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Diego García
Francesco Sangiorgi
Branko Urošević

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Overconfidence and Market Efficiency with Heterogeneous Agents[1]

Diego García[2], Francesco Sangiorgi[3] and Branko Urošević[4]

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[2]Tuck School of Business Administration, Dartmouth College.
[3]Fondazione Collegio Carlo Alberto.
[4]Faculty of Economics, University of Belgrade and Department of Economics and Business, Universitat Pompeu Fabra.

Abstract

We study financial markets in which both rational and overconfident agents coexist and make endogenous information acquisition decisions. We demonstrate the following irrelevance result: when a positive fraction of rational agents (endogenously) decides to become informed in equilibrium, prices are set as if *all* investors were rational, and as a consequence the overconfidence bias does *not* affect informational efficiency, price volatility, rational traders' expected profits or their welfare. Intuitively, as overconfidence goes up, so does price informativeness, which makes rational agents cut their information acquisition activities, effectively undoing the standard effect of more aggressive trading by the overconfident. The main intuition of the paper, if not the irrelevance result, is shown to be robust to different model specifications.

JEL Classification: D80, G10.
Keywords: partially revealing equilibria, overconfidence, rational expectations, information acquisition, price informativeness.

1 Introduction

Bounded rationality of economic agents participating in financial markets has been a subject of intense scrutiny in the last decade (see, for example, Thaler (1992), Thaler (1993), and Shleifer (2000)). One such well-documented behavioral pattern is investor overconfidence.[1] Our paper contributes to the emerging literature on the effects of behavioral biases in financial markets by studying the reaction of rational agents to the degree of overconfidence of a set of irrational traders. To the best of our knowledge, this is the first paper that simultaneously adopts two important features of real financial markets: 1) coexistence of rational and overconfident traders, and 2) endogenous information acquisition by agents.[2] In particular, we extend the existing literature by analyzing the impact that the presence of heterogenous (i.e. rational and overconfident) traders has on informational efficiency of prices, willingness of agents to acquire information, market liquidity, and performance and welfare of rational (and overconfident) agents.

Most of the existing models with overconfidence assume exogenous distribution of information among the economic agents. Such simplification is not innocuous: since traders' overconfidence impacts the market precisely through the incorrect interpretation of their private signals on the fundamental value of the traded asset, the effects of overconfidence in the economy may crucially depend on the distribution of information among the agents. It seems natural, therefore, not to specify a priori the information that different agents possess, but to instead allow it to arise endogenously. We first show that overconfidence will reduce rational agents' incentives to gather information within the standard competitive rational expectations paradigm (Hellwig, 1980). In this setup we show that a simple condition on the primitives of the model exists under which overconfidence has no price impact, and as a consequence has no impact on informational efficiency, price volatility, as well as welfare and expected profits of rational agents. None of these properties are affected by the presence of overconfident traders (and coincide with the values in the purely rational economy) if the degree of overconfidence in the economy is below a certain threshold.

To gain intuition for this result we first recall that overconfident traders, by overestimating the precision of their signal, trade more aggressively on their private signals than rational traders. In doing so, more information is revealed by the price. Rational agents react to such anticipated behavior of the overconfident by scaling down their own demand for information,

[1] For an excellent review on psychological literature on overconfidence see Odean (1998) and references therein. For empirical evidence on overconfidence in financial markets see Barber and Odean (2001), Glaser and Weber (2003), and Statman, Thorley, and Vorkink (2003), among many others.

[2] DeLong, Shleifer, Summers, and Waldmann (1990), DeLong, Shleifer, Summers, and Waldmann (1991), Shleifer and Vishny (1997), and Bernardo and Welch (2001), among others, demonstrate that irrational traders may have long-term viability and can coexist with rational traders. For an opposite result, where behavioral agents are driven out of the market, see Sandroni (2005).

aiming to neutralize the negative externality imposed by overconfidence on the rational agents' expected profits and welfare. This "reaction" can be observed only when rational traders are free to decide whether or not to become informed. Thus, endogeneity of information acquisition is crucial for this result to hold.

Nevertheless, investors heterogeneity does influence other properties of the equilibrium. The presence of overconfidence leads to a *decrease* in the overall informed population as opposed to an increase (as argued elsewhere in the literature). Moreover, overconfident traders earn higher expected profits than rational traders but achieve a worse risk return trade-off, providing a new testable implication. Finally, an economy with overconfident agents will always exhibit a higher trading volume than if all agents were rational, a result well established theoretically as well as empirically (see Barber and Odean, 2001, for example).

Within the class of competitive models, the irrelevance result for informational efficiency is shown to be robust to different assumptions regarding the information gathering technology: when agents can choose the precision of the signal they purchase (as in Verrecchia, 1982), and when the error term in the private signal is perfectly correlated among agents (as in Grossman and Stiglitz, 1980). We further show that the main intuition from the paper, that rational agents will cut down information acquisition activities the more overconfident agents there are in the market, is robust to the competitive assumption. In particular, we extend the Kyle (1985) framework to accommodate for rational and overconfident agents. Within this framework, but with exogenous information structure, Odean (1998) and Benos (1998) show that overconfidence increases price informativeness and liquidity. We show that if information acquisition activities are endogenous this may no longer be the case - a result with a similar flavor to the irrelevance proposition discussed above.[3] Our analysis therefore suggests that the effects of overconfidence are more subtle than what the literature portraits.

Several recent theoretical studies focus on the effects of overconfidence on key features of financial markets, as well as on the performance of overconfident traders.[4] Kyle and Wang (1997), Odean (1998) and Benos (1998) consider models with informed insiders and noise traders submitting market orders and find that overconfidence leads to an increase in trading volume, market depth and price informativeness. Both Kyle and Wang (1997) and Benos (1998) allow for rational agents in their models, but information acquisition decisions are fixed in both models.[5] Odean (1998), heuristically, argues that the introduction of rational

[3]In non-competitive models it is virtually impossible to get the irrelevance result that we uncover in the competitive framework due to the discreteness of strategic models.

[4]See Caballé and Sàkovics (2003), Daniel, Hirshleifer, and Subrahmanyam (1998), Daniel, Hirshleifer, and Subrahmanyam (2001), and Scheinkman and Xiong (2003) for some recent work.

[5]In Model III, Odean (1998) allows traders can decide to purchase a single piece of costly information. The author finds that in an economy with *only* overconfident traders, a greater degree of overconfidence leads to a larger fraction of traders that would decide to become informed in equilibrium. In contrast to our paper, Odean (1998) does not model rational traders.

traders to his model "would mitigate but not eliminate the effects of overconfident traders"(see Odean, 1998, Model I). Rubinstein (2001) summarizes the effects of overconfidence by stating that "[overconfidence] does create a positive externality for passive investors who now find that prices embed more information and markets are deeper than they should be." We show that precisely due to this externality, rational agents will reduce their information gathering activities, and that, indeed, this can eliminate the standard positive effect of overconfidence on price informativeness.

The paper is organized as follows. Section 2 presents a competitive model with endogenous information acquisition. The irrelevance result is developed in detail in section 3. Section 4 considers various extensions, where we argue that the results discussed in the paper are robust to the types of financial market model we consider in the main body of the paper. Section 5 concludes. Proofs are relegated to the Appendix.

2 The model

The basic model in this paper extends the standard one period rational expectations model with endogenous information acquisition (see Hellwig (1980) and Verrecchia (1982)) to the setting in which overconfident (irrational) economic agents coexist with rational ones. In particular, we assume that a measure $m_o \in (0,1)$ of the trader population is of the type o (overconfident), while the measure $m_r = 1 - m_o$ is of the type r (rational). All traders in the economy have CARA preferences with risk aversion parameter τ, i.e. their utility function, defined over the terminal wealth, is $u(W_i) = -e^{-\tau W_i}$. There are two assets in the economy: a riskless asset (the numeraire) in perfectly elastic supply (its gross return is, without loss of generality, normalized to 1), and a risky asset with payoff X and random supply Z. Without loss of generality we normalize initial wealth to zero. Letting θ_i denote the number of units of the risky asset bought by agent i, and letting P_x denote its price, we have that the final wealth of a trader i is given by $W_i = \theta_i(X - P_x)$.

Each trader can decide to purchase a noisy signal about the payoff of the risky asset, which we will denote by $Y_i = X + \epsilon_i$, at a cost $c > 0$. Therefore, the information set of uninformed trader i, which we denote by $\mathcal{F}_i$, consists of the risky asset price P_x, while for the informed the information set contains, also, the signal. Formally, we will denote an informed agent's information set by $\mathcal{F}_I$ (the σ-algebra generated by (Y_i, P_x)) and an uninformed agent's information set by $\mathcal{F}_U$ (the σ-algebra corresponding to the risky asset price P_x). All random variables X, Z and ϵ_i are independent Gaussian random variables, defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, with zero mean and variances equal, respectively, to σ_x^2, σ_z^2 and σ_ϵ^2. We further normalize the payoff of the risky asset X so that $\sigma_x^2 = 1$.

