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Preamble

This doctoral dissertation deals with various areas of research in applied microeconomic theory. The first chapter deals with the emergence of alliances in the market for international roaming calls. Chapter 2 belongs to the field of political economy and offers a dynamic explanation for the emergence of political polarization. Chapter 3 analyzes a procurement problem, where specialized sellers are better informed than the buyer about which of their products suits the buyer's needs. Chapter 4 compares the impact of simple policy interventions in markets where not all customers can discern the actual quality of offered goods. All chapters are self contained and can be read independently.

The market of international roaming is the focus of the first chapter. International roaming provides subscribers with the possibility to use mobile telecommunication services outside the coverage area of their *home* operator's network. While subscribers are traveling in a foreign country, they may use the infrastructure of a so-called *host* operator. For roaming services, the host operator then charges *wholesale* rates to the roaming subscribers' home operator that in turn bills *retail* prices to its customers. By now, technological means have been developed that allow operators to control at which foreign network their roaming subscribers register.[1] Hence, we would expect that the competition to host roaming subscribers results in reasonable wholesale rates.

Yet, in a recent survey on this market, the OECD (2009) finds that compared to the underlying costs, the roaming charges are excessive. In particular, the OECD (2009, p. 5) asserts that "the major contributor to high retail charges is the wholesale rates charged by foreign operators". Based on this evidence, Chapter 1 investigates whether

[1] See e.g. European Commission (2006).

the formation of international roaming alliances may explain this puzzle. Indeed, after the technologies to control the roaming traffic became available by 2003, trans-national alliances emerged that aimed at directing their roaming subscribers preferably to other alliance members. The OECD (2009) acknowledges that these alliances have not been effective in bringing down roaming prices.

The model developed in the first chapter studies the interplay of the wholesale and retail prices in the roaming market. The emphasis lies on the effects of international roaming alliances on prices and on the operators' incentives to form them. As we show, these alliances may serve as a commitment device to soften retail competition. More precisely, mutually directing subscribers to a partner network at high wholesale prices within an alliance induces affiliated operators to offer less attractive retail deals to potential subscribers. Since retail tariffs are strategic complements in our model, the domestic competitor reacts by raising the own retail prices, which has a positive feedback effect on the profit generated by the alliance. Besides, within an alliance each operator exclusively hosts the foreign partner's traveling subscribers and a reciprocal wholesale price is charged. Therefore, additional expenses caused by a higher wholesale price are perfectly recouped from wholesale profits earned with the partner's traveling subscribers. Not surprisingly, the prospect of higher profits due to softer retail competition encourages operators to endogenously form these alliances.

Chapter 1 contributes in two regards to the existing literature. First, our model introduces imperfect competition and non-linear retail tariffs in a setup that exhibits *symbiotic production* according to Carter and Wright (1994): Each operator offers roaming services as intermediate products to foreign operators, and resells roaming services from foreign operators to own subscribers. If operators are monopolists in each country, then cooperation helps to avoid double marginalization as shown by Carter and Wright (1994). In contrast, under imperfect competition cooperation within alliances generally deteriorates welfare. Second, we contribute to the literature on vertical relations. Several papers have shown that firms may soften competition by agreeing on a two part wholesale tariff that includes a fixed payment in order to compensate for an excessive unit price.[2] We point out that firms may also soften competition with-

[2]See e.g. Bonanno and Vickers (1988); Shaffer (1991); Rey and Stiglitz (1995).

out needing to resort to non-linear wholesale tariffs when they commit to mutually providing services.

The second chapter provides an explanation for why political leaders may want to adopt ideological positions and maintain them over time even in the face of conflicting evidence.[3] As documented in the empirical work on Congressional voting behavior of Poole and Rosenthal (2007), McCarty, Poole, and Rosenthal (2006) and others, the belief systems of political elites can often largely be captured with a single dimension, their *ideology*, which almost always mirrors party affiliation. Moreover, ideological positions of individual members are remarkably stable, or as Poole (2007, p. 435) puts it, "members of Congress die in their ideological boots." Partisan politics are a frequent phenomenon even regarding so-called valence issues like foreign policy for which there should be a common agreement among the electorate.[4]

To address these empirical findings, Chapter 2 develops a dynamic model that ties observable characteristics of political representatives, such as their party affiliation, to voters' expectations. In our setup politicians are better informed than the voting public about an underlying state of nature that determines the desirability of a given policy measure. The issue itself is non-partisan, that is everybody has the same policy preferences, but voters attach ideological labels to both candidates and available policy alternatives. Given their beliefs about the prevailing state, voters form expectations about which policy candidates are likely to implement once in office, and which of those is most likely to succeed. We show that politicians may act partisan simply because voters' *expect* them to. Suppose voters expect political candidates to systematically implement policies that are "close" to their own ideology, once in office. These expectations induce voters to elect the representative whose perceived partisan policy (ideology) is most likely to correspond to the underlying state, based on their current information. This may suffice to induce candidates to actually act partisan, in the first place. The specific motivation is one of signal-jamming: an incumbent who sticks to

[3]This chapter is based on joint work with Anke S. Kessler.

[4]In the U.S. Congress, for example, support for the president on matters of foreign policy and defense has largely been along party lines ever since the Vietnam War [Meernik (1993)]. Two recent polls found that Americans share common views on a wide array of foreign policy issues, and would prefer that Democrats and Republicans seek common ground. See the website of Partnership for a Secure America http://www.psaonline.org/.

his partisan policy demonstrates confidence and avoids revealing that current circumstances would favor his opponents' partisan position. As even inefficient policies may turn out to be successful, this behavior potentially allows to hold up the electorates' belief in the adequacy of the incumbent's ideology. If voters expect partisan behavior in the future, this makes his re-election more likely. The result is political failure in the sense that the equilibrium partisan policy outcomes are Pareto dominated.

Thus, our model can explain policy divergence from the fact that voters *perceive* policies to be ideologically tinted and *expect* candidates to act partisan. In contrast to most of the existing literature that requires partisan preferences, we are able to explain polarized and partisan politics on matters where voters commonly agree.[5] Moreover, because in the partisan equilibrium incumbents will tend to enact the partisan policy independent of the prevailing state, policy changes only occur after the incumbent has changed. Since this only occurs when the electorate has found evidence regarding the incumbent's inadequacy, our model also delivers a novel explanation for policy persistence.[6]

Chapter 3 studies the optimal design of the procurement process for credence goods. Darby and Karni (1973) have coined the notion of *credence goods* to refer to situations where an expert seller knows better than the customer which type of good or service suits the buyer's needs. Credence goods comprise for instance medical treatments, repair services or the provision of certain complex goods. In contrast to the literature on credence goods that has concentrated on products where the *seller* typically sets the price, we focus on situations where the *buyer* can design and commit to a procurement mechanism.[7]

A major problem in credence good markets is that sellers may be tempted not to advise truthfully if this improves their prospects to earn profits. If sellers are specialized on different versions of a good or service, then their advice affects with whom a buyer will finally trade. Chapter 3 studies a situation where each potential seller provides a

[5]See e.g. Palfrey (1984), Osborne and Slivinski (1996), Besley and Coate (1997), Van Weelden (2009) or Kartik and McAfee (2007).

[6]Apart from Coate and Morris (1999), we are not aware of further contributions that explain policy persistence.

[7]See for instance Dulleck and Kerschbamer (2006) for a survey of the credence good literature.

distinct alternative. This situation is likely to occur when different products serve the same purpose but differ considerably in their technological nature. For example, when Atlantic Richfield successfully drilled for oil in Alaska, the problem soon became how to ship the oil to global markets. Boeing proposed tanker aircrafts, General Dynamics offered a line of tanker submarines for travel beneath the Arctic ice cap, and another group proposed extending the Alaska Railroad to the oil well.

We investigate optimal procurement schemes that account for the sellers' incentives to glorify the own product. More specifically, sellers have two-dimensional private information on the costs to produce their good and on how well it suits the buyer's needs. If the buyer knew the suitability, he would be willing to procure from sellers with an expensive but adequate product. However, this would allow sellers that produce at low costs to extract rents when exaggerating the adequacy of their products. Therefore, the buyer has to reward these sellers for admitting the inadequacy of their products. The buyer thus faces a trade-off between procuring the adequate good more often and economizing on rents. Because of this trade-off, the optimal procurement scheme depends on the expected value of the sellers' suitability information as compared to the magnitude of cost uncertainty. If the good's adequacy is of little importance, the buyer optimally commits to sometimes trading with a seller that is as expensive as its rivals but offers a *less* suited product. In contrast, if the good's adequacy is important enough, then the principal optimally gives priority to the suitability of the good over the costs although this requires to concede extremely high rents to the sellers.

There are two areas of application where buyers can conceivably commit to the outcome of a procurement scheme. First, in procurement decisions concerning valuable goods such as military equipment, we often observe contractually arranged procurement procedures. If necessary, buyers may even reinforce their commitment power by hiring a third party for designing and enforcing optimal procurement schemes. A second application is the design of laws and provisions for transactions that involve credence goods. For example, in the medical sector the process of identifying the optimal treatment is heavily regulated. Both patients and physicians have to comply with these provisions. Therefore, our results could be fruitfully applied when designing the provisions that regulate the remuneration and the choice of physicians.

Chapter 4 deals with the related problem that some consumers cannot discern the actual quality of the goods. In contrast to the preceding chapter, we now turn to goods that do not warrant a sophisticated design of contracts between sellers and customers. Instead, firms simply post prices and uninformed customers form beliefs regarding the quality of the desired goods. Since uninformed customers cannot react to the *actual* quality of the offered products, their presence reduces firms' incentives to invest in quality. Market solutions like promising warranties or developing a reputation for high quality goods may alleviate this imperfection. Yet, these instruments are often insufficient and policy makers are concerned about poor free-market qualities when the goods' grade is difficult to discern. Recent examples that illustrate the alleged lack of quality are energy efficiency or safety properties of electrical devices, the quality of food, hazardous contents of play-toys or the sustainability in the production of timber. Chapter 4 therefore investigates the welfare consequences of popular policy interventions in these markets.

The adoption of minimum quality standards (MQS) and certification of high grade goods are popular instruments to protect customers from poor quality products. A MQS restricts firms not to sell goods whose quality is below this standard.[8] Certification is a process where a third party verifies if a product fulfills certain criteria. We often observe that the government designs and enforces certification either directly, or promotes the creation of non-governmental organizations for these tasks.[9] A particularly prevalent form of certification is to award a label if a product exceeds some publicly known criterion, but not to disclose the actual product's quality.[10]

Although both instruments aim at improving the quality in the market, they differ in their impact. While the effect of a MQS on equilibrium qualities and prices has already received some attention in the literature, little is known about the impact of

[8]Examples are abundant and include safety standards for manufactured products, contents requirements for textiles or food, occupational licensing for professional services or environmentally related standards.

[9]In contrast to a MQS, certification does not require the power of coercion and can in principle be provided both by private and by public institutions. Yet, firms are often unable to build up a private certifying institution because of credibility issues.

[10]Examples are the German "Blauer Engel" label which is directly awarded by the state (`http://www.blauer-engel.de`) or the Forest Stewardship Council (FSC) which certifies timber from sustainable forestry (`http://www.fsc.org`).

certification on oligopolistic markets.[11] Chapter 4 therefore develops a model in which firms first decide on entry and on the quality of their products and then compete in prices with vertically differentiated products. Before firms move, the government may either adopt a MQS or determine a certification threshold that is required to obtain the certificate. We find that a MQS typically gives rise to less differentiated products and intensifies price competition. In contrast, certification gives firms the choice whether to demonstrate a high quality of its product or not and may lead to more differentiated goods. Although this gives rise to higher quality-weighted prices, we find that suitably chosen certification raises consumer surplus, as the positive effect from higher quality goods dominates.

Since firms have no obligation to comply with the certification standard, certification is only effective if at least one firm earns a higher profit when selling a high quality product with the label compared to offering a lower quality product that lacks the certificate. Hence, the quality investments that are needed to obtain the label may not be unduly high compared to the expected revenues. In addition, the decision to sell a certified product also depends on the profit that a firm expects to earn from selling a good without the certificate. If the informational asymmetry is severe, the profit from selling a product without the label is low, and a firm will comply even with a demanding certification standard. This yields the surprising result that the government has more leeway in manipulating the market qualities when the grade is difficult to discern. Yet, the government may be forced to reduce the certification standard so as to maintain its acceptance, when the proportion of informed consumers increases due to exogenous reasons such as technological developments.

An important aspect when comparing these instruments is their effect on the entry of firms. A MQS tends to reduce the firms' profits due to lower equilibrium prices and higher minimum investments in quality. Especially when the goods' quality is opaque, so that the firms' ability to differentiate their goods is restricted, a MQS may deter the entry of firms. In contrast, our analysis indicates that suitable certification fosters differentiation of goods and does not restrict entry. We thus conclude that suitable certification may improve welfare more than a MQS if only few customers are

[11] See the literature review in Chapter 4.

informed.

Each of the four chapters emphasizes a different role of commitment - or the lack thereof. In the first chapter, we argue that operators form alliances in order to commit to competing less aggressively for subscribers. The ability to commit is detrimental for subscribers, who will be confronted with less favorable retail tariffs. Hence, Chapter 1 studies a situation in which restricting the ability to commit may improve welfare. This is in contrast to Chapter 2, where politicians would commit to efficient behavior if this was possible. Precisely the absence of commitment power, coupled with the presence of asymmetric information, gives rise to coordination problems and may ultimately lead to an unfavorable equilibrium outcome. Commitment plays also a positive, albeit different, role in Chapter 3. As the literature on credence goods has shown, the absence of commitment on the buyer's side often yields equilibrium outcomes in which the seller's expert information is not efficiently used. Hence, the buyer's ability to commit leads to a more informed decision and is therefore welfare increasing. Finally, in Chapter 4, active firms could increase their profit by committing to producing higher quality goods. The lack of commitment gives the government the opportunity to manipulate the equilibrium qualities by help of a suitable certification policy.

Chapter 1

Do International Roaming Alliances Harm Consumers?

1.1 Introduction

International roaming provides subscribers with the possibility to use their mobile phone outside the geographical coverage area of their home operator's network, by means of a visited network. A Mobile Network Operator (MNO) that allows subscribers of a foreign operator to access its network acts as *host operator*. For roaming services, a host operator charges wholesale prices to the roaming subscribers' *home operator* that in turn charges retail prices to its subscribers.[1] MNOs are typically active on two related markets: They offer roaming services to foreign operators and buy roaming services for own traveling subscribers on the *wholesale market*. In addition, they compete in their home country on the *retail market* for subscribers.

The European market for international roaming accounted for approximately €8.5 billion or 5.7% of the estimated total mobile industry revenues in 2005 [European Commission (2006)]. At the same time, roaming contributed almost 12% to the European mobile industry profits. It is thus highly profitable and expected to further grow during the next years. The European Commission (2006) assessed that both the average

[1] Roaming services include the possibility to receive or to place calls as well as to use mobile data services such as SMS.

roaming retail and wholesale prices were unjustifiably high. For example, it estimated that the per-minute costs (including a margin for fixed costs) for originating, transmitting and terminating an outgoing roaming call were approximately 20 cents, while wholesale prices were on average about 75 cents and retail prices were roughly €1.10. This raises the question why competition has not been effective in the roaming market.

In this chapter we argue that *international* alliances of MNOs may result in inefficiently high wholesale prices that would not be sustainable otherwise.[2] Recently, such alliances have been formed, claiming to facilitate the provision of roaming services.[3] Affiliated operators typically agree on special roaming wholesale conditions based on the promise to direct roaming subscribers preferably to other alliance members. In contrast, ordinary roaming agreements usually do not encompass the obligation to direct subscribers to each other.[4] They just specify the roaming wholesale prices that an operator charges when hosting traveling subscribers of another foreign operator. We claim that because of strategic considerations MNOs prefer to form alliances in order to commit to trade roaming services at inefficiently high wholesale prices. As we show, this allows MNOs to soften competition on the retail market and thereby increases total profits.

In our model, in each of two equally sized countries two MNOs compete on the retail market à la Hotelling for subscribers.[5] We ignore nationwide calls and focus instead on subscribers' demand for roaming calls abroad.[6] To provide this service, each operator needs to access the foreign operators' infrastructure. Operators may form international alliances and mutually promise to procure roaming services exclusively from their partner network. In this case they jointly negotiate on a mutual wholesale price. Operators may also post wholesale prices and buy roaming services without being af-

[2] International alliances are formed by operators which own networks in different countries.

[3] One example is the *Freemove* alliance whose web page can be found under http://www.freemovealliance.com.

[4] The technical and contractual conditions for concluding and implementing international roaming agreements between GSM operators have been standardized by the GSM Association. See e.g. Sutherland (2001).

[5] Our model is similar to existing models of telecommunication in this respect. See e.g. Laffont, Rey, and Tirole (1998a,b).

[6] Hence in our model MNOs offer the single service to their subscribers to place roaming calls once they are abroad. However, we believe that the issues discussed in this chapter are specific to roaming calls and orthogonal to other services usually offered by MNOs. At the loss of simplicity other services like nationwide mobile phone calls could be easily integrated.

filiated to an alliance. They first set the wholesale roaming prices and decide from which foreign operator to buy roaming services. Then they offer two-part retail tariffs to potential subscribers in their home country.

In the absence of alliances, competition among foreign operators to host traveling subscribers drives down wholesale prices for roaming services. In contrast to models of network-interconnection, there is no "competitive bottleneck" in the sense that no particular foreign operator has to provide the roaming services.[7] Operators will thus direct their subscribers to the foreign network that offers the lowest wholesale price.

Agreeing in an alliance on wholesale prices above the true marginal costs serves as commitment to compete less aggressively for subscribers. At the retail level, a higher wholesale price is perceived as an increase in the marginal cost and is passed through to customers. Since retail tariffs are strategic complements in our model, this induces the domestic competitor to raise its fixed fee which has a positive feed-back effect on the first operator's profit in turn. Besides, within an alliance each operator exclusively hosts the foreign partner's traveling subscribers and a reciprocal wholesale price is charged. Therefore additional expenses caused by a higher wholesale price are perfectly recouped from wholesale profits earned with the partner's traveling subscribers. Increasing the wholesale price within one alliance also increases the profits of competing operators. This might explain why domestic competitors rarely complained when international alliances were formed.

Our findings are interesting in light of recent technological developments that have increased the strategic importance of roaming alliances. The European Commission (2006) estimated that roughly 80% of the roaming traffic was already actively directed by use of these technologies in 2006. Until recently, operators had limited technical means to determine which foreign network their subscribers would use.[8] Customers that did not manually register in a particular foreign network were almost randomly assigned among foreign operators. Not being able to direct subscribers to networks

[7]In models of interconnected networks subscribers usually become member at one particular network which then becomes monopolist for the access to this subscriber. The fact that there is ex-ante competition for subscribers but a de-facto monopoly of access ex-post is denoted as "competitive bottleneck". See e.g. Armstrong (2002); Armstrong and Wright (2007).

[8]For a detailed technical description, see e.g. Stumpf (2001), Salsas and Koboldt (2004) or European Commission (2005).

that offer cheap roaming services induced MNOs to charge high wholesale prices even without the help of alliances.[9] Hence, the strategic importance of alliances increased with the improvement of network selection technologies.

Further practical issues can be addressed by help of our model. First, the Groupe Speciale Mobile Association (GSMA), to which most of the MNOs are affiliated, created a common framework to simplify the negotiations on roaming agreements between operators.[10] It contains a non-discrimination clause that restricts MNOs to offer similar wholesale terms for roaming services to all foreign operators. We account for this clause by restricting operators to apply the same wholesale price that has been fixed within an alliance also for unilateral roaming agreements. Surprisingly, this clause even amplifies the anti-competitive impact of alliances, since too high wholesale prices within alliances then also obstruct competition for unilateral roaming agreements. Second, the European Commission introduced a price cap both at the retail and at the wholesale level in 2007 and there is an intense debate about the effects of such an intervention. In our setup, introducing a binding price cap only at the retail level decreases the usage prices but typically also reduces the consumer surplus. This may happen because operators compete in two part tariffs at the retail level. If the usage price is bounded above, then operators may increase the unregulated monthly fee even more. This so-called *waterbed effect* may turn seemingly helpful regulatory interventions on its head.

While our setup is tailored to the international roaming market, there are other important applications, such as the market for cash withdrawals. Banks often only own an automated teller machine (ATM) network in their home country and have to rely on the infrastructure of foreign banks in order to allow customers to withdraw money abroad.

Turning to the existing literature, our model exhibits what Carter and Wright (1994) call *symbiotic production*: Each operator offers roaming services as intermediate products to foreign operators, and resells roaming services from foreign operators to own

[9] We show in Section 1.6.2 that in the absence of control regarding the host network the wholesale prices may even exceed the monopoly level.

[10] These Standard Terms for International Roaming Agreement (STIRA) were created in 1996 and received conditional exemptions from the cartel prohibition under Article 81 (3) of the EC Treaty according to Stumpf (2001).

subscribers. Carter and Wright (1994) assume that there is a monopolistic operator in each country and find that double marginalization leads to inefficiently high retail prices. They conclude that both operators and consumers would be better off if operators cooperated and bilaterally reduced their wholesale prices. In contrast, we show that the role of alliances is reversed when there is competition both on the retail and on intermediate product markets. This is because in our model price competition at the wholesale level eliminates a positive markup and thus the problem of double marginalization without alliances.

The role of the wholesale roaming prices in our setup resembles that of the access prices in the two-way network interconnection model of Laffont, Rey, and Tirole (1998a). They find that the collusive power of access prices vanishes if operators compete in two-part tariffs. In our model, higher wholesale prices allow to raise profits even though firms compete in two-part tariffs on their home market. In the roaming market, if an operator enters into an international alliance and agrees on a high wholesale roaming price, the domestic competitor's perceived costs for roaming services remain unchanged. Due to the different impact on competing operators roaming wholesale prices are not neutral in our model.

There are also conceptual similarities to the literature of vertical relationships.[11] In particular, Shaffer (1991) shows that downstream firms might prefer paying higher unit prices for intermediate goods and receiving a fixed compensation to low unit prices if this serves as a commitment device to soften downstream competition. For the same reason operators prefer to commit to a high wholesale price in our model. However, our reasoning does not require fixed payments to compensate higher unit prices since operators mutually provide roaming services in an alliance. In addition, the existing literature has analyzed competition in *linear* prices on the downstream market so far. To our knowledge, this model is the first to show that operators may also exploit strategic complementarity even though competing in *nonlinear* prices in the downstream market.

Recently, a small literature that analyzes the international roaming market emerged. Salsas and Koboldt (2004) as well as Lupi and Manenti (2006, 2009) also consider a

[11] See e.g. Bonanno and Vickers (1988), Shaffer (1991) and Rey and Stiglitz (1995).

setup of two operators in each of two countries. However, Salsas and Koboldt (2004) do not explicitly take into account that each operator is active both on the wholesale market and on the retail market and therefore cannot consider the possibility of international alliances. Another difference of their base setup is their assumption that roaming traffic cannot be directed to a particular foreign network.[12] Lupi and Manenti (2006, 2009) assume that operators act as local monopolists on the retail market. Therefore, they do not analyze operators' incentives to set high wholesale prices in order to soften retail competition. In their setup, alliances optimally set wholesale prices at marginal costs, which is not in line with the current evidence. In addition, Lupi and Manenti (2009) cannot explain why alliances emerge endogenously.

The remainder of the chapter is organized as follows: In the next section, we formally introduce our basic model. Section 1.3 characterizes the equilibrium retail tariffs for given wholesale prices. In Section 1.4 we first show that equilibrium wholesale prices equal marginal cost in the absence of international alliances and typically increase in the number of alliances. Section 1.5 adds a first stage in which alliances can be formed. As a result, two competing alliances endogenously emerge in the absence of regulatory constraints. In Section 1.6, we discuss further issues such as the role of network selection technologies, the impact of a non-discrimination clause and of introducing price caps and we generalize the set of wholesale instruments before we conclude in Section 1.7.

1.2 The Model

There are two countries A and B as well as two MNOs with index 0 and 1 in each country. Operator xi is active in *home* country $x \in \{A, B\}$ and has position $i \in \{0, 1\}$. Each operator's network covers only its home country. Thus, subscribers have to be hosted by another operator while traveling abroad. Initially, we assume that operators dispose of technological means to determine on which foreign network their traveling

[12] In an extension, Salsas and Koboldt (2004) assume that traffic can be (partially) directed to the cheapest foreign operator and they find that this assumption drives wholesale prices down.

subscribers register.[13] We focus on outgoing roaming calls that subscribers may place while traveling abroad and assume that it is the only service which MNOs offer to their subscribers. In particular, we abstract from nationwide calls.[14]

In order to allow own subscribers to place roaming calls abroad, operators have to buy these services on the *wholesale market* from a foreign MNO which then hosts these customers. Thus, each operator competes with its domestic competitor on the *wholesale market* to sell roaming services to foreign operators. They also compete on the *retail market* for subscribers which live in the operator's home country.

Cost structure: Each of the four operators incurs the same marginal cost $c \geq 0$ when a traveling subscriber places a roaming call.[15] In addition, operators have to incur monthly fixed costs C_F per subscriber, e.g. for billing.

Retail market: MNOs offer a two-part tariff: Operator xi charges a usage price $p_{xi} \in \mathbb{R}$ per roaming call and a (monthly) fixed fee $F_{xi} \in \mathbb{R}$. When a consumer places q roaming calls, she has to pay in total $p_{xi}q + F_{xi}$.

As in Laffont, Rey, and Tirole (1998a), networks are differentiated à la Hotelling. In each country, consumers' tastes l are uniformly distributed on the segment $[0, 1]$. The operators are located at the two extremities and the index $i \in \{0, 1\}$ also indicates their position. Each consumer may join at most one network which generates a fixed surplus v_0. Placing q roaming calls generates a *gross surplus* $u(q)$. Consumers have quasilinear preferences in wealth such that the (incremental) utility of a consumer with taste l who joins operator xi and places q roaming calls is

$$-\frac{1}{2\sigma}|i - l| + u(q) - p_{xi}q - F_{xi} + v_0 \,.$$

The term $-\frac{1}{2\sigma}|i - l|$ expresses the loss of utility in case the joined network does not

[13] By 2006, roughly 80% of the European roaming traffic was indeed directed to the desired foreign network.

[14] Further services such as nationwide calls could be included in the model at the cost of tractability. Due to competition in two part tariffs, usage prices would be set equal to perceived marginal costs. The surplus generated by these services is then captured by the parameter v_0 introduced below.

