# Banking and the Determinants of Credit Crunches

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#### Abstract

Why do banks suddenly tighten the criteria needed for credit? Credit crunches are often explained by the implementation of new regulatory rules or by sudden drops in firm quality. We present a novel model of an artificial credit market and show that crunches have a tendency to occur even if firm quality remains constant, as well as when there are no new regulatory rules stipulating lenders capital requirements. We find evidence in line with the asset deterioration hypothesis and results that emphasise the importance of accurate firm quality estimates. In addition, we find that an increase in the debts' time to maturity reduces the probability of a credit crunch and that a conservative lending approach is intrinsically related to the onset of crunches. Thus, our results suggest some up till now partially overlooked components contributing to the financial stability of an economy.

Keywords: lending, screening, agent based model, financial stability

**JEL:** C63, E51, G21

## 1 Introduction

During a timespan of over twenty years, from the early nineties to present date, nearly all developed countries have experienced some form of supply side credit crunch in parts of their economies. During a crunch, seemingly eligible borrowers find it hard to get credit under reasonable terms, forcing firms that rely on external capital to a halt. Why do providers of credit suddenly mobilise their lending strategies in such a way? Existing theories provide a useful platform when building an understanding of the determinants of crunches. According to the Risk-Based Capital hypothesis (RBC),

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the implementation of new risk-based regulatory rules governing lenders' allocation of resources, may have a significant negative impact on the supply of credit. Berger and Udell (1994) tested the RBC hypothesis on the perceived crunch in the US after the implementation of the first Basel Accord in the late eighties/early nineties. They found some support in favor of the RBC hypothesis but refrained from ruling out competing theories. Sharpe (1995) on the other hand claimed that banks reduce credit supply due to unpredicted losses in bank capital. In analogy with the RBC hypothesis, he concluded that the reduction in credit coincides with banks having difficulties in meeting the minimum regulatory capital requirements. Pazarbasioglu (1996) found evidence in line with the asset deterioration hypothesis, suggesting that banks become less willing to supply credit during periods associated with a deterioration in asset quality. In addition, according to the financial instability hypothesis, first discussed by Minsky (1975, 1992), economies have a tendency to naturally evolve into a "Ponzi" phase in which firms are forced to borrow to meet their obligations on existing liabilities. Since lenders judge liability structures subjectively, sudden drops in the supply of credit may occur when corporate debt reaches some unforeseen

A common aspect in these theories is that some "out of lender" controlled event is to blame for sudden drops in credit supply. The driving force is either an evolutionary force in firm liability structures, regulatory changes or some unresolved issue of asymmetric information characterising the credit market. Thus, existing theories pay little attention to the "inlender" determinants of credit crunches. Turning to the existing literature on banking, lenders are forced to deal with excessive information asymmetry problems since borrowers have reason to withhold information in order to gain credit. Lenders seek to resolve this problem by practicing screening, Allen (1990), and monitoring, Winton (1995), thus reducing their exposure to counter party risk. If the estimates used in these procedures are based on subjective judgments of acceptable liability structures or fail to incorporate risks driven by exogenous shocks, such shocks may lead to a reduction in credit supply due to unforeseen losses. This suggests that the estimates used in the lenders' screening procedures influence the credit supply. Hence, the decisions made by suppliers of credit play an important role in the evolution of debt and ultimately on the eruption of credit crunches.

In this paper, we focus on the decisions made by lenders and find that the level of conservatism practiced when banks pursue their internal credit risk goal plays a key part in determining the onset of crunches. We also find that an increase in the debt's time to maturity, reduces the probability of a credit crunch. In addition, we are able to confirm the importance of accurate estimates in the banks' screening procedures and find evidence in line with the asset deterioration hypothesis. The findings are based on simulations of a novel artificial agent based economy derived from a fairly general banking

model in which lenders screen applicants in order to reduce their exposure to default risk. We do not focus on the distinctive nature of asset value deterioration, nor do we focus on the implementation of new risk-based regulatory rules. We track the determinants of crunches from a different angle, by analyzing the banks' profit maximising decisions under various economic conditions. Due to the dynamic and complex nature of credit crunches, we derive an Agent Based Model (ABM) of the credit market and show that sudden drops in the supply of credit will evolve even if lenders have close to perfect information about their debt contract counterparts. The simulations show that the credit market has a natural tendency to evolve into periods characterised by over lending, in which banks acquire riskier debt than what is specified by its profit maximising condition. Such periods are swiftly followed by periods in which banks try to cut back on risky debt, making credit difficult to obtain. If such "cutbacks" are coordinated across banks, the market may experience the eruption of a credit crunch. These crunches are seemingly spontaneous but highly dependent on the speed by which lenders adjust to internal credit risk goals, the debts time to maturity, the spread between lending and deposit rates, accurate firm quality estimates and the parameters defining the evolution of firm assets. We link the speed of adjustment to the level of conservatism within the bank's organizational structure. The real world equivalence can be thought of as lenders' willingness to engage in new risky ventures or their willingness to use new and unexplored debt instruments.

The outline of the paper is as follows. The next section discusses the theoretical underpinnings of the model. This is followed by a description of the artificial economies model environment and the conditions driving the behaviour of the agents. In the final sections we present and discuss the results derived from the simulations and conclude.

## 2 Theoretical underpinnings

A credit crunch is defined as a period of time in which credit and investment capital is hard to obtain. Viewing banks as financial intermediators and providers of investment capital, this definition suggests that the onset of credit crunches are related to the banks' screening and monitoring procedure. The information production in imperfect screening and its adverse pool effects have been studied previously by Broecker (1990), Chiesa (1998) and Gehrig (1998) among others. This section considers the case in which banks practice perfect screening in order to reduce their exposure to credit risk. In contrast with previous studies, we consider a continuum of firm qualities and view screening as truncating the distribution function defining firm quality.

Consider a two-period economy under the supervision of a financial au-

thority. The economy is made up of a finite number of risk-neutral firms, k=1,...,M, and banks, i=1,2,...,N, providing unsecured credit to firms. Firms are assumed to be heterogeneous in terms of quality summarised by  $\theta_k \in [0,1]$ , which is privately known by the firms. At the initial date, firms are given the choice of carrying out a risky project lasting one time period. In order to undertake the project, firms need to raise external capital equivalent to  $l_k$  on the credit market. The gross return of the investment,  $R(\theta_k) \in [0, \infty]$ , is realised after one time period and retrieved with probability  $1 - \theta_k$ . Firm returns are increasing in  $\theta_k$  such that firm quality also represents the riskiness of firm actions. A high quality firm is thus characterised by a low value of  $\theta_k$ . The distribution of firm returns are binary and the success rate of the investment is firm size independent. For simplicity, it is assumed that in case of failure the firm defaults without liquidation value, allowing us to interpret  $\theta_k$  as the firm's probability to default. Thus, firms are protected by limited liability such that they only care about the payoffs when the project succeeds. As such, the firms always implement their projects when granted a loan.

