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## Shadow Sorting

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# Shadow Sorting

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## Abstract

This paper investigates the border between formal employment, shadow employment, and unemployment in an equilibrium model of the labor market with market frictions. From the labor demand side, firms optimally create legal or shadow employment through a mechanism that is akin to tax evasion. From the labor supply side, heterogeneous workers sort across the two sectors, with high productivity workers entering the legal sector. Such worker sorting appears fully consistent with most empirical evidence on shadow employment. The model sheds also light on the "shadow puzzle", the increasing size of the shadow economy in OECD countries in spite of improvements in technologies detecting tax and social security evasion. Shadow employment is correlated with unemployment, and it is tolerated because the repression of shadow activity increases unemployment. The model implies that shadow wage gaps should be lower in depressed labor markets and that deregulation of labor markets is accompanied by a decline in the average skills of the workforce in both legal and shadow sectors. Based on micro data on two countries with a sizeable shadow economy, Italy and Brazil, we find empirical support to these implications of the model. The paper suggests also that policies aimed at reducing the shadow economy are likely to increase unemployment.

*Key Words: Unemployment, Matching, Shadow Activity.*

*JEL classification: J30*

# 1 Introduction

Modern information technologies allowing to cross-check information coming from different administrative sources and to quickly buildup and update inventories of bank accounts, make it relatively easy to detect and repress shadow activity. However, this is not done and Governments' statements of "tolerance zero" vis-à-vis the informal sector do not seem to be taken too seriously by firms and workers who continue to go underground. Indeed, the informal sector is flourishing: available estimates point to an upward trend in the size of shadow economy in OECD countries from high levels. The shadow share of GDP ranges from a low 10 per cent of GDP in the Nordics, UK and Switzerland to peaks of 20 to 30 per cent in Southern Europe and Ireland and 40 per cent in transitional economies of Eastern Europe and Asia.

Why is the informal sector so much tolerated? How do borders between shadow employment, legal employment and unemployment evolve under different macroeconomic conditions and institutional configurations? What does the reduction of the shadow sector imply in terms of labor productivity?

In this paper we address these issues theoretically and empirically, and we offer a simple explanation of the "shadow puzzle": shadow employment and unemployment are two faces of the same coin. Shadow employment is indeed correlated with unemployment. Based on macro, regional as well as microdata in Italy and Brazil we find clear evidence for this claim. Following this result, we argue that shadow employment is tolerated because its repression increases unemployment, with undesirable political consequences.

Our theory endogenises the choice of both, workers *and* firms, to go idle in an equilibrium model of the labor market with market frictions. From the labor demand side, firms optimally create legal or shadow employment through a mechanism that is akin to tax evasion. Being shadow means not paying taxes (including social security contributions) and not being liable to severance pay in case of a breakup of the employment relationship. However, there is a positive probability that irregular employment is detected, in which case the match is immediately dissolved. From the labor supply side, heterogeneous workers sort across the two sectors, with high productivity workers entering the legal sector. Such worker sorting appears fully consistent with most empirical evidence on shadow employment.

Repressing shadow employment, that is, increasing the detection probability, means increasing

job destruction and reducing job creation in the shadow segment. While this regression tends to increase total employment in the legal sector, it also increases unemployment. Available theories of the informal sector – recently reviewed by Schneider and Enste (2000) – do not capture these trade-offs. This is because such theories take a partial equilibrium approach, focus either on labor demand or on labor supply, and do not consider sorting of workers with varying productivity levels in the two pools. Another distinguishing feature of our model is indeed that it self-selects workers in the two pool endogenously, by determining the productivity threshold demarcating the two pools.

The model implies a positive correlation between unemployment and shadow employment that is evident in cross country data as well as in regional data from Brazil and Italy, two countries with large shadow employment. To ensure that such correlation is not a statistical artifact we use a unique Brazilian data set where unemployment and shadow employment are two mutually exclusive states, and we find strong support for the positive correlation.

The model also implies that shadow wage gaps should be lower in depressed labor markets. We find empirical support also for this implication.

The paper proceeds as follows. Section 1 presents few empirical regularities on shadow employment. Section 2 introduces and solves the model, obtaining the various equilibrium configurations. Section 3 evaluates the comparative statics properties of the equilibria and provides some numerical simulations of the model. Section 4 assesses the empirical relevance of the model, drawing on micro data from two countries with a large shadow pool, namely Brazil and Italy. Finally, Section 5 briefly summarizes and concludes.

## 2 Shadow Facts

The consensus definition of the shadow economy is “all economic activities which contribute to the officially calculated (or observed) gross national product, but escape detection in the official estimates of GDP” [Feige, 1989 and 1994; Lubell 1991 and Schneider 1994]. This definition encompasses not only legal, but also illegal activities, such as trade in stolen goods, drug dealing, gambling, smuggling, etc.. In this paper we confine our attention to a subset of the shadow economy, namely to legal activities. As is apparent from the above, our notion of shadow employment is one of a lawful activity were it reported to tax authorities and subject to work regulations. We focus on this (large) subset of the shadow economy as our aim is to contribute to the literature on

the enforcement of labor regulations and to complement research on tax evasion, which has so far overlooked the effects of tax evasion and shadow employment on unemployment<sup>1</sup>.

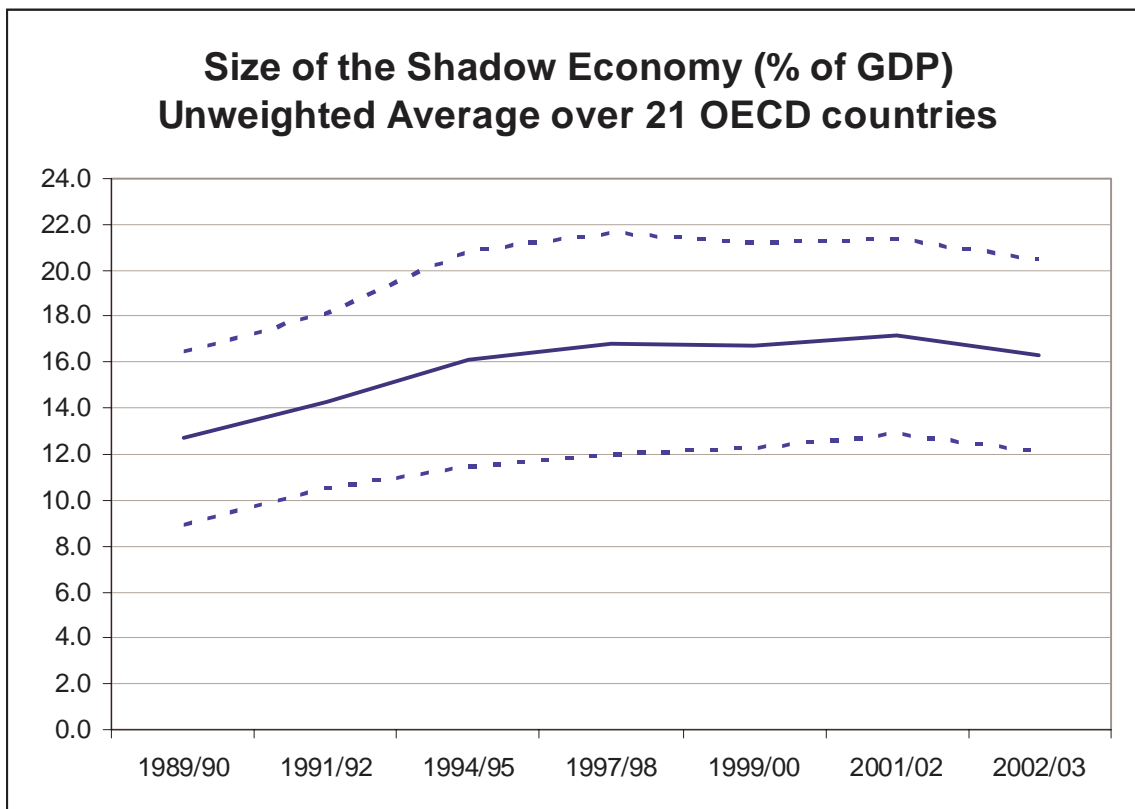
Unfortunately, available estimates of the shadow economy do *not* disentangle legal from illegal shadow economy and rarely provide measures of shadow employment. The methods being used to measure the shadow economy either draw from *direct* inferences, that is surveys trying to elicit involvement of respondents in unregistered activities or estimates based on tax audits, or from *indirect* methods, which basically draw on the inconsistencies between different statistical sources in order to gauge the size of the underground economy. Among the latter methods, discrepancies between national income and expenditure statistics or between physical input (mainly electricity consumption) indicators of economic activity and official GDP statistics or between changes in the volumes of transactions and official GDP-GNP growth or in terms of "excess" currency demand (basically the residuals of a standard currency demand function), are the most frequently used. All the above methods have pros and cons, and the wide variance of estimates being provided is an indication of the limitations of these techniques. With these caveats in mind, let us briefly review the evidence on the size of the shadow economy, as also repeatedly summarized by Schneider (2002,2003,2004).

There are two key findings which are confirmed by all studies we are aware of.

