optimal portfolios and their ex-post performance.

2. **Optimal Asset Allocation**

2.1. *Portfolio Strategies and Overview*

We summarize the portfolio strategies under scrutiny in Table 0. We allow the investor to have alternative preferences over moments of the distribution of final wealth. MV denotes mean-variance preferences, MVS indicates an investor who also likes positive skewness of wealth while a MVSK investor also dislikes fat tails in the wealth distribution.

Returns on stock indexes are allowed to follow two alternative processes. One is the traditional Gaussian-I.I.D. case, which we indicate with $k = 1$. Under such distribution, differences in optimal portfolio strategies across investors disappear, because both the skewness and the excess kurtosis of wealth are zero. However, such return distribution is rejected by both Industry and Book-to-Market data when we perform specification tests (see Table 2). Data support an alternative generating process, which we label $k = 2$, indicating that returns are normal conditional on two states of the market. In one such state, that we name "bull", expected returns are higher and volatilities are lower than in the "bear" state (see Table 3). Furthermore, it is possible to measure the contribution of each stock index not only to the variance of wealth, but to its skewness and its kurtosis as well. To this end, Table 4 reports the co-skewness and co-kurtosis matrices alongside the conventional covariance measures.

We combine alternative preferences with these two return distributions, as displayed in Table 1. And, for each combination, we compute optimal portfolios (Tables 5) and ex-post performance (Tables 6). When regimes are allowed for, optimal portfolios can be studied along different dimensions. First, we have the "average" portfolio composition when the investor does not know which state the market is in, but attributes to each its long-run probability. Then we have portfolio composition when the market is bear and when it is bull. Finally, we can measure how each portfolio share changes as the probability of being in a bear/bull state is updated by the investor (Figures 2 and 3).

We measure ex-post performance through three different indicators. One is the expected return to volatility ratio (Sharpe ratio), which is insensitive to both skewness and kurtosis of wealth. Thus, a portfolio strategy that increases downside risk is given the same grade as another one with the same Sharpe ratio that does not. This does not happen with the Sortino ratio, which falls when downside risk increases. The third performance metric is the certainty equivalent of maximum utility, which - in the case of a MVSK investor - also captures the higher moments of wealth.

Thus, we are able to assess whether the Sharpe/Sortino ratios of a MVSK investor exceeds the one of a MV investor. We can also analyze whether a MV investor would be better off considering time-varying mean and variances, by accounting for regimes. And we quantify the costs for a MVSK investor of using a Gaussian return distribution instead of the regime-switching one.

The following subsections provide technical details, and the reader who is interested only in empirical results may skip it.

2.2. *Investor Preferences*

This section describes the investor's objectives and the return generating process and goes on to characterize the method used to solve for the optimal asset allocation. We are interested in studying the asset allocation problem at time t for an investor with a T-period investment horizon. Suppose that the investor's utility function $U(W_{t+T};\boldsymbol{\theta})$ only depends on wealth at time $t+T$, W_{t+T}, and its shape is captured through the parameters in $\boldsymbol{\theta}$. The investor maximizes expected utility by choosing among h risky assets whose continuously compounded returns are given by the vector $\mathbf{r}_t^s \equiv (r_{1t}\ r_{2t}\ \ldots\ r_{ht})'$. Portfolio weights are collected in the vector $\boldsymbol{\omega}_t \equiv (\omega_{1t}\ \omega_{2t}\ \ldots\ \omega_{ht})'$. The portfolio selection problem solved by a buy-and-hold investor with initial unit wealth becomes[5]

$$\max_{\boldsymbol{\omega}_t} \quad E_t\left[U(W_{t+T}(\boldsymbol{\omega}_t);\boldsymbol{\theta})\right]$$
$$s.t.\ W_{t+T}(\boldsymbol{\omega}_t) = \left\{\boldsymbol{\omega}_t' \exp\left(\mathbf{R}_{t+T}^s\right)\right\}, \tag{1}$$

where $\mathbf{R}_{t+T}^s \equiv \mathbf{r}_{t+1}^s + \mathbf{r}_{t+2}^s + \ldots + \mathbf{r}_{t+T}^s$ is the vector of continuously compounded portfolio returns over the $T-$period investment horizon, and portfolio hares sum to 1. Accordingly, $\exp(\mathbf{R}_{t+T}^s)$ is a vector of cumulated portfolio returns. No short-selling can be imposed through the constraint $\omega_{it} \in [0,1]$ for $i = 1, 2, \ldots, h$.[6]

We approximate a Von-Neumann Morgenstern expected utility function with a function of four moments of the wealth distribution, of the form:

$$\hat{E}_t[U^m(W_{t+T};\boldsymbol{\theta})] = \sum_{n=0}^{m} \kappa_n E_t[(W_{t+T} - v_T)^n], \tag{2}$$

with $\kappa_0 > 0$, and κ_n positive (negative) if n is odd (even). When $n = 2$, the investor has mean-variance preferences (MV): under non-satiation and risk aversion, marginal utility is positive ($U' > 0$) and decreasing ($U'' < 0$) in wealth. Assuming decreasing absolute risk aversion, we further have $U''' > 0$ (investors prefer positive skew) while, as shown by Kimball (1993), decreasing absolute prudence implies that $U'''' < 0$.

2.2.1. **The Return Process**

We assume that the vector of continuously compounded returns, $\mathbf{r}_t = (r_{1t}, r_{2t}, \ldots, r_{ht})'$, is generated by a Markov switching vector autoregressive process driven by a common unobservable state variable, S_t, that takes integer values between 1 and k:

$$\mathbf{r}_t = \boldsymbol{\mu}_{s_t} + \sum_{j=1}^{p} \mathbf{A}_{j,s_t}\mathbf{r}_{t-j} + \boldsymbol{\varepsilon}_t. \tag{3}$$

Here $\boldsymbol{\mu}_{s_t} = (\mu_{1st}, \ldots, \mu_{hs_t})'$ is a vector of intercepts in state s_t, $\mathbf{A}_{j,s_t}$ is an $h \times h$ matrix of autoregressive coefficients associated with the jth lag in state s_t, and $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, \ldots, \varepsilon_{ht})' \sim N(\mathbf{0}, \boldsymbol{\Omega}_{s_t})$ is a vector of Gaussian

[5] Our partial equilibrium framework treats returns as exogeneous, as in Ang and Bekart (2001), Barberis (2000), Campbell et al. (2003), Das and Uppal (2004), and Kandel and Stambaugh (1996).

[6] Classic results by Merton (1969) and Samuelson (1969) obtain in special cases such as power utility with constant investment opportunities. Alternative solution methods to (1) under predictability of returns are described in Ang and Bekaert (2002), Barberis (2000), Brandt (1999), Brennan, Schwarz and Lagnado (1997), Campbell and Viceira (1999, 2001), Campbell, Chan and Viceira (2003), Kandel and Stambaugh (1996), and Lynch (2001).

return innovations with zero mean vector and state-dependent covariance matrix $\mathbf{\Omega}_{s_t}$:

$$\mathbf{\Omega}_{s_t} = E\left[\left(\mathbf{r}_t - \boldsymbol{\mu}_{s_t} - \sum_{j=1}^{p} \mathbf{A}_{j,s_t}\mathbf{r}_{t-j}\right)\left(\mathbf{r}_t - \boldsymbol{\mu}_{s_t} - \sum_{j=1}^{p} \mathbf{A}_{j,s_t}\mathbf{r}_{t-j}\right)' |s_t\right].$$

The state-dependence of the covariance matrix captures the possibility of heteroskedastic shocks to asset returns, which is supported by strong empirical evidence, c.f. Bollerslev et al.(1992). Each state is assumed to be the realization of a first-order, homogeneous Markov chain as the transition probability matrix, $\mathbf{P}$, governing the evolution in the common state variable, S_t, has elements

$$\mathbf{P}[i,j] = \Pr(s_t = j|s_{t-1} = i) = p_{ij}, \quad i,j = 1,..,k. \tag{4}$$

Conditional on knowing the state next period, the return distribution is Gaussian. However, since future states are never known in advance, the return distribution is a mixture of normals with the mixture weights reflecting the current state probabilities and the transition probabilities.

Even in the absence of autoregressive terms, (3)-(4) imply time-varying investment opportunities. For example, the conditional mean of asset returns is an average of the vector of mean returns, $\boldsymbol{\mu}_{s_t}$, weighted by the filtered state probabilities $(\Pr(s_t = 1|\mathcal{F}_t), .., \Pr(s_t = k|\mathcal{F}_t))'$, conditional on information available at time t, $\mathcal{F}_t$. Since these state probabilities vary over time, the expected return will also change. In addition, this setup can readily be extended to incorporate a range of predictor variables such as the dividend yield. This is done simply by expanding the vector $\mathbf{r}_t$ with additional predictor variables, $\mathbf{z}_t$, and modeling the joint process $\mathbf{y}_t = (\mathbf{r}_t' \;\, \mathbf{z}_t')'$.

When regimes are persistent and mean returns, $\boldsymbol{\mu}_{s_t}$, differ across states, expected returns vary over time. Similarly, when the covariance matrices, $\mathbf{\Sigma}_{s_t}$, differ across states, there will be predictability in higher order moments such as volatilities, correlations, skews and tail thickness. Predictability is therefore not confined to mean returns but carries over to the entire return distribution. Effectively, the return distribution is calculated as a weighted average of the individual, state-specific distributions using weights that are updated as new return data arrive.

2.2.2. **The Portfolio Allocation Problem**

We now indicate how to solve the investor's optimal asset allocation problem when preferences are defined over moments of terminal wealth (2) and returns follow the regime switching process (3)-(4). We follow Guidolin and Timmermann (2008) and compute the expected utility as

$$\hat{E}_t[U^m(W_{t+T};\boldsymbol{\theta})] = \sum_{n=0}^{m} \kappa_n \sum_{j=0}^{n} (\text{-}1)^{n-j} v_T^{n-j} \binom{n}{j} E_t\left[\left(\boldsymbol{\omega}_t' \exp\left(\mathbf{R}_{t+T}^s\right)\right)^j\right].$$

In turn, the nth moment of the cumulated return on the portfolio is given by:

$$E_t\left[\left(\boldsymbol{\omega}_t' \exp\left(\mathbf{R}_{t+T}^s\right)\right)^n\right] = \sum_{n_1=0}^{n} \cdots \sum_{n_h=0}^{n} \lambda(n_1, n_2, ..., n_h) \left(\prod_{i=1}^{h} \omega_i^{n_i}\right) M_{t+T}^{(n)}(n_1, ..., n_h),$$

where $\sum_{i=1}^{h} n_i = n$, $0 \leq n_i \leq n$ $(i = 1, ..., h)$,

$$\lambda(n_1, n_2, ..., n_h) \equiv \frac{n!}{n_1! n_2! \; ... \; n_h!}.$$

and $M_{t+T}^{(n)}(n_1, ..., n_h)$ can be evaluated recursively, using equations in the Appendix. The moments of the wealth distribution can thus be obtained by solving a small set of difference equations corresponding to the number of regimes in the return distribution. The otherwise complicated numerical problem of optimal asset allocation is reduced to one of solving for the roots of a low-order polynomial. This solution is closed-form in the sense that it is computable with a finite number of elementary operations.

In our application below we use $m = 4$ moments in the preference specification. The weights on the first four moments of the wealth distribution are determined to ensure that our results can be compared to those in the existing literature that uses power utility functions. For a given coefficient of relative risk aversion, θ, the power utility function serves as a guide in setting values of $\{\kappa_n\}_{n=0}^{m}$ in (2). Expanding the powers of $(W_{t+T} - v_T)$ and taking expectations, we obtain the following expression for the four-moment preference function:

$$\hat{E}_t[U^4(W_{t+T}; \theta)] = \kappa_{0,T}(\theta) + \kappa_{1,T}(\theta) E_t[W_{t+T}] + \kappa_{2,T}(\theta) E_t[W_{t+T}^2] + \kappa_{3,T}(\theta) E_t[W_{t+T}^3] + \kappa_{4,T}(\theta) E_t[W_{t+T}^4], \quad (5)$$

where[7]

$$\begin{aligned}
\kappa_{0,T}(\theta) &\equiv v_T^{1-\theta}\left[(1-\theta)^{-1} - 1 - \frac{1}{2}\theta - \frac{1}{6}\theta(\theta+1) - \frac{1}{24}\theta(\theta+1)(\theta+2)\right] \\
\kappa_{1,T}(\theta) &\equiv \frac{1}{6} v_T^{-\theta}\left[6 + 6\theta + 3\theta(\theta+1) + \theta(\theta+1)(\theta+2)\right] > 0 \\
\kappa_{2,T}(\theta) &\equiv -\frac{1}{4}\theta v_T^{-(1+\theta)}\left[2 + 2(\theta+1) + (\theta+1)(\theta+2)\right] < 0 \\
\kappa_{3,T}(\theta) &\equiv \frac{1}{6}\theta(\theta+1)(\theta+3) v_T^{-(2+\theta)} > 0 \\
\kappa_{4,T}(\theta) &\equiv -\frac{1}{24}\theta(\theta+1)(\theta+2) v_T^{-(3+\theta)} < 0.
\end{aligned}$$

Notice that the expected utility from final wealth increases in $E_t[W_{t+T}]$ and $E_t[W_{t+T}^3]$, so that higher expected returns and more right-skewed distributions lead to higher expected utility. Conversely, expected utility is a decreasing function of the second and fourth moments of the terminal wealth distribution.

A solution to the optimal asset allocation problem can now easily be found from (5) by solving a system of cubic equations in $\hat{\boldsymbol{\omega}}_t$ derived from the first and second order conditions

$$\nabla_{\boldsymbol{\omega}_t} \hat{E}_t[U^4(W_{t+T}; \theta)]\Big|_{\hat{\boldsymbol{\omega}}_t} = \mathbf{0}', \qquad H_{\boldsymbol{\omega}_t} \hat{E}_t[U^4(W_{t+T}; \theta)]\Big|_{\hat{\boldsymbol{\omega}}_t} \text{ is negative definite.}$$

Thus $\hat{\boldsymbol{\omega}}_t$ sets the gradient, $\nabla_{\boldsymbol{\omega}_t} \hat{E}_t[U^4(W_{t+T}; \theta)]$, to a vector of zeros and produces a negative definite Hessian matrix, $H_{\boldsymbol{\omega}_t} \hat{E}_t[U^4(W_{t+T}; \theta)]$.

[7]The notation $\kappa_{n,T}$ makes it explicit that the coefficients of the fourth order Taylor expansion depend on the investment horizon through the coefficient v_T, the point around which the approximation is calculated. We follow standard practice (e.g. Jondeau and Rockinger (2004)) and set the point around which the Taylor series expansion is computed to $v_T = E_t[W_{t+T-1}]$, the expected value of the investor's wealth for a $T-1$ period investment horizon.

Mean-Variance-Skew preferences are given by:

$$\hat{E}_t[U^3(W_{t+T};\theta)] = \kappa_{0,T}(\theta) + \kappa_{1,T}(\theta)E_t[W_{t+T}] + \kappa_{2,T}(\theta)E_t[W^2_{t+T}] + \kappa_{3,T}(\theta)E_t[W^3_{t+T}] \tag{6}$$

where now $\kappa_{0,T}(\theta) \equiv v_T^{1-\theta}\left[(1-\theta)^{-1} - 1 - \frac{1}{2}\theta - \frac{1}{6}\theta(\theta+1)\right]$, $\kappa_{1,T}(\theta) \equiv v_T^{-\theta}\left[1+\theta+\frac{1}{2}\theta(\theta+1)\right] > 0$, $\kappa_{2,T}(\theta) \equiv -\frac{1}{2}\theta v_T^{-(1+\theta)}(2+\theta) < 0$, and $\kappa_{3,T}(\theta) \equiv \frac{1}{6}\theta(\theta+1)v_T^{-(2+\theta)} > 0$.

while MV preferences simplify to:

$$\hat{E}_t[U^2(W_{t+T};\theta)] = \kappa_{0,T}(\theta) + \kappa_{1,T}(\theta)E_t[W_{t+T}] + \kappa_{2,T}(\theta)E_t[W^2_{t+T}] \tag{7}$$

where $\kappa_{0,T}(\theta) \equiv v_T^{1-\theta}\left[(1-\theta)^{-1} - 1 - \frac{1}{2}\theta\right]$, $\kappa_{1,T}(\theta) \equiv v_T^{-\theta}(1+\theta) > 0$, and $\kappa_{2,T}(\theta) \equiv -\frac{1}{2}\theta v_T^{-(1+\theta)} < 0$.

Thus the optimal portfolio composition, which in a standard MV problem with Gaussian return depends only on the variance-covariance matrix of returns and on risk aversion, also depends on:

1. differences between mean returns, μ_1, μ_2, and variances, σ_1, σ_2, (and more generally covariance parameters) across states. For example, skew in the return distribution can only be induced provided that $\mu_1 \neq \mu_2$, c.f. Timmermann (2000).

2. The current state probabilities $(\pi_t, 1-\pi_t)$ which determine moments of returns at all future points provided that either the mean or variance parameters differ across states ($\mu_1 \neq \mu_2$ or $\sigma_1 \neq \sigma_2$).

3. State transition probabilities which also affect the speed of mean reversion in the investment opportunity set towards its steady state.

4. The number of moments of the wealth distribution that matters for preferences, m, in addition to the weights on the various moments.

5. The investment horizon, T.

   Our benchmark results assume that $\theta = 5$, a coefficient of relative risk aversion compatible with the bulk of empirical evidence. Later we present robustness results that allow this coefficient to assume both larger and smaller values.

3. **Empirical Results**

3.1. *Data and Descriptive Statistics*

We analyze three datasets of monthly equity returns. The first one - which we label IE - comprises eight international portfolios, three of which are from emerging markets (1988:01-2008:08). The second one covers ten US industry portfolios (IND), from July 1926 to July 2008, while the third one refers to ten Book-to-Market (BM) sorted portfolios from five geographical areas plus the world market portfolios (1975:01-2007:12). Our choice of data, if anything, distorts against finding a large extent of non-normalities. Indeed, our focus is neither on individual security returns nor on data sampled at higher frequencies, where it is easier to uncover non-normal features. We also avoid analyzing portfolios of size and momentum portfolios, let alone hedge-fund returns, that are already known to display asymmetries that are exploitable in a portfolio setting (see e.g. Guidolin and Nicodano, 2008; Hong et al., 2007).

Table 1 reports summary statistics, with the lower parts displaying the correlation/co-kurtosis matrix and the co-skewness matrix respectively.

Panel A shows a wide dispersion of monthly mean returns and volatilities across international equity portfolios. Also the ratio of expected return to volatility (Sharpe) covers a wide range, from -0.064 and 0.059 for Japan and EM Asia, to 0.193 and 0.174 respectively for EM Europe & Middle East and EM Latin America. Correlations involving the UK and North America are generally higher than those involving Emerging Markets, whose cross-correlations never exceed 0.491.[8]

On the contrary, monthly mean returns are not particularly disperse in the US Industry dataset, ranging from 0.831 for Telecommunication, to 1.097 for Energy (Panel B). Yet their Sharpe ratios differ markedly, from 0.095 for Other Industries to 0.143 for Non Durables. Industry portfolios unsurprisingly display higher cross-correlations, always in excess of 0.5, than other international portfolios being exposed to the same country risk factor.

Dispersion in mean returns and Sharpe ratios is also high in Book-to-Market international portfolios (Panel C). As in Fama and French (1998), higher mean returns on value portfolios are the norm, with the exception of the US market, with a peak of 1.73 for the UK one. Value portfolios usually yield the highest Sharpe ratios as well, e.g. 0.201 for the United Kingdom Value. There are large and significant correlations between Value and Growth portfolios within country. For instance, the correlation between EU ex-UK ex-Scand Value and EU ex-UK ex-Scand Growth is 0.850.

An investor with higher-order preferences cares about higher order moments and co-movements in returns. This is why we also measure the co-skew of a triplet of stock returns $i, j, l = 1, ..., h$ as in Jondeau and Rockinger (2004):

$$S_{i,j,l} \equiv \frac{E[(r_{it} - E[r_{it}])(r_{jt} - E[r_{jt}])(r_{lt} - E[r_{lt}])]}{\{E[(r_{it} - E[r_{it}])^2]E[(r_{jt} - E[r_{jt}])^2]E[(r_{lt} - E[r_{lt}])^2]\}^{1/2}}. \tag{8}$$

When $i = j = l$, $S_{i,j,l}$ reduces to the third central moment of returns on asset i, which captures the traditional measure of skew, $Skew_i = S_{i,i,i}/\sigma_i^3$. reported in the upper panel of Table 1. When $i \neq j \neq l$, $S_{i,j,l}$ gives a signed measure of the strength of the linear association among deviations of returns from their means across triplets of asset returns. A risk-averse investor dislikes negative values of $S_{i,j,l}$ corresponding to cases when returns in different markets are below their mean at the same time.

When only the returns on two assets are involved, $S_{i,j,j}$ reflects the strength of the linear association between squared deviations from the mean and signed deviations from the mean for a pair of assets. A security i with negative $S_{-i,i,i}$ coefficients for the majority of all possible pairs of returns on other securities (denoted as $-i$) is a security that becomes highly volatile when other securities give low returns, and vice-versa. To a risk averse investor this is an unattractive feature since risk rises in periods with low returns. A security i with predominantly negative $S_{i,-i,-i}$ coefficients pays low (high) returns when other securities become highly volatile; again this feature is harmful to a risk-averse investor since the security performs

[8] Diversification benefits for emerging market investments are highlighted in earlier literature (De Santis (1993), Harvey (1995)) that also discusses how integration leads to increased correlations (Bekaert and Harvey (1997)).

poorly when other assets are highly risky. The bottom panel of Table 1 reports the elements $S_{-i,i,i}$ and $S_{i,-i,-i}$ respectively above and below the diagonal.

Turning to fat tails in the return distribution, the co-kurtosis of a set of four stock returns $i, j, l, q = 1, ..., h$ is equal to:

$$K_{i,j,l,b} \equiv \frac{E[(r_{it} - E[r_{it}])(r_{jt} - E[r_{jt}])(r_{lt} - E[r_{lt}])(r_{qt} - E[r_{qt}])]}{\{E[(r_{it} - E[r_{it}])^2]E[(r_{jt} - E[r_{jt}])^2]E[(r_{lt} - E[r_{lt}])^2]E[(r_{qt} - E[r_{qt}])^2]\}^{1/2}}. \tag{9}$$

When $i = j = l = q$, $K_{i,j,l,q}$ becomes proportional to the coefficient of kurtosis, $Kurt_i = K_{i,i,i,i}/\sigma_i^4$ reported in the upper part of Table 1. When $i \neq j \neq l \neq q$, $K_{i,j,l,q}$ gives a signed measure of the strength of the linear association among deviations of returns from their means across four-tuples of asset returns. The term $K_{i,i,j,j}$, which is present in the middle section of Table 1, sheds light on the correlation between volatility shocks across markets. Large positive values are undesirable, reflecting that volatility tends to be large at the same time in market i as in other market, thus increasing the overall portfolio risk. [9]

We can now comment on the relevance of higher order moments in our sample. The Jarque-Bera test referring to the IE dataset (Panel 1) rejects Normality at 1% for five equity portfolios, but it cannot reject the null for two of them.[10] Only two (four) values of the co-kurtosis (co-skewness) differ statistically from zero at 1% significance level. On the contrary, normality is rejected at the 1% level for all equity portfolios in the IND and BM datasets, described in Panel B and C respectively. Co-kurtosis is statistically different from zero at 1% significance level in all (most) portfolios , while co-skew is (almost) always negligible in the IND (BM) datasets. Thus it appears that non-normalities are moderate in the first dataset, intermediate in Book-to-Market sorted portfolios and substantial in the industry portfolios. However, systematic skewness appears in the BM dataset only.

3.2. *The Return Generating Process*

Empirical analyses of portfolio problems often specify an exogenous distribution of returns. We instead perform several specification tests, allowing our data to endogenously determine the number of regimes k. Table 2 reports the results of these tests, for up to $k = 4$ regimes. We also let the test determine whether there should be an autoregressive term of order $p = 1$, as opposed to no lag ($p = 0$). In Table 2, the term MSIA(1,1) is the same as VAR(1) - indicating linear predictability of returns - while MSIA(1,0) indicates the Gaussian model with unpredictable returns. MSIA(2,1) allows for an autoregressive term of order 1 as well as for two regimes, while MSIAH(2,1) adds heteroskedasticity in the error terms in the form of a regime-switching covariance matrix for returns.

The information criteria do not discriminate between alternative return processes in the case of the first dataset (Panel A), confirming that non-normalities are modest. For the industry dataset, the BIC and HQ

[9] $K_{i,i,i,-i}$ measures the signed linear association between cubic and simple deviations from means for a pair of assets. A security i with positive values of $K_{i,i,i,-i}$ becomes skewed to the left when other securities pay below-normal returns and is hence undesirable to risk-averse investors.

[10] These test results differ from those in Guidolin and Timmermann (2008) *(a o b??)*, who find evidence of non normality also for UK and the Jarque-Bera statistics for Pacific ex-Japan and US are higher than in our dataset- namely 5655.6 vs. 40.94 for Pacific and 162.71 vs. 13.309 for US.

criteria indicate that a MSIH(2,0) return process provides a good fit of the data. Other well-fitting models include MSIH(3,0) and MSIH(4,0), but we stick to the more parsimonious specification. [11]

We can now turn to Table 3, which displays estimates of the parameters of the Gaussian i.i.d. return process in Panel A and of the two-regime model in Panel B. In our description below, we focus on the two-regime representation, as the parameters associated with the single-state one mirror closely the sample descriptive statistics. In one state - which we name Bear - most equity portfolios have lower mean returns. The bear regime tends to last from a minimum of 3.43 months in the BM dataset to a maximum of 5.60 month in the Industry dataset (see Panel C of tables 3A, 3B and 3C). The persistence of bull markets is always higher than the bear one- consistent with business cycle evidence - and it is highest in the Industry dataset (19.46 months) and lowest in the International one (9.7 months). The conditional correlation/volatilities matrixes are estimated with high precision under the two-state model, with a level of significance almost always exceeding 1%. Volatilities in the bear state are always larger than in the bull state, with the exception of EM Latin America in International database[12] Finally, mean returns usually (do not) differ from zero in the (bear) bull state. These regularities sharply differentiate the two regimes in all datasets.

In Table 3A, the two-regime representation leads to even greater dispersion across International and Emerging equity portfolios than that already present in the single state representation. In the Bear regime, four out of eight markets have negative Sharpe ratios (JP, Pacific EX JP, Europe EX UK, UK), whereas Emerging Markets still have positive Sharpe ratios with EM Latin America displaying a particularly high Sharpe ratio of 0.428. North American stocks turn out to have the second highest Sharpe ratio, 0.090, with a relatively low volatility of 4.28. Under the Bull regime, volatilities are lower for every stock market but for Emerging Latin America, which is more volatile during Bull than Bear states. Nonetheless, the Sharpe ratio of EM Latin America (and also EM Europe & Middle East) exceeds 0.38, far higher than others – all of which are positive. Correlations involving North American and EU-ex-UK portfolios tend to be higher in bear markets, confirming the insight by Longin and Solnik (2001) that diversification appears more difficult in bear states. Panel C shows that the Bull regime is almost twice as likely as the Bear one (0.348 vs. 0.652).

For industry portfolios in Table 3B, the Bull regime is more than three times as likely as the Bear one (0.224 vs. 0.776).[13]In line with positive mean returns of North American stocks in the IE dataset, every mean US Industry return is positive in the Bear regime, with a wide variation between Energy and Durables on the one side, with respectively 0.760 and 0.751, and Other and Shops at the other end of the spectrum, with 0.111 and 0.081 respectively. Looking at volatilities, we note that Energy has a relatively low volatility (9.318) compared with that of e.g. Durables (13.181). Note that both correlations and volatilities are higher for industry than for country portfolios in the bear regime, suggesting that country diversification is more powerful than industry diversification, as found by Griffin and Karolyi (1998).

Under the Bull regime, the ranking of industry portfolios changes. Considering Sharpe ratios, the highest

[11]Catao and Timmermann (2007) construct pure country and pure industry factor mimicking portfolios out of firm level data. By comparison, they reject both linearity and normality in both country and industry returns. A two-regime specification is the most suitable according to three information criteria (BIC, AIC, HQ).

[12]This evidence is consistent with previous findings. For instance, Schwert (1989) and Hamilton and Lin (1996) indicate that the volatility of stock returns is higher during recessions than during expansions.

[13]This is broadly consistent with Catao and Timmermann (2007), where returns stay for 40 months in bear states and 42 in the normal one for country indexes vs 8 and 26 for industry indexes.

are Non Durables,Telecommunications and Health, all exceeding 0.284. All volatilities are lower under Bull than under Bear regimes, while correlations are comparable across stock portfolio, with correlations being higher in Bull rather than Bear regimes - even for defensive industries like Utilities.[14]

In Book-to-Market international portfolios, EU ex-UK ex-Scand Value has the highest Sharpe ratio (0.448). Bull-market correlations across Value portfolios range from 0.319 (United States/Scandinavia) to 0.562 (EU ex-UK ex-Scand/UK). Correlations across Growth portfolios are generally higher, ranging from 0.327 (United States/Asia & Pacific) to 0.626 (EU ex-UK ex-Scand/United States). Conditional correlations are similar to those of the single state model.

When Bear, UK Value and Asia & Pacific Value turn out to have the highest Sharpe ratios, respectively 0.246 and 0.222; whereas EU ex-UK ex-Scand and Asia & Pacific growth the lowest, -0.035 and -0.018.

Ang and Chen (2002) study the changing correlation across bull and bear states in US BM portfolios. We do confirm their result that US Value has higher correlation with the world portfolio than US Growth in bear markets. However, the same finding does not carry over to other international BM portfolios: value and growth stocks display a similar pattern of correlations across states, with all correlations being larger in bear states. Petkova and Zhang (2005) argue that the beta risk of value–minus–growth is higher in bear states: given that Value stocks have far higher volatilities than Growth stocks in bear markets, their finding carries over to our international dataset. This also rationalizes why the expected return differential between value and growth stocks appears to be largest in Bear markets. For instance, the difference in mean returns between Asia & Pacific Value (1.168) and Asia & Pacific Growth (0.612) is 0.556 when Bull, increasing to 1.741 in Bear markets when mean returns respectively equal 1.623 and -0.118.[15]

Against this background, we can now study how these equity indexes enter optimal portfolios.

3.3. *The Composition of Optimal Equity Portfolios*

The left-hand side panel of Table 5 reports portfolio composition when short sales are not possible, which we comment upon in this section. The panel is further divided into columns according to the investor horizon. Each page is divided horizontally according to investors' preferences. In the upper panel there is the classic $MV(1)$ case, followed by the unconditional allocations when there are two regimes $k = 2$ and preferences are defined over $m = 2, 3$ and 4 moments. The lowest two parts of each table describe portfolio allocations conditional on the two states. In this commentary, we will mostly comment on the classic mean variance case, on the case where the mean variance investor predicts changing moments (MV(2)) and on four moment preferences, MVSK(2).

3.3.1. **International**

A classic mean-variance investor cares only about the trade-off between means and volatilities and invests only in those portfolios with the highest Sharpe ratios, for given correlation. As to the International data set,

[14]This last observation does not align with previous findings (on weekly data) showing that defensive US industries have lower correlations in bear markets (Ang and Chen, 2002).

[15]The risk premium on value firm may be related to irreversibility of fixed investment. They may be stuck with unproductive capital in bad states of the world and hence underperform with respect to growth firms when the price of risk is high (Zhang, 2005).

the MV(1) investor weights only EM Latin America (from 0.383 to 0.415 depending on T) and EM Europe & Middle East (from 0.617 to 0.585) which provide the highest Sharpe ratios across every investment horizon and exhibit a relatively low cross-correlation (0.479).

When allowing for regimes, a mean-variance investor weighs positively North America stocks (from 0.413 to 0.243) along with Latin America and Middle East. This occurs unconditionally, but it is especially true in the Bear state, where portfolio composition is shifted away from emerging markets towards North America (from 0.997 to 0.690, depending on the investor horizon T). In line with findings of Guidolin and Timmermann (2008), this is explained by the high Sharpe ratio of this country index in Bear states. Portfolio composition does not significantly differ between Bear and Bull states in other respects, with only North America receiving more weight when Bear and Latin America and Middle East when Bull.

We observe a further increase in diversification when the investor has four-moment preferences, MVSK. In fact, weights are positive for every country index when the investment horizon is one month and for 7 out of 8 when the horizon reaches one year. The co-kurtosis between stock portfolios helps rationalizing portfolio weights: Japan, which receives a weight of 0.253 in the Bull state, has the lowest co-kurtosis with EM EU & Middle East (1.253), which in turn has the second highest weight (0.239). On the contrary, Pacific ex-Japan and EM Asia, which display the highest co-kurtosis (2.833), are the portfolios with the lowest weights, respectively 0 and 0.035.

