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Timing of Investment and Dynamic Pricing in Privatized Sectors*

Ornella Tarola† *and Sandro Trento*‡

Abstract

Firms in equipment-intensive sectors, where investment in production is performed at diminishing marginal cost, spend billions of dollars in equipment and production capacity. Typically, this expenditure is induced by either the replacement of existing equipment, which deteriorates with age and can result in higher operating costs and lower production capacity, or further investment, to benefit from any technological improvement embedded in new equipment. We identify the optimal price policy, and the ensuing optimal sequence of investment timing a privatized firm selects through time and compare them with choices made at the time when such a type of firm was under public-ownership.

JEL Classification: D21, L21, L23.

Keywords*:* Planning investment, dynamic programming, economic behavior, privatization.

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1 Introduction

Firms in equipment-intensive sectors spend billions of dollars in equipment and production capacity. Typically, this expenditure is induced by either the replacement of existing equipment, which deteriorates with age and can result in higher operating costs and lower production capacity, or further investment, to benefit from any technological improvement embedded in new equipment. In these sectors, where investment in production is performed at diminishing marginal cost, firms are traditionally faced with a trade-off, when defining their production capacity: either invest more and take advantage of economies of scale, or invest less, without committing substantial resources for large capacity investments. Of course, when this investment is not well defined and the production capacity does not suffice to meet the current demand, supply shortages may result in lost profit for the firm.

In recent years, with the aim of improving the supply chain and avoid losses, companies have started combining with investment plans innovative pricing policy to affect consumers' demand. In order to increase its demand (shifting its demand curve rightward) a firm may price its products aggressively and this in turn can enable the firm to move along its average cost curve. The joint determination of investment and pricing policy has been quite useful and has generated relevant benefits: not only higher profit, but also reduction in production variability and more efficiency in supply (Chen and Levi, 2004; McGill and van Rjzin, 1999; Cook, 2000).

In the literature, a problem which partially resembles the joint determination of investment and price has been considered by the inventory theory. Admittedly, in the inventory problem, posed by Arrow et al. (1951) and Scarf (1958), the main focus is on *inventory replenishment*, rather than on *equipment replacement.* Also, in the traditional approach the demand pattern is assumed to be exogenously given. Nevertheless, recent developments in this field study the problem of coordinating price and inventory replenishment under several different assumptions and define the optimal policy in terms of the inventory level and pricing (see e.g. Federgruen and Heching 1997, Karakul 2008, and Webster and Weng 2008)[1].

[1]Admittedly, this joint planning is a long-standing problem. Whitin (1955) was the first to add pricing decisions to inventory problem. In his model for there is a fixed cost for each order, and the per unit order cost and inventory holding cost are also given. The demand is deterministic and linear in price. Starting from this pioneristic model and taking into account that in the current reengineering these two planning decisions, pricing and investment, belong to the same area of a firm, a lot of studies have deepened analysis. The problem has been tackled in severel

In this paper, we try to nest the replacement decision on the production-inventory problem. Indeed, we analyse a similar inventory problem, but now identifying *how frequently* a stock of old durables should be *replaced* by a stock of new ones under the assumption that the instantaneous demand varies with price at each point of time. Thus, we define the optimal policy in terms of *dynamic pricing* and *replacement timing*[2].

2 The Modeling Framework

Such a type of problem is particularly relevant in the equipment-intensive sectors, such as water service sector, power generation sector, gas sector and s.o., where under pressures of high quality standards, technical complexities and decaying infrastructure, governments with reduced means to satisfy the new requirements may decide to privatizatize the state-owned firms traditionally in charge of operating the service. Examples of this privatization trend can be found nearly everywhere: the water service is operated by privatized local monopolies in the US; power generation is usually managed by private firms in Europe where private monopolies are even engaged in gas transmission and distribution; during the nineties in several Latin American countries like Argentina and Brazil, generation, transmission and distribution of public utilities - water, transport, telecommunication *inter alia* - were vertically disintegrated, and transmission and distribution became regulated by private monopolies. As private entities, these firms started to adopt a profit-maximization criterion when planning investment — which normally consist in replacing either machines or some components, namely a *fixed stock* of equipment – and use a *tariff policy* in order to manipulate the demand function over time, on the basis of their available production capacity.