As in Diamond (1984), banks act as information producers about the firms' investment projects. Here, it is assumed that banks observe the density of firm quality,  $f(\theta)$ , from which they make a noisy firm estimate  $\theta_{k,i}^b$ . We let the interest rate on external capital, r, and the deposit rate,  $\rho$ , be exogenous to the model and assume that lending is the banks' only source of profit. Given the above, the representative bank's unconditional expected profit function is:

$$\pi_b^e = \sum_{k}^{m \in M} \left[ (1 + r_l)(1 - \theta_k^b) - 1 \right] l_k - \rho D$$
 (2.1)

where m is the subgroup of firms facing their demand towards the representative bank and D is the bank's deposits. To purely study the effects of lending while ignoring the bank's exposure to deposit risks, it is assumed that the bank finances lending using a stock of own capital, i.e. equity. The bank's equity is given by E such that  $\sum_{k}^{\hat{m}\in M} l_k \leq E$  and  $E - \sum_{k}^{\hat{m}\in M} l_k \geq \hat{E}$  where  $\hat{E}$  is the minimum capital requirement as decided by the financial authorities and  $\hat{m}$  is the number of firms granted credit. As such, the deposit costs in (2.1) can be ignored. Thus, the interest rate on external capital, r, is interpreted as the spread between the lending and the deposit rate. Since banks observe the distribution of firm quality, the banks' beliefs about  $\theta_k$  are taken on M. Using this, we rewrite the representative bank's unconditional expected profit function in (2.1) as:

$$\pi_b^e = [(1+r)(1-\theta^e) - 1] \sum_{k=0}^{m \in M} l_k$$
 (2.2)

where  $\theta^e$  is the expected default rate (quality). From the bank's expected

profit function in (2.2), it is fairly obvious that above some value of  $\theta^e$ , expected bank-profit turns negative. More specifically, in the unconditional case the bank only participates on the credit market if:

$$\theta^e \le r/(1+r)$$

However, as discussed by Gehrig (1998), when a contract is negotiated, banks may prefer to screen applicants in order to assess their credit risks. As such, it is assumed that the bank resolves the possibility of negative profits by screening applicants to identify risky firms which are removed from the bank's credit portfolio. We assume for the remainder of this section that the bank has the ability to practice perfect and costless screening such that  $\theta_k^b = \theta_k$ . This allows us to study the bank's first best solution and to derive the determinants of the requirements needed for credit.

The process of screening loan applicants can be thought of as discriminating between firms and only picking firms that live up to some minimum requirements for credit. Thus, we view screening as choosing a suitable value of a truncating function  $\lambda$ , constructed to be the function that solves:

$$E[\theta|\theta_k \le \theta^*(\lambda)] = \lambda \theta^e, \quad 0 \le \lambda \le 1 \tag{2.3}$$

where  $\theta^*(\lambda)$  is the truncation point on  $f(\theta)$ , monotonically increasing in  $\lambda$ . The criterion needed for credit is represented by  $\theta^*(\lambda)$  and the expression in (2.3) states the expected default rate (quality) in the subpopulation of firms below the truncation point, i.e. the conditional expected default rate. The distribution of firm quality for some general distribution is displayed in Figure 1 in which we see that the bank screens applicants and reduces its exposure to default risk by truncating the distribution of firm quality. However, given any probability density function of firm quality for  $\theta_k \in [0,1]$  and a finite sample of firms (M), by truncation of  $f(\theta)$ , the bank will reduce the sample size of eligible firms forcing a reduction of the bank's credit supply. To see this, we acknowledge that  $\hat{m} = \hat{m}(\theta^*) \in m$  is the number of firm's with  $\theta_k \leq \theta^*$  such that  $\partial \hat{m}/\partial \theta^* > 0$ 0. Since  $\sum_{k}^{\hat{m}(\theta^*) \in m} l_k \leq \sum_{k}^{m \in M} l_k$ , there exists some profit maximising value of  $\lambda$  and the bank's optimisation problem boils down to a decision between quality and quantity of credit. Hence, if the bank tightens the criterion needed for credit, i.e. it reduces  $\theta^*$ , fewer firms will default on their loans but the supply of credit will drop, reducing the bank's potential profits. This crucial link between the bank's credit supply and the screening procedure of loan applicants provides a useful platform when forming a understanding of the determinants of credit crunches.

To explore this link we acknowledge that the bank's expected credit supply function can be written as the product of the m firms' demand for credit

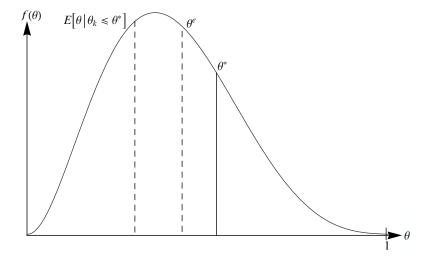


Figure 1: Screening reduces the expected default rate.

and the probability that a firm meets the requirements of the bank:

$$L(\lambda) = \sum_{k}^{m \in M} l_k \int_0^{\theta^*} f(\theta) \ d\theta \tag{2.4}$$

For tractability, let the expected credit supply function be based on the profit maximising value of  $\theta^*$ . This allows us to define a weight,  $\omega$ , that scales the now constant probability in (2.4). Since  $\theta^*$  is monotonically increasing in  $\lambda$  and since  $E[\theta|\theta_k \leq \theta^*(\lambda)]$  is linear in  $\lambda$ , we solve the bank's expected credit supply function by scaling  $\omega$  with  $\lambda$ , restricting the weight to positive values. This allows us to rewrite (2.4) as:

$$L(\lambda) = \lambda \omega \sum_{k}^{m \in M} l_k \tag{2.5}$$

Combining (2.5) with the definition of the conditional expected default rate in (2.3) and the bank's expected profit function in (2.2) gives us the bank's *conditional* expected profit function:

$$E[\pi_b | \theta_k \le \theta^*(\lambda)] = [(1+r)(1-\theta^e \lambda) - 1] \lambda \omega \sum_{k=0}^{m \in M} l_k$$
 (2.6)

Maximising (2.6) with respect to  $\lambda$  and simplifying results in the bank's first order condition<sup>1</sup>:

$$\partial E[\pi_b | \theta_k \le \theta^*] / \partial \lambda = \omega \left[ r - 2(1+r)\theta^e \lambda \right] \sum_{k=0}^{m \in M} l_k = 0$$
 (2.7)

 $<sup>^{1}\</sup>mathrm{For}$  illustrative reasons the regulatory bodies restriction is ignored.