In the basic setup, the only difference between the two types of traders is that type o incorrectly believe that the variance of the signal σ_ϵ^2 is equal to $b_\epsilon^{-1}\sigma_\epsilon^2$, where $b_\epsilon > 1$. Thus, traders of type o overestimate the precision of the signal, and higher values of b_ϵ are associated with higher degrees of overconfidence. In contrast, traders of type r correctly estimate the precision of the signal (for such traders $b_\epsilon = 1$). Type $j = o, r$ expectations are denoted as $\mathbb{E}^j$. Here, agents of type r compute the expectations vis-a-vis the true measure (we denote $\mathbb{E}^r$ as $\mathbb{E}$ for brevity), while the agents of the type o, those with a behavioral bias, compute their expectations, denoted by $\mathbb{E}^o$, using the probability measure that underestimates the variance of the signal (i.e. that uses $b_\epsilon^{-1}\sigma_\epsilon^2$ instead of σ_ϵ^2).[6]

Every trader in the economy is a price-taker and knows the structure of the market. In particular, each type $j = o, r$ knows that the other type has different beliefs about the precision of the signal.[7] The timing in the model is as follows. For each type $j = o, r$, a fraction λ_j of the respective population decides to acquire a signal. Once that decision is made, each trader submits the demand schedule for the risky asset conditional on her information set ($\mathcal{F}_I$ or $\mathcal{F}_U$). The price is set to clear the market. Finally, the fundamental value of the risky asset is revealed and the endowments consumed.

The next definition is standard.

Definition 1 *An equilibrium in the economy is defined by a set of trading strategies θ_i and a price function $P_x : \Omega \to \mathbb{R}$ such that:*

1. *Each agent i of type j chooses her trading strategy so as to maximize her expected utility given her information set $\mathcal{F}_i$:*

$$\theta_i \in \arg\max_\theta \mathbb{E}^j \left[u\left(W_i\right) | \mathcal{F}_i \right]. \tag{1}$$

2. *The market clears:*

$$m_o \Theta_o + m_r \Theta_r = Z; \tag{2}$$

where $\Theta_j = \frac{1}{m_j} \int_0^{m_j} \theta_i di$ is the per capita (average) trade by the type j agents ($j = o, r$).

The setup thus far closely parallels Diamond (1985), which is a special variation of the

[6] We treat the overconfidence bias of agents as exogeneous. In principle, if the overconfident could participate in multiple trading rounds they could update their estimate of the precision of the signal by observing past performance. In this case rational learning could eliminate their bias. See Hirshleifer and Luo (2001) for a discussion of this point; Daniel, Hirshleifer, and Subrahmanyam (1998) and Gervais and Odean (2001) for models in which agents learn about their own abilities; and Záboníc (2004) for a rational model in which a bias in self-assessment arises endogenously.

[7] In equilibrium, traders properly deduce the fraction of the population of each trader type that, in equilibrium, becomes informed. This is consistent with the bulk of the literature in rational expectations models (see Squintani, 2006, and the references therein).

model discussed in Verrecchia (1982).[8] For expositional simplicity we introduce two basic assumptions regarding the information technology.

Definition 2 *We call an information technology non-trivial if* $C(\tau)^{-1}b_\epsilon > \sigma_\epsilon^2$*, where* $C(\tau) \equiv e^{2c\tau} - 1$.

Definition 3 *We say that the information technology satisfies the no free lunch condition if* $\Lambda^* \leq 1$*, where*

$$\Lambda^* = \frac{1}{m_r}\left(\tau\sigma_\epsilon\sigma_z\sqrt{C(\tau)^{-1} - \sigma_\epsilon^2} - m_o b_\epsilon\right). \tag{3}$$

Definition 2 requires that the information technology has a sufficiently high price-to-quality ratio so that some traders find it optimal to invest in information acquisition activities. If the condition did not hold no agent would ever become informed in equilibrium. Definition 3 plays the opposite role. In particular, when $\Lambda^* \geq 1$ the equilibrium at the information acquisition stage will be such that all agents, rational and overconfident, find it optimal to acquire information. The label "free-lunch" comes from a slightly different interpretation of the source of information. In particular, consider a model where a seller of information charges some price c for the signal (see Admati and Pfleiderer, 1986). From the definition of the equilibrium in the next section it will become clear that such seller of information will never choose c that would violate $\Lambda^* \leq 1$.[9] The variable Λ^* will play a crucial role in the discussion that follows. In essence, the equilibrium in the model will depend crucially on whether the constant Λ^* is positive or not. We further discuss the role of these assumptions on the model's primitives in the next section.

3 Equilibrium prices

This section solves for the competitive equilibrium with information acquisition, and derives main results of the paper including the irrelevance result. Throughtout this section, we assume that the information technology is non-trivial and does not allow free lunch.

[8]The main difference from those models is that we relax their assumption that there are only rational agents in the economy. In section 4.1 we further argue that the reduced-form model of Diamond (1985) is isomorphic to the model of Verrecchia (1982) for an open set of the model's primitives.

[9]Indeed, it can be seen that charging c such that $\Lambda^* > 1$ would be strictly dominated by charging $\hat{c}$ such that $\Lambda^* = 1$. Thus, such seller of information would be "leaving money on the table," and Definition 3 rues out this case.

3.1 The competitive equilibrium with information acquisition

As is customary in models with endogenous information acquisition, the model is solved in two stages: we first determine the equilibrium asset price function by taking λ_j as exogenously fixed; then we go back to the information acquisition stage and find the equilibrium values for λ_j, thus completing the specification of equilibrium.

Lemma 1 *For given values of $\lambda_j \geq 0$, the competitive equilibrium price P_x is given by the expression $P_x = \hat{a}X - \hat{d}Z$, where the coefficients $\hat{a}$ and $\hat{d}$ satisfy:*

$$\frac{\hat{a}}{\hat{d}} \equiv \gamma = \frac{1}{\tau\sigma_\epsilon^2}\left(\lambda_o m_o b_\epsilon + \lambda_r m_r\right); \tag{4}$$

$$\hat{d} = \frac{1 + \dfrac{\gamma}{\tau\sigma_z^2}}{\gamma + \dfrac{\gamma^2}{\tau\sigma_z^2} + \dfrac{1}{\tau}}. \tag{5}$$

The informational content of price, or simply market efficiency, is measured by the conditional variance of the fundamental asset value given the market price. From Lemma 1 it follows that this quantity is given by:

$$\mathrm{var}\left(X|P_x\right) = \left(1 + \frac{\gamma^2}{\sigma_z^2}\right)^{-1}. \tag{6}$$

The smaller the conditional variance (6), the more information is revealed by the price in equilibrium. Since the information revealed by the price monotonically increases in γ, comparative statics of γ encapsulate everything we need to know about the dependence of (6) on the parameters measuring the overconfidence in the economy. When λ_j are exogenously fixed we obtain

$$\frac{d\gamma}{db_\epsilon} = \frac{m_o\lambda_o}{\tau\sigma_\epsilon^2} \geq 0. \tag{7}$$

From (7) it follows that, when λ_o is exogenous and positive, an increase in the intensity of overconfidence b_ϵ raises the amount of information revealed by the price. The intuition for this result is the same as in Odean (1998), namely, the more overconfident traders are, the more aggressively they trade on their information, which makes the price more informative.

The next Lemma characterizes the equilibrium with endogenous information acquisition.

Lemma 2 *The equilibrium with information acquisition belongs to one of the following two classes:*

(a) If the parameters of the model are such that $\Lambda^* > 0$, a fraction (possibly all) of the rational agents and all overconfident agents become informed: in equilibrium $\lambda_o^* = 1$ and $\lambda_r^* = \Lambda^*$.