Taking into account this scenario, we try to answer the following questions: *What is the optimal price policy, and the ensuing optimal sequence of investment the firm should select through time ? How does this planning decision change when technological advancements take place?*

different scenarios, e.g single selling period vs multi periods, deterministic demand vs stochastic demand, price-dependent demand vs stock, price and time dependent function. Excellent survey papers on joint pricing and inventory control problem are by Yano and Gilbert (2003) and Elmaghraby and Keskinocak (2003).

[2] Quite interestingly, our problem is also related to the macroeconomic literature on vintage, introduced by Terborgh (1949) and Smith (1961) and then developed by Malcomson (1975). In his seminal paper, Malcomson (1975) analyses the investment problem in a vintage model that embeds an obsolescence effect caught by a decreasing cost pattern, and defines a policy of scrapping and replacement of obsolete equipment. Further developments in this field come from Van Hilten (1991), Boucekkine, Germain and Licandro (1997), Boucekkine, del Rio and Licandro, (1999).

We analyse two different cases. First we assume that equipment is affected by a physical deterioration which reduces its production capacity over time. Then, we discuss an extension of the paper that embeds technological advancements. Traditionally, in inventory models equipment is equally productive regardless of the time when it is installed, as technological progress does not take place. However, in reality, capital goods of later date embed new technologies and thus become more and more productive over time (more precisely, newer capital goods are either more productive – process innovation – or produce goods of better quality – product innovation). As a consequence, investment's plans even consist in scrapping and replacement, not only in purely replenishment. With the aim of representing this further case, we assume that each time the firm invests in new equipment, it is *as if* an expansion capacity investment would be undertaken due to the fact that technological advancements increase the production capacity of newer equipment[3].

We characterize the optimal policy which is adopted when the monopolist can manipulate the price trajectory and the resulting sequence of investment timing, taking into account the constraint of meeting demand at each point of time. In both the above described scenarios, we find that the optimal price policy, through which the monopolist can manipulate demand through time, leads to a sequence of time points when to install new equipment which are equally spaced. Interestingly, we find that the distance between two successive dates increases with the technological progress embedded in the equipment. This is due to the fact that each time an investment is undertaken, the production capacity deriving from the new stock of durables is higher than the one associated to the previous vintage capital. As a consequence, *ceteris paribus*, the monopolist can meet the demand for a period which is longer, the more advanced the equipment. Furthermore, the optimal price pattern between two dates at which new equipment is installed falls into two categories: either the price pattern is such that total existing capacity is fully used at each instant between these dates; or the instantaneous monopoly price is used for some period within these dates and, thereafter, until the point of time when new equipment is installed, the price which dampens instantaneous demand at the level of the available capacity is adopted.

Finally, we show that the time between the points when new invest-

[3]Taking into account the impact of technological progress on equipment, whose productive capacity is assumed to increase with technical advancements, this paper brings together the inventory problem literature and the expansion capacity literature, initiated by Manne (1962, 1967) and developed afterward by Srinivasan (1967), D'Aspremont, Gabszewicz and Vial (1972) and Gabszewicz and Vial (1972).

ment is undertaken is longer than the one which would have been selected by a planner facing demand levels corresponding to the instantaneous monopoly price. This results seems to be quite surprisingly. Indeed, when the demand can be affected through price, the profit-maximizing firm finds profitable to replace (or renewal, when technological progress takes place) its production capacity less frequently than a state-owned entities would do.

These findings are *a priori* far from being evident. Why should a stationarity property derive from a profit-maximizing behavior under the constant investment constraint? Also, why should a private monopolist invest less frequently than a state-owned firm? Even obliged to add constant equipment, depending on the interest rate the monopolist could have preferred to manipulate the price from period to period in order to increase or decrease the time between two successive points when to install new equipment, rather than using a constant-cycle policy. Furthermore, keeping old equipment decreases the production capacity and thus reduce the chance to meet the demand over time. This, in turn, could determine lost in profit as said before.

The model is described in section 3. Then (section 4), we fully characterize both the optimal price and investment policies. We summarize our findings in the conclusion.