The first common denominator of these "*consensus guesses*" is a marked upward trend in the size of the shadow economy. Figure 1 reproduces the (unweighted) average "shadow share" of GDP in all OECD countries for which estimates, *based on the same methodology*, are available for a relatively long-series. As revealed by the dotted lines (plotting one standard deviation above and below the unweighted cross-country average), there is no sign that this trend has increased the cross-country dispersion in the size of the shadow economy. The coefficient of variation of the shadow shares actually decreased from 1989-2000 to 2002-3 and there is not a single country with a declining shadow share. The upward trend in the shadow share is consistent across methods: it is found to hold not only in estimates based on currency demand, but also on the so-called DYMIMIC method (dynamic multiple indicators multiple causes, Giles, 1999) which estimates a set of structural equations within which the size of the shadow economy cannot be measured directly and then uses this predicted structural dependence in estimating the size of the shadow economy. Also estimates of the shadow economy in terms of headcounts point to an upward trend: Schneider

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<sup>1</sup>See Burdett, Lagos and Wright (2000) for an analysis of the relationship between crime and unemployment.



*Countries:* Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Great Britain, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, USA  
*Sources:* Currency demand approach, Friedrich Schneider

Figure 1: The upward trend of the shadow economy

(2000) estimated that in the European area the number of persons working in the unofficial economy doubled within the two decades from 1978 to 1998.

The second fact is the relatively low productivity of shadow jobs documented by studies relying on micro-level data. In particular, Gonzaga (2003), Hoek (2004), Almeida and Carneira (2005), drawing on data on the informal sector in Brazil, Lacko (2000), Bernabe (2005) and Commander and Rodionova (2005), focusing on transitional economies, as well as Boeri and Garibaldi (2002) and Brandolini and D'Alessio (2004), drawing on Italian data consistently document that workers engaged in shadow employment have, on average, lower educational attainments than regular workers and or hold jobs requiring unskilled workers. The way in which shadow jobs are identified in

these studies may not be neutral with respect to the productivity content of jobs in the two pools. However, the fact that low-skilled workers (or occupations) are represented in shadow employment is consistent across alternative measures of shadow employment.

Table 2 displays the distribution of employment by educational attainment for shadow and non-shadow segments of the labor force in Italy, according to different data sources and definitions. In particular, the top panel draws on Bank of Italy data and identifies shadow employment by looking at self-reported social security records: shadow employees are those who either reported to have never paid social security contributions throughout their career (definition 1) or who report the same number of months of contributions (definition 2) during the same employment spell two years apart (which implies that they have not been paying contributions in between the two interviews)<sup>2</sup>. Clearly definition 1 is more restrictive than definition 2. The mid-panel of Table 2 draws on labor Force Survey data and identifies as shadow employees those individuals who are employed according to internationally agreed, objective, definitions, but who define themselves as non-employed. Finally, the bottom panel draws on data collected by an ad-hoc Istat-Fondazione Curella survey carried out in Sicily in 1995 (Busetta and Giovannini, 1998). In this context, shadow employment is identified in the individuals reporting to hold an irregular job, where irregular means not paying social security contributions, understating the actual pay in order to pay lower taxes and contributions or being altogether without a labor contract.

All data sources and measures of shadow employment suggest that workers with lower educational attainments are over-represented in the shadow pool.

Overall, shadow employment has mainly the characteristics of “marginal shadow employment”, that is, employment in low productivity jobs, rather than “development shadow employment”, i.e., new jobs having the potential to become highly productive after some gestation period. In other words, “infant industry” arguments cannot be applied to justify tolerance vis-à-vis the informal sector. We are looking for deeper and empirically more relevant (“development shadow employment” seems to involve a tiny fraction of unregistered employment) explanations for the weak repression of shadow employment.

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<sup>2</sup>Clearly this second definition requires exploiting the longitudinal features of the Bank of Italy Survey. For a description see Boeri and Brandolini (2004).



**Shadow Employment by Educational Attainment of the Workforce**  
*a) Bank of Italy survey, average 1995-2002*

Education	Shadow (Def.1)	Shadow (Def. 2)	Control (Def.1 and 2)	Shadow (Def. 3)	Control (Def.3)
	$\Delta\text{contrib}=0$	$\Delta\text{contrib}=0 + \Delta\text{contrib} < 0$	$\Delta\text{contrib}=2$	No contribution at all	At least 1 year of contribution
Primary or lower	13.5	14.7	7.5	32.1	30.5
Lower secondary	35.4	33.6	27.8	31.5	27.7
Lower vocational (3 years)	6.8	6.5	9.1	4.0	6.3
Secondary school	33.8	32.0	40.8	23.9	26.4
Tertiary education	10.5	13.1	14.8	8.5	9.1

*b) LFS data, Italy average 1995-2002*

Education	Shadow	Regular employment
Primary or lower	38.4	15.0
Lower secondary	25.6	36.1
Lower vocational (3 years)	4.3	7.8
Secondary school	24.5	29.9
Tertiary education	7.2	11.2

*c) Istat-Fondazione Curella, Sicily 1995*

Education	Main job		Secondary job	
	Shadow	Regular employment	Shadow employment	Regular employment
Primary or lower	24.0	13.5	19.5	8.8
Lower secondary	27.3	26.1	20.7	17.6
Secondary school	40.3	41.9	39.0	44.1
Tertiary education	8.4	18.4	20.7	29.4

Figure 2: Shadow Employment by Educational Attainment of the Labor Force

### 3 A Two Sectors Model with Sorting

#### 3.1 Shadow Employment and Worker's Sorting

We consider an economy with a measure one of heterogeneous workers and two sectors. The worker type is indicated by  $x$ , where  $x$  refers to labor market productivity and its value is drawn from a continuous cumulative distribution function  $F$  with support  $[x_{\min}, x_{\max}]$ .  $x$  is a fixed time invariant worker characteristic, with  $x_{\min} > 0$ .

There are two sectors in the labor market: the regular sector and the shadow sector. The gross value of production of each worker is indicated with  $px$  where  $p$  is a productivity component common to all jobs and  $x$  is an idiosyncratic component. To keep the notation simple, we initially assume that  $p = 1$ , and we consider changes in  $p$  in the numerical simulations. In the regular sector firms pay a production tax  $\tau$  in every period in which they employ a worker. In the shadow sector the tax is evaded and there is an instantaneous monitoring rate equal to  $\rho$ . Conditional on being monitored in the shadow sector, the shadow job is destroyed. Both regular and shadow jobs are exogenously destroyed at rate  $\lambda$ <sup>3</sup>.

Firms can freely post a vacancy in either sector. We focus on single jobs, and each firm is made of one job. Posting a vacancy in the regular sector costs  $k_g$  per period while in the shadow sector costs  $k_b$ . There is free entry of firms in both sectors and the equilibrium value of a vacancy is driven down to zero. Job creation characterizes the labor demand side of the model.

The labor supply is governed by the workers' sorting behavior. Workers are endowed with a unit of time and freely decide whether it is optimal to search and work in the shadow sector or in the legal sector. Entering a sector is a full time activity, and workers can not simultaneously work and/or search in both sectors. In the legal sector there is a specific unemployed income (the unemployment benefits) which is not available in the shadow sector.

Labor markets are imperfect, and there are market frictions in each sector. We follow the main matching literature (Pissarides, 2000) and assume that the meeting of vacant jobs and unemployed workers is regulated by a matching function with constant returns to scale. Different matching functions exist in different sectors. In what follows we let with  $v_g$  and  $v_b$  the number of vacancies in both sectors, and  $u_g$  and  $u_b$  the number of unemployed job seekers. The matching function in

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<sup>3</sup>In the simulations we also assume that conditional on  $\lambda$  striking, regular jobs need to pay a firing tax  $T$ .

each sector is indicated with

$$m^i(u^i, v^i) \quad i = g, b$$

with positive first derivative and negative second derivative. As in the traditional matching models with constant returns to scale, the transition rate depends on the relative number of traders and it is indicated with  $\theta^i = \frac{v^i}{u^i}$ . Specifically, the transition rate for firm is indicated with  $q^i(\theta^i) = \frac{m(u^i, v^i)}{v^i}$  with  $q'(\theta^i) < 0$ , while the transition rate for workers is indicated with  $\alpha'^i(\theta^i) = \theta^i q(\theta^i)$  with  $\alpha' > 0$ .

Successful matches in each sector enjoy a pure economic rent, and we let wages be the outcome of a Nash bargaining problem, with workers getting a fraction  $\beta$  of the total surplus. We assume, for simplicity, that  $\beta$  is identical in the two sectors.

We solve the model in three steps. First we present the value functions and the asset equations, and define the key equilibrium conditions. Next, we solve the workers' sorting behaviour in partial equilibrium, taking as given job creation (the labor demand side of the model) and the transition rate in each market. We then focus on job creation taking worker behaviour as given. Finally we discuss the general equilibrium of the model, and we perform a set of numerical simulations.

### 3.2 Discussion

Before proceeding to the solution of the model, few important issues need to be discussed. Our theory does not deal with the optimal enforcement of legal activity. Within the model, enforcement takes place through the combination of random detection (the monitoring rate  $\rho$ ) and finite punishment (in the form of job destruction). The influential analysis of Becker (1968) has shown that, from the social welfare standpoint, it is always optimal to substitute a higher fine for a lower probability of detection, and that fines should be optimally set at their maximum level. In such optimal enforcement setting, shadow employment would not be observed in equilibrium. While the Becker argument is clear and convincing, we rarely observe such harsh punishment, possibly because important market imperfections reduce the size of the optimal fine. Davidson et al. (2004) have recently shown that with capital market imperfections and/or asymmetric information, the optimal fine lies below the maximum level. Even though we do not explicitly take into account these features, we believe that our realistic enforcement rule can be rationalized in such more complex models, which are nevertheless left to further research.