(b) If the parameters of the model are such that $\Lambda^* \leq 0$, a fraction (possibly all) of the overconfident traders becomes informed and no rational trader becomes informed: in equilibrium $\lambda_r^* = 0$ and (in the interior solution)

$$\lambda_o^* = \frac{\tau \sigma_z}{m_o} \sqrt{k_\epsilon \sigma_\epsilon^2 \left(C(\tau)^{-1} - k_\epsilon \sigma_\epsilon^2\right)} \tag{8}$$

Lemma 2 shows that depending on the values of the primitives that characterize the economy, different types of equilibria may endogenously arise: traders who decide to acquire the signal and become informed can be either only a fraction of overconfident traders, all overconfident but no rational traders, all overconfident and a fraction of rational traders, or all traders in the economy. The relevant property of the equilibrium is that rational traders become informed only if all overconfident traders are informed.[10] This result is intuitive since overconfident overestimate the precision of the signal, and therefore it cannot be that some rational trader decides to become informed and an overconfident does not.[11]

Fixing other parameter values, region $\Lambda^* > 0$ arises when: (i) degree of overconfidence $m_o b_\epsilon$ is sufficiently small; (ii) information acquisition costs c are sufficiently low and/or the variability of the aggregate supply shock σ_z is large; (iii) values of the risk-aversion τ and signal precision σ_ϵ^2 are intermediate. The first two conditions are rather intuitive: if there are many overconfident agents, or their bias is too high, they will crowd out the rational agents, and we are back to the setting where the overconfident are the marginal buyers of information. If the cost is low or the noise large, traders find information acquisition activities more attractive, eventually making the rational traders (marginal) buyers of information. The third result comes from the dual role that those two parameters, risk-aversion and signal precision, play in this type of competitive models. On one hand they affect the value of becoming informed: more risk-tolerant agents are willing to pay more for a signal, and more precise signals are more valuable to agents. At the same time these parameter values affect the information revealed by prices: more risk-tolerant agents, or agents with more precise signals, trade more aggressively thereby exacerbating the negative externality of their trades. It can be shown that

[10] The fact that the overconfident will always buy the signal before the rational agents do is independent of the strong parametric assumptions of this paper. It follows from Blackwell's theorem on comparisons of information structures that overconfident agents will assign a higher value to a given signal. We thank an anonymous referee from highlighting this.

[11] In the existing literature with overconfidence and asymmetric information, it is typically argued that those traders that do not buy the information are those that value it properly (see, for instance, Odean (1998), page 1907 and Daniel, Hirshleifer, and Subrahmanyam (2001), page 928). Lemma 2 formalizes this argument in the class of models we study.

this second effect dominates for small values of τ and σ_ϵ^2, which pushes down the fraction of informed agents towards zero. At the same time, as both τ and σ_ϵ^2 grow without bound agents eventually have no incentives to buy information, and again we do not satisfy the $\Lambda^* > 0$ condition.

3.2 Irrelevance result and comparative statics

In the following Proposition we state the main irrelevance result on overconfidence.

Proposition 1 *If $\Lambda^* > 0$ then overconfidence is irrelevant for the parameters of the equilibrium price function, and as a consequence for informational efficiency, price volatility and rational traders expected profits and welfare. These quantities are equal to those that would endogenously arise in a fully rational economy, i.e. the equilibrium is independent of the overconfidence parameters b_ϵ and m_o.*

We can interpret $\Lambda^* = 0$ as an irrelevance threshold and think of this result in the following way. Compare two economies characterized by a common set of primitives (variances and risk aversion): one in which $m_o = 0$ (fully rational economy) and one in which $m_o > 0$, i.e., in which a positive measure of overconfident traders interacts with rational traders. The above Proposition states that as long as the degree of overconfidence in the economy, as measured by $m_o b_\epsilon$, is not too large[12] the two economies will have identical asset prices. While previous studies argue that overconfidence is costly to society, (see, for instance, Odean, 1998), Proposition 1 gives the conditions under which the process of competitive trading itself is a mechanism able to prevent overconfidence from affecting the informational efficiency of the price, *and* the welfare and profits of the rational traders. In this case overconfidence can be costly only to the overconfident.

This result obtains because of the reaction on the part of rational traders to the presence of overconfidence. From the equilibrium equation for γ in (4), we have that for $\Lambda^* > 0$

$$\frac{d\gamma}{db_\epsilon} = \frac{1}{\tau\sigma_\epsilon^2}\left(m_o + m_r\frac{d\lambda_r^*}{db_\epsilon}\right). \tag{9}$$

The first term, $m_o/\tau\sigma_\epsilon^2$, is the standard term stemming from more aggressive trading by the overconfident agents as b_ϵ increases. The second term, which measures the (negative) reaction of the rational population to the increase in overconfidence, is what drives the irrelevance result. A simple inspection of (3), and noting that $\lambda_r^* = \Lambda^*$, yields that γ is indeed independent of

[12] Note that the condition $\Lambda^* > 0$ is equivalent to requiring $m_o b_\epsilon$ to be below the threshold value $\tau\sigma_\epsilon\sigma_z\sqrt{C(\tau)^{-1} - \sigma_\epsilon^2}$.

the overconfidence parameter b_ϵ.[13] In turn, this implies that the parameters of the equilibrium price function (see equations (4) and (5)) do not depend on overconfidence parameters and are given by the same quantities as in the fully rational economy. As a consequence, the same is true for the unconditional variance, expected utilities and the expected profits of the rational traders.

To gain some intuition on why the reaction of rational traders *exactly* offsets overconfidence, notice that when $\Lambda^* > 0$, the rational traders are the marginal buyers of information, and the equilibrium fraction of informed rational traders (λ_r^*) is set to equate informed and uninformed expected utilities. In the Appendix it is shown that this condition is equivalent to

$$e^{-2\tau c} \text{var} \left(X | P_x, Y_i \right)^{-1} = \text{var} \left(X | P_x \right)^{-1}; \tag{10}$$

where the two conditional variances only depend on the amount of noise of the economy σ_z^2, the precision of agents' signals σ_ϵ, and the equilibrium parameter γ. When the rational agents are the marginal buyers of information (10) needs to hold as an equality, and therefore it must be that $d\gamma/db_\epsilon = d\gamma/dm_o = 0$, which in turn implies the reaction in λ_r^* described above. The presence of overconfidence is perceived by rational traders as an "exogenous" effect on price informativeness, which in turn affects the relative expected utility of informed versus uninformed. Since in equilibrium expected utilities must be equal, and the overconfidence parameters (m_o, b_ϵ) enter into (10) only *indirectly* via γ, the equilibrium condition on information acquisition requires λ_r^* to adjust in such a way that the net effect on γ is identically zero.[14] In contrast, a marginal change in one of the other "fundamental" primitives of the model $(\sigma_z^2, \sigma_\epsilon^2, \tau, c)$, does imply an adjustment in λ_r^* to equate expected utilities, but because these parameters enter *directly* into (10), this adjustment will affect the equilibrium price coefficients.

On the other hand, as long as $\Lambda^* > 0$ is satisfied, the two economies (the fully rational and the one with overconfidence) will exhibit some interesting differences, described in the next Proposition.

Proposition 2 *If $\Lambda^* > 0$ then: (i) the measure of informed traders is lower that what would be observed in a fully rational economy; (ii) overconfident traders earn higher expected profits than rational traders, although the Sharpe ratios of their portfolios are lower; and (iii) expected trading volume is increasing in parameters of overconfidence.*

[13]Similarly, differentiating (3) with respect to m_o one can see that γ does not depend on m_o either.

[14] For the same reason, the same result can be generated in an economy with agents with two different risk-aversion parameters, say $\bar{\tau} > \underline{\tau}$. If the high risk-aversion agents are the marginal buyers of information, then changes in the risk-aversion parameter $\underline{\tau}$ will not affect price informativeness. Therefore, these results can be viewed as a precise statements under which the weak inequalities in Verrecchia (1982), in terms of the effects of risk-aversion on price informativeness, hold as equalities.

We will discuss these three results in order. Result (i) is surprising. In fact, it goes in the opposite direction of what previous literature finds: Odean (1998), for example, considers a model where overconfident traders can decide to acquire a single piece of information, and finds that too many of them are willing to buy it. We find that the measure of informed traders, both rational and overconfident, is lower than in the corresponding rational economy. This is rather intuitive: when m_o or b_ϵ increases, γ remains constant, but since the overconfident reveal more of their signal than rational traders, now a smaller measure of informed is sufficient to sustain a given level of γ.

Result (ii) follows by noting that the overconfident take higher risks (without realizing it) by trading more aggressively on their information, which in turn yields higher expected profits.[15] Differently from an agent who is simply less risk averse, the overconfident incorrectly weights the market price in his trading strategy, which yields a portfolio with higher volatility and a lower Sharpe ratio (with respect to a rational agent). The result that overconfident achieve a worse risk return trade-off provides a new testable implication, and is in contrast to models in which the overconfident are better off, using the true probability measure, than the rational agents.[16]

Result (iii) confirms the robustness of previous findings on the effect of overconfidence on trading volume. Namely, an increase in the degree of overconfidence $m_o b_\epsilon$ enhances expected trading volume. On one hand the trading volume of the overconfident goes up, due to their higher responsiveness to their information. The rational agents, as a group, trade less as overconfidence rises: even though the trading strategies of informed and uninformed rational agents are unchanged, the fraction of informed rational agents is decreasing in overconfidence, and thereby total trading volume for the rational agents is reduced. The proposition shows that the effect on the overconfident dominates the later effect, and trading volume is indeed increasing in $m_o b_\epsilon$. Our conclusions are consistent with the bulk of the empirical evidence on trading volume and overconfidence, while at the same time showing that some properties of asset prices may actually be independent of overconfidence.