3 The sketch of the model

We consider a monopolist facing a demand function $D(p)$ as follows:

$$D(p) = A - p$$

where p is the instantaneous price. Due to a deterioration process, the production capacity of the existing equipment decreases linearly over time. Then, this equipment has to be replaced at time when its capacity does not suffice to meet the current level of demand. Indeed, while the existing capacity may exceed the current demand level $D(p)$, no under-capacity is admitted. The sequence of replacements may only consists of lumpy investment in new equipment whose size is constant and equal to $\bar{X}$. The time horizon is unbounded and no technological progress takes place. To sum up, we assume that *(i) the sequence of replacement plans consists of a constant investment size* $\bar{X}$; *(ii) the monopolist sets the price regime* $p(t)$ *for* $t \in [t_i, t_{i+1}[$ *, and any period* $[t_i, t_{i+1}[$ *and, accordingly, selects the timing of investment* $\{t_i\}, t_i = 0, 1, 2...$ *when to install the exogenously given capacity* $\bar{X}$. Dates selected by the monopolist in order to install new equipment are called *regeneration points,* and the interval of time $[t_i, t_{i+1}[$ between two regeneration points is called *cycle.* In each cycle $[t_i, t_{i+1}[$, the deterioration of the equipment $\bar{X}_{t_i}$ installed at the regeneration point t_i makes the production capacity $X_{t_i}(t)$ decreasing over time namely:

$$X_{t_i}(t) = \bar{X} - \alpha(t - t_i), \tag{1}$$

where $\bar{X} = X_{t_i} = X_{t_{i+1}}$.

The investment cost for new equipment of size $\bar{X}$ is equal to:

$$f(\bar{X}) = c\bar{X}.$$

It is worth stressing that while an exogenous sequence of capacity investment would automatically determine the corresponding sequence of regeneration points, when the monopolist can manipulate the price through time, this correspondence does not hold anymore since it depends on the price policy selected by the firm. A *price policy* is a function $p(t)$ which specifies the price fixed by the monopolist at each instant t. Given any price policy, a sequence of regeneration points $(t_1; t_2; ....t_i...) = \mathbf{t}(p(t))$ may be associated to it. Formally, the problem is to find $p(t)$ and, accordingly $\mathbf{t}(p(t)) = (t_1; t_2; ....t_i...)$, so that the objective function $V(\mathbf{t}, p(t))$

$$V(\mathbf{t}, p(t)) = \int_0^\infty p(t)\, D(p)\, e^{-rt} dt - \sum_{t_i=0}^{\infty} c\bar{X}e^{-rt_i}$$

is maximized subject to the capacity constraint

$$D(p(t)) \leq X(t).$$

A policy is said to be *optimal* when it consists of an optimal price pattern through time and, as a consequence, an optimal sequence of dates when to install capacity in order to satisfy demand at each instant of time.

4 The optimal policy

4.1 The optimal price policy within a cycle

We start by considering the optimal price policy within a cycle. Notice that, within any cycle $[t_i, t_{i+1}[$, the objective function $V(\mathbf{t}, p(t))$ achieves its maximum for $p(t)$ given by

$$p(t) = \max(A/2, A - \bar{X} + \alpha(t - t_i))$$

for $i = \max[n \mid t_n < t]$ the integer part of t. This follows from the maximization problem

$$\underset{p(t)}{max} = p(t)D(t; p(t))$$

s.t.

$$D\left(t;p\left(t\right)\right)\leq X_{t_i},t_i=\left[t\right].$$

Whatever the selected sequence of regeneration points, during the cycle $[t_i, t_{i+1}[$, two price regimes may arise, depending on the production capacity $\bar{X}$ with respect to the demand. Assume that at some point of time t, where $t_i \leq t < t_{i+1}$, the capacity constraint is not binding, namely $D\left(t;p\left(t\right)\right) < X_{t_i}$. Then, at the optimal policy, $p(t)$ should be set equal to the maximizing price $p^M = \frac{A}{2}$, as the demand does not need to be dampened. Yet, the demand corresponding to the monopoly price $p^M = \frac{A}{2}$ does not change over time while the current capacity decreases during the cycle $[t_i, t_{i+1}[$. When it happens that the capacity constraint turns out to be binding, namely $D\left(t;p\left(t\right)\right) = X_{t_i}$, then the firm has to choose the price $p^C(t) = A - \bar{X} + \alpha(t - t_i)$ so as to contract the demand $D\left(t;p\left(t\right)\right)$ within the limits imposed by the production capacity. These two price patterns $p^M(t)$ and $p^C(t)$ are called, respectively, *monopoly price regime* and *constrained price regime.* Further, we denote by t_i^* the point of time when the monopoly price regime $p^M(t)$ becomes equal to the constrained price regime $p^C(t)$ and we label this point as the *switching point* t_i^*. It is easy to verify that $t_i^* = \frac{2\bar{X}-A}{2\alpha} + t_i$. Then, three scenarios may arise. In the first scenario the switching point is exactly equal to t_{i+1}, so the optimal price pattern coincides with the monopoly price regime during the whole cycle (scenario A). In the second scenario, the switching point lies between the two investment dates of a cycle $[t_i, t_{i+1}[$; then both these two regimes are used at the optimal price pattern, the first one between t_i and t_i^* and the second between t_i^* and t_{i+1} (scenario B). In the third scenario the switching point lies before t_i or coincides with it; then, the monopolist is forced to use the constrained price regime during the whole cycle in order to meet the capacity constraint (scenario C). We prove below that the switching point t_i^* can never be *exactly equal* to the regeneration point t_{i+1}, excluding thereby scenario A.