The difference between legal and shadow jobs considered in the model focuses only on tax compliance, and does not consider the possibility that jobs in the two sectors differ along other

important dimensions, such as capital intensity, health insurance, and firm sponsored training. In reality, workers' sorting decision takes probably into account of various job characteristics, and there is evidence that legal jobs provide more training. We believe that it is technically possible to provide such key extensions, without affecting the main results of the paper.

Our model considers shadow employment as a full time activity and does not allow workers to hold multiple jobs (i.e. a regular job alongside a shadow job). In terms of flows, the model ignores on the job search and direct transitions from shadow to legal employment without intervening unemployment spells. Some of these features were considered by Boeri and Garibaldi (2002) in a matching model with fixed labor supply, without any scope for worker sorting, the key feature of this paper.

### 3.3 Value Functions

The value of a filled job in the legal sector with productivity  $x$  reads

$$rJ^g(x) = x - w^g(x) - \tau + \lambda[V^g - J^g(x)]$$

where  $\tau$  is the tax rate,  $V^g$  is the value of a vacancy and  $r$  is the pure discount rate. Jobs are destroyed at the exogenous rate  $\lambda$ , and  $w_g(x)$  is the wage rate.

Unemployment is a full time activity, and workers can not work in the shadow sector during an unemployment spell. The value of unemployment in the legal sector for a worker of type  $x$  is

$$rU^g(x) = b + \alpha^g(\theta)[W^g(x) - U^g(x)]$$

where  $b$  is the specific unemployed income (the unemployment benefits), and  $W^g(x)$  is the value of the job for a type  $x$ . The value of a job in the legal sector is

$$rW^g(x) = w^g(x) + \lambda[U^g(x) - W^g(x)].$$

Posting vacancies in the legal sector is costly, and yields a per period return equal to  $-k_g$ . Conditional on meeting a worker, at rate  $q^g(\theta^g)$ , the firms gets the expected value of a job. In formula, its expression reads

$$rV = -k_g + q^g(\theta^g) [E[J(z) | z \in \Omega] - V]$$

where the expectation is taken with respect to the productivity of workers that search in the legal sector. The expression  $\Omega$  refers to the support of workers that search in the legal sector.

The value functions for jobs in the shadow sector are similarly defined. The main differences is that in the shadow sectors firms do not pay the production tax  $\tau$  and the job is monitored and destroyed at rate  $\rho$ . Further, there is no specific unemployed income  $b$ . The four value functions read

$$\begin{aligned} rJ^b(x) &= x - w^b(x) + (\lambda + \rho)[V^b - J^b(x)] \\ rW^b(x) &= w^b(x) + (\lambda + \rho)[U^b(x) - W^b(x)] \\ rU^b(x) &= \alpha^b(\theta^b)[W^b(x) - U^b(x)] \\ rV^b &= -k_b + q^b(\theta^b) \left[ E \left[ J^b(z) \mid z \in \Omega^c \right] - V^b \right] \end{aligned}$$

where  $\Omega^c$  is support of workers that search in the shadow sector.

Wages in each sector and in each job are the outcome of a bilateral matching problem and workers get a fraction  $\beta$  of the total surplus so that

$$[W^i(x) - U^i(x)] = \beta[W^i(x) - U^i(x) + J^i(x) - V^i] \quad i = b, g$$

for simplicity we have assumed that the fraction of the surplus is the same in both sectors.

### 3.4 Equilibrium Conditions

There are three key equilibrium conditions

- Free entry and job creation in the legal sector ( $JC^g$ ), which implies that the value of a vacancy be zero

$$V^g = 0$$

This equation will determine market tightness in the legal sector  $\theta^g$

- Free entry and job creation in the shadow sector ( $JC^b$ ), which implies that the value of a vacancy be zero

$$V^b = 0$$

This equation will determine market tightness in the shadow sector  $\theta^b$

- Workers' sorting (*Sort*). If we assume that workers' sorting satisfies the reservation property, (a feature that holds in equilibrium) the labor supply is described by the marginal worker

with productivity  $R$ , where  $R$  is the productivity level for which the worker is indifferent between the two sectors, so that

$$U^g(R) = U^b(R)$$

Using the reservation property, the three key conditions are

$$\alpha^b(\theta^b)[W^b(R) - U^b(R)] = b + \alpha^g(\theta^g)[W^g(R) - U^g(R)] \quad (\text{Sort})$$

$$\frac{k_g}{q^b(\theta^b)} = \frac{\int_R^{x^u} J^g(z) dF(z)}{1 - F(R)} \quad (\text{JC}^g)$$

and

$$\frac{k_b}{q^b(\theta^b)} = \frac{\int_{x_l}^R J^b(z) dF(z)}{F(R)} \quad (\text{JC}^b)$$

The first condition says that the marginal worker is indifferent between searching for a job in the legal or the shadow sector. The second condition says that the total search costs in the legal sector are identical to the expected value of a job. The last condition has a similar interpretation, but refers to the shadow sector. The system determines the three endogenous variables  $\theta^g$ ,  $\theta^b$  and  $R$

### 3.5 Stocks

The model is closed by determining the stock of workers into the four possible labor market states: unemployment and employment in each of the two sectors. If we indicate with  $u^i$  the stock of unemployed in each sector and with  $n^i$  the stock of employed, we have

$$u^g + u^b + n^g + n^b = 1$$

Workers' sorting implies that the share of workers in the shadow sectors is  $F(R)$  while the remaining  $1 - F(R)$  workers search in the legal sector. Employed workers in the shadow sector lose their job at rate  $\lambda + \rho$  while they find jobs at a rate  $\alpha^b(\theta^b)$  so that the balance flow condition for unemployment in the shadow sector is

$$\alpha^b(\theta^b)u^b = (\lambda + \rho)(F(R) - u^b)$$

where  $n^b = F(R) - u^b$ . Unemployment and employment in the shadow sector read respectively

$$\begin{aligned} u^b &= \frac{(\lambda + \rho)F(R)}{\lambda + \rho + \alpha^b(\theta^b)} \\ n^b &= \frac{\alpha^b(\theta^b)F(R)}{\lambda + \rho + \alpha^b(\theta^b)} \end{aligned}$$

In the legal sector, the unemployment and the employment rate are respectively

$$\begin{aligned} u^g &= \frac{\lambda(1 - F(R))}{\lambda + \alpha^b(\theta^b)} \\ n^g &= \frac{\alpha^b(\theta^b)(1 - F(R))}{\lambda + \alpha^b(\theta^b)} \end{aligned}$$

We are now in a position to formally define the equilibrium of the model.

**Definition 1** *Equilibrium.* The equilibrium is obtained by a triple  $R, \theta^g$  and  $\theta^b$  and a vector of stock variables that satisfy the value functions  $J^i, W^i, U^i, V^i$  ( $i = g, b$ ), Nash Bargaining, and i) Workers' sorting, ii) Job Creation in the legal sector, iii) Job Creation in the shadow sector, iv) balance flow conditions.

### 3.6 Solving the worker's sorting behavior

The surplus of a job in each sector is defined as the sum of the worker's and firm value of being on the job, net of the respective outside options, so that

$$S^i(x) = J^i(x) - V^i + W^i(x) - U^i(x)$$

Using the value functions previously defined, as well as the free entry condition (which drives the value of a vacancy down to zero), the surplus of a match for a legal job with productivity  $x$  is

$$(r + \lambda)S^g(x) = x - \tau - b - \alpha^g(\theta^g)[W^g(x) - U^g(x)]$$

Recalling that wages get a fraction  $\beta$  of the total surplus, the previous expression reads

$$S^g(x) = \frac{x - \tau - b}{r + \lambda + \beta\alpha^g(\theta^g)}$$

with  $S^g = \frac{1}{r + \lambda + \beta\alpha^g(\theta^g)}$ . Proceeding similarly, the surplus in the shadow sector is

$$S^b(x) = \frac{x}{r + \lambda + \rho + \beta\alpha^b(\theta^b)}$$

In partial equilibrium, the job finding rates  $a^i$  are constant, and the surplus from the job is an increasing linear function of the match specific productivity  $x$ .

The surplus from the job can be used to obtain an expression for the value of unemployment, whose expression is given by

$$\begin{aligned} U^b(x) &= \frac{\alpha^b(\theta^b)\beta x}{r + \lambda + \rho + \beta\alpha^b(\theta^b)} \\ U^g(x) &= b + \frac{\alpha^g(\theta^g)\beta[x - \tau - b]}{r + \lambda + \beta\alpha^g(\theta^g)} \end{aligned}$$

Figure 3 shows the two value functions in partial equilibrium. The differences in the two curves are driven by the intercept (which is negative in the legal sector) and the slope. We make two key assumptions in this respect:

- **Taxation is large enough relative to unemployment benefits.** We formally assume that  $b(r + \lambda) < \tau\alpha^g\beta$ . This implies that the intercept of  $U^g$  is negative in Figure 3.
- **Monitoring is large enough.** We formally assume that  $\alpha^g\rho\beta + (r + \lambda)\beta(\alpha^g - \alpha^b) > 0$ . This implies that the value function of  $U^g$  is steeper than  $U^b$ .