[15] The result in the Proposition refers to the comparison between overconfident and rational *informed* traders. Rational uninformed trade on less precise information, and achieve lower expected profits but the same expected utility of their informed colleagues. This makes the comparison between informed and uninformed expected profits of risk averse agents uninteresting.

[16] See Kyle and Wang (1997) and Dubra (2004) for some examples from the literature, as well as the discussion in section 4.3.

Hirshleifer and Luo (2001) propose an evolutionary model in which the replication of rational and overconfident is assumed to be increasing in the profitability (expected profits) of their strategies. According to this evolutionary mechanism, overconfident always survive in the long run. In their model traders are risk averse and assumed to be all informed. But when some traders find it optimal not to become informed, the comparison of expected profits might not be the appropriate measure of performance (risk matters for expected utility). Hence, the result that overconfident earn higher expected profits but lower Sharpe ratios could provide a new (negative) argument for the evolutionary selection of overconfident traders in financial markets.

Above the irrelevance threshold,[17] only a fraction of overconfident and no rational traders become informed in equilibrium. Going back to the expression for γ, which measures price informativeness, we see that in that case:

$$\frac{d\gamma^*}{db_\epsilon} = \frac{1}{\tau\sigma_\epsilon^2}\left(m_o \frac{d\left(\lambda_o^* b_\epsilon\right)}{db_\epsilon}\right).$$

Now there are two effects that influence γ, the direct effect through higher information revelation by the informed (overconfident) agents, plus the change in the fraction of informed agents. It can be easily verified from (8) that the product $\lambda_o^* b_\epsilon$ is increasing in b_ϵ, therefore increasing information revelation.[18] A higher value of γ in turn implies that the impact of noise on the equilibrium price is reduced, and so are noise traders expected losses (and therefore other traders' expected profits and welfare). This illustrates the fact that in order to capture the effects that we described in Propositions 1 and 2 it is necessary to consider a model with heterogeneous agents, where rational agents coexist together with overconfident traders.

4 Extensions

In this section of the paper we consider several models in which we illustrate the robustness of the previous results. We study more general information acquisition technologies, a version of the Grossman and Stiglitz (1980) model, and an imperfectly competitive market (as in Kyle (1985)). We argue that the main results of the previous section, in particular the fact that price informativeness is unaffected by overconfidence, is robust across these three rational expectations models.

4.1 General information acquisition technologies

Consider now the following variation of the basic model. Agents can obtain signals of the type $Y_i = X + \epsilon_i$, with $\epsilon_i \sim \mathcal{N}(0, 1/p)$. In order to obtain such signals traders need to pay the price, in units of the numeraire, equal to $c(p)$. We assume that $c(p) \geqslant 0$, $c'(p) > 0$ and $c''(p) \geq 0$, $\forall\ p > 0$. Thus, the cost of their signal is increasing and convex in its precision. In this way we extend the basic model to allow for more general information gathering technologies. The overconfident, as before, erroneously believe to receive signals, after paying the cost $c(p_o)$, with precision $b_\epsilon p_o$ for some $b_\epsilon > 1$.

[17] That is, when $m_o b_\epsilon \geq \tau\sigma_\epsilon\sigma_z\sqrt{C(\tau)^{-1} - \sigma_\epsilon^2}$.

[18] It should be noted that in general λ_o^* may not be increasing in b_ϵ. For large values of b_ϵ the negative externality imposed by the informed on price informativeness may actually make λ_o^* decreasing in b_ϵ. See the discussion on non-monotonicity relationships in this type of REE models following Lemma 2.

The competitive equilibrium in this variation of the model is defined as in section 2. The equilibrium in information acquisition is characterized by fractions of informed agents λ_r^* and λ_o^*, and precision levels p_r^* and p_o^*, such that: (1) no uninformed agent would want to become informed; (2) no informed agent would be better off by choosing other precision levels $p \neq p^*$, or by becoming uninformed.[19] The equilibrium in information acquisition follows Verrecchia (1982), with the additional considerations that may arise if $\lambda_r^* \neq 1$.[20]

For the purpose of characterizing the equilibrium, define the following function of the primitives:

$$\Lambda_{GI}^* = -\frac{m_o b_\epsilon p_o^*}{m_r p_r^*} + \frac{1}{m_r p_r^*}\sqrt{\frac{\tau\sigma_z^2\left(e^{-2c(p_r^*)\tau} - 2\tau c'(p_r^*)\right)}{c'(p_r^*)}};$$

where p_o^* and p_r^* are defined in the Appendix. The next Proposition describes the equilibrium in such economy.

Proposition 3 *When traders can choose a signal of arbitrary precision, then the fraction of rational informed traders is given by: a) $\lambda_r^* = \Lambda_{GI}^*$ if $\Lambda_{GI}^* \in (0,1)$; b) $\lambda_r^* = 1$ if $\Lambda_{GI}^* \geq 1$; c) $\lambda_r^* = 0$ if $\Lambda_{GI}^* \leq 0$. The irrelevance result in Proposition 1 holds if $\Lambda_{GI}^* \in (0,1)$.*

If the parameters of the model are such that $\Lambda_{GI}^* \in (0,1)$, then an interior fraction of rational agents becomes informed. The interpretation of Λ_{GI}^* as an irrelevance threshold is similar to the basic model: for Λ_{GI}^* to be positive it must be that

$$m_o b_\epsilon < \frac{1}{p_o^*}\sqrt{\tau\sigma_z^2\left(e^{-2c(p_r^*)\tau} - 2\tau c'(p_r^*)\right) c'(p_r^*)^{-1}},$$

where the left-hand side of the above expression can be interpreted as the degree of overconfidence, and the term on the right as some threshold level. The intuition of the irrelevance result goes back to the usual expression for the relative price coefficients γ, which in this case takes on the form

$$\gamma = \frac{m_o b_\epsilon p_o^*}{\tau} + \frac{m_r \lambda_r^* p_r^*}{\tau}. \tag{11}$$

so that the impact of overconfidence is given by

$$\frac{d\gamma}{db_\epsilon} = \frac{m_o p_o^*}{\tau} + \frac{m_o b_\epsilon}{\tau}\frac{dp_o^*}{db_\epsilon} + \frac{m_r p_r^*}{\tau}\frac{d\lambda_r^*}{db_\epsilon} + \frac{m_r \lambda_r^*}{\tau}\frac{dp_r^*}{db_\epsilon}. \tag{12}$$

The impact of overconfidence on price revelation is driven by the standard first two terms (more aggressive trading by the overconfident plus more information acquisition on their part),

[19] Note that since in principle we do not exclude the case $c(0) > 0$ we must allow for this possibility separately in the analysis.

[20] The assumptions in Verrecchia (1982) imply that equation (30) in the Appendix never binds. In our symmetric model this means that either all agents become informed, or none does, as we show in the proof.

plus the two other terms which measure the response by rational agents to the higher levels of overconfidence. In the Appendix we show that when $\lambda_r^* \in (0,1)$, then rational traders react by scaling down the demand for information via the second term (response in the equilibrium fraction of informed traders) in a way that offsets the first two terms given by the increase of overconfidence, and the fourth term (response in the equilibrium precision) is equal to zero. On the other hand, if $\lambda_r^* = 1$, then the third term is equal to zero and the offsetting effect comes from the fourth term, i.e. $dp_r^*/db_\epsilon < 0$, but is smaller in magnitude than the positive effect resulting from more aggressive trading by the uninformed, and therefore overconfidence will increase price informativeness.

4.2 Correlated signals

To inspect the robustness of our main result on overconfidence and informational efficiency, we further consider the case in which every informed agent gets a signal $Y_i = X + \epsilon_i$ with $\epsilon_i = \epsilon$, $\forall i$, i.e. a competitive economy where agents get signals whose errors are perfectly correlated. All other assumptions regarding the structure of the market are unchanged with respect to section 2. This variation of the model is a direct extension of the model of Grossman and Stiglitz (1980), and allows us to argue that independence of the signals does not drive any of the results derive thus far.[21]

Prices are conjectured to be of the form $P_x = \hat{a}\left(Y - \gamma^{-1}Z\right)$. Prices now transmit information, but do not aggregate it, and therefore the noise of the signal appears in the equilibrium price. Notice that in this model γ is again the relevant parameter for market efficiency, since

$$\mathrm{var}(X|P_x) = \frac{\sigma_\epsilon^2 + \dfrac{\sigma_z^2}{\gamma^2}}{1 + \sigma_\epsilon^2 + \dfrac{\sigma_z^2}{\gamma^2}} \tag{13}$$

and that (13) is monotonically decreasing in γ. Furthermore, as we show in the proof of Proposition 4, in equilibrium we have that

$$\gamma = \frac{1}{\tau\sigma_\epsilon^2}\left(\lambda_o m_o b_\epsilon + \lambda_r m_r\right); \tag{14}$$

where λ_j denotes, as before, the fractions of agents that are informed. Equation (13) and (14) immediately imply that when λ_j are exogenous, an increase in overconfidence b_ϵ raises the amount of information revealed by the price.