Proposition 1 *During any cycle* $[t_i, t_{i+1}[$ *the switching point* t_i^* *belongs either to the interior of the cycle* t_i*, or it is strictly smaller than* t_i*.*

Proof. Assume that, for an exogenously given capacity $\bar{X}$, $t_{i+1}^{'}$, $t_{i+1}^{'} \neq t_{i+1}$, would be the corresponding optimal decision point, and $t_{i+1}^{'} = t_i^*$. Then, the monopolist can quote the monopoly price regime during the whole cycle. Indeed, the production capacity of the equipment installed at time t_i while decreasing over time, can still satisfy the demand level at time $t_{i+1}^{'}$, namely $X_{t_i}^{'} = D_{t_{i+1}^{'}}^M$. Then, the present value of the discounted flow of revenues R during the cycle $[t_i, t'_{i+1}[$ obtains as:

$$R = \int_{t_i}^{t'_{i+1}} \frac{A^2}{4} e^{-rt} dt. \tag{2}$$

Assume now to postpone the investment of the new equipment $\bar{X}$ by δ, where $t'_{i+1} + \delta > t^*_i$. The firm gains the discounted cost $G = c_i \bar{X}(e^{-rt_{i+1}} - e^{-r(t_{i+1}+\delta)})$ saved by postponing the investment. Yet, the demand is not completely satisfied. This induces to switch from the monopoly price regime $p^M_{t_i}$ to the constrained price regime $p^C_{t_i}$. Accordingly, the present value of the discounted flow of revenues R during the cycle $[t_i, t'_{i+1} + \delta[$ turns into

$$R' = \int_{t_i}^{t'_{i+1}} \frac{A^2}{4} e^{-rt} dt + \int_{t'_{i+1}}^{t'_{i+1}+\delta} \left[A(\bar{X} - \alpha(t - t_i)) - (\bar{X} - \alpha(t - t_i))^2\right] e^{-rt} dt \tag{3}$$

where the second integral denotes the revenue stemming from using the constrained price regime between t'_{i+1} and $t'_{i+1} + \delta$. Substracting (3) from (2) yields the loss L resulting from the switch between the two price regimes in the cycle $[t_i, t'_{i+1} + \delta[$:

$$L = \int_{t'_{i+1}}^{(t'_{i+1}+\delta)} \left[\frac{A^2}{4} - \left[A(\bar{X} - \alpha(t - t_i)) - (\bar{X} - \alpha(t - t_i))^2\right]\right] e^{-rt} dt.$$

This loss L is a function of the order of δ^2 over an interval of length δ, so its order is of magnitude δ^3, while the gain G is of first order in δ. Accordingly, for δ small enough the net loss is negative. Then if the investment in new equipment would be postponed, the corresponding discounted profit would increase, which is the desired contradiction. **Q.E.D.**

Also, it is worth noting that the switching point t^*_i can never exceed the point t_{i+1}: in this case, it would be sufficient to postpone the date t_{i+1} when to install the new equipment up to the point where this capacity is equal to $D^M\left(t_{i+1}, p^M(t_{i+1})\right)$, say $t_{i+1} + \delta$, in order to gain the discounted costs, without incurring any reduction of revenue. Thus an investment policy implying excess capacity is never profitable. It follows from above that only two scenarios may be observed at the optimal price policy within a cycle. Either (i) the level of demand at some $t \geq t_i$ can be satisfied by the existing equipment X_{t_i}, for any t_i, then the optimal price policy consists in quoting between the regeneration point t_i and the switching point t^*_i the monopoly price regime and, after the switching point up to the next regeneration point t_{i+1}, the constrained price regime;

or (ii) the level of demand at t_i cannot be met by the existing equipment and then the constrained price regime is quoted within the whole cycle. Notice that for $\bar{X} = \frac{A}{2}$, then $t_i^* = t_i$. Accordingly, the first scenario holds for any $\bar{X} > \frac{A}{2}$, while the second one for any $\bar{X} < \frac{A}{2}$.