Not only do MVSK preferences lead to the highest level of diversification, but also to the most stable portfolio holding dynamics. Indeed, comparing MV(2) with MVSK holdings in Figure 2, we see that the two-regime mean-variance model is characterized by infrequent spikes, which, on the contrary, do not appear in the case of MVSK preferences. Additionally, portfolio weights for classic mean-variance investors MV(1) are more stable than MV(2) ones, as MV(1) investors update expected returns and volatility estimates only slightly when including new data.

When the horizon increases up to T=120, the composition of MV(1) investors does not vary significantly. On the contrary, MV(2) and MVSK investors concentrate their portfolios in three countries only, namely North America, EM Latin America and EM Europe and Middle East in roughly the same proportions. This indirectly suggests that non-normalities, which were moderate according to sample based tests, disappear as the horizon lengthens.[16]

3.3.2. **Industry**

Turning to the Industry data set, the optimal portfolio of a classic mean-variance investor, MV(1), is similar to that of an investor who also accounts for regimes , MV(2). In fact, both of them invest in Energy, Health and Non-Durables sectors, which have the highest Sharpe ratios. Furthermore the correlation between Energy and Health does not exceed 0.5. Considering separately the Bear and Bull state, we note that Energy is overweighted in Bear states, as it has the highest mean return and a low volatility, whereas positions in Health and Non-Durables increase in Bull markets when they have the highest Sharpe ratios.

When allowing for preferences on third and fourth moments (MVSK), the level of the portfolio diversification slightly increases. Investors hold Energy, Health, Telecommunications and Durables, probably due to

[16] Both MV(2) and MVS investors hold such a three-country portfolio provided their horizon is not lower than 3 months.

their low co-kurtosis (e.g. 2.744 between Energy and Telecommunications) and the positive co-skew between Energy and Durables (0.096). Differently from MV investors, MVSK ones choose almost the same allocation between the two regimes, with only Energy and Telecommunications receiving slightly higher weights when Bear, as opposed to Health and Non-Durables when Bull.

As to the portfolio holding dynamics, MV(1) has the most stable one, whereas both MV(2) and MVSK have infrequent spikes. The inclusion of higher moments reduces the size of the spikes, which is lower for MVSK.

Increasing the investment horizon to T=120 alters the portfolio composition of MV(1) investors, tilting it towards Telecommunications and away from Health. Also MV(2) investors tilt their portfolio allocation away from Health, but towards Non Durables. The MVSK allocation lies in the middle of these two patterns, as it overweights Non-Durables and underweights both Health and Telecommunications.

3.3.3. **Book-to-Market**

Turning now to our last dataset, portfolio weights for a MV investor are concentrated in Value stocks for $T = 1, 12$ - namely UK Value and Scandinavia Value - irrespective of regime considerations.

In the Bull regime, both EU Ex-UK ex-Scandinavia Value in $T = 1$ and United States Growth for $T > 1$ receive a large weight having a very large conditional Sharpe ratio (0.448 and 0.497 respectively). When Bear, the first portfolio receives a zero weight having a conditional Sharpe ratio of 0.006, while UK Value (0.246) is overweighted and Scandinavia (0.192) and US Growth (0.188) are also included in the investor's portfolios.

The highest degree of diversification is achieved when the investor has four-moment preferences (MVSK). Specifically, an investor who accounts also for skewness and kurtosis would diversify in 6 on 11 portfolios, with Value stocks being more heavily weighted. This behavior is not only explained by the low correlations between Asia & Pacific Value and UK Value (0.399) and between Asia & Pacific Value and Scandinavia Value (0.370) but also by the low co-kurtosis - for instance 1.833 between UK Value and Scandinavia Value and 1.902 between Asia & Pacific Value and Scandinavia Value.

When Bear, 7 portfolios are included, with Growth stocks receiving higher weight with respect to the ergodic case, since they have much lower volatility than Value stocks. Such high degree of diversification holds in the Bull state as well.

Turning to the dynamics of portfolio weights, there is a striking difference between MV(1) and MV(2) investors in the stability of their holdings. Indeed, MV(1) appear to have more stable weights, due to the absence of spikes that, on the contrary, characterize MV(2) holdings - especially in the long run. Furthermore, portfolio holdings are more stable with MVSK than with the other models, despite the presence of some spikes of small size.

When the horizon increase to $T = 60$, US Value and US Growth increase in importance to totally replace value portfolios for $T = 120$ - when returns are considered gaussian. This pattern is present but less marked with MVSK model: when $T = 120$ the portfolio composition involves also Scandinavia Value, along with US Value and US Growth.

3.3.4. **Co-Skew and Co-Kurtosis Properties**

Table 4 reports the $T = 1$ steady-state skew and kurtosis properties of each stock portfolio, implied by the Markov-switching return process. We can first observe that the higher order moments implied by our regime-switching model closely match their sample counterparts, offering further evidence that the model is not misspecified. We can also investigate whether higher order properties explain why some stocks enter prominently in MV portfolios but have marginal, if any, role in MVS and MVSK portfolios. Candidates are both EM Latin America and EM EU & Middle East in the IE dataset.

Table 4 shows that the EM Latin America portfolio has large negative values of both own-market skew ($S_{US,US,US}$), and co-skews $S_{US,US,j}$, $S_{US,j,j}$, producing either the largest negative or second largest negative sample estimates of these moments across all regions. Hence EM Latin America stock returns tend to be negative when volatility is high in other markets and they are more volatile when other markets experience negative returns. The implication is that they provide little or no hedge against adverse return or volatility shocks in other markets. A similar limitation affects the desirability of EM EU & Middle East stocks. These effects allow us to explain why aversion to skew in the distribution of final wealth reduces the weights of these stock portfolios.

In the Industry data set, the portfolio becomes concentrated in Energy when moving from MV to MVS preferences for T=1. This can be traced back to the positive skewness of Energy returns (0.034) as well as to the negative skew of both Non-Durables (-0.147) and Health (-0.037). Co-skewness between portfolios are -0.092 and -0.067 (Energy/ Non-Durables), -0.010 and -0.012 (Energy/ Health) and -0.101 and -0.066 (Health/ Non-Durables). Thus, there is a similarity between EM Latin America and EM EU & Middle East and Health/ Non-Durables, as well as for North American shares and Energy.

When considering MVSK, the importance of both own-kurtosis and co-kurtosis help increase the number of portfolio held. Indeed, Telecommunications and Energy, which are the most demanded, have the lowest own-kurtosis -respectively 5.761 and 5.315 - and the lowest co-kurtosis, 2.744.

In the BM dataset, we saw that investors with MVS preferences do not invest in the Scandinavian Value stocks that enter MV(1) portfolios together with UK Value. This can be probably traced back to the own implied skewness of this equity portfolio, which exceeds by far the one of UK Value (0.065 versus 0.0385), as well as to their negative co-skewness coefficients (-0.068 and -0.049).

Table 4 shows that implied estimates of the co-kurtosis are highest for UK Value/UK Growth stock indexes (6.853). This region also produces high estimates of own-market kurtosis (6.919 and 10.34). The high value of own-market kurtosis for UK Value may explain why the allocation to this region does not increase further when shifting from preferences defined over skew ($m = 3$) to those over kurtosis as well ($m = 4$).

In general, stocks entering optimal portfolios of four-moment preferences investors have good co-kurtosis properties, with co-kurtosis coefficients ranging from 1.864 to 2.357. For instance, Asia Pacific Value stocks - that are not demanded at all under MV preferences - enter optimal portfolios of both MVK and MVSK investors having a co-kurtosis of 1.902 with Scandinavian value and 2.357 with UK value. By comparison, US Growth, which dominates Asia Pacific Value in terms of implied Sharpe ratio (0.267 versus 0.184), has higher co-kurtosis (1.936 with Scandinavian and of 3.677 with UK Value).

3.3.5. **Regularities across datasets**

This subsection highlights the regularities found in all datasets.

A first regularity is that an investor with preferences defined over four moments (MVSK) always reaches the highest degree of diversification. A MV(1) investor has a concentrated portfolio, holding only two indexes in IE and Book-to-Market dataset and three indexes in the Industry one. A MV(2) investor respectively includes six, three and two portfolios in the IE, Industry and Book-to-Market datasets. On the contrary, under MVKS, five Industry, six Book-to-Market and every country index have positive weight. Portfolio holdings in MV models are focussed on portfolios having higher Sharpe ratios and lower correlations. Adding higher moment preferences as well results in the addition of those stock portfolios displaying lower implied co-kurtosis and higher co-skewness. However, increased diversification is especially attached to kurtosis aversion. This is clear from the comparison of MVS and MVK portfolios in the case of the International dataset: a strictly greater number of indexes receives non-zero portfolio weights when the investor has MVK instead of MVS preferences for every time horizon (except for T=120). Thus, it appears that skewness aversion induces concentration in a subset of assets with good skewness properties - as already known from the literature. In the International dataset, MVS investors choose only the 3 highest Sharpe-ratio portfolios, giving more weight to EM EU and Middle East, which has the lowest skew. Moreover, co-skew between those three portfolios is moderate.

Another regularity concerns the volatility of portfolio holdings over time. Indeed, MV(2) models always entail more volatile weights, with infrequent spikes, whose size increases with increasing horizon. On the other hand, with MVSK, spikes vanish in IE dataset and are small in both Industry and Book-to-Market data. Therefore, transactions costs are more likely to adversely affect a mean-variance investor rather than a four-moment one and could possibly reverse the ranking of models in term of performance.

Ang and Bekaert (2004) suggest that RS strategies are relatively robust to transaction costs because they are designed to exploit changes in expected returns and volatilities that are associated with infrequent changes of regimes is relatively high. Our findings qualify this observation: the very contribution to the stability of portfolio shares is offered by the higher moments, given that they exhibit higher volatility under MV(2) than MV(1), i.e. when we allow for two regimes.

It is also the case that a shorter horizon increases the sensitivity of portfolio composition with respect to the current state of the market. For instance, a T=1 investor in the Industry dataset weights heavily stock indexes that perform well in the bear state when the probability of being in a bear state is high, because the bear state is persistent. A MV investor thus gives 0.612 weight to the Energy portfolio for T=1. On the contrary, a T=120 investor, believing to be in a bear state, cares also about stock portfolios that outperform in bull markets as she knows that the chances of shifting to a bull regimes are higher. Non-Durable stocks are then weighted 0.516, thanks to their attractiveness in bull states and despite their low mean return (0.162) in bear markets, while the weight on Energy falls to 0.253.

Last but not least, in each dataset there are several stock portfolios that are defensive in a traditional way, i.e. display relatively low correlations with other portfolios. Three portfolios that appear to be defensive for investors that recognize changes in mean returns and volatilities across regimes are North American, Energy and, to much lesser extent, US and Scandinavia Growth.

4. **The Ex-Post Performance of Equity Portfolios**

We know that the expected utility of an investor who cares about higher moments falls when s/he overlooks predictability in returns and/or higher order moments of the return distribution. This is, for instance, the case in Ang and Bekaert (2002) in an international portfolio problem - provided that the asset menu includes a short-term bond allowing investors to abandon equities in the bear state. It is also the case when dealing with size-sorted equity portfolios, as in Guidolin and Nicodano (2008), due to the dismal performance of small caps in bear states. Out-of-sample analyses confirm that gains are large when diversifying internationally if the possibility of shifting into cash in bear states is allowed (Ang and Bekaert (2004) and Guidolin and Timmermann (2008)). In these papers the benchmark is rather extreme, being the mean-variance allocation with no predictability. However, we know that ex-post gains from timing both volatility (Fleming *et al.*, 2001) and higher order moments (Jondeau and Rockinger, 2009) can be large. These papers find positive gains in a model where expected returns are constant and there are no regimes. We now turn to an assessment of out-of-sample performance gains in our three datasets, extending previous evidence along several dimensions.

We recursively estimate all the parameters of the models described in Table 0 and proceed to calculate the portfolio performance figures at all points in time. For the Markov-switching model this implies re-estimating all parameters and the state probability vector on an expanding window of data using the EM algorithm. For other models, only the parameters are estimated recursively by MLE. The out-of-sample period for our International, Industry and Book-to-Market International data run from 1998:01-2008:07, 1980:01-2008:07, and 1995:01-2007:12 respectively.[17]

First, we use three different indicators of portfolio performance (see Table 6). The Sharpe ratio does not capture any effect on the skewness or kurtosis of wealth, while the Sortino ratio falls when downside risk increases. This enables to compare the performance of investors endowed with different preferences, by checking - for instance - whether the Sharpe/Sortino ratios of a 3 or 4 moment-preference investor exceeds the one of a MV investor. We can similarly analyze whether an investor with mean-variance preferences achieves better performance by considering time varying mean and variances across market regimes, i.e. by becoming what we label a MV(2) investor. And we also compute the certainty equivalent of maximum utility, CEQ, associated with different investor preferences [18]. Finally, Table 7 will report two other indicators of performance. The Treynor ratio indicates excess return on one unit of systematic risk, β, while Jensen's alpha captures excess-returns that are not associated to such systematic risk. It may well be that such measures, as well as the Sharpe ratio, increase with more negative skew in portfolio returns[19], which would instead reduce welfare of a MVS investor.

A first observation concerns the relative performance of the equally weighted strategy vis-à-vis optimizing ones. Recently, the literature suggested that the equally weighted strategy would be the appropriate benchmark to evaluate the relative performance of active strategies (DeMiguel et al., 2009). From this point

[17] In our comments below we do not emphasize results concerning the 10 year horizon. This is because we produce only 7 (36) out of sample performances for the International (Book-to-Market) experiment. We are in the process of producing results for T=60 horizon, which will deliver a more meaningful comparison.

[18] We follow Ang and Bekaert (2002) and Guidolin and Timmerman (2008a) to obtain estimates of the CEQ.

[19] This is the idea behind he manipulation of Sharpe ratios discussed by Goetzmann et al.

of view, we observe that 1/N never consistently outperforms optimizing strategies. According to 5%-95% confidence bounds, the equally weighted strategy is always equivalent or dominated by the other ones. It achieves its best performance in the International dataset for a long time horizon (T=120), since it ranks first according to both Sharpe and Sortino Ratio. Yet, the other performance measures do not support this result even in this case[20]. In the other two datasets the equally weighted strategy is never the first best and, for T$\geq$ 60 months, it is almost always dominated according to all the five measures (Sharpe, Sortino and Treynor ratios, CEQ and Jensen's Alpha).

We now turn to the relative performance among optimizing strategies. Panel A of Tables 6 and 7 reveals that MV models are often the best performers in International diversification problems according to any measure. Such good ex-post performance of Mean Variance portfolios is perhaps not surprising, as we saw that non-normalities are moderate in this data-set. Furthermore, we already know from Ang and Bekaert (2004) that the out-of-sample cost of adopting i.i.d. mean-variance strategies is low when there is no risk-free rate. However, when T=12,60, predicting regimes seems to matter, since MVS and MV(2) are either equivalent or dominate the simple MV(1) strategy.[21]For instance, when T=60, the 2-regimes strategies dominate MV(1) according to Sharpe Ratio, Treynor Ratio and Jensen's Alpha. In particular, unreported figures for MV(1) show that Jensen's α confidence interval is negative: $[-82.828, -58.636]$ while it is strictly positive for the other strategies. When T=12, following a MV(2) strategy as opposed to a MV(1) delivers a CEQ of 4.993 rather than 4.713 and a strictly preferable Jensen's α (22.530, with confidence bounds [7.441,37.445] vs. -22.495 with bounds [-38.883,-4.464]). Thus, accounting for regimes is rewarding for longer horizons even in this dataset with moderate non-normalities. This complements prior results by Fleming et. al. (2001), who study a short-horizon mean-variance investor with daily data. It also adds to the evidence in Jondeau and Rockinger (2009), who use a DCC specification for the return process.

In the Industry dataset both MV and MVSK models rank high among optimizing strategies. In particular, MVSK - as well as MV(2) - perform better for shorter horizons (T$\leq$ 12). For instance, MVSK yields a higher CEQ (13.755 vs. 10.867) and higher Jensen's alpha (124.699 vs. 73.429) than the MV(1) strategy for T=1. Moreover, it ranks at least second for both T=1 and T=12 according to all the performance measures, even if no optimizing strategy is clearly the best on all counts [22]. The importance of predicting both stock market regimes and higher order moments fades away with longer time horizons. MV(1) ranks first according to most metrics for T=60 and according to all of them for T=120. This result is very strong for T=120, since MV(1) dominates all the other strategies when we consider Treynor Ratios or Jensen's alphas (see Table 7 Panel B). Its 5%-95% confidence bounds, [1.373,1.839] (Treynor Ratio) and [49.970, 69.109] (Jensen's Alpha), are strictly greater than the ones for the MVS strategy, which ranks second, [1.033,1.197] and [24.127,35.387]. All in all, a MV strategy deals well with industry diversification irrespective of the investment horizon - despite substantial non-normalities both in our sample statistics as well as in the characterization of the return generating process.

[20]We do not actually report performance for T=120 in the International and Emerging dataset, as they get nonsensically large due to the few out-of-sample observations.

[21]This pattern is not confirmed only when T=120, as MV(1) outperforms both MVS and MV(2). Yet the very few (7) out-of-sample observations in this specific experiment make this evidence unreliable.

[22]Such good performance of a MVSK strategy is not necessarily associated wih better *ex-post* higher order moments: the kurtosis of wealth is actually higher for MVSK than, say, for MV(2) for both T=1 and T=12.

By contrast, accounting for higher order preferences and regimes adds economic value when investing in Book-to-Market international portfolios, as evident from panel C in both Table 6 and 7. Both the MVS and the MVSK strategy outperform the MV(1) model. Even restricing attention to MV preferences, it turns out that the CEQ of a MV(2) investor always exceeds that of a MV(1) investor.

The welfare of an investor following the MV(2) strategy is lower than both MVSK (18.643 vs. 17.701) when T=1 and MVS for T= $12, 60, 120$ (8.586 vs. 7.328, 12.868 vs. 3.378 and 30.206 vs. 14.964 respectively). Note that for T=60, 120, confidence bounds are not overlapping ([11.278, 14.762] vs. [2.360, 4.575] and [29.524,30.906] vs. [14.135,15.881] respectively) and MVS strictly dominates MV(2) in a statistical sense. Hence, MV investors would prefer to delegate MVS(K) ones when dealing with Book-to-Market portfolios, opposite to what happens with the Industry ones. This is confirmed by the certainty equivalent of the best MV strategy, which is also lower than the MVS(K) one for all time horizons.

A final observation concerns the relevance of skewness for longer investment horizons. Indeed, the MVS strategy dominates MVSK for T≥ 60.[23] For $T = 60$, for instance, the Sharpe (Sortino) Ratio of a MVS investor, 0.571 (12.868), strictly exceeds the one of MVSK managers, 0.186 (3.603), who represent the second best alternative among the optimizing strategies. [24].More generally, it appears that predicting skewness rather than kurtosis is more important in all datasets, since MVS performances are never dominated by MVSK ones.

4.1. *Robustness*

So far we commented on results referring to the case of a risk aversion coefficient equal to 5 and no short sale. We now turn to a situation when short sales are allowed. In this case, all three performance measures are generally lower than with short sales constraint, particularly for long horizons. This is not surprising, as the typical extreme long or short positions involved by short selling are able to exacerbate any misspecification or imprecise estimation, especially in the longer run. This result confirms previous research, which points out the importance of restricting the volatility of portfolios weights to achieve higher *out-of-sample* performance. (among others, DeMiguel et al., 2009, Diris et al. 2008, Jagannathan and Ma, 2003).

Turning to the ordering of portfolio choice models, we see that allowing for short sales does not alter the relative ranking of performance when T=1 in International data according to the performance measures reported in Table 6. Specifically, MV(1) is still the best model according to both Sharpe ratio and Sortino ratio, whereas MVSK yields the highest certainty equivalent. Moreover, the relative performance of the second best and third best model is also consistent with the previous case. Results are mixed when T increases to 12, with the ranking being the same only according to the Sharpe ratio, as MV(1) gives the highest risk-return trade-off also with short sales.

Industry data do not show the same patterns when we remove short sale constraints. When T=1, the ranking changes both according to the Sharpe ratio and Certainty equivalent, whereas MVSK is consistently the best under the Sortino ratio. When T=12, MVSK and MV(1) are still the best and second best model

[23]An exception concerns the equally weighted strategy. While ranking second to MVS - according to Sortino Ratio when T=60 and to both Sharpe and Sortino Ratios for T=120, the confidence bounds of 1/N and MVS overlap.

[24]MVSK is dominated by the equally weighted strategy for T=120 according to all the performance measures - the Sharpe Ratios being, for instance, equal to [2.692,5.323] and [0.848,1.306] respectively.

according to the Sharpe ratio, while the ranking is completely changed for Sortino and CEQ.

Book-to-market data show more stable rankings, with MVSK being the best model according to every measure for T=1. Furthermore, the whole ranking is unchanged when short sales are allowed.in cases sich as T=60 in Table 6.[25]

We assess ex-post performance also when investors have risk aversion coefficients equal to 2 and 10 (not reported here for sake of brevity). Generally, the relative performance of the models is not affected by the change in risk aversion. For instance, MV(1) is still the best model in International data both with risk aversion 2 and 10 when T=1. What changes is the absolute value of the certainty equivalent, that is higher for low risk aversion coefficients, as the investor weighs less portfolio volatility.

One final remark concerns the effect of transaction costs when short sales are allowed. It is well-known that transaction costs are larger, reducing portfolio ex-post performances more with than without short sales, due to the large long and short positions entailed. Should this be the case, both MVSK and 1/N are likely to improve their relative performance with respect to MV models which yield less stable portfolio holdings.

5. **Concluding comments**

We find that the benchmark, equally weighted strategy never outperforms the ex-post performance of the optimizing ones in the three datasets under investigation.

We also uncover large ex-post gains from exploiting predictable moments up to the fourth order in international stock portfolios ranked according to Book-to-Market values. This result is the mirror image, cast in a portfolio choice setting, of previous finance literature highlighting the bad performance of Value portfolios in bear states. Importantly, regime-switching models deliver gains not only to an investor who cares about higher order moments, but also to an agent with mean-variance preferences.

The latter observation holds also when dealing with Industry portfolios, at least for short horizons. Descriptive statistics point to large co-kurtosis across Industry portfolios. Despite this evidence, mean variance strategies perform better than those considering predictability in higher order moments as well. This fact begs for an explanation. Guidolin and Nicodano (2008) indicate that third (fourth) moments yield considerable (little) additional welfare in sample over a dataset characterized by both types of non-normalities. We conjecture that MVSK strategies perform poorly in the Industry dataset because these portfolio returns display large co-kurtosis - i.e. high volatility when other stocks are also highly volatile - but little co-skewness. Further work may scrutinize whether the type of non-normality in the data affects ex-post gains from predicting higher order moments.

Our analysis also confirms previous results by Ang and Bekaert (2004): the out-of-sample cost of adopting mean-variance strategies is low when the investor diversifies across international equities, with no opportunity to shift into bonds in bear states. We see, however, that even in this dataset accounting for regimes and skewness, especially for long horizons, can improve risk-adjusted performances.

All datasets suggest that - to some extent - modelling the regime-switching nature of stock returns is

[25] Both the Treynor ratio and Jensen's alpha show lower ranking stability than the other performance measures. Despite this, Table 7 reveals the presence of some regularities for longer time horizons (T$\geq$12 months).

beneficial, but higher order moments matter only in two out of three datasets. This ought to be considered as a lower bound on the relevance of higher order moments for portfolio strategies, as it is conditional on the specific parametrization of our MVSK preferences. Their economic importance may be much larger for other types of preferences: for instance, allowing for investors' disappointment aversion, as in Hong et al. (2007), may boost gains from timing higher-order moments relative to the case of power utility.

In conclusion, allowing for a simple and parsimonious 2-state representation of the return distribution improves on ex-post performance always in the BM dataset, and for only a subset of investment horizons in the other two datasets. Predicting third and fourth moments, on top of the first two, need not always deliver gains even when descriptive statistics indicate the presence of sizeable non-normalities. The inclusion of ex-post transaction costs, which we leave for future work, may further increase the relative attractiveness of MVSK strategies because portfolio shares appear to be less sensitive to variations in expected returns.

References

[1] Ang A., and G., Bekaert, 2002, "International Asset Allocation with Regime Shifts", *Review of Financial Studies*, 15, 1137-1187.

[2] Ang A., and G., Bekaert, 2004, "How do Regimes Affect Asset Allocation?," *Financial Analysts Journal*, 60, 2, 86-99.

[3] Ang A., and G., Bekaert, 2007, "Stock Return Predictability: Is it There?," *Review of Financial Studies*, 20, 3, 651-707.

[4] Ang A. and J., Chen, 2002, "Asymmetric Correlations of Equity Portfolios," *Journal of Financial Economics,* 63, 443-494.

[5] Avramov, D., T. Chordia, 2006, "Predicting Stock Returns," *Journal of Financial Economics* 82, 387–415.

[6] Bekaert, G., and C. R. Harvey, 1997, "Emerging equity market volatility ," *Journal of Financial Economics,* 43, 1, 29-77.

[7] Barberis, N., 2000, "Investing for the Long Run When Returns Are Predictable", *Journal of Finance*, 55, 225-264.

[8] Bollerslev, T., R.Y. Chou and K.F. Kroner, 1992, "ARCH Modeling in Finance: A Review of the Theory and Empirical Evidence," *Journal of Econometrics*, 52, 5-59.

[9] Brandt, M. W., 1999, "Estimating Portfolio and Consumption Choice: A Conditional Euler Equations Approach," *Journal of Finance,* 54, 1609-1646.

[10] Bossaerts, P. and P., Hillion, 1999. "Implementing Statistical Criteria to Select Return Forecasting Models: What Do We Learn?," *Review of Financial Studies*, 12, 2, 405-428.

[11] Brennan, M., E. S. Schwartz, and R. Lagnado, 1997, "Strategic Asset Allocation," *Journal of Economic Dynamics and Control,* 21, 8-9, 1377-1403.

[12] Brennan, M., and Y., Xia, 2001, "Assessing Asset Pricing Anomalies," *Review of Financial Studies,* 14, 905–942.

[13] Campbell, J., and L.,Viceira, 1999, "Consumption and Investment Decisions When Expected Returns are Time Varying", *Quarterly Journal of Economics*, 114, 2, 433-495.

[14] Campbell, J., and L.,Viceira, 2001, "Strategic Asset Allocation," *Oxford University Press.*

[15] Campbell, J., Chan, and L., Viceira, 2003, "A Multivariate Model of Strategic Asset Allocation," *Journal of Financial Economics*, 67, 41-80.

[16] Catao, L., and Timmermann, A., 2007, “Volatility Regimes and Global Equity Returns”, Forthcoming in *T. Bollerslev, J. Russell, and M. Watson, (eds.), “Volatility and Time Series Econometrics: Essays in Honour of Robert F. Engle”* , Oxford University Press.

[17] Das, S., and R., Uppal, 2004, “Systemic Risk and International Portfolio Choice,” *Journal of Finance*, 59, 6, 2809-2834.

[18] DeMiguel, V., L. Garlappi, and R. Uppal, 2009, “Optimal Versus Naive Diversification: How Inefficient is the 1/N Portfolio Strategy?,” *Review of Financial Studies*, 22, 5, 1915-1953.

[19] Diris, B., Palma, F. and P. Schotman, “Long-term strategic asset allocation: an out-of-sample evaluation,” Netspar w.p., 2008.

[20] Dittmar, R., 2002, “Nonlinear Pricing Kernels, Kurtosis Preference, and Evidence from the Cross Section of Equity Returns,” *Journal of Finance*, 57, 369-403.

[21] Fama, E., and K. French, 1988, “Dividend Yields and Expected Stock Returns,” *Journal of Financial Economics*, 22, 3-25.

[22] Fleming, J., C.Kirby, and B. Ostdiek, 2001, “The Economic Value of Volatility Timing,” *Journal of Finance,* 56, 329-352.

[23] Fugazza, C., M. Guidolin, and G. Nicodano, 2009, “Time and Risk Diversification in Real Estate Investment: the Ex-Post Performance,” forthcoming *Real Estate Economics.*

[24] Garlappi L., R. Uppal and T. Wang, 2007, Portfolio Selection with Parameter and Model Uncertainty: a Multiprior Approach, *Review of Financial Studies* 20, 1, 41-81.

[25] Goyal, A. and I., Welch, 2008, “A Comprehensive Look at The Empirical Performance of Equity Premium Prediction,” *Review of Financial Studies* 21, 4, 1455-1508.

[26] Griffin JM, and G. A. Karolyi, 1998, “Another look at the role of the industrial structure of markets for international diversification strategies”, *Journal of Financial Economics*, Volume 50, Issue 3, 1 December 1998, Pages 351-373.

[27] Guidolin, M. and G., Nicodano, 2008, “Small Caps in International Equity Portfolios: The Effects of Variance Risk,” *Annals of Finance*, 5, 1, 15-48.

[28] Guidolin, M. and A., Timmermann, 2008a, “Size and Value Anomalies under Regime Switching,” *Journal of Financial Econometrics*, 6, 1, 1-48.

[29] Guidolin, M. and A., Timmermann, 2008b, “International Asset Allocation under Regime Switching, Skew and Kurtosis Preferences,” *Review of Financial Studies*, 21, 2, 889-935.

[30] Hamilton, J. D., and G. Lin, 1996, “Stock Market Volatility and the Business Cycle,” *Journal of Applied Econometrics*, 11, 5, 573-593.

[31] Harvey, C., 1995, “Predictable risk and returns in emerging markets,” *Review of Financial Studies,* 8, 773 - 816.

[32] Harvey, C., and A., Siddique, 2000, “Conditional Skewness in Asset Pricing Tests,” *Journal of Finance*, 55, 1263-1295.

[33] Hong Y., J. Tu, and G. Zhou, 2007, Asymmetries in Stock Returns: Statistical Tests and Economic Evaluation, *Review of Financial Studies,* 20, 1547 - 1581

[34] Jagannathan, R., and T. Ma, 2003, “Risk Reduction in Large Portfolios: Why Imposing the Wrong Constraints Helps,” *Journal of Finance*, 58, 4, 1651-1683.

[35] Jondeau, E., and M., Rockinger, 2004, “Optimal Portfolio Allocation Under Higher Moments,” EFMA 2004 Basel Meetings.

[36] Jondeau, E., and M., Rockinger, 2009, “Time-Variability in Higher Moments Is Important for Asset Allocation,” submitted to *Journal of Financial Econometrics.*

[37] Kandel, S., and R., Stambaugh, 1996, “On the Predictability of Stock Returns: An Asset Allocation Perspective,” *Journal of Finance*, 51, 385-424.

[38] Keim, D., and R., Stambaugh, 1986, “Predicting Returns in the Stock and Bond Markets,” *Journal of Financial Economics,* 17, 357-390.

[39] Kimball, M. S., 1993, “Standard Risk Aversion,” *Econometrica*, 61, 589–611.

[40] Kumar, A., Institutional Skewness Preferences and the Idiosyncratic Skewness Premium, *University of Notre Dame*, 2005

[41] Lettau, M., and S. Van Nieuwerburgh, 2008, Reconciling the Predictability Evidence, *Review of Financial Studies,* 21, 1607-1652.

[42] Lynch A. W., 2001, “Portfolio choice and equity characteristics: characterizing the hedging demands induced by return predictability,” *Journal of Financial Economics,* 62, 67–130.

[43] Longin, F., and B., Solnik, 2001, “Correlation Structure of International Equity Markets During Extremely Volatile Periods,” *Journal of Finance*, 56, 649-676.

[44] Merton R. C., 1969, “Lifetime Portfolio Selection under Uncertainty: The Continuous-Time Case,” *The Review of Economics and Statistics*, 51, 3, 247-257.

[45] Pesaran H., and A., Timmermann, 1995, “Predictability of Stock Returns: Robustness and Economic Significance,” *Journal of Finance,* 50, 1201-1228.

[46] Petkova R., and L. Zhang, 2005, “Is Value Riskier than Growth?,” *Journal of Financial Economics*, 78, 187–202.

[47] Samuelson, P. A., 1969, "Lifetime Portfolio Selection By Dynamic Stochastic Programming," *The Review of Economics and Statistics*, 51, 3, 239-246.

[48] Schwert, G. William, 1989, "Business Cycles, Financial Crises, and Stock Volatility," *Carnegie-Rochester Conference Series on Public Policy*, Elsevier, 31, 1, 83-125.

[49] Timmermann, A., 2000, "Moments of Markov Switching Models," *Journal of Econometrics*, 96, 75-111.

[50] Turner, C., R., Startz, and C., Nelson, 1989, "A Markov Model of Heteroskedasticity, Risk, and Learning in the Stock Market," *Journal of Financial Economics*, 25, 3-22.

[51] Zhang, L., 2005. "The Value Premium," *Journal of Finance*, 60, 1, 67-103.