such that  $\lambda^* = \lambda^* (\theta^e, r)$  conditioned on the profit maximising value of  $\theta^*$ . More specifically, we use the first order condition in (2.7) and solve for the profit maximising value of the truncating function which is stated as:

$$\lambda^* = \frac{r}{2\theta^e(1+r)} \tag{2.8}$$

Since  $\partial \lambda^*/\partial r > 0$  and since  $\theta^*$  is monotonically increasing in  $\lambda$ , the model predicts an increase in  $\theta^*$  if the spread between the lending and the deposit rate is increased. In addition, since  $\partial \lambda^*/\partial \theta^e < 0$ , we conclude that the bank tightens the criterion needed for credit if the unconditional expected default rate is increased. Combining (2.8) with the definition of the bank's conditional expected default rate in (2.3), we get the bank's profit maximising conditional expected default rate, expressed only as a function of the interest rate spread:

$$E[\theta|\theta_k \le \theta^*(\lambda^*)] = \frac{r}{2(1+r)}$$
(2.9)

The expression in (2.9) highlights the importance of interest rates on the criterion needed for credit and since  $\partial E[\theta|\theta_k \leq \theta^*(\lambda^*)]/\partial r > 0$ , we conclude that an increase in the spread between the lending and the deposit rate increases the amount of credit risk undertaken by banks. In addition, by substituting for (2.8) and (2.9) in (2.6), we express the bank's profit maximising conditional expected profit function in terms of the models exogenous variables:

$$E[\pi_b | \theta_k \le \theta^*(\lambda^*)] = \omega \sum_{k=0}^{m \in M} l_k \frac{r^2}{4\theta^e (1+r)} > 0$$
 (2.10)

with  $\partial E[\pi_b|\theta_k \leq \theta^*(\lambda^*)]/\partial r > 0$  and  $\partial E[\pi_b|\theta_k \leq \theta^*(\lambda^*)]/\partial \theta^e < 0$ . Studying the implications of (2.10), we see that the bank expects positive profits by screening out unwanted firms, conditioned on perfect estimates of firm quality. However, by viewing the bank's expected profits as expected revenue, (2.10) also corresponds to the bank's expected deposit costs in a perfectly competitive economic environment.

Summing up our findings so far, in this section we have derived a simple theoretical banking model in which banks maximise profits by screening applicants and removing risky firms from its credit portfolio. Despite its simple construct, the model is able to highlight the importance of firm quality and the spread between lending and deposit rates on the criterion needed for credit. Since a credit crunch is defined as a period in time in which credit and investment capital are hard to obtain, we argue that a tightening of the criterion needed for credit, and its determinants, is intrinsically related to the onset of a credit crunch. Despite this however, the theoretical model fails to capture the distinctive nature of credit crunches. Credit crunches are

by definition dynamic phenomena since the tightening of the criterion needs to be coordinated across banks throughout a period of time. By viewing the credit market as a complex adaptive system, we proceed with constructing an artificial credit market based on the insights from the theoretical model presented in this section.

## 3 An artificial credit market

Through the theoretical two-period model in the previous section, we found variables that influence the representative bank's decision regarding the criterion needed for credit. However, the model fails to capture the dynamics of a credit market. In addition, it provides us with few new insights concerning the distinctive nature of credit crunches. To cope with these issues, this section expands the model by viewing the credit market as a complex adaptive system, as defined in Tesfatsion (2006). As such, we construct an Agent Based Model (ABM) of a credit market based on repeated debt contracts. The theoretical model in the previous section is used as the base on which we build the new model. This allows us to compare and validate the results derived in this section with the theoretical results derived in the previous section. We first discuss the details of the artificial economy and then derive the decision rules governing the agents' behaviour, making the model suitable for a complex dynamic economic environment with repeated outcomes.

### 3.1 The model

The ABM is set on a finite spaced torus populated with an initial number of firms (k=1,...,M) and banks (i=1,...,N) spread out on a grid at random. By situating the agents on a torus we are able to simulate the liquidity of the credit market as discussed and defined in detail later in this section. Time is discrete and represents new possible debt-contracts and/or maturity dates. Banks are governed by a financial authority stipulating a regulatory rule requiring banks to hold own capital based on the Capital Adequacy Ratio (CAR) such that for any given bank and time:

$$CAR_{i,t} \ge K, \quad 0 \le K \le 1 \tag{3.1}$$

where K represents the minimum capital requirements. All debt owned by the bank is unweighted and the sum of a bank's Tier-capital is equivalent to the bank's equity capital, henceforth referred to as the banks equity. In analogy with the theoretical model in the previous section, firm quality,  $\theta_k$ , is measured as the firms' ability to repay the debt, i.e. the probability of default and banks truncate firm quality to maximise profits. Firm quality is drawn from a truncated two parameter beta distribution,  $\theta \sim \text{Beta}(\alpha, \beta)_{|\theta_k < \mathcal{T}}$ , where the beta distribution is chosen for its ability to replicate bounded distributions of firm quality.<sup>2</sup> At every time step, firms search the torus for external capital through a  $360^0$  random walk. The torus is of size  $b^2$  where  $b \in \mathbb{Z}$  divisible with remainder.

When a firm encounters a bank, the firm states its demand for credit which the bank evaluates according to the regulatory rule in (3.1). If the bank lives up to the requirement, it makes a noisy estimate of the firm's probability of default,  $\theta_{k,i}^b = \theta_k + \phi_{k,i}$  with support  $[0, \mathcal{T}]$  where  $\phi_{k,i}$  is a random draw from a normal distribution with mean 0 and standard deviation  $\sigma^f$ . Estimates outside of the support region are re-estimated. If the bank's estimate of firm quality is below the truncation point, a debt contract is formed. The debt lasts for, at least,  $\kappa$  time periods and is repaid upon the firm-bank encounter, making the maturity date of the contract stochastic. Thus, we allow for different maturity dates without specifying the details in the contract. In addition, firms are restricted from lending until it repays the debt with interest. If the bank rejects the firm's demand for credit, the firm continues its search for a debt contract.

Given the above, the probability of a firm-bank encounter depends on the debts minimum time to maturity  $(\kappa)$ , the size of the torus (b) as well as on the number of firms  $(M_t)$  and banks  $(N_t)$  active on the credit market. Since a firm previously in debt is restricted from signing a new debt contract until the previously acquired debt is repaid and since a debt contract is formed upon a firm-bank encounter, these parameters implicitly define the liquidity of the credit market. As such, we denote market liquidity as  $\psi(\kappa, b, M_t, N_t)$ where the last three parameters defines the density of the credit market. When exploring the properties of liquidity as defined above, we acknowledge that a sparsely populated credit market, relative to the size of the torus, may experience random demand-side drops in credit reducing the overall indebtedness of firms. However, if  $\psi$  is large, sudden drops in the aggregate debt level only reflects the decisions made by the suppliers of credit allowing us to vary  $\psi$  when simulating variations in the liquidity of the credit market. Given the above, we state the probability of a debt contract being formed by bank i at any given date as:

$$Pr(\text{Contract}_{i,t}) = h(\psi(\kappa, b, M_t, N_t), Pr(\theta_k \le \theta_{i,t}^*), Pr(\text{CAR}_{i,t} \ge K))$$
 (3.2)

The first term in (3.2) determines how the frequency in the debt contract formation is affected by market liquidity. The second two terms determine how the probability of a debt contract is affected by the supply side of credit.