[21]One can show that the irrelevance result holds for imperfectly correlated signals, i.e. signal structures of the form $Y_i = X + \epsilon + \epsilon_i$, where ϵ denotes a common error term, and the ϵ_i's are i.i.d., which subsumes the model in section 2 and the one currently being discussed.

We next turn to describing the equilibrium at the information acquisition stage. Define Λ^*_{GS} as

$$\Lambda^*_{GS} = \frac{1}{m_r}\left(\tau\sigma_\epsilon\sigma_z\sqrt{\frac{(1-C(\tau)\sigma^2_\epsilon)}{(1+\sigma^2_\epsilon)C(\tau)}} - m_o b_\epsilon\right). \tag{15}$$

The next Proposition characterizes the equilibrium with endogenous information acquisition of perfectly correlated signals.

Proposition 4 *The equilibrium with information acquisition belongs to one of the following two classes:*

(a) If the parameters of the model are such that $\Lambda^*_{GS} > 0$, a fraction (possibly all) of the rational agents and all overconfident agents become informed. In particular $\lambda^*_o = 1$ and $\lambda^*_r = \Lambda^*_{GS}$.

(b) If the parameters of the model are such that $\Lambda^*_{GS} \leq 0$, a fraction (possibly all) of the overconfident traders becomes informed, but none of the rational agents, $\lambda^*_r = 0$.

If $\Lambda^*_{GS} > 0$ then overconfidence is irrelevant for informational efficiency, that is, γ is equal to what would endogenously arise in a fully rational economy.

The equilibrium with endogenous information acquisition shares the same properties of the basic model: rational traders will become informed only if all overconfident are informed. The intuition for the irrelevance result is identical to the case where signals were independent: the rational traders, when they are the marginal buyers of information, scale back their information acquisition activities (less of them become informed), and this exactly offsets the standard effect of higher price informativeness stemming from more overconfidence.

This shows that the result on the irrelevance of overconfidence for market efficiency is robust to other types of information structure in the market. It should be remarked that other variables of interest, and in particular the price function itself, do depend on the level of overconfidence b_ϵ, in contrast to the case studied in section 3. This dependence goes much along the same lines as in Odean (1998) (Model III) and will not be reported here for brevity.

4.3 An imperfectly competitive model

In order to further analyze the effects of overconfidence in markets populated by both rational and overconfident agents we now turn to study a multi-agent version of the Kyle (1985) model. The main departure point from the previous section is the fact that all agents are "large", in the sense that their trades affect prices. We recall that Odean (1998) and Benos (1998) showed that the introduction of overconfidence increases market depth.[22] We show below that this result depends critically on the fact that informed agents are overconfident: once we allow for rational traders and endogenous information acquisition a higher degree of overconfidence can make some rational agents drop out of the market, thereby decreasing market liquidity.

We consider a finite-agent economy, where all traders observe a signal of the form $Y_i = X + \epsilon_i$, where $X \sim \mathcal{N}(0,1)$ denotes the final payoff of the risky asset, and $\epsilon_i \sim \mathcal{N}\left(0, \sigma_\epsilon^2\right)$. For simplicity all signals' errors ϵ_i are assumed to be independent. There are m overconfident agents, who erroneously believe that the variance of their signal's estimation error is actually $k_\epsilon \sigma_\epsilon^2$, where $k_\epsilon < 1$.[23] In addition to overconfident agents, n rational traders exist in the economy. These agents estimate the precision of their private signal correctly. In order to abstract from risk-aversion effects we let both overconfident and rational traders be expected profits maximizers. On top of these two types of agents, there are also noise traders in the market, who submit orders that we denote by U, where $U \sim \mathcal{N}\left(0, \sigma_u^2\right)$.

As usual in this type of models, prices are set by a risk-neutral market maker, who is assumed to be competitive (i.e. earns zero expected profits in equilibrium). Namely, the market maker sets prices equal to the expected value of the fundamental, conditional on total order flow. We let θ_i denote the trading strategy of agent i. All traders and the market maker are assumed to know the structure of the market, in particular they rationally anticipate the trading strategies of other types of traders, given their exogenously specified biases. The following definition formalizes the notion of an equilibrium in this type of model.

Definition 4 *An equilibrium in the economy is defined by a set of trading strategies θ_i and a price function $P_x : \Omega \rightarrow \mathbb{R}$ such that:*

1. *Each agent i chooses her trading strategy so as to maximize her expected profits given her signal Y_i:*
$$\theta_i \in \arg\max_{\theta} \pi_i = \mathbb{E}^i\left[\theta_i(X - P_x)|Y_i\right]; \tag{16}$$
where if agent i is overconfident the expectation is taken under the beliefs that $\epsilon_i \sim \mathcal{N}(0, k_\epsilon \sigma_\epsilon^2)$, whereas if agent i is rational $\epsilon_i \sim \mathcal{N}(0, \sigma_\epsilon^2)$.

2. *The market maker breaks even:*
$$P_x = \mathbb{E}[X|\omega], \tag{17}$$
where ω denotes the total order flow, i.e. $\omega = \sum_{i=1}^{n+m} \theta_i + U$.

[22] The analysis is also similar to Kyle and Wang (1997), although the emphasis in that paper is on the commitment benefits of overconfidence.

[23] In the previous notation, $b_\epsilon = 1/k_\epsilon$

The following lemma characterizes the equilibrium price and trading strategies.[24]

Lemma 3 *The equilibrium price and trading strategies are linear in ω and Y_i respectively, i.e. price is given by $P_x = \lambda\omega$, rational agents' trading strategies are $\theta_i = \beta_r Y_i$ and those of the overconfident are $\theta_i = \beta_o Y_i$, where*

$$\beta_r = \frac{\eta}{1+2\sigma_\epsilon^2}; \qquad \beta_o = \frac{\eta}{1+2k_\epsilon\sigma_\epsilon^2}; \tag{18}$$

$$\lambda^{-1} = \eta\left(1 + \frac{n}{1+2\sigma_\epsilon^2} + \frac{m}{1+2k_\epsilon\sigma_\epsilon^2}\right); \tag{19}$$

$$\eta^2 = \sigma_u^2\left(\frac{n(1+\sigma_\epsilon^2)}{(1+2\sigma_\epsilon^2)^2} + \frac{m(1+(2k_\epsilon-1)\sigma_\epsilon^2)}{(1+2k_\epsilon\sigma_\epsilon^2)^2}\right)^{-1}. \tag{20}$$

A necessary and sufficient condition for an equilibrium to exist is that (20) defines a positive real number.[25]

As expected, the overconfident agents trade more aggressively than the rational. This is simply due to the fact that these agents believe their information to be more precise than that of the rational. It should nonetheless be noted that the trading aggressiveness of the overconfident is no longer a simple function of their behavioral bias: it now depends, through the market maker price setting, on the market wide variable η, which is itself a non-monotonic function of the bias measure b_ϵ. The following proposition is immediate.

Proposition 5 *If the number of informed agents m and n are exogenously fixed, then market depth is increasing in overconfidence.*

The proposition highlights the robustness of the positive effect of overconfidence on market liquidity, when information is exogenously fixed, reported elsewhere in the literature (Odean, 1998; Benos, 1998). Compared to a purely rational economy, financial markets with overconfident will exhibit higher market depth.

We now turn to study the incentives to acquire information by rational agents. In particular, we fix the number (and information) of the overconfident, and allow a large number of rational agents to purchase a signal of precision $1/\sigma_\epsilon^2$ for a cost c. We let n^* denote the largest n^* such that $\pi_r(n^*) \geq c$, i.e. n^* denotes the largest number of rational agents such that if n^* of them are informed it is still profitable for them to acquire information. This is the natural outcome of a standard Nash equilibrium in information acquisition in this type of setting.

[24] The Lemma extends Benos (1998), who considers the extreme case in which $k_\epsilon = 0$.

[25] In the analysis that follows we will always assume this condition to be satisfied.

The following proposition shows that the same forces that were in action in the competitive models play a role in this version of the Kyle (1985) model for moderate levels of overconfidence.