6. **Appendix**

Moments of the wealth distribution

This appendix displays equations for moments of the wealth distribution, when there are autoregressive terms in the return process, and the number of regimes is set to $k = 2$. Using (3) the n-th noncentral moment satisfies the recursions

$$\begin{aligned} M_{i,t+T}^{(n)} &= M_{i,t+T-1}^{(n)}(n) p_{ii} \exp\left(n\mu_i + n\sum_{j=1}^{p} a_{j,i} E_t[r_{t+T-j}] + \frac{n^2}{2}\sigma_i^2 \right) + \\ & \quad + M_{-i,t+T-1}^{(n)}(n)(1-p_{-i-i}) \exp\left(n\mu_i + n\sum_{j=1}^{p} a_{j,i} E_t[r_{t+T-j}] + \frac{n^2}{2}\sigma_i^2 \right) \end{aligned}$$

or

$$\begin{aligned} M_{1,t+1}^{(n)} &= \tilde{\alpha}_1^{(n)} M_{1,t}^{(n)} + \tilde{\beta}_1^{(n)} M_{2,t}^{(n)} \\ M_{2,t+1}^{(n)} &= \tilde{\alpha}_2^{(n)} M_{1,t} + \tilde{\beta}_2^{(n)} M_{2,t}^{(n)}, \end{aligned}$$

where now

$$\begin{aligned} \tilde{\alpha}_1^{(n)} &= p_{11} \exp\left(n\mu_1 + n\sum_{j=1}^{p} a_{j,1} E_t[r_{t+T-j}] + \frac{n^2}{2}\sigma_1^2 \right) \\ \tilde{\beta}_1^{(n)} &= (1-p_{22}) \exp\left(n\mu_1 + n\sum_{j=1}^{p} a_{j,1} E_t[r_{t+T-j}] + \frac{n^2}{2}\sigma_1^2 \right) \\ \tilde{\alpha}_2^{(n)} &= (1-p_{11}) \exp\left(n\mu_2 + n\sum_{j=1}^{p} a_{j,2} E_t[r_{t+T-j}] + \frac{n^2}{2}\sigma_2^2 \right) \\ \tilde{\beta}_2^{(n)} &= p_{22} \exp\left(n\mu_2 + n\sum_{j=1}^{p} a_{j,2} E_t[r_{t+T-j}] + \frac{n^2}{2}\sigma_2^2 \right). \end{aligned}$$

Subject to these changes, the earlier methods can be used with the only difference that terms such as $\exp\left(n\mu_i + \frac{n^2}{2}\sigma_i^2\right)$ have to be replaced by

$$\exp\left(n\mu_1 + n\sum_{j=1}^{p} a_{j,i} E_t[r_{t+T-j}] + \frac{n^2}{2}\sigma_1^2 \right).$$

The term $\sum_{j=1}^{p} a_{j,i} E_t[r_{t+T-j}]$ may be decomposed in the following way:

$$\sum_{j=1}^{p} a_{j,i} E_t[r_{t+T-j}] = I_{\{j>T\}} \sum_{j=1}^{p} \left(I_{\{j\geq T\}} a_{j,i} r_{t+T-j} + I_{\{j<T\}} a_{j,i} E_t[r_{t+T-j}] \right),$$

where $E_t[r_{t+1}], ..., E_t[r_{t+T-1}]$ can be evaluated recursively, c.f. Doan et al. (1984):

$$\begin{aligned}
E_t[r_{t+1}] &= \pi_{1t}\left(\mu_1 + \sum_{j=1}^{p} a_{j,1} r_{t-j}\right) + (1-\pi_{1t})\left(\mu_2 + \sum_{j=1}^{p} a_{j,2} r_{t-j}\right) \\
E_t[r_{t+2}] &= \boldsymbol{\pi}_t' \mathbf{P} \mathbf{e}_1 \left(\mu_1 + \sum_{j=1}^{p} a_{j,1} E_t[r_{t+1}]\right) + (1-\boldsymbol{\pi}_t' \mathbf{P} \mathbf{e}_1)\left(\mu_2 + \sum_{j=1}^{p} a_{j,2} E_t[r_{t+1}]\right) \\
& ... \\
E_t[r_{t+T-1}] &= \boldsymbol{\pi}_t' \mathbf{P}^{T-1} \mathbf{e}_1 \left(\mu_1 + \sum_{j=1}^{p} a_{j,1} E_t[r_{t+T-2}]\right) + (1-\boldsymbol{\pi}_t' \mathbf{P}^{T-1} \mathbf{e}_1)\left(\mu_2 + \sum_{j=1}^{p} a_{j,2} E_t[r_{t+T-2}]\right).
\end{aligned}$$

Table 0

List of asset-allocation models considered

This table lists the asset-allocation models we consider. The last column of the table gives the abbreviation used to refer to the strategy in the tables where we compare the performance of the optimal portfolio strategies. We consider six different time horizons, listed in the second column and the levels of relative risk aversions in the third column.

	Models	Horizons	Risk Aversions	Abbreviation
		T=1,3,6,12,24,60,120	2,5,10	
	Naive			
1	Equal weighted			1/N
	No predictability, no higher moments			MV
2	Mean-variance (associated with MSIA(1,0))			MV(1)
3	Mean-variance, no shortsales			MV(1)-c
	Predictability and higher moments preferences			
4	2 Moment Pref with MSIH(2,0) returns			MV(2)
5	2 Moment Pref with MSIH(2,0) returns, no shortsales			MV(2)-c
	Predictability and higher moments preferences			
6	3 Moment Pref with MSIH(2,0) returns			MVS
7	3 Moment Pref with MSIH(2,0), no shortsales			MVS-c
8	3 Moment Pref with MSIH(2,0) returns			MVK
9	3 Moment Pref with MSIH(2,0) returns, no shortsales			MVK-c
10	4 Moment Pref with MSIH(2,0) returns			MVSK
11	4 Moment Pref with MSIH(2,0) returns, no shortsales			MVSK-c

Table 1

Summary Statistics for Equity Returns

The table reports basic moments for monthly equity total return series for international portfolios from January 1988 to July 2008 (Panel A), Industries indices from July 1926 to July 2008 (Panel B) and International Book-to-Market portfolios from January 1975 to December 2007(Panel C) in the upper part of each panel. All returns are expressed in local currencies. Means, Median and Standard Deviations are annualized. The column Jarque-Bera reports the value of the Jarque-Bera statistics for normality, while LB(12) reports the 12th-order Ljung-Box statistic. The middle part of each panel reports the correlation and co-kurtosis matrices, the lower part the co-skewness matrix. In the co-skewness matrix, coefficients above the main diagonal refer to the sample covariance between the square of the returns of the row portfolio and the level of returns of the column portfolio; coefficients below the main diagonal refer to the sample covariance between the level of the returns of the column portfolio/index and the square of returns of the row portfolio/index. In the correlation/co-kurtosis matrix, correlations are reported above the main diagonal and sample covariances between squared portfolio returns appear below the main diagonal.The symbols ** and * respectively denote statistical significance at 1% and 5%.

Panel A (International MSCI USD Returns, 1988:01 - 2008:07)

	Mean	St. Dev.	Sharpe ratio	Median	Min.	Max.	Skewness	Kurtosis	Jarque-Bera	LB(12)	LB(12)-squares
Pacific ex-Japan	0.746*	5.428**	0.072	0.927	-23.1	15.3	-0.530*	4.685*	40.94**	18.62	28.59**
Japan	-0.051	6.310**	-0.064	-0.272	-21.6	21.7	0.101	3.696	5.43	10.94	52.51*
Europe ex-UK	0.766*	4.928**	0.084	1.103	-15.6	13.8	-0.542*	4.059	23.73**	15.48	25.22*
United Kingdom	0.707	4.397**	0.080	0.673	-10.9	14.1	0.038	3.178	0.389	11.23	57.88**
North America	0.756**	3.924**	0.103	1.093	-14.3	10.4	-0.441*	3.714	13.309**	7.77	34.88**
EM Latin America	1.906**	8.933**	0.174	2.680	-35.4	27.3	-0.594*	4.536*	38.985**	10.97	20.72
EM Asia	0.774	7.111**	0.059	1.078	-19.7	22.1	-0.181	3.717	6.657*	33.06**	45.27**
EM Europe & Middle East	1.846**	7.747**	0.193	2.450	-29.0	38.8	0.272	5.888**	89.256**	14.75	5.40

Correlation and (variance) co-kurtosis matrices

	Pacific ex-JP	Japan	EU ex-UK	UK	North Amr.	EM Latin Amr.	EM Asia	EM EU & Middle East
Pacific ex-JP		0.444**	0.592**	0.621**	0.601**	0.545**	0.785**	0.424**
Japan	1.292		0.462**	0.480**	0.368**	0.321**	0.406**	0.221**
EU ex-UK	1.894	1.715		0.744**	0.669**	0.410**	0.508**	0.467**
UK	1.533	1.459	2.317*		0.664**	0.394**	0.442**	0.353*
North Amr.	1.766	1.346	2.801*	2.138**		0.500**	0.551**	0.404**
EM Latin Amr.	1.923	1.806	2.012*	1.379	2.255		0.491**	0.479**
EM Asia	3.149**	1.440	2.114	1.488	1.979	1.826		0.475**
EM EU & Middle East	1.407	1.503	2.000	1.384	2.250	2.286	1.586	

Co-skewness matrix

	Pacific ex-JP	Japan	EU ex-UK	UK	North Amr.	EM Latin Amr.	EM Asia	EM EU & Middle East
Pacific ex-JP		-0.273	-0.360	-0.203	-0.352*	-0.385	-0.364	-0.299*
Japan	-0.159		-0.052	0.071	-0.078	-0.307	-0.160	-0.115
EU ex-UK	-0.374**	-0.306		-0.351	-0.496*	-0.354	-0.470**	-0.344
UK	-0.043	-0.090	-0.178		-0.080	-0.117	-0.118	-0.187
North Amr.	-0.393**	-0.231	-0.520**	-0.238		-0.475*	-0.410*	-0.419
EM Latin Amr.	-0.400**	-0.420*	-0.213	-0.271	-0.380		-0.269	-0.337
EM Asia	-0.292	-0.263	-0.372*	-0.209	-0.273	-0.277		-0.288*
EM EU & Middle East	-0.220	-0.131	-0.126	-0.218	-0.344	-0.565	-0.260	

Table 1 (cont'ed)

Summary Statistics for Equity Returns

Panel B (CRSP Industry Returns, 1926:07 - 2008:07)

	Mean	St. Dev.	Sharpe ratio	Median	Min.	Max.	Skewness	Kurtosis	Jarque-Bera	LB(12)	LB(12)-squares
Non Durables	0.978**	4.691**	0.143	1.090	-24.5	34.4	-0.029	8.843**	1401.5**	36.17**	333.9**
Durables	1.074**	7.593**	0.101	1.000	-34.8	79.7	1.203	18.42*	9993.4**	51.38**	266.4**
Manufacturing	1.034**	6.322**	0.115	1.330	-29.8	57.4	0.978	15.59**	6761.2**	39.407**	440.3**
Energy	1.097**	5.983**	0.132	0.860	-26.0	33.5	0.238	6.183**	425.02**	23.27*	251.4**
Hi Tech	1.094**	7.437**	0.106	1.220	-33.8	53.4	0.296	9.030**	1506.7**	26.91**	562.3**
Telecommunications	0.831**	4.594**	0.115	0.880	-21.6	28.2	0.056	6.277**	441.18**	29.49**	342.2**
Shops/Distribution	0.975**	5.884**	0.114	1.130	-30.2	37.1	-0.016	8.501**	1242.0**	56.72**	458.6**
Health	1.089**	5.766**	0.136	1.070	-34.7	38.7	0.171	10.210**	2136.2**	53.59**	605.9**
Utilities	0.902**	5.685**	0.105	1.050	-33.0	43.2	0.095	10.61**	2379.7**	52.12**	622.3**
Other	0.921**	6.473**	0.095	1.260	-30.0	58.7	0.971	16.83**	8006.3**	64.55**	490.0**

Correlation and (variance) co-kurtosis matrices

	Non Durables	Durables	Manufacture	Energy	Hi Tech	Telecom	Shops	Health	Utilities	Other
Non Durables		0.754**	0.851**	0.616**	0.736**	0.671**	0.866**	0.801**	0.707**	0.847**
Durables	10.490*		0.873**	0.607**	0.779**	0.618**	0.798**	0.649**	0.635**	0.802**
Manufacturing	10.150**	15.873*		0.723**	0.862**	0.671**	0.841**	0.762**	0.703**	0.905**
Energy	5.297**	7.095**	7.328**		0.607**	0.495**	0.576**	0.562**	0.617**	0.689**
Hi Tech	7.546**	11.109*	10.872**	5.273**		0.676**	0.785**	0.723**	0.624**	0.798**
Telecommunications	4.614**	5.257**	5.930**	3.265**	5.248**		0.670**	0.600**	0.635**	0.695**
Shops/Distribution	7.880**	9.697**	9.688**	4.810**	7.499**	4.878**		0.740**	0.655**	0.824**
Health	8.130**	9.952**	10.163**	4.987**	7.842**	5.079**	7.860**		0.625**	0.741**
Utilities	6.660**	8.297**	9.188**	4.686**	7.514**	5.839**	7.502**	7.707**		0.740**
Other	8.496**	10.936**	13.017**	6.500**	9.662**	6.876*	8.927**	9.863**	10.436**	

Co-Skewness matrices

	Non Durables	Durables	Manufacture	Energy	Hi Tech	Telecom	Shops	Health	Utilities	Other
Non Durables		0.248	0.196	-0.032	0.058	-0.153	-0.054	-0.021	-0.077	0.046
Durables	0.633		1.042	0.554	0.754	0.164	0.577	0.615	0.397	0.663
Manufacturing	0.505	0.981		0.459	0.684	0.259	0.477	0.600	0.472	0.777
Energy	-0.013	0.248	0.239		0.128	-0.052	-0.064	0.118	0.029	0.180
Hi Tech	0.172	0.451	0.460	0.156		-0.017	0.167	0.234	0.155	0.333
Telecommunications	-0.093	-0.085	0.043	-0.126	-0.030		-0.096	-0.041	0.064	0.109
Shops/Distribution	-0.051	0.228	0.187	-0.058	0.069	-0.104		-0.051	-0.024	0.119
Health	0.061	0.325	0.363	0.150	0.192	0.030	0.037		0.065	0.311
Utilities	-0.046	0.120	0.228	-0.032	0.098	0.089	-0.020	0.009		0.302
Other	0.337	0.658	0.802	0.380	0.555	0.370	-0.020	0.555	0.579	

Table 1 (cont'ed)
Summary Statistics for Equity Returns

Panel C (International Book-to-Market Sorted Portfolio Local Returns, 1975:01 - 2007:12)

	Mean	St. Dev.	Sharpe ratio	Median	Min.	Max.	Skewness	Kurtosis	Jarque-Bera	LB(12)	LB(12)-squares
World	0.694**	3.844**	0.055	1.000	-22.0	12.8	-0.985**	6.832	306.4**	13.88	6.851
EU ex-UK ex-Scand Value	1.326**	4.740**	0.178	1.640	-18.7	16.4	-0.486*	5.130*	90.47**	24.06*	22.72*
EU ex-UK ex-Scand Growth	0.967**	4.294**	0.112	1.280	-24.9	14.9	-0.733	6.915	288.4**	18.78	15.89
United Kingdom Value	1.725**	6.187**	0.201	1.700	-27.0	45.5	0.845	10.95	1088.9**	13.80	44.02**
United Kingdom Growth	1.353**	5.851**	0.148	1.325	-27.9	53.8	1.610*	20.44*	5186.8**	12.60	11.83
Asia & Pacific Value	1.300**	5.123**	0.159	1.000	-25.0	19.1	-0.095	5.615*	113.4**	11.57	56.82**
Asia & Pacific Growth	0.401	4.936**	-0.017	0.525	-18.4	25.1	-0.003	5.214	80.88**	11.21	81.93**
Scandinavia Value	1.676**	6.428**	0.185	1.765	-22.1	25.8	0.175	4.358*	32.46**	29.48**	25.14*
Scandinavia Growth	1.486**	6.270**	0.160	1.770	-21.4	25.5	0.037	4.763**	51.36**	22.75*	80.34**
United States Value	1.081**	4.769**	0.125	1.255	-24.3	14.2	-0.473	4.901	74.41**	7.301	11.83
United States Growth	1.448**	4.302**	0.224	1.660	-20.4	23.7	-0.177	7.616**	353.7**	19.06	29.92**

Correlation and (variance) co-kurtosis matrices

	World	EU ex-UK ex-Scand Value	EU ex-UK ex-Scand Growth	UK Value	UK Growth	Asia Pacific Value	Asia Pacific Growth	Scandinavia Value	Scandinavia Growth	United States Value	United States Growth
World		0.742**	0.792**	0.632**	0.644**	0.593**	0.681**	0.525**	0.625**	0.859**	0.804**
EU ex-UK ex-Scand Value	4.624**		0.850**	0.573**	0.506**	0.470**	0.441**	0.584**	0.542**	0.532**	0.627**
EU ex-UK ex-Scand Growth	6.160**	4.682**		0.550**	0.584**	0.434**	0.513**	0.499**	0.619**	0.621**	0.593**
United Kingdom Value	4.901**	3.204**	4.632**		0.786**	0.403**	0.389**	0.419**	0.377**	0.464**	0.556**
United Kingdom Growth	5.620**	3.397**	5.560*	13.637*		0.317**	0.357**	0.370**	0.407**	0.546**	0.563**
Asia & Pacific Value	4.452**	3.157**	4.368*	3.107**	2.962*		0.649**	0.412**	0.332**	0.337**	0.360**
Asia & Pacific Growth	2.995**	2.047**	2.811**	1.837**	1.806**	3.071**		0.344**	0.454**	0.409**	0.350**
Scandinavia Value	2.827**	2.515**	2.820**	2.149**	2.162**	2.325**	1.620**		0.643**	0.372**	0.432**
Scandinavia Growth	3.330**	2.453**	3.569**	2.132**	2.200**	2.267**	1.989**	2.160**		0.546**	0.434**
United States Value	5.050**	3.166**	4.520*	3.428**	3.819*	3.078	1.873**	2.223**	2.789**		0.784**
United States Growth	5.582**	3.727**	4.887*	6.251*	8.375*	3.082*	1.748**	2.256**	2.219**	4.460**	

Co-Skewness matrices

	World	EU ex-UK ex-Scand Value	EU ex-UK ex-Scand Growth	UK Value	UK Growth	Asia Pacific Value	Asia Pacific Growth	Scandinavia Value	Scandinavia Growth	United States Value	United States Growth
World		-0.837*	-0.976*	-0.516	-0.499	-0.829*	-0.572*	-0.650*	-0.681*	-0.797	-0.733
EU ex-UK ex-Scand Value	-0.661*		-0.595*	-0.359	-0.416	-0.501*	-0.335*	-0.437*	-0.433*	-0.564*	-0.552*
EU ex-UK ex-Scand Growth	-0.881	-0.685		-0.536	-0.499	-0.771	-0.445	-0.642*	-0.552	-0.783	-0.741
United Kingdom Value	-0.040	-0.116	-0.119		0.994	-0.129	-0.049	-0.249	-0.208	-0.097	0.340
United Kingdom Growth	-0.138	-0.028	0.048	1.236		-0.199	-0.041	-0.168	-0.056	0.015	0.589
Asia & Pacific Value	-0.583	-0.410	-0.528	-0.278	-0.404		-0.239	-0.436	-0.437	-0.524	-0.533
Asia & Pacific Growth	-0.301	-0.205	-0.197	-0.097	-0.148	-0.154		-0.329**	-0.307*	-0.259*	-0.285*
Scandinavia Value	-0.339	-0.317	-0.366	-0.191	-0.280	-0.216	-0.123		-0.109	-0.309	-0.300
Scandinavia Growth	-0.389	-0.349*	-0.353	-0.308	-0.231	-0.399*	-0.243	-0.134		-0.225	-0.324
United States Value	-0.644	-0.635**	-0.790*	-0.376	-0.407	-0.583	-0.360*	-0.437	-0.424		-0.451
United States Growth	-0.501	-0.559	-0.668	0.017	0.006	-0.489	-0.343	-0.424	-0.412	-0.415	

** Statistical significance at 1%; * Statistical significance at 5%.

Table 2

Model Selection Statistics

The table reports estimates for the multivariate Markov switching conditionally heteroskedastic VAR model:

$$r_t = \mu_{s_t} + \sum_{j=1}^{p} A_{j,s_t} r_{t-j} + \varepsilon_t$$

where $\mu_{s\,t}$ is the intercept vector in state s_t, A_{j,s_t} is the matrix of autoregressive coefficients associated with leg $j \geq 1$ in state s_t and $\varepsilon_t = (\varepsilon_{1t},...\varepsilon_{ht})' \sim N(\mathbf{0}, \Omega_{s_t})$. The unobserved state variable s_t Is governed by a first-order Markov chain that can assume k distinct values. P autoregressive terms are considered. The sample period is 1988:01-2008:08 for Panel A (International porfolios), 1926:07-2008:07 for Panel B (Industries) and 1975:01-2007:12 for Panel C (Book-to-Market). MISIAH(k,p) stands for Markov Switching Intercept Autoregressive Heteroskedasticity Model with k states and p autoregressive lags.

Panel A (International MSCI USD Returns, 1988:01 - 2008:08)

Model (K,p)	Log-likelihood	LR Statistic	Davies' approx. p-value	BIC	HQ	AIC	Number of parameters	Number of obs.	Saturation ratio	Tests
International MSCI USD Returns, 1988:01 - 2008:07										
Single-state models										
MSIA(1,0)	3350.92	__	__	**-25.978**	-26.450	-26.701	44	1984	45.1	
MSIA(1,1)	3530.31	__	__	-23.723	-25.573	-26.820	108	1976	18.3	
Two-state models										
MSI(2,0)	3375.97	50.089	0.000	**-26.025**	-26.482	-26.790	54	1984	36.7	
MSIH(2,0)	3450.75	199.665	0.000	-25.828	**-26.790**	-27.103	90	1984	22.0	H: 149.58 (0.000)
MSH(2,0)	3334.33	138.826	0.000	**-26.075**	-26.769	**-27.237**	82	1984	24.2	I: 232.84 (0.000)
MSIA(2,1)	3501.38	219.044	0.000	-24.292	-25.837	-26.878	182	1976	10.9	VAR: 250.82 (0.000)
MSIAH(2,1)	3530.31	276.913	0.000	-23.723	-25.573	-26.820	218	1976	9.1	H: 57.87 (0.012)
Three-state models										VAR: 159.12 (0.032)
MSI(3,0)	3385.22	68.591	0.000	-25.665	-26.424	-26.800	66	1984	30.1	
MSIH(3,0)	3570.60	439.350	0.000	-25.727	**-26.895**	**-27.682**	138	1984	14.4	
MSIA(3,1)	3576.25	368.785	0.000	-23.203	-25.393	-26.868	258	1976	7.7	
MSIAH(3,1)	3666.89	550.075	0.000	-22.331	-25.132	-27.019	330	1976	6.0	
Four-state models										
MSI(4,0)	3415.33	128.813	0.000	-25.764	-26.442	-26.898	80	1984	24.8	
MSIH(4,0)	3647.38	592.912	0.000	-25.235	**-26.826**	**-27.898**	188	1984	10.6	
MSIA(4,1)	3669.26	554.801	0.000	-22.216	-25.068	-26.990	336	1976	5.9	

Table 2 (cont'ed)

Model Selection Statistics

Panel B (CRSP Industry Returns, 1926:07 - 2008:07)

Model (K,p)	Log-likelihood	LR Statistic	Davies' approx. p-value	BIC	HQ	AIC	Number of parameters	Number of obs.	Saturation ratio	Tests
					CRSP Industry Returns, 1926:07 - 2008:07					
Single-state models										
MSIA(1,0)	19354.00	__	__	-38.843	-39.043	-39.166	65	9850	151.5	
MSIA(1,1)	19481.34	__	__	-38.441	-38.949	-39.261	165	9850	59.7	
Two-state models										
MSI(2,0)	19428.38	148.767	0.000	-38.910	-39.147	-39.292	77	9850	127.9	H: 2028.35 (0.000)
MSIH(2,0)	20442.56	2177.118	0.000	**-40.391**	**-40.797**	-41.047	132	9850	74.6	
MSH(2,0)	20242.57	1777.147	0.000	**-40.344**	-40.720	-40.950	122	9850	80.7	I: 399.97 (0.000)
MSIA(2,1)	19943.31	923.937	0.000	-38.595	-39.448	-39.972	277	9840	35.5	
MSIAH(2,1)	20514.06	2065.444	0.000	-39.370	-40.393	-41.020	332	9840	29.6	
Three-state models										
MSI(3,0)	19532.68	357.374	0.000	-39.024	-39.476	-39.304	91	9850	108.2	
MSIH(3,0)	20504.22	2300.450	0.000	**-40.226**	-40.845	**-41.225**	201	9850	49.0	
MSIA(3,1)	20106.58	1250.476	0.000	-38.129	-39.333	-40.072	391	9840	25.2	
MSIAH(3,1)	20704.65	2446.620	0.000	-38.574	-40.117	-41.064	501	9840	19.6	
Four-state models										
MSI(4,0)	19607.79	507.578	0.000	-39.064	-39.393	-39.596	107	9850	92.1	
MSIH(4,0)	20423.45	2138.898	0.000	-40.136	**-40.973**	**-41.487**	272	9850	36.2	
MSIH(4,0)-VAR(1)	20826.51	2690.348	0.000	-39.725	**-40.871**	**-41.574**	372	9840	26.5	

Table 2 (cont'ed)

Model Selection Statistics

Panel C (International Book-to-Market Sorted Portfolio Local Returns, 1975:01 - 2007:12)

Model (K,p)	Log-likelihood	LR Statistic	Davies' approx. p-value	BIC	HQ	AIC	Number of parameters	Number of obs.	Saturation ratio	Tests
International Book-to-Market Sorted Portfolio Local Returns, 1975:01 - 2007:12										
Single-state models										
MSIA(1,0)	8772.32	__	__	-43.142	-43.609	-43.916	77	4356	56.6	
MSIA(1,1)	8929.72	__	__	-41.171	-43.190	-44.516	198	4345	21.9	
Two-state models										
MSI(2,0)	8830.76	116.877	0.000	**-43.240**	-43.787	-44.145	90	4356	48.4	H: 505.68 (0.000)
MSIH(2,0)	9083.60	622.555	0.000	**-43.521**	**-44.468**	-45.089	156	4356	27.9	
MSH(2,0)	9038.61	532.590	0.000	**-43.513**	**-44.389**	-44.965	145	4356	30.0	I: 89.98 (0.000)
MSIA(2,1)	9123.80	388.169	0.000	-41.171	-43.190	-44.516	332	4345	13.1	
MSIAH(2,1)	9240.63	621.826	0.000	-40.764	-43.184	-44.773	398	4345	10.9	H: 233.66 (0.000)
Three-state models										
MSI(3,0)	8854.06	163.463	0.000	-43.132	-43.769	-44.187	105	4356	41.5	
MSIH(3,0)	9212.01	879.363	0.000	-42.946	-44.384	**-45.328**	237	4356	18.4	
MSIA(3,1)	9344.81	830.185	0.000	-40.232	-43.078	-44.946	468	4345	9.3	
MSIAH(3,1)		No converge achieved (too many parameters)					600	4345	7.2	
Four-state models										
MSI(4,0)	8894.16	243.666	0.000	-43.077	-43.818	-44.304	122	4356	35.7	
MSIH(4,0)	9375.96	1207.270	0.000	-42.520	**-44.463**	**-45.737**	320	4356	13.6	
MSIH(4,0)-VAR(1)	9452.93	1046.411	0.000	-41.188	-43.870	**-45.630**	441	4345	9.9	

Table 3

Estimated Markov Switching Models

The table shows estimation results for the regime switching model

$$r_t = \mu_{s_t} + \varepsilon_t .$$

r_t is the vector collecting monthly total return series, μ_{s_t} is the intercept vector in state s_t and $\varepsilon_t = (\varepsilon_{1t},...\varepsilon_{ht})'$~ $N(\mathbf{0}, \Omega_{s_t})$. Panel A shows the estimation results for the International portfolios dataset, Panel B for the Industries one, Panel C for the Book-to-Market portfolios one. Each Panel reports the results for the single-state model, k=1, (Panel A), for the two-state model k=2 (Panel B) and its Markov chain properties (Panel C) (ergodic probabilites and average state duration).

Panel A (International MSCI USD Returns, 1988:01 - 2008:08)

Panel A - SINGLE STATE MODEL								
	Pacific EX JP	Japan	Europe EX UK	UK	North America	EM Latin America	EM Asia	EM Europe & Middle East
1. Mean returns	0.746*	-0.051	0.766*	0.707*	0.756**	1.906**	0.774	1.846**
2. Correlations/Volatilities								
Pacific EX JP	5.428**							
JP	0.444**	6.310**						
Europe EX UK	0.592**	0.462**	4.928**					
UK	0.621**	0.480**	0.744**	4.397**				
North America	0.601**	0.368**	0.669**	0.664**	3.924**			
EM Latin America	0.545**	0.321**	0.410**	0.394**	0.500**	8.932**		
EM Asia	0.785**	0.406**	0.508**	0.442**	0.551**	0.491**	7.111**	
EM Europe and Middle East	0.424**	0.221**	0.467**	0.352**	0.404**	0.479**	0.475**	7.747**
Panel B - TWO-STATE MODEL								
	Pacific EX JP	JP	Europe EX UK	UK	North America	EM Latin America	EM Asia	EM Europe & Middle East
1. Mean returns								
Bear/High Volatility State	-0.386*	-1.325**	-0.087	-0.051	0.384	0.496	0.336	0.227
Bull/Low Volatility State	1.371**	0.653	1.238**	1.125**	0.961**	2.685**	1.017*	2.740**
2. Correlations/Volatilities								
Bear/High Volatility State								
Pacific EX JP	7.081**							
JP	0.399**	7.524**						
Europe EX UK	0.498**	0.473**	5.850**					
UK	0.554**	0.514**	0.772**	4.478**				
North America	0.561**	0.355**	0.621**	0.509**	4.284**			
EM Latin America	0.514**	0.336*	0.234*	0.365*	0.475**	1.159**		
EM Asia	0.779**	0.365**	0.435**	0.364**	0.550**	0.478**	9.173**	
EM Europe and Middle East	0.333*	0.002	0.375**	0.448**	0.483**	0.407**	0.411**	8.406**
Bull/Low Volatility State								
Pacific EX JP	4.095**							
JP	0.478**	5.376**						
Europe EX UK	0.706**	0.429**	4.244**					
UK	0.712**	0.443**	0.727**	4.281**				
North America	0.665**	0.372**	0.713**	0.767**	3.681**			
EM Latin America	0.574**	0.276*	0.620**	0.427**	0.537**	6.888**		
EM Asia	0.803**	0.456**	0.600**	0.534**	0.565**	0.509**	5.615**	
EM Europe and Middle East	0.512**	0.386**	0.532**	0.267*	0.335*	0.554**	0.554**	7.176**
3. Transition probabilities	Bear/High Volatility State				Bull/Low Volatility State			
Bear/High Volatility State	0.807**				0.193			
Bull/Low Volatility State	0.103				0.897**			
Panel C - MARKOV CHAIN PROPERTIES, TWO-STATE MODEL								
	Bear	Bull			**Average duration (in months)**		Bear	Bull
Ergodic Probs	0.348	0.652					5.18	9.70

** Statistical significance at 1%; * Statistical significance at 5%.

Table 3 (cont'ed)

Estimated Markov Switching Models

Panel B (CRSP Industry Returns, 1926:07 - 2008:07)

Panel A - SINGLE STATE MODEL

	Non-Durables	Durables	Manufacture	Energy	Hi Tech	Telecom	Shops/ Distrib.	Health	Utilities	Other
1. Mean returns	0.978**	1.074**	1.034**	1.097**	1.094**	0.831**	0.975**	1.089**	0.902**	0.921**
2. Correlations/Volatilities										
Non-Durables	4.691**									
Durables	0.754**	7.593**								
Manufacture	0.851**	0.873**	6.322**							
Energy	0.616**	0.607**	0.723**	5.983**						
Hi Tech	0.735**	0.779**	0.862**	0.609**	7.437**					
Telecom	0.671**	0.618**	0.671**	0.495**	0.676**	4.594**				
Shops/ Distrib.	0.866**	0.798**	0.841**	0.576**	0.785**	0.670**	5.884**			
Health	0.801**	0.649**	0.762**	0.562**	0.723**	0.600**	0.740**	5.766**		
Utilities	0.707**	0.635**	0.703**	0.617**	0.624**	0.635**	0.655**	0.625**	5.685**	
Other	0.847**	0.802**	0.905**	0.689**	0.798**	0.695**	0.824**	0.741**	0.740**	6.473**

Panel B - TWO STATE MODEL MSIH(2,0)