[15] This marginal cost includes origination, transfer and termination. For simplicity, we assume that all roaming calls are terminated at some third party fixed network so that we can abstract from traffic generated by the termination of roaming calls.

correspond exactly to the consumers taste where $\sigma > 0$ parametrizes the degree of taste differentiation. A consumer that does not join either network receives utility that is normalized to 0. For technical convenience, we assume that joining a network is sufficiently attractive (i.e. v_0 is high enough) so that all subscribers join a network on the relevant range of prices.[16] Preferences are the same in both countries. Note that consumers care only about their domestic operator, not about which foreign operator handles their roaming calls.[17]

The optimal individual demand and the resulting consumers' value from roaming calls are defined as

$$\begin{aligned} q(p) &\equiv \arg\max_q \{u(q) - pq\} , \\ v(p) &\equiv u(q(p)) - pq(p) . \end{aligned}$$

Since subscribers have quasilinear preferences concerning wealth, the value function v satisfies the envelope condition $v'(p) = -q(p)$. We maintain the following mild assumption throughout the chapter:

Assumption 1.1. *Per customer demand $q(p)$ is non-negative, continuously differentiable and non-increasing on $\mathbb{R}$: $q(\cdot) \in \mathbb{R}_+,\ q'(\cdot) \leq 0$. Subscribers have a strictly positive demand for roaming services at the true marginal cost: $q(c) > 0$.*

For future reference we define the *net surplus* of a tariff as

$$w(p, F) \equiv v(p) - F . \tag{1.1}$$

Economically, the net surplus indicates how much of the value $v(p)$ created by placing roaming calls retains with the subscriber.

If the difference between the net surpluses offered by competing retail contracts in

[16] This assumption is commonly made the literature on network interconnection. See e.g. Laffont, Rey, and Tirole (1998a, p. 7) for further discussion.

[17] The assumption that consumers do not care which foreign network provides the roaming services can be justified in several ways. One plausible reasoning relies on a heterogeneous coverage. Since a subscriber usually lives and works at a priori known places, she prefers to join a network that offers good coverage at these focal points. However, when signing a mobile phone contract, a subscriber is usually less aware of the foreign places where she will use roaming services.

country x is not too large ($|w_{xi} - w_{xj}| < \frac{1}{2\sigma}$), both operators achieve a strictly positive market share.[18] In this case, the market share of operator i in country x is

$$n_{xi} = n(w_{xi}, w_{xj}) \equiv \frac{1}{2} + \sigma\left(w_{xi} - w_{xj}\right) \ . \tag{1.2}$$

If instead operator i offers a contract that is far more attractive than its competitor's tariff ($w_{xi} \geq w_{xj} + \frac{1}{2\sigma}$), it corners the whole market.

Wholesale market: In order to allow subscribers to use foreign networks, operators may either conclude *roaming agreements* or form *international alliances*.

A *roaming agreement* specifies that operator xi hosts subscribers of operators yj but does not contain any obligation that operator xi also buys roaming services from operator yj. MNOs compete to become host operator for foreign subscribers by simultaneously posting a wholesale price per roaming call.[19] If operator yj accepts the offer of operator xi, then they conclude a roaming agreement which fixes the wholesale price $\tilde{a}_{xi}$.

Mobile operators with different home countries may also form *international alliances*.[20] Within an alliance, operators negotiate on a wholesale price at which they *mutually* provide roaming services. Alliance members commit to direct their subscribers to the partner network abroad. It will become clear that the appeal of alliances lies precisely in the commitment that subscribers are possibly not hosted by the cheapest operator abroad. After a wholesale price has been negotiated, it becomes public knowledge.[21] Note that members of an alliance may *sell* roaming services to foreign operators that are outside of an alliance.

Figure 1.1 summarizes the structure of the model with an example. Operators $A0$ and $B0$ are affiliated to an alliance and host each other's subscribers at the wholesale price a_0. In addition, operator $A0$ also hosts subscribers of $B1$ while $A1$ buys roaming

[18]See e.g. Laffont, Rey, and Tirole (1998a).

[19]Note that the restriction to linear wholesale prices prevents the use of two-part tariffs to soften competition as in Shaffer (1991).

[20]We suspect that domestic regulation agencies would prohibit alliances that would involve more than one MNO of a country. Members of these alliances could then collude on their domestic retail prices as well, thereby weakening competition.

[21]This assumption reflects that the wholesale prices, which are also called Inter-Operator-Tariffs, are published by the GSM Association.

services from operator $B1$.

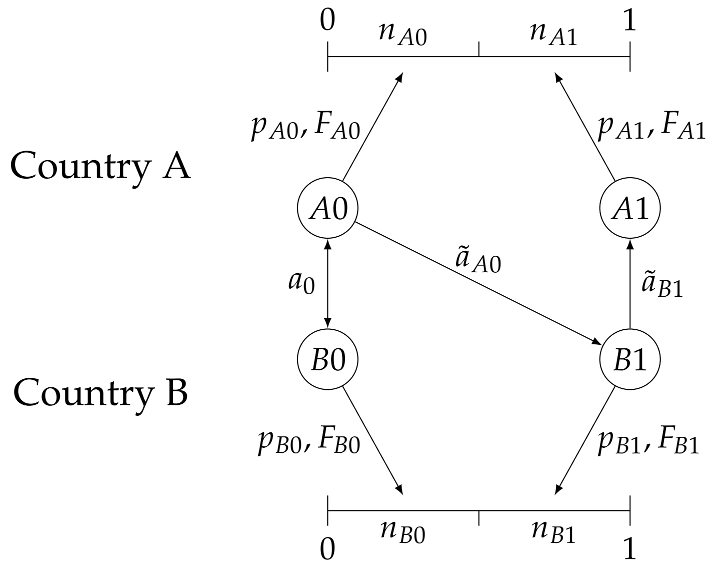


Figure 1.1: Model setup - overview

Timing: The base model consists of the following stages:

1. Members of an alliance negotiate on the wholesale price for roaming calls within their alliance.

2. MNOs simultaneously post wholesale roaming prices for operators that are not affiliated with an alliance.[22] MNOs that do not pertain to an alliance choose which foreign operator hosts their traveling subscribers.

3. Operators set retail tariffs. Consumers subscribe to their preferred network and place their roaming calls.

The sequential structure allows MNOs to set their wholesale prices strategically. It reflects that due to legal and practical reasons, wholesale prices can be changed less easily than retail tariffs.[23] We look for subgame perfect Nash equilibria and solve the model by backward induction.

[22]The results remain unchanged if wholesale prices that are set within alliances are not publicly known before MNOs post the wholesale prices for unilateral roaming agreements.

[23]In Europe, the Standard Terms for International Roaming Agreement (STIRA) issued by the GSM Association provide guidelines how wholesale prices have to be set. They prescribe that wholesale prices have a validity of at least six months.

1.3 Retail Equilibrium

In this section, we take as given the choice of the foreign host operator and characterize the equilibrium retail tariffs, market shares and retail profits.

The perceived marginal cost of the reselling operator xi, which we denote as c_{xi}, equals the wholesale price of its host operator. For example, if roaming services for traveling subscribers of operator Ai are provided by operator Bj then the perceived marginal cost of operator Ai is $c_{Ai} = a_{Bj}$. Per subscriber, an operator earns $\pi^R_{xi} = q(p_{xi})(p_{xi} - c_{xi}) + F_{xi} - C_F$.

To derive the profit maximizing retail tariff, it is convenient to express the profit in terms of the retail per call price p_{xi} and the net surplus w_{xi} rather than in terms of p_{xi} and F_{xi}.[24] The implied fixed fee can then be retrieved by the identity $F_{xi} = v(p_{xi}) - w_{xi}$. The retail profit is thus $\Pi^R_{xi} = n(w_{xi}, w_{xj})\,(q(p_{xi})(p_{xi} - c_{xi}) + v(p_{xi}) - w_{xi} - C_F)$.

The availability of two-part tariffs yields pricing at perceived marginal cost, that is $p^*_{xi} = c_{xi}$.[25] Intuitively, setting the usage price equal to the perceived marginal cost avoids any dead-weight loss (from the viewpoint of the reselling operator).[26] The fixed fee is then used to extract $v(c_{xi}) - w_{xi}$ without causing any inefficiencies. Using the optimal per call price, the retail profit of operator xi simplifies to

$$\Pi^R_{xi} = \Pi^R(w_{xi}, w_{xj}, c_{xi}) \equiv n(w_{xi}, w_{xj})\,(v(p_{xi}) - w_{xi} - C_F)\ . \tag{1.3}$$

When both domestic operators serve the market, the corresponding first order condition determines the profit maximizing level of net surplus

[24] After this transformation, the usage price p_{xi} does not enter the market share any more, so that the optimal tariff does not depend on the competitor's usage price.

[25] This finding is by now well understood. See e.g. Laffont, Rey, and Tirole (1998a); Armstrong (2002). This claim is formally proved in Lemma 1.1.

[26] If $q'(c_{xi}) = 0$, then $p^*_{xi} = c_{xi}$ is not a strict maximizer of $\pi^R(p_{xi}, a_{xi}, c_{xi})$, and its maximum is also attained by other per call prices. However, the usage prices do not affect the best response of the retail competitor. As all retail per call prices that attain the maximum retail profits are economically equivalent, we treat them as one equivalence class.

$$w^*(c_{xi}, w_{xj}) = \frac{1}{2}[v(c_{xi}) + w_{xj} - C_F - \frac{1}{2\sigma}]. \quad (1.4)$$

Solving the system of best responses allows us to characterize the retail equilibrium as follows.

Lemma 1.1. *A retail equilibrium always exists. If the difference between perceived marginal costs is not too big, namely $|v(c_{x0}) - v(c_{x1})| \leq \frac{3}{2\sigma}$, the retail equilibrium is uniquely characterized by*

$$w^*(c_{xi}, c_{xj}) = \frac{2}{3}v(c_{xi}) + \frac{1}{3}v(c_{xj}) - \frac{1}{2\sigma} - C_F, \quad (1.5)$$

$$n^*(c_{xi}, c_{xj}) = \frac{1}{2} + \frac{\sigma}{3}[v(c_{xi}) - v(c_{xj})], \quad (1.6)$$

$$\Pi^{R*}(c_{xi}, c_{xj}) = \frac{\left(n^*(c_{xi}, c_{xj})\right)^2}{\sigma}. \quad (1.7)$$

If instead $v(c_{xi}) - v(c_{xj}) > \frac{3}{2\sigma}$, then there exists a unique equilibrium in weakly undominated strategies[27] *where operator xi serves the whole market and offers $w^*_{xi} = \frac{1}{2\sigma} + v(c_{xj}) - C_F$, while its competitor sets $w^*_{xj} = v(c_{xj}) - C_F$.*

Proof. See Appendix A1.1. ■

Increasing an operator's perceived marginal cost has two effects. First, it directly reduces operator xi's retail profit. Second, it softens retail competition.[28] Intuitively, the competitor anticipates that operator xi optimally reduces its subscribers' net surplus when its marginal cost increases. Since net surpluses are strategic complements by equation (1.4), competitor xj optimally also offers less attractive contracts to its own subscribers.[29] The total impact of an increase of operator xi's perceived marginal cost on its retail profit is

[27] See Palfrey and Srivastava (1991) for a definition of the undominated Nash Equilibrium concept. An undominated NE may not consist of strategies that are weakly dominated.

[28] By Lemma 1.1, if the difference in perceived per call costs is too big so that the competitor stays out of the market, a marginal increase in own per call costs triggers no strategic effect of softer competition.

[29] This conclusion relies also on the stability of the retail equilibrium. For a comprehensive discussion of strategic complementarity, see e.g. Bulow, Geanakoplos, and Klemperer (1985).

$$\frac{\mathrm{d}\Pi_{xi}^{R*}}{\mathrm{d}c_{xi}} = \frac{\partial \Pi_{xi}^{R}}{\partial w_{xj}} \frac{\mathrm{d}w_{xj}^{*}}{\mathrm{d}c_{xi}} + \frac{\partial \Pi_{xi}^{R}}{\partial c_{xj}} = -\frac{2n_{xi}^{*}}{3} q(c_{xi}) \,. \tag{1.8}$$

Since the negative direct effect of a cost increase dominates the positive strategic effect, an operator *unilaterally* prefers lower wholesale roaming prices. However, as we show in the next section, within an alliance the negative direct effect will be offset by gains at the wholesale level, while the strategic effect remains.

1.4 Wholesale Equilibrium

This section analyzes the equilibrium wholesale prices that obtain for a given number of alliances. We suppose that operators with the same index i form alliances, which is without loss of generality due to our symmetry assumptions.

Wholesale prices of unilateral roaming agreements

We first derive the equilibrium wholesale prices for roaming services that will be offered to MNOs which have not formed an alliance. Recall that joining an alliance does not preclude MNOs from *selling* roaming services to foreign operators that do *not* pertain to this alliance.[30] So each operator xi may offer (simultaneously with its domestic competitor xj) to act as host operator for subscribers of country y at the wholesale price $\tilde{a}_{xi}$.[31] By the results of the previous section, any operator that is not member of an alliance optimally buys roaming services from the foreign operator which offers the lowest wholesale price.

In the absence of alliances, operators thus compete in a standard Bertrand way to serve as host operator. It is profitable to undercut the wholesale price of the domestic competitor as long as the wholesale margin $\tilde{a}_{xi} - c$ is strictly positive. By the usual Bertrand reasoning, any operator offers roaming services at wholesale price $\tilde{a}_{xi}^{*} = c$ in equilibrium.

[30]Regulation authorities might prohibit alliances that force members not to sell to outsiders as this behavior might be perceived illegal.

[31]For simplicity and without loss of generality, we suppose that operators cannot discriminate the wholesale price according to which foreign operator buys roaming services.

A similar reasoning holds if one alliance has been formed. Suppose operators xi and yi belong to an alliance while xj and yj remain without alliance. As before, by undercutting slightly any rival's price $\tilde{a}_{xj} > c$ above the true marginal cost, operator xi additionally earns strictly positive wholesale profits from selling to yj.[32] Since undercutting $\tilde{a}_{xj}$ reduces the retail market of the partner network, it also lowers the wholesale profits generated with these subscribers. However, when undercutting slightly, this effect is negligible since the perceived marginal cost of operator yj and hence the retail market shares stay almost constant. As operator xj also undercuts any $\tilde{a}_{xi} > c$, the unique equilibrium prices are again $\tilde{a}^*_{xi} = \tilde{a}^*_{xj} = c$.

If two alliances have been created, then all operators are committed to buy roaming services from their partner network, so that unilateral roaming agreements play no role. We can thus summarize:

Proposition 1.1. *In unilateral roaming agreements, the equilibrium wholesale price for roaming services equals the cost of providing a roaming call c. In particular, this applies if international alliances are not feasible.*

Proof. In the text. ■

Wholesale prices within alliances

Both members of an alliance commit to buying roaming services exclusively from the foreign partner network, even in case another foreign operator offers cheaper wholesale prices for roaming services. We assume that each alliance negotiates on a single bilateral wholesale roaming price that maximizes the joint profit and applies for roaming calls in both directions: $a_{Ai} = a_{Bi} \equiv a_i$.[33] We later consider richer sets of wholesale agreements in Section 1.6.4.

Since the negotiated wholesale prices become public knowledge, the ensuing retail equilibrium tariffs are as described in Section 1.3, treating the own wholesale price as

[32] Operator xi's retail profit is unaffected by the wholesale price that it charges to deliver roaming services outside the alliance, since the retail pricing decision of operator xj is independent of its wholesale profits.

[33] In any symmetric equilibrium, operators would deliberately choose $a_{Ai} = a_{Bi}$ even if they were allowed to set possibly differing wholesale prices (a_{Ai}, a_{Bi}).

a perceived marginal cost: $c_{xi} = a_i$. Indeed, after wholesale prices have been set in an alliance, an operator cannot affect the retail market share of its foreign partner any more. Hence the level of wholesale profit is treated as constant when deciding on the own retail tariff for domestic subscribers.

Operator xi's overall profit comes from reselling roaming calls to subscribers in its home country x and from selling roaming services to operator yi.[34] Because of symmetric costs and demand across countries all members of one alliance achieve equal market shares $n^*_{Ai} = n^*_{Bi} \equiv n^*_i$ and equal retail profits $\Pi^{R*}_{Ai} = \Pi^{R*}_{Bi} \equiv \Pi^{R*}_i$.[35] Therefore, each member of alliance i earns the total profit

$$\Pi_i = \Pi(a_i, c_j) \equiv n^*(a_i, c_j) \left[\pi^W(a_i) + \pi^{R*}(a_i, c_j)\right] \tag{1.9}$$

where

$$\pi^W(a_i) \equiv q(a_i)[a_i - c]$$

denotes the per customer wholesale profit. Suppose now that all operators obtain a positive market share. Then, using the results of Section 1.3, the marginal profit generated by an increase in the wholesale price of alliance i is

$$\frac{\partial \Pi}{\partial a_i}(a_i, c_j) = q(a_i)\left[\frac{1}{3}n^*_i - \epsilon(a_i)n^*_i - \frac{\sigma}{3}\pi^W(a_i)\right] \tag{1.10}$$

where $\epsilon(p) \equiv \frac{-(p-c)q'(p)}{q(p)}$ is the *markup* elasticity of per customer demand.[36]

Three effects determine the marginal profit (1.10). These arise *indirectly* through a change of the ensuing equilibrium retail tariffs. The first term in (1.10) represents the positive *strategic* effect of softer competition discussed in Section 1.3. The last two terms refer to inefficiencies that arise when the wholesale price diverges from the true

[34] Recall that no profits can be generated by unilateral roaming agreements.

[35] Both operators j have the same perceived marginal cost since they either form an alliance and negotiate on a reciprocal wholesale price a_j or remain without alliance and buy roaming services at the true marginal cost c.

[36] Note that the demand elasticity in markup terms is closely related to the price elasticity of demand which is defined as $\eta(p) \equiv \frac{-pq'(p)}{q(p)}$. The following relationship holds: $\epsilon(p) = \eta(p)\frac{(p-c)}{p} < \eta(p)$. In case of $c = 0$, the markup elasticity coincides with the price elasticity of per customer demand. See also Anderson, de Palma, and Nesterov (1995).

marginal cost. By Section 1.3, an increase of the wholesale price will be passed on to customers directly and causes undesired deadweight loss from the viewpoint of an alliance.[37] In addition, increasing the wholesale price induces the operators to offer less attractive retail tariffs. This reduces the customer base and therefore the wholesale profit. Note that equation (1.10) does not contain any *direct* price effect. Since each member sells the same quantity of roaming calls to the foreign partner that it buys for own subscribers, any additional expenses for roaming services at the retail level are perfectly recouped at the wholesale level.

Setting marginal profits (1.10) to zero and rearranging, we obtain the Lerner condition

$$\frac{a_i^* - c}{a_i^*} = \frac{1}{3\left[\eta_q(a_i^*) + \eta_{n^*}(a_i^*, c_j)\right]} \tag{1.11}$$

where $\eta_q(a_i) \equiv -\frac{a_i q^{'}(a_i)}{q(a_i)}$ is the *price* elasticity of per customer demand and $\eta_{n^*}(a_i, c_j) \equiv -\frac{\mathrm{d}n_i^*}{\mathrm{d}a_i}\frac{a_i}{n_i^*} = \frac{\sigma a_i q(a_i)}{3n_i^*}$ is the price elasticity of the equilibrium retail market share.[38]

We need the following technical assumption to guarantee existence and uniqueness of a wholesale equilibrium:

Assumption 1.2. *The markup elasticity of per customer demand $\epsilon(p)$ is non-decreasing for all prices above marginal costs whenever $\epsilon(p) \leq 1$.*

Assumption 1.2 assures that the marginal impact of deadweight loss is non-decreasing in the wholesale price. It is satisfied by many commonly used demand functions, including constant demand, linear demand or constant (price) elasticity demand.

Let $a^*(c_j)$ denote the wholesale price that maximizes alliance i's profits when the competing operators have the perceived marginal cost c_j. Based on the optimality condition (1.11), the following lemma establishes that wholesale prices are strategic complements on the relevant range.[39]

[37] While the envelope theorem states that an increase in the retail per call price has no marginal effect on retail profits, it has a negative effect on wholesale profits.

[38] In case of two alliances and a symmetric wholesale price, each alliance achieves a market share of $n_i^* = \frac{1}{2}$ and the price elasticity of the market share simplifies to $\eta_{n^*}(a_i) \equiv \frac{2}{3}\sigma a_i q(a_i)$.

[39] Formally, the relevant range is $\mathcal{E} = \left\{p \in \mathbb{R} | \epsilon(p) < \frac{1}{3} \wedge p \geq c \wedge q(p) > 0 \wedge v(p) > v(c) - \frac{3}{2\sigma}\right\}$ as shown in Appendix A1.1.

Lemma 1.2. *If Assumption 1.2 holds, then a best response $a^*(c_j)$ uniquely exists and is strictly increasing in c_j on the relevant range.*

Proof. See Appendix A1.1. ■

The own market share increases in the perceived marginal cost of the competing operators. A higher market share amplifies the strategic effect of softer competition, which results in a higher own profit maximizing wholesale price. We now turn to our main result:

Proposition 1.2. *Suppose that Assumption 1.2 holds.*

i) If a single alliance i is created, then the unique equilibrium wholesale price a^{1*} within this alliance is characterized by equation (1.11) *using $c_j = c$ and exceeds the true marginal cost: $a^{1*} > c$.*

ii) If two alliances are formed, a unique equilibrium in weakly undominated strategies exists in which both alliances set the symmetric wholesale price $a_0 = a_1 = a^*$.[40] *This equilibrium price is characterized by equation* (1.11) *using $c_j = a^*$ and exceeds the bilateral wholesale price in case only one alliance is formed: $a^* > a^{1*} > c$.*

Proof. See Appendix A1.1. ■

Besides existence and uniqueness, Proposition 1.2 confirms that alliances set higher wholesale prices for roaming calls than would be socially optimal.[41] Assumption 1.2 assures existence and uniqueness but is not needed to derive that a strictly positive markup on the wholesale level necessarily occurs.

To understand the intuition for part i), let us compare the situation without alliances to that in which one alliance has emerged. Without alliances Bertrand competition be-

[40] This refinement is only needed in case demand is constant below c to rule out implausible equilibria. In this case, there exist corner equilibria in which alliance i sets wholesale prices far below c and corners the whole market while the rival alliance j sets a wholesale price above c and is driven out of the market. The equilibrium price a_i^* is then weakly dominated by $a_i = c$. This class of equilibria is implausible since alliance j sets a_j^* far above c, knowing that lower prices would also guarantee non-negative profits and yield strictly higher profits if alliance i would adjust its price, too.

[41] In contrast to Laffont, Rey, and Tirole (1998a), candidate equilibria are robust to big deviations. According to equation (1.10), the marginal profit becomes negative when the per customer wholesale profit π_i^W is large. Together with Assumption 1.2, this implies that equilibrium prices cannot exceed the marginal costs c by too much. Hence, a deviation as to corner the market would require wholesale prices below the true marginal costs and would not be profitable.

tween foreign operators pushes the wholesale price down to marginal cost. Given this wholesale price each operator offers a two part tariff setting the price per call equal to the true marginal cost and extracting some of the consumer surplus via the fixed fee. When operators i form an alliance, competition still keeps the wholesale prices for the remaining two operators at the efficient level c. Within an alliance, members can jointly decide on the wholesale price. Raising the wholesale price of the alliance above marginal cost induces competing operators to offer less attractive retail contracts by the strategic complementarity discussed in Section 1.3. This strategic effect increases profits and is of first order. The additional expenses needed to procure roaming services for own subscribers are fully recouped since the foreign partner buys the same quantity of these services for its subscribers. A higher wholesale price also leads to a distorted retail tariff which is set as to maximize the retail profit instead of the total profit. However, for wholesale prices close to the true marginal cost c the optimal retail tariff from the viewpoint of the alliance is almost attained, so that the impact from distortions on the total profit is of second order.[42] Hence, starting out from a wholesale price equal to marginal cost, it is always optimal to raise the wholesale price at least somewhat once an alliance is formed.

According to part ii) of Proposition 1.2, the equilibrium wholesale prices further increase if a second alliance is formed. When the rivals j also form an alliance, they will negotiate on a wholesale price above c by the same reasoning as above. Since the optimal wholesale price is upward sloping in the competitor's price by Lemma 1.2, each alliance will set a higher wholesale price as would do a single alliance. Note however, that the equilibrium wholesale price remains below the level that maximizes the industry profits, characterized by $\frac{a^M - c}{a^M} = \frac{1}{\eta_q(a^M)}$.[43]

Relaxing the assumption of homogeneous customers does not qualitatively change our main result of harmful alliances as we show in Appendix A1.2. There, we allow for light and heavy users, assuming that the mean demand for roaming calls of the pop-

[42]This follows from the envelope theorem.

[43]The role of wholesale prices differs from that of access-prices in Laffont, Rey, and Tirole (1998a). In their model of network interconnection, even the industry monopoly profits can be attained provided the retail equilibrium exists since the access price equally applies to both domestic competitors. In our model, taking a_j as given and increasing the bilateral wholesale price of alliance i decreases its market share. The danger of losing too many subscribers keeps wholesale prices below the level that maximizes industry profits.

ulation is unchanged and that subscribers in both segments have the same degree of taste differentiation $1/\sigma$. We find that heavy users, which are particularly valuable for operators, are more inclined to switch to the competitor after an increase of the usage price. We show that even though the fear of losing heavy users reduces the equilibrium wholesale price somewhat, it remains strictly above the true marginal cost, where the marginal loss from distorted retail tariffs is still of second order.

Comparative Statics

We now present some comparative statics of the equilibrium wholesale price when two alliances are in place, which will be shown to be the configuration that obtains if alliances are endogenously formed.[44]

Proposition 1.3. *Suppose that Assumption 1.2 holds.*

i) The equilibrium wholesale price a^* decreases in the degree of competition on the retail market σ.

ii) The equilibrium wholesale price a^* decreases if the per customer demand is multiplied by some constant $\lambda > 1$.

iii) Suppose that the per customer demand function $\tilde{q}$ is more elastic than q: $\eta_{\tilde{q}}(p) > \eta_q(p)\ \forall p$. Denote the associated symmetric equilibrium wholesale prices by $\tilde{a}^*$ and a^*. If the per customer demand $\tilde{q}$ is weakly higher than q at the equilibrium price a^* (i.e. $\tilde{q}(a^*) \geq q(a^*)$), then the wholesale equilibrium price decreases in the elasticity of customer demand: $\tilde{a}^* < a^*$.

Proof. See Appendix A1.1. ■

Part i) of Proposition 1.3 states that wholesale equilibrium prices are lower if taste differences of customers ($1/\sigma$) are small. In this case the negative effect of losing market share when increasing the wholesale price is strong compared to the competition softening effect.

According to part ii), the equilibrium price decreases if the per customer demand rises uniformly. Intuitively, a higher demand implies that the usage price becomes more

[44] The same comparative statics obtain in case of only one alliance.

important relative to the differences in taste so that the market share becomes more elastic. Thus, increasing the wholesale price leads to a stronger reduction in market share and the forgone wholesale profit per customer increases due to a higher demand per customer. Due to the amplified negative effects from distorted retail tariffs, the equilibrium price decreases.[45]

Part iii) compares differences in the elasticity of demand. When demand is more elastic, the dead-weight loss invoked by setting the wholesale price above marginal costs becomes more pronounced and thus disciplines alliances. The proposition also requires that the more elastic demand function $\tilde{q}$ exceeds the demand q at the equilibrium price a^*. This condition assures that operators have no countervailing incentive to raise the wholesale price due to a reduced elasticity of the market share.