From the value functions, we can get an expression for the reservation productivity. The reservation value  $R$ , if it exists, is the crossing point of the two lines. Its formal expression, when considering  $\alpha^g$  and  $\alpha^b$  exogenous and constant is

$$R = \frac{[\tau\alpha^g\beta - b(r + \lambda)](r + \lambda + \rho + \beta\alpha^b)}{\alpha^g\rho\beta + (r + \lambda)\beta(\alpha^g - \alpha^b)}$$

Existence in partial equilibrium requires  $R > 0$ , and the two key assumptions above ensure that  $R$  is positive. The equilibrium we are considering implies that shadow jobs are occupied by workers with low skills, in line with the evidence discussed in Section 2 of this paper. This is a key premise of our theoretical analysis.

**Remark 2** *Shadow jobs are occupied by relatively low skilled workers.*

There are several results in the partial equilibrium setting, and are graphically obtained by shifts and movements of the two lines

- An *increase in unemployment benefits* reduces the reservation productivity  $R$ , so that more people search in the legal market. At given job finding rates, an increase in unemployment benefits increases legal employment. This is the standard entitlement effects of unemployment benefits, a labor supply phenomenon that was first noted by Burdett and Mortensen (1982) and Atkinson (1991) and recently received a lot of attention (Fredrikson and Holmlund, 2002; Garibaldi Wasmer, 2005; Boeri, 2000). Formally, it is obtained by noting that

$$\frac{\partial R}{\partial b} = -\frac{(r + \lambda)(r + \lambda + \rho + \beta\alpha^b)}{\alpha^g\rho\beta + (r + \lambda)\beta(\alpha^g - \alpha^b)} < 0$$



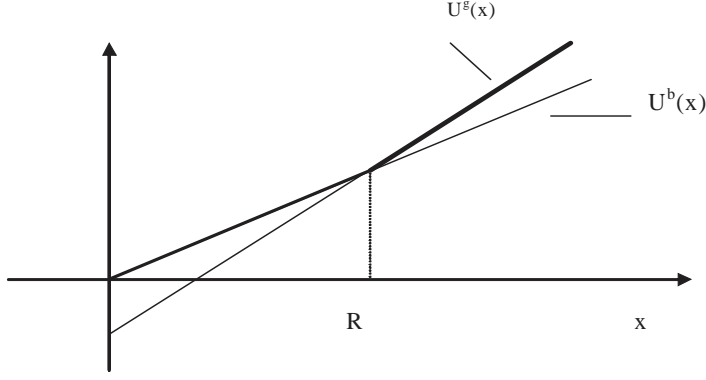


Figure 3: Workers' sorting in partial equilibrium (with constant job finding rate)

- An *increase in taxation* increases shadow employment. This is the standard mechanism that taxation moves away people from the regular sector into the shadow employment, as noted by the work of Schneider (2002) and recently by Davis and Henrekson (2004). Formally, it is obtained by observing that

$$\frac{\partial R}{\partial \tau} = \frac{\alpha^g \beta (r + \lambda + \rho + \beta \alpha^b)}{\alpha^g \rho \beta + (r + \lambda) \beta (\alpha^g - \alpha^b)} > 0$$

- An *increase in the monitoring rate* reduces shadow employment. An increase in the monitoring rate reduces the return from shadow employment and induces people to search in the legal market. Formally, this result is obtained by noting that

$$\frac{\partial R}{\partial \rho} = - \frac{[b(r + \lambda) - \tau \alpha^g \beta] \alpha^b \beta (r + \lambda + \beta \alpha^g)}{[(r + \lambda) \beta (\alpha^b - \alpha^g) - \alpha^g \rho \beta]^2} < 0$$

### 3.7 Labor Demand and Job Creation

To solve for job creation we need to evaluate the expected value of a job. We first focus on legal jobs. After an integration by parts, and making use of the sharing rule, the integral in equation

$JC^g$  can be written as

$$\int_R^{x^u} S(z)dF(z) = S(x^u) - S(R) + (1 - F(R))S(R) - S'(R) \int_R^{x^u} F(z)dz$$

$$\frac{\int_R^{x^u} (1 - F(z))dz}{r + \lambda + \beta\theta^g q(\theta^g)} + \frac{(1 - F(R))[R - \tau - b]}{r + \lambda + \beta\theta^g q(\theta^g)}$$

so that the job creation condition is

$$\frac{k_g[r + \lambda + \beta\alpha^g(\theta^g)]}{q(\theta^g)(1 - \beta)} = \frac{\int_R^{x^u} (1 - F(z))dz}{1 - F(R)} + [R - \tau - b] \quad (1)$$

Proceeding similarly for the expected value of bad jobs, the free entry condition reads

$$\frac{k_b[r + \lambda + \beta\alpha^b(\theta^b)]}{q(\theta^b)(1 - \beta)} = R - \frac{\int_{x^l}^R F(z)dz}{F(R)} \quad (2)$$

Market tightness  $\theta^i$  and the associated job finding rates  $\alpha_i$  depend on the various parameters, as well as on the workers' sorting behavior. Most parameters have a direct effect on job creation, plus an indirect effect via the reservation productivity  $R$ . Formally, we can write

$$\alpha^g(\theta^g) = \alpha^g(R(\cdot), b, r, \lambda, \beta)$$

$$\alpha^b(\theta^b) = \alpha^b(R(\cdot), \rho, \lambda, \beta)$$

where the symbol  $R(\cdot)$  suggests that  $R$  is itself an endogenous variable. Some important comparative static results follows

- An *increase in the reservation productivity  $R$*  increases market tightness and the job finding rates in both sectors. An increase in  $R$  increases the average quality of the workforce in both sectors, so that firms naturally respond by posting more vacancies per unemployed. This result is important, and shows how sorting affects job creation. Formally, it is obtained by noting that  $\frac{\partial\theta^g}{\partial R} > 0$  and  $\frac{\partial\theta^b}{\partial R} > 0$  since

$$\frac{k_g}{(1 - \beta)} \frac{\beta\alpha'^g(\theta^g)q^g(\theta^g) - q'^g(\theta^g)(r + \lambda + \beta\alpha^g(\theta^g))}{q^g(\theta^g)^2} \frac{\partial\theta^g}{\partial R} = \frac{f(R) \int_R^{x^u} F(z)dz}{(1 - F(R))^2}$$

$$\frac{k_b}{(1 - \beta)} \frac{\beta\alpha'^b(\theta^b)q^b(\theta^b) - q'^b(\theta^b)(r + \lambda + \beta\alpha^b(\theta^b))}{q^b(\theta^b)^2} \frac{\partial\theta^b}{\partial R} = \frac{f(R) \int_{x^l}^R F(z)dz}{F(R)^2}$$

where the LHS is positive since  $q' < 0$ .

- An *increase in unemployment benefits  $b$* , at given reservation productivity  $R$ , reduces job creation in the legal sector. This is the standard adverse effect of unemployment income on job creation, an effect that works mainly through the wage rule.

- An increase in taxation, at given reservation productivity  $R$ , reduces job creation in the legal sector. This is also a textbook adverse labor demand effect of taxation
- An increase in the monitoring rate  $\rho$ , at given reservation productivity  $R$ , reduces job creation in the shadow sector. Higher monitoring rate acts as an increase in the destruction rate on shadow jobs.

### 3.8 General Equilibrium

The general equilibrium of the model is obtained by solving for the triple  $R, \theta^g, \theta^b$  that simultaneously satisfy Sort  $JC^b$  and  $JC^g$ . One way to solve for the general equilibrium result is to consider the workers' sorting condition by explicitly considering the relationship between the job finding rates and the reservation productivity. This is equivalent to solving

$$\frac{\alpha^b(R, \cdot)\beta R}{r + \lambda + \rho + \beta\alpha^b(R, \cdot)} = b + \frac{\alpha^g(R, \cdot)\beta[R - \tau - b]}{r + \lambda + \beta\alpha^g(R, \cdot)} \quad (3)$$

where the expression  $\alpha^b(R, \cdot)$  and  $\alpha^g(R, \cdot)$  are consistent with the job creation conditions. Both sides of the expression are increasing functions of  $R$ . The difference with respect to the partial equilibrium result is that the expressions for the value of unemployment in equations (3) are no longer simple linear function, but they are both increasing functions of  $R$ . To understand this, consider the effects of an increase in  $R$  on the value of unemployment in both sectors, there are two effects at work.

- First, there is a positive *surplus effect*. This is analogous to the effect analysed in partial equilibrium. An increase in  $R$  increases the value of unemployment in both sectors, but has a larger effect on the legal sector in light of the difference in the slope and the presence of  $\rho$  in the shadow sector.
- Second, there is a *job creation effect*. An increase in  $R$  increases the job finding rate in both sector, since the average value of the workforce increases.

As both effects reinforce each other in a non linear fashion, multiple equilibria can not be ruled out ex-ante. This should not be surprising, since multiple equilibria in matching models with double heterogeneity are a standard feature (Albreth and Vroman, 2002).