Proposition 6 *Given m, let n^* be determined endogenously. For moderate levels of overconfidence, n^* is weakly decreasing in overconfidence. As a result, market depth can decrease as a function of overconfidence.*

The result in Proposition 6 highlights the robustness of the main effect which drives the irrelevance result of previous sections:[26] rational agents' incentives to gather information are reduced when overconfidence appears. As discussed in Benos (1998), an increase in overconfidence (given m and n) has two opposite effects on the aggressiveness of rational traders: a market liquidity effect and a strategic substitution effect. The first one is related to the increase in market depth, which causes rational traders be more aggressive; the second is related to the increase in the aggressiveness of the overconfident, which leads rational traders to trade less. When overconfidence is not too severe the second effect dominates, reducing expected trading profits of rational traders.[27] This can in turn force some of them to drop out of the market and reduce market depth.[28] One can view this result in light of the benefits of overconfidence as a commitment device, discussed in Kyle and Wang (1997) and Benos (1998). Namely, if there is heterogeneity with respect to commitment power, those agents that lack commitment will have less incentives to invest in information, compared to the economy where all agents lack this commitment power. This in turn can make the market less liquid.

5 Conclusion

This paper considers a model in which rational traders coexist with overconfident ones. We have shown that endogenizing the information acquisition decision generates new predictions on the effects of overconfidence on asset prices, with respect to models with exogenous information distribution. In particular, there exist economies in which the equilibrium price corresponds to what would endogenously arise in a rational expectations equilibrium. The rational agents react to the presence of overconfident agents by reducing their information acquisition activities, since the returns to informed trading are reduced when overconfident agents trade more aggressively and thereby reveal more of their information through prices. This reaction offsets the impact of the overconfident on asset prices. On the other hand, we show that other asset

[26] In the finite-agent economies, such an irrelevance result is impossible to obtain, due to the discreteness of the model.

[27] In particular, a sufficient condition for n^* to be weakly decreasing in overconfidence is that $2k_\epsilon\sigma_\epsilon^2 > 2\sigma_\epsilon^2 - 1$, which is clearly satisfied as $k_\epsilon \to 1$ or when $2\sigma_\epsilon^2 - 1 < 0$.

[28] Consider the following numerical example: $\sigma_\epsilon^2 = 1/5$; $\sigma_u^2 = 2$; $c = 0.1$; $m = 2$. One can easily verify that for $k_\epsilon = 0.5$ the model implies $n^* = 3$ and $\lambda^{-1} \approx 3.8$, while for $k_\epsilon = 0.4$ the model implies $n^* = 2$ and $\lambda^{-1} \approx 3.6$.

pricing variables are impacted be overconfidence: trading volume is higher in the presence of overconfident traders, confirming empirical findings in the literature. Our results yield further insights into the interaction of overconfidence, information acquisition and price revelation in financial markets.

Appendix

Proof of Lemma 1.

By standard techniques, it is straightforward to see that the average trade by the overconfident can be written as

$$\Theta_o = m_o \lambda_o \frac{b_\epsilon}{\tau \sigma_\epsilon^2} X + (\lambda_o q_o + (1 - \lambda_o) w) P_x$$

where $w = (1/\tau)\left(\gamma (1/d - \gamma) / \sigma_z^2 - 1\right)$ and $q_o = w - (1/\tau) b_\epsilon / \sigma_\epsilon^2$. Similarly the average trade by the rational agents is given by

$$\Theta_r = m_r \lambda_r \frac{1}{\tau \sigma_\epsilon^2} X + (\lambda_r q_r + (1 - \lambda_r) w) P_x$$

where $q_r = w - (1/\tau)/\sigma_\epsilon^2$ Using the market clearing condition (2) we obtain two equilibrium conditions from which (4) and (5) follow. □

Proof of Lemma 2.

An informed overconfident agent t gets ex ante expected utility[29]

$$\mathbb{E}^o [u(W_t)] = -\sqrt{\frac{\mathrm{var}^o(X|Y_t, P_x)}{\mathrm{var}^o(X - P_x)}} e^{\tau c} \tag{21}$$

and an informed rational t agent has expected utility

$$\mathbb{E} [u(W_t)] = -\sqrt{\frac{\mathrm{var}(X|Y_t, P_x)}{\mathrm{var}(X - P_x)}} e^{\tau c}. \tag{22}$$

On the other hand, an uninformed t agent (rational or overconfident)[30] expected utility is given by

$$\mathbb{E} [u(W_t)] = \mathbb{E}^o [u(W_t)] = -\sqrt{\frac{\mathrm{var}(X|P_x)}{\mathrm{var}(X - P_x)}}. \tag{23}$$

For each class of traders (rational or overconfident), the equilibrium fraction of informed traders is set to equate informed and uninformed expected utilities. If such equality does not hold for any value of λ between zero and one, then the equilibrium fraction of informed traders corresponds to the corner solution of one (zero) if the informed (uninformed) achieves higher

[29] The ex-ante utility expressions follow from Admati and Pfleiderer (1987).

[30] Notice that unconditional variances in (23) do not involve the random variable ϵ, hence are equal for rational and overconfident

expected utility. From (22) and (23), it follows that a rational agent will buy information if

$$-\text{var}(X|Y_t, P_x)^{1/2} e^{\tau c} \geq -\text{var}(X|P_x)^{1/2}. \tag{24}$$

If this inequality is satisfied, then it must be that (21) is greater than (23), since $\text{var}^o(X|Y_t, P_x) < \text{var}(X|Y_t, P_x)$. This in turn implies the corner solution $\lambda_o^* = 1$. Condition (24) can be expressed more explicitly as

$$\left(1 + \frac{\gamma^2}{\sigma_z^2}\right) e^{2\tau c} \leq \left(1 + \frac{\gamma^2}{\sigma_z^2} + \frac{1}{\sigma_\epsilon^2}\right).$$

In the interior solution $\lambda_r^* \in (0,1)$, the above inequality holds as an equality. Substituting γ from (4), using $\lambda_o^* = 1$ and solving for λ_r^* we find the expression in the Lemma.[31]

For parameter values such that $\Lambda^* \leq 0$, none of the rational agents would choose to be informed,[32] so $\lambda_r^* = 0$. An overconfident agent will buy information if

$$-\text{var}^o(X|Y_t, P_x)^{1/2} e^{\tau c} \geq -\text{var}^o(X|P_x)^{1/2}. \tag{25}$$

When the above inequality binds as an equality, using γ from (4), the fact that $\lambda_r^* = 0$, writing explicitly (25) and solving for λ_o^* gives the expression in the Lemma. When the inequality in (25) is strict, then $\lambda_o^* = 1$. Finally, notice that Definition 2 rules out the case in which condition (25) is violated. □

Proof of Proposition 1.

Substituting λ_r^* and λ_o^* from Lemma 2, and using (3) in expression (4) for γ, we have that

$$\begin{aligned}\gamma &= \frac{1}{\tau\sigma_\epsilon^2}\left(\lambda_o^* m_o b_\epsilon + \lambda_r^* m_r\right) = \frac{1}{\tau\sigma_\epsilon^2}\left(m_o b_\epsilon + m_r \frac{1}{m_r}\left(\tau\sigma_\epsilon\sigma_z\sqrt{C(\tau)^{-1} - \sigma_\epsilon^2} - m_o b_\epsilon\right)\right) \\ &= \frac{\sigma_z}{\sigma_\epsilon}\sqrt{C(\tau)^{-1} - \sigma_\epsilon^2}.\end{aligned}$$

Therefore, γ is independent of the overconfidence parameters $(m_o,\, b_\epsilon)$. Further note that the price coefficient d only depends on b_ϵ through γ (see equation (5)). Therefore the price function is independent of $(m_o,\, b_\epsilon)$. Price volatility (simply defined as $\text{var}(P_x) = \hat{a}^2 + \hat{d}^2\sigma_z^2$) and rational traders expected utilities ((22) and (23)), only depend on $(m_o,\, b_\epsilon)$ via the price coefficients. The same can be shown for rational expected profits, defined (net of the cost of information) for agent i as $\mathbb{E}[\theta_i(X - P_x)]$. This completes the proof. □

Proof of Proposition 2.

[31] Notice that Definition 3 rules out the the case in which the inequality in (24) is strict, but it does not rule out the limiting case in which $\lambda_r^* = 1$.

[32] In particular, if $\Lambda^* < 0$, then condition (24) would be violated for any $\lambda_r \geq 0$, implying $\lambda_r^* = 0$.

The measure of informed traders, $m_o\lambda_o^*+m_r\lambda_r^*$, is decreasing in overconfidence when $\Lambda^* > 0$, since in this case $\lambda_o^* = 1$ and from expression (3) we have that

$$m_o + m_r\Lambda^* = m_o + \left(\tau\sigma_\epsilon\sigma_z\sqrt{C(\tau)^{-1} - \sigma_\epsilon^2} - m_o b_\epsilon\right)$$

The above expression valued at $b_\epsilon = 1$ corresponds to the measure of informed traders in a fully rational economy, and is decreasing in b_ϵ.