	Non-Durables	Durables	Manufacture	Energy	Hi Tech	Telecom	Shops/ Distrib.	Health	Utilities	Other
1. Mean returns										
Bear/High Volatility State	0.162	0.751	0.718	0.760	0.650	0.238	0.081	0.483	0.447	0.111
Bull/Low Volatility State	1.212**	1.167**	1.125**	1.194**	1.221**	1.001**	1.232**	1.264**	1.032**	1.154**
2. Correlations/Volatilities										
Bear/High Volatility State										
NoDur	7.233**									
Durbl	0.666**	13.181**								
Manuf	0.810**	0.810**	10.848**							
Enrgy	0.473**	0.492**	0.624**	9.318**						
HiTec	0.695**	0.711**	0.837**	0.490**	12.657**					
Telcm	0.653**	0.511**	0.590**	0.393*	0.488**	7.395**				
Shops	0.832**	0.718**	0.796**	0.417**	0.732**	0.591**	9.543**			
Hlth	0.775**	0.543**	0.739**	0.434**	0.686**	0.522**	0.682**	8.903**		
Utils	0.671**	0.499**	0.600**	0.532**	0.470**	0.621**	0.528**	0.514**	10.021**	
Other	0.832**	0.746**	0.890**	0.605**	0.770**	0.625**	0.801**	0.706**	0.662**	10.987**
Bull/Low Volatility State										
NoDur	3.607**									
Durbl	0.826**	4.913**								
Manuf	0.893**	0.905**	4.186**							
Enrgy	0.739**	0.689**	0.797**	4.586**						
HiTec	0.772**	0.814**	0.875**	0.691**	4.999**					
Telcm	0.683**	0.686**	0.724**	0.574**	0.795**	3.360**				
Shops	0.894**	0.850**	0.872**	0.698**	0.821**	0.723**	4.247**			
Hlth	0.823**	0.729**	0.787**	0.670**	0.756**	0.660**	0.785**	4.453**		
Utils	0.744**	0.698**	0.754**	0.681**	0.700**	0.649**	0.729**	0.706**	3.550**	
Other	0.866**	0.832**	0.914**	0.752**	0.814**	0.738**	0.838**	0.771**	0.778**	4.351**

3. Transition probabilities	Bear/High Volatility State	Bull/Low Volatility State
Bear/High Volatility State	0.822**	0.178
Bull/Low Volatility State	0.051	0.949**

Panel C - MARKOV CHAIN PROPERTIES, TWO-STATE MODEL

	Bear	Bull		Bear	Bull
Ergodic Probs	0.224	0.776	**Average duration (in months)**	5.60	19..46

Table 3 (cont'ed)

Estimated Markov Switching Models

Panel C (International Book-to-Market Sorted Portfolio Local Returns, 1975:01 - 2007:12)

Panel A - SINGLE STATE MODEL	World	EU ex-UK ex-Scand Value	EU ex-UK ex-Scand Growth	UK Value	UK Growth	Asia & Pacific Value	Asia & Pacific Growth	Scandinavia Value	Scandinavia Growth	US Value	US Growth
1. Mean returns	0.694**	1.326**	0.967**	1.725**	1.353**	1.300**	0.401	1.676**	1.486**	1.081**	1.448**
2. Correlations/Volatilities											
World	3.844**										
EU ex-UK ex-Scand Value	0.742**	4.740**									
EU ex-UK ex-Scand Growth	0.792**	0.850**	4.294**								
United Kingdom Value	0.632**	0.573**	0.549**	6.187**							
United Kingdom Growth	0.644**	0.506**	0.584**	0.786**	5.851**						
Asia & Pacific Value	0.593**	0.470**	0.434**	0.403**	0.317**	5.123**					
Asia & Pacific Growth	0.681**	0.441**	0.513**	0.389**	0.357**	0.649**	4.936**				
Scandinavia Value	0.525**	0.584**	0.500**	0.419**	0.370**	0.412**	0.344**	6.428**			
Scandinavia Growth	0.625**	0.542**	0.619**	0.377**	0.407**	0.332**	0.454**	0.643**	6.270**		
United States Value	0.859**	0.532**	0.621**	0.464**	0.550**	0.337**	0.409**	0.372**	0.546**	4.769**	
United States Growth	0.804**	0.623**	0.593**	0.556**	0.563**	0.360**	0.350**	0.432**	0.434**	0.784**	4.302**

Panel B - TWO STATE MODEL MSIH(2,0)	World	EU ex-UK ex-Scand Value	EU ex-UK ex-Scand Growth	UK Value	UK Growth	Asia & Pacific Value	Asia & Pacific Growth	Scandinavia Value	Scandinavia Growth	US Value	US Growth
1. Mean returns											
Regime 1 (Bull Word/Low Vol.)	0.956**	1.851**	1.446**	1.512**	1.296**	1.168**	0.612*	1.690**	1.392**	1.317**	1.557**
Regime 2 (Bear Word/High Vol.)	0.049	0.036	-0.209	2.247*	1.494	1.623*	-0.118	1.643*	1.716	0.501	1.182
2. Correlations/Volatilities											
Regime 1 (Bull Word/Low Vol.)											
World	2.910**										
EU ex-UK ex-Scand Value	0.714**	4.128**									
EU ex-UK ex-Scand Growth	0.787**	0.860**	3.268**								
United Kingdom Value	0.613**	0.562**	0.557**	4.431**							
United Kingdom Growth	0.636**	0.490**	0.552**	0.760**	3.961**						
Asia & Pacific Value	0.605**	0.432**	0.466**	0.335*	0.306*	3.886**					
Asia & Pacific Growth	0.720**	0.425**	0.501**	0.439**	0.410**	0.718**	3.997**				
Scandinavia Value	0.507**	0.532**	0.466**	0.496**	0.431**	0.383*	0.394**	5.387**			
Scandinavia Growth	0.595**	0.495**	0.518**	0.446**	0.457*	0.282*	0.385**	0.651**	4.740**		
United States Value	0.844**	0.454**	0.574**	0.389**	0.493**	0.325**	0.459**	0.319*	0.510**	3.664**	
United States Growth	0.817**	0.614**	0.626**	0.518**	0.517**	0.327**	0.420**	0.403**	0.507**	0.786**	3.133**
Regime 2 (Bear Word/High Vol.)											
World	5.438**										
EU ex-UK ex-Scand Value	0.777**	5.767**									
EU ex-UK ex-Scand Growth	0.791**	0.845**	5.948**								
United Kingdom Value	0.663**	0.637**	0.576**	9.131**							
United Kingdom Growth	0.661**	0.563**	0.628**	0.801**	8.915**						
Asia & Pacific Value	0.598**	0.544**	0.435**	0.446**	0.324*	7.297**					
Asia & Pacific Growth	0.648**	0.454**	0.518**	0.365*	0.331*	0.605**	6.663**				
Scandinavia Value	0.550**	0.663**	0.543**	0.369**	0.338**	0.440**	0.298*	8.436**			
Scandinavia Growth	0.656**	0.622**	0.716**	0.333*	0.380*	0.365**	0.514**	0.642**	8.959**		
United States Value	0.868**	0.609**	0.651**	0.524**	0.587**	0.353*	0.363*	0.422**	0.577**	6.705**	
United States Growth	0.797**	0.663**	0.577**	0.584**	0.591**	0.386*	0.299*	0.460**	0.389**	0.785**	6.293**

3. Transition probabilities	Regime 1 (Bull Word/Low Volatility)	Regime 2 (Bear Word/High Volatility)
Regime 1 (Bull Word/Low Vol.)	0.901**	0.099
Regime 2 (Bear Word/High Volati	0.292	0.708**

Panel C - MARKOV CHAIN PROPERTIES, TWO-STATE MODEL	Regime 1	Regime 2			Regime 1	Regime 2
Ergodic Probs	0.747	0.253		**Average duration (in months)**	10.10	3.43

Table 4

Moments Implied by Estimated Two-State Markov Switching Model

This table reports moment implied by the estimated Two-State Model for returns. In the co-skewness matrix, coefficients above the main diagonal refer to the sample covariance between the square of the returns of the row portfolio/index and the level of returns of the column portfolio/index; coefficients below the main diagonal refer to the sample covariance between the level of the returns of the column portfolio/index and the square of returns of the row portfolio/index. In the correlation/co-kurtosis matrix, correlations are reported above the main diagonal and sample covariances between squared portfolio returns appear below the main diagonal. Panels A, B and C respectively refer to the International, the Industry and the International Book-to-Market Portfolios.

Panel A (International MSCI USD Returns, 1988:01 - 2008:08)

	Mean	St. Dev.	Sharpe ratio	Median	Min.	Max.	Skewness	Kurtosis
Pacific ex-Japan	0.713	4.381	0.082	0.830	-21.8	17.8	-0.308	4.222
Japan	-0.060	5.084	-0.081	0.037	-12.1	12.3	-0.058	3.592
Europe ex-UK	0.748	3.964	0.100	0.818	-13.1	12.2	-0.247	3.672
United Kingdom	0.703	3.558	0.098	0.723	-9.8	13.4	-0.005	3.279
North America	0.719	3.192	0.115	0.708	-9.5	9.1	-0.135	3.442
EM Latin America	1.889	7.237	0.212	2.066	-27.3	21.9	-0.286	4.106
EM Asia	0.708	5.747	0.062	0.771	-19.7	17.0	-0.100	3.789
EM Europe & Middle East	1.843	6.290	0.237	1.875	-22.1	22.0	-0.055	4.101

Correlation and (variance) co-kurtosis matrices

	Pacific ex-JP	Japan	EU ex-UK	UK	North Amr.	EM Latin Amr.	EM Asia	EM EU & Middle East
Pacific ex-JP		0.447	0.594	0.621	0.603	0.545	0.784	0.426
Japan	1.457		0.459	0.476	0.369	0.318	0.416	0.223
EU ex-UK	1.873	1.635		0.740	0.671	0.417	0.513	0.469
UK	1.731	1.494	2.241		0.666	0.397	0.445	0.353
North Amr.	1.826	1.313	2.333	2.001		0.501	0.552	0.401
EM Latin Amr.	1.862	1.350	1.788	1.322	1.861		0.492	0.480
EM Asia	2.833	1.671	1.960	1.430	1.799	1.716		0.481
EM EU & Middle East	1.428	1.253	1.665	1.272	1.732	1.914	1.535	

Co-skewness matrix

	Pacific ex-JP	Japan	EU ex-UK	UK	North Amr.	EM Latin Amr.	EM Asia	EM EU & Middle East
Pacific ex-JP		-0.164	-0.195	-0.124	-0.178	-0.210	-0.213	-0.166
Japan	-0.104		-0.062	-0.008	-0.054	-0.152	-0.084	-0.048
EU ex-UK	-0.193	-0.168		-0.181	-0.227	-0.164	-0.212	-0.172
UK	-0.030	-0.055	-0.092		-0.021	-0.055	-0.048	-0.090
North Amr.	-0.172	-0.107	-0.219	-0.093		-0.203	-0.175	-0.186
EM Latin Amr.	-0.218	-0.215	-0.112	-0.144	-0.180		-0.135	-0.179
EM Asia	-0.170	-0.142	-0.182	-0.111	-0.136	-0.142		-0.154
EM EU & Middle East	-0.103	-0.045	-0.068	-0.114	-0.163	-0.247	-0.114	

Table 4 (cont'ed)

Moments Implied by Estimated Two-State Markov Switching Model

Panel B (CRSP Industry Returns, 1926:07 - 2008:07)

	Mean	Std. Dev.	Sharpe ratio	Median	Min.	Max.	Skewness	Kurtosis
Non-Durables	0.969	4.198	0.158	1.050	-20.7	21.0	-0.147	6.187
Durables	1.056	6.759	0.111	1.104	-31.2	40.6	0.338	10.426
Manufacture	1.017	5.631	0.126	1.074	-24.8	33.3	0.263	9.314
Energy	1.095	5.369	0.147	1.147	-18.5	27.5	0.034	5.315
Hi Tech	1.086	6.624	0.118	1.139	-27.0	37.3	0.036	7.112
Telecom	0.827	4.114	0.127	0.908	-11.0	18.4	-0.147	5.761
Shops/ Distrib.	0.965	5.243	0.126	1.047	-25.8	26.0	-0.126	6.472
Health	1.069	5.149	0.148	1.138	-23.8	24.0	-0.037	6.688
Utilities	0.891	5.113	0.115	0.959	-15.8	24.6	-0.093	8.157
Other	0.905	5.792	0.104	1.003	-19.6	31.5	0.174	9.673

Correlation and (variance) co-kurtosis matrices

	Non-Durables	Durables	Manuf.	Energy	Hi Tech	Telecom	Shops/ Distrib.	Health	Utilities	Other
Non-Durables		0.754	0.849	0.616	0.732	0.671	0.866	0.803	0.709	0.846
Durables	6.005		0.873	0.610	0.778	0.619	0.799	0.652	0.636	0.802
Manufacture	6.145	8.597		0.728	0.861	0.673	0.840	0.760	0.706	0.904
Energy	3.664	4.456	4.849		0.609	0.501	0.577	0.562	0.616	0.690
Hi Tech	4.869	6.595	6.749	3.806		0.677	0.785	0.721	0.627	0.797
Telecom	3.561	4.013	4.341	2.744	4.311		0.672	0.600	0.636	0.695
Shops/ Distrib.	5.285	6.052	6.130	3.530	5.211	3.849		0.744	0.660	0.823
Health	5.008	5.526	5.804	3.432	4.925	3.582	4.909		0.629	0.739
Utilities	4.607	5.344	5.831	3.696	5.077	4.230	4.999	4.822		0.740
Other	5.535	6.608	7.628	4.405	6.082	4.707	5.749	5.651	6.344	

Co-skewness matrix

	Non-Durables	Durables	Manuf.	Energy	Hi Tech	Telecom	Shops/ Distrib.	Health	Utilities	Other
Non-Durables		-0.013	-0.036	-0.092	-0.069	-0.147	-0.132	-0.101	-0.129	-0.101
Durables	0.126		0.294	0.145	0.199	-0.025	0.115	0.143	0.063	0.144
Manufacture	0.082	0.274		0.111	0.174	0.006	0.082	0.136	0.086	0.178
Energy	-0.067	0.046	0.040		0.005	-0.071	-0.081	-0.012	-0.039	0.003
Hi Tech	-0.023	0.099	0.099	0.015		-0.089	-0.016	0.014	-0.021	0.037
Telecom	-0.143	-0.129	-0.087	-0.115	-0.119		-0.147	-0.107	-0.087	-0.085
Shops/ Distrib.	-0.127	-0.010	-0.025	-0.092	-0.060	-0.135		-0.103	-0.104	-0.067
Health	-0.066	0.042	0.055	-0.010	-0.004	-0.066	-0.071		-0.064	0.021
Utilities	-0.128	-0.039	-0.007	-0.078	-0.053	-0.079	-0.114	-0.091		-0.014
Other	-0.011	0.125	0.165	0.048	0.095	0.010	-0.122	0.088	0.077	

Table 4 (cont'ed)

Moments Implied by Estimated Two-State Markov Switching Model

Panel C (International Book-to-Market Sorted Portfolio Local Returns, 1975:01 - 2007:12)

	Mean	St. Dev.	Sharpe ratio	Median	Min.	Max.	Skewness	Kurtosis
World	0.717	3.244	0.072	0.764	-18.2	14.6	-0.440	5.207
EU ex-UK ex-Scand Value	1.391	4.055	0.224	1.487	-11.4	15.7	-0.269	4.122
EU ex-UK ex-Scand Growth	1.024	3.619	0.149	1.131	-15.2	15.9	-0.441	5.205
United Kingdom Value	1.689	5.171	0.233	1.609	-19.4	36.0	0.385	6.919
United Kingdom Growth	1.335	4.870	0.175	1.300	-22.0	42.3	0.591	10.34
Asia & Pacific Value	1.285	4.338	0.184	1.253	-13.0	16.0	0.040	4.894
Asia & Pacific Growth	0.405	4.172	-0.019	0.441	-14.5	22.0	-0.060	4.418
Scandinavia Value	1.680	5.505	0.217	1.653	-12.5	24.6	0.065	4.042
Scandinavia Growth	1.443	5.315	0.180	1.415	-21.9	26.6	0.048	4.641
United States Value	1.091	4.028	0.151	1.143	-22.4	16.3	-0.247	4.549
United States Growth	1.447	3.606	0.267	1.456	-15.7	21.4	-0.086	5.689

Correlation and (variance) co-kurtosis matrices

	World	EU ex-UK ex-Scand Value	EU ex-UK ex-Scand Growth	UK Value	UK Growth	Asia Pacific Value	Asia Pacific Growth	Scandinavia Value	Scandinavia Growth	United States Value	United States Growth
World		0.741	0.792	0.626	0.634	0.601	0.683	0.528	0.626	0.857	0.803
EU ex-UK ex-Scand Value	3.218		0.851	0.567	0.496	0.478	0.442	0.580	0.538	0.524	0.623
EU ex-UK ex-Scand Growth	4.030	3.488		0.543	0.574	0.447	0.513	0.498	0.613	0.617	0.594
United Kingdom Value	3.324	2.427	3.011		0.779	0.399	0.386	0.424	0.379	0.453	0.548
United Kingdom Growth	3.633	2.402	3.477	6.853		0.315	0.350	0.370	0.408	0.535	0.552
Asia & Pacific Value	3.021	2.237	2.677	2.357	2.170		0.653	0.416	0.337	0.345	0.365
Asia & Pacific Growth	2.599	1.742	2.217	1.740	1.716	2.572		0.352	0.454	0.413	0.357
Scandinavia Value	2.245	2.137	2.204	1.833	1.822	1.902	1.521		0.647	0.371	0.431
Scandinavia Growth	2.749	2.118	2.866	1.883	2.034	1.864	1.920	2.254		0.544	0.442
United States Value	3.886	2.328	3.088	2.513	2.836	2.128	1.743	1.820	2.403		0.783
United States Growth	3.964	2.700	3.165	3.677	4.441	2.220	1.645	1.936	1.993	4.824	

Co-Skewness matrices

	World	EU ex-UK ex-Scand Value	EU ex-UK ex-Scand Growth	UK Value	UK Growth	Asia Pacific Value	Asia Pacific Growth	Scandinavia Value	Scandinavia Growth	United States Value	United States Growth
World		-0.382	-0.439	-0.193	-0.205	-0.295	-0.250	-0.244	-0.261	-0.350	-0.306
EU ex-UK ex-Scand Value	-0.316		-0.306	-0.177	-0.195	-0.215	-0.162	-0.204	-0.205	-0.268	-0.260
EU ex-UK ex-Scand Growth	-0.425	-0.363		-0.228	-0.247	-0.291	-0.232	-0.270	-0.273	-0.367	-0.326
United Kingdom Value	-0.018	-0.084	-0.078		0.397	0.014	-0.025	-0.068	-0.044	-0.034	0.137
United Kingdom Growth	-0.072	-0.078	-0.049	0.470		-0.022	-0.031	-0.042	0.004	-0.016	0.203
Asia & Pacific Value	-0.204	-0.179	-0.222	-0.044	-0.109		-0.076	-0.125	-0.122	-0.186	-0.168
Asia & Pacific Growth	-0.153	-0.132	-0.141	-0.030	-0.055	-0.055		-0.108	-0.116	-0.122	-0.108
Scandinavia Value	-0.135	-0.147	-0.160	-0.049	-0.082	-0.059	-0.050		-0.023	-0.119	-0.105
Scandinavia Growth	-0.147	-0.166	-0.162	-0.070	-0.059	-0.105	-0.085	-0.027		-0.086	-0.107
United States Value	-0.295	-0.298	-0.356	-0.130	-0.165	-0.193	-0.158	-0.163	-0.164		-0.202
United States Growth	-0.216	-0.266	-0.298	0.025	0.004	-0.150	-0.139	-0.151	-0.134	-0.179	

Table 5

Portfolio Weights as a Function of the Initial State

This table displays average optimal portfolio shares. The out-of-sample period for our International (Panel A), Industry (Panel B) and Book-to-Market International (Panel C) data runs from 1998:01-2008:07, 1980:01-2008:07, and 1995:01-2007:12 respectively. The l.h.s. (r.h.s) refers to portfolios subject (free) from short-sales constraints. The first six columns refer to the investor horizon. The upper (lower) part of each panel refers to the allocation associated with the single-state (two-state) model. In the latter case, we highlight the "ex-ante" portfolio shares computed using the ergodic probabilities, and the shares conditional on the bear and the bull states. Each row, within a given case, is associated with investor preferences ranging from mean-variance to four-moments.

Panel A (International MSCI USD Returns, 1988:01 - 2008:08)

		T=1	T=3	T=12	T=24	T=60	T=120	"Slope"	T=1	T=3	T=12	T=24	T=60	T=120	"Slope"
		No-short sales							Unconstrained						
		Single-State Model (Unconditional Allocation)													
	Pacific EX JP	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.108	-0.136	-0.096	-0.072	-0.116	-0.222	-0.114
	Japan	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.948	-0.969	-0.994	-1.067	-1.216	-1.535	-0.587
	Europe EX UK	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.084	-0.017	-0.037	-0.049	-0.057	-0.034	0.050
	UK	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.631	0.608	0.613	0.643	0.679	0.793	0.163
	North America	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.316	0.304	0.307	0.279	0.263	0.143	-0.173
	EM Latin America	0.383	0.391	0.398	0.400	0.407	0.415	0.033	0.642	0.683	0.676	0.706	0.810	1.038	0.397
	EM Asia	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.313	-0.309	-0.329	-0.343	-0.381	-0.418	-0.105
	EM Europe and Middle East	0.617	0.609	0.602	0.600	0.593	0.585	-0.033	0.865	0.836	0.861	0.902	1.018	1.235	0.370
		Two-State Model (Current State: Ergodic/Unconditional Probabilities)													
Mean-Variance Preferences	Pacific EX JP	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.787	-0.562	-0.870	-0.923	-1.052	-1.318	-2.105
	Japan	0.162	0.000	0.000	0.000	0.000	0.000	-0.162	-1.306	-1.219	-1.230	-1.285	-1.482	-1.848	-0.542
	Europe EX UK	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.426	-0.207	-0.127	-0.088	-0.104	-0.104	1.323
	UK	0.122	0.000	0.000	0.000	0.000	0.000	-0.122	2.063	1.133	1.071	1.099	1.195	1.358	-0.705
	North America	0.413	0.306	0.288	0.276	0.260	0.243	-0.170	-0.087	0.768	1.058	1.054	1.146	1.246	1.333
	EM Latin America	0.134	0.289	0.291	0.289	0.305	0.331	0.197	1.148	0.588	0.540	0.554	0.644	0.802	-0.346
	EM Asia	0.018	0.000	0.000	0.000	0.000	0.000	-0.018	-1.007	0.077	0.320	0.346	0.410	0.520	1.527
	EM Europe and Middle East	0.150	0.405	0.421	0.435	0.435	0.426	0.276	0.828	0.423	0.238	0.243	0.243	0.344	-0.485
Mean-Var-Skew Preferences	Pacific EX JP	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-4.185	-2.865	-2.401	-1.895	-1.162	-0.695	3.490
	Japan	0.165	0.000	0.000	0.000	0.000	0.000	-0.165	-3.622	-2.672	-2.065	-1.357	-1.625	-1.492	2.130
	Europe EX UK	0.000	0.000	0.000	0.000	0.000	0.000	0.000	5.864	3.288	1.839	-0.182	-0.104	-0.123	-5.987
	UK	0.127	0.000	0.000	0.000	0.000	0.000	-0.127	-3.740	-1.240	-0.275	0.940	1.063	1.144	4.884
	North America	0.415	0.349	0.320	0.277	0.265	0.239	-0.175	4.789	3.219	2.053	1.407	1.387	1.115	-3.675
	EM Latin America	0.123	0.217	0.274	0.300	0.304	0.325	0.202	1.415	0.831	1.323	0.841	0.752	0.870	-0.546
	EM Asia	0.023	0.000	0.000	0.000	0.000	0.000	-0.023	-4.385	-2.241	-1.468	-0.613	0.430	0.587	4.972
	EM Europe and Middle East	0.147	0.434	0.406	0.423	0.432	0.436	0.289	4.864	2.680	1.994	1.857	0.259	-0.405	-5.269
Mean-Var-Skew-Kurtosis	Pacific EX JP	0.010	0.001	0.000	0.000	0.000	0.000	-0.010	-0.071	-0.019	-0.081	-0.153	-0.316	-0.560	-0.489
	Japan	0.223	0.204	0.098	0.000	0.000	0.000	-0.223	0.286	0.254	0.112	-0.026	-0.333	-0.735	-1.021
	Europe EX UK	0.017	0.022	0.044	0.031	0.000	0.000	-0.017	-0.052	0.035	0.055	0.044	0.052	0.031	0.083
	UK	0.130	0.088	0.111	0.138	0.023	0.000	-0.130	0.173	0.096	0.163	0.235	0.374	0.587	0.414
	North America	0.244	0.258	0.249	0.253	0.269	0.225	-0.019	0.087	0.092	0.124	0.169	0.277	0.407	0.320
	EM Latin America	0.148	0.151	0.218	0.263	0.316	0.342	0.194	0.207	0.180	0.252	0.301	0.428	0.595	0.389
	EM Asia	0.033	0.061	0.030	0.019	0.003	0.000	-0.033	0.090	0.088	0.097	0.135	0.185	0.284	0.194
	EM Europe and Middle East	0.195	0.215	0.250	0.297	0.388	0.433	0.238	0.280	0.274	0.277	0.296	0.331	0.390	0.111
Mean-Var-Kurtosis Preferences	Pacific EX JP	0.013	0.002	0.000	0.000	0.000	0.000	-0.013	-0.079	-0.018	-0.082	-0.153	-0.313	-0.540	-0.461
	Japan	0.247	0.225	0.099	0.000	0.000	0.000	-0.247	0.272	0.244	0.108	-0.032	-0.336	-0.740	-1.012
	Europe EX UK	0.028	0.032	0.050	0.043	0.000	0.000	-0.028	-0.035	0.031	0.054	0.055	0.055	0.044	0.078
	UK	0.125	0.098	0.120	0.147	0.034	0.000	-0.125	0.163	0.116	0.160	0.228	0.377	0.585	0.423
	North America	0.153	0.158	0.180	0.204	0.240	0.214	0.061	0.081	0.081	0.123	0.162	0.269	0.393	0.312
	EM Latin America	0.174	0.172	0.230	0.270	0.316	0.344	0.170	0.221	0.192	0.250	0.304	0.430	0.594	0.373
	EM Asia	0.036	0.070	0.058	0.029	0.006	0.000	-0.036	0.088	0.088	0.119	0.141	0.188	0.284	0.195
	EM Europe and Middle East	0.225	0.243	0.263	0.307	0.404	0.442	0.217	0.288	0.266	0.269	0.296	0.330	0.381	0.092
		Two-State Model (Current State: Bear/High Volatility)													
Mean-Variance Preferences	Pacific EX JP	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-2.070	-1.678	-1.625	-1.715	-1.898	-2.350	-0.280
	Japan	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.822	-1.574	-1.535	-1.616	-1.832	-2.273	-0.451
	Europe EX UK	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.775	-0.464	-0.382	-0.351	-0.389	-0.389	0.387
	UK	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.642	2.024	1.811	1.878	1.997	2.319	-0.323
	North America	0.997	0.867	0.820	0.789	0.747	0.690	-0.307	2.397	1.866	1.859	1.857	2.033	2.319	-0.078
	EM Latin America	0.003	0.072	0.090	0.097	0.125	0.160	0.157	0.338	0.374	0.393	0.423	0.478	0.620	0.282
	EM Asia	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.298	1.013	0.948	0.992	1.118	1.374	0.076
	EM Europe and Middle East	0.000	0.061	0.090	0.114	0.129	0.150	0.150	-1.008	-0.560	-0.469	-0.467	-0.507	-0.620	0.388
Mean-Var-Skew Preferences	Pacific EX JP	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-3.501	-2.482	-2.863	-3.110	-2.280	-1.368	2.133
	Japan	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-4.092	-2.740	-1.698	-1.035	-2.264	-2.881	1.211
	Europe EX UK	0.000	0.000	0.000	0.000	0.000	0.000	0.000	6.481	5.422	2.221	-1.780	-0.348	-0.486	-6.967
	UK	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.503	-0.642	1.259	4.216	2.069	1.656	2.159
	North America	1.000	1.000	0.907	0.788	0.751	0.686	-0.314	5.573	4.342	2.690	2.292	2.156	1.904	-3.668
	EM Latin America	0.000	0.000	0.055	0.095	0.115	0.166	0.166	-5.916	-3.440	0.525	0.442	0.758	0.781	6.697
	EM Asia	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-2.099	-0.608	-0.226	0.202	1.419	1.623	3.722
	EM Europe and Middle East	0.000	0.000	0.038	0.117	0.134	0.148	0.148	5.058	1.148	-0.908	-0.226	-0.509	-0.230	-5.288

Panel A (International MSCI USD Returns, 1988:01 - 2008:08)

		T=1	T=3	T=12	T=24	T=60	T=120	"Slope"	T=1	T=3	T=12	T=24	T=60	T=120	"Slope"
		Two-State Model (Current State: Bear/High Volatility)													
		No-short sales							Unconstrained						
Mean-Var-Skew-Kurtosis	Pacific EX JP	0.028	0.002	0.000	0.000	0.000	0.000	-0.028	0.083	0.003	-0.136	-0.278	-0.570	-0.982	-1.065
	Japan	0.155	0.135	0.039	0.000	0.000	0.000	-0.155	0.324	0.270	0.077	-0.071	-0.403	-0.844	-1.168
	Europe EX UK	0.048	0.062	0.048	0.027	0.000	0.000	-0.048	0.139	0.125	0.087	0.064	0.025	-0.033	-0.171
	UK	0.000	0.000	0.057	0.078	0.024	0.000	0.000	-0.075	-0.002	0.165	0.292	0.565	0.920	0.995
	North America	0.511	0.527	0.534	0.527	0.625	0.638	0.127	0.012	0.053	0.174	0.282	0.495	0.794	0.782
	EM Latin America	0.068	0.078	0.130	0.144	0.153	0.181	0.113	0.140	0.157	0.221	0.242	0.327	0.457	0.317
	EM Asia	0.029	0.047	0.053	0.053	0.010	0.000	-0.029	0.035	0.094	0.188	0.294	0.485	0.739	0.705
	EM Europe and Middle East	0.162	0.150	0.140	0.172	0.188	0.182	0.020	0.343	0.300	0.225	0.175	0.076	-0.052	-0.394
Mean-Var-Kurtosis Preferences	Pacific EX JP	0.037	0.007	0.000	0.000	0.000	0.000	-0.037	0.078	0.010	-0.141	-0.278	-0.573	-0.977	-1.055
	Japan	0.226	0.193	0.060	0.000	0.000	0.000	-0.226	0.316	0.250	0.079	-0.084	-0.411	-0.860	-1.175
	Europe EX UK	0.080	0.090	0.072	0.039	0.000	0.000	-0.080	0.165	0.117	0.090	0.072	0.032	-0.014	-0.179
	UK	0.000	0.006	0.086	0.117	0.033	0.000	0.000	-0.092	0.014	0.166	0.295	0.566	0.919	1.011
	North America	0.261	0.294	0.342	0.392	0.561	0.614	0.353	0.002	0.063	0.161	0.271	0.492	0.787	0.785
	EM Latin America	0.110	0.123	0.164	0.171	0.175	0.188	0.078	0.158	0.168	0.215	0.251	0.339	0.451	0.293
	EM Asia	0.048	0.070	0.083	0.081	0.016	0.000	-0.048	0.036	0.094	0.201	0.300	0.485	0.747	0.711
	EM Europe and Middle East	0.239	0.217	0.193	0.199	0.215	0.197	-0.041	0.338	0.284	0.229	0.174	0.070	-0.053	-0.391
		Two-State Model (Current State: Bull/Low Volatility)													
Mean-Variance Preferences	Pacific EX JP	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.225	0.040	-0.457	-0.472	-0.583	-0.744	-2.969
	Japan	0.247	0.000	0.000	0.000	0.000	0.000	-0.247	-1.017	-0.969	-1.040	-1.093	-1.270	-1.577	-0.560
	Europe EX UK	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.750	-0.065	0.013	0.053	0.051	0.048	1.798
	UK	0.187	0.000	0.000	0.000	0.000	0.000	-0.187	1.710	0.618	0.663	0.643	0.734	0.806	-0.904
	North America	0.093	0.000	0.000	0.000	0.000	0.000	-0.093	-1.395	0.158	0.615	0.604	0.638	0.661	2.057
	EM Latin America	0.201	0.395	0.382	0.383	0.395	0.402	0.201	1.562	0.695	0.588	0.615	0.704	0.878	-0.684
	EM Asia	0.028	0.000	0.000	0.000	0.000	0.000	-0.028	-2.214	-0.414	-0.016	-0.003	0.030	0.061	2.276
	EM Europe and Middle East	0.245	0.605	0.618	0.617	0.605	0.598	0.353	1.880	0.936	0.633	0.653	0.697	0.866	-1.014
Mean-Var-Skew Preferences	Pacific EX JP	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-4.352	-2.954	-2.096	-1.166	-0.537	-0.315	4.037
	Japan	0.253	0.000	0.000	0.000	0.000	0.000	-0.253	-3.203	-2.596	-2.193	-1.504	-1.246	-0.730	2.473
	Europe EX UK	0.000	0.000	0.000	0.000	0.000	0.000	0.000	5.457	2.034	1.611	0.668	0.025	0.070	-5.388
	UK	0.193	0.000	0.000	0.000	0.000	0.000	-0.193	-5.308	-1.557	-1.084	-0.794	0.514	0.835	6.143
	North America	0.093	0.000	0.000	0.000	0.000	0.000	-0.093	4.182	2.556	1.634	0.904	0.926	0.663	-3.519
	EM Latin America	0.188	0.328	0.390	0.393	0.387	0.395	0.207	5.134	3.128	1.700	1.007	0.721	0.895	-4.239
	EM Asia	0.035	0.000	0.000	0.000	0.000	0.000	-0.035	-5.460	-3.036	-2.031	-1.002	-0.100	0.026	5.486
	EM Europe and Middle East	0.239	0.672	0.610	0.607	0.613	0.605	0.366	4.549	3.425	3.460	2.888	0.698	-0.444	-4.993
Mean-Var-Skew-Kurtosis	Pacific EX JP	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.152	-0.029	-0.050	-0.083	-0.167	-0.322	-0.170
	Japan	0.253	0.240	0.125	0.000	0.000	0.000	-0.253	0.253	0.237	0.124	-0.002	-0.292	-0.642	-0.895
	Europe EX UK	0.000	0.000	0.039	0.031	0.000	0.000	0.000	-0.148	-0.013	0.037	0.032	0.063	0.065	0.213
	UK	0.193	0.130	0.139	0.165	0.022	0.000	-0.193	0.301	0.147	0.157	0.193	0.258	0.383	0.082
	North America	0.093	0.105	0.090	0.100	0.075	0.000	-0.093	0.122	0.112	0.093	0.105	0.157	0.191	0.069
	EM Latin America	0.188	0.187	0.259	0.317	0.392	0.418	0.230	0.230	0.191	0.266	0.327	0.471	0.653	0.424
	EM Asia	0.035	0.066	0.017	0.000	0.000	0.000	-0.035	0.117	0.080	0.046	0.046	0.024	0.037	-0.080
	EM Europe and Middle East	0.239	0.272	0.331	0.387	0.511	0.582	0.343	0.277	0.275	0.328	0.382	0.486	0.634	0.357
Mean-Var-Kurtosis Preferences	Pacific EX JP	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.159	-0.032	-0.049	-0.084	-0.168	-0.301	-0.141
	Japan	0.247	0.235	0.119	0.000	0.000	0.000	-0.247	0.248	0.233	0.118	-0.004	-0.288	-0.647	-0.895
	Europe EX UK	0.000	0.000	0.037	0.042	0.000	0.000	0.000	-0.138	-0.015	0.034	0.044	0.066	0.071	0.209
	UK	0.187	0.144	0.133	0.160	0.034	0.000	-0.187	0.295	0.162	0.150	0.188	0.271	0.387	0.093
	North America	0.093	0.083	0.092	0.099	0.065	0.000	-0.093	0.120	0.090	0.096	0.103	0.143	0.172	0.052
	EM Latin America	0.201	0.194	0.253	0.317	0.389	0.425	0.224	0.242	0.198	0.258	0.328	0.465	0.659	0.416
	EM Asia	0.028	0.066	0.042	0.000	0.000	0.000	-0.028	0.113	0.082	0.071	0.052	0.028	0.030	-0.082
	EM Europe and Middle East	0.245	0.278	0.323	0.381	0.511	0.575	0.330	0.279	0.281	0.321	0.374	0.482	0.628	0.348