Examples. The results of this section can be illustrated by some common demand functions that admit explicit solutions. First, we assume that the per customer demand q is constant: $q(p) = \bar{q}$. Clearly, in this case there is no concern of deadweight loss and an alliance trades off solely the benefits from softer competition with the loss of market share. The elasticity of the retail market share becomes $\eta_n(a_i) = \frac{\sigma a_i}{3n_i^*}\bar{q}$ and the equilibrium wholesale price can be explicitly determined by solving condition (1.11): $a^*_{\bar{q}} = c + \frac{1}{2\sigma\bar{q}}$. This formula confirms that the equilibrium price is decreasing in the degree of competition σ and in the demand $\bar{q}$.

Another example that admits an explicit solution is the commonly used constant elasticity demand $\tilde{q}(p) = \frac{A}{p}$. Using this specification, the equilibrium wholesale price is $a^*_{\tilde{q}} = c + \frac{c}{2+2\sigma A}$. If $A \geq \left(c\bar{q} + \frac{1}{2\sigma}\right)$ then $\tilde{q}(a^*_{\bar{q}}) \geq \bar{q}$ and the hypothesis of Proposition 1.3, part iii) is satisfied. Indeed, for $A = \left(c\bar{q} + \frac{1}{2\sigma}\right)$, we have $a^*_{\tilde{q}} = c + \frac{c}{3+2\sigma\bar{q}c} < a^*_{\bar{q}}$.

[45] Note that from condition (1.11), the wholesale equilibrium price depends only on the elasticity of per customer demand η_q and on the elasticity of the market share $\eta_{n^*} = \frac{\sigma a_i q(a_i)}{3n_i^*}$. Since multiplying the demand $q(\cdot)$ by some constant $\lambda > 0$ leaves the demand elasticity unchanged, it has the same effect on the market equilibrium price as multiplying σ by λ.

1.5 Endogenous Formation of Alliances

We now endogenize the choice of MNOs to form alliances. Operators whose home network is in the same country may not collaborate within an alliance, for example due to legal constraints.[46] Therefore any alliance consists of exactly one MNO with home country A and another of country B.

Formally, we introduce a formation stage that takes place before wholesale prices are set. For simplicity, we assume that operator $A0$ may form an international alliance with $B0$ and $A1$ with $B1$.[47] Competing operators simultaneously decide on creating an alliance. In order to circumvent coordination issues we assume that operators form an alliance whenever this increases the total profit of its members.[48] Thus, to analyze how many alliances are created in equilibrium, we simply have to compare the equilibrium profits of each configuration.

Creating an alliance dominates staying alone. Suppose first that operators j do not create an alliance and therefore buy roaming services at a wholesale price of c. Forming an alliance allows operators i to commit to a wholesale price that exceeds the true marginal cost. Since marginally increasing the wholesale price is profitable at c, this raises the total profit: $\Pi(a^{1*}, c) > \Pi(c, c)$. Suppose now that operators j form an alliance. Then, creating an additional alliance is even more profitable, since it additionally induces operators j to further increase their wholesale price to $a^* > a^{1*}$, which makes setting a high wholesale price within an alliance even more profitable: $\Pi(a^*, a^*) > \Pi(c, a^{1*})$.[49] This yields the following prediction:

Proposition 1.4. *Suppose that Assumption 1.2 holds. Then a unique subgame perfect equilibrium exists with two competing alliances being formed. In every country, the market is equally split between both alliances. Both alliances set the equilibrium wholesale price a^* characterized by Proposition 1.2, part ii).*

[46]Otherwise, all operators would agree on a wholesale price that maximizes joint *industry* profits.

[47]By the symmetry assumptions, this restriction is without loss of generality.

[48]Putting aside coordination issues, this formulation generates the same results as a more complicated formation stage in which operators announce their choice and alliances are only formed if two operators agree to form an alliance.

[49]Formally, this inequality follows from $\frac{\partial \Pi}{\partial a_j}(a_i, a_j) = \frac{1}{3} q(a_j) \left[2n_i^* + \sigma \pi^W(a_i)\right] > 0$.

Proof. In the text. ■

Decomposing the total equilibrium profit shows that alliances increase the wholesale profit without lowering the equilibrium retail profit. Due to our simple Hotelling framework, the retail equilibrium profit $\Pi_i^{R*} = \frac{(n_i^*)^2}{\sigma}$ depends only on the market share but not on the absolute level of retail prices. Since the retail market is equally shared when either all operators stay alone or two alliances have been created, the retail equilibrium profit remains unchanged. However, with alliances, operators additionally earn a strictly positive wholesale margin which makes them better off in total. Subscribers are unambiguously worse off once alliances are introduced since the equilibrium retail usage price increases while the equilibrium fixed fee remains unchanged.

Note that the strategic effect cannot be achieved if operators Ai and Bi merge rather than form an alliance. A merged operator i possesses a network in both countries. It therefore sets the retail prices in each country as to maximize the sum of retail *and* wholesale profits of both countries. Within a merged operator, the perceived marginal cost when setting the retail tariff remains at c, independently of the chosen (virtual) wholesale price. Hence, conducting a merger generates no strategic effects and leads to the same profits as staying alone. As a policy implication, if creating an international alliance or an international merger generates additional positive effects beyond this model, then competition authorities should promote international mergers instead of alliances. Indeed, the OECD (2009) observes that the prices for roaming services drop, once operators with networks in different countries merge and therefore cannot credibly commit to excessive wholesale prices any longer.

1.6 Extensions

1.6.1 Non-Discrimination Clause

The STIRA framework which was introduced by the GSMA in 1996 contains a so called non-discrimination clause. According to this clause, an operator should apply the same terms and conditions on the wholesale market to all foreign operators when providing access to its network. In this section, we show that the non-discrimination clause impairs competition for unilateral roaming agreements and allows alliances to raise the rivals' marginal cost. Compared to the results of our base model, this leads to even higher usage prices and further increases equilibrium profits.

In the spirit of this clause, we now assume that operators have to charge the same wholesale price that has been negotiated within an alliance whenever they sell roaming services to non-affiliated operators. Thus, in contrast to the timing considered above, only operators that have not joined an alliance may post a wholesale price.

Since the non-discrimination clause only affects the equilibrium wholesale prices if exactly one alliance has emerged, suppose now that only operators i have formed an alliance.[50] Then operators j generate positive wholesale revenues only if they offer a lower wholesale price than alliance i.[51] For a given a_i, the wholesale profit of operator xj that charges a wholesale price of a_{xj} is

$$\tilde{\Pi}^W(a_{xj}, a_i) \equiv \begin{cases} 0 & \text{if} \quad a_{xj} > a_i \\ n^*(a_{xj}, a_i)\pi^W(a_{xj}) & \text{if} \quad a_{xj} \le a_i \end{cases}$$

where $n^*(a_i, a_{xj}) \equiv \frac{1}{2} + \frac{\sigma}{3}\left(v(a_i) - v(a_{xj})\right)$ and $\pi^W(a_i) \equiv q(a_i)[a_i - c]$ remain as already defined in Section 1.3 and 1.4, respectively. Denote by $\tilde{a}^*(a_i)$ the best response of the non-affiliated operators j as a function of the wholesale price a_i set by alliance i. We show in Proposition 1.5 below that $\tilde{a}^*(a_i)$ equals any wholesale price a_i up to some uniquely defined threshold $\bar{a}^\dagger$: $\tilde{a}_j^*(a_i) = a_i \quad \text{if } a_i \in \left[c, \bar{a}^\dagger\right]$. This threshold lies above the wholesale equilibrium price a^* defined by Proposition 1.2.[52] This is because operator xj sets $\tilde{a}_{xj}$ in order to maximize its *wholesale* profit and only internalizes that increasing the wholesale price reduces the retail market share but not that it also lowers the retail per customer profit of the reselling operator yj.

[50] In case of two alliances, their members are committed to buy roaming services only within the same alliance. So, wholesale prices for non-affiliated operators play no role.

[51] As a tie-breaking rule that assures equilibrium existence, we assume that whenever all operators of one country offer the same wholesale price, operators that do not pertain to an alliance buy all roaming services from a non-alliance operator.

[52] We formally prove this result in Proposition 1.5 below.

We now illustrate why the equilibrium price[53] a^{ND*} that obtains when only one alliance i has been formed unambiguously exceeds a^*. Alliance i cannot earn profits from selling roaming services to any operator j since non-affiliated operators weakly undercut the negotiated wholesale price whenever $a_i > c$. Moreover, any wholesale price a_i below $\bar{a}^\dagger$ yields a retail market share for alliance i of 1/2 since the non-affiliated operators j will exactly match this price. Raising the wholesale price from a^* to $\bar{a}^\dagger > a^*$ thus allows the alliance to increase its wholesale profit without losing market share.[54] Therefore, if only one alliance has emerged, the equilibrium wholesale prices of all operators are at least $\bar{a}^\dagger$.[55]

The following proposition establishes that indeed all operators' profits are highest when only one alliance is formed and that this configuration obtains in equilibrium.

Proposition 1.5. *Suppose that assumption 1.2 holds and a non-discrimination clause is in place. Then a single alliance that sets the unique profit-maximizing wholesale price a^{ND*} emerges in equilibrium. The wholesale price a^{ND*} strictly exceeds the price a^* characterized by Proposition 1.2. Introducing a non-discrimination clause unambiguously increases all operators' profits and decreases both customer surplus and welfare.*

Proof. See Appendix A1.1. ■

Intuitively, a non-discrimination clause allows operators in an alliance to commit not to undercut the wholesale prices of non-affiliated operators.[56] Similar to Ordover, Saloner, and Salop (1990), this commitment assures that rival operators will have to pay high wholesale prices for roaming services. The clause thus severely restricts competition to provide non-affiliated operators with roaming services and essentially allows

[53]The superscript ND refers to non-discrimination

[54]In the proof of Proposition 1.5 we show that $\epsilon(\bar{a}^\dagger) > 1$ so that lower per customer demand is more than offset by a higher margin.

[55]Note that higher wholesale prices partially obtain since the alliance sets its wholesale price before the non-affiliated operators. Since wholesale prices are strategic complements as shown by Lemma 1.2, if two alliances chose their wholesale prices sequentially, higher wholesale prices than a^* would obtain. But whenever $a^{ND*} = \bar{a}^\dagger$, then a^{ND*} even exceeds the wholesale price that would be set by the alliance i, if another alliance j observed a_i before negotiating on a_j. A sufficient condition for $a^{ND*} = \bar{a}^\dagger$ is $\sigma\pi^W(\bar{a}^\dagger) \leq 1$.

[56]In a different setup with secret contracts, Rey and Tirole (2007) recently reported that a non-discrimination clause may be harmful, since it confers commitment against opportunistic but socially desirable behavior. We have thus discovered another reason why commitment obtained by help of a non-discrimination clause may be advantageous for firms.

an alliance to soften competition both at the retail *and* at the wholesale level.[57] Interestingly, the prediction that some operators join alliances while others do not seems to be in line with the actual behavior of European MNOs.

1.6.2 The Role of Host Network Selection

This section serves to analyze the competitive impact of recent technological developments that have improved the home operators' control over the choice of foreign host networks for roaming.[58] Departing from our assumption of perfect network selection technologies, we now consider the other polar case of operators having no control which foreign network their subscribers use.[59] Appendix A1.3 covers intermediate levels of control. As we show, the possibility of traffic direction increases the competitive pressure in the wholesale market. We find that alliances are without bite if the host network is randomly determined and conclude that the importance of international alliances has increased with recent technological improvements.

We assume that operators cannot discriminate the retail usage price contingent on which foreign network is used. If price discrimination was feasible, subscribers would always choose the cheapest network.[60] Hence, operators could perfectly control the network selection by setting the price of the preferred foreign network lower than that of the non-desired network. The outcome would then be economically equivalent to our base model.

When buying roaming calls from foreign MNOs on the wholesale market, operator xi's perceived marginal cost is:

$$c_{xi} = \frac{1}{2}\left(a_{y0} + a_{y1}\right) \tag{1.12}$$

[57]These results are partially driven by our assumption that only two networks operate in each country. However, we suspect that in a symmetric setup of N operators, the non-discrimination clause would induce the creation of $N-1$ alliances.

[58]Salsas and Koboldt (2004) offer a more extensive treatment of recent technological developments.

[59]In the past, operators indeed accepted that their traveling subscribers were assigned almost randomly to foreign networks, for several reasons like e.g. coverage. For a more extensive discussion, see Salsas and Koboldt (2004).

[60]However, according to European Commission (2005) there is empirical evidence that few subscribers are aware or engaged in manual network selection.

Again, the optimal per call price equals the perceived marginal cost: $p^*_{xi} = c_{xi}$. The retail equilibrium net surplus, market share and the equilibrium profits remain as established in Lemma 1.1. Since each operator has to procure half of the roaming services from each foreign operator, $c_{xi} = c_{xj}$. Thus the retail market is perfectly shared and the equilibrium profit is constant in a_{yi} by our results of Section 1.3: $\hat{\Pi}^{R*}_{xi} = \frac{1}{4\sigma}$.

No international alliances. In the absence of alliances the total wholesale demand of operator xi is $\hat{Q}_{xi} \equiv \frac{1}{2}q\left(\frac{1}{2}(a_{x0}+a_{x1})\right)$. The demand does not depend on the actual market share of the reselling operators, since *both* purchase half of their traffic from operator xi. The total profit of operator xi is:[61]

$$\hat{\Pi}^{NA}_{xi} = \hat{\Pi}^{R*}_{xi} + \frac{1}{2}(a_{xi}-c)\, q\left(\frac{1}{2}(a_{x0}+a_{x1})\right) \tag{1.13}$$

Similar to Section 1.4, operator xi sets its wholesale price so as to maximize its wholesale profits $(a_{xi}-c)\, Q_{xi}$. The following mild technical assumption assures that the per customer demand is elastic enough for an equilibrium to exist:[62]

Assumption 1.3. *The markup elasticity of per customer demand $\epsilon(p)$ is increasing for all prices above marginal costs whenever $q(p) > 0$ and there exists some $\tilde{p} > c$ with $\epsilon(\tilde{p}) = 2$.*

Proposition 1.6. *Suppose that Assumption 1.3 holds and that operators cannot select the host network of their subscribers. If no alliances are feasible there exists a unique symmetric equilibrium wholesale price a^{NA*}, characterized by*

$$\frac{a^{NA*}-c}{a^{NA*}} = \frac{2}{\eta_q(a^{NA*})} \tag{1.14}$$

where $\eta_q(\cdot)$ is the price elasticity of per customer demand.

Proof. Rearranging the first order condition that is necessary for maximization of $\hat{\Pi}^{NA}_{xi}$ yields condition (1.14). Rewriting the marginal profit in terms of markup-elasticity and evaluating at $a_{xj} = a_{xi}$ yields $\frac{\partial \hat{\Pi}^{NA}}{\partial a_{xi}} = \frac{1}{2}q(a_{xi})\left[1-\frac{1}{2}\epsilon(a_{xi})\right]$. Thus the first order condition is satisfied at $\tilde{p}$ which uniquely exists by Assumption 1.3. The profit is strictly quasiconcave since $\epsilon'(p) > 0$ whenever $q(p) > 0$ by assumption. ■

[61] The superscript NA refers to “no alliance”.

[62] Since now the market share does not decrease in the own wholesale price, Assumption 1.3 has to be stronger than Assumption 1.2.

By Proposition 1.6, if operators cannot influence which foreign network their subscribers use to place roaming calls, the resulting equilibrium wholesale price is extremely high. Unilaterally increasing the wholesale price a_{xi} causes a negative externality on the rival, since the wholesale demand of operator xj is reduced while only the margin of operator xi increases. As operators do not take this externality into account, the resulting equilibrium price even exceeds the monopoly price.

Two international alliances. We now analyze the equilibrium outcome after operators with the same location have formed two competing alliances and omit the country index for brevity. Operators have to offer roaming services on the wholesale market to all foreign operators for the same price a_i that is negotiated within an alliance.[63] Thus, the only remaining virtue of alliances is to set the wholesale price cooperatively instead of competitively.

If both alliances have negotiated wholesale prices a_i and a_j, the profit of each operator in alliance i is

$$\hat{\Pi}_i = \hat{\Pi}_i^{R*} + \frac{1}{2}(a_i - c)\, q\left(\frac{1}{2}(a_0 + a_1)\right). \tag{1.15}$$

Since both the retail and the wholesale profit is the same as in the case of no alliances treated above, we conclude:

Proposition 1.7. *Suppose that Assumption 1.3 holds and that operators cannot select the host network of their subscribers. The formation of two alliances does not affect the wholesale equilibrium price, which remains characterized by* (1.14). *Ceterus paribus, with two alliances the equilibrium wholesale price under random network selection lies above that under perfect network selection given by Proposition 1.2, part ii).*

[63] This restriction facilitates the comparison with the results of the base model. When allowing MNOs to discriminate between members of the alliance and non-members, then the wholesale price $\hat{a}_i$ that applies to non-members will be set extremely high and in many cases there is no equilibrium. Intuitively, as foreign operator j that is not in alliance i has to buy half of its roaming calls from operator i, setting a high $\hat{a}_i$ increases the perceived marginal costs of operator j and therefore increases the retail market share of alliance i.

Proof. The proof of existence and uniqueness parallels that of Proposition 1.6, since the same objective function is maximized. For Proposition 1.11 in Appendix A1.3 we prove that the equilibrium price decreases with the quality of network selection. ■

Intuitively, there are two reasons why equilibrium prices are now higher than in the base model. Due to random network selection, the perceived marginal costs c_i of operators within alliance *i and* those of the rival alliance *j* equally depend on the wholesale price a_i. First, this makes an alliance's retail market share insensitive to increases of the own wholesale price. Second, raising the wholesale price a_i may increase the wholesale profit generated from sales to operators of the competing alliance.

The insight that without network control the presence of alliances does not affect the wholesale prices is at first glance surprising. One might be tempted to conjecture that alliances mitigate the problem of double marginalization as in Carter and Wright (1994).[64] Indeed, assuming linear retail and wholesale prices, Lupi and Manenti (2009) find that even without control of network selection, alliances negotiate reciprocal wholesale prices equal to marginal costs. However, as we analyze competition on the retail market with two part tariffs, no deadweight loss is caused at the retail level and double marginalization is not an issue. Hence, there is no externality that an alliance could internalize when coordinating on a wholesale price. Our model therefore provides an explanation why in Europe international roaming alliances were formed mainly after powerful network selection technologies have become available.

1.6.3 Policy Intervention

We now investigate the effects of imposing a retail price cap when two alliances have emerged. In practice, implementing a retail price cap does not require collaboration with foreign regulators since it directly affects the country in which it is imposed. In contrast, a wholesale price cap clearly increases both welfare and consumer surplus in

[64]In contrast to our model, Carter and Wright (1994) assume that there is a monopolist in each country and that the monopolists set linear tariffs both at the wholesale and retail market. They find that if operators cooperatively set wholesale prices to maximize their profits, then both consumer surplus and profits exceed the uncooperative outcome since the double-marginalization problem is circumvented.

our model but usually requires international cooperation between regulators. Knowing the precise effects of a retail price cap appears thus necessary in order to select the optimal policy. The interest in this question is exemplified by the intense debate that took place before the European Commission introduced a price cap both at the retail and at the wholesale level in 2007. Indeed, our results suggest that *solely* restricting the *retail usage* price is likely to have a detrimental effect on consumer surplus even in the absence of any informational asymmetries.

We first analyze the impact of a retail usage price cap $\overline{p}$ on the retail equilibrium tariffs for given wholesale prices. Remember that each operator xi optimally sets the retail usage price p_{xi} so as to maximize the retail surplus. If the wholesale price a_i exceeds the price cap, then the optimal choice is to set the usage price as high as possible, namely $p^*_{xi} = \overline{p}$. The maximized surplus generated on the retail stage is therefore:[65]

$$\overline{v}(a_i) \equiv \begin{cases} v(\overline{p}) + q(\overline{p})\,(\overline{p} - a_i) & \text{if } a_i > \overline{p} \\ v(a_i) & \text{if } a_i \leq \overline{p} \end{cases} \tag{1.16}$$

Clearly, restricting the usage price not to exceed $\overline{p}$ reduces the surplus created at the retail level whenever $a_i > \overline{p}$ and when the demand is decreasing at $\overline{p}$. The retail profit per customer is now $\overline{\pi_i}^R = \overline{v}(a_i) - \overline{w}_i - C_F$ where $\overline{w}_i$ is the subscriber's net surplus.

Since the retail equilibrium tariffs derived in Section 1.3 depend on a_i only through $v(a_i)$, they remain valid when a price cap is in place after replacing $v(a_i)$ by the function $\overline{v}(a_i)$. Whenever the wholesale prices of the competing alliances are close enough, namely $|\overline{v}(a_0) - \overline{v}(a_1)| < \frac{3}{2\sigma}$, both operators achieve a positive market share given by $\overline{n}^*(a_i, a_j) = \frac{1}{2} + \frac{\sigma}{3}[\overline{v}(a_i) - \overline{v}(a_j)]$. In this case the equilibrium level of net surplus $\overline{w}^*_i$ conceded to consumers reads as follows:

$$\overline{w}^*_i = \frac{2}{3}\overline{v}(a_i) + \frac{1}{3}\overline{v}(a_j) - C_F - \frac{1}{2\sigma} \tag{1.17}$$

In particular, for symmetric wholesale prices the equilibrium per customer profit is $\overline{\pi_i}^{R*} = \frac{1}{2\sigma}$ as in Section 1.3.[66]

[65]We make the realistic assumption that operators cannot restrict the quantity of roaming calls per subscriber.

[66]Since a price cap usually reduces the retail surplus, the fact that retail profits remain constant implies

Concerning the equilibrium wholesale prices, we assume that the cap is imposed before operators negotiate on wholesale prices. For wholesale prices above $\overline{p}$ that give rise to a shared market, the total marginal profit of an operator is

$$\frac{\partial \overline{\Pi}^*}{\partial a_i}(a_i, a_j) = \frac{q(\overline{p})}{3}\left[\overline{n}_i^* - \sigma \overline{\pi}_i^W\right] \tag{1.18}$$

where $\overline{\pi}_i^W \equiv q(\overline{p})(a_i - c)$ denotes the per customer wholesale profit in case of a binding price cap. Since increasing the wholesale price above $\overline{p}$ leaves the retail usage price unchanged, the deadweight loss is not exacerbated. Setting the marginal profit equal to zero yields that the per customer wholesale profit is $\overline{\pi}^{W*} = \frac{1}{2\sigma}$ in any symmetric equilibrium. It exceeds its counterpart without price cap $\pi^{W*} = \frac{1-3\epsilon(a^*)}{2\sigma}$ derived from condition 1.10. Using $\overline{\pi}^{W*} = \frac{1}{2\sigma}$ with (1.16) and (1.17) yields that the equilibrium net surplus per customer is $\overline{w}^* = \overline{v}(\overline{a}^*) - \overline{\pi}^{W*} - \frac{1}{2\sigma} - C_F = v(\overline{p}) + q(\overline{p})(\overline{p} - c) - \frac{1}{\sigma} - C_F$ which is clearly maximal for $\overline{p} = c$. The next proposition establishes that even introducing the optimal retail price cap $\overline{p} = c$ usually decreases consumer surplus.

Proposition 1.8. *Denote by a^* the equilibrium wholesale price without price cap according to Proposition 1.2. Suppose that Assumption 1.2 holds and that demand is decreasing at a^*: $q'(a^*) < 0$. Then introducing a retail per call price cap $\overline{p} \leq a^*$ decreases consumer surplus and increases industry profits. If the price cap is not set below the true marginal cost and $\overline{p} < a^*$, total welfare increases. If the price cap is sufficiently close to the unrestricted equilibrium wholesale price (i.e. $q(\overline{p}) - q(a^*) < \frac{3\epsilon(a^*)}{2\sigma(a^*-c)}$), then the equilibrium wholesale price increases.*

Proof. See Appendix A1.1. ■

If the mild conditions of Proposition 1.8 are satisfied, restricting the retail per call price decreases deadweight-loss and thus increases total welfare since the market remains covered.

Two countervailing effects determine how a price cap influences the wholesale equilibrium price. A retail price cap prevents operators from passing through high wholesale prices to subscribers. Therefore, increasing the wholesale price does not aggravate the deadweight-loss, which renders higher wholesale prices more attractive. On the other

that even if the regulator could impose a cap on the usage retail prices *and* fix the symmetric wholesale prices, the consumer surplus would be generally reduced.

hand, a cap on the retail price guarantees that each subscriber places at least $q(\overline{p})$ calls. This increases the wholesale profit per customer and renders subscribers more valuable, thereby inducing alliances to set lower wholesale prices. Whenever the condition $q(\overline{p}) - q(a^*) < \frac{3\epsilon(a^*)}{2\sigma(a^*-c)}$ holds, the first effect dominates and higher wholesale prices obtain. If $\epsilon(a^*) > 0$, a price cap which is set close enough to a^* satisfies this condition and thus increases the wholesale price.

Our results suggest that in order to protect subscribers, price caps should preferably be imposed on the wholesale level. This might explain why national regulation authorities have mostly chosen not to regulate retail roaming prices prior to the intervention of the European Commission.

1.6.4 Wholesale Fees per Roaming Subscriber

So far, we have assumed that operators can only charge linear prices at the wholesale level. This assumption reflects roughly the wholesale price structure that is used in practice at the moment. However, in this section we show that two-part tariffs on the upstream level render alliances even more profitable. Now, operators may both charge a per call wholesale-price and a fee that has to be paid for any foreign customer that visits the network.[67] As in Section 1.4, we assume that operators with same position have formed alliances and omit the country index for brevity.

The per customer fee enters as perceived fixed cost and therefore renders customers less attractive at the retail level. The optimal retail per call price remains equal to the wholesale per call price of the alliance. Thus, the per customer profit is now $\tilde{\pi}_i^R = v(p_i) - w_i - \phi_i - C_F$. The retail profit of operator i conditional on alliance i having agreed on the wholesale price a_i and the per customer fee ϕ_i reads $\Pi_i^R = n(w_i, w_j)\tilde{\pi}_i^R$. Solving for the retail equilibrium as in Section 1.3 yields the retail equilibrium net surplus $w_i^* = \frac{2}{3}\left[v(c_i) - \phi_i\right] + \frac{1}{3}\left[v(c_j) - \phi_j\right] - \frac{1}{2\sigma} - C_F$.

Denote the per customer wholesale profit by $\tilde{\pi}_i^W = q(a_i)\,(\tilde{a}_i - c) + \phi_i$. The first order conditions which characterize the optimal per call wholesale price $\tilde{a}_i^*$ and the optimal

[67] Note that this pricing structure differs from two-part tariffs used for example as franchise fees. In our setup, the fixed fee ϕ_{xi} is paid for any customer. In contrast, a franchise fee is paid only once.

per customer wholesale fee ϕ_i^* are

$$\frac{\sigma}{3}\tilde{\pi}_i^{R*} = \frac{\sigma}{3}\tilde{\pi}_i^{W*} + n_i^*\epsilon(\tilde{a}_i^*)\,, \quad (1.19)$$
$$\tilde{\pi}_i^{R*} = \tilde{\pi}_i^{W*} \quad (1.20)$$

where $\epsilon(\cdot)$ refers to the per customer demand elasticity in terms of markup as before. Inserting condition (1.20) into condition (1.19) yields $n_i^*\epsilon(\tilde{a}_i^*) = 0$ which for $n_i^* \neq 0$ is only satisfied for $\tilde{a}_i^* = c$. Hence, as long as operator i expects to achieve a strictly positive retail market share, it is optimal to set the wholesale per call price equal to the true marginal costs.