For expected profits, a direct computation shows that for an overconfident informed agent i's trading strategy can be expressed as $\theta_i = b_\epsilon\kappa(Y_i - P_x) + wP_x$, with $\kappa = 1/(\tau\sigma_\epsilon^2)$. It is immediate that we can write the expected profits of an overconfident informed agent as $\pi_o \equiv \mathbb{E}[\theta_i(X - P_x)] = \kappa D + \pi_u$, where $\pi_u = \mathbb{E}[wP_x(X - P_x)]$ are the expected profits of uninformed agents, and $D = \mathbb{E}[(X - P_x)^2]$.[33] Setting $b_\epsilon = 1$ recovers the trading strategy and expected profits for rational informed agents. It is immediate that overconfident agents earn higher expected profits than the rational traders. Furthermore, note that the variance of the profits of the overconfident agents can be expressed as $v_o \equiv \mathrm{var}[\theta_i(X - P_x)] = v_u + b_\epsilon^2\kappa^2 F + 2b_\epsilon\kappa G$, where $G = \mathrm{cov}[(X - P_x)^2, wP_x(X - P_x)]$, and $F = \mathrm{var}[(Y_i - P_x)(X - P_x)]$. Making the dependence of π_o and v_o on b_ϵ explicit, the statement in the proposition reduces to showing that $S(b_\epsilon) \equiv \pi_o(b_\epsilon)/\sqrt{v_o(b_\epsilon)}$ satisfies $S(1) > S(b_\epsilon)$ for all $b_\epsilon > 1$. Some tedious but straightforward calculations show that $S(b_\epsilon)$ actually achieves a maximum at $b_\epsilon = 1$, which is sufficient for the claim in the proposition.

Trading volume is measured in ex-ante terms, as the number of shares that are expected to be traded in the market. Each trader's expected trading volume, T_i, is given by the expectation of the absolute value of his trading strategy, i.e. $T_i = \mathbb{E}\left[|\theta_i|\right]$. Expected trading volume is defined as $V = \int_i T_i di$, where the index of integration runs through all agents (overconfident and rational). Some simple calculations[34] show that

$$V = \sqrt{\frac{2}{\pi}}\left[m_o\sqrt{w^2\mathrm{var}(P_x) + 2Ab_\epsilon + Bb_\epsilon^2} + m_r\left(\lambda\sqrt{w^2\mathrm{var}(P_x) + 2A + B} + (1-\lambda)\sqrt{w^2\mathrm{var}(P_x)}\right)\right];$$

where $A = w^2 d^2\sigma_z^2/\sigma_\epsilon^2$ and $B = \left(1/(\tau\sigma_\epsilon^2)\right)^2\left(\sigma_\epsilon^2 + \mathrm{var}(X - P_x)\right)$. Noting that the trading strategies of the rational agents, in the equilibrium under consideration, are independent of b_ϵ, we have that

$$\frac{\partial V}{\partial b_\epsilon} = \sqrt{\frac{2}{\pi}}m_o\left[\frac{A + Bb_\epsilon}{\sqrt{w^2\mathrm{var}(P_x) + 2Ab_\epsilon + Bb_\epsilon^2}} - \sqrt{\mathrm{var}(w^2\mathrm{var}(P_x) + 2A + B)} + \sqrt{\mathrm{var}(w^2\mathrm{var}(P_x))}\right].$$

In order to see that the above quantity is positive for all b_ϵ the reader can verify (after some

[33] Notice that we abstract from the cost of information, which does not affect any of the results that follow

[34] Using the fact that if $x \sim N(0, \sigma^2)$, then $\mathbb{E}\left[|x|\right] = \sqrt{\frac{2\sigma^2}{\pi}}$.

tedious calculations) that $\frac{\partial V}{\partial b_\epsilon}$ is indeed positive when evaluated at $b_\epsilon = 1$, and that $\frac{\partial^2 V}{\partial b_\epsilon^2} > 0$. This completes the proof. □

Proof of Proposition 3.

An informed rational agent will choose p_r so as to maximize

$$\mathbb{E}\left[u(W_t)\right] = -\sqrt{\frac{\text{var}(X|Y_t, P_x)}{\text{var}(X - P_x)}} e^{\tau c(p_r)} \tag{26}$$

where the above conditional variance depends on p_r, namely

$$\text{var}(X|Y_t, P_x) = \left(1 + \frac{\gamma^2}{\sigma_z^2} + p_r\right)^{-1}. \tag{27}$$

When maximizing (26) agents take the parameters of the price function as given. The first-order condition of (26) with respect to p_r yields

$$2\tau c'(p_r^*)\left[1 + \frac{\gamma^2}{\sigma_z^2} + p_r^*\right] = 1 \tag{28}$$

Similarly, an informed overconfident agent will choose p_o^* such that

$$2\tau c'(p_o^*)\left[1 + \frac{\gamma^2}{\sigma_z^2} + b_\epsilon p_o^*\right] = 1 \tag{29}$$

It is straightforward to show, as in Lemma 2, that no rational agent will become informed unless all overconfident choose to do so. As in the main body of the text we focus then on the case where $\lambda_o^* = 1$. Equating the expected utilities of a rational informed (26) and a rational uninformed agent we get

$$e^{2\tau c(p_r^*)}\left(1 + \frac{\gamma^2}{\sigma_z^2}\right) = \left(1 + \frac{\gamma^2}{\sigma_z^2} + p_r^*\right) \tag{30}$$

where γ is given by (11). Substituting (11) and (28) into (30) we get a quadratic equation for λ_r, whose unique non negative solution yields $\lambda_r^* = \Lambda_{GI}^*$.

The above argument yields the equilibrium value for λ_r^* as long as $\Lambda_{GI}^* \in (0, 1)$. Otherwise the equilibrium λ_r^* is characterized by corner solutions ($\lambda_r^* = 0$ if $\Lambda_{GI}^* \leq 0$ and $\lambda_r^* = 1$ if $\Lambda_{GI}^* \geq 1$). Assume now that the parameters are such that $\Lambda_{GI}^* \in (0, 1)$ and therefore $\lambda_r^* = \Lambda_{GI}^*$. Substituting (4.1) into (11) it is easy to see that (11) is not a direct function of b_ϵ since the first term of (11) cancels out with the first term in (4.1). Therefore $d\gamma/db_\epsilon = 0$ as long as $dp_r/db_\epsilon = 0$. The last condition can be verified by substituting (11) into (28): since γ is not directly a function of b_ϵ then the first-order condition for p_r is not a function of b_ϵ neither. This yields the result that if $\lambda_r^* = \Lambda_{GI}^*$ then $d\gamma/db_\epsilon = 0$.

On the other hand, now suppose that $\lambda_r^* = 1$, i.e. constraint (30) does not bind and all rational agents find it optimal to become informed. Applying the implicit function theorem to (28) we have

$$\frac{dp_r^*}{db_\epsilon} = -\frac{m_o}{m_r\sigma_\epsilon^2}\left(\frac{4c'(p_r^*)\gamma/\sigma_z^2}{4c'(p_r^*)\gamma/\sigma_z^2 + 2\tau c'(p_r^*)/m_r + 2\tau c''(p_r^*)\left(\mathrm{var}(X|Y_i,P_x)m_r\right)^{-1}}\right). \qquad (31)$$

Given the assumption on the cost function, i.e. $c'(p^*) > 0$ and $c''(p^*) \geq 0$, the fraction in parenthesis in the above expression is less than 1. Then, it can be easily checked by substituting (31) into (12) that in this case $d\gamma/db_\epsilon > 0$. □

Proof of Proposition 4.

The proof closely follows those of Lemma 1 and 2. The aggregate trade by the overconfident is

$$\Theta_o = m_o\left(\lambda_o\frac{\mathbb{E}^o(X|Y,P_x) - P_x}{\tau\mathrm{var}^o(X|Y,P_x)} + (1-\lambda_o)\frac{\mathbb{E}^o(X|P_x) - P_x}{\tau\mathrm{var}^o(X|P_x)}\right);$$

whereas for the rational agents

$$\Theta_r = m_r\left(\lambda_r\frac{\mathbb{E}(X|Y,P_x) - P_x}{\tau\mathrm{var}(X|Y,P_x)} + (1-\lambda_r)\frac{\mathbb{E}(X|P_x) - P_x}{\tau\mathrm{var}(X|P_x)}\right).$$

Substituting for the conditional expectations and variances (in particular note that for the informed agents their signal Y is now a sufficient statistic for X, i.e. they do not condition their trade on price) and using the market clearing condition $\Theta_o + \Theta_r = Z$ yields (14).

The description of the equilibrium at the information acquisition stage follows as in Lemma 2, where $\mathrm{var}(X|P_x)$ is now given by (13), and $\mathrm{var}(X|Y,P_x) = \mathrm{var}(X|Y) = 1 + 1/\sigma_\epsilon^2$. Solving for λ_r^* and λ_o^* yields the statements in the Proposition.