Table 5 (cont'ed)

Portfolio Weights as a Function of the Initial State

Panel B (CRSP Industry Returns, 1926:07 - 2008:07)

		No-short sales							Unconstrained						
		T=1	**T=3**	**T=12**	**T=24**	**T=60**	**T=120**	**"Slope"**	**T=1**	**T=3**	**T=12**	**T=24**	**T=60**	**T=120**	**"Slope"**
		Single-State Model (Unconditional Allocation)													
	Non Durables	0.106	0.000	0.000	0.095	0.159	0.194	0.088	0.868	1.055	0.820	0.846	0.834	0.838	-0.031
	Durables	0.000	0.000	0.000	0.021	0.000	0.000	0.000	0.288	0.446	0.304	0.145	0.039	0.046	-0.242
	Manufacturing	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.154	-0.104	-0.138	-0.134	-0.129	-0.138	0.016
	Energy	0.461	0.503	0.585	0.653	0.596	0.604	0.143	0.699	1.073	0.506	0.404	0.407	0.411	-0.288
	Hi Tech	0.000	0.000	0.145	0.070	0.045	0.040	0.040	0.213	0.459	0.259	0.022	0.029	0.025	-0.188
	Telecommunications	0.000	0.000	0.000	0.000	0.115	0.113	0.113	-0.090	-0.632	-0.140	0.160	0.255	0.309	0.399
	Shops/Distribution	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.172	-0.351	-0.107	-0.110	-0.103	-0.099	0.073
	Health	0.433	0.497	0.177	0.161	0.085	0.049	-0.384	0.544	0.918	0.326	0.274	0.284	0.227	-0.317
	Utilities	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.174	-0.266	-0.080	-0.013	-0.014	-0.014	0.160
	Other	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.023	-1.598	-0.749	-0.593	-0.602	-0.606	0.417
		Two-State Model (Current State: Ergodic/Unconditional Probabilities)													
Mean-Variance Preferences	Non Durables	0.034	0.143	0.359	0.564	0.673	0.678	0.644	0.819	1.104	0.851	0.875	0.857	0.880	0.061
	Durables	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.195	0.468	0.095	0.104	0.106	0.106	-0.089
	Manufacturing	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.139	-0.947	-0.474	-0.516	-0.510	-0.526	-0.387
	Energy	0.463	0.393	0.314	0.290	0.261	0.261	-0.202	0.718	1.095	0.448	0.446	0.443	0.446	-0.271
	Hi Tech	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.189	0.459	0.107	0.116	0.112	0.124	-0.065
	Telecommunications	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.021	-0.857	-0.004	0.133	0.127	0.118	0.140
	Shops/Distribution	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.155	0.059	0.117	0.113	0.128	0.128	0.284
	Health	0.503	0.464	0.327	0.146	0.066	0.061	-0.442	0.612	1.076	0.398	0.251	0.249	0.245	-0.368
	Utilities	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.167	-0.462	-0.047	-0.050	-0.037	-0.050	0.116
	Other	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.050	-0.995	-0.491	-0.471	-0.474	-0.471	0.579
Mean-Var-Skew Preferences	Non Durables	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.326	0.616	0.804	1.016	0.977	0.986	0.660
	Durables	0.046	0.000	0.000	0.000	0.000	0.000	-0.046	0.319	0.440	0.109	0.106	0.110	0.103	-0.216
	Manufacturing	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.252	-0.753	-0.538	-0.560	-0.581	-0.621	-0.369
	Energy	0.954	0.709	0.581	0.345	0.314	0.305	-0.649	0.670	0.816	0.483	0.407	0.445	0.536	-0.134
	Hi Tech	0.000	0.038	0.145	0.290	0.345	0.367	0.367	0.117	0.307	0.126	0.105	0.091	0.106	-0.011
	Telecommunications	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.181	-0.381	0.083	0.151	0.131	0.092	-0.089
	Shops/Distribution	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.076	0.022	0.092	0.117	0.116	0.143	0.219
	Health	0.000	0.253	0.274	0.365	0.341	0.328	0.328	0.328	0.626	0.486	0.287	0.225	0.173	-0.155
	Utilities	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.022	-0.123	-0.016	-0.043	-0.052	-0.045	-0.023
	Other	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.591	-0.569	-0.630	-0.587	-0.461	-0.473	0.118
Mean-Var-Kurtosis Preferences	Non Durables	0.054	0.000	0.145	0.356	0.679	0.686	0.632	0.255	0.456	0.879	0.884	0.867	0.871	0.616
	Durables	0.000	0.053	0.000	0.000	0.000	0.000	0.000	0.264	0.224	0.104	0.095	0.110	0.107	-0.157
	Manufacturing	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.619	-0.420	-0.475	-0.504	-0.524	-0.519	0.100
	Energy	0.358	0.358	0.335	0.300	0.259	0.260	-0.097	0.470	0.461	0.440	0.446	0.445	0.441	-0.029
	Hi Tech	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.089	0.165	0.110	0.120	0.122	0.119	0.030
	Telecommunications	0.297	0.210	0.175	0.098	0.000	0.000	-0.297	0.294	0.201	0.138	0.139	0.125	0.125	-0.169
	Shops/Distribution	0.000	0.047	0.040	0.024	0.000	0.000	0.000	0.029	0.030	0.097	0.108	0.128	0.132	0.102
	Health	0.272	0.323	0.305	0.222	0.062	0.054	-0.218	0.272	0.171	0.227	0.235	0.248	0.244	-0.027
	Utilities	0.019	0.009	0.000	0.000	0.000	0.000	-0.019	0.057	0.014	-0.031	-0.051	-0.044	-0.043	-0.101
	Other	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.111	-0.300	-0.489	-0.472	-0.477	-0.477	-0.366
Mean-Var-Skew-Kurtosis	Non Durables	0.000	0.000	0.144	0.405	0.663	0.679	0.679	0.094	0.156	0.460	0.886	0.873	0.859	0.766
	Durables	0.046	0.079	0.032	0.000	0.000	0.000	-0.046	0.327	0.316	0.185	0.093	0.101	0.108	-0.219
	Manufacturing	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.457	-0.405	-0.465	-0.513	-0.505	-0.525	-0.067
	Energy	0.362	0.414	0.360	0.297	0.254	0.258	-0.104	0.477	0.531	0.485	0.452	0.450	0.451	-0.027
	Hi Tech	0.000	0.016	0.018	0.045	0.005	0.000	0.000	0.061	0.149	0.121	0.114	0.114	0.112	0.051
	Telecommunications	0.286	0.151	0.084	0.045	0.000	0.000	-0.286	0.326	0.184	0.159	0.134	0.126	0.127	-0.199
	Shops/Distribution	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.047	-0.037	0.078	0.129	0.131	0.138	0.185
	Health	0.291	0.339	0.362	0.208	0.078	0.063	-0.228	0.343	0.379	0.463	0.231	0.233	0.250	-0.093
	Utilities	0.015	0.000	0.000	0.000	0.000	0.000	-0.015	0.069	0.078	0.004	-0.049	-0.051	-0.051	-0.120
	Other	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.193	-0.350	-0.490	-0.477	-0.471	-0.470	-0.276
		Two-State Model (Current State: Bear/High Volatility)													
Mean-Variance Preferences	Non Durables	0.000	0.000	0.054	0.267	0.405	0.516	0.516	-0.045	-0.217	0.632	0.748	0.844	0.945	0.990
	Durables	0.000	0.000	0.056	0.103	0.094	0.000	0.000	0.188	0.199	0.002	0.004	0.003	0.007	-0.181
	Manufacturing	0.000	0.000	0.000	0.000	0.023	0.026	0.026	1.662	2.059	0.278	0.150	0.009	-0.140	-1.802
	Energy	0.612	0.817	0.679	0.397	0.251	0.253	-0.359	0.713	0.958	0.506	0.380	0.378	0.380	-0.333
	Hi Tech	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.329	-0.048	-0.130	-0.126	-0.117	-0.122	0.207
	Telecommunications	0.239	0.000	0.000	0.000	0.000	0.005	-0.234	0.852	0.413	0.549	0.478	0.406	0.367	-0.485
	Shops/Distribution	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.144	-0.730	-0.358	-0.354	-0.306	-0.253	0.891
	Health	0.148	0.183	0.211	0.233	0.227	0.201	0.053	0.251	0.109	0.209	0.310	0.330	0.341	0.090
	Utilities	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.246	0.194	0.065	0.060	0.054	0.024	-0.222
	Other	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.393	-1.936	-0.752	-0.650	-0.600	-0.549	0.844
Mean-Var-Skew Preferences	Non Durables	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.010	-0.056	0.401	0.576	1.079	1.314	1.304
	Durables	0.246	0.088	0.113	0.224	0.184	0.145	-0.101	0.290	0.328	0.103	0.062	0.007	-0.024	-0.314
	Manufacturing	0.000	0.025	0.000	0.000	0.000	0.000	0.000	0.962	1.779	0.298	0.265	0.185	0.015	-0.947
	Energy	0.209	0.766	0.804	0.676	0.603	0.559	0.350	0.550	0.943	0.427	0.451	0.470	0.297	-0.253
	Hi Tech	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.210	-0.014	-0.100	-0.139	-0.135	-0.155	0.055
	Telecommunications	0.000	0.000	0.039	0.000	0.000	0.000	0.000	0.855	0.482	0.608	0.570	0.485	0.559	-0.296
	Shops/Distribution	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.931	-0.779	-0.477	-0.287	-0.296	-0.214	0.717
	Health	0.545	0.121	0.001	0.066	0.203	0.296	-0.249	0.213	0.010	0.057	0.242	0.172	0.272	0.059
	Utilities	0.000	0.000	0.043	0.034	0.010	0.000	0.000	0.216	0.196	0.084	0.075	0.073	0.004	-0.212
	Other	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.038	-1.621	-0.566	-0.558	-0.647	-0.541	0.496

Panel B (CRSP Industry Returns, 1926:07 - 2008:07)

		T=1	T=3	T=12	T=24	T=60	T=120	"Slope"	T=1	T=3	T=12	T=24	T=60	T=120	"Slope"
		Two-State Model (Current State: Bear/High Volatility)													
		No-short sales							Unconstrained						
Mean-Var-Kurtosis Preferences	Non Durables	0.000	0.000	0.095	0.356	0.502	0.515	0.515	0.088	0.259	0.450	0.630	0.736	0.750	0.662
	Durables	0.104	0.113	0.125	0.140	0.124	0.084	-0.020	0.321	0.275	0.084	-0.002	0.007	0.008	-0.313
	Manufacturing	0.000	0.000	0.000	0.000	0.037	0.033	0.033	-0.406	0.139	0.258	0.276	0.268	0.261	0.667
	Energy	0.276	0.398	0.432	0.495	0.310	0.251	-0.025	0.372	0.468	0.375	0.284	0.156	0.094	-0.278
	Hi Tech	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.048	0.004	-0.137	-0.128	-0.134	-0.074	-0.122
	Telecommunications	0.296	0.213	0.103	0.008	0.000	0.000	-0.296	0.320	0.405	0.541	0.495	0.458	0.375	0.055
	Shops/Distribution	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.041	-0.257	-0.358	-0.357	-0.348	-0.300	-0.259
	Health	0.167	0.216	0.245	0.001	0.026	0.117	-0.051	0.197	0.169	0.444	0.494	0.334	0.209	0.012
	Utilities	0.157	0.060	0.000	0.000	0.000	0.000	-0.157	0.191	0.174	0.084	0.055	0.062	0.062	-0.129
	Other	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.090	-0.637	-0.741	-0.746	-0.540	-0.385	-0.295
Mean-Var-Skew-Kurtosis	Non Durables	0.000	0.000	0.000	0.345	0.501	0.518	0.518	0.085	0.143	0.379	0.550	0.694	0.736	0.651
	Durables	0.107	0.091	0.000	0.000	0.000	0.000	-0.107	0.335	0.285	0.214	0.074	0.007	-0.034	-0.369
	Manufacturing	0.000	0.000	0.000	0.000	0.030	0.018	0.018	-0.445	0.110	0.276	0.278	0.260	0.257	0.702
	Energy	0.267	0.406	0.535	0.614	0.305	0.287	0.020	0.360	0.483	0.382	0.356	0.295	0.178	-0.182
	Hi Tech	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.066	0.047	-0.064	-0.125	-0.128	-0.126	-0.193
	Telecommunications	0.297	0.230	0.074	0.000	0.000	0.000	-0.297	0.321	0.368	0.544	0.538	0.543	0.526	0.205
	Shops/Distribution	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.028	-0.285	-0.358	-0.360	-0.305	-0.140	-0.112
	Health	0.163	0.228	0.391	0.041	0.165	0.177	0.014	0.200	-0.041	-0.086	0.084	0.124	0.173	-0.026
	Utilities	0.166	0.045	0.000	0.000	0.000	0.000	-0.166	0.206	0.140	0.064	0.061	0.042	-0.023	-0.229
	Other	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.099	-0.251	-0.351	-0.456	-0.533	-0.548	-0.448
		Two-State Model (Current State: Bull/Low Volatility)													
Mean-Variance Preferences	Non Durables	0.171	0.115	0.095	0.476	0.675	0.683	0.512	1.616	2.515	0.950	0.963	0.962	0.974	-0.642
	Durables	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.316	0.497	0.114	0.111	0.103	0.106	-0.210
	Manufacturing	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-2.011	-2.570	-0.609	-0.581	-0.586	-0.588	1.422
	Energy	0.218	0.147	0.049	0.154	0.261	0.260	0.042	0.960	1.302	0.466	0.447	0.449	0.449	-0.512
	Hi Tech	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.355	0.528	0.124	0.129	0.135	0.138	-0.218
	Telecommunications	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.531	-1.471	-0.405	0.053	0.102	0.112	0.643
	Shops/Distribution	0.195	0.049	0.000	0.000	0.000	0.000	-0.195	0.698	0.795	0.350	0.215	0.175	0.166	-0.531
	Health	0.416	0.689	0.856	0.370	0.064	0.058	-0.359	0.708	1.084	0.510	0.154	0.153	0.142	-0.567
	Utilities	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.667	-1.003	-0.062	-0.043	-0.049	-0.056	0.611
	Other	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.445	-0.678	-0.438	-0.448	-0.444	-0.443	0.002
Mean-Var-Skew Preferences	Non Durables	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.234	0.976	0.773	0.906	1.180	1.126	-0.108
	Durables	0.865	0.259	0.000	0.000	0.000	0.000	-0.865	0.359	0.613	0.117	0.094	0.023	-0.046	-0.406
	Manufacturing	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.087	-1.542	-0.706	-0.552	-0.624	-0.549	0.538
	Energy	0.000	0.245	0.453	0.311	0.333	0.295	0.295	1.037	1.374	0.431	0.401	0.232	0.257	-0.781
	Hi Tech	0.000	0.000	0.035	0.075	0.146	0.205	0.205	0.453	0.377	0.174	0.146	0.083	0.158	-0.295
	Telecommunications	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.087	-0.630	-0.165	0.050	0.034	0.017	0.105
	Shops/Distribution	0.135	0.049	0.005	0.000	0.000	0.000	-0.135	0.566	0.610	0.358	0.266	0.277	0.207	-0.359
	Health	0.000	0.447	0.507	0.614	0.521	0.500	0.500	0.601	0.613	0.508	0.156	0.236	0.243	-0.358
	Utilities	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.248	-0.507	-0.068	-0.051	-0.070	-0.082	0.166
	Other	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.322	-0.496	-0.532	-0.336	-0.410	-0.556	-0.234
Mean-Var-Kurtosis Preferences	Non Durables	0.000	0.000	0.000	0.240	0.604	0.677	0.677	-0.086	0.142	0.892	0.864	0.879	0.871	0.957
	Durables	0.126	0.063	0.004	0.000	0.000	0.000	-0.126	0.275	0.285	0.118	0.108	0.117	0.110	-0.165
	Manufacturing	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.469	-0.703	-0.608	-0.583	-0.585	-0.594	-0.125
	Energy	0.263	0.331	0.378	0.324	0.288	0.267	0.004	0.349	0.457	0.447	0.456	0.448	0.456	0.108
	Hi Tech	0.075	0.020	0.000	0.000	0.000	0.000	-0.075	0.237	0.248	0.140	0.125	0.127	0.144	-0.094
	Telecommunications	0.212	0.154	0.064	0.014	0.000	0.000	-0.212	0.222	0.141	0.093	0.112	0.104	0.105	-0.117
	Shops/Distribution	0.045	0.120	0.084	0.033	0.000	0.000	-0.045	0.205	0.209	0.154	0.177	0.172	0.169	-0.036
	Health	0.167	0.284	0.470	0.389	0.108	0.056	-0.110	0.280	0.344	0.250	0.238	0.249	0.249	-0.031
	Utilities	0.113	0.028	0.000	0.000	0.000	0.000	-0.113	0.169	0.060	-0.051	-0.059	-0.058	-0.053	-0.222
	Other	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.181	-0.183	-0.437	-0.438	-0.454	-0.456	-0.276
Mean-Var-Skew-Kurtosis	Non Durables	0.000	0.000	0.000	0.359	0.674	0.686	0.686	-0.099	0.133	0.350	0.604	0.864	0.870	0.969
	Durables	0.130	0.070	0.000	0.000	0.000	0.000	-0.130	0.292	0.304	0.112	0.059	-0.094	-0.145	-0.437
	Manufacturing	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.505	-0.669	-0.568	-0.350	-0.246	-0.146	0.359
	Energy	0.258	0.394	0.356	0.285	0.262	0.259	0.001	0.339	0.510	0.446	0.145	0.004	-0.024	-0.363
	Hi Tech	0.091	0.013	0.000	0.000	0.000	0.000	-0.091	0.253	0.226	0.168	0.115	0.076	0.054	-0.199
	Telecommunications	0.201	0.166	0.085	0.029	0.000	0.000	-0.201	0.226	0.162	0.096	0.034	-0.024	-0.095	-0.321
	Shops/Distribution	0.046	0.071	0.039	0.000	0.000	0.000	-0.046	0.218	0.186	0.183	0.165	0.174	0.174	-0.045
	Health	0.154	0.269	0.520	0.327	0.064	0.056	-0.099	0.287	0.335	0.250	0.243	0.253	0.249	-0.038
	Utilities	0.119	0.018	0.000	0.000	0.000	0.000	-0.119	0.176	0.042	-0.042	-0.047	-0.050	-0.056	-0.232
	Other	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.188	-0.229	-0.357	-0.360	-0.305	-0.270	-0.082

Table 5 (cont'ed)

Portfolio Weights as a Function of the Initial State

Panel C (International Book-to-Market Sorted Portfolio Local Returns, 1975:01 - 2007:12)

		T=1	T=3	T=12	T=24	T=60	T=120	"Slope"	T=1	T=3	T=12	T=24	T=60	T=120	"Slope"
		No-short sales							Unconstrained						
		Single-State Model (Unconditional Allocation)													
	World	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-4.900	-3.654	-3.659	-3.483	-2.956	-2.669	2.231
	EU ex-UK ex-Scand Value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.372	1.254	0.649	0.570	0.528	0.485	-0.887
	EU ex-UK ex-Scand Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.910	-0.954	0.018	0.495	0.869	0.984	2.894
	United Kingdom Value	0.599	0.677	0.781	0.596	0.240	0.000	-0.599	1.725	1.405	0.749	0.675	0.304	0.251	-1.475
	United Kingdom Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.282	-0.957	-0.382	-0.362	-0.383	-0.370	-0.087
	Asia & Pacific Value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.731	3.236	1.059	0.845	0.784	0.753	-1.978
	Asia & Pacific Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-2.678	-1.495	-0.658	-0.345	-0.084	0.149	2.827
	Scandinavia Value	0.401	0.323	0.219	0.094	0.000	0.000	-0.401	-0.044	-0.694	-0.442	-0.433	-0.374	-0.305	-0.261
	Scandinavia Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.057	1.743	1.345	0.850	0.274	0.129	-1.928
	United States Value	0.000	0.000	0.000	0.304	0.594	0.653	0.653	0.066	-1.234	-0.244	-0.094	0.384	0.576	0.510
	United States Growth	0.000	0.000	0.000	0.006	0.166	0.347	0.347	2.862	2.351	2.565	2.282	1.654	1.017	-1.846
		Two-State Model (Current State: Ergodic/Unconditional Probabilities)													
Mean-Variance Preferences	World	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-5.059	-3.954	-3.145	-2.759	-2.460	-2.049	3.010
	EU ex-UK ex-Scand Value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.347	1.848	0.897	0.845	0.794	0.746	-0.601
	EU ex-UK ex-Scand Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.934	-1.450	-0.748	-0.240	0.004	0.137	2.070
	United Kingdom Value	0.649	0.633	0.329	0.085	0.000	0.000	-0.649	1.728	1.143	0.648	0.539	0.375	0.204	-1.524
	United Kingdom Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.290	-1.238	-0.847	-0.704	-0.633	-0.516	-0.226
	Asia & Pacific Value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.754	2.095	1.560	1.048	0.986	0.934	-1.820
	Asia & Pacific Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-3.113	-1.795	-1.059	-0.745	-0.548	-0.375	2.738
	Scandinavia Value	0.351	0.367	0.671	0.405	0.047	0.000	-0.351	-0.074	-0.495	-0.565	-0.575	-0.598	-0.603	-0.529
	Scandinavia Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.974	1.243	0.746	0.249	-0.249	-0.262	-2.236
	United States Value	0.000	0.000	0.000	0.435	0.847	0.906	0.906	0.127	-0.204	-0.495	0.345	0.859	1.743	1.616
	United States Growth	0.000	0.000	0.000	0.075	0.106	0.094	0.094	3.540	3.807	4.008	2.997	2.469	1.041	-2.499
Mean-Var-Skew Preferences	World	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-3.243	-3.038	-3.043	-2.974	-2.756	-2.635	0.608
	EU ex-UK ex-Scand Value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.575	0.461	0.742	0.771	0.815	0.868	0.293
	EU ex-UK ex-Scand Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.123	-0.935	-0.483	-0.043	0.065	0.141	1.264
	United Kingdom Value	1.000	1.000	0.435	0.349	0.174	0.058	-0.942	1.269	1.055	0.836	0.650	0.379	0.220	-1.048
	United Kingdom Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.055	-0.608	-0.536	-0.454	-0.339	-0.218	-0.163
	Asia & Pacific Value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.899	2.293	1.304	1.145	0.951	0.886	-1.013
	Asia & Pacific Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.587	-1.077	-0.495	-0.363	-0.148	-0.106	1.481
	Scandinavia Value	0.000	0.000	0.259	0.506	0.648	0.704	0.704	-0.055	-0.288	-0.458	-0.532	-0.600	-0.634	-0.578
	Scandinavia Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.177	0.623	0.346	0.050	-0.242	-0.295	-1.472
	United States Value	0.000	0.000	0.000	0.059	0.150	0.230	0.230	0.465	0.503	0.646	0.982	1.918	2.085	1.620
	United States Growth	0.000	0.000	0.306	0.086	0.028	0.008	0.008	1.679	2.011	2.142	1.768	0.956	0.687	-0.992
Mean-Var-Kurtosis Preferences	World	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.745	-2.469	-3.059	-3.596	-3.857	-3.950	-2.205
	EU ex-UK ex-Scand Value	0.003	0.000	0.000	0.000	0.000	0.000	-0.003	0.246	0.720	0.881	0.875	0.890	0.897	0.651
	EU ex-UK ex-Scand Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.093	0.104	0.136	0.154	0.184	0.193	0.100
	United Kingdom Value	0.097	0.343	0.294	0.154	0.035	0.000	-0.097	0.171	0.676	0.150	0.067	0.049	0.028	-0.143
	United Kingdom Growth	0.095	0.000	0.000	0.000	0.000	0.000	-0.095	0.110	-0.078	-0.385	-0.490	-0.514	-0.530	-0.640
	Asia & Pacific Value	0.146	0.000	0.000	0.000	0.000	0.000	-0.146	0.244	0.705	1.015	0.999	0.989	0.998	0.754
	Asia & Pacific Growth	0.169	0.084	0.000	0.000	0.000	0.000	-0.169	0.492	0.227	0.214	0.209	0.206	0.195	-0.297
	Scandinavia Value	0.240	0.420	0.553	0.394	0.145	0.095	-0.145	0.166	0.085	-0.405	-0.591	-0.609	-0.634	-0.800
	Scandinavia Growth	0.080	0.000	0.000	0.000	0.000	0.000	-0.080	0.102	0.035	-0.182	-0.209	-0.208	-0.242	-0.345
	United States Value	0.078	0.000	0.000	0.384	0.794	0.884	0.806	0.782	1.084	1.843	2.045	2.305	2.475	1.693
	United States Growth	0.090	0.153	0.153	0.068	0.026	0.021	-0.069	0.338	-0.089	0.793	1.537	1.566	1.570	1.231
Mean-Var-Skew-Kurtosis	World	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-2.218	-3.045	-3.345	-3.875	-3.985	-4.140	-1.922
	EU ex-UK ex-Scand Value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.045	0.212	0.670	0.748	0.894	0.913	0.958
	EU ex-UK ex-Scand Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.164	-0.809	-0.059	0.095	0.116	0.161	0.325
	United Kingdom Value	0.229	0.541	0.274	0.134	0.034	0.000	-0.229	0.548	1.359	0.854	0.745	0.459	0.274	-0.274
	United Kingdom Growth	0.082	0.000	0.000	0.000	0.000	0.000	-0.082	0.197	0.082	-0.145	-0.257	-0.450	-0.491	-0.688
	Asia & Pacific Value	0.275	0.084	0.000	0.000	0.000	0.000	-0.275	0.772	1.827	1.018	1.094	1.064	1.007	0.234
	Asia & Pacific Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.147	-0.330	-0.084	0.074	0.156	0.187	0.040
	Scandinavia Value	0.151	0.259	0.515	0.294	0.114	0.037	-0.114	-0.041	-0.175	-0.305	-0.475	-0.567	-0.607	-0.566
	Scandinavia Growth	0.228	0.047	0.000	0.000	0.000	0.000	-0.228	0.463	0.174	-0.183	-0.236	-0.217	-0.255	-0.718
	United States Value	0.000	0.037	0.200	0.560	0.674	0.749	0.749	0.734	1.145	1.748	1.904	2.456	2.955	2.221
	United States Growth	0.035	0.032	0.011	0.012	0.178	0.214	0.179	0.606	0.693	0.830	0.894	0.953	0.997	0.390
		Two-State Model (Current State: World Bull/Low Volatility)													
Mean-Variance Preferences	World	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-3.405	-3.004	-2.748	-2.244	-2.009	-1.780	1.625
	EU ex-UK ex-Scand Value	0.807	0.495	0.245	0.094	0.035	0.024	-0.783	4.277	3.884	2.364	1.303	0.984	0.967	-3.310
	EU ex-UK ex-Scand Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.868	-0.847	-0.394	-0.034	0.064	0.146	2.014
	United Kingdom Value	0.000	0.063	0.000	0.000	0.000	0.000	0.000	1.381	0.840	0.374	0.073	0.021	-0.072	-1.453
	United Kingdom Growth	0.000	0.034	0.000	0.000	0.000	0.000	0.000	-0.909	-1.450	-0.563	-0.602	-0.549	-0.548	0.361
	Asia & Pacific Value	0.109	0.084	0.063	0.005	0.000	0.000	-0.109	2.708	1.330	1.093	1.080	1.063	1.009	-1.699
	Asia & Pacific Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-4.535	-2.048	-0.749	-0.394	-0.093	0.074	4.609
	Scandinavia Value	0.074	0.056	0.000	0.000	0.000	0.000	-0.074	0.762	2.096	-0.727	-0.734	-0.696	-0.718	-1.480
	Scandinavia Growth	0.009	0.034	0.000	0.000	0.000	0.000	-0.009	-0.423	0.044	-0.453	-0.440	-0.441	-0.434	-0.011
	United States Value	0.000	0.047	0.103	0.348	0.684	0.847	0.847	2.009	0.050	3.370	3.074	2.794	1.745	-0.264
	United States Growth	0.001	0.187	0.589	0.553	0.281	0.129	0.128	1.002	0.693	-0.567	0.894	0.953	0.611	-0.391
Mean-Var-Skew Preferences	World	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-2.902	-3.018	-2.703	-1.919	-2.252	-1.932	0.970
	EU ex-UK ex-Scand Value	0.204	0.603	0.365	0.184	0.145	0.084	-0.120	2.012	2.614	1.869	1.052	0.850	0.560	-1.452
	EU ex-UK ex-Scand Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.946	-0.554	-0.099	0.074	0.250	0.483	1.429
	United Kingdom Value	0.000	0.084	0.345	0.249	0.135	0.074	0.074	0.643	0.705	0.288	0.077	-0.019	-0.304	-0.947
	United Kingdom Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.371	-0.643	-0.383	-0.569	-0.249	-0.086	0.285
	Asia & Pacific Value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.772	1.287	1.094	1.086	0.952	0.958	-0.814
	Asia & Pacific Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-2.188	-1.083	-0.583	-0.202	-0.001	0.099	2.287
	Scandinavia Value	0.000	0.256	0.290	0.453	0.506	0.629	0.629	0.371	0.953	-0.658	-0.636	-0.812	-0.727	-1.098
	Scandinavia Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.165	0.103	-0.594	-0.272	-0.339	-0.409	-0.244
	United States Value	0.000	0.000	0.000	0.084	0.194	0.204	0.204	1.580	0.677	2.323	2.675	2.480	1.999	0.419
	United States Growth	0.796	0.000	0.000	0.030	0.020	0.009	-0.787	1.193	0.693	0.446	0.894	0.953	0.358	-0.834

Panel C (International Book-to-Market Sorted Portfolio Local Returns, 1975:01 - 2007:12)