Using the definition of a credit crunch as a period in time in which credit and investment capital is hard to obtain, sudden reductions in the supply of credit can be tracked back to the frequency in which new debt

<sup>&</sup>lt;sup>2</sup>The truncation is motivated by the equation of motion defining firm asset values, discussed in detail below.

contracts are formed. Since the probability in (3.2) depends on the capital adequacy ratio as well as the acceptable level of credit risk, the model has the ability to capture effects on credit crunches caused by the implementation of new regulatory rules as well as the effects caused by a deterioration in firm quality. However, since reductions in credit supply needs to be coordinated across banks in order for a credit crunch to erupt, we state the probability of a debt contract being formed by any bank at time t as:

$$Pr\left(\operatorname{Contract}_{t}\right) = Pr\left(\bigcup_{i=1}^{N_{t}} \operatorname{Contract}_{i}\right)$$
 (3.3)

Hence, the complement of (3.3) defines the probability that no debt contract will be signed at time t, arguably an important component determining the probability of a credit crunch. Since the probability that a contract will be signed at time t depends on  $Pr(CAR_{i,t} \leq K)$ , the probability in (3.3) relates to the bank's ability to build up equity, dependent on the bank's expected profit and choice of criterion needed for credit. As we will see, this in turn depends on the bank's previous encounters. In addition, due to the finite number of firms, the bank's debt portfolio is indirectly dependent on the debt portfolios of its competitors due to random spill-over effects of counter party risk. Returning to (3.2) and acknowledging that the probabilities by this reasoning are dependent, we see that the model is complex, motivating the use of simulation techniques when determining the determinants of crunches.

### 3.2 The Firms

Firms are assumed to be born debt free with a pre-specified initial value of equity,  $E_0^f$ , identically distributed across firms. Firms are defined by the balance sheet identity allowing us to write the asset value of a representative firm as:

$$A_t^f = E_t^f + L_t^f, \quad t \ge 1$$

where  $A_t^f$  is the firm's asset value,  $E_t^f$  is the firm's equity value and  $L_t^f$  is the value of firm liabilities at time t. As discussed in the previous section, firms always implement their projects when granted credit. However, the demand for investment capital may vary between time periods. Using this, we let a random draw from the firm's equity value represent the firm's demand for credit:

$$l_t = \eta_t E_t^f$$

where  $\eta_t \sim U(0,1)$  resulting in  $0 \leq l_t \leq E_t^f$ . If the bank's estimate of firm quality lies below the truncation point and the bank meets the requirements made by the model's regulatory body, the firm is granted credit from the bank to fund a risky project. The project lasts until the loan's maturity date on which the firm generates a gross return of  $R_{T^m}$  if the project succeeds, where  $T^m = t + \tau(\kappa)$  denotes the loan's maturity date with  $\partial \tau / \partial \kappa > 0$ .

When determining the equation of motion defining the evolution of the firms' asset values, we use the results in the work of Black and Scholes (1973) and Merton (1974) such that the probability that the firm defaults on its loan can be derived from the firm's asset value. Assuming that the firm fails to meet its obligations to the bank if  $A_t^f < L_t^f$ , we write the equation of motion defining the representative firm's asset value as:<sup>3</sup>

$$A_{T^m}^f = A_{T^m - 1} + \frac{E_{T^m - 1}}{\Phi^{-1}(\theta)} \Delta W_{T^m}$$
(3.4)

where  $\Phi^{-1}(\theta)$  is the inverse of the standard normal distribution taken at firm quality and where  $\Delta W_{T^m} \sim N(0,1)$ . Note that (3.4) requires  $\mathcal{T} < 0.5$  such that  $\theta \in [0, 0.5]$  due to the symmetry of the standard normal distribution. Given (3.4), the asset value of the firm remains constant between maturity dates and the firm defaults with probability  $\theta$  when the project's profit is realised. If the asset value of the firm drops below zero, the firm files for bankruptcy and fails to meet its obligations to the bank. Thus, we have a steady flow of firms exiting the credit market through bankruptcy. The firm-entry process is governed by a simple rule requiring the number of firms active in the credit market at time t to be approximately equal to the constant and pre-specified finite number of firms,  $M_t \approx M$ . Hence, in every time period the model gives birth to  $d_{t-1}$  new firms, where  $d_{t-1}$  is the number of firm defaults in the previous time period. Since firms with a high value of  $\theta_k$  have a high probability of default and since  $\theta_k$  is drawn from the truncated beta distribution, a consequence of the firm-entry process is that  $\partial \theta_t^e / \partial t < 0$ , i.e. the economic environment grows "safer" with time.

#### 3.3 The Banks

In analogy with the theoretical model in the previous section, banks use equity to provide firms with loans. The equity value of the banks at the initial date,  $E_0^b$ , is pre-specified and identically distributed across banks. At the end of each time period, banks will have accumulated profits from matured loans, funded new projects using its equity and suffered from defaulted loans. Using this, we construct the equation of motion defining the representative bank's asset value from the balance sheet identity such that:

$$A_t^b = \left(E_{t-1}^b + \pi_t^b\right) + \left(L_{t-1}^b + \sum_{k}^{m_t^n \in m_t} l_k - \sum_{k}^{m_t^d \in \hat{m}_t} l_k\right), \quad t \ge 1$$

where  $A_t^b$  is the bank's asset value,  $E_t^b$  is the bank's equity,  $L_t^b$  is the value of the bank's outstanding debt,  $m_t \in M_t$  is the number of firms facing their demand towards the representative bank,  $\hat{m}_t^n$  is the number of firms granted

<sup>&</sup>lt;sup>3</sup>See Appendix A for details.

credit at time t,  $\hat{m}_t$  is the number of firms in the bank's debt portfolio at time t and where  $m_t^d$  is the number of firms repaying their debt at time t.

As in the previous section, banks screen applicants to maximise profits by truncating the distribution function defining firm quality. The functional form of the truncating function can be specified in various ways reflecting the decision making process within the bank. This makes the model flexible for variations in corporate structure. Here, we assume that the bank's management has absolute control over the truncating function allowing us to treat  $\lambda$  as the bank's decision variable. As such, the solution to the bank's optimisation problem in the artificial economy bears obvious resemblance to the results derived in the previous section. To see this, define the value of the truncating function at time t as  $\lambda_{i,t}$ . Using the results in the previous section and acknowledging that the banks now rely on noisy estimates of firm quality, we rewrite the representative bank's objective function as:<sup>4</sup>

$$E[\pi_{b,t}|\theta_{k,t}^b \le \theta_t^*] = [(1+r)(1-\theta_t^e \lambda_t) - 1] \lambda_t \omega_t \sum_{k=0}^{m_t \in M_t} l_k$$
 (3.5)

We condition on the profit maximising value of  $\theta^*$  and maximise (3.5) with respect to  $\lambda_t$ , including the regulatory bodies constraint (3.1). This gives us the optimal value of the truncating function for the representative bank in the artificial economy:

$$\lambda_t^* = \begin{cases} \frac{r}{2\theta_t^e(1+r)} & \text{if } CAR_t > K \\ 0 & \text{if } CAR_t \le K \end{cases}$$

indicating that in an economic environment with fixed interest rates, the criterion needed for credit only varies with the estimate of  $\theta^e_t$ . Since  $\partial \theta^e_t/\partial t < 0$  and since  $\partial \lambda^*_t/\partial \theta^e_t < 0$  it follows that  $\partial \theta^*_t/\partial t > 0$ , using that  $\theta^*_t$  is monotonically increasing in  $\lambda_t$ . Stated differently, banks tend to take on more risky debt as the economy evolves.