Using the expression for γ from (14) we have that when $\Lambda_{GS}^* > 0$

$$\begin{aligned}\gamma &= \frac{1}{\tau\sigma_\epsilon^2}\left(\lambda_o m_o b_\epsilon + \lambda_r m_r\right) = \frac{1}{\tau\sigma_\epsilon^2}\left(m_o b_\epsilon + m_r\frac{1}{m_r}\left(\tau\sigma_\epsilon\sigma_z\sqrt{\frac{(1-C(\tau)\sigma_\epsilon^2)}{(1+\sigma_\epsilon^2)C(\tau)}} - m_o b_\epsilon\right)\right)\\ &= \frac{\sigma_z}{\sigma_\epsilon}\sqrt{\frac{(1-C(\tau)\sigma_\epsilon^2)}{(1+\sigma_\epsilon^2)C(\tau)}}\end{aligned}$$

Therefore, γ is independent of the overconfidence parameters (m_o, b_ϵ). This completes the proof. □

Proof of Lemma 3.

Each agent maximizes his expected trading profits, $\pi_i = \theta_i \mathbb{E}[(X - P_x)]$, i.e. for the rational agents

$$\max_{\theta_i} \quad \theta_i \mathbb{E}(X|Y_i) - \lambda\theta_i^2 - \theta_i\lambda[(n-1)\beta_r + m\beta_o]\mathbb{E}(X|Y_i);$$

which yields the optimal trading strategies

$$\theta_i = \frac{\left(\lambda^{-1} - (n-1)\beta_r - m\beta_o\right)}{2(1+\sigma_\epsilon^2)} Y_i \equiv \beta_r Y_i. \tag{32}$$

Similarly for the overconfident traders we have

$$\theta_i = \frac{\left(\lambda^{-1} - n\beta_r - (m-1)\beta_o\right)}{2(1+k_\epsilon\sigma_\epsilon^2)} Y_i \equiv \beta_o Y_i. \tag{33}$$

Some simple manipulations of (32) and (33) yields (18) for some constant η that satisfies

$$\eta + n\beta_r + m\beta_o = \lambda^{-1}. \tag{34}$$

It is straightforward to see, given the standard properties of normally distributed random variables, that $\mathbb{E}[X|\omega] = \lambda\omega$, where

$$\lambda = \frac{n\beta_r + m\beta_o}{(n\beta_r + m\beta_o)^2 + (n\beta_r^2 + m\beta_o^2)\sigma_\epsilon^2 + \sigma_u^2}. \tag{35}$$

Using (35) with (34), (32) and (33) yields the expression for the equilibrium value for λ, namely equation (19). □

Proof of Proposition 5.

If we let $\lambda^{-1}(k_\epsilon)$ denote the market depth as a function of the overconfidence bias, we have that $\lambda^{-1}(1) < \lambda^{-1}(k_\epsilon)$, for all $k_\epsilon < 1$.[35] The result directly follows from partially differentiating (19) with respect to k_ϵ. Taking into account the condition for the existence of the equilibrium, it is easy to verify that $d\lambda^{-1}/dk_\epsilon < 0$. □

Proof of Proposition 6.

It is straightforward to compute the expected trading profits at equilibrium, $\pi_r = \mathbb{E}\left[\theta_i(X - P_x)\right]$, by the informed rational agents, which are given by

$$\pi_r = \frac{(1+\sigma_\epsilon^2)}{(1+2\sigma_\epsilon^2)^2}\eta\xi \tag{36}$$

[35] This generalizes Benos (1998), who showed $\lambda^{-1}(1) < \lambda^{-1}(0)$.

where

$$\xi = \left(1 + \frac{n}{1+2\sigma_\epsilon^2} + \frac{m}{1+2k_\epsilon\sigma_\epsilon^2}\right)^{-1}$$

Therefore we have

$$\frac{d\pi_r}{dk_\epsilon} = \frac{(1+\sigma_\epsilon^2)}{(1+2\sigma_\epsilon^2)^2}\left(\frac{d\eta}{dk_\epsilon}\xi + \eta\frac{d\xi}{dk_\epsilon}\right)$$

It is easy to verify that

$$\text{sign}\left(\frac{d\eta}{dk_\epsilon}\right) = \text{sign}\left(-1 + 2\sigma_\epsilon^2(1-k_\epsilon)\right);$$

and that $d\xi/dk_\epsilon > 0$. It follows that for $d\pi_r/dk_\epsilon > 0$ a sufficient condition is $2k_\epsilon\sigma_\epsilon^2 > 2\sigma_\epsilon^2 - 1$. The second result follows immediately by considering small changes in the overconfidence parameter when the constraint $\pi_r(n^*) \geq c$ binds. □

References

Admati, A. R., and P. Pfleiderer, 1986, "A monopolistic market for information," *Journal of Economic Theory*, 39, 400 – 438.

Admati, A. R., and P. Pfleiderer, 1987, "Viable allocations of information in financial markets," *Journal of Economic Theory*, 43, 76–115.

Barber, B., and T. Odean, 2001, "Boys will be boys: gender, overconfidence and common stock investment," *Quarterly Journal of Economics*, 116, 261–292.

Benos, A., 1998, "Aggressiveness and survival of overconfident traders," *Journal of Financial Markets*, 1, 353–383.

Bernardo, A., and I. Welch, 2001, "On the evolution of overconfidence and entrepreneurs," working paper, Cowles Foundation.

Caballé, J., and J. Sàkovics, 2003, "Speculating against an overconfident market," *Journal of Financial Markets*, 6, 199–225.

Daniel, K., D. Hirshleifer, and A. Subrahmanyam, 1998, "A theory of overconfidence, self attribution, and security market under- and over-reaction," *Journal of Finance*, 53, 1839–1885.

——— , 2001, "Overconfidence, arbitrage, and equilibrium asset pricing," *Journal of Finance*, 53, 921–965.

DeLong, J., A. Shleifer, L. Summers, and R. Waldmann, 1990, "Noise trader risk in financial markets," *Journal of Political Economy*, 98, 703–738.

——— , 1991, "The survival of noise traders in financial markets," *Journal of Business*, 64, 1–19.

Diamond, D., 1985, "Optimal release of information by firms," *Journal of Finance*, 40, 1071–1094.

Dubra, J., 2004, "Optimism and overconfidence in search," *Review of Economic Dynamics*, 7, 198–218.

Gervais, S., and T. Odean, 2001, "Learning to be overconfident," *Review of Financial Studies*, 14, 1–27.

Glaser, M., and M. Weber, 2003, "Overconfidence and trading volume," working paper, CEPR.

Grossman, S., and J. Stiglitz, 1980, "On the impossibility of informationally efficient markets," *American Economic Review*, 70, 393–408.

Hellwig, M. F., 1980, "On the aggregation of information in competitive markets," *Journal of Economic Theory*, 22, 477–498.

Hirshleifer, D., and G. Y. Luo, 2001, "On the survival of overconfident traders in a competitive securities market," *Journal of Financial Markets*, 4, 73–84.

Kyle, A. S., 1985, "Continuous auctions and insider trading," *Econometrica*, 53, 1315–1336.

Kyle, A. S., and F. A. Wang, 1997, "Speculation duopoly with agreement to disagree: can overconfidence survive the market test?," *Journal of Finance*, 52, 2073–2090.

Odean, T., 1998, "Volume, volatility, price and profit when all traders are above average," *Journal of Finance*, 53, 1887–1934.

Rubinstein, M., 2001, "Rational markets: yes or no? The affirmative case," *Financial Analysts Journal*, 57, 15–29.

Sandroni, A., 2005, "Efficient markets and Bayes' rule," *Economic Theory*, 26, 714–764.

Scheinkman, J., and W. Xiong, 2003, "Overconfidence and speculative bubbles," *Journal of Political Economy*, 111, 1183–1219.

Shleifer, A., 2000, *Inefficient markets: an introduction to behavioral finance.* Oxford University Press, New York, NY.

Shleifer, A., and R. Vishny, 1997, "The limits of arbitrage," *Journal of Finance*, 52, 35–55.

Squintani, F., 2006, "Mistaken self-perception and equilibrium," *Economic Theory*, 27, 615–641.

Statman, M., S. Thorley, and K. Vorkink, 2003, "Investor overconfidence and trading volume," working paper, Santa Clara University.

Thaler, R., 1992, *The winner's curse.* Princeton University Press, Princeton, NJ.

——— , 1993, *Advances in behavioral finance.* Russell Sage Foundation, New York, NY.

Verrecchia, R., 1982, "Information acquisition in a noisy rational expectations economy," *Econometrica*, 50, 1415–1430.

Zábonı́c, J., 2004, "A model of rational bias in self-assessments," *Economic Theory*, 23, 259–282.