Then, we can state the following:

Lemma 2 *When $\bar{X} > \frac{A}{2}$, then $t_i^* > t_i$ and the optimal policy consists in quoting both the monopoly and the constrained price regimes within the cycle $[t_i, t_{i+1}[$; while for $\bar{X} < \frac{A}{2}$, then $t_i^* < t_i$ and the constrained price regime applies during the whole of the cycle.*

We prove now that both in the case when $\bar{X} > \frac{A}{2}$ and in the reverse one, the optimal investment time policy is unique and stationary. However, according to the above lemma, the corresponding optimal price policy differs in both these cases.

Proposition 3 *In the case when the production capacity $\bar{X}$ exceeds the value for which $t_i = t_i^*$, namely $\bar{X} > \frac{A}{2}$, the optimal price policy involves both the monopoly and constrained price regimes; furthermore, the optimal investment timing is unique and stationary.*

Proof. First, notice that the first part of the proposition follows immediately from the first part of the lemma. Consider a specific cycle $[t_i, t_{i+1}[$ with $t_i^* \in [t_i, t_{i+1}[$, and assume that the firm invests at time $t_i + \delta$ instead of time t_i. Then, the cycle $[t_{i-1}, t_i[$ grows longer by δ, while the cycle $[t_i, t_{i+1}[$ becomes shorter and equal to $[t_i + \delta, t_{i+1}[$. Accordingly, the change in the revenue function R due to postponing the investment for δ is between t_i and $t_i + \delta$, and writes:

$$\Delta R = \int_{t_i}^{t_i+\delta} \left(\frac{A^2}{4} - A(\bar{X}_{t_{i-1}} - \alpha(t - t_{i-1})) + \left(\bar{X}_{t_{i-1}} - \alpha(t - t_{i-1})\right)^2 \right) e^{-rt} dt. \tag{4}$$

$\frac{A^2}{4}$ is the equation of a straight line denoting the revenue function if the investment is undertaken at time t_i, while $A \begin{pmatrix} \left(\bar{X}_{t_{i-1}} - \alpha(t - t_{i-1}\right) + \\ -A\left(\bar{X}_{t_{i-1}} - \alpha(t - t_{i-1})\right)^2 \end{pmatrix}$ is the equation of a parabola representing the revenue function if the investment is undertaken at time $t_i + \delta$ and thus, given the capacity $X_{t_{i-1}}$ installed at time $_{t_{i-1}}$ and still available, the constrained price regime is quoted from t_{i-1}^* up to the end of the cycle $t_i + \delta$. Taking into account that $X_{_{t_{i-1}}}(t) = \bar{X} - \alpha(t_{i-1}^* - t_{i-1})$, (4) can be re-expressed as follows:

$$\Delta R = \int_{t_i}^{t_i+\delta} \alpha^2 (t_{i-1}^* - t)^2 e^{-rt} dt$$

The difference in the cost function C writes as

$$\Delta C = c\bar{X}(e^{-rt_i} - e^{-r(t_i+\delta)}) \sim \delta rc\bar{X}e^{-rt_i}$$

up to a term quadratic in δ. Then, the net change writes as the difference between the change in the cost function and the change in the revenue function, namely:

$$\Delta C - \Delta R = \left[cr\bar{X} - \alpha^2(t_i - t_{i-1}^*)^2\right]\delta e^{-rt_i}$$

It is immediate to show that given $\bar{X}$, there is a unique t_i such that the net change cancels out. **Q.E.D.** ■

Now, we move to consider the alternative scenario where the production capacity is such that it does not satisfy the demand level at the regeneration point t_i. We show that even in this case the optimal policy is unique and stationary, although the price policy consists of quoting the constrained price regime within the whole of the cycle.