**Remark 3** *Multiple equilibria can not be ruled out, and depend on the distribution of productivity.*

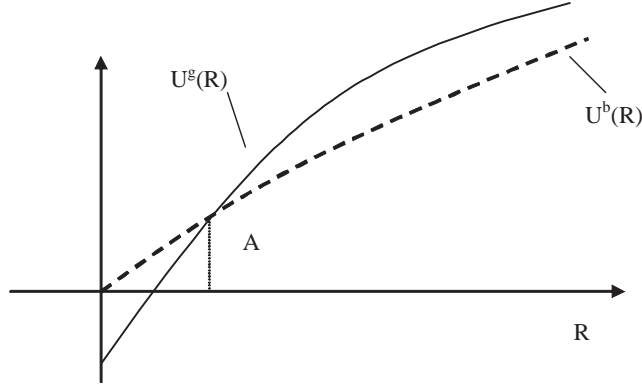


Figure 4: The General Equilibrium

Since both sides are increasing and non linear functions of  $R$ , there is no guarantee that the equilibrium is unique.

In the simulations that follow, where we use a distribution for the productivity  $x$  that is negative exponential, there is a unique equilibrium. In any case, if there were two equilibria, there would be different implications for the distribution of skills across the two sectors, with a perverse equilibrium that implies that high productivity workers enter the shadow sector. In figure 4, the equilibrium of point  $A$  is consistent with the skilled distribution that we highlighted in the comparative static section. The feature of such an equilibrium can be described as follows

$$\begin{aligned}
 U^g(R^*) &= U^b(R^*) \\
 U'^g(R^*) &> U'^b(R^*)
 \end{aligned}$$

where the second condition ensures that the value function of the legal sector is the steepest one in the equilibrium point.

## 4 Simulations and Comparative Static

The comparative static results in the general equilibrium are not straightforward, since they combine the effect of each parameter on the labor demand and the labor supply of the model.

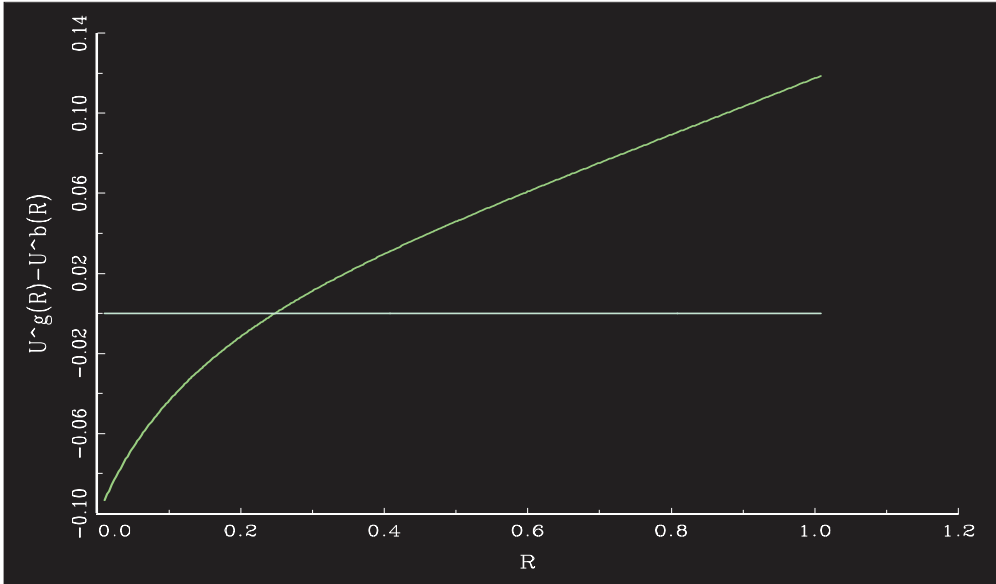


Figure 5: General Equilibrium with Exponential Distribution

Consider the effects of taxation. An increase in taxes tend to push jobs into the shadow sector, and to decrease the value of each job. This is a standard result that reduces job creation. Yet, the resulting increase in  $R$  improves the average quality of the workforce in the legal sector, with a positive effect on job creation. As a result, the total effect on job creation may be ambiguous.

Consider an increase in the monitoring rate. On the one hand, it reduces  $R$  from the labor supply standpoint and reduces job creation in the shadow sector. Both effects reinforce each other, and tend to reduce  $R$ . On the other hand, the reduction in  $R$ , by increasing the average productivity of workers in the legal sector, feeds back on job creation in the legal sector, and tends to reduce  $R$ . This suggests that an increase in the monitoring rate can reduce job creation in the good sector.

Similar logical arguments follow for the other comparative static exercise. The increase in unemployment benefits reduces (in partial equilibrium) the number of people in the shadow sector by reducing  $R$ . The fall in  $R$  induces a feed back effect on the average quality of the workforce in the legal sector and, from the labor demand side, a reduction in job creation.

#### 4.0.1 Baseline Specification

The baseline specification of the model is described in Table 1. With respect to the model presented in the equations, the empirical specification of the productivity is  $px$ , where  $x$  is the idiosyncratic

component of productivity and  $p$  is an aggregate component. Further, in addition to a production tax  $\tau$ , the simulations consider also a firing tax  $T$ , to be paid only in the legal sector conditionally on a job separation (when the shock  $\lambda$  strikes).

The distribution is negative exponential. Figure 5 reports the difference between  $U^g(R) - U^b(R)$  for different values of the reservation productivity. The general equilibrium is described by the crossing of such difference with the zero line. The figure clearly shows that there is a single crossing and that the equilibrium is unique. The baseline parameterization is described and reported in Table 1. Most parameters are standard in the literature (notably a 0.5 value for the bargaining share and the matching elasticity). The search costs correspond to 25 percent of the value of the labor product, a value that is roughly consistent with the structural estimates provided by Yashiv (2000).

The shadow rate, defined as the ratio between employment in the shadow sector and total employment (including both  $n^g$  and  $n^b$  at the denominator) is around 14 percent. We perform various comparative static exercises

Table 1: Calibration

<i>Parameters</i>	Notation	Legal	Shadow
Discount Rate	$r$	0.03	
Separation Rate	$\lambda$	0.15	0.15
Unemployed Income	$b$	0.10	0.00
Firing Tax	$F$	0.10	0.00
Matching Elasticity	$\eta^i$	0.50	0.50
Monitoring Rate	$\rho$	0.00	0.06
Production Tax	$\tau$	0.20	0.00
Matching Function Constant	$A^i$	0.50	0.50
Workers' Surplus Share	$\beta$	0.50	0.50
Common Productivity	$p$	1.50	1.50
Search Costs	$k^i$	0.40	0.40
<i>Equilibrium Values</i>			
Sorting Productivity	$R$	0.24	
Market Tightness	$\theta^i$	2.70	0.16
Job Finding Rate	$\alpha^i$	0.82	0.28
<i>Aggregate Statistics</i>			
Unemployment	$u^i$	12.10	7.52
Employment	$n^i$	66.23	14.15
Shadow Rate	$s$	17.60	
Average Wage	$w^i$	1.37	0.12
(a), Distribution is Exponential with parameter $B = 1.00$			
<i>Source: Authors' calculation</i>			

#### 4.0.2 Changes in Aggregate Conditions

We study the effects of the increase in  $p$  on the general equilibrium of the model. The results are reported in Table 2. With the exception of  $p$ , all the other parameters are identical to those of Table 1.

An increase in aggregate productivity increases employment and reduces unemployment in the legal sector. Further, it reduces employment in the shadow sector. This is one of the key macroeconomic results of the paper. Unemployment and shadow employment are positively correlated across different states of the macroeconomy

- **Remark 4** *Unemployment and shadow employment are two faces of the same coin. Worse aggregate conditions induce an increase in both unemployment and shadow employment (as well as its shadow rate)*

Table 2: Changes in Aggregate Conditions

$p$	$R$	$\theta_b$	$\theta_g$	$u_b$	$u_g$	$n_b$	$n_g$	$s$	$x_g$	$x_b$	$\bar{w}_g$	$\bar{w}_b$
1.50	0.24	0.16	2.70	7.52	12.10	14.15	66.23	17.60	1.24	0.12	1.37	0.12
1.61	0.23	0.16	2.92	7.03	11.90	13.22	67.85	16.31	1.23	0.11	1.46	0.12
1.73	0.21	0.16	3.15	6.60	11.70	12.40	69.30	15.18	1.21	0.10	1.55	0.12
1.84	0.20	0.16	3.39	6.22	11.51	11.68	70.60	14.19	1.20	0.10	1.64	0.12
1.95	0.19	0.16	3.62	5.88	11.32	11.03	71.77	13.33	1.19	0.09	1.73	0.12

$u_g$  and  $u_b$  are the unemployment rates respectively in the legal and shadow sector

$n_g, E_{n_b}$ , are respectively legal and shadow employment.

$x_g$  and  $x_b$  are the average idiosyncratic productivity in the legal and shadow employment

$\bar{w}_g$  and  $\bar{w}_b$  are the average wages legal and shadow employment

*Source:* Authors' calculation

The logic of this result can be expressed as follows. The increase in  $p$  tends to increase job creation and market tightness. Simultaneously, the increase in  $p$  induces a fall in the marginal productivity  $R$ , so that average quality worsens in both sectors. This tends to reduce job creation. The second effect appears to be quantitatively more important in the legal sector, since the productivity is proportional to  $x$ .

Table 2 shows that wage differentials between the legal and the shadow sector (the shadow wage gap) are quantitatively more important when aggregate business conditions are good.

- **Remark 5** *Wage differentials should be larger in less depressed regions.*

There are two adjustment mechanisms behind this result. First, a larger  $p$  directly affects match productivity inducing an increase in wages per any given  $x$ . Second, the rise in aggregate productivity involves a reduction of the productivity threshold so that the average quality of matches in both sectors decline. This tend to depress average wages in both sectors. As the aggregate shock is multiplicative, its direct (positive) effects on wages are quantitatively more important in the legal sector than in the shadow sector, whilst the indirect effects are nearly symmetric due to the common threshold,  $R$ .