Proposition 1.9. *Suppose that Assumption 1.2 holds, that $q^{'}(c) < 0$ and that operators have formed two competing alliances. If each alliance can negotiate both on a wholesale per call price and on a per customer fee, there exists a unique symmetric equilibrium. The equilibrium wholesale per call price equals the true marginal cost c and the wholesale profit is $\tilde{\pi}_i^{W*} = \phi_i^* = \frac{1}{2\sigma}$. Compared to the symmetric equilibrium without per customer fees, characterized by Proposition 1.2, each operator's wholesale profit and welfare is higher.*

Proof. First note that in any symmetric equilibrium, each operator has market share $n_i^* = \frac{1}{2}$ and hence earns the retail profit $\tilde{\pi}_i^{R*} = \frac{1}{2\sigma}$. Inserting these values and $\tilde{a}_i^* = c$ into equation (1.20) yields $\phi_i^* = \frac{1}{2\sigma}$. Furthermore, this critical point is a maximum, since $\frac{\partial^2\Pi}{\partial\phi_i^2}(\phi_i, \phi_j) = -\frac{1}{3} - \frac{\sigma}{9} < 0$ for (ϕ_i, ϕ_j) such that $n_i^* \in (0, 1)$. It can be easily verified that $\Pi(\phi_i, \phi_j) \leq \Pi(\phi_j - \frac{3}{2\sigma}, \phi_j)$ for all $\phi_i < \phi_j - \frac{3}{2\sigma}$, so that cornering the market is never optimal. If wholesale per customer fees are not feasible, by Proposition 1.2, $\pi^W(a_i^*) = \frac{1}{2\sigma} - \frac{3}{2}\sigma\epsilon(a_i^*) < \frac{1}{2\sigma} = \tilde{\pi}_i^{W*}$. The difference in welfare is $-\int_c^{a_i^*}(x - c)\,q'(x)\mathrm{d}x > 0$. ■

Intuitively, increasing the per customer fee reduces the per customer retail profit and thus softens retail competition. Starting from $\phi_i = 0$ and $a_i = c$, raising the per customer fee avoids deadweight loss and is thus more attractive than raising the wholesale price from the viewpoint of an alliance.

1.7 Conclusion

This chapter presents a tractable model of international roaming in which operators compete both at the wholesale and at the retail level. We have shown that operators have incentives to form alliances and to commit to mutually providing roaming services at inefficiently high wholesale prices. As Section 1.6.2 points out, these alliances serve to alleviate the competitive pressure that has lately increased due to recent improvements in network selection technologies.

Our analysis yields a number of policy implications. International alliances that are often claimed to improve efficiency, might reduce welfare and harm consumers. If operators mutually sell roaming services, it is difficult for regulatory agencies to discover whether wholesale prices are set for strategical reasons. As we have shown, in the roaming market, fixed fees as suggested by Shaffer (1991) are not needed in order to soften competition. From the perspective of a regulatory agency this means that the absence of two-part tariffs as often observed in the roaming wholesale market does not imply that wholesale prices are not set at a inefficiently high level for strategical reasons.

Another important insight is that the so-called *waterbed effect* might render seemingly helpful regulatory interventions useless or even detrimental. As is shown in Section 1.6.3, when regulators impose a binding retail price cap but leave the monthly fees unregulated, the waterbed effect might even cause consumer surplus to decrease. Our analysis suggests that whenever regulators restrict one price instrument, then reactions of operators concerning their remaining instruments should be taken into account. If the regulation of all price instruments is not desired, then other measures might be more effective. For example, according to our model, a ban of international alliances might bring roaming prices down and increase welfare. Our suggestion might have constituted an alternative approach than the price cap on roaming prices which was introduced by the European Parliament in 2007.

Our model also illustrates that non-discrimination clauses that look innocent at first sight might have detrimental effects once the interaction with international alliances is taken into account. Therefore we advise to carefully review the rules of conduct that have been introduced by organizations as the GSM Association with respect to their competitive impact.[68]

Notably, central predictions of Lupi and Manenti (2009) who also analyze the international roaming market are almost reversed in our model.[69] However, their model differs in important characteristics such as the retail price structure and the degree of retail competition. Therefore, regulators should carefully analyze which of the currently available models captures best the key characteristics of a given roaming market.

There remain further open questions that merit future research, even though we have already extended our base model in several directions. While our assumption of a balanced demand for roaming services helped to simplify the setup and to keep the model tractable, it would be certainly interesting to explore to what extend our results carry over in a less balanced setting. Even though demand imbalances would render a more sophisticated negotiation game necessary, we suspect that operators could still rely on inefficiently high wholesale prices to soften competition. Furthermore, our model delivers testable predictions for future empirical work.

A1.1 Appendix - Proofs of Lemmas & Propositions

Proof of Lemma 1.1

We omit the country index for brevity in what follows.

Suppose that $\left|v(c_i) - v(c_j)\right| < \frac{3}{2\sigma}$. We first show that (1.4) indeed maximizes retail profits given w_j^*. Since $\frac{\partial \Pi^R}{\partial p_i}(p_i, w_i, w_j, c_i) = n_i q^{'}(p_i)(p_i - c_i)$, $\Pi^R(p_i, w_i, c_i) - \Pi^R(c_i, w_i, c_i) = n_i \int_{c_i}^{p_i} q^{'}(p)(p - c_i)\mathrm{d}p \leq 0$ with strict inequality whenever $n_i > 0$ and $q(c_i) \neq q(p_i)$. Thus $p_i^* = c_i$ maximizes Π_i^R independently of w_i and w_j. Moreover, $\frac{\partial \Pi^R}{\partial w_i}(c_i, w_i, w_j^*, c_i) = 2\sigma \left(w_i^* - w_i\right)$ so that $\Pi^R \left(c_i, w_i^*, w_j^*, c_i\right) > \Pi^R \left(c_i, w_i, w_j^*, c_i\right)$.

[68]See www.gsmworld.com. Interestingly, the GSMA consists of almost all MNOs, so that possibly rules of conduct have been developed in order to increase the industry profit.

[69]Assuming linear prices and monopolistic demand on the retail level, Lupi and Manenti, 2009 find that alliances improve efficiency since they serve to circumvent the double marginalization problem. However, they predict that alliances do not emerge in equilibrium.

Solving simultaneously the reaction functions (1.4) for both operators yields equation (1.5). Being a system of linearly independent equations, the solution is unique. The condition $\left|v(c_i) - v(c_j)\right| < \frac{3}{2\sigma}$ assures that the market share stays between zero and one.

We now show that whenever $v(c_i) - v(c_j) \geq \frac{3}{2\sigma}$ there exists a unique equilibrium in pure weakly undominated strategies, which entails $n_i^* = 1$ and $n_j^* = 0$.

We first establish that any such corner equilibrium necessarily involves $p_i^* = c_i$, $w_i^* = \frac{1}{2\sigma} - C_F + v(c_j)$, $p_j^* = c_j$ and $w_j^* = v(c_j) - C_F$. Define $\tilde{w}_i$ such that given $\left(w_j, v(c_i), v(c_j)\right)$, operator i just serves the whole market: $\frac{1}{2} + \sigma\left(\tilde{w}_i - w_j\right) = 1$. Note that whenever $n_i^* = 1$ then necessarily $w_i^* = \tilde{w}_i$ as setting $w_i > \tilde{w}_i(w_j)$ would yield strictly lower profits.

We now show that whenever $n_j^* = 0$, then necessarily $w_j^* = v(c_j) - C_F$: Any strategy with $w_j > v(c_j) - C_F$ entails $\pi_i^R < 0$ and is weakly dominated by $p_j = c_j$ and $w_j = w_j^*$. Now suppose that $w_j < v(c_j) - C_F$ was an equilibrium. By the preceding discussion, necessarily $w_i = \tilde{w}_i(w_j)$. Then player j could achieve a strictly positive retail profit by deviating to $w_j + \frac{v(c_j) - C_F - w_j}{2}$ which contradicts equilibrium.

We now show that a unique corner equilibrium arises iff $v(c_i) - v(c_j) \geq \frac{3}{2\sigma}$. *If-Existence*: Given, $w_j^* = v(c_j) - C_F$ and $w_i^* = \frac{1}{2\sigma} - C_F + v(c_j)$, it can be directly verified that $\frac{\partial \Pi^R}{\partial w_i}(w_i, w_j^*, c_i) > 0$ for $w_i < w_i^*$ and $\frac{\partial \Pi^R}{\partial w_j}(w_j, w_i^*, c_i) < 0$ for $w_j > w_j^*$ which together with the preceding paragraphs confirms that w_i^* and w_j^* are mutually profit maximizing. *If-Uniqueness*: There exists no interior equilibrium since inserting $v(c_i) - v(c_j) \geq \frac{3}{2\sigma}$ into (1.6) yields $n_i^* \geq 1$ which is not interior. *Only-if*: Suppose that $0 \leq v(c_i) - v(c_j) < \frac{3}{2\sigma}$: For $w_j^* = v(c_j) - C_F$ as required in any corner equilibrium, the best response of player i is $w_i^* < \tilde{w}_i$ which implies $n_i^* < 1$ and therefore causes a contradiction.

Proof of Lemma 1.2

Define $\mathcal{E} = \left\{p \in \mathbb{R} | \epsilon(p) < \frac{1}{3} \wedge p \geq c \wedge q(p) > 0 \wedge v(p) > v(c) - \frac{3}{2\sigma}\right\}$. First we establish some auxiliary lemmas that will be also useful for other proofs.

Lemma 1.3. *Define* $\psi(p) \equiv x(p)\,[z - \epsilon(p)] - y\,\pi^W(p)$ *with* $x(p) \geq 0$, $x(c) > 0$, $x'(p) \leq 0$, $z \in (0,1]$ *and* $y > 0$. *If Assumptions 1.1 and 1.2 hold, then the equation* $\psi(p) = 0$ *has a unique solution* $p^* > c$. *This solution satisfies* $\psi'(p^*) < 0$.

Proof. There are three cases: a) There exists some $\hat{p}$ with $\epsilon(\hat{p}) = 1$; b) $\lim_{p\to\infty} \epsilon(p) = 1$ which implies that $\lim_{p\to\infty} \pi^W(p) > 0$; c) $\lim_{p\to\infty} \epsilon(p) = \bar{\epsilon} < 1$ which implies that $\lim_{p\to\infty} \pi^W(p) = \infty$.[70] In the first case, $\psi(\hat{p}) < 0$, while in the other two cases $\lim_{p\to\infty} \psi(p) < 0$. Since $\psi(c) = x(c)z > 0$, by continuity there exists a $p^* > c$ s.t. $\psi(p^*) = 0$. As $\psi'(p) = -x(p)\epsilon'(p) + \left(x'(p) - yq(p)\right)(z - \epsilon(p)) < 0$ whenever $\psi(p) \geq 0$, p^* is unique. ■

Lemma 1.4. *If Assumption 1.2 holds, then:*

i) $\pi^W(a_i)$ *is concave on* $\mathcal{E}$ *in* a_i.

ii) Given $a_j \in \mathcal{E}$, *any* $a_i \in \mathcal{E}$ *that satisfies the first order necessary conditions for being a local maximum of* $\Pi(a_i, a_j)$ *strictly maximizes* $\Pi(a_i, a_j)$ *in* $\mathcal{E}$.

Proof. Part i) $\frac{\partial \pi^W}{\partial p}(p) = (p - c)q'(p) + q(p) = q(p)\,(1 - \epsilon(p))$. Hence $\frac{\partial^2 \pi^W}{\partial p^2}(p) = q'(p)\,(1 - \epsilon(p)) - q(p)\,\epsilon'(p) < 0$ as $\epsilon'(p) > 0$ by Assumption 1.2 and $1 - \epsilon(p) > 0$ for $p \in \mathcal{E}$.

Part ii) By definition of $\mathcal{E}$, $\forall a_i, a_j \in \mathcal{E}$, since $|v(c) - v(a_i)| < \frac{3\sigma}{2}$ we have $n^*\left(a_i, a_j\right) \in (0,1)$. Define $\varphi(a_i, a_j) \equiv (1 - 3\epsilon(a_i))\,n_i^* - \sigma\pi^W(a_i)$ and note that by (1.10), $\frac{\partial \Pi(a_i,a_j)}{\partial a_i} = \frac{1}{3}q(a_i)\varphi(a_i, a_j)$. The result follows from $\frac{\partial \varphi(a_i,a_j)}{\partial a_i} = -2\sigma q(a_i)\left[\frac{2}{3} - \epsilon(a_i)\right] - 3\epsilon'(a_i)n_i^* < 0$, which is true since $\sigma > 0$, $\epsilon(a_i) < \frac{1}{3}$ and $\epsilon'(a_i) \geq 0$ by Assumption 1.2. ■

Lemma 1.5. *For all* (a_i, a_j) *s.t.* $n^*(a_i, a_j) \in (0,1)$ *the following inequalities hold:*

i) If $a_i < c$ *then* $\frac{\partial \Pi(a_i,a_j)}{\partial a_i} > 0$.

ii) If $q(a_i) = 0$ *then* $\Pi(c, a_j) > \Pi(a_i, a_j)$.

iii) If $a_i > c$, $q(a_i) > 0$ *and* $\epsilon(a_i) \geq \frac{1}{3}$ *then* $\frac{\partial \Pi(a_i,a_j)}{\partial a_i} < 0$.

iv) If Assumption 1.2 holds and $a_i > c$, $q(a_i) > 0$, $v(a_i) < v(c) - \frac{3}{2\sigma}$ *then* $\frac{\partial \Pi(a_i,a_j)}{\partial a_i} < 0$.

Proof. Part i) By Assumption 1.1, $q(a_i) \geq q(c) > 0$ which implies that $\pi^W(a_i) < 0$ for $a_i < c$ and thus by equation (1.10), $\frac{\partial \Pi}{\partial a_i}(a_i, a_j) > 0$.

[70] Integrating up $\frac{-(p-c)q'(p)}{q(p)} \leq 1 - \bar{\epsilon}\ \forall p \geq c$ yields $\int \frac{q'(p)}{q(p)}\mathrm{d}p \geq -\bar{\epsilon} \int \frac{1}{(p-c)}\mathrm{d}p$. Using $p > \underline{p} > c$, we get $\pi(p) \geq \pi(\underline{p}) \left[\frac{p-c}{\underline{p}-c}\right]^{\bar{\epsilon}}$ which goes to infinity as $p \to \infty$.

Part ii) Any a_i with $q(a_i) = 0$ implies that $a_i > c$ and $q^{'}(a_i) = 0$ by Assumption 1.1. As $q(a^{'}) = 0 \quad \forall a^{'} \geq a_i$, we have $v(a_j) \geq v(a_i)$ and hence $n^*(a_i, a_j) \leq \frac{1}{2}$. In addition, $q(a_i) = 0$ implies $q(a_i)(a_i - c) = 0$. Hence $\Pi(a_i, a_j) = \frac{1}{\sigma}n^*(a_i, a_j)^2 < \frac{1}{\sigma}n^*(c, a_j)^2 \leq \Pi(c, a_j)$ holds which contradicts a_i being optimal. To see that $\Pi(c, a_j) \geq \frac{1}{\sigma}n^*(c, a_j)^2$, distinguish two cases: if $v(c) - v(a_j) \leq \frac{3}{2\sigma}$, then $\Pi(c, a_j) = \frac{1}{\sigma}n^*(c, a_j)^2$ by Lemma 1.1. If $v(c) - v(a_j) > \frac{3}{2\sigma}$, then by the same Lemma $\pi_i^W > \frac{1}{\sigma}$ and hence $\Pi(c, a_j) > \frac{1}{\sigma}n^*(c, a_j)^2$.

Part iii) Since $\epsilon(a_i) \geq \frac{1}{3}$ and $q(a_i)(a_i - c) > 0$, $\frac{\partial \Pi}{\partial a_i}(a_i, a_j) = q(a_i)\left[\frac{-\sigma}{3}q(a_i)(a_i - c) + n^*(a_i, a_j)\left(\frac{1}{3} - \epsilon(a_i)\right)\right] < 0$.

Part iv) If $\epsilon(a_i) \geq \frac{1}{3}$ then by part iii) the claim follows. If $\epsilon(a_i) < \frac{1}{3}$ then by Assumption 1.2, for all $\tilde{a}_i \in [c, a_i]$, $\epsilon(\tilde{a}_i) \leq \epsilon(a_i)$. By definition $v^{'}(p) = -q(p)$ and the condition $v(c) - v(a_i) < \frac{3}{2\sigma}$ is equivalent to $\int_c^{a_i} q(a)\mathrm{d}a < \frac{3}{2\sigma}$. By Assumption 1.2, $\epsilon^{'}(\tilde{a}_i) \geq 0$ for $\tilde{a}_i \in [c, a_i]$ and thus $\pi^W(a_i) = \int_c^{a_i}(1 - \epsilon(a))\, q(a)\mathrm{d}a \geq (1 - \epsilon(a_i))\int_c^{a_i} q(a)\mathrm{d}a$. Therefore, $\int_c^{a_i} q(a)\mathrm{d}a \geq \frac{3}{2\sigma}$ implies $\pi^W(a_i) \geq (1 - \epsilon(a_i))\frac{3}{2\sigma}$. From (1.10) we have $\frac{\partial \Pi}{\partial a_i}(a_i, a_j) \leq \left[\frac{1}{3} - \epsilon(a_i) - \frac{\sigma}{3}\pi^W(a_i)\right] q(a_i) \leq \left[\frac{1}{3} - \epsilon(a_i) - \frac{1}{2}(1 - \epsilon(a_i))\right] q(a_i) = \frac{1}{2}\left[-\frac{1}{3} - \epsilon(a_i)\right] q(a_i) < 0$ where the first inequality is because $\left(\frac{1}{3} - \epsilon(a_i)\right) n_i^* \leq \frac{1}{3} - \epsilon(a_i)$. ■

Proof of Lemma 1.2.

Note that for all $a_i, a_j \in \mathcal{E}$, $n^*(a_i, a_j) \in (0, 1)$ by definition of $\mathcal{E}$.

Existence & Uniqueness: By Lemma 1.3, for any $a_j \in \mathcal{E}$ there exists a unique $\hat{a} \in \mathcal{E}$ such that $(1 - 3\epsilon(a_i))\, n^*(\hat{a}, a_j) - \sigma\pi^W(a_i) = 0$. Since $\frac{\partial \Pi(a_i, a_j)}{\partial a_i} = \frac{1}{3}q(a_i)\varphi(a_i, a_j)$ and by Lemma 1.4, part ii), $a_i = \hat{a}$ strictly maximizes $\Pi(a_i, a_j)$ in $\mathcal{E}$. By Lemma 1.5, $\hat{a}$ remains a strict maximizer in $\mathbb{R}$.

Monotonicity in a_j: Any profit maximizing wholesale price $a^*(a_j)$ involves $\frac{\partial \Pi}{\partial a_i}(a^*(a_j), a_j) = 0$. By Lemma 1.4, part ii), any critical point is also a strict maximum which implies $\frac{\partial^2 \Pi}{\partial a_i^2}(a^*(a_j), a_j) < 0$. Therefore, by the implicit function theorem, the claim is true if $\frac{\partial^2 \Pi}{\partial a_i \partial a_j}(a^*(a_j), a_j) > 0$. Differentiating (1.10) with respect to a_j yields $\frac{\partial^2 \Pi}{\partial a_i \partial a_j}(a^*(a_j), a_j) = \frac{\sigma}{3}q(a_i)^2\left(\frac{1}{3} - \epsilon(a_i)\right) > 0$. ■

Proof of Proposition 1.2

We first prove the following auxiliary lemma:

Lemma 1.6. *In any equilibrium in weakly undominated strategies both alliances have a positive market share: $n^*(a_i^*, a_j^*) \in (0,1)$.*

Proof. Suppose to the contrary that $n^*(a_i^*, a_j^*) = 1$ which implies $\Pi(a_j^*, a_i^*) = 0$. Define the highest wholesale price that allows to corner the market $\bar{a}_i$ implicitly by $v(\bar{a}_i) = v(a_j^*) + \frac{3}{2\sigma}$. We show that any $a_i < c$ is weakly dominated by $a_i^* = c$: Whenever a_j^* is such that $\bar{a}_i < c$, then for $a_i \in (\bar{a}_i, c)$, by equation (1.10), $\frac{\partial \Pi}{\partial a_i}(a_i, a_j) = q(a_i)\left[\left(\frac{1}{3} - \epsilon(a_i)\right) n_i^* - \frac{\sigma}{3}\pi^W(a_i)\right] > 0$ since $\pi^W(a_i) < 0$ and $\epsilon(a_i) \leq 0$. For $a_i < \bar{a}_i$, $\frac{\partial \Pi}{\partial a_i}(a_i, a_j) = -q(a_i)\epsilon(a_i) \geq 0$. Thus for $\bar{a}_i < c$ and for any $a_i < c$, $\Pi(c, a_j^*) > \Pi(a_i, a_j^*)$. If $\bar{a}_i \geq c$, then $\Pi(c, a_j^*) \geq \Pi(a_i, a_j^*)$.

Since $a_i^* \geq c$, the corner equilibrium involves $\Pi(a_i^*, a_j^*) \geq \Pi^R(a_i^*, a_j^*) \geq \frac{1}{\sigma}$. Then deviating to $\hat{a}_j = a_i^*$ yields $\Pi(\hat{a}_j, a_i^*) \geq \frac{1}{4\sigma}$ contradicting optimality of a_j^*. ■

Proof of Proposition 1.2.

Part i) By Proposition 1.1 and Lemma 1.2, $a^{1*} = a^*(c)$ uniquely exists.

Part ii) Recall that $\mathcal{E} = \left\{p \in \mathbb{R} | \epsilon(p) < \frac{1}{3} \wedge p \geq c \wedge q(p) > 0 \wedge v(p) > v(c) - \frac{3}{2\sigma}\right\}$. We first show *existence* of a symmetric equilibrium $a_0^* = a_1^* = a^*$ and consequently $n_0^* = n_1^* = \frac{1}{2}$. By Lemma 1.5 of Section A1.1 this equilibrium involves $a^* \in \mathcal{E}$. Define $\psi(p) \equiv (1 - 3\epsilon(p)) - 2\sigma\pi^W(p)$. By Lemma 1.3 of Section A1.1, there is a unique $\hat{a} > c$ with $\psi(\hat{a}) = 0$.

It remains to show that the candidate $\hat{a}$ is indeed a symmetric equilibrium. By definition of ψ , $a_i = \hat{a}$ satisfies the necessary first order condition when $a_j = \hat{a}$. By Lemma 1.4 of Section A1.1, the first order conditions are also sufficient for being a global maximum on $\mathcal{E}$. By Lemma 1.5, $a_i = \hat{a}$ remains a maximizer on the set of all $a_i \in \mathbb{R}$ such that $n(a_i, a_j) \in (0,1)$. Setting a_i high enough so that $n_i = 0$ cannot be optimal either, as this gives zero profits.

It remains to show that $\Pi(\hat{a}, \hat{a}) \geq \Pi(\tilde{a}_i, \hat{a})$ for $\tilde{a}_i$ such that $n(\tilde{a}_i, \hat{a}) = 1$. Since $\hat{a} \in \mathcal{E}$, the inequality $v(c) < v(\hat{a}) + \frac{3}{2\sigma}$ holds. Cornering the market requires $v(\tilde{a}_i) \geq v(\hat{a}) + \frac{3}{2\sigma}$, and thus $\tilde{a}_i < c$. For any $a_i < c$ such that $v(a_i) > v(a_j) + \frac{3}{2\sigma}$, marginal profits are $\frac{\partial \Pi}{\partial a_i}(a_i, a_j) = -q(a_i)\epsilon(a_i) \geq 0$ since $\epsilon(a_i) \leq 0$. Thus $\Pi(\tilde{a}_i, \hat{a}) \leq \Pi(c, \hat{a}) < \Pi(\hat{a}, \hat{a})$.

Uniqueness: There is no other symmetric equilibrium since any interior equilibrium must belong to $\mathcal{E}$ and since in $\mathcal{E}$ the necessary first order condition is uniquely satisfied at $\hat{a}$ by the previous discussion.

We now show that no asymmetric equilibrium exists. Suppose to the contrary that an asymmetric equilibrium with $a_i^* > a_j^*$ and hence $n_i^* < n_j^*$ exists. By Assumption 1.2, $a_i^* > a_j^*$ implies $\epsilon(a_i^*) \geq \epsilon(a_j^*)$. By Lemma 1.6, this equilibrium must involve a strictly positive market share for both alliances and a strictly positive per customer demand. The necessary first order conditions are:

$$
\begin{aligned}
\left(\frac{1}{3} - \epsilon(a_i^*)\right) n_i^* - \frac{\sigma}{3}\pi^W(a_i^*) &= 0 \\
\left(\frac{1}{3} - \epsilon(a_j^*)\right) n_j^* - \frac{\sigma}{3}\pi^W(a_j^*) &= 0
\end{aligned}
$$

But $\epsilon(a_i^*) \geq \epsilon(a_j^*)$ and $n_i^* < n_j^*$ implies $\left(\frac{1}{3} - \epsilon(a_i^*)\right) n_i^* < \left(\frac{1}{3} - \epsilon(a_j^*)\right) n_j^*$. Furthermore, by Lemma 1.5, $a_i^*, a_j^* \in \mathcal{E}$. Hence $\frac{1}{3} \geq \epsilon(a_i^*) \geq \epsilon(a_j^*)$ and thus $\pi^W(a_i^*) > \pi^W(a_j^*)$. Taken together this implies $\left(\frac{1}{3} - \epsilon(a_i^*)\right) n_i^* - \frac{\sigma}{3}\pi^W(a_i^*) < n_j^* \left(\frac{1}{3} - \epsilon(a_j^*)\right) - \frac{\sigma}{3}\pi^W(a_j^*) = 0$ which contradicts the first order necessary conditions.

Finally, we show that $a^* > c$: The necessary condition for $a_i^* = c$ is $n^*(c, a_j^*)\frac{q(c)}{3} = 0$ which is never true as $q(c) > 0$ by Assumption 1.1.

Rearranging the equilibrium condition $\psi(a^*) = 0$ yields the equilibrium per customer profits. ■

Proof of Proposition 1.3

Rewriting condition (1.11) for a symmetric equilibrium yields

$$2\sigma q(a^*)\,(a^* - c) + 3\epsilon(a^*) - 1 = 0 \tag{1.21}$$

Part i and ii) Applying the implicit function theorem on this condition, the claim is true if $\frac{\partial}{\partial a}\left(2\sigma q(a^*)\left(a^*-c\right)+3\epsilon(a^*)\right)>0$. By Assumption 1.2, $\epsilon^{'}(a^*)>0$. In addition, $\frac{\partial}{\partial a}q(a^*)\left(a^*-c\right)>0$ since $\epsilon(a^*)<\frac{1}{3}$ which completes the proof.