Proposition 4 *In the case when the production capacity $\bar{X}$ is lower than the value for which $t_i = t_i^*$, the optimal price policy consists of quoting the constrained price regime; furthermore, the optimal investment timing is unique and stationary.*

Proof. First, notice that the first part of the proposition follows immediately from the second part of the lemma.

Consider again a specific cycle $[t_i, t_{i+1}[$ with $t_i^* \in [t_i, t_{i+1}[$, and assume that the firm invests at time $t_i + \delta$ instead of time t_i. Accordingly, the change in the revenue function R due to postponing the investment is between t_i and $t_i + \delta$, and writes:

$$\Delta R = \int_{t_i}^{t_i+\delta} \begin{pmatrix} A(\bar{X}_{t_{i-1}} - \alpha(t - t_{i-1})) - \left(\bar{X}_{t_{i-1}} - \alpha(t - t_{i-1})\right)^2 + \\ -\left(A(\bar{X}_{t_i} - \alpha(t - t_i)) - \left(\bar{X}_{t_i} - \alpha(t - t_i)\right)^2\right) \end{pmatrix} e^{-rt}dt \tag{5}$$

where the first term $A(\bar{X}_{t_{i-1}} - \alpha(t - t_{i-1})) - \left(\bar{X}_{t_{i-1}} - \alpha(t - t_{i-1})\right)^2$ denotes the revenue function when the new equipment is installed at time $t_i + \delta$ while the second term $\left(A(\bar{X}_{t_i} - \alpha(t - t_i)) - \left(\bar{X}_{t_i} - \alpha(t - t_i)\right)^2\right)$ denotes the revenue function when the equipment is installed at time t_i. From the definition of $\bar{X}$ it follows that the above expression can be written as follows:

$$\Delta R = \int_{t_i}^{t_i+\delta} \left(\alpha^2(t_i^* - t_{i-1}^*)(2t - t_i^* - t_{i-1}^*)\right) e^{-rt}dt.$$

Again, the difference in the cost function is

$$\Delta C = c\bar{X}(e^{-rt_i} - e^{-r(t_i+\delta)}) \sim \delta rc\bar{X}e^{-rt_i}$$

up to a term quadratic in δ. Then, the net change writes as:

$$\Delta C - \Delta R = \left[cr\bar{X} - \alpha^2(t_i^* - t_{i-1}^*)(2t_i - (t_i^* + t_{i-1}^*)\right] \delta e^{-rt_i}$$

So, given $\bar{X}$, there is a unique t_i such that the net change cancels out. **Q.E.D.** ■

Starting with this sequence of cycles, we can examine whether the optimal price policy which is adopted by the monopolist leads to cycles which are larger or smaller than the ones resulting from the planning solution

Proposition 5 *The optimal price policy entails a sequence of cycles which are longer than the ones which would have been selected by a planner facing demand levels corresponding to the instantaneous monopoly price.*

Proof. First, notice that at the optimal price regime, the instantaneous monopoly price is never quoted for the whole cycle. Then, the demand function can be either constant from the beginning of the cycle up to the switching point, and then dampened up to the end of the cycle, when the optimal price policy entails alternating both the monopoly and the constrained regimes, or dampened for the whole cycle when the constrained regime is only quoted. Similarly, we have seen that during the cycle resulting from the optimal regime, the production capacity cannot exceed or coincide with the demand level corresponding to the monopoly price. Then, at the optimal cycle the production capacity must be strictly smaller than demand level corresponding to the monopoly price while, at the social planner solution, it is exactly equal to this magnitude. Accordingly, given the production capacity, at the optimal price policy, the cycle during which demand levels are satisfied is longer then the one for which demand levels resulting from the instantaneous monopoly price are served. **Q.E.D**

5 Technological progress

Let us relax now the assumption of no technological progress and assume that owing to technical change, the production capacity associated to the equipment $\bar{X}$ installed at the regeneration point t_i, is $\beta_{t_i}\bar{X}$,with $\beta_{t_i} > 1$, namely:

$$X_{t_i}(t) = \beta_{t_i}\bar{X} - \alpha(t - t_i). \tag{6}$$

Further, let us assume that

$$\beta_{t_i} < \beta_{t_{i+1}}.$$

So, the production capacity $\beta_{t_i}\bar{X}$ of the equipment installed at time t_i is lower than that of the equipment installed at time t_{i+1}[4]. Thus, within any cycle $[t_i, t_{i+1}[$, the objective function $V(\mathbf{t}, p(t))$ achieves its maximum for $p_i(t)$ given by