However, the economy will suffer from short term fluctuations around the time path of the criterion needed for credit due to noisy estimates of firm quality. To see this, we acknowledge that  $E[\theta|\theta_k \leq \theta_t^*(\lambda_t^*)] \neq E[\theta|\theta_k^b \leq \theta_t^*(\lambda_t^*)]$  where the inequality is due to imperfect estimates of firm quality.<sup>5</sup> It is reasonable to assume that banks learn about the quality of firms by interim information production, Besanko and Kanatas (1993) and Holmström and Tirole (1997). Thus, we assume that the bank observes the true quality of firms for the subpopulation of firms currently in its debt portfolio. Using this, we let the bank have adaptive expectations of (2.3) such that  $E[\theta|\theta_k^b \leq \theta_t^*(\lambda_t^*)] = \sum_k \hat{m}_{t-1}^{\hat{m}_{t-1}} \theta_{k,t-1}/\hat{m}_{t-1}$ . Relating this to the profit maximising conditional default rate in (2.9), we let the bank solve for the point of

<sup>&</sup>lt;sup>4</sup>Since  $E[\theta^b] = E[\theta] + E[\phi] = \theta^e$ .

<sup>&</sup>lt;sup>5</sup>See Appendix B for details.

truncation by an iterative procedure stated as:

$$\theta_t^* = \begin{cases} \theta_{t-1}^* - c, & \text{if } \sum_{k}^{\hat{m}_{t-1}} \theta_{k,t-1} / \hat{m}_{t-1} > \frac{r}{2(1+r)} \\ \theta_{t-1}^*, & \text{if } \sum_{k}^{\hat{m}_{t-1}} \theta_{k,t-1} / \hat{m}_{t-1} = \frac{r}{2(1+r)} \\ \theta_{t-1}^* + c, & \text{if } \sum_{k}^{\hat{m}_{t-1}} \theta_{k,t-1} / \hat{m}_{t-1} < \frac{r}{2(1+r)} \end{cases}$$

where  $0 \le c \le r/(2(1+r))$  is a parameter representing the speed by which banks move towards the optimal truncation point. In addition, the bank is refrained from lending if  $CAR_t \le K$ , honouring the regulatory rule in (3.1).

Examining the iterative procedure above, defining  $\theta_t^*$ , we acknowledge four things. First, since the optimal truncation point,  $\theta_t^*$ , represents the criterion needed for credit and since the truncation point determines the riskiness of the bank's credit portfolio; movements towards the optimal truncation point can be thought of as movements towards the bank's internal credit risk goal. Thus, c represents the speed of adjustment to the bank's internal credit risk goal. Second, given the parameter space of c, the bank may "overshoot" its own credit risk goal and acquire a debt portfolio characterized by more risk then as stated in (2.9). This opens up for periods characterised by "over lending" in which over lending banks try to reduce their exposure to credit risk by tightening the criterion needed for credit. Third, if such a tightening occurs simultaneously across banks, the economy may move into a time period in which credit and investment capital is hard to obtain. Fourth, the bank's initial debt contracts may influence the bank's future decision regarding  $\theta^*$ . To reduce this effect, we set  $\theta_0^* = 0$  allowing the bank to steadily build up the riskiness of its credit portfolio using the iterative procedure as stated above.

Banks with a low value of c take small steps towards the optimal level of credit risk. Hence, the bank's speed of adjustment to its internal credit risk goal reflects the level of conservatism within the bank's organisational structure where conservative banks have a relatively low value of c. Relating c to the real world, the speed of adjustment to the bank's internal credit risk goal can be thought of as a parameter reflecting the bank's willingness to engage in new risky ventures or as its willingness to use new and unexplored debt instruments characterised by more or unexplored risk. Since we are interested in the determinants of credit crunches, we study the case in which all banks are equally conservative. This allows us interpret c as a parameter reflecting the general level of conservatism in the economy.

### 4 Simulations

In order to find the determinants of credit crunches, we simulate the artificial economy in different states, implementing the framework discussed in the

previous sections.<sup>6</sup> We first define a restrictive measure of a credit crunch within the context of the model and then explore the properties of the artificial economy through a selected simulation. The selected simulation is chosen as to illustrate the features of a progressive economy populated with many credit worthy firms.

When defining a restrictive and measurable variable of a credit crunch, we first acknowledge that a credit crunch is defined as a period in time in which credit and investment capital is difficult to obtain. By focusing on the decisions made by the suppliers of credit, i.e. the banks, we see that a credit crunch is intrinsically related to the banks' decisions regarding  $\theta_{i,t}^*$ . If the average truncation point drops below some threshold, investment capital becomes hard to obtain since only a small sample of firms are eligible for credit. Using this, in the absence of a stringent formal definition, we define an indicator variable of a credit crunch as:

Crunch = 
$$\begin{cases} 1, & \text{if } \theta_{i,t}^* = 0, \ \forall i, \quad t > 0 \\ 0, & \text{if else} \end{cases}$$

which by all means of measurement is a tightening of the criterion needed for credit. Arguably, this definition relates to the probability in (3.3) since  $\sum_{i}^{N} \theta_{i,t}^{*} \to 0 \Rightarrow Pr(\text{Contract}_{t}) \to 0$ . However, the definition above neglects the potential effects on the supply of credit caused by (i) the banks' inability to live up to the capital requirements, and (ii) the potential effects on crunches caused by risk based regulatory changes affecting (3.3) through  $Pr(\text{CAR}_{i,t} \geq K)$ . Remembering that all debt is unweighted in this version of the model, we neglect these issues.

The properties of the model are illustrated through a selected simulation of the credit market in which banks have close to perfect firm quality estimates ( $\sigma^f = 0.0001$ ) and where the unconditional expected default rate ( $\theta_t^e$ ) is lower than the banks' optimal expected default rate as stipulated in (2.9). Thus, since  $\mathcal{T} = 0.5$ , ex-ante we may expect banks with almost perfect firm quality estimates to set  $\theta_{i,t}^* = 0.5$ . However, since banks may oversample from the pool of risky firms, occasional and periodic decreases in the average truncation point is expected. The parameters of the beta distribution are chosen to be  $\alpha = 2.6$  and  $\beta = 150$  such that firm quality is distributed with a heavy tail to the right. Given this, the unconditional expected default rate at the initial time period is  $\theta_0^e \approx 1.7$  percent. The interest rate on external capital is set to r = 4 percent such that the *optimal* conditional expected default rate is 1.92 percent, i.e. 22 basis points higher than the unconditional expected default rate. We set the minimum capital requirements to K=8 percent, replicating the capital requirements enforced by the bank for international settlements in Basel, assuming that banks are refrained from holding capital to mitigate future risks. Figure 2 illustrates the evolution of

 $<sup>^6</sup>$ The NetLogo environment is used for the simulations. The code is available on request.