Part iii) Consider any pair of demand functions q and $\tilde{q}$ with $\eta_{\tilde{q}}(a)>\eta_q(a)\ \forall a\in\mathbb{R}$ and $\tilde{q}(a^*)\geq q(a^*)$. Since $\epsilon(a)=\eta_q(a)\frac{a-c}{a}$, $\eta_{\tilde{q}}(a)>\eta_q(a)$ implies $\epsilon_{\tilde{q}}(a)>\epsilon_q(a)$ for $a-c>0$. We show that the equilibrium wholesale price $\tilde{a}^*$ that corresponds to per customer demand $\tilde{q}$ is higher than the equilibrium price a^* for demand q. By the proof of Proposition 1.2, the function $\psi_q(a)\equiv 2\sigma q(a)\left(a-c\right)+3\epsilon_q(a)-1$ is increasing in a for $a\in\mathcal{E}$ and $\psi_q(c)=-1$. Define $\psi_{\tilde{q}}(a)$ likewise for demand $\tilde{q}$. To show that $\tilde{a}^*<a^*$, just note that $\psi_{\tilde{q}}(a^*)>\psi_q(a^*)=0$ where the inequality comes from the hypothesis $\tilde{q}(a^*)\geq q(a^*)$ and $\epsilon_{\tilde{q}}(a)-\epsilon_q(a)>0$ and the last equality is the equilibrium condition of a^* being an equilibrium for demand q. Since $\psi_{\tilde{q}}(a^*)>0$, by continuity there exists an $\tilde{a}^*<a^*$ such that $\psi_{\tilde{q}}(\tilde{a}^*)=0$. This equilibrium candidate is indeed an equilibrium for demand $\tilde{q}$ by the proof of Proposition 1.2.

Proof of Proposition 1.5

For brevity we omit the country index whenever possible. Suppose w.l.o.g. that operators i have formed an alliance. The marginal wholesale profit of non-alliance operators j for $a_j<a_i$ is

$$\frac{\partial\tilde{\Pi}^W}{\partial a_j}(a_j,a_i)=q(a_j)\left[n^*(a_j,a_i)\left[1-\epsilon(a_j)\right]-\frac{\sigma}{3}\pi^W(a_j)\right]. \tag{1.22}$$

If assumption 1.2 holds, then the wholesale profit is strictly quasiconcave since $-\frac{2\sigma q(a_j)}{3}\left(1-\epsilon(a_j)\right)-n^*(a_j,a_i)\epsilon^{'}(a_j)<0$. In addition, by Lemma 1.3 of Section A1.1, there exists an unique $a^\dagger(a_i)$ such that $n^*(a^\dagger(a_i),a_i)\left[1-\epsilon(a^\dagger(a_i))\right]-\frac{\sigma}{3}\pi^W(a^\dagger(a_i))=0$. Quasiconcavity assures that the best response of operators j is

$$\tilde{a}_j^*=\tilde{a}^*(a_i)\equiv\begin{cases}a_i & \text{if}\quad a_i\leq\bar{a}^\dagger\\ a^\dagger(a_i) & \text{otherwise}\end{cases}$$

where $\bar{a}^\dagger$ is the highest value of a_i such that the operators j find it optimal to offer the same wholesale price. $\bar{a}^\dagger$ is uniquely defined by

$$\frac{1}{2}\left[1-\epsilon(\bar{a}^\dagger)\right]-\frac{\sigma}{3}\pi^W(\bar{a}^\dagger)=0 \tag{1.23}$$

due to Lemma 1.3. Clearly for all $c < a_i < \bar{a}^\dagger$, $\frac{\mathrm{d}\tilde{a}_j^*}{\mathrm{d}a_i}=1$. Denote the equilibrium wholesale price that obtains with two alliances according to Proposition 1.2 by a^*. Comparing equation (1.22) and (1.23), shows that whenever Assumption 1.2 holds, then $\bar{a}^\dagger > a^*$. For later use, we show that $\tilde{a}^*(a_i) > a^*$ whenever $a_i > a^*$. This property is clearly satisfied for $\bar{a}^\dagger \geq a_i > a^*$. For $a_i > \bar{a}^\dagger$, note that $a^\dagger(a_i) > a^*(a_i) > a^*$ where the first inequality is because $n^*(a_j,a_i)\left[1-\epsilon(a_j)\right]-\frac{\sigma}{3}\pi^W(a_j) > n^*(a_j,a_i)\left[\frac{1}{3}-\epsilon(a_j)\right]-\frac{\sigma}{3}\pi^W(a_j)$ and both sides are decreasing in a_j whereas the second inequality comes from the monotonicity of $a^*(a_i)$ and the fact that $a^*(a^*)=a^*$.

We now analyze the equilibrium price a_i^{ND*} (the superscript ND refers to non-discrimination) that obtains when only one alliance has been formed. Taking into account the best response of operators j, the marginal profit of a member of alliance i reads now as follows:

$$\begin{aligned}\frac{\partial\Pi^{ND}}{\partial a_i}(a_i)=q(a_i)\left[\left(\frac{1}{3}-\epsilon(a_i)\right)n^*(a_i,\tilde{a}_j{}^*)\right.\\ \left.+\frac{\sigma}{3}\left(\frac{q(\tilde{a}_j{}^*)}{q(a_i)}\frac{\mathrm{d}\tilde{a}_j{}^*}{\mathrm{d}a_i}\left(\frac{2n^*(a_i,\tilde{a}_j{}^*)}{\sigma}+\pi^W(a_i)\right)-\pi^W(a_i)\right)\right]\end{aligned} \tag{1.24}$$

For $c < a_i < \bar{a}^\dagger$, equation (1.24) simplifies to $\frac{\partial\Pi^{ND}}{\partial a_i}(a_i)=q(a_i)\frac{1}{2}\left(1-\epsilon(a_i)\right)$ because $\tilde{a}^*(a_i)=a_i$ implies that the market share and thus the retail profits remain constant as a_i is slightly increased.

Now we show that a maximizer a^{ND*} exists. By equation (1.22), $\epsilon(\tilde{a}_j^*) < 1$ which implies $\epsilon'(\tilde{a}_j^*) \geq 0$ by Assumption 1.2. For $a_i > \bar{a}^\dagger$, applying the implicit function theorem yields

$$\frac{\mathrm{d}\tilde{a}_j^*}{\mathrm{d}a_i}=\frac{\sigma q(a_i)\left(1-\epsilon(\tilde{a}_j^*)\right)}{2\sigma q(\tilde{a}_j^*)\left(1-\epsilon(\tilde{a}_j^*)\right)+3\epsilon'(\tilde{a}_j^*)n^*(\tilde{a}_j^*,a_i)}$$

and thus $0 \leq \frac{q(\tilde{a}_j^*)}{q(a_i)} \frac{\mathrm{d}\tilde{a}_j^*}{\mathrm{d}a_i} < \frac{1}{2}$. Inserting this into equation (1.24) yields

$$\begin{aligned}\frac{\partial \Pi^{ND}}{\partial a_i}(a_i) &\leq q(a_i)\left[\left(\frac{2}{3} - \epsilon(a_i)\right) n^*(a_i, \tilde{a}^*(a_i)) - \frac{\sigma}{6}\pi^W(a_i)\right] \\ &\leq q(a_i)\left[\left(\frac{2}{3} - \epsilon(a_i)\right)\frac{1}{2} - \frac{\sigma}{6}\pi^W(a_i)\right]\end{aligned}$$

for $a_i > \bar{a}^\dagger$.

If $\frac{\partial \Pi^{ND}}{\partial a_i}(\bar{a}^\dagger) \leq 0$, define $\hat{a} = \bar{a}^\dagger$. Otherwise, define $\hat{a}$ as the solution to $\left(\frac{2}{3} - \epsilon(a_i)\right)\frac{1}{2} - \frac{\sigma}{6}\pi^W(a_i) = 0$ which uniquely exists according to Lemma 1.3. By Assumption 1.2, $\forall a_i > \hat{a}$, $\frac{\partial \Pi^{ND}}{\partial a_i}(\hat{a}) \leq 0$. Since $[c, \hat{a}]$ is a compact interval, by the Weierstrass-Theorem, there exists some $a^{ND*} \in [c, \hat{a}]$ that maximizes $\Pi^{ND}(a_i)$ and which is also a global maximum by the preceding paragraph .

To see that $a^{ND*} > a^*$, note that for all $a_i < \bar{a}^\dagger$, $\tilde{a}^*(a_i) = a_i$ and therefore $\frac{\partial \Pi^{ND}}{\partial a_i}(a_i) = \frac{\partial \Pi}{\partial a_i}(a_i, a_i) + q(a_i)\left(\frac{1}{3} + \frac{\sigma}{3}\pi^W(a_i)\right)$. Since $a^* < \bar{a}^\dagger$, $\frac{\partial \Pi}{\partial a_i}(a^*, a^*) = 0$ and $\frac{\partial \Pi}{\partial a_i}(a_i, a_i) > 0 \quad \forall a_i \in [c, a^*]$ implies $\frac{\partial \Pi^{ND}}{\partial a_i}(a_i) > 0 \quad \forall a_i \in [c, a^*]$. Hence $a^{ND*} > a^*$.

Given the equilibrium prices, each operator sells roaming services to exactly one foreign operator, so that the total profits remain as defined in equation (1.9). To see that $\Pi(\tilde{a}_j{}^*(a^{ND*}), a^{ND*}) > \Pi(a^*, a^*)$, note that $\tilde{a}_j{}^*(a^{ND*}) > a^*$ as shown above. Since also $\tilde{a}_j{}^*(a^{ND*}) \leq a^{ND*}$, $\Pi(\tilde{a}_j{}^*(a^{ND*}), a^{ND*}) \geq \Pi(\tilde{a}_j{}^*(a^{ND*}), \tilde{a}_j{}^*(a^{ND*})) > \Pi(a^*, a^*)$ where the last inequality is due to $\epsilon(a^{ND*}) < 1$. Since a^{ND*} maximizes Π^{ND}, $\Pi(a^{ND*}, \tilde{a}^*(a^{ND*})) \geq \Pi(\bar{a}^\dagger, \tilde{a}^*(\bar{a}^\dagger)) = \Pi(\bar{a}^\dagger, \bar{a}^\dagger) > \Pi(a^*, a^*)$. Since also $\Pi(a^*, a^*) > \Pi(c, c)$, it is straight forward to show the following: If $\Pi(a^{ND*}, \tilde{a}^*(a^{ND*})) > \Pi(\tilde{a}^*(a^{ND*}), a^{ND*})$ then only one operator in country B and both operators in country A announce to form an alliance in the unique (up to relabeling) equilibrium. If $\Pi(a^{ND*}, \tilde{a}^*(a^{ND*})) < \Pi(\tilde{a}^*(a^{ND*}), a^{ND*})$ then exactly one operator in country A and both operators in country B announce to form an alliance. If $\Pi(a^{ND*}, \tilde{a}^*(a^{ND*})) = \Pi(\tilde{a}^*(a^{ND*}), a^{ND*})$, then either one operator in country A or one operator in country B do not announce to form an alliance. Anyways, exactly one alliance emerges in equilibrium.

Proof of Proposition 1.8

We first prove the following auxiliary Lemma:

Lemma 1.7. *If assumption 1.2 holds and $q'(a^*) < 0$, then $v(c) - v(a^*) < \frac{1}{2\sigma}$, where a^* is the equilibrium wholesale price defined by Proposition 1.2.*

Proof. The equilibrium condition $\frac{\partial \Pi}{\partial a_i}(a^*, a^*) = 0$ yields $\pi^{W*} \equiv q(a^*)(a^* - c) = \frac{1-3\epsilon(a^*)}{2\sigma}$. Assumption 1.2 implies that $v(c) - v(p) \leq \frac{\pi^W(p)}{1-\epsilon(p)}$ for any $p \in \mathcal{E}$. Both results together yield $v(c) - v(a^*) \leq \frac{\pi^W(a^*)}{1-\epsilon(a^*)} = \frac{1-3\epsilon(a^*)}{2\sigma(1-\epsilon(a^*))} < \frac{1}{2\sigma}$ where the last inequality is due to $\frac{1}{3} \geq \epsilon(a^*) > 0$. ■

We now show *existence* of a unique symmetric equilibrium. Denote the wholesale price that obtains after the retail price cap has been introduced by $\overline{a}^*$ and the equilibrium net surplus as $\overline{w}^*$. By the same reasoning as in Lemma 1.4, the first order condition is sufficient for a (local) maximum. Similar to the proof of Proposition 1.2, we define $\overline{\psi}(a) \equiv \frac{6}{q(\overline{p})} \frac{\partial \overline{\Pi}^*}{\partial a_i}(a, a) = 1 - 2\sigma q(\overline{p})(a - c)$. We claim that that wholesale prices $a_0 = a_1 = \overline{a}^*$ with $\overline{a}^*$ being uniquely characterized by $\overline{\psi}(\overline{a}^*) = 0$ support an equilibrium. By definition of $\overline{\psi}$, the equilibrium price $\overline{a}^*$ locally strictly maximizes both alliances' profits.

Next we show that $\overline{\Pi}(a_i, \overline{a}^*)$ is strictly quasiconcave in a_i if both alliances have a positive market share: Define $\overline{n}^*(a_i, a_j) \equiv \frac{1}{2} + \frac{\sigma}{3}\left[\overline{v}(a_i) - \overline{v}(a_j)\right]$ using the generalized value $\overline{v}(\cdot)$ of (1.16). For $a_i \geq \overline{p}$, $\frac{\partial \overline{\Pi}}{\partial a_i}(a_i, \overline{a}^*) = \frac{q(\overline{p})}{3}\left[\overline{n}^*(a_i, \overline{a}^*) - \frac{\sigma}{3}\overline{\pi}^W(a_i)\right]$ with $\overline{\pi}^W(a_i) \equiv q(\overline{p})(a_i - c)$. Since $\frac{\partial \overline{\Pi}}{\partial a_i}(\overline{a}^*, \overline{a}^*) = 0$ and $\overline{n}^*(a_i, \overline{a}^*)$ decreases in a_i while $\overline{\pi}^W(a_i)$ increases in a_i, we have $(\overline{a}^* - a_i)\frac{\partial \overline{\Pi}}{\partial a_i}(a_i, \overline{a}^*) > 0$ for $a_i > \overline{p}$ and $a_i \neq \overline{a}^*$. For $a_i < \overline{p}$, $\frac{\partial \overline{\Pi}}{\partial a_i}(a_i, \overline{a}^*) = \frac{q(a_i)}{3}\left[(1 - 3\epsilon(a_i))\,\overline{n}^*(a_i, a_j) - \sigma\pi^W(a_i)\right]$which differs from (1.10) only by the market share $\overline{n}^*(a_i, \overline{a}^*)$ instead of $n^*(a_i, a^*)$. We show below that $\overline{v}(\overline{a}^*) < v(a^*)$ which implies $\overline{n}^*(a_i, \overline{a}^*) > n^*(a_i, a^*)$ for $a_i < \overline{p}$. Since by hypothesis $\overline{p} \leq a^*$, we have $\frac{\partial \overline{\Pi}}{\partial a_i}(a_i, \overline{a}^*) > \frac{\partial \Pi}{\partial a_i}(a_i, a^*) > 0$ where the last inequality is due to Lemma 1.4.

It remains to prove that drastic deviations in order to corner the market are unprofitable. We first show that given $\overline{p} \leq a^*$, any deviation wholesale price $\tilde{a}_i$ to corner the market requires that $\tilde{a}_i < c$ or equivalently $v(\tilde{a}_i) > v(c)$. To derive a lower bound for $\overline{v}^* \equiv \overline{v}(\overline{a}^*)$, note that $\overline{v}^* = v(\overline{p}) - q(\overline{p})\,(\overline{a}^* - \overline{p}) = v(c) - \overline{\pi}^{W*} - \int_c^{\overline{p}} \epsilon(p)q(p)\mathrm{d}p$ with

$\overline{\pi}^{W*} \equiv q(\overline{p})(\overline{a}^* - c)$. The equilibrium condition $\overline{\psi}(\overline{a}^*) = 0$ implies $\overline{\pi}^{W*} = \frac{1}{2\sigma}$. Besides, $\overline{p} \leq a^* \in \mathcal{E}$ guarantees that $\int_c^{\overline{p}} \epsilon(p)q(p)\mathrm{d}p \leq \epsilon(\overline{p})\left(v(c) - v(\overline{p})\right) \leq \frac{1}{3}\left(v(c) - v(a^*)\right) < \frac{1}{6\sigma}$, where the last inequality is due to Lemma 1.7. Taken together, $\overline{v}^* > v(c) - \frac{4}{6\sigma}$. Cornering the market requires $v(\tilde{a}_i) \geq \overline{v}^* + \frac{3}{2\sigma} > v(c) + \frac{5}{6\sigma} > v(c)$. For any $a_i < c$ such that $v(a_i) > \overline{v}^* + \frac{3}{2\sigma}$, marginal profits are $\frac{\partial \overline{\Pi}}{\partial a_i}(a_i, a_j) = -q(a_i)\epsilon(a_i) \geq 0$ since $\epsilon(a_i) \leq 0$. Thus $\overline{\Pi}(\tilde{a}_i, \overline{a}^*) \leq \overline{\Pi}(c, \overline{a}^*) < \overline{\Pi}(\overline{a}^*, \overline{a}^*)$.

The preceding two paragraphs establish that there is no profitable deviation, which completes the proof of existence.

We now show that $\overline{v}^* < v(a^*)$, which suffices to prove that any binding price cap reduces the consumer surplus since $\overline{w}^* - w^* = \overline{v}^* - v(a^*)$. The condition $\overline{v}^* = v(\overline{p}) - q(\overline{p})\left(\overline{a}^* - \overline{p}\right) < v(a^*)$ can be rewritten as $v(\overline{p}) + q(\overline{p})\left(\overline{p} - c\right) - \overline{\pi}^{W*} < v(a^*)$ and is satisfied if $v(c) - \overline{\pi}^{W*} < v(a^*)$ since $v(\overline{p}) + q(\overline{p})\left(\overline{p} - c\right) \leq v(c)$. Reordering this condition and using $\overline{\pi}^{W*} = \frac{1}{2\sigma}$ yields $v(c) - v(a^*) < \frac{1}{2\sigma}$ which is true by Lemma 1.7.

If $\overline{p} < a^*$, then clearly $v(\overline{p}) + q(\overline{p})\left(\overline{p} - c\right) > v(a^*) + q(a^*)\left(a^* - c\right)$ and total welfare increases.

Comparing $\overline{\psi}(a)$ to $\psi(a)$ defined in the proof of Proposition 1.2 yields $\overline{\psi}(a) - \psi(a) = 3\epsilon(a) + 2\sigma\left(q(a) - q(\overline{p})\left(a - c\right)\right)$. Therefore, the condition $\overline{\psi}(a^*) > \psi(a^*) = 0$ holds by the hypothesis $q(\overline{p}) - q(a^*) < \frac{3\epsilon(a^*)}{2\sigma(a^* - c)}$. Since $\overline{\psi}'(a) = -\sigma q(\overline{p}) < 0$, $\overline{\psi}(a^*) > 0$ implies $\overline{\psi}(\overline{a}^*) = 0$ for $\overline{a}^* > a^*$.

A1.2 Appendix - Heterogeneous Consumers

Our main result of this section is that heterogeneous consumers lead to unambiguously lower profits in equilibrium. However, alliances still allow to raise equilibrium profits. We assume that operators of both countries with same position in their home market have formed alliances and omit the country index for brevity of notation. We focus on candidate symmetric equilibria that satisfy the necessary first order conditions of profit maximization.

Retail demand structure. In contrast to our main setup, there are two types of consumers indicated by θ_k with $k \in \{L, H\}$ and $\theta_L < \theta_H$.[71] A consumer of type θ_k values roaming calls according to $v_k(p) \equiv \theta_k v(p)$ with $v(p)$ defined as in Section 1.2. Likewise, $u_k(q)$ denotes the utility that a subscriber of type θ_k obtains from consuming q roaming calls.[72] Subscribers still have quasilinear preferences so that the demand of an θ_k subscriber is given by $q_k(p) \equiv \theta_k q(p)$. The measure of subscribers remains normalized to 1 in every country. A proportion β of these are *light users* with type θ_L and relatively low demand. The remaining fraction of $1-\beta$ are *heavy users* characterized by θ_H. Without loss of generality, we normalize $\theta_L < 1 < \theta_H$ such that $\beta\theta_L + (1-\beta)\,\theta_H \equiv 1$.[73] For future reference, we define the heterogeneity of consumers as the variance of their type: $\rho \equiv \beta\,(\theta_L - 1)^2 + (1-\beta)\,(\theta_H - 1)^2$. The base model with homogeneous consumers corresponds to $\rho = 0$. All consumers have the same degree of differentiation σ and the consumers' location is stochastically independent from their type. The consumers' type is observable by the MNOs. We discuss below the implications of relaxing this assumption.

Retail pricing structure. Similar to Section 1.3, operator i sets the retail per call price p_{ki} and the fixed fee F_{ki} for a type θ_k subscriber. We equivalently express the problem in terms of price per call p_{ki} and net surplus $w_{ki} \equiv v_k(p_{ki}) - F_{ki}$.

Wholesale pricing structure. MNOs cannot discriminate the wholesale prices according to which type of customer the roaming calls are sold finally. They still charge a linear wholesale price a_i to foreign operators.

Retail equilibrium. By the same reasoning as in Section 1.3, it is optimal to set the usage price equal to marginal cost. Given the perceived marginal cost c_i and the per customer cost C_F, the retail profits of operator i are then

$$\Pi_i^R = \beta n_{Li}\pi_{Li}^R + (1-\beta)\,n_{Hi}\pi_{Hi}^R \tag{1.25}$$

with $\pi_{ki}^R = \pi_k^R(w_{ki}, c_i) \equiv v_k(c_i) - w_{ki} - C_F$ being the per customer retail profit and $n_{ki} = n_k(w_{ki}, w_{kj}) \equiv \frac{1}{2} + \sigma(w_{ki} - w_{kj})$ being the market share in segment $k \in \{L, H\}$.

[71] In a model of network interconnection, Dessein (2003) uses a similar setup.

[72] Note that due to our specification, $u_k(q) \neq \theta_k u(q)$ in general.

[73] This normalization allows us to interpret $q(p)$ as the mean demand per consumer at the per call price p.

Solving for the equilibrium net surplus and market share yields

$$w_{ki}^{*} = \theta_k \left(\frac{2}{3} v(c_i) + \frac{1}{3} v(c_j) \right) - \frac{1}{2\sigma} - C_F \tag{1.26}$$

$$n_{ki}^{*} = \frac{1}{2} + \frac{\theta_k \sigma}{3} \left(v(c_i) - v(c_j) \right) \tag{1.27}$$

The further results of this section can be conveniently expressed in terms of the equilibrium *share of roaming calls* (as opposed to the market share of *subscribers*), defined as $\tilde{n}_i^* \equiv \beta n_{iL}^* \theta_L + (1-\beta) n_{iH}^* \theta_H$. Inserting the equilibrium retail market shares (1.27) yields $\tilde{n}_i^* = \frac{1}{2} + \frac{\sigma}{3} \left(v(c_i) - v(c_j) \right) (1+\rho)$. The factor $1+\rho$ indicates that the equilibrium share of roaming calls $\tilde{n}_i^*$ reacts more sensitively to differences in the perceived marginal costs compared to the equilibrium *share of subscribers* n_i^*. According to (1.26), an operator that faces higher unit costs offers a less attractive tariff especially to heavy users. Since the degree of differentiation $1/\sigma$ is independent of the type, the market shares in the heavy user segment are less balanced than in the light user segment. Inserting the optimal tariffs in (1.25) and rearranging yields the retail equilibrium profit

$$\Pi_i^{R*} = \Pi^{R*}(c_i, c_j) \equiv \frac{\sigma\rho}{9} \left(v_i - v_j \right)^2 + \frac{1}{\sigma} \left(\frac{1}{2} + \frac{\sigma}{3} \left(v_i - v_j \right) \right)^2 \tag{1.28}$$

with $v_i \equiv v(c_i)$. The marginal retail equilibrium profit with respect to the perceived unit cost is

$$\frac{\partial \Pi^{R*}}{\partial c_i}(c_i, c_j) \equiv -\frac{2q(c_i)}{3} \tilde{n}_i^* \, . \tag{1.29}$$

Wholesale equilibrium. When setting the retail tariffs, operators consider the negotiated wholesale prices as perceived marginal costs. Thus, the profit per member of alliance i is now $\Pi_i = \Pi(a_i, a_j) \equiv \tilde{n}_i^* q(a_i) (a_i - c) + \Pi^{R*}(a_i, a_j)$. Whenever the wholesale prices a_0 and a_1 do not differ too much, that is $|v(a_0) - v(a_1)| < \frac{3}{2\sigma\theta_H}$, the marginal profit is:

$$\frac{\partial \Pi}{\partial a_i}(a_i, a_j) = q(a_i)\left[\left(\frac{1}{3} - \epsilon(a_i)\right)\tilde{n}_i^* - \frac{\sigma}{3}\pi^W(a_i)\,(1+\rho)\right]$$

Rearranging the first order condition yields the Lerner formula

$$\frac{a^* - c}{a^*} = \frac{1}{3\left[\eta_q(a^*) + \eta_{\tilde{n}_i^*}(a^*)\right]} \tag{1.30}$$

where $\eta_q(a_i)$ is the price elasticity of the mean per customer demand and $\eta_{\tilde{n}_i^*}(a_i) \equiv -\frac{\mathrm{d}\tilde{n}_i^*}{\mathrm{d}a_i}\frac{a_i}{\tilde{n}_i^*}$ refers to the price elasticity of the equilibrium *share of calls*. In particular, a symmetric equilibrium entails $\tilde{n}_i^* = \frac{1}{2}$ and thus $\eta_{\tilde{n}_i}(a_i) = \frac{2\sigma}{3}(1+\rho)\,a_i q(a_i)$. Now we can identify the effect of consumer heterogeneity on the candidate equilibrium wholesale price:

Proposition 1.10. *Suppose that equation* (1.30) *uniquely characterizes the equilibrium wholesale price and that Assumption 1.2 holds. Then an increase in consumer heterogeneity ρ, holding everything else constant, reduces the symmetric wholesale equilibrium price.*

Proof. In any symmetric equilibrium, the condition $\left(\frac{1}{3} - \epsilon(a^*)\right)\frac{1}{2} - \frac{\sigma}{3}\pi^W(a^*)\,(1+\rho) = 0$ must be satisfied. The left hand side is clearly decreasing in ρ and if Assumption 1.2 holds, it is decreasing in a^*. Application of the implicit function theorem on this condition yields $\frac{da^*}{d\rho} < 0$. ■

Intuitively, consumer heterogeneity renders increasing the wholesale price less profitable relative to the gains from softer retail competition, since this leads to a loss of disproportionately many heavy users.