$$p_i(t) = \max(A/2, A - \beta_{t_i}\bar{X} + \alpha(t - t_i))$$

It is worth noting that the switching point T_i^* writes now as $T_i^* = \frac{2\beta_{t_i}\bar{X} - A}{2\alpha_i} + t_i$. So, *ceteris paribus*, the higher the value of β – that is the more drastic the technological change –, the later the time of the switch between the monopoly price and the constrained regime within any cycle. That it to say that:

$$T_i^* - t_i = \frac{2\beta_{t_i}\bar{X} - A}{2\alpha} > t_i^* - t_i = \frac{2\bar{X} - A}{2\alpha}.$$

It is easy to verify that in this new scenario, the main results which have been proved above still hold. In particular, it can be easily shown that the optimal policy is still unique and stationary, both in the case when $T_i^* > t_i$ and in the reverse one, namely when $T_i^* < t_i$ (it suffices to repeat the proof of Propositions 3 and 4 using $\beta_{t_i}\bar{X}$ instead of $\bar{X}$). However, when technological advancements take place, then the sequence of equipment installed over time entails a production capacity which increases from a cycle to another, namely: $\beta_{t_i - i}\bar{X} < \beta_{t_i}\bar{X}$. Accordingly, even if at the regeneration point t_{i-1}, the production capacity $\beta_{t_{i-1}}\bar{X}$ does not suffice to meet the level of demand corresponding to the monopoly price at that time t_{i-1}, namely $\beta_{t_{i-1}}\bar{X} < \frac{A}{2}$, nevertheless it may happen that the successive investment in new equipment allows to meet the level of demand corresponding to the monopoly price at the new regeneration point t_i, that is $\beta_{t_i}\bar{X} > \frac{A}{2}$. Thus, Proposition 4 is reformulated as follows. Let t_m be the first regeneration point at which the production capacity can meet the demand corresponding to the monopoly price at that time, namely $\beta_{t_m}\bar{X} = \frac{A}{2}$. Then:

Lemma 6 *The optimal price policy consists up to the cycle $[t_{m-1}, t_m[$ in quoting the constrained price regime, adjusted in each cycle; and then, from the cycle $[t_m, t_{m+1}[$ on, in a monopoly and a constrained regimes within each cycle. The resulting distance between two successive regeneration points tends to increase from cycle to cycle, as the production capacity increases due to technological advancements.*

[4]It is immediate to see that (6) coincides with (1) when $\beta_{t_i} = \beta_{t_{i+1}} = \beta = 1$.

6 Conclusion

In this paper, we have considered an alternative version of the inventory problem, and identified the optimal investment timing and pricing policies when the size of the investment is fixed. Further, taking into account the impact of technological progress on equipment, whose production capacity is assumed to increase with technical advancements, this paper brings together the inventory problem literature and the expansion capacity literature, where it is assumed that demand grows exogenously over time. Our results are in line with both the (S,s) policy which has been proved to hold in several different variants of the inventory problem and with the optimal stationary policy which typically arises in the capacity expansion models. Also, we have found that a private firm rather than investing at the time when the production capacity does not suffice to meet the demand level at that time, prefers – as this choice is more profitable – to dampen the demand function and postpone the investment. Indeed, when a market environment is considered, the time between two investments is longer than the one spent under public ownership. It is worth noting that from a profit-maximization view point, the above result is in line with the well known problem of over-investment under public ownership. Indeed, overinvestment has been a common characteristic of most public utilities in the past few decades. As a consequence of this, an overcapacity phenomenon has been observed wideworld, with severe economic costs. Quite interestingly however, we also show that a privatized firm in order to follow a profit-maximizing rationale is likely to not catch all the existing technological opportunities, as it invests less frequently than a public enterprise would do. As advances in technology tend to be embodied in the latest vintages of capital, (namely new capital is better than old capital), the longer the time between successive investments, the slower the technological advancements for this firm.

There is at least a further direction on which to build upon our model, namely considering a firm facing a demand function which first increases and then decreases over time and thus verifying the optimal firm's behavior in an economy characterized by a sort of cycle in the economy. Although there is a large body of macroeconomic literature analyzing this topic, nevertheless it could be interesting to nest this problem in our model as specific case in order to evaluate it from a microeconomic view point.

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