		T=1	T=3	T=12	T=24	T=60	T=120	"Slope"	T=1	T=3	T=12	T=24	T=60	T=120	"Slope"
		Two-State Model (Current State: World Bull/Low Volatility)													
		No-short sales							**Unconstrained**						
Mean-Var-Kurtosis Preferences	World	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.972	-2.561	-2.994	-3.204	-3.385	-3.417	-1.445
	EU ex-UK ex-Scand Value	0.092	0.424	0.355	0.264	0.104	0.048	-0.044	0.412	1.075	0.977	0.962	0.923	0.864	0.452
	EU ex-UK ex-Scand Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.118	0.195	0.178	0.162	0.147	0.142	0.024
	United Kingdom Value	0.110	0.085	0.039	0.000	0.000	0.000	-0.110	0.180	0.524	0.333	0.084	0.054	0.006	-0.174
	United Kingdom Growth	0.090	0.039	0.000	0.000	0.000	0.000	-0.090	0.143	-0.053	-0.377	-0.487	-0.532	-0.560	-0.703
	Asia & Pacific Value	0.244	0.149	0.084	0.009	0.000	0.000	-0.244	0.379	0.722	0.855	0.980	1.048	1.070	0.691
	Asia & Pacific Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.346	0.229	0.112	0.078	0.044	0.009	-0.337
	Scandinavia Value	0.150	0.254	0.428	0.301	0.173	0.095	-0.055	0.080	0.134	-0.294	-0.648	-0.698	-0.721	-0.801
	Scandinavia Growth	0.107	0.049	0.004	0.000	0.000	0.000	-0.107	0.089	0.145	-0.371	-0.400	-0.416	-0.428	-0.517
	United States Value	0.178	0.000	0.000	0.305	0.594	0.659	0.481	0.907	1.901	2.405	2.894	2.914	3.074	2.167
	United States Growth	0.029	0.000	0.090	0.121	0.129	0.198	0.169	0.318	0.693	0.176	0.894	0.953	0.961	0.642
Mean-Var-Skew-Kurtosis	World	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-3.006	-2.748	-2.405	-2.124	-1.984	-1.874	1.132
	EU ex-UK ex-Scand Value	0.097	0.310	0.094	0.048	0.005	0.000	-0.097	0.404	0.826	0.924	0.970	0.981	0.988	0.584
	EU ex-UK ex-Scand Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.117	-0.158	0.154	0.181	0.240	0.265	0.148
	United Kingdom Value	0.121	0.167	0.048	0.013	0.000	0.000	-0.121	0.182	0.506	0.235	0.083	-0.070	-0.141	-0.323
	United Kingdom Growth	0.090	0.024	0.000	0.000	0.000	0.000	-0.090	0.143	0.066	-0.204	-0.374	-0.498	-0.569	-0.712
	Asia & Pacific Value	0.254	0.135	0.084	0.056	0.005	0.000	-0.254	0.381	1.168	1.064	1.009	0.964	0.904	0.523
	Asia & Pacific Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.342	-0.133	-0.043	0.067	0.090	0.117	-0.225
	Scandinavia Value	0.146	0.320	0.731	0.485	0.294	0.085	-0.061	0.078	-0.245	-0.609	-0.677	-0.710	-0.725	-0.803
	Scandinavia Growth	0.109	0.044	0.000	0.000	0.000	0.000	-0.109	0.111	0.197	0.074	-0.204	-0.316	-0.404	-0.515
	United States Value	0.184	0.000	0.038	0.384	0.624	0.684	0.500	0.905	1.304	1.758	2.084	2.156	2.460	1.555
	United States Growth	0.000	0.000	0.005	0.014	0.072	0.231	0.231	1.342	0.693	0.052	0.894	0.953	-0.021	-1.363
		Two-State Model (Current State: World Bear/High Volatility)													
Mean-Variance Preferences	World	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-2.954	-2.045	-1.450	-1.004	-0.964	-0.924	2.030
	EU ex-UK ex-Scand Value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.859	-1.349	-0.795	-0.204	0.004	0.158	2.017
	EU ex-UK ex-Scand Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-2.963	-1.649	-1.456	-1.094	-0.579	-0.205	2.758
	United Kingdom Value	0.795	0.927	0.785	0.649	0.359	0.124	-0.671	2.954	1.559	1.240	0.974	0.740	0.573	-2.381
	United Kingdom Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.232	0.571	-0.035	-0.154	-0.213	-0.404	-0.636
	Asia & Pacific Value	0.059	0.020	0.000	0.000	0.000	0.000	-0.059	2.549	2.094	1.649	1.395	1.147	1.059	-1.490
	Asia & Pacific Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.821	-0.958	-0.675	-0.539	-0.475	-0.438	1.383
	Scandinavia Value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.427	1.594	0.940	0.224	-0.153	-0.378	-0.805
	Scandinavia Growth	0.146	0.048	0.004	0.000	0.000	0.000	-0.146	2.054	1.748	1.104	0.749	0.473	0.173	-1.881
	United States Value	0.000	0.000	0.074	0.145	0.264	0.495	0.495	-1.180	-2.149	-1.049	-0.648	-0.174	0.299	1.479
	United States Growth	0.000	0.005	0.137	0.206	0.377	0.381	0.381	3.561	1.584	1.527	1.301	1.194	1.087	-2.474
Mean-Var-Skew Preferences	World	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-2.310	-1.742	-1.304	-1.291	-1.083	-1.029	1.281
	EU ex-UK ex-Scand Value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.045	-1.099	-0.707	-0.304	-0.043	0.113	1.158
	EU ex-UK ex-Scand Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.291	-0.893	-0.749	-0.707	-0.292	-0.088	1.202
	United Kingdom Value	0.668	0.856	0.804	0.645	0.386	0.174	-0.494	1.940	1.230	0.872	0.724	0.438	0.339	-1.601
	United Kingdom Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.136	0.235	-0.110	-0.206	-0.317	-0.382	-0.518
	Asia & Pacific Value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.621	1.609	1.330	1.217	1.191	1.155	-0.466
	Asia & Pacific Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.664	-0.584	-0.048	-0.003	-0.068	-0.159	0.505
	Scandinavia Value	0.169	0.074	0.114	0.148	0.358	0.564	0.395	0.319	0.782	0.411	0.043	-0.139	-0.235	-0.554
	Scandinavia Growth	0.163	0.065	0.000	0.000	0.000	0.000	-0.163	1.108	1.464	0.827	0.551	0.372	0.105	-1.003
	United States Value	0.000	0.000	0.000	0.063	0.174	0.230	0.230	-0.265	-0.779	0.270	0.797	0.799	1.050	1.315
	United States Growth	0.000	0.005	0.082	0.144	0.082	0.032	0.032	1.450	0.778	0.209	0.179	0.143	0.131	-1.319
Mean-Var-Kurtosis Preferences	World	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.733	-2.403	-2.954	-3.184	-3.404	-3.451	-1.718
	EU ex-UK ex-Scand Value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.268	-0.688	-0.548	-0.383	-0.143	0.088	0.356
	EU ex-UK ex-Scand Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.092	-0.182	-0.129	-0.114	-0.050	0.033	-0.059
	United Kingdom Value	0.160	0.653	0.806	0.685	0.453	0.215	0.055	0.398	1.285	0.639	0.583	0.403	0.327	-0.071
	United Kingdom Growth	0.116	0.053	0.024	0.000	0.000	0.000	-0.116	0.077	-0.094	-0.211	-0.192	-0.190	-0.194	-0.271
	Asia & Pacific Value	0.205	0.073	0.011	0.000	0.000	0.000	-0.205	0.363	1.039	1.218	1.234	1.217	1.200	0.837
	Asia & Pacific Growth	0.068	0.014	0.000	0.000	0.000	0.000	-0.068	0.380	-0.056	-0.003	0.032	0.059	0.084	-0.296
	Scandinavia Value	0.150	0.104	0.094	0.124	0.099	0.095	-0.055	0.192	0.083	-0.125	-0.145	-0.194	-0.353	-0.545
	Scandinavia Growth	0.186	0.102	0.034	0.005	0.000	0.000	-0.186	0.355	0.940	0.735	0.435	0.084	-0.040	-0.395
	United States Value	0.000	0.000	0.024	0.135	0.345	0.684	0.684	0.598	0.873	1.640	1.943	2.094	2.140	1.542
	United States Growth	0.114	0.001	0.007	0.051	0.103	0.006	-0.108	0.545	0.205	0.738	0.792	1.124	1.166	0.621
Mean-Var-Skew-Kurtosis	World	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.782	-1.596	-1.430	-1.329	-1.284	-1.253	0.529
	EU ex-UK ex-Scand Value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.259	-0.655	-0.512	-0.375	-0.104	0.087	0.346
	EU ex-UK ex-Scand Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.074	-0.262	-0.194	-0.132	-0.094	0.034	-0.040
	United Kingdom Value	0.163	0.524	0.723	0.695	0.595	0.524	0.361	0.401	1.300	0.651	0.450	0.164	0.084	-0.317
	United Kingdom Growth	0.121	0.074	0.023	0.000	0.000	0.000	-0.121	0.077	-0.085	-0.200	-0.314	-0.384	-0.420	-0.497
	Asia & Pacific Value	0.214	0.184	0.099	0.074	0.013	0.000	-0.214	0.361	1.123	1.205	1.104	1.074	1.003	0.642
	Asia & Pacific Growth	0.079	0.045	0.008	0.000	0.000	0.000	-0.079	0.384	-0.108	0.449	0.453	0.419	0.214	-0.170
	Scandinavia Value	0.148	0.114	0.080	0.074	0.045	0.037	-0.111	0.175	0.065	-0.125	-0.135	-0.147	-0.094	-0.269
	Scandinavia Growth	0.183	0.043	0.014	0.000	0.000	0.000	-0.183	0.384	1.350	0.742	0.495	0.174	0.008	-0.376
	United States Value	0.000	0.009	0.053	0.147	0.204	0.415	0.415	0.618	1.044	1.759	1.984	2.044	2.093	1.475
	United States Growth	0.093	0.007	0.000	0.010	0.143	0.024	-0.069	0.566	-1.177	-1.346	-1.200	-0.862	-0.756	-1.322

Table 6
Out-of-Sample Performance

This table reports the best, second best and third best model for stock portfolios, in the case of No Short Sales, over four investment horizons: T=1,12,60,120. The last column reports the performance of the equally weighted strategy. We rank models according to the Sharpe Ratio (SR), the Sortino Ratio and the Certainty Equivalent (CEQ). The symbol * indicates that the model has the same ranking when short sales are allowed.

Panel A (International)	Best	Second Best	Third Best	1/N
		T=1		
SR	MV(1) *	MVSK *	MVS *	1/N
	0.528	0.473	0.333	0.389(third best)
	[-0.077,1.201]	[-0.125,1.127]	[-0.281,0.990]	[-0.225,1.039]
SO RT	MV(1) *	MVSK *	MVS *	1/N
	0.777	0.662	0.460	0.561 (third best)
	[-0.117,1.776]	[-0.174,1.710]	[-0.376,1.593]	[-0.346,1.521]
CEQ	MVSK *	MV(1) *	MV(2) *	1/N
	3.910	0.183	-1.344	3.291 (second best)
	[-8.668,15.999]	[-17.651,18.236]	[-17.202,13.763]	[-8.095,14.243]
		T=12		
SR	MV(1) *	MVSK *	MV(2)	1/N
	0.653	0.546	0.485	0.467
	[0.469,0.865]	[0.362,0.755]	[0.308,0.685]	[0.281,0.675]
SO RT	MV(1)	MVSK	MV(2)	1/N
	1.244	0.925	0.841	0.843 (third best)
	[1.011,1.524]	[0.695,1.221]	[0.539,1.209]	[0.566,1.079]
CEQ	MV(2)	MV(1)	MVSK	1/N
	4.993	4.713	4.580	2.947
	[1.251,9.163]	[-0.132,10.569]	[0.107,9.461]	[-1.254,7.627]
		T=60		
SR	MVS	MV(2)	MVSK	1/N
	0.671	0.667	0.496	0.374
	[0.557,0.824]	[0.554,0.819]	[0.411,0.608]	[0.291,0.472]
SO RT	MV(1)	MVSK	MV(2)	1/N
	1.437	1.386	1.345	1.392 (second best)
	[1.193,1.896]	[1.076,1.954]	[1.060,1.976]	[1.021,1.829]
CEQ	MVS	MV(2)	MV(1)	1/N
	12.3122	12.1958	9.6215	6.4525
	[9.799,15.350]	[9.757,15.256]	[6.141,15.086]	[4.643,8.786]
		T=120		
SR	MV(1)	MVS	MV(2)	1/N *
	1.709	1.483	1.479	2.675 (best)
SO RT	MV(1) *	MVS	MV(2) *	1/N *
	4.463	2.977	2.965	5.268 (best)
CEQ	MV(1) *	MVSK	MVS	1/N
	49.921	20.330	19.573	18.728

Table 6 (cont'ed)
Out-of-Sample Performance

Panel B (Industry)	**Best**	**Second Best**	**Third Best**	**1/N**
		T=1		
SR	MV(2)	MVSK	MV(1) *	1/N
	1.138	0.984	0.874	0.754
	[0.744,1.553]	[0.634,1.347]	[0.490,1.283]	[0.451,1.244]
SO RT	MVSK *	MV(2)	MV(1) *	1/N
	1.751	1.703	1.276	1.095
	[1.079,2.563]	[1.032,2.590]	[0.674,2.059]	[0.609,1.958]
CEQ	MV(2)	MVSK	MV(1) *	1/N
	15.086	13.755	10.867	12.655 (third best)
	[8.813,21.207]	[7.257,20.415]	[5.209,16.413]	[4.573,15.742]
		T=12		
SR	MVSK *	MV(1) *	MV(2)	1/N
	0.943	0.919	0.901	0.750
	[0.822,1.082]	[0.803,1.048]	[0.794,1.016]	[0.724,0.962]
SO RT	MV(2)	MVSK *	MV(1)	1/N
	1.745	1.570	1.527	1.395
	[1.550,1.971]	[1.370,1.789]	[1.335,1.791]	[1.219,1.634]
CEQ	MV(2)	MVSK *	MV(1)	1/N *
	12.772	12.645	12.501	11.369
	[11.137,14.454]	[10.932,14.444]	[10.793,14.248]	[9.604,13.227]
		T=60		
SR	MV (1) *	MVS	MV(2)	1/N
	0.882	0.864	0.819	0.754
	[0.803,0.983]	[0.788,0.957]	[0.751,0.904]	[0.689,0.834]
SO RT	MVSK	MV(2)	MV(1)	1/N
	1.718	1.705	1.628	1.314
	[1.594,1.874]	[1.587,1.868]	[1.493,1.809]	[1.242,1.414]
CEQ	MV(2)	MVSK	MV(1)	1/N
	18.511	18.158	17.828	15.326
	[17.294,19.859]	[16.963,19.476]	[16.561,19.244]	[14.072,16.729]
		T=120		
SR	MV(1) *	MVS	MV(2) *	1/N *
	1.071	0.972	0.796	0.894 (third best)
	[0.989,1.179]	[0.902,1.061]	[0.737,0.871]	[0.824,0.982]
SO RT	MV(1)	MVS	MVSK	1/N
	2.184	1.897	1.878	1.655
	[2.009,2.480]	[1.694,2.174]	[1.703,2.094]	[1.482,1.893]
CEQ	MV(1) *	MV(2)	MVS	1/N
	30.922	30.247	30.040	27.602
	[29.832,32.113]	[28.499,32.190]	[28.557,31.719]	[25.793,29.680]

Table 6 (cont'ed)
Out-of-Sample Performance

Panel C (Book-to-Market)	Best	Second Best	Third Best	1/N
		T=1		
SR	MVSK *	MV(2)	MVS*	1/N
	1.322	1.185	0.652	0.593
	[0.741,1.973]	[0.618,1.843]	[0.087,1.265]	[0.032,1.215]
SO RT	MVSK *	MV(2)	MVS	1/N
	1.857	1.586	0.897	0.790
	[1.005,2.993]	[0.811,2.689]	[0.122,1.891]	[0.043,1.704]
CEQ	MVSK *	MV(2)	MVS	1/N
	18.643	17.701	9.858	9.126
	[11.136,25.988]	[9.127,26.252]	[1.098,18.436]	[1.246,16.785]
		T=12		
SR	MVS	MV(2)	MVSK *	1/N
	0.606	0.546	0.543	0.542
	[0.415,0.836]	[0.360,0.770]	[0.367,0.750]	[0.352,0.770]
SO RT	MVS	MVSK	MV(2)	1/N
	0.791	0.751	0.666	0.686 (third best)
	[0.607,1.028]	[0.577,1.024]	[0.500,0.901]	[0.560,0.882]
CEQ	MVS	MVSK	MV(2)	1/N
	8.586	7.432	7.328	6.926
	[5.258,12.138]	[4.053,11.215]	[3.944,10.949]	[3.308,10.802]
		T=60		
SR	MVS *	MVSK *	MV(2) *	1/N
	0.571	0.186	0.169	0.280 (second best)
	[0.492,0.672]	[0.118,0.251]	[0.097,0.236]	[0.212,0.353]
SO RT	MVS *	MVSK *	MV(2) *	1/N
	1.253	0.811	0.711	0.964 (second best)
	[1.089,1.571]	[0.487,1.025]	[0.403,0.959]	[0.793,1.186]
CEQ	MVS *	MVSK *	MV(2) *	1/N
	12.868	3.603	3.378	6.469 (second best)
	[11.278,14.762]	[2.512,4.959]	[2.360,4.575]	[5.319,7.870]
		T=120		
SR	MVS	MV(2)	MVSK	1/N
	3.836	1.069	1.007	3.451 (second best)
	[3.258,4.953]	[0.902,1.396]	[0.848,1.306]	[2.692,5.323]
SO RT	MVS *	MV(2)	MVSK	1/N
	6.040	2.216	2.028	3.711 (second best)
	[4.992,9.236]	[1.810,3.284]	[1.659,2.994]	[3.040,7.034]
CEQ	MVS *	MVSK *	MV(2)	1/N
	30.206	16.165	14.964	20.768 (second best)
	[29.524,30.906]	[15.139,17.363]	[14.135,15.881]	[20.223,21.278]

Table 7

This table reports the best, second best and third best model for stock portfolios, in the case of No Short Sales, over four investment horizons: T=1,12,60,120. The last column reports the performance of the equally weighted strategy. We rank models according to the Treynor Ratio (TR) and Jensen's Alfa (JA). The symbol * indicates that the model has the same ranking when short sales are allowed.

Panel A (International)	**Best**	**Second Best**	**Third Best**	**1/N**
		T=1		
TR	MV(1)	MVS	MVSK	1/N
	0.367	0.146	0.126	0.130 (third best)
	[-3.607,3.426]	[-1.690,1.867]	[-0.469,1.022]	[-1.026,1.247]
JA	MV(1)	MVS	MV(2)	1/N
	313.260	296.129	287.387	297.290 (second best)
	[198.878,411.150]	[191.352,393.037]	[205.655,365.680]	[223.143,364.240]
		T=12		
TR	MV(1)	MV(2)	MVSK*	1/N
	0.140	0.109	0.103	0.089
	[0.103,0.180]	[0.069,0.152]	[0.070,0.136]	[0.056,0.123]
JA	MV(2)*	MVS	MVK	1/N
	22.530	12.255	-6.755	-3.540 (third best)
	[7.441,37.445]	[-2.878,27.317]	[-19.506,5.660]	[-15.744,8.068]
		T=60		
TR	MVS	MV(2)	MVSK	1/N
	0.284	0.282	0.190	0.139
	[0.223,0.356]	[0.223,0.354]	[0.146,0.237]	[0.098,0.183]
JA	MVS*	MV(2)*	MVSK*	1/N
	31.946	31.735	7.768	-10.883
	[24.979,38.988]	[25.047,38.570]	[1.563,13.793]	[-16.591,-5.717]
		T=120		
TR	MV(1)	MVK	MVSK	1/N
	-0.405	-0.604	-0.609	-2.952
JA	MV(1)*	MVK	MVSK	1/N
	286.784	109.252	109.246	82.477

Table 7
Out-of-Sample Performance

Panel B (Industry)	**Best**	**Second Best**	**Third Best**	**1/N**
		T=1		
TR	MVSK	MV(2)	MVS	1/N
	0.080	0.055	0.054	0.037
	[0.054,0.113]	[0.037,0.073]	[0.025,0.087]	[0.021,0.054]
JA	MVSK	MVS	MV(1)*	1/N
	124.699	108.774	73.429	35.731
	[83.205,165.834]	[64.180,155.209]	[51.097,97.218]	[24.945,47.499]
		T=12		
TR	MVSK	MV(1)	MV(2)	1/N
	0.186	0.185	0.184	0.144
	[0.161,0.213]	[0.161,0.212]	[0.160,0.208]	[0.126,0.162]
JA	MV(1)	MVSK	MV(2)	1/N
	31.555	31.347	28.063	12.571
	[25.616,37.712]	[25.191,37.402]	[21.258,34.635]	[8.878,16.430]
		T=60		
TR	MV(1)*	MVSK	MV(2)*	1/N
	0.632	0.571	0.557	0.456
	[0.569,0.709]	[0.504,0.657]	[0.506,0.617]	[0.422,0.491]
JA	MV(1)*	MV(2)	MVS	1/N
	31.672	22.395	21.431	10.315
	[26.159,37.330]	[16.472,28.219]	[18.344,24.605]	[7.768,12.835]
		T=120		
TR	MV(1)*	MVS	MVSK	1/N
	1.5786	1.110	1.080	0.994
	[1.373,1.839]	[1.033,1.197]	[0.884,1.262]	[0.930,1.063]
JA	MV(1)*	MVS	MV(2)	1/N
	59.871	29.693	26.274	17.707
	[49.970,69.109]	[24.127,35.387]	[13.506,38.705]	[13.441,22.102]

Table 7
Out-of-Sample Performance

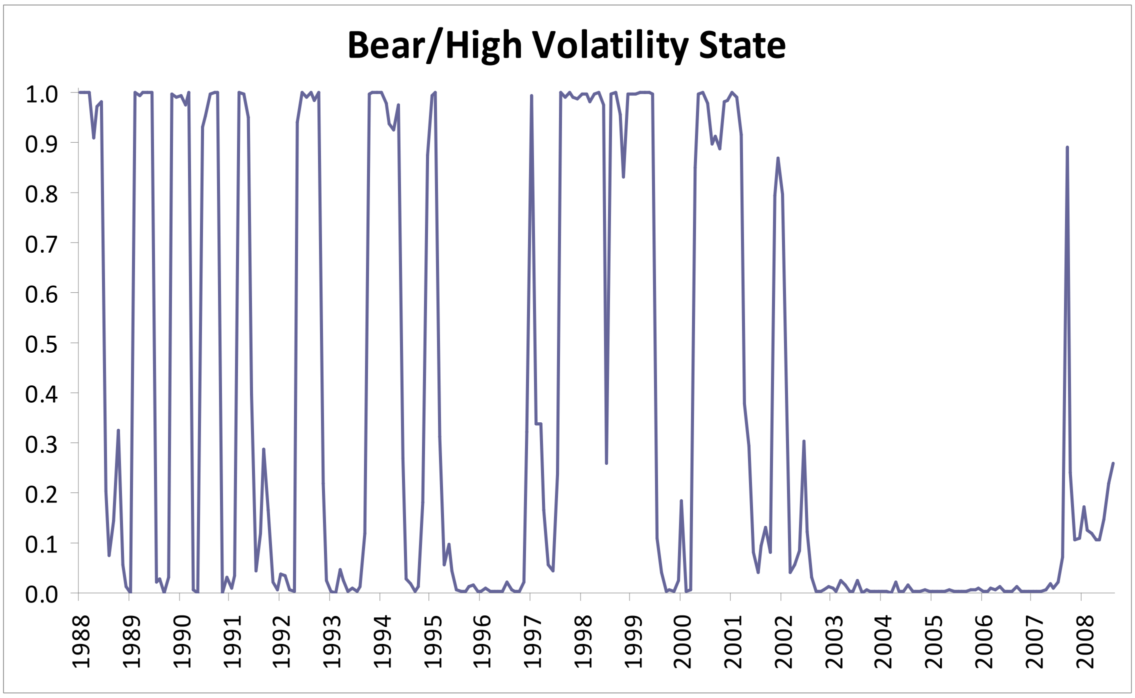
Panel C(BM)	**Best**	**Second Best**	**Third Best**	**1/N**
		T=1		
TR	MVSK	MV(2)	MVS	1/N
	0.055	0.054	0.029	0.023
	[0.032,0.077]	[0.029,0.081]	[0.004,0.054]	[0.001,0.045]
J'S A	MVSK	MV(1)	MV(2)	1/N
	39.152	32.557	29.587	16.719
	[18.426,58.609]	[11.988,55.292]	[-10.620,71.605]	[3.111,29.012]
		T=12		
TR	MVS*	MV(2)	MVSK*	1/N
	0.114	0.099	0.094	0.089
	[0.081,0.150]	[0.068,0.131]	[0.066,0.120]	[0.061,0.115]
JA	MVS	MV(2)	MV(1)	1/N
	22.040	17.132	5.400	2.790
	[14.044,30.922]	[9.911,25.261]	[1.768,9.213]	[0.337,5.086]
		T=60		
TR	MVS*	MVSK	MV(2)	1/N
	0.339	0.098	0.087	0.140(second best)
	[0.266,0.419]	[0.056,0.141]	[0.044,0.130]	[0.096,0.186]
JA	MVS*	MV(2)*	MV(1)	1/N
	36.935	-4.299	-5.287	8.321(second best)
	[31.744,41.297]	[-7.003,-0.927]	[-6.729,-3.573]	[7.532,9.108]
		T=120		
TR	MVS*	MV(2)	MVSK	1/N
	0.721	0.120	0.062	0.375 (second best)
	[0.493,1.877]	[0.099,0.138]	[0.031,0.092]	[0.329,0.464]
JA	MVS*	MV(1)	MV(2)	1/N
	69.354	-17.285	-33.823	32.067 (second best)
	[52.828,96.072]	[-27.667,-10.887]	[-62.339,-17.071]	[26.540,40.516]

Figure 1

Smoothed State Probabilities from Two-State Markov Switching Models – International Data

The graphs plot the smoothed state probabilities for the two-state switching model. Panels A, B and C respectively refer to the International, the Industry and the International Book-to-Market Portfolios.

Panel A (International MSCI USD Returns, 1988:01 - 2008:08)



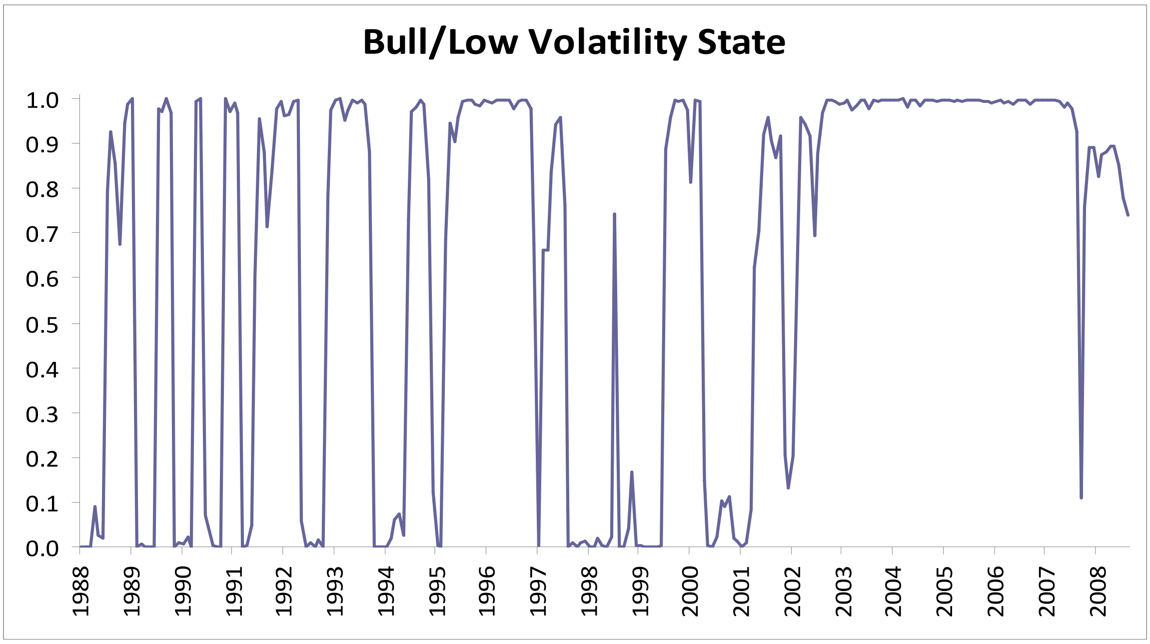
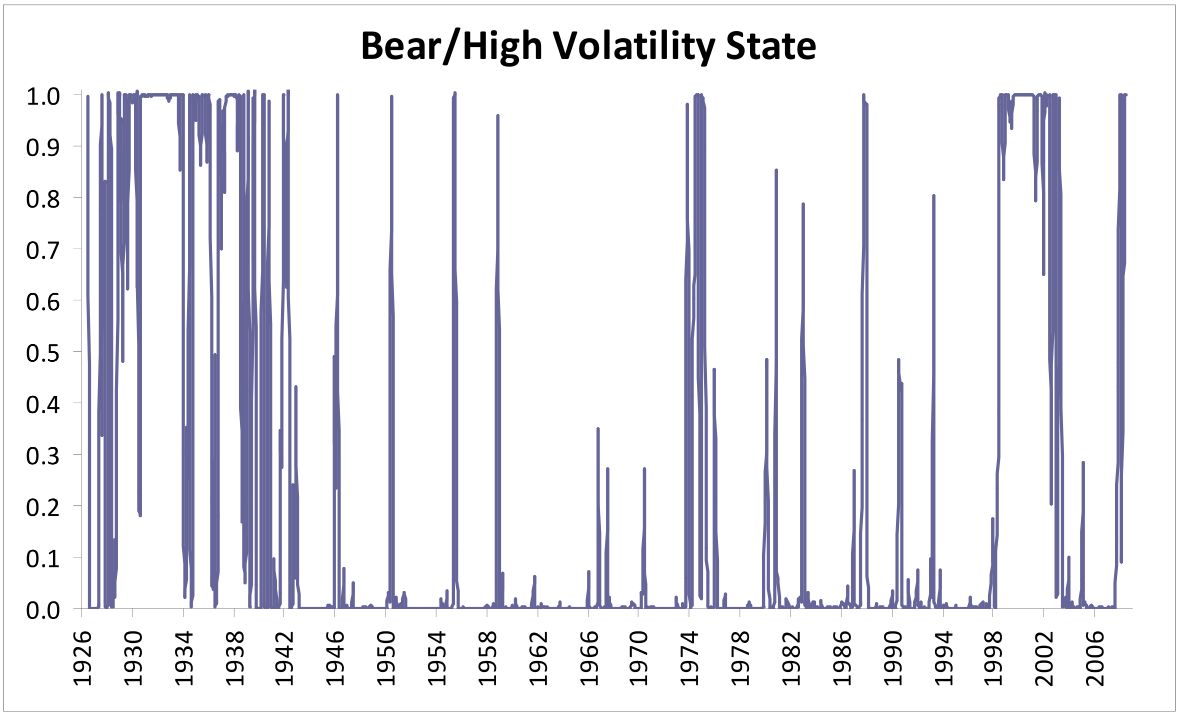


Figure 1 (cont'ed)

Smoothed State Probabilities from Two-State Markov Switching Models – Industry Data

Panel B (CRSP Industry Returns, 1926:07 - 2008:07)