Non observable customer types: Even when customer types are unobservable for the MNOs, the results of this section are likely to carry over. In this case, MNOs have to elicit this information by offering incentive compatible contracts. However, it is easy to verify that for any symmetric wholesale price, the retail tariffs (1.26) indeed satisfy the incentive constraints for truth telling.[74] This somewhat surprising finding is in line with the observation of Armstrong and Vickers (2001) and Rochet and Stole (2002) that

[74]However, after a deviation from a symmetric equilibrium wholesale price, the incentive conditions may bind.

private information of consumers may not cause any quantity distortions in certain competitive environments.[75]

A1.3 Appendix - Continuous Network Selection

We assume that at most the proportion $\bar{\gamma} \in [0.5, 1]$ of roaming calls can be directed to a particular foreign network.[76] This bound on the proportion reflects the fact that the restriction does not come from capacity constraints (which would render an absolute constraint more plausible) but rather from an unreliable technology that cannot guarantee that a subscriber registers in the preferred network. We have analyzed the polar cases of perfect network selection ($\bar{\gamma} = 1$) and of no control ($\bar{\gamma} = 0.5$) in the base model and in Section 1.6.2, respectively.

For clarity, we present the results from the viewpoint of operators with home network in country A. When buying roaming calls from foreign MNOs on the wholesale market, operator Ai may decide to buy proportion γ_{Ai} from operator $B0$ and proportion $1 - \gamma_{Ai}$ from operator $B1$. Operator Ai's perceived marginal cost is:

$$c_{Ai} = \gamma_{Ai} a_{B0} + (1 - \gamma_{Ai})\, a_{B1} \tag{1.31}$$

Assuming that operators cannot discriminate the retail prices according to which host network provides the roaming services, the optimal per call price equals the perceived marginal cost: $p^*_{Ai} = c_{Ai}$. The equilibrium net surplus, market shares and the retail equilibrium profits remain as established in Lemma 1.1.

We now turn to the wholesale market.

No international alliances. As discussed in Sections 1.3 and 1.4, operators prefer to buy roaming calls from the cheapest foreign operator.

[75] They also discuss the sensitivity of this result with respect to assumptions like symmetry.

[76] This specification is equivalent to the following assumption: Operators can direct their subscribers to the desired foreign network only with probability $\tilde{\gamma} \in [0, 1]$. The remaining subscribers are assigned randomly to the host networks. Then one immediately sees that $\overline{\gamma} = \tilde{\gamma} + \frac{1}{2}(1 - \tilde{\gamma}) = \frac{1}{2}\,(1 + \tilde{\gamma})$. See also Salsas and Koboldt (2004), Section 3.5 for a slightly different assumption.

$$\gamma^*_{Ai} = \begin{cases} \bar{\gamma} & \text{if } a_{B0} < a_{B1} \\ 1-\bar{\gamma} & \text{if } a_{B0} > a_{B1} \end{cases}$$

We define the optimized perceived marginal cost of operator Ai as the cheapest possible mean cost for roaming calls, given the posted prices of foreign operators:

$$c^*_{Ai} = c^*(a_{B0}, a_{B1}) \equiv \bar{\gamma} \min\{a_{B0}, a_{B1}\} + (1-\bar{\gamma}) \max\{a_{B0}, a_{B1}\}$$

The main implication of imperfect host network selection is that operators may generate positive demand even when not offering the cheapest wholesale price. We assume for simplicity that foreign operators divide the traffic evenly among both domestic networks if these offer equal wholesale prices. Using the results of the retail equilibrium, in the absence of alliances the total wholesale demand of operator Ai (where the superscript NA means "no alliance") is:

$$Q^{NA}_{Ai} = Q^{NA}(a_{Ai}, a_{Aj}) \equiv \begin{cases} \bar{\gamma} q\left((1-\bar{\gamma})\, a_{Aj} + \bar{\gamma} a_{Ai}\right) & \text{if } a_{Ai} < a_{Aj} \\ \frac{1}{2} q(a_{Ai}) & \text{if } a_{Ai} = a_{Aj} \\ (1-\bar{\gamma})\, q\left((1-\bar{\gamma})\, a_{Ai} + \bar{\gamma} a_{Aj}\right) & \text{if } a_{Ai} > a_{Aj} \end{cases}$$

The demand is independent of the actual market share of the reselling operators, since for all price combinations, both foreign operators purchase the same part of their traffic at operator Ai. The overall profit of operator Ai is therefore:

$$\Pi^{NA}_{Ai} = \Pi^{NA}(a_{Ai}, a_{Aj}) \equiv \Pi^{R*}(c_{Ai}, c_{Aj}) + (a_{Ai} - c)\, Q^{NA}(a_{Ai}, a_{Aj})$$

Operator Ai sets its wholesale price in order to maximize its wholesale profit $(a_{Ai} - c)\, Q^{NA}(a_{Ai}, a_{Aj})$.

Lemma 1.8. *Suppose that Assumption 1.3 holds. For $\bar{\gamma} \in (0.5, 1)$, there is no pure strategy equilibrium.*

Proof. We first show that there is no symmetric equilibrium. Suppose to the contrary that $a^*_{A0} = a^*_{A1}$. If $a^*_{A0} = c$, then increasing the own price increases wholesale profits. If $a^*_{A0} > c$, then undercutting slightly increases the profit.

We now show that there is no asymmetric equilibrium. Let p^* denote the maximizer of $(p-c)\,q(p)$.[77] Suppose to the contrary w.l.o.g. that $a^*_{A0} \neq a^*_{A1}$. Then there exists an operator Ai such that $a^*_{Ai} \neq p^*$. But then there exists an $\hat{a}_{Ai}$ such that $\text{sign}(\hat{a}_{Ai} - a_{Aj}) = \text{sign}(a^*_{Ai} - a_{Aj})$ and $|\hat{a}_{Ai} - p^*| < |a^*_{Ai} - p^*|$. By assumption 1.3, this implies that $(\hat{a}_{Ai} - c)\,Q^{NA}(\hat{a}_{Ai}, a^*_{Aj}) > (a^*_{Ai} - c)\,Q^{NA}(a^*_{Ai}, a^*_{Aj})$ and therefore contradicts equilibrium. ■

Under imperfect network selection the fully competitive equilibrium of Section 1.4 vanishes and there is no other equilibrium in which both operators set higher wholesale prices. Intuitively, there is no equilibrium with $a^*_{A0} = a^*_{A1} = c$ because deviating upwards generates strictly positive wholesale profits.

Two international alliances. We now analyze the equilibrium outcome after operators with the same location have formed two competing alliances and omit the country index for brevity. We maintain all assumptions of Section 1.6.2, except that now, the proportion $\bar{\gamma} \in [0.5, 1]$of an operator's subscribers are directed to foreign partner network to place roaming calls.

If both alliances have negotiated the wholesale prices a_i and a_j, the equilibrium wholesale demand for roaming calls of operator i is

$$Q_i = Q(a_i, a_j) \equiv \bar{\gamma} n^*_i q\left(\bar{\gamma} a_i + (1-\bar{\gamma})\, a_j\right) + (1-\bar{\gamma})\,(1-n^*_i)\, q\left(\bar{\gamma} a_j + (1-\bar{\gamma})\, a_i\right)$$

where

$$n^*_i = \frac{1}{2} + \frac{\sigma}{3}\left[v\left(\bar{\gamma} a_i + (1-\bar{\gamma})\, a_j\right) - v\left(\bar{\gamma} a_j + (1-\bar{\gamma})\, a_i\right)\right]$$

is the equilibrium retail market share. The profit of each operator in alliance i is:

[77] Which exists by Assumption 1.3.

$$\Pi_i = \Pi(a_i, a_j) \equiv \Pi^{R*}(c_i, c_j) + (a_i - c)\left[\bar{\gamma} n_i^* q\left(c_i\right) + \left(1-\bar{\gamma}\right)\left(1-n_i^*\right) q\left(c_j\right)\right] \quad (1.32)$$

If both firms realize a strictly positive market share, the marginal profit with respect to the own wholesale price is:

$$\begin{aligned} \frac{\partial \Pi}{\partial a_i}(a_i, a_j) &= Q(a_i, a_j) + \frac{\mathrm{d} n_i^*}{\mathrm{d} a_i}\left[2\frac{n_i^*}{\sigma} + (a_i - c)\left(\bar{\gamma} q(c_i) + \left(1-\bar{\gamma}\right) q(c_j)\right)\right] \\ &\quad + (a_i - c)\left[\bar{\gamma}^2 n_i^* q^{'}(c_i) + \left(1-\bar{\gamma}\right)^2\left(1-n_i^*\right) q^{'}(c_j)\right] \end{aligned} \quad (1.33)$$

with

$$\frac{\mathrm{d} n_i^*}{\mathrm{d} a_i} = \frac{\sigma}{3}\left(\left(1-\bar{\gamma}\right) q(c_j) - \bar{\gamma} q(c_i)\right) .$$

Considering a symmetric equilibrium with $a_i^* = a_j^* = a^*$ and therefore $c_i^* = c_j^* = a^*$ as well as $n_i^* = \frac{1}{2}$ yields

$$\frac{a^* - c}{a^*} = \frac{1 - \frac{2}{3}\left(2\bar{\gamma} - 1\right)}{\left[\left(\bar{\gamma}^2 + \left(1-\bar{\gamma}\right)^2\right)\eta_q(a^*) + \left(2\bar{\gamma}-1\right)^2 \eta_n(a^*)\right]} \quad (1.34)$$

where $\eta_q(\cdot)$ is the price elasticity of the per customer demand and $\eta_n(a^*) \equiv \frac{2}{3}\sigma a^* q(a^*)$ is the price elasticity of the retail market share for $a_j = a_i = a^*$ in case of perfect traffic direction.[78]

Comparing (1.34) with the equilibrium characterization (1.11) of the base model reveals that for the same wholesale price a_i, the right hand side of (1.34) is always larger than that of (1.11) since $1 - \frac{2}{3}\left(2\bar{\gamma} - 1\right) \geq \frac{1}{3}$, $\bar{\gamma}^2 + \left(1-\bar{\gamma}\right)^2 \leq 1$ and $\left(2\bar{\gamma}-1\right) \leq 1$ hold. These observations allow to establish that imperfect traffic steering leads to higher equilibrium wholesale prices:

[78] Both $\eta_q(\cdot)$ and $\eta_n(a^*)$ are defined as in Section 1.4.

Proposition 1.11. *Suppose that assumption 1.2 holds. Then the equilibrium wholesale price a^* in any symmetric equilibrium is decreasing in the quality of the traffic steering technology $\bar{\gamma}$.*

Proof. Using (1.33) with $a_i = a_j$ and $\frac{dn_i^*}{da_i}|_{a_i=a_j} = \frac{\sigma}{3}q(a_i)(1-2\bar{\gamma})$ and reordering, yields the first order condition

$$1 - \frac{2}{3}(2\bar{\gamma}-1)\left[1+(2\bar{\gamma}-1)\sigma(a^*-c)\,q(a^*)\right] - \epsilon(a^*)\left(\bar{\gamma}^2+(1-\bar{\gamma})^2\right) = 0$$

As the the middle term is strictly negative for $\bar{\gamma} > 0.5$ and 0 for $\bar{\gamma} = 0.5$, it follows that $\epsilon(a^*)\left(\bar{\gamma}^2+(1-\bar{\gamma})^2\right) < 1$. Applying the implicit function theorem yields

$$\frac{da^*}{d\bar{\gamma}} = \frac{2\left[1+2\sigma q(a^*)(a^*-c)\right] + 2\epsilon(a^*)(2\bar{\gamma}-1)}{-(2\bar{\gamma}-1)^2\sigma q(a^*)\left(1-\left(\bar{\gamma}^2+(1-\bar{\gamma})^2\right)\epsilon(a^*)\right) - \frac{3}{2}\left(\bar{\gamma}^2+(1-\bar{\gamma})^2\right)\epsilon'(a^*)}$$

Clearly, the denominator of the right hand side is strictly negative since $1 - \left(\bar{\gamma}^2+(1-\bar{\gamma})^2\right)\epsilon(a^*) > 0$ and $\epsilon'(a^*) \geq 0$ by assumption 1.2. The numerator is strictly positive. Taken together $\frac{da^*}{d\bar{\gamma}} < 0$. ■

Intuitively, there are two channels that cause a higher equilibrium price when network selection is imperfect ($\bar{\gamma} < 1$). First, compared to the base model ($\bar{\gamma} = 1$), the retail market share is less sensitive to increases of the wholesale price. This is because the perceived marginal costs c_i of operators within alliance i depend less on the own wholesale price a_i while the perceived marginal costs of operators of the rival alliance j depend partly on a_i. Second, under imperfect traffic direction, operators of alliance j have to procure a proportion $1 - \bar{\gamma}$ of their roaming calls from alliance i. When selling to non-alliance operators, the alliance does not take lower retail profits that are implied by a higher wholesale price into account, which renders a high wholesale price more attractive.

Chapter 2

Ideologues: Explaining Partisanship and Persistence in Politics*

2.1 Introduction

Political leaders often define themselves in terms of a set of beliefs and values that they adhere to, and consistently base their political action on that set. Such leaders, who place greater weight on ideology as a collection of ideas about how society should work and the best way to achieve this goal, can be referred to as ideological leaders or *ideologues*. One has to look no further than to contemporary American politics to find plenty of ideologues: "liberal", "conservative", "moderate", "leftist" – politicians routinely use ideological labels to describe themselves and their opponents, and the American public, led by journalists and political activists, are happy to join in. Of course, one may wonder what's in a name. Surprisingly much as it turns out.

As documented in the empirical work on Congressional voting behavior of Poole and Rosenthal (2007), McCarty, Poole, and Rosenthal (2006) and others, the belief systems of political elites can often largely be captured with a single dimension, their *ideology*,

*This Chapter is based on joint work with Anke S. Kessler.

which almost always mirrors party affiliation: with just the label "conservative" (Republican), for example, one can fairly accurately predict a politician's stance on policy issues as disparate as taxes, gun control, affirmative action, health care, and abortion. Moreover, ideological positions of individual members are remarkably stable. That is, based upon the roll call voting record, once elected to Congress, members adopt an ideological position and maintain that position throughout their careers – once a liberal or a conservative or a moderate, always a liberal or a conservative or a moderate.[1] As Poole (2007, p. 435) puts it, "members of Congress die in their ideological boots." Clearly, this phenomenon is neither exclusive to the U.S., nor is it confined to positional (divisive) issues that voters have different preference over, depending on their socio-economic status, race, gender, or religion. Partisan politics are a frequent phenomenon even regarding so-called valence issues for which there should be a common agreement among the electorate (such as crime, foreign policy, corruption and economic growth).[2]

To analyze these issues, this chapter suggests a theory of ideology for public leaders. We seek to answer three questions. First, why do political elites adopt ideological labels and play them out in partisan politics, especially on policies where voters would prefer their representative to seek common ground? Second, why ideological views are so persistent, even in the face of changing circumstances to the point where they are at odds with the facts? Third, what are the cost of such behavior? To this end, we develop a dynamic model that closely ties observable characteristics of political representative (such as their gender, their party affiliation, or their district) to voters'

[1]What is more, members of Congress seem to remain ideologically consistent even in the face of changing personal or electoral conditions: members' voting records remain essentially the same, regardless of whether they plan to retire, plan to run for a higher office, serve in a higher office, or have their districts redrawn. See Poole and Rosenthal (1997) and Poole (2007) and the references therein.

[2]In the U.S. Congress, for example, support for the president on matters of foreign policy and defense has largely been along party lines ever since the Vietnam War [Meernik (1993)]. On a more general note, empirical evidence from the U.S. Congress support the view that partisanship of political representatives often does not simply mirror equally divided constituents. Rather than representing the district voters, a representative's own ideology is the primary determinant of roll-call voting patterns [Lee, Moretti, and Butler (2004) and Levitt (1996)]. In either case, voter polarization is presumably a lesser danger for valence issues. Polling data on foreign policy confirm this presumption. Two recent pools conducted by the Program on International Policy Attitudes (PIPA) and the Chicago Council on Foreign Relations (CCFR) found that Americans share common views on a wide array of foreign policy issues, and would prefer that Democrats and Republicans seek common ground. For details, see the website of Partnership for a Secure America (http://www.psaonline.org/), an organization dedicated to recreating the bipartisan center in American national security and foreign policy.

expectations. Our model starts from the observation that voters are often uncertain about how policy instruments map into policy outcomes. To capture this idea, we assume that the electorate does not observe external circumstances that make a specific policy more desirable than others. Given their beliefs about the prevailing state, voters therefore form expectations about which policy candidates are likely to implement once in office, and which of those is most likely to succeed. Importantly, voters attach ideological labels *both* to the various policy alternatives that are available *and* to the political candidates running for office. To develop our argument in the strongest manner possible, we assume that candidates derive the exact same utility from the policy measure as the electorate at large,[3] so that their ideological characterization is truly nothing more than a label.

Our main finding is that, politicians may take a persistent ideological stance and act partisan simply because voters' *expect* them to.[4] The argument is as follows. Suppose voters expect political candidates to act partisan once in office, i.e., to remain "true to their colors", implementing policies that are "close" to their own ideology as perceived by the voting public. Given these expectations, voters have a straightforward incentive to elect the representative whose perceived partisan policy (ideology) corresponds to what they think is in their best interest based on their current information. As we show, this may suffice to induce candidates to actually act partisan, i.e., according to their ideology, in the first place. The specific motivation is one of signal-jamming: an incumbent who sticks to his partisan policy avoids revealing that current circumstances would favor his opponents' partisan position, making his re-election more likely if voters expect partisan behavior in the future.[5] As even inefficient policies may turn out to be successful, this behavior potentially allows to hold up the electorates' belief in the

[3]It should be emphasized that the theory also applies for non-valence (positional) issues. There already is an extensive literature on these type of policies, however, which provides a range of complementary explanations for why candidates diverge in platforms and voting records. We refer to this literature in more detail below.

[4]The theory implies, for instance, that a female Democrat from California is likely to take a liberal stance on most issues, not because her true preferences or her belief system necessarily reflects this view, but because her constituents expect a female Democrat from California to be a liberal (and elected her for this very reason).

[5]Alesina and Cukierman (1990) study an environment in which voters are unsure about the ideological position of candidates (as opposed to the state of the economy as in this chapter). Akin to the signal-jamming effect we find, they show that politicians may want to deliberately choose "ambiguous" policies in order to conceal their true preferences, thereby keeping their ideological advantage.

incumbent's ideology. The result is political failure in the sense that the equilibrium partisan policy outcomes are Pareto dominated. Thus, the model can explain policy bias and divergence from the fact that voters *perceive* policies to be ideologically tinted and *expect* candidates to act partisan. Both sides are caught in an *ideology trap*: because voters expect the ideology of office holders to determine their political actions, an officials (re-)election chances will vary with his or her perceived ideology. In their desire to influence the outcome of the election, these expectations induce the officials to act partisan. Importantly, the issue itself can be non-partisan, meaning that neither voters nor politicians have to display any intrinsic preferences for either policy: a leader does not have to be a "true believer" to be an ideologue. Because incumbents will tend to enact the partisan policy independent of the prevailing state in equilibrium, our analysis also explains why office holders will maintain their ideology and deny conflicting evidence, resulting in policies that are likely to persist.[6]

Our theory is related – and contributes to – three different strands of the literature. First, there is a growing economic literature on the origins of ideologies as a collection of ideas and firmly held beliefs. Bénabou and Tirole (2006) and Bénabou (2008) study voters' perceptions about a fundamental property of the underling economy, and show that maintaining beliefs that contradict reality can be an equilibrium phenomenon. In forming their beliefs, individuals optimally trade off the benefit of being able to motivate themselves (or their children) toward effort and the costs of misinformed decisions. While these papers can explain ideology as a collectively held belief system, our contribution focuses on leaders and political elites who *publicly act* upon – rather than genuinely entertain – certain beliefs in order to maintain their power and leadership role.[7]

Second, our argument also bears on the important question of why political parties and politicians seeking office diverge in their positions on critical issues, contrary to what

[6]The resilience of economic policies that benefit (target) a specific groups of voters has been studied by Coate and Morris (1999) who use a dynamic model to formalize the intuition that implementation of a policy increases the political effectiveness of its beneficiaries in lobbying. The main difference between Coate and Morris (1999) and our approach is that we focus on *non-partisan* (valence) issues, which do not target specific groups.

[7]At the same time, our setting is not ideology free, since we require the electorate to attach ideological labels to policies and politicians alike, e.g, the Military Commissions Act (which effectively excluded U.S. prisoners of war from protection of the Geneva Conventions) is universally perceived to be "conservative", as is a male Republican candidate from Texas.

the Downsian model would predict. In the past two decades, scholars in economics and political science have identified a number of factors that contribute to policy divergence, including the multi-dimensional issues [Ansolabehere and Snyder (2000)], the threat of third-party entry [Palfrey (1984)], citizen candidates [Osborne and Slivinski (1996), Besley and Coate (1997)], improved electoral control [Van Weelden (2009)], and an electorate that is imperfectly informed about candidates' types [Kartik and McAfee (2007), Callander and Wilkie (2007) and Callander (2008)]. All of these explanations, however, require *partisan preferences*. Moreover, since enacted policies in these models directly reflect the preferences of the electorate, they are silent on why policies can persist over time even in the face of new (and conflicting) evidence. Indeed, the only other contribution known to us that is able to explain polarized and partisan politics on matters where voters commonly agree is Carrillo and Castanheira (2008).[8] In their paper, candidates choose polarized positions in order to commit to investing in the quality of their platform. In contrast, our explanation relies on the dynamic consideration that an incumbent may tow the party line in order to improve the prospects of reelection.

Finally, our model is closely related to the literature on political failure. In a model similar to ours, Cukierman and Tommasi (1998) show that if voters are imperfectly informed about an incumbent ideology, an incumbent's electoral prospects may increase the more atypical is the policy he proposes to implement. Harrington (1993), Canes-Wrone, Herron, and Shotts (2001) and more recently, Maskin and Tirole (2004) emphasize a negative incentive effect of elections: if the office-holding motive is sufficiently strong, politicians may choose the most popular (rather than the optimal) alternative. In a similar vein, Stasavage (2007) shows that if debates are held under the public eye, candidates may ignore their private information about the true desirability of various policy measures and instead promote policies popular among their constituents, leading to deeper polarization and dissent. Our analysis goes beyond these contributions by emphasizing how the inefficiency can depend solely on voters' *expectations* about a candidate's future policy intentions, rather than on a true discrepancy between the ideal policy of a candidate and that of the electorate at large.

[8]Another line of research has focused on explaining the prevailing polarization on "moral" issues, such as abortion or gay marriage. Glaeser, Ponzetto, and Shapiro (2005) identify a form of strategic extremism, which helps politicians to induce their core constituents to vote.

The remainder of this chapter is organized as follows. The basic framework is developed in Section 2.2. Section 2.3 provides an in-depth analysis of the model, and shows that both partisanship and non-partisanship can arise in equilibrium. Section 2.4 considers two extensions. We first demonstrate that our model uniquely predicts which of these equilibria occurs if candidates have arbitrary small biases towards their partisan policy. Second, we show that partisan behavior becomes even more plausible if the prospects of inefficient policies are themselves uncertain. Section 2.5 concludes this chapter.

2.2 A Dynamic Model of Partisanship

2.2.1 Preferences and Economic Environment

Consider an infinite-horizon economy in discrete time. The economy is populated by an infinite number of risk-neutral consumer-voters who derive the same per-period benefit $b_t = b(a_t, s_t) \in \{0, b\}$ from a policy decision a_t. For simplicity, we take a_t to be binary; in particular, there is a "left-wing" alternative $a_t = l$ and a "right-wing" alternative $a_t = r$.[9] Consumers know the set of feasible policies (and have common views on which they perceive as being left-wing and right-wing, respectively) but are uncertain about the underlying state of the economy $s_t \in \{l, r\}$.[10] As an example, take the issue of state versus market provision of public services (such as health care and education): here, the underlying state s_t captures the relative efficacy of government provision and the policy decision is whether or not the service is publicly provided, where public provision is commonly viewed as the "left-wing" alternative and private

[9] Assuming a binary political decision also has some appeal in that voters may find it difficult to make subtle distinctions between policies, e.g., they may only take note of whether government spending goes up or down. In this sense, policies may be quite broadly defined and fit well into the ideological spectrum of "left" and "right". The presumption of one-dimensionality is supported by empirical evidence from the US Congress: in well-known study using data on roll-call votes from the House and the Senate, Poole and Rosenthal (1997, 2007) show that more than 80 percent of representatives' voting records over the past 40 years can be explained solely on the basis of a one-dimensional variable (i.e., their "ideology").

[10] Even if there was a small i.i.d probability ν that consumers observe the true state at the end of each period, our results would be qualitatively robust. If $\nu > 0$, then signal jamming by implementing an inappropriate policy becomes less attractive. Yet, for small ν there still exists a non-empty set of parameters that admits the partisan equilibrium discussed below.

provision is universally perceived as a "right-wing" policy.

Voters' per period payoff stochastically depends on the unobserved state s_t as follows:

$$b(a_t = s_t) = \quad b \quad \text{with probability } 1$$

$$b(a_t \neq s_t) = \begin{cases} b & \text{with probability } \pi \\ 0 & \text{with probability } 1-\pi \end{cases}$$

In other words, if the policy choice matches the state, the policy is *successful* with probability one and voters receive a certain payoff of b. Otherwise, the policy *fails* with probability $1 - \pi > 0$ in which case we normalize payoffs to zero. [11]

The state of the economy evolves over time according to a symmetric transition function

$$\Pr(s_{t+1} = s_t) = \gamma = 1 - \Pr(s_{t+1} \neq s_t), \tag{2.1}$$

independent of the policy chosen. We assume that the state is persistent, in the sense that $\gamma \in (0.5, 1)$. Letting μ_t denote the likelihood voters attach to the left-state $s_t = l$, we can write individual preferences as in period t

$$\mathrm{E} \sum_{j=0}^{\infty} \beta^j b_{t+j} = \mathrm{E} \sum_{j=0}^{\infty} \beta^j b\left(a_{t+j}, s_{t+j}\right). \tag{2.2}$$

where $\beta < 1$ is the discount factor. Note that, by construction, the issue is *non-partisan* (ideologically neutral) in the sense that all voters unanimously agree on the best alternative: if they knew the state to be s, they unanimously preferred the policy that is appropriate for the state, i.e., $a = s$. Since they do not know s but share a common belief μ, voters prefer policy l over policy r in any given period t if and only if $\mu_t \geq \frac{1}{2}$.

Political decisions are not taken in direct democratic vote. Instead, voters elect an office holder as their representative in each period, who selects and implements the policy alternative a_t. Unlike voters, politicians observe the state s, which may simply reflect their greater expertise, better access to resources, or their greater incentive to become

[11] Our results do not hinge on the simplifying assumption that a political failure perfectly reveals that a non-matching policy has been implemented. Assuming instead that a policy that matches the state is successful with probability $\zeta > \pi$ would not change our analysis qualitatively as long as $\zeta > 1 - \frac{(1-\pi)(1-\gamma)}{\gamma}$. Moreover, the expressions in this chapter are the limit for $\zeta \to 1$.

informed.[12]

There are two observable types of politicians, left-wing L and right-wing R. We interpret the type $i \in \{L, R\}$ as politicians' "ideology" or "party affiliation", but any other observable characteristic such as the candidates' gender, their home district, or their previous position on a different (unrelated) policy issue would work equally well. Consistent with our notion that the issue is non-partisan, politicians derive the *same* utility from the policy a given state s as the voters, *independent* of their type i. However, they also care about holding office. We formalize this second motive in the usual fashion by a rent ϕ that politicians receive from being elected to office in period t. In summary, the per-period utility of an incumbent of type i in period t when the state is s_t is

$$u_t^i = b(a_t, s_t) + \phi. \tag{2.3}$$

When not in office, politicians receive a continuation utility of zero.[13] We thus assume that not being re-elected is an absorbing state, i.e., a once defeated incumbent never returns to holding office.