Table 3: Changes in Total Taxation Conditions

$\tau$	$R$	$\theta_b$	$\theta_g$	$u_b$	$u_g$	$n_b$	$n_g$	$s$	$x_g$	$x_b$	$\bar{w}_g$	$\bar{w}_b$
0.200	0.24	0.16	2.70	7.52	12.10	14.15	66.23	17.60	1.24	0.12	1.366	0.120
0.205	0.26	0.18	2.73	7.70	11.87	15.08	65.35	18.75	1.26	0.12	1.380	0.128
0.210	0.27	0.19	2.77	7.88	11.62	16.05	64.44	19.94	1.27	0.13	1.395	0.136
0.215	0.29	0.21	2.80	8.06	11.38	17.06	63.50	21.18	1.29	0.14	1.411	0.144
0.220	0.31	0.22	2.84	8.24	11.12	18.10	62.53	22.45	1.31	0.15	1.427	0.153

$u_g$  and  $u_b$  are the unemployment rates respectively in the legal and shadow sector  
 $n_g, E_{n_b}$ , are respectively legal and shadow employment.  
 $\bar{w}_g$  and  $\bar{w}_b$  are the average wages legal and shadow employment  
*Source:* Authors' calculation

Table 4: Changes in Firing Taxes

$T$	$R$	$\theta_b$	$\theta_g$	$u_b$	$u_g$	$n_b$	$n_g$	$s$	$x_g$	$x_b$	$\bar{w}_g$	$\bar{w}_b$
0.100	0.24	0.16	2.70	7.52	12.10	14.15	66.23	17.60	1.24	0.12	1.366	0.120
0.113	0.25	0.17	2.71	7.59	12.01	14.50	65.90	18.03	1.25	0.12	1.372	0.123
0.125	0.25	0.17	2.72	7.66	11.92	14.85	65.57	18.46	1.25	0.12	1.378	0.126
0.138	0.26	0.18	2.73	7.72	11.84	15.20	65.24	18.90	1.26	0.12	1.384	0.129
0.150	0.27	0.18	2.75	7.79	11.75	15.56	64.90	19.34	1.27	0.13	1.390	0.132

$u_g$  and  $u_b$  are the unemployment rates respectively in the legal and shadow sector  
 $n_g, E_{n_b}$ , are respectively legal and shadow employment.  
 $\bar{w}_g$  and  $\bar{w}_b$  are the average wages legal and shadow employment  
*Source:* Authors' calculation

### 4.0.3 Changes in Taxation and Regulations

We study the effects of the increase in  $\tau$  on the general equilibrium of the model. The results are reported in Table 3. All the other parameters are identical to those of Table 1. More taxes and regulations increase shadow employment and reduce legal employment. This is the standard result of Schneider (2002). It is also consistent with the work of Davis and Henrekson (2005).

The effect of taxation on unemployment is quantitatively very modest, since there are two countervailing effects at work. There is the indirect effect on job creation via the increase in the reservation productivity (reducing unemployment) plus the direct effect of taxes on market tightness in the legal sector (increasing unemployment).

Changes in regulation (through the firing tax) are qualitatively analogous to the effects of taxation.

Table 5: Changes in Monitoring Intensity

$\rho$	$R$	$\theta_b$	$\theta_g$	$u_b$	$u_g$	$n_b$	$n_g$	$s$	$x_g$	$x_b$	$\bar{w}_g$	$\bar{w}_b$
0.06	0.24	0.16	2.70	7.52	12.10	14.15	66.23	17.60	1.24	0.12	1.37	0.12
0.07	0.22	0.13	2.62	7.31	12.53	12.48	67.67	15.57	1.22	0.11	1.33	0.11
0.09	0.20	0.11	2.57	7.17	12.86	11.24	68.73	14.05	1.20	0.10	1.31	0.10
0.10	0.19	0.10	2.53	7.08	13.11	10.27	69.54	12.87	1.19	0.09	1.29	0.09
0.11	0.18	0.08	2.50	7.01	13.31	9.50	70.18	11.92	1.18	0.09	1.28	0.08

$u_g$  and  $u_b$  are the unemployment rates respectively in the legal and shadow sector

$n_g, E_{n_b}$ , are respectively legal and shadow employment.

$\bar{w}_g$  and  $\bar{w}_b$  are the average wages legal and shadow employment

*Source:* Authors' calculation

#### 4.0.4 Changes in the Monitoring Rate

We study the effects of the increase in  $\rho$  on the general equilibrium of the model. The results are reported in Table 5. An increase in monitoring intensity reduces the shadow rate, but it increases unemployment

We view this result as extremely important, since it highlights one of the key reasons why governments may be reluctant to repress the shadow sector. The associated increase in unemployment is politically costly and thus avoided by utility maximizing politicians.

#### 4.0.5 Changes in unemployed income

We now consider the effects of an increase in  $b$ . An increase in unemployed income reduces the shadow rate, and increases unemployment. Yet, the increase in participation in the legal sector increases legal employment and reduces shadow employment. Note that market tightness falls in both sectors.

The increase in unemployed income can be considered as a policy for uncovering (as opposed to repression) shadow activities. Various difficulties are likely to exist in reality in enforcing this policy (unemployment income requires larger taxation and very good monitoring). Yet, it can be quite effective.

Table 6: Changes in Unemployed Income

$b$	$R$	$\theta_b$	$\theta_g$	$u_b$	$u_g$	$n_b$	$n_g$	$s$	$x_g$	$x_b$	$\bar{w}_g$	$\bar{w}_b$
0.100	0.24	0.16	2.70	7.52	12.10	14.15	66.23	17.60	1.24	0.12	1.366	0.120
0.104	0.24	0.16	2.67	7.46	12.20	13.85	66.49	17.24	1.24	0.12	1.360	0.118
0.108	0.24	0.15	2.65	7.40	12.29	13.56	66.75	16.88	1.24	0.11	1.355	0.116
0.111	0.23	0.15	2.63	7.34	12.39	13.27	67.01	16.53	1.23	0.11	1.349	0.113
0.115	0.23	0.15	2.61	7.27	12.49	12.97	67.27	16.17	1.23	0.11	1.343	0.111

$u_g$  and  $u_b$  are the unemployment rates respectively in the legal and shadow sector  
 $n_g, E_{n_b}$ , are respectively legal and shadow employment.  
 $\bar{w}_g$  and  $\bar{w}_b$  are the average wages legal and shadow employment  
*Source:* Authors' calculation

## 5 Empirical Relevance

Our model implies: i) a positive cross-sectional and time-series correlation between the size of the shadow sector and unemployment (the two phenomena are just two faces of the same coin), ii) a "shadow wage gap" that is larger in countries-regions and years in which unemployment is lower, iii) a shadow employment that is increasing in taxation and labor market regulations, and iv) that tighter monitoring increases unemployment. From a political economy perspective, the latter result implies a lax enforcement of regulations in high-unemployment regions.

The purpose of this section is to evaluate the empirical relevance of i), ii) and iv). Implication iii) is common to other models of the shadow economy and holds in many cross-sectional studies, as reviewed by Schneider (2002).

### 5.1 Two faces of the same coin?

Figure 6 documents the correlation between the size of the shadow economy and the non-employment rate across countries and Figure 7 across Italian regions, in both cases over average period data. In particular, Figure 6 displays, on the vertical axis, the cross-country comparable estimates of the shadow economy over GDP provided by Schneider (2004) and, on the horizontal axis non-employment rates (unemployed and inactive as a fraction of the working age population) obtained from harmonised Labour Force Survey (LFS) data. Regional non-employment rates are also obtained from the (Italian) LFS, while the regional estimates of shadow employment are drawn from Istat. The latter are provided in terms of full-time equivalents (ULA, "unità di lavoro equivalenti") and are estimated building on the difference between survey-based employment and