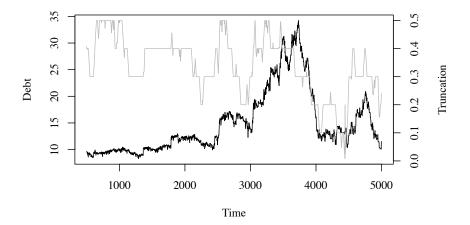


Figure 2: Sum of firm debt and the average truncation point (gray).

debt and the average truncation point in an artificial economy lasting 5000 time periods where the first 500 observations have been removed in order to get rid of transients. The model is simulated with  $M_0=2000$  firms,  $N_0=5$  banks and the torus constructed from b=11. The initial equity of the banks is set to  $E_0^b=2$  and firms are born with  $E_0^f=1$ . The speed of adjustment to the banks' internal credit risk goals is set to c=0.02 and the debts minimum time to maturity is set to  $\kappa=10$ .

From Figure 2 we see that the aggregated debt level has a positive trend, exhibiting cyclical tendencies. In addition, we acknowledge that firm debt is closely related to variations in the average truncation point (the criterion needed for credit). The average truncation point occasionally deviates from the profit maximising solution and at t=4440 the economy evolved into a two period credit crunch. The crunch and the preceding decrease in the average truncation point caused a 58.37 percent drop in debt, compared to the aggregate debt's local maximum at t=3640. Since all parameters are held constant during the simulation period, this indicates that crunches have a natural tendency to occur even if banks have near to perfect estimates of firm quality, in the absence of new regulatory rules or sudden variations in firm quality.

During the time period preceding the credit crunch, the aggregate debt level experienced a heavy growth, despite occasional periodic decreases in the average truncation point. The sudden downturn in debt and the coordinated tightening of the criterion needed for credit (reduced  $\theta_{i,t}^*$ ) forcing the onset of a credit crunch, indicates that banks tend to over lend, acquiring a debt portfolio characterised by more risk then the profit maximising level of credit risk. When realised, the banks seek to "wash out" previously acquired bad

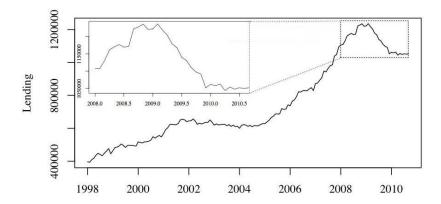


Figure 3: Value of lending to non-financial firms by Swedish banks in billion SEK. Financial crisis enlarged for comparison. Source: Statistics Sweden.

debt by a tightening of the criterion needed for credit. For comparison, Figure 3 exhibits the evolution of lending made by Swedish banks to Swedish non financial firms from January 1998 to September 2010. The series shows a reduced growth in lending after the internet bubble of 2001 and a sharp drop in lending during the aftermath of the financial crisis of 2008. By enlarging the evolution of lending during the aftermath of the financial crises and comparing the figures, we see an obvious resemblance.

The evolution of firm assets are displayed in Figure 4. The series is characterised by a positive trend, growing on average with 5 basis points per time period. The positive trend is frequently broken by sequential downturns, possibly due to reduced lending and sequential firm defaults. Such "busts" to the economy are suspected to be highly dependent on the criterion needed for credit since the equation of motion defining the evolution of firms' asset values is defined by firm quality. Time periods characterised by little or no lending reduces the supply of investment capital. As such, firms have no means of funding potentially fruitful projects, reducing the aggregate growth level of firm assets. In addition, the series is characterised by seemingly random "booms" possibly caused by an increase in project funding and numerous successful projects. Furthermore, we acknowledge that the series shows signs of increased volatility after the onset of the credit crunch at t = 4440, possibly due to the sparse number of new debt contracts.

<sup>&</sup>lt;sup>7</sup>We only measure firms active on the credit market, i.e. firms granted credit at least once, since non-participants have a constant asset value defined only by  $E_0^f$ .

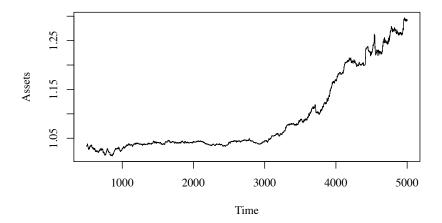


Figure 4: Average firm assets.

#### 4.1 The determinants of credit crunches

From the selected series, we acknowledge that the artificial economy has a natural tendency to spontaneously evolve into a credit crunch. However, the determinants of crunches remain undetermined. In order to find the parameters of the model that can be held accountable for supply side drops in credit, data is collected from simulations of the artificial economy, limited to a sequence of 5000 time periods. The experimental plan used in the study is presented in Table 1.

Since the liquidity of the credit market,  $\psi$ , defining the probability of sudden demand side drops in credit, is jointly determined by the minimum time to maturity  $(\kappa)$ , the number of firms  $(M_t)$ , the number of banks  $(N_t)$  and the size of the torus (b), we choose to hold the size of the torus constant throughout the simulation periods. Remembering that all debt is unweighted in this version of the model, we deem it unlikely that regulatory changes between states will affect the criterion needed for credit. Thus, we keep K constant at 8 percent in all simulations. Since the parameters of the beta distribution defines the evolution of firm assets as well the probability of firm default, these parameters represents the state of the economy. The parameters of the beta distribution are varied in two states such that  $\theta_0^e$  takes on the same value for different values of  $\alpha$  and  $\beta$  on a subset of the simulations.

Given the experimental plan in Table 1, we simulate the artificial economy in 256 different states with 100 replications resulting in a total of 25,600 observations. If the economy experiences a crunch during a simulation period, the result is documented and a new simulation is initiated. Thus, the onset of a credit crunch is defined as a dichotomous variable with one obser-

Table 1: Experimental plan.

Initial Values & Constants	Variable of Interest		
$\theta_{i,0}^* = 0$	Crunch = 1		
$E_{i,0}^{b} = 2$			
$E_{j,0}^f = 1$			
b = 11			
K = 8%			
	Initial Values & Constants $\theta_{i,0}^* = 0$ $E_{i,0}^b = 2$ $E_{j,0}^f = 1$ $b = 11$		

vation per simulation run. We acknowledge that the variable of interest is dependent on the vector of observables such that the probability of a crunch can be estimated using a standard logit model. To determine the effects on crunches caused by the parameters of the beta distribution we estimate two models. The estimates from the logit models are displayed in Table 2 from which we only seek to interpret the signs of the estimates due to the theoretical nature of the model.