The timing of the stage game is as follows. First, nature draws the state s_t, which is immediately revealed to politicians but not to ordinary citizens. Next, elections are held in which voters decide whether to re-elect the incumbent or whether to newly elect the challenger for office (a period defines a term of office). Throughout, we restrict attention to the case where the challenger has a different ideology or party-affiliation than the incumbent. Once elected, the office holder chooses a policy alternative a_t. Finally, voters and politicians observe whether the policy was a success ($b_t = b$) or a failure ($b_t = 0$).

[12]The natural assumption that politicians are generally better informed than the electorate at large is often evoked in the literature. See, e.g., Cukierman and Tommasi (1998) or Maskin and Tirole (2004). Kessler (2005) provides an analysis where officials to endogenously acquire competence on the issues they oversee and specialize in policy formation.

[13]Thus, a politician cares for the legacy tied to successful policies. In particular a politician does not benefit from the legacy of policies that are enacted by a successor. Under the alternative assumption that politicians remain policy-motivated after after having been ousted from office (that is $u_t^i = b(a_t, s_t)$), our results carry over as long as the office rent ϕ is large enough.

2.2.2 Equilibrium Definition

As is common in these types of models, we will restrict attention to pure strategy, stationary and symmetric Markov perfect equilibria of this game. In those equilibria, players ignore all details of the history (including its length) and condition their strategies only on the pay-off relevant information. Note that because there is no link between periods other than the information revealed by politicians about the underlying state and the evolution of that state, the latter can be summarized for the electorate by its belief μ_t at time t. A strategy for a representative voter specifies the probability $P^i(\mu_t) \in [0,1]$ with which candidate i is elected, based on μ_t, with $P^L(\mu_t) + P^R(\mu_t) = 1$.[14] When voters are indifferent between two candidates, we assume either stands equal chances of winning the election. Similarly, a strategy for a type-i candidate $a^i(\mu_t, s_t)$ maps voters' beliefs μ_t (and hence, election outcomes) as well as the current state s_t into a policy choice $a \in \{l, r\}$. In equilibrium, strategies must be mutual best responses and beliefs evolve in a way consistent with Bayes rule whenever possible. Strategies are optimal if they maximize the value functions of candidates and voters. The value function for a representative voter can be written as

$$U(\mu_t) = \max_{P^i(\mu_t)} \ \mathrm{E}\left[\sum_i P^i(\mu_t) b(a^i(\mu_t, s_t), s_t) + \beta U(\mu_{t+1})\right] \tag{2.4}$$

where the expectation is taken over b_t and s_t given current beliefs μ_t. Note that in general, beliefs μ_{t+1} at time $t+1$ will depend on the elected candidate, the equilibrium strategy, the implemented policy and the success or failure of the policy in t. The value function of a type i candidate is

$$V^i(\mu_t, s_t) = \max_{a^i(\mu_t, s_t)} \ P^i(\mu_t)\mathrm{E}\left[b(a^i(\mu_t, s_t), s_t) + \phi + \beta V^i(\mu_{t+1}, s_{t+1})\right], \tag{2.5}$$

where the expectation is over b_t and s_{t+1}, given s_t.

[14] There will be unanimity among electorate, of course, but since no single (infinitesimally small) voter can influence the outcome of an election, every voting strategy is consistent with equilibrium. To eliminate this artificial multiplicity, we will throughout consider a representative voter whose optimal strategy maximizes (2.4) below, i.e, a strategy that would be optimal in case the voter was decisive (the unique weakly undominated strategy if there is a finite number of citizens).

2.3 Equilibrium Analysis

In the following we will use the term *non-partisan* politics to characterize the Pareto-optimal policy choice, i.e., the office holder implements $a_t = s_t$, regardless of her type i. *Partisan politics*, in contrast, involves politicians selecting the alternative that corresponds to their ideology, i.e., $a_t = l$ if $i = L$ and $a_t = r$ if $i = R$, irrespective of the state s_t. Recall from (2.3) that an office holder's per-period utility is independent of her ideology or party affiliation. Consequently, the sole channel through which ideology can possibly influence the choice of policy is through voters' *expectations*, which for the politicians will translate into the likelihood they are (re-)elected to office. It is this link between actual policy choices and voters' expectations about candidates' post-election behavior – partisan or non-partisan – we are most interested in. What matters, as we will see below, are solely voters' perceptions as to a) what constitutes a left-wing and a right-wing policy alternative, and b) who is a left-wing and a right-wing politician. To highlight the interdependencies, we have eliminated all other well-studied determinants of partisan politics (partisan voters, partisan politicians etc.), not because we consider them implausible but simply because they would only serve to disguise the true effects at work here.

2.3.1 The Non-Partisan (Efficient) Equilibrium

As a benchmark, we first construct an equilibrium in which candidates choose policies in a Pareto efficient manner along the equilibrium path, and voters – because they correctly expect non-partisan behavior from their representatives – have no preferences for either type of politician. Thus, suppose incumbents always choose $a_t^i = s_t$, irrespectively of their ideology or party affiliation i. Since both types of politicians implement the same Pareto efficient alternative in every period, voters hold no preference for the incumbent or the challenger and elect either with probability $1/2$.[15] Let $U(i, \mu_t)$ be voter's utility from electing an i-type candidate in period t along the equilibrium path.

[15] The important property of the voting behavior to support this equilibrium is that the prospect of re-election is independent of the implemented policy. Without the restriction that each candidate is elected with equal probability in case the voters are indifferent, there would be a continuum of efficient equilibria with $P^i(\mu_t)$ being constant in μ_t.

We have

$$U(L, \mu_t) = U(R, \mu_t) \quad \text{and} \quad P^i(\mu_t) = \frac{1}{2} \qquad \forall \mu_t, t, i.$$

The implementation of an efficient policy alternative – precisely because it is necessarily conditional on the current state – provides voters with additional information about s_t. Indeed, since the choice of $a_t = s_t$ perfectly reveals s_t, the only uncertainty about the underlying economy stems from the fact that the conditions may change from one period to the next according to (2.1). For any initial belief μ_0, beliefs in this equilibrium therefore evolve according to

$$\mu_{t+1}(a_t, \mu_t) = \begin{cases} \gamma & \text{if } a_t = l \\ 1 - \gamma & \text{if } a_t = r \end{cases} \qquad \forall \mu_t, t.$$

In what follows, we will for notational simplicity focus on left-wing politicians $i = L$, dropping the index i whenever possible. The argument for right-wing politicians $i = R$ is analogous. Recalling that $b_t \equiv b$ if $a_t = s_t$ the value function of an incumbent politician if he or she implements the efficient alternative is

$$V(s_t) = \frac{1}{2} \left\{ b + \phi + \beta \, \mathrm{E} \left[V(s_{t+1}) \right] \right\}.$$

Note that $V(s_t)$ is independent of μ_t, because given the electorate's voting rule any incumbent faces equal chances of being re-elected and defeated, respectively, regardless of beliefs. If the incumbent deviates by choosing $a_t \neq s_t$ in some t, the value function becomes

$$\hat{V}(s_t) = \frac{1}{2} \left\{ \pi b + \phi + \beta \mathrm{E} \left[V(s_{t+1}) \right] \right\},$$

which by inspection is strictly less than $V(s_t)$ for any $\pi < 1$. Hence, $a_t = s_t$ is indeed the utility-maximizing choice for incumbents in each period. We can thus conclude that non-partisan politics and an electoral rule that assigns equal election chances to incumbents and challengers in all periods form an equilibrium. In fact, it is the Markov perfect equilibrium with the highest payoff to the electorate,

$$U^{\max} = \sum_{t=0}^{\infty} \beta^t b = \frac{1}{1 - \beta} b.$$

Proposition 2.1. *[Non-Partisan Equilibrium] There always exists an equilibrium in which elected office holders act non-partisan and are re-elected with probability* $1/2$*. In this equilibrium, voters have full information about the prevailing state following the policy choice in each period, and receive the highest possible utility.*

Proof. In the text. ■

While the non-partisan equilibrium always exists and Pareto-dominates all other equilibria for the voters, it is not the only possible outcome. In the following sections, we will not only demonstrate that partisan politics can be supported in equilibrium as well, but also that non-partisan politics are fragile in the sense that they cannot survive if citizens' expectations about office holders' behavior are subject to (small) uncertainty.

2.3.2 The Partisan Equilibrium

We next study the possibility of a partisan equilibrium. Intuitively, suppose voters' expect office holders to play partisan and choose $a_t = i$ in every period, independent of the current state s_t. The key to observe is that voters are no longer indifferent across politicians with distinct ideologies. In particular, if a voter knew the state to be $s_t = l$, he or she would *strictly prefer* a type-L candidate to a type-R candidate, because only the former's partisan behavior coincides with the efficient policy choice in period t. A direct consequence of this strict preference ordering is that period-t incumbents now face a dilemma whenever their ideology does not match the state. A type-L office holder who selects the non-partisan choice of $a_t = r$ would reveal the state to be $s_t = r$, and would not be re-elected. Similarly, a type R-incumbent who implemented the efficient left-wing alternative $a_t = l$ because the state was $s_t = l$ would face certain defeat. A partisan choice of $a_t \equiv i \neq s_t$, on the other hand, will *conceal* the true state and thus may ensure – conditional on the observed success of the policy – re-election. It is then intuitive that this effect can induce partisan behavior provided politicians care sufficiently strong about their (re-)election prospects. The remainder of this section establishes this result formally.

To this end, consider a type-i candidate whose strategy is to choose the partisan policy whenever in office in period t. Given $\mu_0 \in [1-\gamma, \gamma]$, the voters' belief along the equilibrium path then evolves as follows

$$\mu^L_{t+1}(a_t = l, \mu_t) = \begin{cases} 1-\gamma + (2\gamma - 1)\frac{\mu_t}{\mu_t + (1-\mu_t)\pi} & \text{if policy } a_t = l \text{was a success} \\ 1-\gamma & \text{if policy } a_t = l \text{was a failure} \end{cases} \quad (2.6)$$

$$\mu^R_{t+1}(a_t = r, \mu_t) = \begin{cases} \gamma - (2\gamma - 1)\frac{1-\mu_t}{1-\mu_t + \mu_t\pi} & \text{if policy } a_t = r \text{was a success} \\ \gamma & \text{if policy } a_t = r \text{was a failure.} \end{cases}$$

Note that the office holders' policy choice reveals no new information about the current state on the equilibrium path since the implemented policy always corresponds to the politcians' affiliation. Formally, the beliefs satisfy the property $\mathrm{E}[\mu^L_{t+1} | a_t = l, \mu_t] = \mathrm{E}[\mu^R_{t+1} | a_t = r, \mu_t] = \gamma\mu_t + (1-\gamma)(1-\mu_t)$. Thus, the electorate only learns by observing whether the policy has been successful or not.

As usual, beliefs are not defined off the equilibrium path, i.e., when the electorate observes the non-partisan policy being implemented. Off equilibrium, we make the natural assumption that non-partisan politics are perfectly revealing

$$\mu^L_{t+1}(a_t = r) = 1-\gamma \quad \text{and} \quad \mu^R_{t+1}(a_t = l) = \gamma, \quad (2.7)$$

i.e., if the electorate unexpectedly observes a left-wing office holder to select $a_t = r$, it assumes that the non-partisan state $s_t = r$ must have occurred, and vice versa.[16]

Now suppose voters elect the left-wing (right-wing) candidate for beliefs $\mu_t > 1/2$ ($\mu_t < 1/2$) and give both candidates equal chances of winning for $\mu_t = 1/2$. The value function of the electorate is then

$$U(\mu_t) = \begin{cases} (\mu_t + (1-\mu_t)\pi)\left(b + \beta U(\mu^L_{t+1})\right) + (1-\mu_t)(1-\pi)\beta U(1-\gamma) & \mu_t \geq \frac{1}{2} \\ (1-\mu_t + \mu_t\pi)\left(b + \beta U(\mu^R_{t+1})\right) + \mu_t(1-\pi)\beta U(\gamma) & \mu_t < \frac{1}{2}. \end{cases} \quad (2.8)$$

Closer inspection of (2.8) reveals that $U(\mu_t)$ is increasing in μ_t for values $\mu_t > 1/2$ and

[16] After adapting the Cho&Kreps intuitive criterion to our dynamic framework, it is easy to verify that this out of equilibrium belief is the unique belief satisfying the corresponding concept of equilibrium dominance, on which the intuitive criterion is based.

decreasing in μ_t otherwise (at $\mu_t = 1/2$, the function assumes a minimum). Intuitively, more extreme beliefs increase the benefit of electing the appropriate politician. Related to this property is that voters would never want to 'experiment', i.e., elect a candidate who subsequently is *less* likely to implement the efficient policy in order to receive more precise information about the state.[17] Doing so would only increase the chances of a policy failure, in which case voters would be even more convinced that the elected candidate was not appropriate. Put differently, the electorate would dispose of a more accurate belief only if the implemented policy goes awry. In the unlikely case of success on the other hand, the resulting belief is less precise than the one that would have resulted from having the appropriate candidate successfully implement his partisan policy.

Turning now to candidates, we will without loss of generality again consider the behavior of left-wing candidates, omitting the index L whenever possible. Anticipating the voting behavior of the electorate, the equilibrium value of acting partisan for a left-wing candidate is

$$V(\mu_t, s_t) = \begin{cases} P(\mu_t)\left\{b + \phi + \beta \mathrm{E}\left[\gamma V(\mu_{t+1}, l) + (1-\gamma)V(\mu_{t+1}, r)\right]\right\} & \text{if } s_t = l \\ P(\mu_t)\left\{\pi b + \phi + \beta \mathrm{E}\left[(1-\gamma)V(\mu_{t+1}, l) + \gamma V(\mu_{t+1}, r)\right]\right\} & \text{if } s_t = r \end{cases}$$

where the expectation is taken over b_t (and, consequently, μ_{t+1}) given s_t, and

$$P(\mu_t) = \begin{cases} 1 & \text{if } \mu_t > \frac{1}{2} \\ \frac{1}{2} & \text{if } \mu_t = \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}. \tag{2.9}$$

A candidate who deviates by setting $a_t = r$ in period t, in contrast, would reveal the true state to be $s_t = r$. Voters' beliefs at the beginning of the next period are therefore $\mu_{t+1} < 1/2$, resulting in certain defeat and a utility normalized to zero. Hence, we can

[17]See Lemma A2.1 in Appendix A2.1, which formally establishes how $U(\mu_t)$ depends on μ_t and shows that experimentation does not improve voters' payoffs.

write the office holder's utility $\hat{V}(\mu_t, s_t)$ from such a deviation as

$$\hat{V}(\mu_t, s_t) = \begin{cases} P(\mu_t)\{\pi b + \phi\} & \text{if } s_t = l \\ P(\mu_t)\{b + \phi\} & \text{if } s_t = r. \end{cases}$$

Obviously, no rational incumbent would ever want to select an opponent's partisan policy in a state where in fact her own partisan policy is myopically optimal. Thus, the strategy $a_t = l$ is trivially utility maximizing in the 'partisan' state $s_t = l$. It remains to study when politicians are willing to sacrifice the utility from the Pareto-optimal choice of $a_t = r$ by choosing $a_t = l$ in state $s_t = r$. Comparing $V(\mu_t, r)$ with $\hat{V}(\mu_t, r)$, we see that the answer is yes if $V(\mu_t, r) \geq \hat{V}(\mu_t, r)$ or

$$\beta \mathrm{E}\left[\gamma V(\mu_{t+1}, l) + (1-\gamma) V(\mu_{t+1}, r)\right] \geq (1-\pi) b. \tag{2.10}$$

On the right-hand side of (2.10) are the short-term gains from deviating, as reflected in the additional expected benefit from the optimal non-partisan choice over the suboptimal partisan choice. The left-hand side captures the utility lost by facing certain defeat in this case; it is the future value from remaining in office, which naturally increases in the discount factor β and office rents ϕ (see below). But another, and perhaps less apparent, factor also plays a crucial role: by acting partisan, the candidates must also be able to *improve* their (re-)election chances by a sufficient margin. For the remainder of this section, we will therefore assume that the success probability π of a sub-optimally chosen partisan policy is small enough, such that an office holder who chooses the partisan policy has a chance of being re-elected for any belief $\mu \in [1-\gamma, \gamma]$. In other words, even for $\mu_t = 1-\gamma$, the electorate's updated belief satisfies $\frac{(1-\gamma)}{(1-\gamma)+\gamma\pi} > \frac{1}{2}$, which is equivalent to

Assumption 2.1.

$$\pi < \frac{1-\gamma}{\gamma}. \tag{A2.1}$$

Under Assumption 2.1, a success guarantees re-election (and failure results in sure defeat) irrespective of the state s_t or of the belief μ_t. In this case, $V(\mu_t, s_t)$ assumes a particularly simple form. It is constant (and equal to zero) for beliefs $\mu_t \in [1-\gamma, \frac{1}{2})$ where the candidate is not elected in equilibrium, takes on a single intermediate value for $\mu_t = 1/2$, and is constant again for all higher beliefs $\mu_t \in (\frac{1}{2}, \gamma]$, where the candidate is elected with probability one. Formally, $\forall \mu_t \in (\frac{1}{2}, \gamma]$ we have $P(\mu_t) = 1$ and $\mu_{t+1} > 1/2$ if the policy was successful and $\mu_{t+1} = 1-\gamma < \frac{1}{2}$ otherwise. $V(\mu_t, s_t) \equiv \bar{V}(s_t)$ for all values in this interval. Similarly, $\forall \mu_t, \in [1-\gamma, \frac{1}{2})$, $P(\mu_t) = 0$, implying $V(\mu_t, s_t) \equiv 0$. Selecting the non-partisan policy in state r then will not be optimal if

$$b + \phi \leq \pi b + \phi + \pi\beta[(1-\gamma)\bar{V}(l) + \gamma\bar{V}(r)]$$

or

Assumption 2.2.

$$(1-\pi)b \leq \pi\beta[(1-\gamma)\bar{V}(l) + \gamma\bar{V}(r)] \tag{A2.2}$$

where $\bar{V}(r)$ and $\bar{V}(l)$ can explicitly be computed to read

$$\begin{aligned} \bar{V}(r) &= \frac{b\pi(1+\beta(1-2\gamma\beta)) + (\pi\beta(1-\gamma) + 1 - \beta\gamma)\phi}{\pi\beta(\beta(2\gamma-1)-\gamma) + 1 - \beta\gamma} \\ \bar{V}(l) &= \frac{b(\pi\beta(1-2\gamma)+1) + (1-\beta(\pi\gamma+\gamma-1))\phi}{\pi\beta(\beta(2\gamma-1)-\gamma) + 1 - \beta\gamma}. \end{aligned} \tag{2.11}$$

We can conclude:

Proposition 2.2. *[Partisan Equilibrium] Under Assumptions 2.1 and 2.2, there exists an equilibrium in which elected office holders act partisan regardless of the state. In this equilibrium, politicians are re-elected with probability one if their implemented policy was a success and face certain defeat if it was a failure, and voters receive no information about the prevailing state from the choice of policy (other than ex post from its success or failure).*

Proof. In the text. ■

It is important to contrast the equilibrium behavior in Proposition 2.2 to the well-known danger of office-motivated representatives "pandering to public opinion". Harrington (1993) and Maskin and Tirole (2004) investigate this phenomenon, which turns the accountability role of elections on its head. The authors show that, because the electorate is unable to evaluate the official's actions directly, the desire to be (re-)elected may lead representatives to pursue the most popular, rather than the welfare maximizing, course of action. While similar in its consequences, the policy choice in a partisan equilibrium does not follow the most popular course of action. Instead, incumbents in our model stick to their once enacted policies so as not to reveal that "times have changed". Moreover, what is at the heart of the resulting policy bias is a perceived – as opposed to a real – non-congruency: ideology is a social perception not an innate characteristic of the candidates.

In particular, comparing Proposition 2.1 and 2.2, the blame for the policy bias can be squarely laid on the fact that voters *perceive* policies to be ideologically tinted and *expect* candidates to act partisan. If any one of these conditions is missing, i.e., policies are perceived to be ideologically neutral or candidates are expected to act non-partisan, even the most office-minded politician has no incentive to deviate from what is optimal for the electorate [Proposition 2.1]. Only if voters expect partisan politics in the future will they have an incentive to elect the candidates whose perceived position corresponds to what they think is in their best interest given their current information. And it is the voters' expectations, in turn, which induce candidates to actually act partisan in the first place. Put differently, voters and representatives are caught in an *ideology trap*: because voters expect the ideology of office holders to determine their political actions, an official's (re-)election chances will vary with his or her perceived ideology. In their desire to influence the outcome of the election, these expectations induce the officials to act partisan. Shifts from non-partisan politics to partisan politics confirm the electorate's assessed likelihood of the latter, cementing the polarization even further. Ideologues emerge who are *not* true believers. Instead, ideology is purely a social perception based on observable characteristics of candidates. Thus, issue bundling occurs not because preferences are bundled, but because voters' expectations tie candidates' policy intentions to their observed characteristics such as their party affiliation.

Similarly, the model also should be contrasted with the widely-used adverse selection approach of reputation in repeated games, initially formalized by Kreps, Milgrom, Roberts, and Wilson (1982) and Kreps and Wilson (1982). In these models, small amounts of imperfect information regarding their payoff can induce players to attempt to build a reputation for being of a certain type, as to trigger more favorable responses from others.[18] Translated into our framework, this approach would assume that politicians can be of two unobservable (payoff) types, a "partisan" type and a "non-partisan" type, where the latter is strictly preferable to the electorate. In such a world, candidates with partisan preferences would be tempted to implement an efficient policy so as to appear non-partisan. Obviously, one could not possibly explain ideologically tinted behavior with this line of argument. In contrast, there is no uncertainty about the candidates' type in our model. Thus, implementing efficient policies in the partisan equilibrium cannot serve as a signal for being an efficient type. Rather, the electorate is unsure about the current state of the world, and an incumbent who implements a non-partisan policy will at most signal that a certain state prevails, which in turn makes it desirable to out him from power.

There are two possible misgivings one could have against this line of reasoning. First, voters are strictly better off in the non-partisan equilibrium than in the partisan equilibrium, and thus there may *a priori* be little reason to expect partisan behavior to prevail. Second, non-partisan behavior is not observed on the equilibrium path in the partisan equilibrium: by assumption, if voters unexpectedly see candidates acting non-partisan, they infer that the state must be unfavorable to their ideological position. As we will see, both concerns are rooted in the simple nature of the model and can easily be addressed. We do so in Section 2.4 below, where we develop a) a straightforward refinement that selects the partisan equilibrium whenever it exists, and b) a natural extension of the model in which incumbents act non-partisan on the equilibrium path.[19]

The qualitative results of this section in no way depend on our assumption that there is no uncertainty in the voting behavior of the electorate, which makes competition between candidates especially fierce. In particular, a standard probabilistic voting model where candidates face uncertain electoral prospects and cater to the swing voter would

[18] In a recent application of this approach to a related question, Morris (2001) for example assumes that political advisers can be either good or bad. A priori, both types of adviser would like being perceived as good, which may prompt them to keep their advice "politically correct" (against better knowledge).

[19] In general, the model may have further equilibria. Assuming myopic voters and $\beta = 1$, however, it is possible to show that generically in any symmetric pure strategy Markov equilibrium both parties either always act partisan or their actions converge to efficient play.

yield similar conclusions.[20]

We close this section by studying the set of parameters that supports partisan behavior as an equilibrium phenomenon. First, note that Assumption 2.1 is satisfied for small values of either π or γ, or both. Ceteris paribus, a partisan equilibrium is thus more likely to exist if either i) the electorate is sufficiently uncertain about the underlying state or ii) the success and failure of policies is a sufficiently accurate signal of the state. Intuitively, these conditions ensure that challengers do not credibly deviate to non-partisan behavior (which in turn would make their election optimal for voters). If the state persists over long time horizons ($\gamma \to 1$) or if the signal of a policy's success or failure is very inaccurate ($\pi \to 1$), a challenger who unexpectedly (i.e., off the equilibrium path) won an election would have no incentive to act partisan because even if her partisan choice was successful, the electorate would not be sufficiently convinced of an underlying state change to re-elect her.

Second, to better understand the restrictions embodied in Assumption 2.2, we can substitute for $\bar{V}(r)$ and $\bar{V}(l)$ in condition (A2.2) using (2.11), which yields

$$\frac{b}{\phi + b} \leq \frac{\pi\beta\left(1 - \beta(2\gamma - 1)\right)}{(1 - \pi)\left(1 - \beta\gamma\right)} \tag{2.12}$$

Not surprisingly, partisan behavior is more likely to arise whenever politicians have a strong office holding motive: their rent from holding onto power ϕ, relative to the the payoff b they forgo by not choosing the correct policy must be sufficiently high. Moreover, the incumbent will be more inclined to play partisan for high values of π, i.e., whenever the efficiency cost of inappropriate policies is low because they are still likely to succeed (note the tension to Assumption 2.1 though, which requires π to be low enough for a successful partisan policy to be convincing). Less obviously, the left hand side of (2.12) decreases in γ. Intuitively, since the incumbent faces the trade-off between reelection and efficiency only if the state is unfavorable ($s_t = r$), a more persistent state lowers the chances that the partisan policy will become efficient in the near future. The prospect of repeatedly having to implement inefficient policies lowers the expected value from staying in the office when the state is more persistent. We can

[20] If $P(\mu_t)$ is an arbitrary increasing function of μ_t then the partisan equilibrium exists when the office motive ϕ is sufficiently high.

thus conclude:

Corollary 2.1. *The partisan equilibrium is more likely to exist whenever the office holding motive is strong (ϕ high), the environment is volatile (γ low) and whenever inappropriate policies are unlikely to fail but successful policies are still convincing (intermediate values of π).*

2.3.3 Properties of the Partisan Equilibrium

As explained above, the specific motivation for acting partisan given voters' expectations is one of "signal-jamming". An efficient policy choice conveys information about the state of the world, making it less likely that the incumbent office holder is re-elected if he is expected to act partisan in the future. To improve his chances of re-election, the incumbent thus "jams" the voters' inference problem by instead using the partisan policy, which is both inefficient and less responsive to current circumstances.

The latter fact is noteworthy, not only because it can explain the emergence of "ideologues" but also because, by definition, an ideologue's preferred policy choice does not vary with the underlying state. Thus the model can also provide a possible explanation for inefficient policy persistence: along the equilibrium path, there will not be a deviation from a given policy unless voters oust a politician from office. Moreover, the probability that the policy (ideology of the office holder) varies with the state and changes from one period to the next is smaller than in the non-partisan equilibrium.