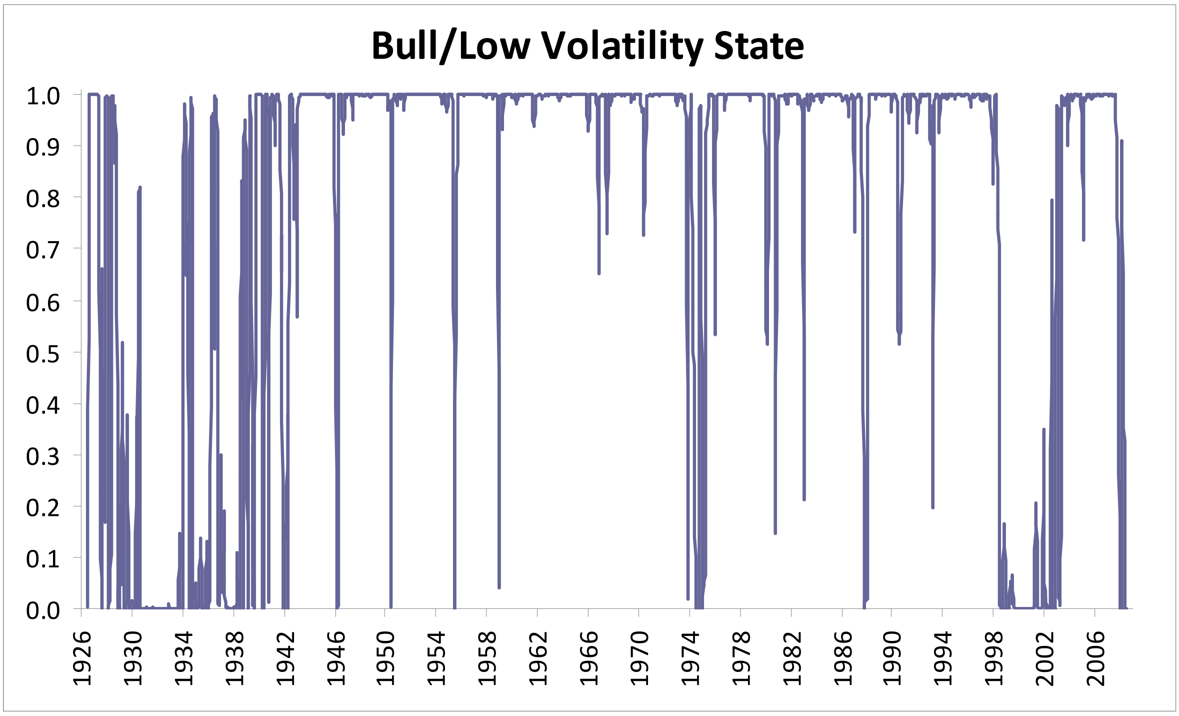
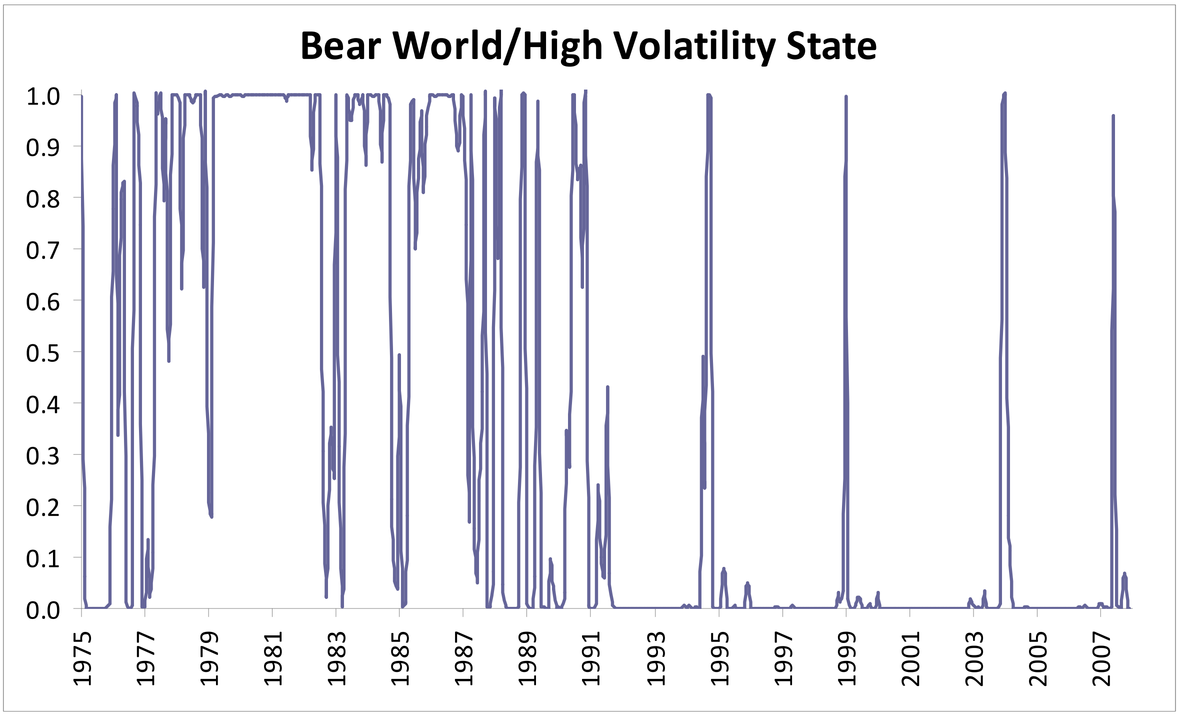


Figure 1 (cont'ed)

Smoothed State Probabilities from Two-State Markov Switching Models – Book-to-Market International Data

Panel C (International Book-to-Market Sorted Portfolio Local Returns, 1975:01 - 2007:12)



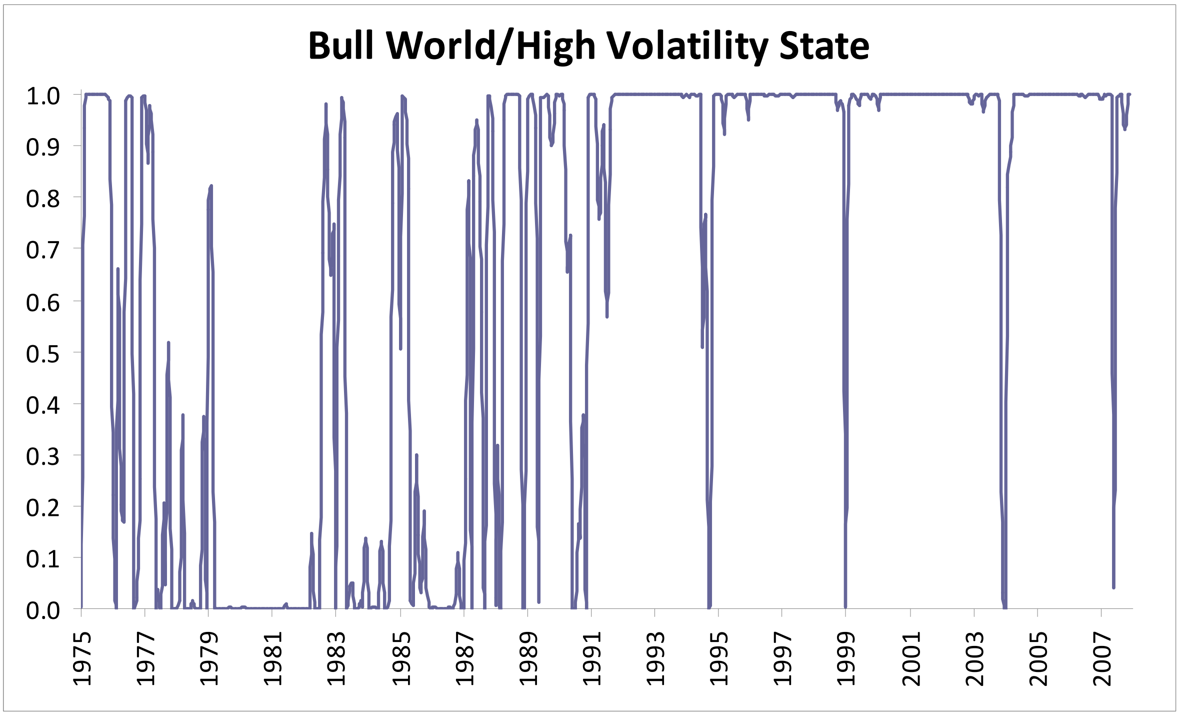
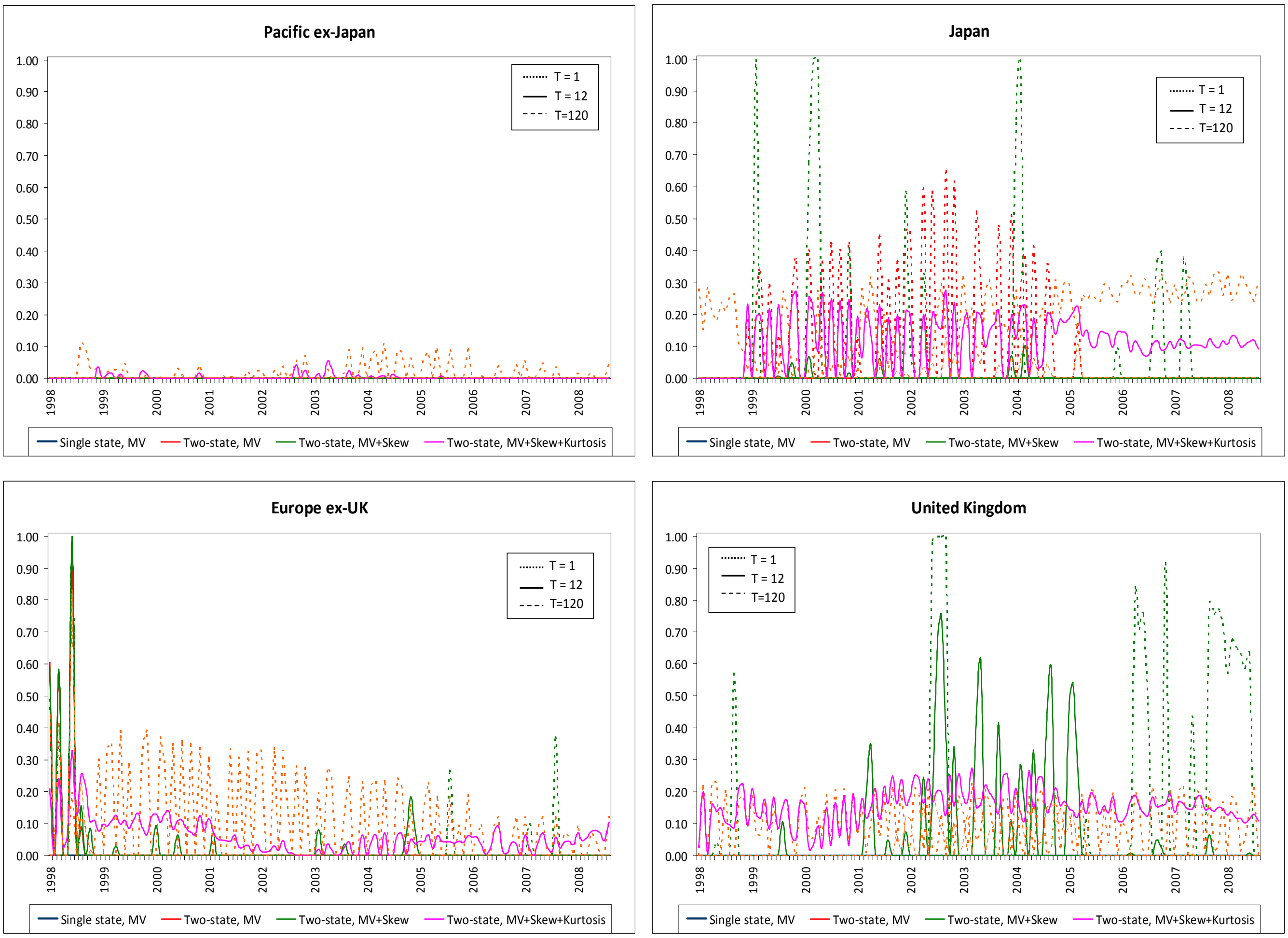


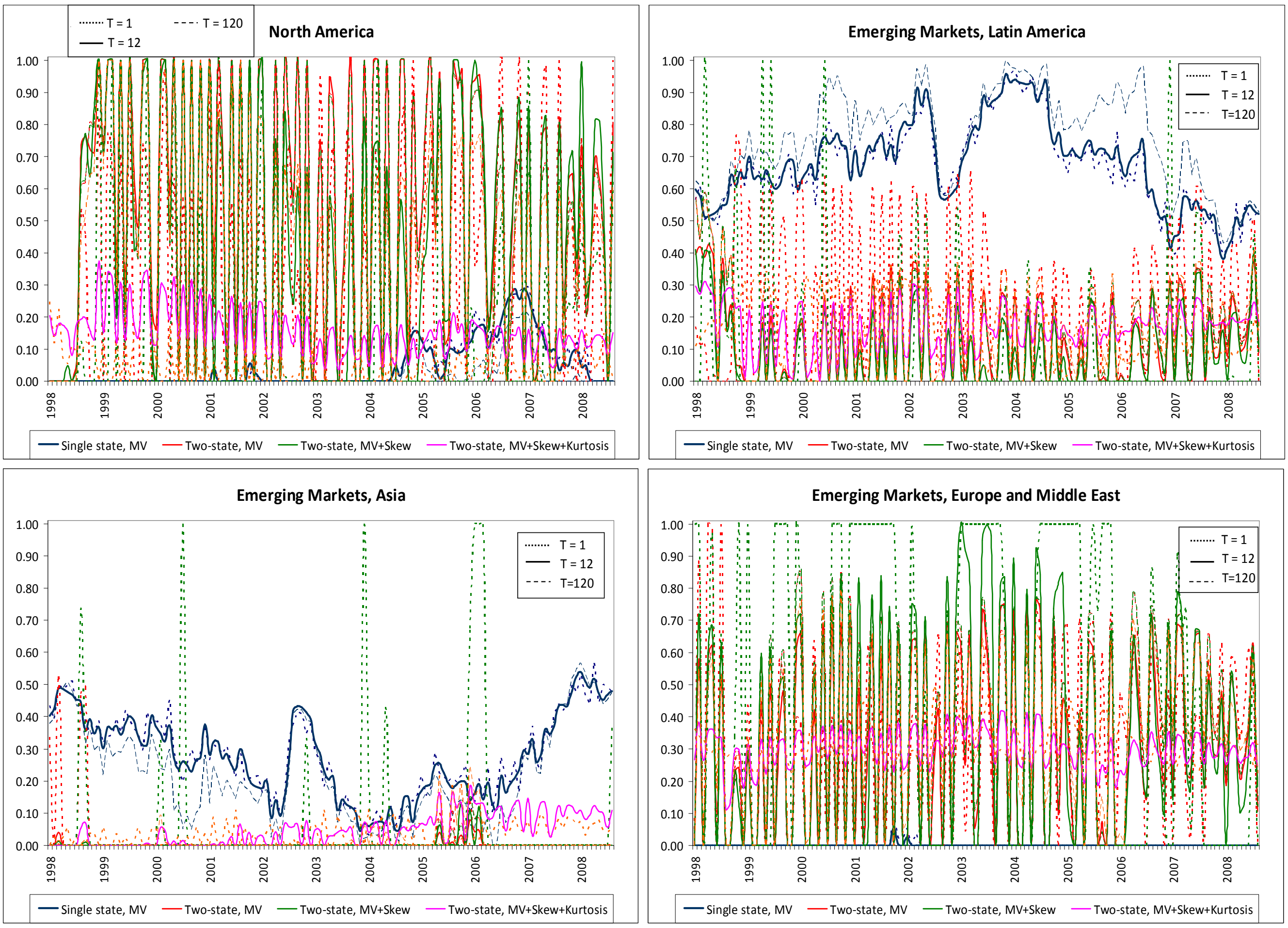
Figure 2

Dynamics of Portfolio Weights over Time, No Short Sales Admitted & CRRA = 5

This figure displays how each portfolio share changes as the probability of being in a bear/bull state is updated by the investor. The dotted/solid/ dashed lines respectively identify investor horizons of 1, 12 and 120 months. Colors refer to investor preferences over moments of the return distribution. Panels A, B and C respectively refer to the International, the Industry and the International Book-to-Market Portfolios.

Panel A (International MSCI USD Returns, 1988:01 - 2008:08)



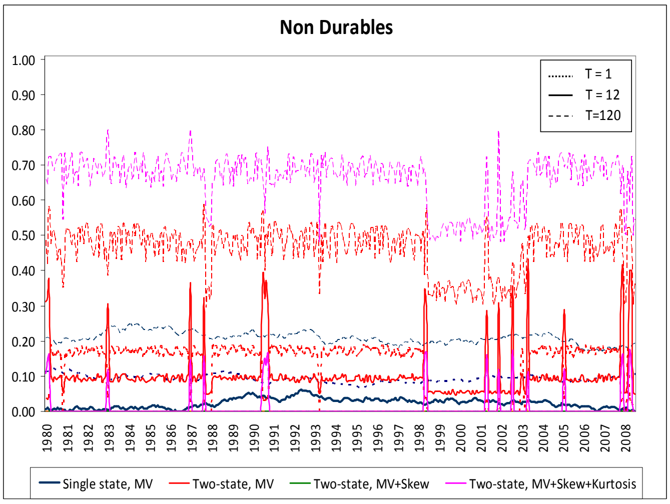


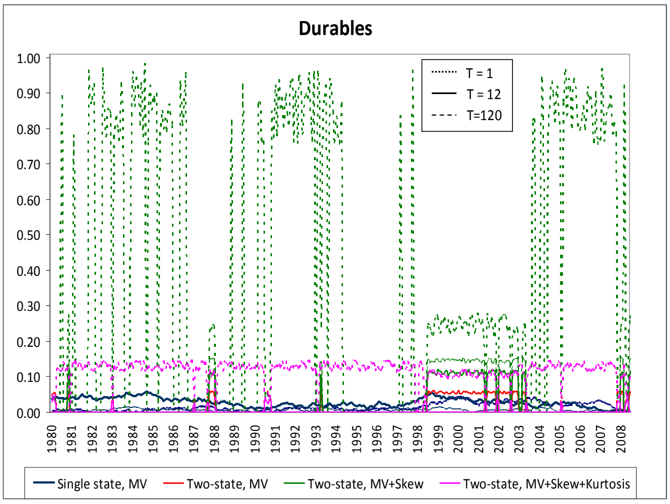
North America
T = 1
T = 12
T = 120
Emerging Markets, Latin America
T = 1
T = 12
T=120
Emerging Markets, Asia
T = 1
T = 12
T=120
Emerging Markets, Europe and Middle East
T = 1
T = 12
T=120
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008
Single state, MV
Two-state, MV
Two-state, MV+Skew
Two-state, MV+Skew+Kurtosis

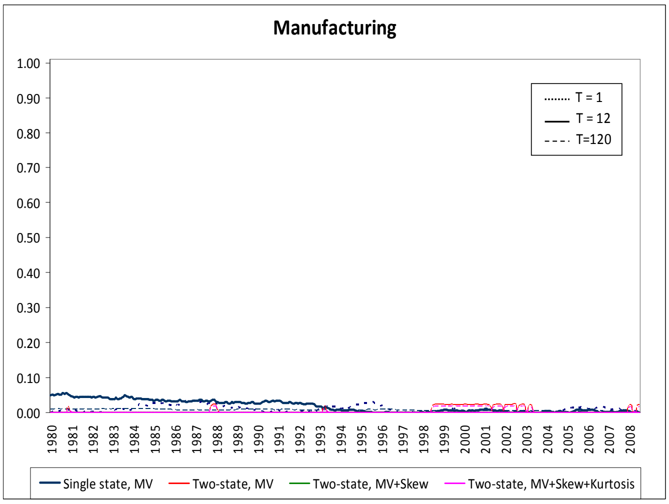
Figure 2 (cont'ed)

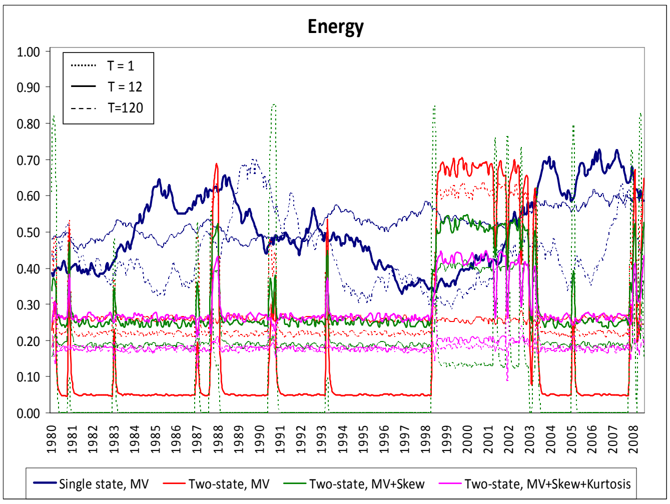
Dynamics of Portfolio Weights over Time, No Short Sales Admitted & CRRA = 5

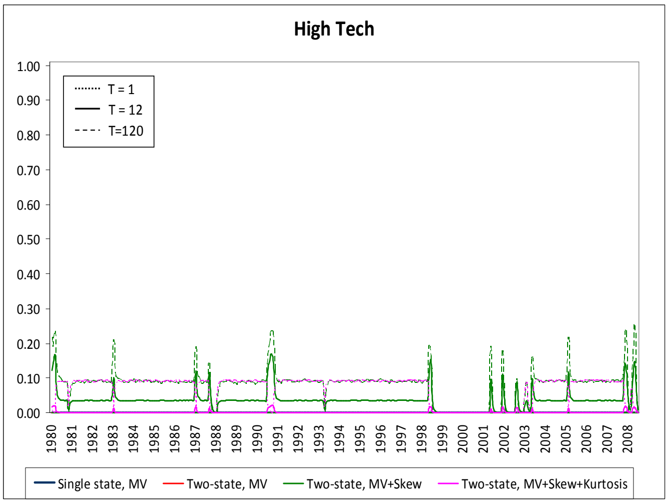
Panel B (CRSP Industry Returns, 1926:07 - 2008:07)

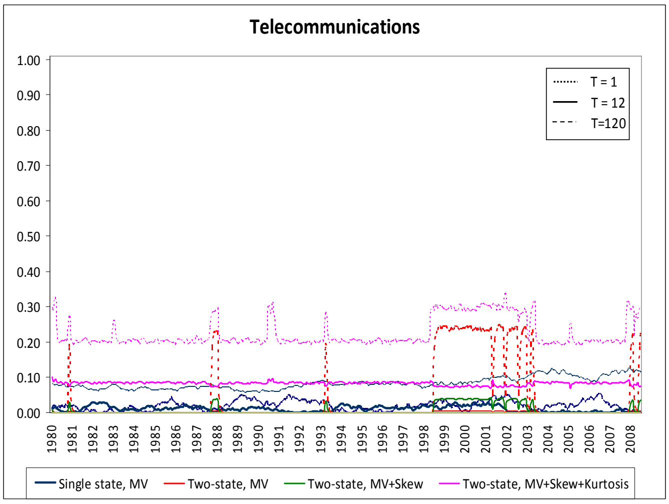


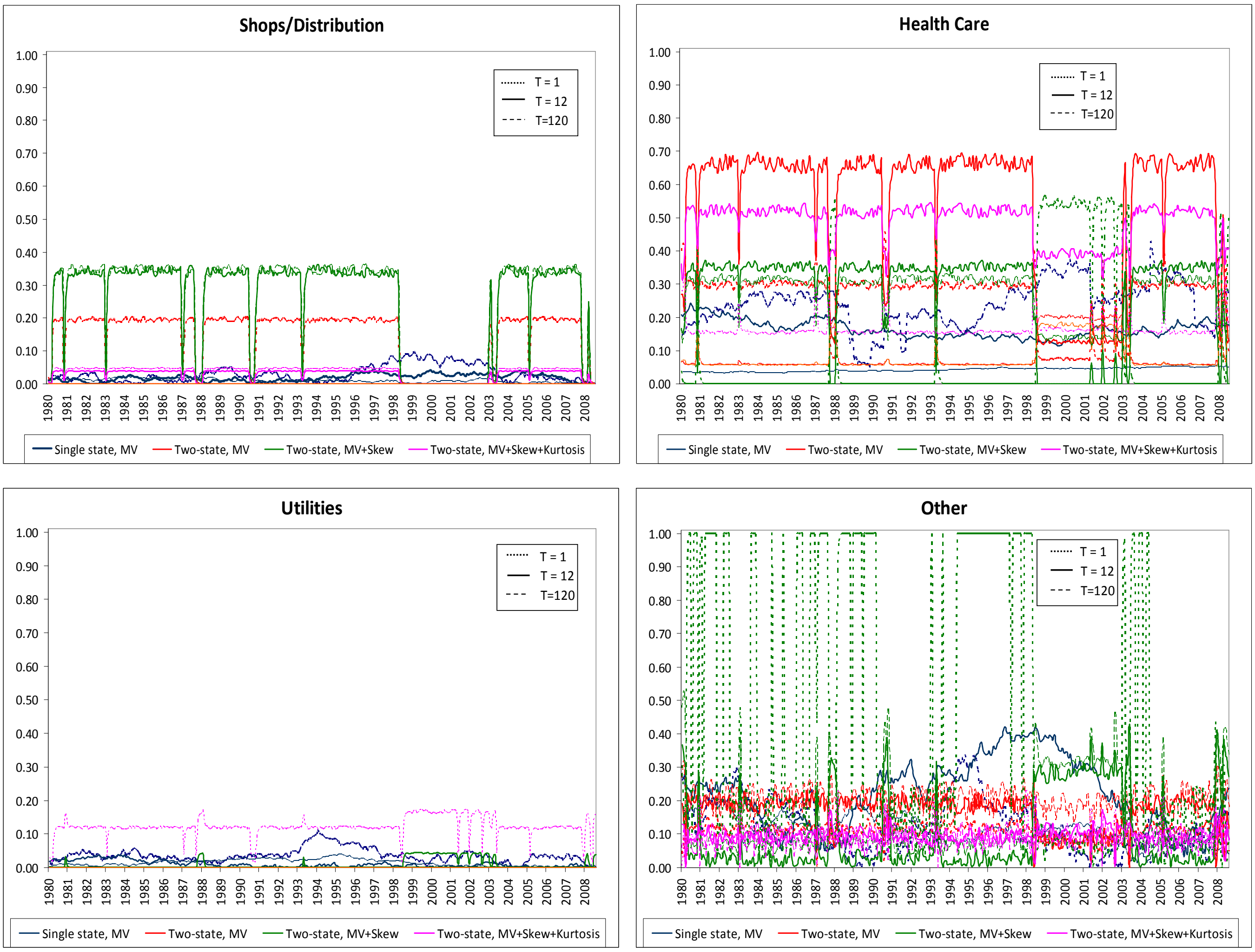










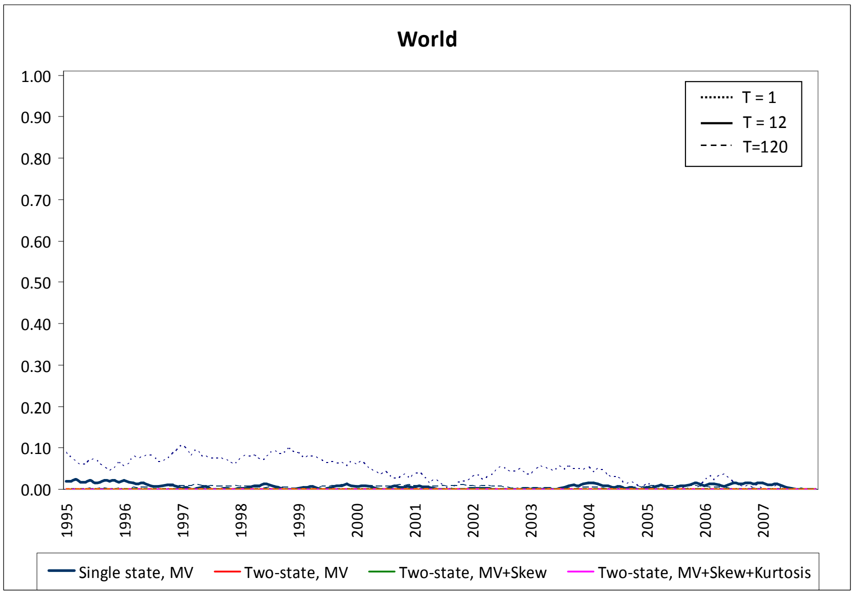


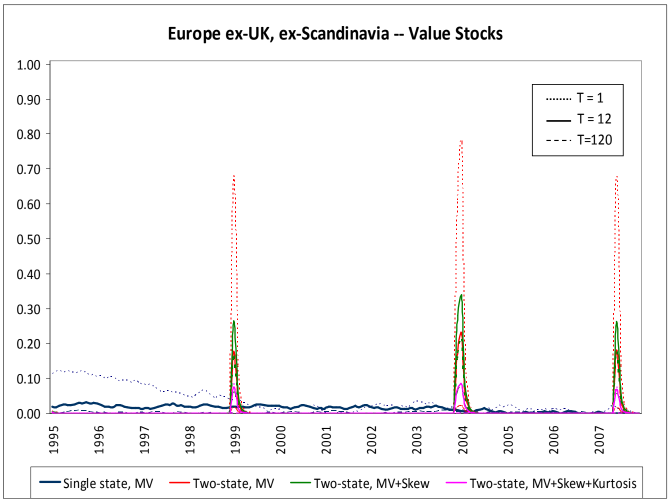
Shops/Distribution
Health Care
Utilities
Other
T = 1
T = 12
T=120
Single state, MV
Two-state, MV
Two-state, MV+Skew
Two-state, MV+Skew+Kurtosis

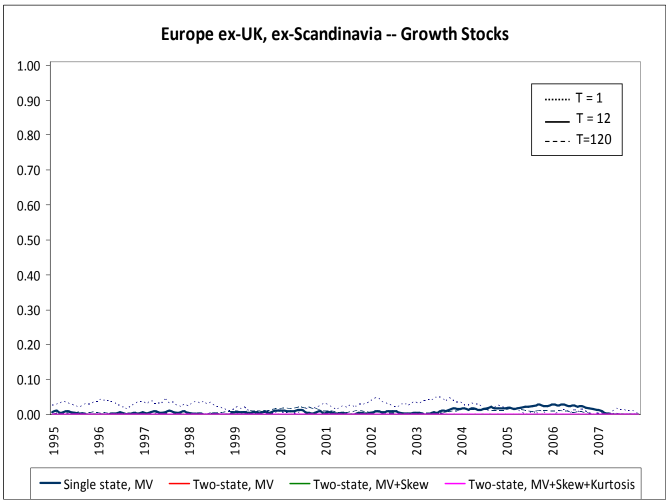
Figure 2 (cont'd)

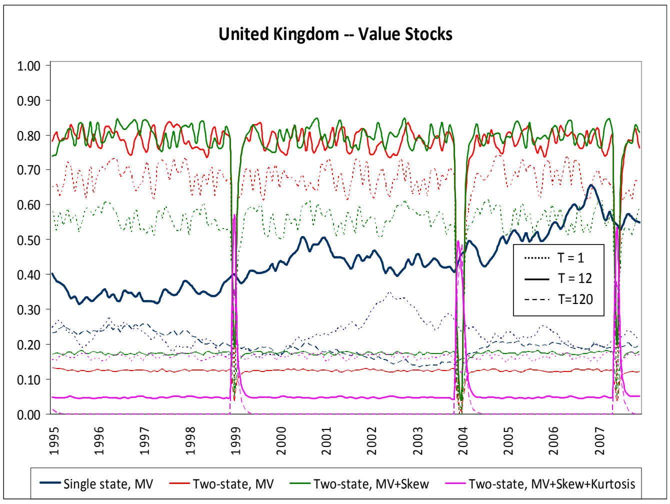
Dynamics of Portfolio Weights over Time, No Short Sales Admitted & CRRA = 5

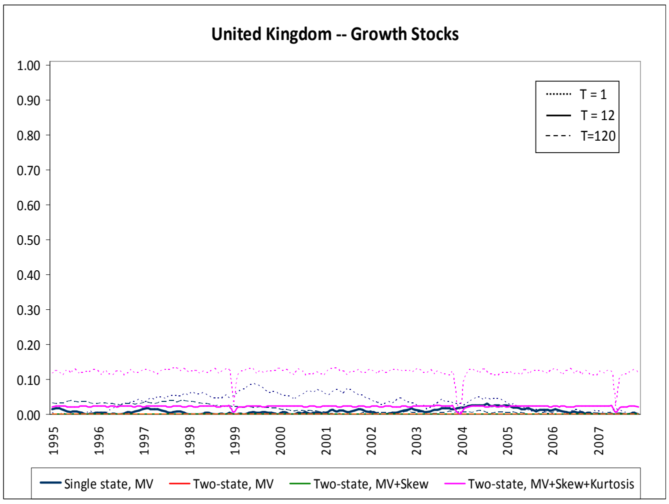
Panel C (International Book-to-Market Sorted Portfolio Local Returns, 1975:01 - 2007:12)