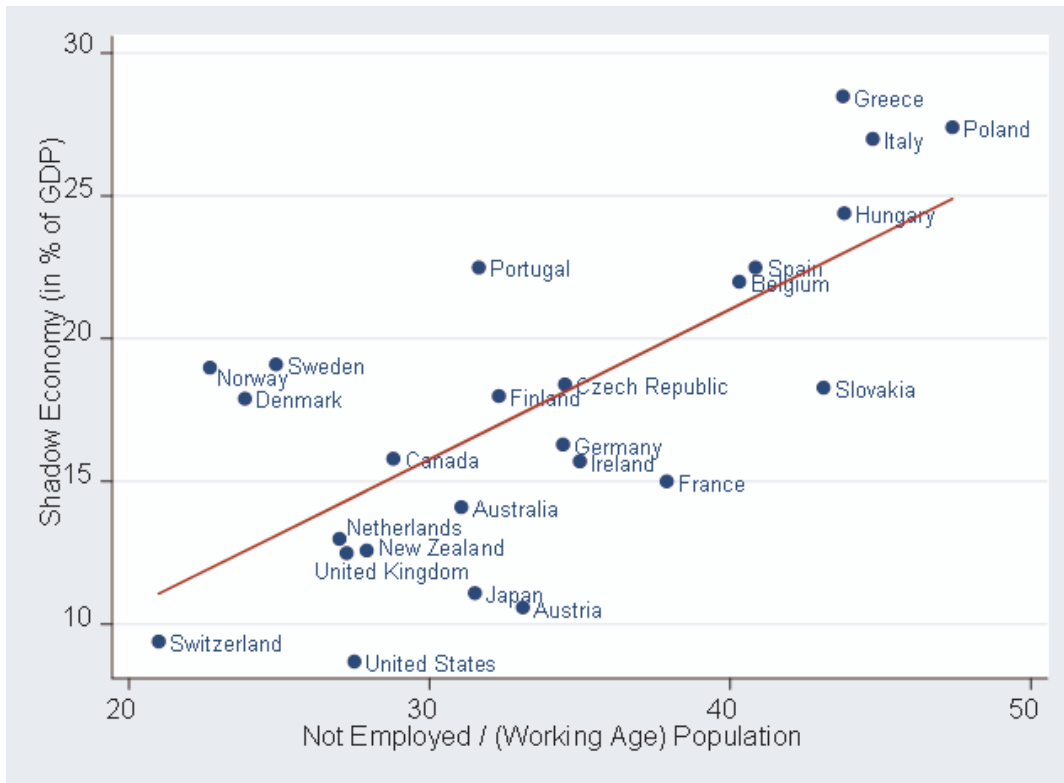


Figure 6: The Size of the Shadow Economy and Non Employment

employment levels, as computed on the basis of administrative (social security records) as well as estimates of illegal employment of foreign workers<sup>4</sup>.

The correlation is striking in both cases: the cross-country correlation is .7 with a t-statistics of 4.76; the cross-regional correlation is .94 with a t-statistics of 11.79. It holds also when shadow employment is broken down by broad sectors, e.g., it is not a byproduct of the specialisation of Southern regions in sectors (e.g., agriculture) where shadow employment is larger. There is also no tendency over time to a reduction in regional differentials in shadow rates: they were in 1995 roughly as large as 10 years earlier.

Unfortunately, there are no long series of shadow employment and unemployment enabling to assess their pairwise correlation over time. Figure 8 hints at co-movements between the shadow rate and unemployment in Italy. The shadow rate initially rose with unemployment and then, more recently declined together with unemployment.

<sup>4</sup>See Calzaroni and Pascarella (1998) for details on the estimates of shadow employment in Italian macro-regions.

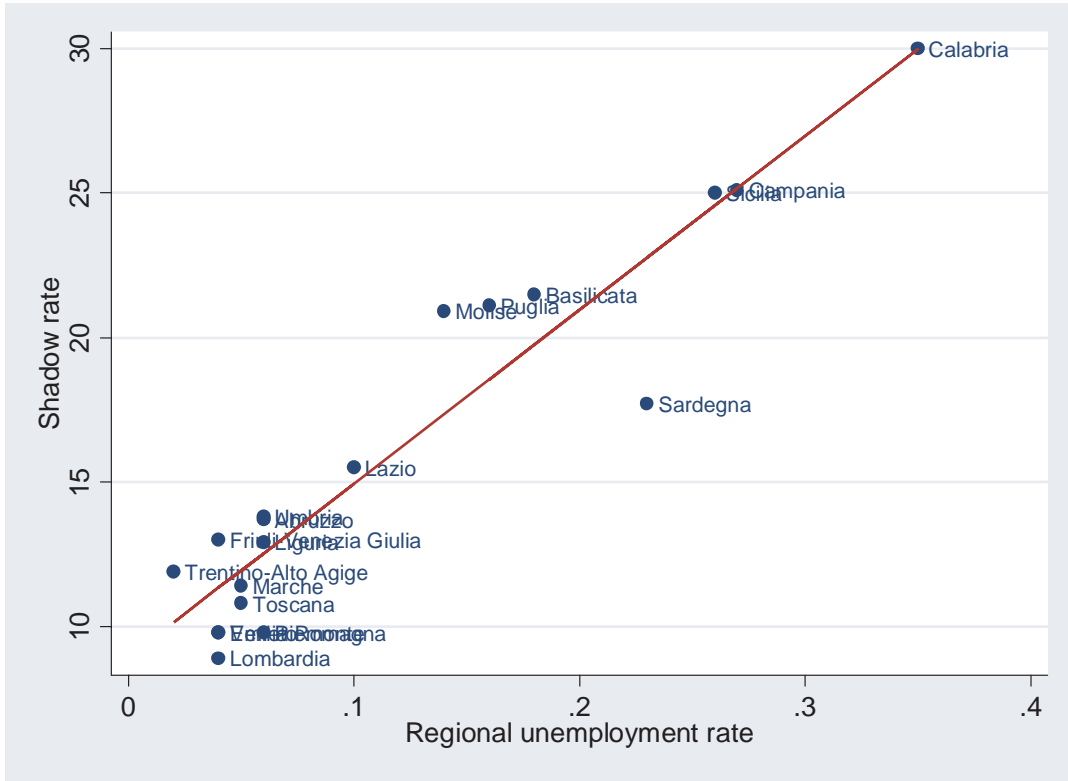


Figure 7: Shadow employment and unemployment across Italian regions

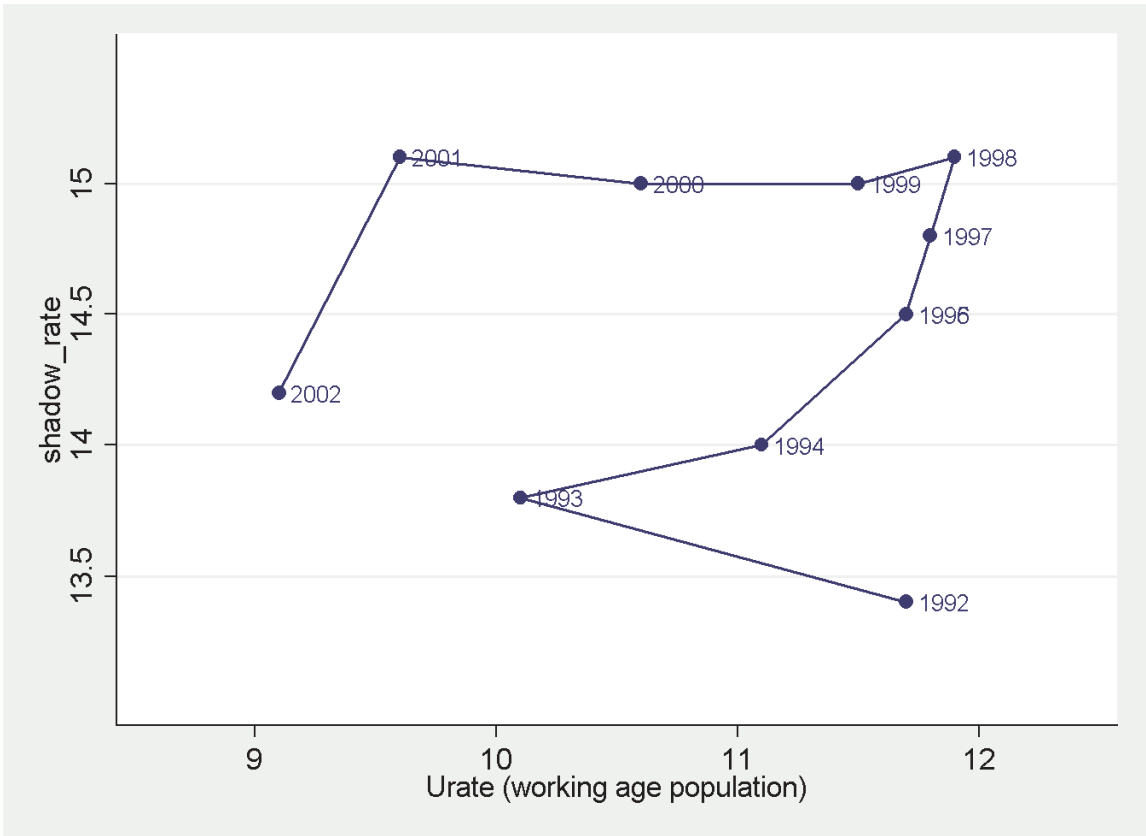


Figure 8: Unemployment and Shadow Employment Over time in Italy

All these correlations are consistent with the implications of our model and can be rationalized by the fact that positive macroeconomic shocks or greater efficiency in a region increases job creation and reduces the reservation productivity level at which jobs turn into formal jobs. However, given the size and statistical significance of correlations, one may think that they are a mere statistical artifact, related to the way in which the two measures are defined. As discussed in the Annex, a spurious correlation may be induced between shadow employment and the unemployment rate, when shadow employment is wrongly classified as unemployment by Labor Force Statistics. The large unemployment rates observed also among prime-age men in Southern Italian regions suggest that LFS data may indeed mis-classify jobs in the shadow sector. Unfortunately, estimates of the shadow economy generally come from statistical sources which are silent on labor market aggregates. When LFS data are used to measure shadow employment (e.g., as done in Table 2), they either just scrap the surface of the phenomenon (the number are too small to achieve regional representation) or concentrate only on the subset of shadow employment which is not mis-classified by LFS statistics. Hence, there is no way to map shadow employment into the different LFS aggregates.