Examining Table 2, we conclude that an increase in the speed of adjustment to the banks' internal credit risk goals (c) has a positive effect on the probability of a credit crunch. This implies that a more conservative approach to lending reduces the probability of sudden supply side drops in credit, even in the absence of variations in the economic conditions; this being a partially overlooked component contributing to the financial stability of an economy.

In addition, we find that an increase in the interest rate on external capital (r), interpreted as the spread between the lending and the deposit rate, has a significant and negative impact on the probability of a crunch. Relating a credit crunch to the criterion needed for credit, this result is fully in line with the findings in the theoretical part of this paper. We also find that an increase in the expected default rate at the initial date  $(\theta_0^e)$ increases the probability of a credit crunch, as previously suggested in the theoretical part of this paper. Turning to the estimates on the parameters of the beta distribution, we see that an increase in  $\alpha$  increases the probability of a credit crunch. This corresponds to an increase in the unconditional expected default rate  $(\theta^e)$  since an increase in  $\alpha$  moves the mode of the truncated beta distribution to the right. In contrast, an increase in  $\beta$  reduces the probability of a credit crunch. Such an increase centres the probability density mass around the mode of the distribution increasing the "distance" to riskier loans. This can be thought of as homogenising firm quality which reduces the probability of a crunch. Since  $\theta_k$  is drawn from the truncated beta distribution and since  $\theta_k$  defines the evolution of firm assets, this result

Table 2: Maximum Likelihood estimates from the logit model on credit crunches.

Variables	Model 1	Model 2
Intercept	2.8792***	10.726***
c	540.31***	527.28***
r	-383.66***	-375.20***
$ heta_0^e$	505.65***	-
$\alpha$	-	4.0634***
$\beta$	-	$-0.0643^{***}$
$\sigma^f$	139.80***	137.38***
$\kappa$	-0.0264***	-0.0259***
$M_0$	-0.0004***	-0.0004***
$N_0$	-0.6529***	$-0.6415^{***}$
Nagelkerke $R^2$ index	0.8254	0.8213

Significance codes: 0.001: "\*\*", 0.01: "\*\*", 0.05: "\*"

is fully in in line with predictions from the asset deterioration hypothesis. Turning to the banks' estimates of firm quality, the results indicate that an increase in the noise affiliated with firm specific estimates of firm quality increases the probability of a credit crunch. Thus, we are able to confirm the importance of firm quality estimates as implicitly suggested by the literature in banking.

In addition, an increase in the debts' minimum time to maturity  $(\kappa)$  decreases the probability of a credit crunch. This result suggests that an increase in the average time to maturity reduces the probability of a credit crunch. To our knowledge, this is an up till now overlooked component determining the onset of credit crunches. Looking at the additional parameters defining market liquidity, we see that an increase in market density, working through an increase in the numbers of firms (M) and banks (N), reduces the probability of a credit crunch.

The results presented in this section are to be interpreted in the light of how the artificial economy is constructed. Due to random movements of a finite number of firms on a torus, banks do not meet the full distribution of eligible firms at every instant. Since banks have adaptive expectations about the credit risk in their debt portfolio, they continue to increase  $\theta_{i,t}^*$  until the credit risk in its debt portfolio equals/or overrides their profit maximising level of credit risk. Acknowledging that firms are allowed to make repayments on matured debt in every time period, the risk associated with a bank's debt portfolio can increase rapidly if the bank grants credit to risky firms at the same instant as less risky firms meet their obligations to the bank. Thus, the faster the bank adjusts to internal credit risk goals, the higher the probability of retrieving a debt portfolio defined by a suboptimal expected default rate. Simultaneous reductions in truncation points due to

spontaneous wash outs of bad debt may then lead to an absolute tightening of the criterion needed for credit forcing the onset of a supply side credit crunch. On the other hand, if the lending capacities of banks are locked in contacts with long maturity dates, the probability of issuing credit to numerous risky firms at the same instant is decreased. This may offset some of the negative side effects caused by rapid variations in the banks' truncation points.

The effects on crunches caused by the parameters of the beta distribution, is more easily understood if we view them in the light of this new insight. If  $\alpha$  is increased, the mode of the distribution defining firm quality is moved to the right, reducing the proportion of firms afflicted with an acceptable default risk. Hence, an increase of  $\alpha$  can be thought of reducing the sample size of eligible firms. As such, rapid increases in the truncation point conditioned on relatively large values of  $\alpha$  may result in an oversampling of risky firms from the bank's perspective, forcing a tightening of criterion needed for credit. An increase of  $\beta$ , on the other hand, increases the distance to risky firms, dampening the negative side effect caused by rapid variations in the banks' truncation points. In addition, if the interest rate is lowered, i.e. the spread between the lending and the deposit rate is decreased, a smaller sample of firms have the ability to bear a positive contribution to the banks' expected profits. The banks react to this by only granting credit to a subgroup of firms that add positive value to the banks' expected profits. Rapid variations in the banks' truncation points will then lead to an oversampling from the segment of value reducing firms. Oversampling from this segment may then lead to a downturn in lending and possibly, to the onset of a credit crunch.

## 5 Conclusion

This paper analyses the determinants and causes of credit crunches. We start by deriving a theoretical banking model in which lenders screen applicants in order to reduce their exposure to default risk. We use the theoretical model and highlight the importance of interest rates and expected default rate on the criterion needed for credit. By viewing the credit market as a complex adaptive system, we proceed by constructing a novel Agent Based Model (ABM) of a credit market based on the results from the theoretical model. The results from the simulations indicate that crunches have a tendency to occur even if banks have close to perfect estimates of firm quality and when firm quality remains constant in the absence of new regulatory rules. We then use the ABM and find that an increase in the speed by which banks adjust to their internal credit risk goal, increases the probability of a credit crunch. We link this parameter to the level of conservatism in the market and conclude that a more conservative approach to lending leads to

fewer credit crunches, an up till now partially overlooked component contribution to the financial stability of an economy. In addition, we are able to show that the onset of crunches are affected by variations in the market conditions defining the evolution of firm assets. If the economy is in a state characterised by few credit worthy firms, the probability of a credit crunch is increased, fully in line with the asset deterioration hypothesis. The simulations also show that an increase in the debts time to maturity reduces the probability of a credit crunch. This is to our knowledge an overlooked component contributing to the probability of a credit crunch. Thus, this paper is able to give new insights in existing theory as well as to highlight the importance of a conservative approach to lending and time to maturity if policy makers seek to reduce the probability of a credit crunch.