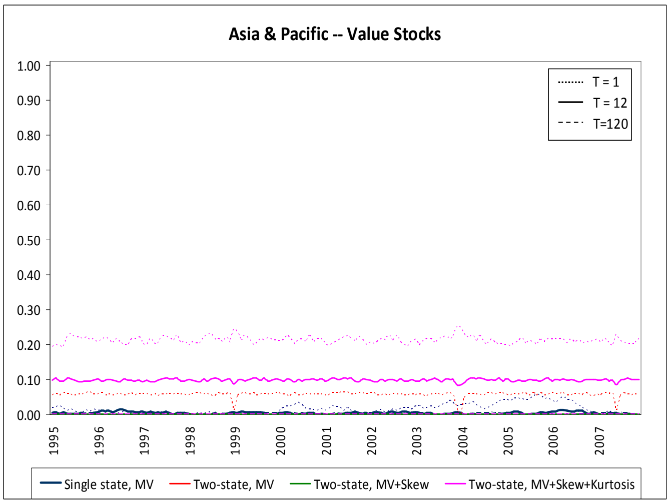


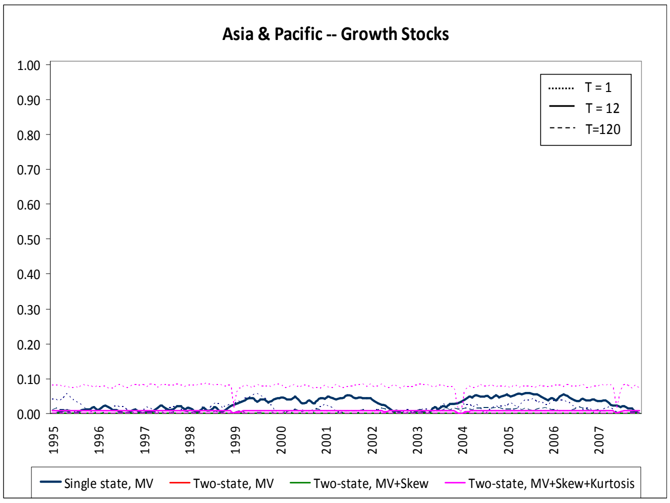


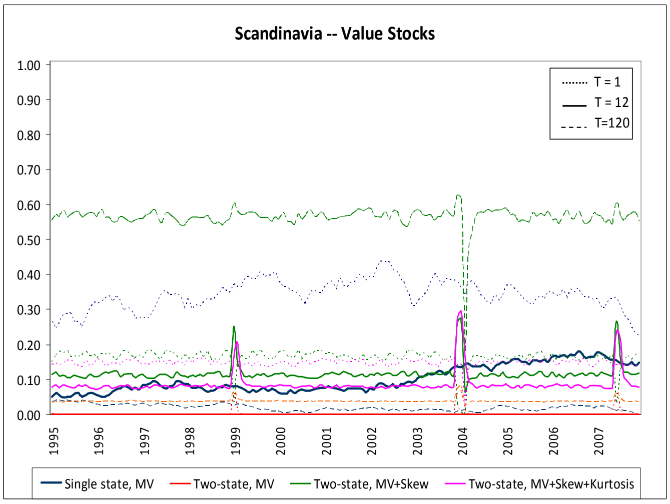


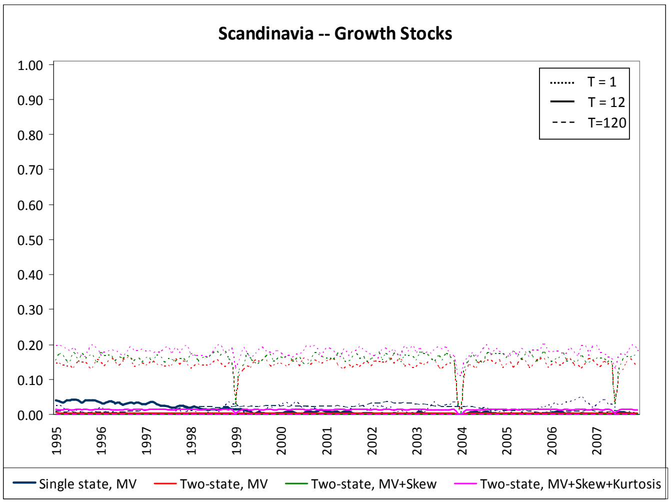


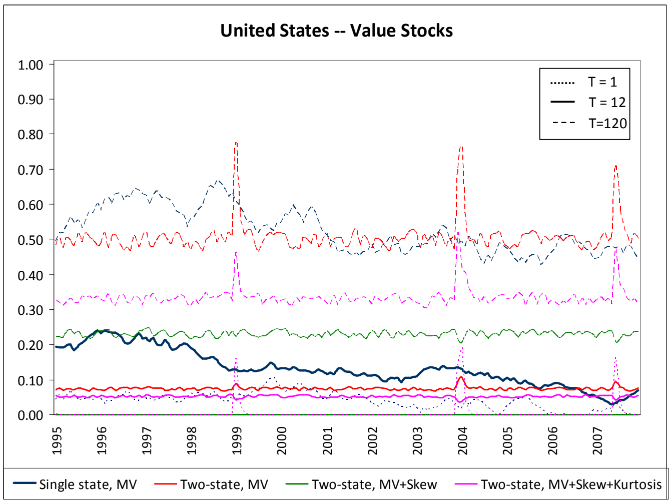



Asia & Pacific -- Value Stocks
T = 1
T = 12
T=120
Single state, MV
Two-state, MV
Two-state, MV+Skew
Two-state, MV+Skew+Kurtosis


Asia & Pacific -- Growth Stocks
T = 1
T = 12
T=120
Single state, MV
Two-state, MV
Two-state, MV+Skew
Two-state, MV+Skew+Kurtosis


Scandinavia -- Value Stocks
T = 1
T = 12
T=120
Single state, MV
Two-state, MV
Two-state, MV+Skew
Two-state, MV+Skew+Kurtosis


Scandinavia -- Growth Stocks
T = 1
T = 12
T=120
Single state, MV
Two-state, MV
Two-state, MV+Skew
Two-state, MV+Skew+Kurtosis


United States -- Value Stocks
T = 1
T = 12
T=120
Single state, MV
Two-state, MV
Two-state, MV+Skew
Two-state, MV+Skew+Kurtosis

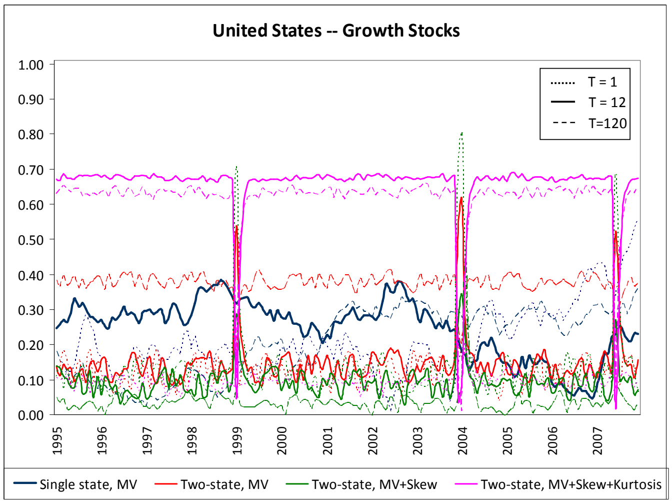
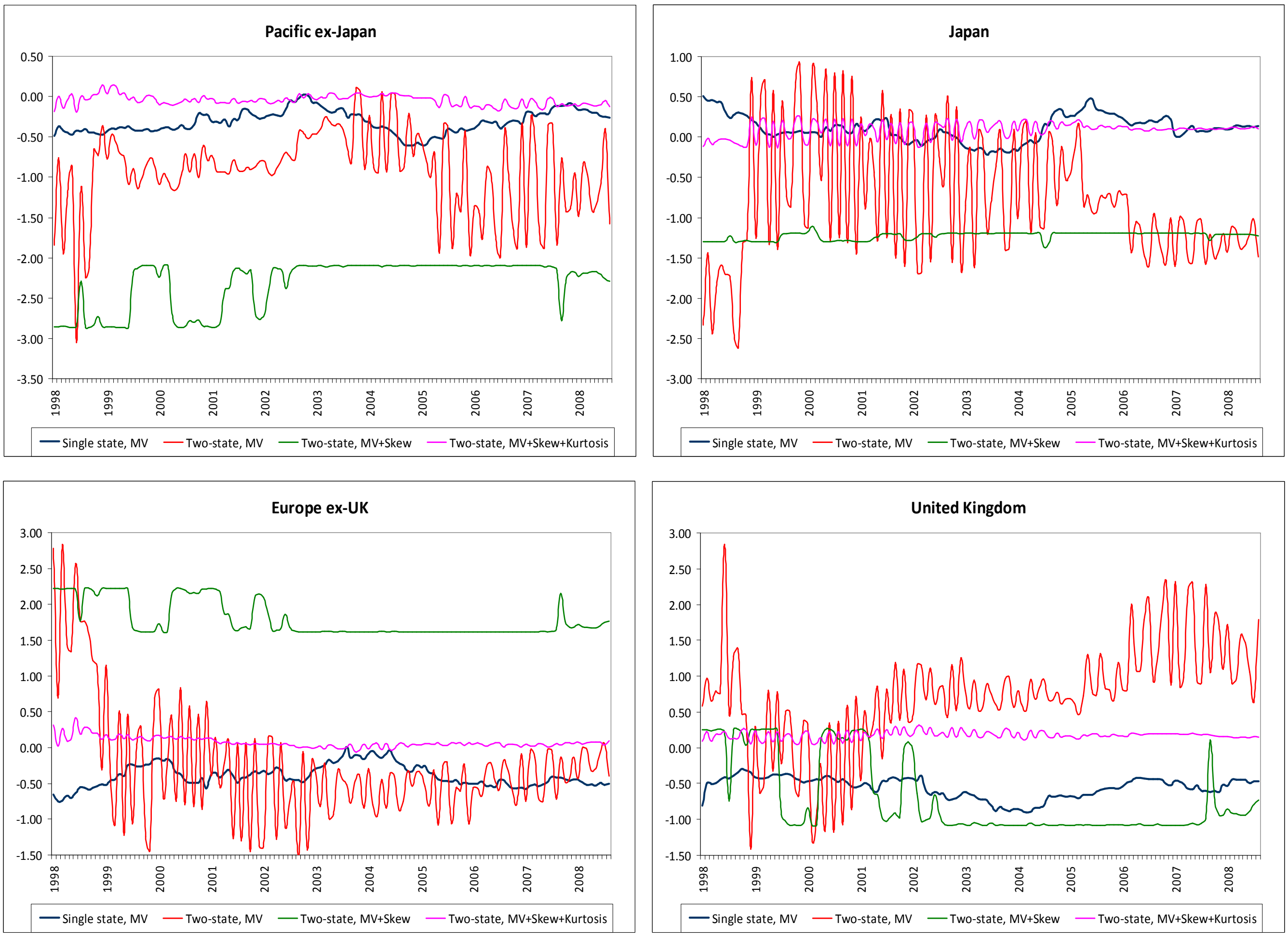

United States -- Growth Stocks
T = 1
T = 12
T=120
Single state, MV
Two-state, MV
Two-state, MV+Skew
Two-state, MV+Skew+Kurtosis

Figure 3

Dynamics of Portfolio Weights over Time, CRRA = 5 & Investment Horizon of 12 Months

This figure displays how each portfolio share changes as the probability of being in a bear/bull state is updated by the investor with a 1-year horizon. The red/green/ pink lines respectively identify investor preferences over mean and variance, mean variance and skew, mean variance and kurtosis, when returns follows a two-state switching model. The blue line depicts the dynamics of the mean-variance allocations when returns follow a single-state model. Colors refer to investor preferences over moments of the return distribution. Panels A, B and C respectively refer to the International, the Industry and the International Book-to-Market Portfolios.

Panel A (International MSCI USD Returns, 1988:01 - 2008:08)



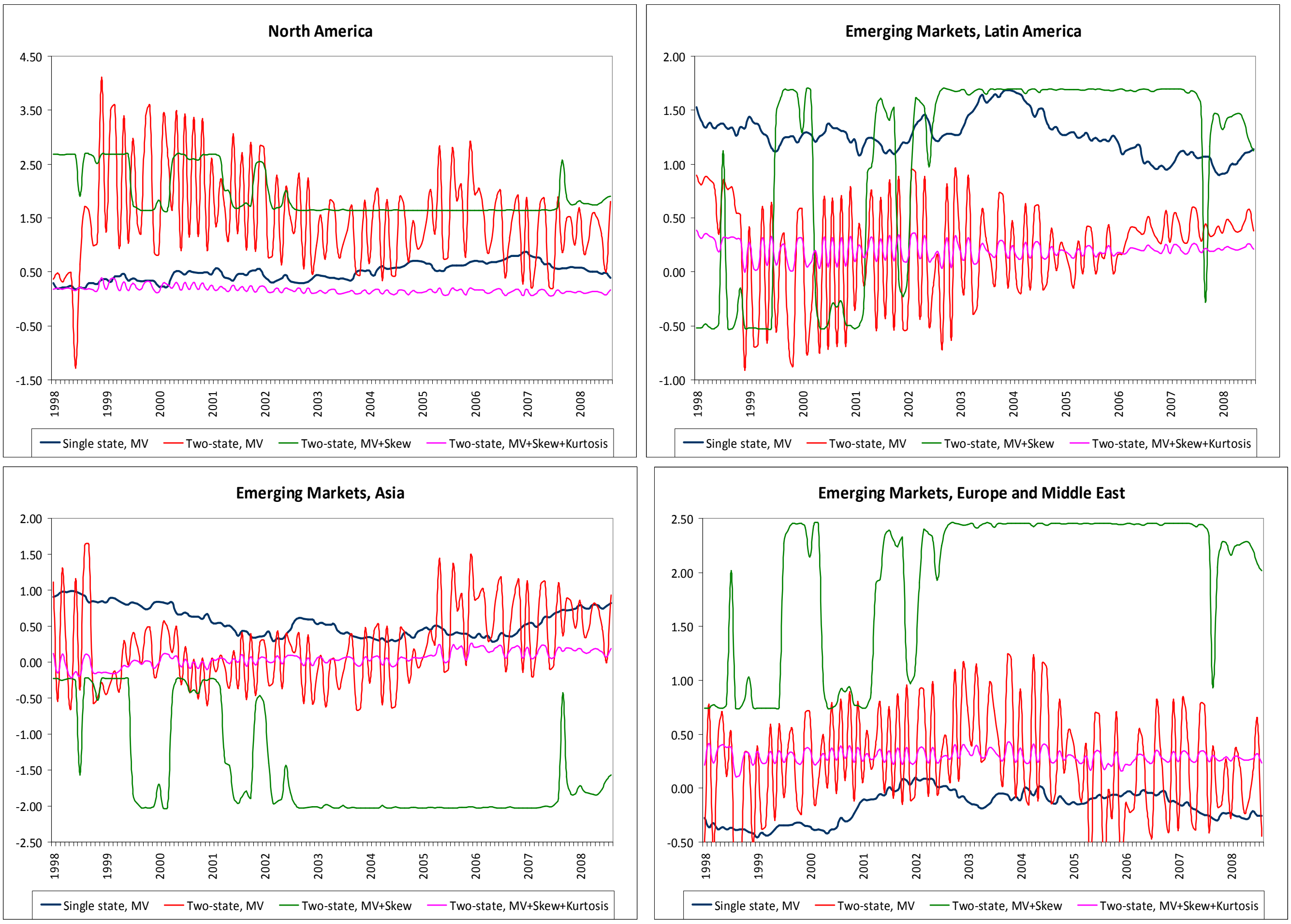
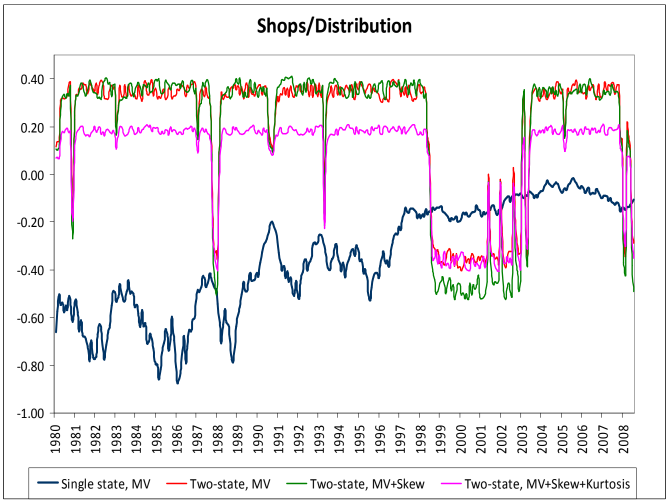
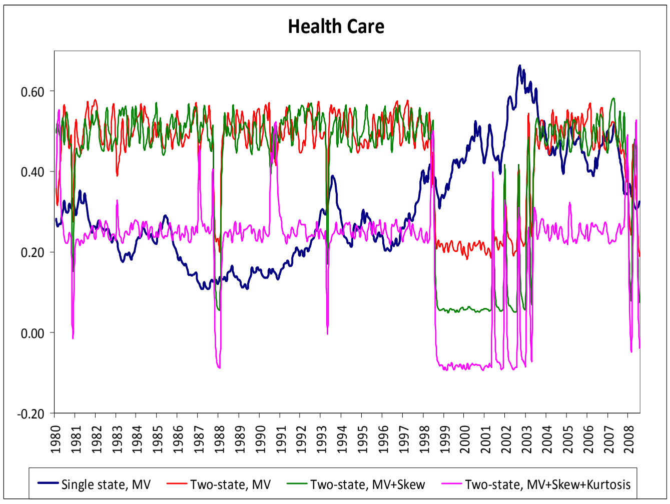

North America
Emerging Markets, Latin America
Emerging Markets, Asia
Emerging Markets, Europe and Middle East
Single state, MV
Two-state, MV
Two-state, MV+Skew
Two-state, MV+Skew+Kurtosis

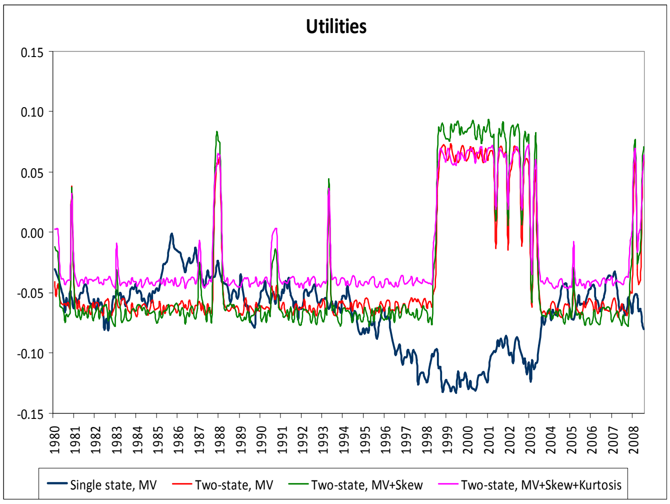
Figure 3 (cont'ed)

Dynamics of Portfolio Weights over Time, CRRA = 5 & Investment Horizon of 12 Months

Panel B (CRSP Industry Returns, 1926:07 - 2008:07)


Shops/Distribution
0.40
0.20
0.00
-0.20
-0.40
-0.60
-0.80
-1.00
1980 1981 1982 1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008
Single state, MV
Two-state, MV
Two-state, MV+Skew
Two-state, MV+Skew+Kurtosis


Health Care
0.60
0.40
0.20
0.00
-0.20
1980 1981 1982 1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008
Single state, MV
Two-state, MV
Two-state, MV+Skew
Two-state, MV+Skew+Kurtosis


Utilities
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
1980 1981 1982 1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008
Single state, MV
Two-state, MV
Two-state, MV+Skew
Two-state, MV+Skew+Kurtosis

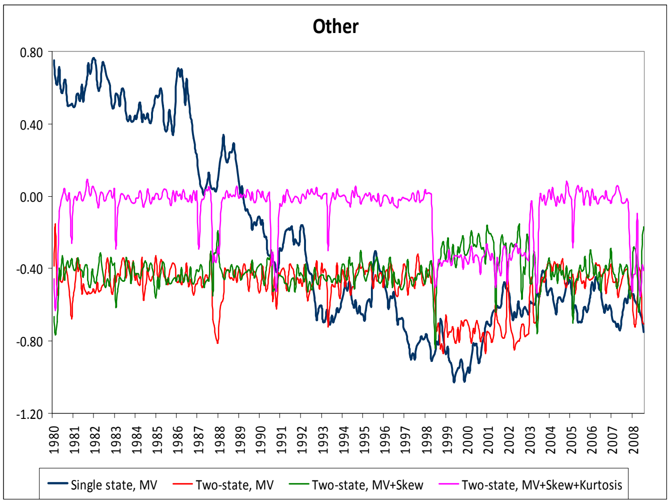
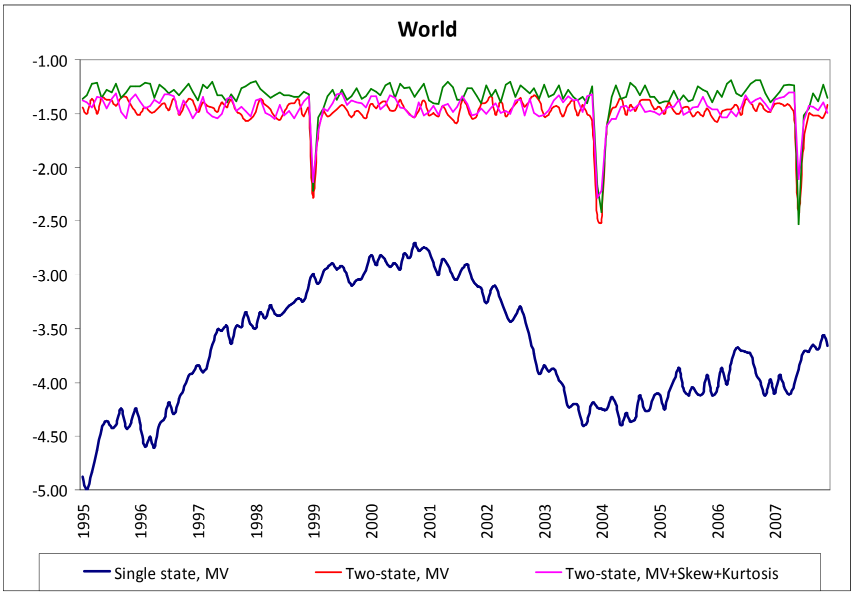
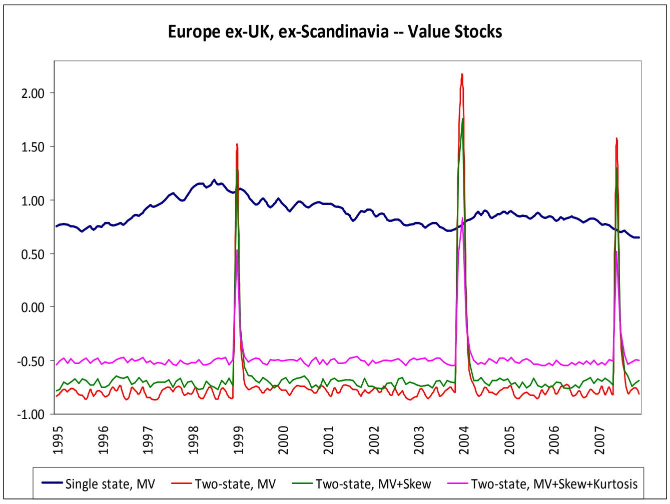

Other
0.80
0.40
0.00
-0.40
-0.80
-1.20
1980 1981 1982 1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008
Single state, MV
Two-state, MV
Two-state, MV+Skew
Two-state, MV+Skew+Kurtosis

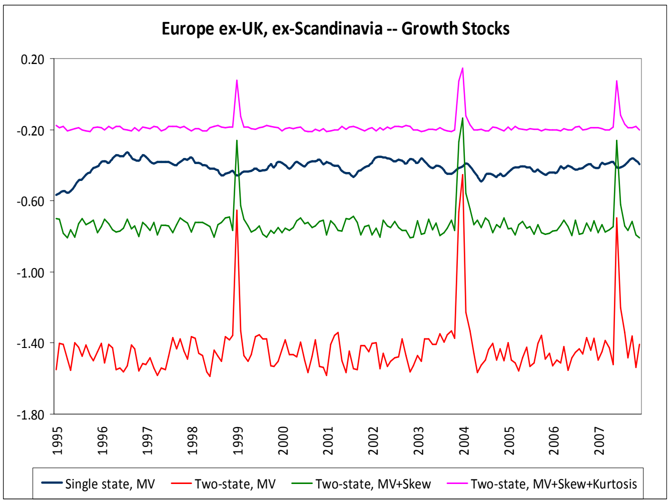
Figure 3 (cont'ed)

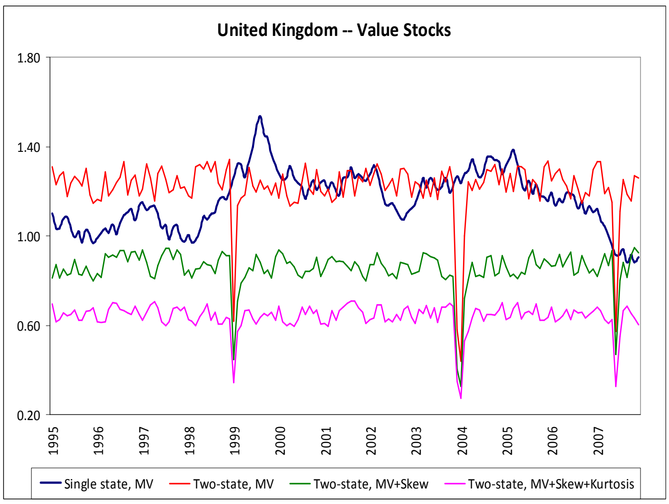
Dynamics of Portfolio Weights over Time, CRRA = 5 & Investment Horizon of 12 Months

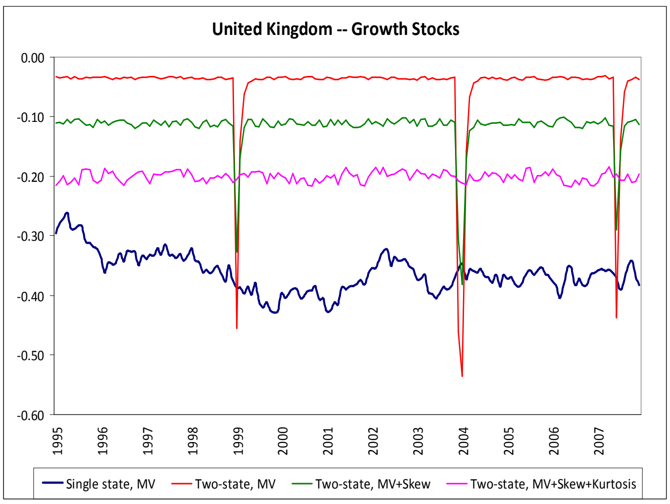
Panel C (International Book-to-Market Sorted Portfolio Local Returns, 1975:01 - 2007:12)

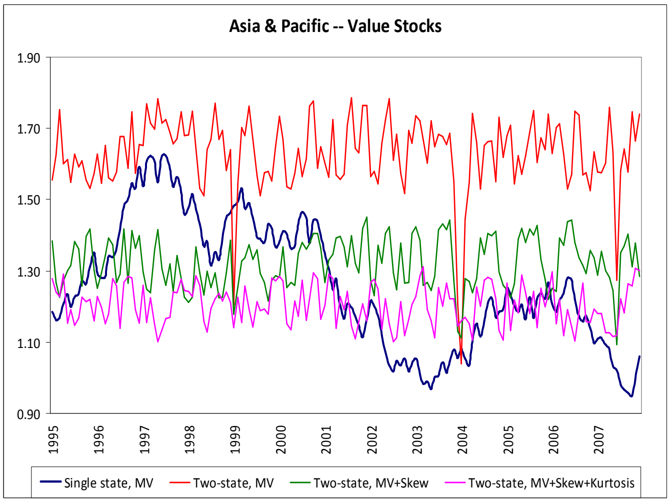


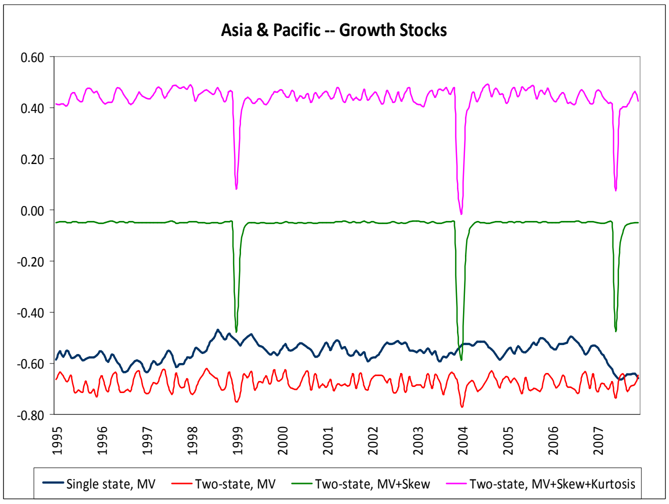


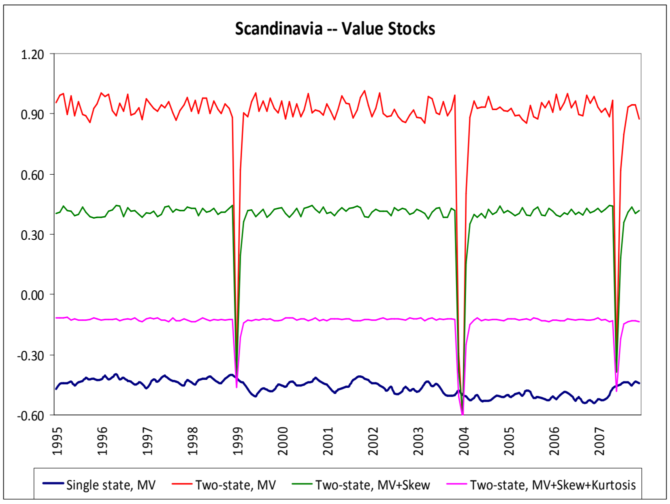


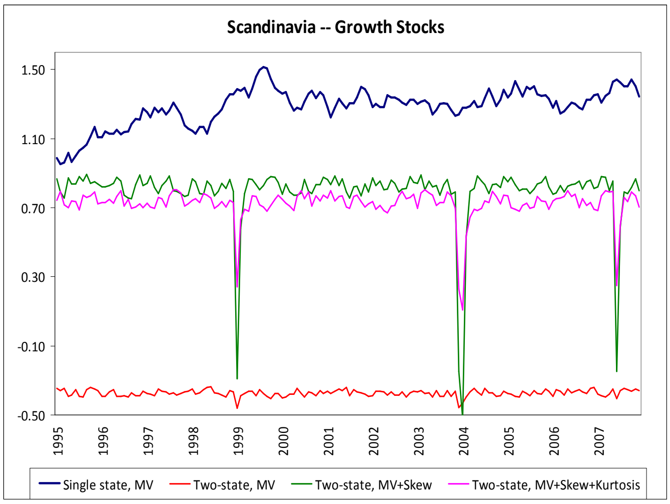


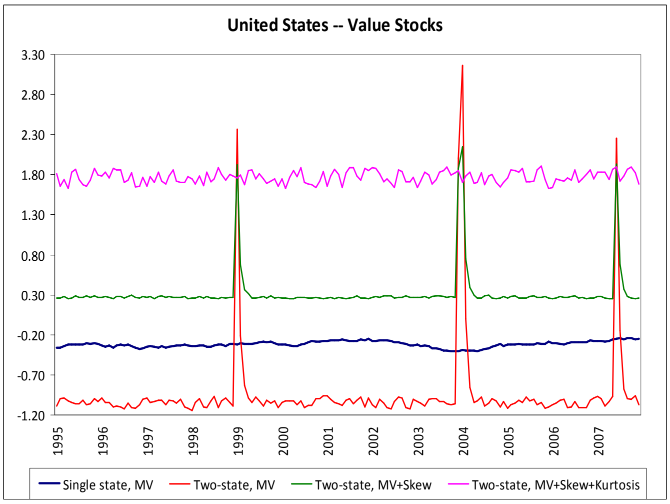


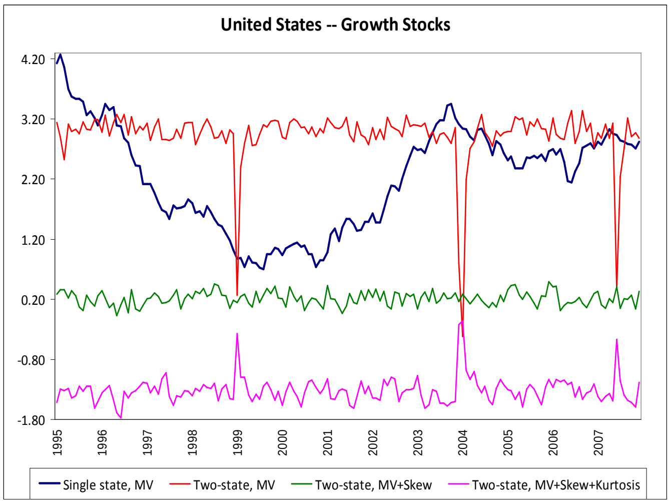

Asia & Pacific -- Value Stocks
1.90
1.70
1.50
1.30
1.10
0.90
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
Single state, MV
Two-state, MV
Two-state, MV+Skew
Two-state, MV+Skew+Kurtosis


Asia & Pacific -- Growth Stocks
0.60
0.40
0.20
0.00
-0.20
-0.40
-0.60
-0.80
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
Single state, MV
Two-state, MV
Two-state, MV+Skew
Two-state, MV+Skew+Kurtosis


Scandinavia -- Value Stocks
1.20
0.90
0.60
0.30
0.00
-0.30
-0.60
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
Single state, MV
Two-state, MV
Two-state, MV+Skew
Two-state, MV+Skew+Kurtosis


Scandinavia -- Growth Stocks
1.50
1.10
0.70
0.30
-0.10
-0.50
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
Single state, MV
Two-state, MV
Two-state, MV+Skew
Two-state, MV+Skew+Kurtosis


United States -- Value Stocks
3.30
2.80
2.30
1.80
1.30
0.80
0.30
-0.20
-0.70
-1.20
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
Single state, MV
Two-state, MV
Two-state, MV+Skew
Two-state, MV+Skew+Kurtosis


United States -- Growth Stocks
4.20
3.20
2.20
1.20
0.20
-0.80
-1.80
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
Single state, MV
Two-state, MV
Two-state, MV+Skew
Two-state, MV+Skew+Kurtosis