An important exception is the PME (Monthly Employment) survey carried out in six Brazilian metropolitan areas since 1982. The survey design is similar to the CPS in the US and includes a question on the payment of social security contributions. Following Almeida and Carneiro (2005), Gonzaga (2003) and Hoeke (2005), we identify shadow workers as those individuals reporting to work but stating that they do not have a social security card. It is a relatively large component of the labor force: the shadow rate can be as high as roughly 1/3. By construction, these shadow workers cannot be classified as unemployed. Figure 9 displays the yearly shadow and unemployment rates in six Brazilian metropolitan areas since the inception of the survey. There is a remarkable positive correlation (ranging from .31 in Rio to .82 in Salvador with t-statistics in the range 3.4 to 6.1). This correlation cannot be a statistical artifact, and provides genuine evidence of our empirical implications.

## 5.2 The Shadow Wage Gap

Our model predicts that improvements in aggregate conditions increase the shadow wage gap.

Table 10 displays the shadow wage gap and a simple Oaxaca decomposition of this gap in Italy over time and across two macro-regions characterised by very different aggregate condition, such



Figure 9: Shadow employment and unemployment in six Brazilian cities

as the North and the Mezzogiorno. In particular, drawing on the Bank of Italy SHIW we run two standard wage regressions for the legal and the shadow sector (individuals stating that they are working but they never paid social security contributions)

$$\bar{w}^g = \bar{X}^g \beta^g$$

and

$$\bar{w}^b = \bar{X}^b \beta^b$$

where  $\bar{X}^i$  denotes average "personal-demographic" characteristics (educational attainments, gender, age, family status, etc.) of sector "i" and  $\beta^i$  the returns to these characteristics. Then we can decompose the shadow wage gap as the sum of a difference in quantities (explained part) and differences in returns (unexplained part), e.g.:

$$\bar{w}^g - \bar{w}^b = (\bar{X}^g - \bar{X}^b) \frac{1}{2} (\beta^g + \beta^b) + (\beta^g - \beta^b) \frac{1}{2} (\bar{X}^g + \bar{X}^b) \quad (4)$$

An advantage of this decomposition is that it isolates the component which drives the changes



<i>Oaxaca Decomposition of the Shadow Wage Gap</i>				
		Shadow wage gap	Explained	Unexplained
<i>All sample</i>	1995	0.94	0.24	0.70
	1998	0.79	0.40	0.39
	2000	0.92	0.26	0.66
	2002	1.04	0.23	0.81
<i>North</i>	all years	0.95	0.30	0.65
<i>South</i>	all years	0.78	0.31	0.48

Notes: Controls include age,gender,family status and educational attainments

Source: Bank of Italy SHIW various years

Figure 10:

in the shadow wage gap according to our model: it is the unexplained (or difference in returns) component, that is, the second term in equation (4). The decomposition is akin to the partial equilibrium comparative statics exercise above, in that it assumes that differences in returns are uncorrelated with changes in the characteristics of the two pools. It should be interpreted as an approximation of the first-round effects of changes in the aggregate shock. Our exercise suggests that the shadow gap has been widening since 1998, at times in which unemployment was declining, and that it is larger in the dynamic North than in the depressed Southern labor markets. The key factor behind these differences is the unexplained (returns) component of the gap.

Hoeke (2005) also reports an increase in the shadow wage gap in Brazil during cyclical upturns.

### 5.3 Enforcement

Modern information technologies allow tax administrations to easily collect and cross-check information from a variety of source. For instance, the Spanish tax administration built-up an inventory of bank accounts which is particularly useful in tracking the shadow sector. The Italian "Agenzie delle Entrate" is developing an inventory of electricity, gas, telephone and water bills of contributors, which can be readily cross-checked with tax records.

There are plenty of anecdotes about poor enforcement in high-unemployment regions, although it is very hard to document this. There are documents of the Italian Agenzia delle Entrate stating that enforcement should be milder in small units and in agriculture, where shadow employment is over-represented. Almeida et al. (2005) report a negative correlation between unemployment and worksite inspections in Brazil. Broadly similar is the conclusions of the Osservatorio Veneto, al-

though shadow employment in Veneto is very much related to immigration. A negative relationship between shadow employment and monitoring is driven in our model by the effects of controls on job creation in the shadow sector. But there can also be political economy argument for observing less repression of the shadow sector in high unemployment regions.

## 6 Final Remarks

An equilibrium search model of the labor market, with workers' sorting, contributes to explain the "shadow puzzle", the increasing size of the shadow economy in OECD countries in spite of improvements in technologies detecting tax and social security evasion. Our model has implications which are broadly supported by the, admittedly scant, evidence on shadow labor markets. In particular, we consistently find a positive cross-sectional and time-series correlation between the shadow rate and unemployment, and this correlation cannot be attributed to a statistical artifact.

Our model delivers also some policy implications. The most important is quite simple: *in order to reduce shadow employment, it is necessary to deregulate the labor market. Deregulation reduces unemployment, and shadow employment is reduced as a by-product.* In this context, the model confirms the traditional wisdom on labor market reforms, and suggests that any policy that fosters job creation and enhances aggregate productivity will induce a reduction in shadow employment. What about specific policies, aimed at discouraging the emergence of shadow activity? Our simple theory suggests that a very cautious approach in this area is warranted, since an increase in the monitoring rate may backfire: in equilibrium, higher monitoring reduces job creation, and increase unemployment. Tight enforcement of entitlement rules to unemployment benefits can be a better option acting on the supply side (when unemployment benefits are collected only by workers with a regular employment history, and cannot be cumulated to income from shadow jobs, the workers' incentive to enter the shadow sector are reduced) and hence has better job creation properties.

In further work we plan to investigate combinations of shadow and regular jobs, both in labor demand and supply. Although this extension will significantly increase the complexity of our model, we are aware that the choice to go shadow is not merely a dichotomic choice. Multiple job holding allows workers, for instance, to allocate hours across the two sectors. And firms can react to idiosyncratic productivity shocks by crossing borders between shadow and regular jobs.

## 7 References

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## 7.1 Annex

### 7.2 A statistical artifact?

According to the labor force statistics, the working age population is classified as  $E^{lf}$ ,  $U^{lf}$ , and  $N^{lf}$  where the values refer respectively to labor force employment, unemployment and out of the labor force. If the labor force is indicated with  $wap$  the function reads

$$E^{lf} + U^{lf} + N^{lf} = wap$$

The unemployment rate is than defined as

$$u^{lf} = \frac{U^{lf}}{E^{lf} + U^{lf}}$$

The official istat definition of the shadow rate,  $s$ , is given by the estimate of shadow employment (lavoro irregolare) over the sum of regular employment  $E^r$ , and shadow employment  $E^s$

$$s = \frac{E^r}{E^s + E^r}$$

The key issue concerns the relationship between  $E^s$  and  $E^{lf}$  or whether shadow employment is part of the labor force employment. The answer depends on various assumptions regarding the position of shadow employment in the labor force statistics

**Assumption 1: shadow employment within the employment measured in the labor force surveys.**

This implies that

$$E^{lf} = E^s + E^r$$

Therefore

$$u^{lf} = \frac{U^{lf}}{E^s + E^r + U^{lf}}$$

from which it follows that

$$\begin{aligned} \frac{\partial u^{lf}}{\partial E^s} &< 0 \\ \frac{\partial s}{\partial E^s} &> 0 \end{aligned}$$

In other words, an increase in shadow employment  $E^s$  leads to an increase in the shadow rate and to a decrease in the unemployment rate. The empirical correlation, in this case is *not* a statistical artifact

**Remark 6** *If shadow employment is part of labor force employment, the correlation between  $s$  and  $u$  is not a statistical artifact*

**Assumption 2: shadow employment is within the out of the labor force measured in the labor force surveys.**

This implies that

$$N^{lf} = \tilde{N} + E^s$$

where  $\tilde{N}$  is a pure measure of out of the labor force (not observed in labor force statistics)

Therefore

$$u^{lf} = \frac{U^{lf}}{E^{lf} + U^{lf}}$$

from which it follows that

$$\begin{aligned} \frac{\partial u^{lf}}{\partial E^s} &= 0 \\ \frac{\partial s}{\partial E^s} &> 0 \end{aligned}$$

In other words, an increase in shadow employment leads to an increase in the shadow rate and has no impact on the unemployment rate. Also in this case, the empirical correlation is *not* a statistical artifact.

**Remark 7** *If shadow employment is part of out of the labor force in labor force surveys, the correlation between  $s$  and  $u$  is not a statistical artifact*

**Assumption 3: shadow employment is within unemployment measured in labor force surveys**

This implies that

$$U^{lf} = \tilde{U} + E^s$$

where  $\tilde{U}$  is a pure unemployment rate while  $E^s$  is shadow employment. In this case the unemployment rate derived from labor force statistics is

$$u^{lf} = \frac{\tilde{U} + E^s}{E^{lf} + \tilde{U} + E^s}$$

from which it follows that

$$\begin{aligned} \frac{\partial u^{lf}}{\partial E^s} &> 0 \\ \frac{\partial s}{\partial E^s} &> 0 \end{aligned}$$

**Remark 8** *If shadow employment is part of labor force unemployment, the correlation between  $s$  and  $u$  is a statistical artifact*

In this latter scenario one should try to correct the official unemployment statistics. Is there a fraction of unemployed people that looks suspicious? Unfortunately there is no mapping from estimates of shadow employment to LFS definitions of employment, unemployment and inactivity. In order to devise some method to track the labor market status of shadow employment we need to introduce some identifying restrictions. This requires some theoretical guidance.