## A The firms' asset values

Assume that the representative firm has a calendar-time counterpart that acts on a credit market where the time horizon is represented by  $T^m$ . Fix a probability space  $(\Omega, \mathcal{F}, P)$  on which there is a standard Brownian motion W. Let  $(\mathcal{F}_t)_{t \in T^m}$  be a filtration on the probability space such that the  $\sigma$ -algebra  $\mathcal{F}_t$  represents the collection of observable events up to time t. Given the above, it is assumed that the asset value of the firm's calendar-time counterpart follows a geometric Brownian motion:

$$dA_t^f = \mu A_t^f dt + \sigma A_t^f dW_t \tag{A.1}$$

where W is a standard Brownian motion under the probability measure P. Moving over to the agent based model's sequential evolution of time, we rewrite (A.1) as:

$$\Delta A_t^f = A_t^f \left( \mu \Delta t + \sigma \Delta W_t \right) \tag{A.2}$$

Since the evolution is bounded by the endpoint,  $T^m$  we let  $W_{t-\tau} = W_{t-1}$  such that the firm's asset value remains constant between maturity dates. Given this, we let time evolve in multiples of one such that  $\Delta W_{T^m} = W_{T^m} - W_{T^m-1} \sim N(0,1)$ . By rearranging (A.2) we get:

$$A_{T^m}^f = A_{T^m-1}^f + \sigma_{T^m}^* \Delta W_{T^m}$$

where  $\sigma_{T^m}^* = A_{T^m}^f(\mu/\Delta W_{T^m} + \sigma)$ . Since  $\Delta W_{T^m} \sim N(0,1)$  it follows that  $A_{T^m}^f \sim N(A_{T^{m-1}}^f, \sigma_{T^m}^*)$ . Thus, the drift terms enter by asymmetric shocks. Acknowledge that  $A_{T^m}^f = A_{T^{m-1}}^f + \sigma_{T^m}^* \Delta W_{T^m} = E_{T^{m-1}}^f + L_{T^{m-1}}^f + \sigma^* \Delta W_{T^m}$ . Use that  $L_{T^{m-1}}^f$  is constant between maturity dates and let the firm default if  $A_{T^m}^f < L_{T^{m-1}}^f$  with probability  $\theta$ . It follows that  $Pr(A_{T^m}^f < L_{T^{m-1}}^f) = Pr(A_{T^m}^f - L_{T^{m-1}}^f < 0) = Pr(E_{T^{m-1}} + \sigma^* \Delta W_{T^m} < 0) = \theta$ . Solve for the  $\sigma_{T^m}^*$  that forces the firm to default with probability  $\theta$  at the maturity

date and it follows that  $\sigma_{T_m}^* = E_{T^m-1}^f/\Phi^{-1}(\theta)$  where  $\Phi^{-1}(\theta)$  is the inverse of the standard normal distribution taken at firm quality. Thus, we rewrite the representative firm's equation of motion as:

$$A_{T^m}^f = A_{T^m - 1} + \frac{E_{T^m - 1}}{\Phi^{-1}(\theta)} \Delta W_{T^m}$$

where  $\theta \in [0, 0.5]$  due to the symmetry of the standard normal distribution.

## B Expected default rates

Let  $\theta_k$  represent realisations of  $f(\theta)$  and let  $\phi_k$  represent realisations from  $f(\phi)$ . We seek the conditional expected default rate conditioned on a measurement error in expectations, i.e.  $E[\theta|\theta^b \leq \theta^*] = E[\theta|\theta + \phi \leq \theta^*] = E[\theta|\theta \leq \theta^* - \phi] = E[\theta|\theta \leq \hat{\theta}^*]$ . As such, we have a random truncation, selected out of a density  $f(\hat{\theta}^*)$ . Since  $\phi \sim N(0, \sigma^f)$  it follows that  $\hat{\theta}^* \sim N(\theta^*, \sigma^f)$ . However, we truncate the distribution such that  $\hat{\theta}^* \in [0, \mathcal{T}]$ . Given this, the expected truncation point is:

$$E[\hat{\theta^*}|0 \le \hat{\theta^*} \le \mathcal{T}] = \frac{\int_0^{\mathcal{T}} \hat{\theta^*} f(\hat{\theta^*}) d\hat{\theta^*}}{F_{\hat{\theta^*}}(\mathcal{T}) - F_{\hat{\theta^*}}(0)}$$

From this it follows that:

$$E[\theta|\theta_k \leq \theta^*] = \frac{\int_0^{\theta^*} \theta f(\theta) \ d\theta}{F_{\theta}(\theta^*)} \neq E[\theta|\theta_k^b \leq \theta^*] = \frac{\int_0^{E[\hat{\theta^*}|0 \leq \hat{\theta^*} \leq \mathcal{T}]} \theta f(\theta) \ d\theta}{F_{\theta}(\theta^*)}$$

where  $F_{\theta}(x)$  is the cumulative distribution function of  $\theta$ . Hence, the bank fails to find the optimal expected default rate.

## References

Allen, F. (1990): "The Market for Information and the Origin of Financial Intermediation," *Journal of Financial Intermediation*, 1, pp. 3–30.

Berger, A. N. and G. F. Udell (1994): "Did Risk-Based Capital Allocate Bank Credit and Cause a 'Credit Crunch' in the U.S.?" *Journal of Money, Credit and Banking*, 26, pp. 585–628.

Besanko, D. and G. Kanatas (1993): "Credit Market Equalibrium with Bank Monitoring and Moral Hazard," *Review of Financial Studies*, 6, 213 – 232.

Black, F. and M. Scholes (1973): "The Pricing of Options and Corporate Liabilities," *The Journal of Political Economy*, 81, pp. 637–654.

- BROECKER, T. (1990): "Credit-worthiness Test and Interbank Competition," *Econometrica*, 58, 429 452.
- Chiesa, G. (1998): "Adverse Selection and Intermediaries," Research in Economics, 52.
- DIAMOND, D. W. (1984): "Financial Intermediation and Delegated Monitoring," The Review of Economic Studies, 51.
- Gehrig, T. (1998): "Screening, cross-boarder banking, and the allocation of credit," Research in Economics, 52, 387 407.
- HOLMSTRÖM, B. AND J. TIROLE (1997): "Financial Intermediation, Loanable Funds and the Real Sector," *The Quarterly Journal of Economics*, 112, 663 691.
- MERTON, R. C. (1974): "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *Journal of Finance*, 29, 449 470.
- MINSKY, H. P. (1975): John Maynard Keynes, Columbia University Press.
- ——— (1992): "The Financial Instability Hypothesis," SSRN eLibrary.
- PAZARBASIOGLU, C. (1996): "A Credit Crunch? A Case Study of Finland in the Aftermath of the Banking Crisis," IMF Working Papers 96/135, International Monetary Fund.
- Sharpe, S. A. (1995): "Bank Capitalization, Regulation, and the Credit Crunch: A Critical Review of the Research Findings," Finance and Economics Discussion Series pp. 95-20, Board of Governors of the Federal Reserve System (U.S.).
- Tesfatsion, L. (2006): Handbook of Computational Economics: Agent-based Computational Economics, Elsevier/North-Holland (Handbooks in Economics Series), vol. 2, chap. Agent-Based Computational Economics: A Constructive Approach to Economic Theory.
- WINTON, A. (1995): "Delegated Monitoring and Bank Structure in a Finite Economy," *Journal of Financial Intermediation*, 4, pp. 158–187.