Finally, despite the fact that incumbents who "stick to their political colors" and do not change policies enact inefficient policies, the political failure does not result in lower election chances. In fact, it is easy to show that – relative to the efficient equilibrium – incumbents enjoy an advantage in the partisan equilibrium: their chances of winning another term in office are strictly higher than even.[21]

These observations are summarized in

[21] One may object to this assertion that since voters are indifferent between candidates in the non-partisan equilibrium, any probability of re-election is consistent with equilibrium behavior (including perfect incumbency advantages with re-election probabilities equal to one). Note, however, that such outcomes would require voters to co-ordinate their voting strategies, an implausible scenario when the electorate is large.

Proposition 2.3. *In a partisan equilibrium:*

i) voters receive strictly less utility than in the non-partisan equilibrium [Policy Failure]

ii) incumbents' policies do not vary with the current state and in the long run policies are less likely to be changed than would be efficient [Policy Persistence], and

iii) the long run probability that an incumbent wins another term in office is strictly greater than one half [Incumbency Advantage].

Proof. See Appendix A2.1. ■

The implication of policy persistence is particularly interesting for two reasons. First, it shows that policies may be resilient not just because they are targeted and thus allow for the formation of powerful interest groups who subsequently lobby for their continued enactment as in Coate and Morris (1999). Persistence may also be a problem for non-targeted (valence) issues, simply because incumbent politicians may be reluctant to abandon their previously enacted policies so as to not openly admit that "times have changed". Second, this persistence gives rise to political failure. Rather than the result of a struggle between powerful interest groups and the public at large, the inefficient inertia in the political process is driven by the fact that, in a world on partisanship, office holders are reluctant to admit that new circumstances warrant a new policy and, therefore, new leaders in the eyes of the electorate.

Both policy persistence and incumbency advantage distinguish our model from other models of policy divergence (such as the citizen-candidate model) and can potentially be tested for empirically. While a full-fledged empirical analysis of these phenomena is beyond the scope of the present chapter, we confine ourselves to point out that these implications are consistent with empirical observations regarding democratic two-party systems. As stated in the Introduction, studies of voting behavior in the U.S. Congress in particular confirm our theoretical predictions of ideological positioning and polarization along party lines [McCarty, Poole, and Rosenthal (2006)]. Using data from roll call voting records, Poole (2007) presents a variety of evidence showing that, once elected, members adopt a consistent ideological position and maintain it over time. Moreover, in spite of (or perhaps even because of) their stubborn behavior, re-election rates for senators and House members are regularly above 80 percent.

In 2002, for instance, 398 House members ran for reelection, of which only 16 were defeated. In the Senate, a mere three out of 26 senators running for reelection lost.

2.4 Extensions

2.4.1 Candidate Behavior

As mentioned above, one possible objection to the partisan equilibrium is that it is Pareto dominated by the non-partisan equilibrium for the voters (though not for the politicians). Arguably, this could make sub-optimal partisan behavior less likely to be observed: if the electorate collectively benefits from expecting representatives to act in its best interest, then why should it expect otherwise? We will show in this section that there are compelling arguments in favor of the partisan equilibrium. Specifically, the non-partisan equilibrium is fragile (unstable) in the sense that it does not survive small perturbations in voters' expectations. Formally, suppose that the electorate expects the office holder to choose the partisan policy with some small probability $\epsilon > 0$.[22]

Proposition 2.4. *Suppose there is an arbitrarily small and i.i.d. probability $\epsilon > 0$ that office holders follow their ideology in each period and that Assumption 2.1 holds. If Assumption 2.2 holds with strict inequality, then the partisan equilibrium continues to exist and there is no equilibrium in which each candidate plays non-partisan (with probability $1 - \epsilon$) along the equilibrium path. Conversely, if Assumption 2.2 is violated, then there exists an equilibrium in which each candidate plays non-partisan (with probability $1 - \epsilon$) along the equilibrium path and there is no partisan equilibrium.*

Proof. See Appendix A2.1. ■

Proposition 2.4 shows that generically, a small amount of voter uncertainty regarding candidate behavior suffices to select the inefficient, partisan equilibrium whenever it

[22] One explanation for why voters could expect partisan behavior to arise with positive probability is party pressure [see Cukierman and Tommasi (1998)]. The possibility of a "partisan shock" could then formalized by a probability ϵ with which the office holder realizes an additional benefit B whenever he chooses the policy a corresponding to her ideology or party affiliation i, and assuming that the per-period payoff from a partisan choice is sufficient to compensate for the expected loss from not choosing the efficient alternative.

exists according to Proposition 2.2. Intuitively, non-partisan behavior is unstable because everyone is equally good as long as he or she is expected to act non-partisan. In such a situation even small amounts of uncertainty regarding candidates' subsequent behavior will make voters strictly prefer the candidate whose ideological position is more likely to succeed given their beliefs about the current state.[23]

While we use the result in Proposition 2.4 primarily to select among equilibria, the fragility of non-partisan equilibria has obvious implications concerning how shifts in voters' expectations translate into policy changes. Consider a situation where non-partisanship has historically prevailed along the equilibrium, so voters have no reason to suspect politicians to enact (inefficient) ideological policies. Yet, a relatively small change in the perception of voters concerning an increased likelihood of partisan behavior would be sufficient to trigger a major trend towards partisanship and polarization. On matters of foreign policy, for example, partisanship as measured by the lack of support for the President by members of the U.S. congress increased dramatically following the Vietnam war (an event that may well have changed peoples' expectations about partisan behavior).[24] Conversely, a seemingly extraneous act such as a public appeal for non-partisanship could revert voters' expectations, thus helping political actors to coordinate on the efficient equilibrium. For this reason, the result is also consistent with – and can possibly account for – occurrences of within-party polarization and convergence, such as the split between Southern and Northern Democrats during the Civil War area and its diminishing importance in the past decades.

2.4.2 Policy Prospects

In this section we allow voters to be uncertain as to the prospect of an inefficiently chosen policy. Apart from capturing reality, the extension serves two purposes. First, since candidates will prefer to implement efficient (non-partisan) policies whenever their partisan policy is unlikely to succeed, voters will observe non-partisan behavior

[23]For the same reason, even in an equilibrium where the incumbents implement non-partisan policies with probability $1 - \epsilon$, the voting behavior now depends on the voters' beliefs, in contrast to the equilibrium discussed in Section 2.3.1.

[24]Using data on foreign policy and defense roll-call votes in the U.S. House and Senate, Meernik (1993) documents that the Vietnam War had a significant impact on bipartisan presidential support: whereas substantial consensus existed prior to the War, is has become much more infrequent afterwards.

on the equilibrium path, eliminating out-of equilibrium beliefs. Second, the partisan equilibrium will exist for a wider range of parameters.

Specifically, assume that the probability of success of an inefficient policy choice π_t evolves stochastically over time in the following way: in each period t, it is either $\pi > 0$, as before, or zero. The latter case captures a situation where it is very important to pick the right policy: inefficient policy choices never succeed and, consequently, the electorate always learns when the wrong policy was implemented. To fix ideas, we will refer to such a period as a *crisis*. Let q be the probability of a normal period (with success probability π), so a crisis occurs with probability $1-q$, independent of the state $s_t \in \{r, l\}$. Candidates learn π_t at the beginning of each period, together with the state of the world. Voters do not observe π_t.[25] Since a crisis doesn't persist by assumption, voters' beliefs over π_t are the same each period, and we can w.l.o.g. condition the election probabilities exclusively on the belief over the state, as before.

Turning to equilibria, observe first that the non-partisan equilibrium still exists since deviating to a partisan policy is even less attractive in a crisis. As in the baseline model, though, a 'partisan' equilibrium where politicians act partisan in normal times and efficient in a crisis is also supported. In this equilibrium, voters again elect the left-wing (right-wing) candidate for beliefs $\mu_t > 1/2$ ($\mu_t < 1/2$) and give both candidates equal chances of winning for $\mu_t = 1/2$. To begin with, suppose the left-wing candidate has been elected in a crisis period and $s_t = r$. A partisan policy $a_t = l$ will surely fail, leading to a current payoff of ϕ and next period's belief of $\mu_{t+1} = 1-\gamma$. A non-partisan choice $a_t = r$ on the other hand will be successful, yielding a higher current payoff of $b+\phi$ with the same next period's belief $\mu_{t+1} = 1-\gamma$. Therefore, non-partisan politics are optimal in a crisis.

As before, a candidate is only willing to implement the partisan policy if this assures reelection in case of success; in particular this must be true if the electorate holds the worst possible beliefs, $\mu_t = 1-\gamma$. However, since the partisan policy is less often implemented than in the base model, observing a successful partisan policy now contains more information and therefore has a larger effect on the posterior belief. Specifically,

[25] The assumption that voters do not observe the success probability at all is made to simplify matters. Our qualitative argument remains valid as long as there is some residual uncertainty with regard to π_t.

Assumption 2.1 becomes

$$\pi < \frac{(1-\gamma)}{\gamma q}. \tag{2.13}$$

Next, let $V^c(\mu_t, s_t)$ denote the left-wing candidate's expected discounted value if state s_t occurs, the electorate has belief μ_t and he follows the equilibrium strategy for the rest of the game. We can adapt the condition (2.10) of the base model that supports partisan behavior in any *normal* period,

$$\beta E\left[\gamma V^c(\mu_{t+1}, l) + (1-\gamma)V^c(\mu_{t+1}, r)\right] \geq (1-\pi)b \tag{2.14}$$

where the value functions are slightly modified to account for the additional uncertainty induced by π_t:

$$V^c(\mu_t, s_t) = \begin{cases} P(\mu_t)\left\{b + \phi + \beta\left[\gamma V^c(\mu_{t+1}, l) + (1-\gamma)V^c(\mu_{t+1}, r)\right]\right\} & \text{if } s_t = l \\ P(\mu_t)\{q\pi b + (1-q)b + \phi + q\beta\pi\left[(1-\gamma)V^c(\mu_{t+1}, l) + \gamma V^c(\mu_{t+1}, r)\right]\} & \text{if } s_t = r \end{cases}$$

It is easy to show that $V^c(\mu_t, s_t) = 0$ for $\mu_t < 1/2$ and $V^c(\mu_t, s_t) \equiv \overline{V}^c(s_t)$ for $\mu_t > 1/2$, as in section 2.3.2.

Simple algebra shows that condition (2.14) is equivalent to condition (2.10) from section 2.3.2. To intuitively understand why condition (2.10) remains unchanged, assume for the moment that (2.10) is satisfied with equality. Then, the office holder is indifferent between implementing his inefficient partisan policy and the efficient one whenever $\pi_t = \pi$. In this case, the value of being in office in the non-partisan state and following the equilibrium strategy equals that of implementing the efficient policy after observing $\pi_t = 0$ (and not getting reelected afterwards): $\overline{V}^c(r) = \phi + b$. Since the value in state r equals that of the base model, and both the strategy and the payoff in state l remain as in section 2.3.2, we must also have $\overline{V}(l) = \overline{V}^c(l)$. Now suppose that condition (2.10) holds with strict inequality, which renders holding office more attractive. By the preceding paragraph, both in the base model and in this section, an incumbent would prefer to implement the partisan policy whenever $\pi_t = \pi$. Since incumbents implement the efficient policy if $\pi_t = 0$, the possibility of a crisis *ceteris paribus* decreases the value of office holders in the partisan equilibrium whenever it exists, i.e. $\overline{V}^c(l) \leq \overline{V}(l)$ and $\overline{V}^c(r) \leq \overline{V}(r)$.

Proposition 2.5. *Under condition* (2.13) *and Assumption 2.2, there exists an equilibrium in which elected office holders act partisan in normal times and efficient in times of crisis. In this equilibrium, politicians are re-elected with probability one if their implemented policy was a success and face certain defeat if it was a failure or they implemented the non-partisan policy.*

Proof. See Appendix A2.1. ■

In summary, we find that the possibility of a crisis renders the partisan equilibrium more plausible. Intuitively, if the electorate is uncertain about the prospects of inefficient policies, it expects the candidates sometimes to implement the non-partisan policy. If voters observe that a politician has abandoned his ideology, they know that he did so to avoid a certain political failure - as a result, they (correctly) do not interpret this behavior as a sign of honesty and therefore do not draw inferences regarding the politician's future strategy. Finally observe that the partisan equilibrium continuously converges to the equilibrium in the basic framework as $q \to 0$, thereby justifying the off-equilibrium beliefs of section 2.3.2: upon observing the non-partisan policy being implemented, the electorate assumes that the incumbent has been forced to abandon his ideology, simply because the conflicting evidence was too strong.

2.5 Discussion and Concluding Remarks

This chapter proposes a theory of ideology for public leaders. We have shown that there are circumstances under which elected officials may adopt ideologically opposed positions, resulting in inefficient partisan policies even in areas that are generally perceived to be non-partisan. In contrast to existing explanations of partisanship, equilibrium polarization can emerge in our model despite the fact that voters and their representatives are in complete agreement as to which is the optimal course of action. The problem the parties face can be viewed as an "ideology trap", which emerges because voters perceive alternative policy measures to be ideologically tinted, and expect candidates to remain 'true to their ideology' which itself is a social perception grounded in observable characteristics (such as their gender, their party affiliation, or their position on a different policy issue).

Thus, the model can explain policy bias and divergence from the fact that voters *perceive* policies to be ideologically tinted and *expect* candidates to act partisan. Moreover, such partisan politics are persistent in the sense that equilibrium polices are less volatile and less responsive to changes in the underlying state than efficient policies. Importantly, the inertia is not driven by a fear of appearing incompetent. Rather, in a partisan world, leaders are reluctant to abandon previously enacted policies and admit that 'times have changed' because new circumstances will warrant a new policy and, therefore, new leaders in the eyes of the electorate.[26]

The key insight from our analysis provides a plausible explanation for a range of empirical regularities that, collectively, the previous literature on polarization cannot account for. In particular, the theory shows why ideology plays a role on matters that should be non-partisan (e.g., national security), why differences in observed characteristics such as party affiliation, gender, or electoral district can lead to differences in the political platforms of candidates that otherwise share similar policy preferences, why these differences can lead to issue bundling, and why bad, ideology driven policies can

[26]Using the US relations to Iraq as an example, take George W. Bush's reluctance to admit that his strategy in Iraq failed. According to our model, it is not the gain from appearing competent (or the loss from appearing incompetent) that causes the political failure. Instead, admitting mistakes would imply that the Democrats' strategy to deal with the situation in Iraq was preferable, which in turn implies that a Democrat could do better when in office.

persist. The model is also consistent with the observation that the ongoing polarization in the U.S. (which is largely driven by increased extremism within the Republican Party) is not mirrored by an equal shift in the American public.

Finally, we believe that our model could fruitfully be applied to other settings where leadership and ideology are tied together. While we have cast the discussion within the framework of policy formation in a representative democracy for obvious reasons, it is important to note that our basic line of argumentation is valid in a broader context: as long as a leader needs supporters to stay in power and is challenged in his leadership (implicitly and explicitly) on occasion, he will have an incentive to live up to his supporters expectations. If those expectations are ideologically biased, then ideologues will emerge irrespective of whether the context is one of political, religion, or ethnic affiliation.

A2.1 Appendix

The following lemma establishes that there is no "experimenting" in equilibrium

Lemma A2.1. *Suppose that both candidates implement their partisan policy in each period and that $\beta < 1$. Then the electorate's value function $U(\cdot)$ is unique and*

i) is axially symmetric around 0.5, i.e $U(\mu_t) = U(1-\mu_t)$ for $\mu_t \in [1-\gamma, \gamma]$,

ii) satisfies $U'(\mu_t) \leq -(1-\gamma)(1-\pi)^2 b$ for $\mu_t < 0.5$ and $U'(\mu_t) \geq (1-\gamma)(1-\pi)^2 b$ for $\mu_t > 0.5$,

iii) satisfies $b + \beta U(\mu_t) - \beta U(1-\gamma) \geq (1-\gamma)(1-\pi) b \quad \forall \mu_t \in [1-\gamma, \gamma]$.

The electorate's optimal voting strategy is identical to that of a myopic electorate.

Proof. Let the function $\varphi^L(\mu_t) \equiv 1-\gamma+(2\gamma-1)\frac{\mu_t}{\mu_t+(1-\mu_t)\pi}$ map the the belief μ_t in period t into the belief that results in $t+1$ when incumbent L successfully implements the policy l. Define $\varphi^R(\mu_t) \equiv \gamma-(2\gamma-1)\frac{1-\mu_t}{1-\mu_t+\mu_t\pi}$ similarly for incumbent R.

Step 1: We prove uniqueness and properties i)-iii) by use of the Contraction Mapping Theorem: Define the functional operator $T : \mathcal{U} \mapsto \mathcal{U}$ that maps the space of bounded continuous functions $\mathcal{U}$ defined on $[1-\gamma, \gamma]$ with range $\mathbb{R}^+$into itself as follows:

$$TU(\mu) = \begin{cases} (\mu+(1-\mu)\pi)\left(b+\beta U(\varphi^L(\mu)\right)+(1-\mu)(1-\pi)\beta U(1-\gamma) & \mu \geq 0.5 \\ (1-\mu+\mu\pi)\left(b+\beta U(\varphi^R(\mu)\right)+\mu(1-\pi)\beta U(\gamma) & \mu < 0.5 \end{cases}$$

Since T is a contraction, there exists a unique electorate's value function $U(\cdot)$.[27] We will prove properties i), ii) and iii) of U by use of Corollary 1 of Stokey and Lucas (1989, Theorem 3.2). We have to show that if U satisfies these properties, then TU also satisfies them. Suppose that U satisfies properties i), ii) and iii).

[27] It can be easily verified that this operator is a *contraction* since it satisfies the Blackwell's sufficient conditions of *discounting* and *monotonicity* according to Stokey and Lucas (1989, Theorem 3.3). As $\mathcal{U}$ together with the *sup*-Norm is a complete metric space, the contraction mapping Theorem Stokey and Lucas (1989, Theorem 3.2) applies.

i) Since $\varphi^L(0.5+x) = 1-\varphi^R(0.5-x)$, TU also satisfies $TU(0.5-x) = TU(0.5+x)$ for $x \in [0, \gamma - 0.5]$.

ii) For $\mu_t > 0.5$,

$$\begin{aligned} TU'(\mu_t) &= (1-\pi)\left(b+\beta U(\varphi^L(\mu_t) - \beta U(1-\gamma)\right) + (\mu_t + (1-\mu_t)\pi)\,\beta U'(\varphi^L(\mu_t))\varphi^{L'}(\mu_t) \\ &\geq (1-\gamma)\,(1-\pi)^2\, b \end{aligned}$$

where the inequality is because of $b + \beta U(\mu_t) - \beta U(1-\gamma) \geq (1-\gamma)\,(1-\pi)\,b$ by property iii) and because the second term is non negative by property ii). For $\mu_t < 0.5$, an analogous argument applies.

iii) For $\mu_t > 0.5$ we have $TU(\mu_t) = (\mu_t + (1-\mu_t)\pi)\left(b + \beta U(\varphi^L(\mu_t)) - \beta U(1-\gamma)\right) + \beta U(1-\gamma)$ which implies

$$\begin{aligned} b + \beta TU(\mu_t) - \beta TU(1-\gamma) &= b - (\gamma + (1-\gamma)\pi)\left(b + \beta U(\varphi^L(\gamma)) - \beta U(1-\gamma)\right) \\ &\quad + (\mu_t + (1-\mu_t)\pi)\left(b + \beta U(\varphi^L(\mu_t)) - \beta U(1-\gamma)\right) \\ &\geq b - (\gamma + (1-\gamma)\pi)\left(b + \beta U(\varphi^L(\gamma)) - \beta U(1-\gamma)\right) \\ &\geq (1-\gamma)\,(1-\pi)\, b \end{aligned}$$

where we used property i) repeatedly. The first inequality is due to property iii) and the last one due to property ii).

Step 2: Now we show that it is indeed optimal to vote for the left party if $\mu_t > 0.5$ (an analogous argument holds for $\mu_t < 0.5$). Deviating once and electing the right party yields $\hat{U}(\mu_t) = (1-\mu_t+\mu_t\pi)\left(b + \beta U(\varphi^R(\mu_t))\right) + \mu_t(1-\pi)\beta U(\gamma)$. Hence

$$\begin{aligned} U(\mu_t) - \hat{U}(\mu_t) &= (2\mu - 1)(1-\pi)\left(b + \beta U(\varphi^L(\mu_t)) - \beta U(\gamma)\right) \\ &\quad + (1-\mu_t+\mu_t\pi)\left(\beta U(\varphi^L(\mu_t)) - \beta U(\varphi^R(\mu_t))\right) \geq 0 \end{aligned}$$

where the inequality follows because the first term is positive due to property iii) and the second is positive due to $\varphi^L(\mu_t) - 0.5 > |0.5 - \varphi^R(\mu_t)|$ and property ii). To see that

$\varphi^L(\mu_t) - 0.5 > 0.5 - \varphi^R(\mu_t)$ when $\varphi^R(\mu_t) < 0.5$, inserting the formulas from above and rearranging yields

$$\frac{1}{1+(1-\mu_t)\pi\mu_t^{-1}} > \frac{1}{1+(1-\mu_t)^{-1}\pi\mu_t}$$

which is true for $\mu_t > 0.5$. ■

Proof of Proposition 2.3

Part i) is trivial. To show part ii), define the random variable $\tilde{s}_t \in \{m, n\}$ whose two realizations are "match" $\tilde{s}_t = m$ when equilibrium play prescribes $a_t = s_t$ in a given period and "non-match" n whenever $a_t \neq s_t$. In the partisan equilibrium, the transition probabilities between these "states" are:

$$T = \begin{pmatrix} t_{mm} & t_{mn} \\ t_{nm} & t_{nn} \end{pmatrix} = \begin{pmatrix} \gamma & 1-\gamma \\ (1-\pi)\gamma + \pi(1-\gamma) & (1-\pi)(1-\gamma) + \pi\gamma \end{pmatrix}$$

where the element t_{ij} of the transition matrix T denotes the transition probability from state i to state j. In the partisan equilibrium, a change in the implemented policy (i.e. $a_t \neq a_{t+1}$) only occurs if the implemented policy in period t was $a_t \neq s_t$ and failed. Hence the probability of a policy change between period t and $t+1$ is $Pr(\tilde{s}_t = n)(1-\pi)$. In the efficient equilibrium a policy change occurs whenever the true state changes, i.e. with probability $1-\gamma$. By definition, the partisan equilibrium involves more persistence in a given period t whenever the probability of a change in policies between period t and $t+1$ is lower than the probability of change in the efficient equilibrium which is $1-\gamma$. This condition is satisfied whenever $Pr(\tilde{s}_t = n)\,(1-\pi) \leq 1-\gamma$.

We proceed to show that for any initial belief and state, the long run probability of having a non-match is small enough to satisfy this condition. The (generically unique) stationary stationary distribution corresponds to the eigenvector which is associated to the unit eigenvalue of T'. It is $\bar{f}' = \left(\frac{-2\pi\gamma+\gamma+\pi}{1-2\gamma\pi+\pi}, \frac{1-\gamma}{1-2\gamma\pi+\pi}\right)$, where the first (second) element denotes the stationary probability that a match (non-match) occurs. The long run probability that a non-match occurs is thus $\lim_{t\to\infty} Pr(\tilde{s}_t = n) = \frac{1-\gamma}{1-2\gamma\pi+\pi}$. Due to $\gamma < 1$, we have

$$(1-\pi)\lim_{t\to\infty} Pr(\tilde{s}_t = n) = \frac{(1-\pi)(1-\gamma)}{1-(2\gamma-1)\pi} < (1-\gamma)$$

which completes the proof.

To show part c), recall that in the partisan equilibrium, an incumbent is not re-elected only in the event of a political failure. From the proof of part b), this occurs with probability $Pr(\tilde{s}_t = n)(1-\pi)$, which is in the long run equal to

$$(1-\pi)\lim_{t\to\infty} Pr(\tilde{s}_t = n) = \frac{(1-\pi)(1-\gamma)}{1-2\gamma\pi+\pi} < (1-\gamma) < \frac{1}{2}$$

where the last inequality follows from $\gamma > \frac{1}{2}$.

Proof of Proposition 2.4

Since ϵ restricts the *minimum* probability for implementing the partisan policy, for all $\epsilon \in (0,1)$, strategies and re-election probabilities *in the partisan equilibrium* are unchanged. Moreover, neither voters' nor office holders' payoffs are affected. Thus, partisan behavior continues to be an equilibrium under Assumptions 2.1 and 2.2.

Turning to the *most efficient equilibrium* (or ϵ-efficient equilibrium, indicated by the superscript ϵE), recall that voters' optimally vote as if they were myopic by Lemma A2.1. Hence for any $\epsilon > 0$ the reelection probabilities are now

$$P^{\epsilon E}(\mu) = \begin{cases} 1 & \text{if } \mu > 0.5 \\ 0.5 & \text{if } \mu = 0.5 \\ 0 & \text{else} \end{cases}$$

and equal those of the partisan equilibrium. The evolution of beliefs in the non-partisan equilibrium is

$$\mu^L_{t+1}(a_t, \mu_t) = \begin{cases} 1-\gamma + (2\gamma-1)\frac{\mu_t}{\mu_t + (1-\mu_t)\epsilon\pi} \equiv \varphi^{L,\epsilon E}(\mu_t) & \text{if } a_t = l \text{was a success} \\ 1-\gamma & \text{if } a_t = l \text{failed or } a_t = r \end{cases}.$$

The value for a left wing politician in the ϵ-efficient equilibrium is $V^{\epsilon E}(\mu_t) = 0$ for $\mu_t < 0.5$ and

$$\bar{V}^{\epsilon E}(s) \equiv \begin{cases} b + \phi + \beta\left[\gamma \bar{V}^{\epsilon E}(l) + (1-\gamma)\bar{V}^{\epsilon E}(r)\right] & \text{if } s = l \\ (1-\epsilon)b + \phi + \epsilon\pi\left[b + \beta\left(\gamma\bar{V}^{\epsilon E}(r) + (1-\gamma)\bar{V}^{\epsilon E}(l)\right)\right] & \text{if } s = r, \end{cases}$$

for $\mu_t > 0.5$ since L-type incumbents are not re-elected following the efficient choice of $a_t = r$ in state $s_t = r$.

Now suppose that the partisan equilibrium exists. Then, generically, (2.10) is satisfied with strict inequality,

$$(1-\pi)b < \pi\beta[(1-\gamma)\bar{V}^P(l) + \gamma\bar{V}^P(r)], \tag{2.15}$$

where $\bar{V}^P(l) > 0$ and $\bar{V}^P(r) > 0$ denotes the values in the partisan equilibrium [Proposition 2.2]. Because the reelection probabilities are the same in the ϵ-efficient equilibrium as in the partisan equilibrium, a repeated deviation by playing $a_t = l$ in states $s_t = r$ guarantees an expected payoff of $\bar{V}^P(s)$. We want to show that whenever (2.15) holds, then $\bar{V}^P(s) > \bar{V}^{\epsilon E}(s)$, i.e. a repeated deviation is profitable. We use the same contraction argument as in Lemma A2.1 of Appendix A2.1. According to this reasoning, it suffices to show that if $\bar{V}^P(s) > \bar{V}^{\epsilon E}(s)$, $s \in \{l, r\}$ then also

$$\bar{V}^P(r) > (1-\epsilon)b + \phi + \epsilon\pi\left[b + \beta\left(\gamma\bar{V}^{\epsilon E}(r) + (1-\gamma)\bar{V}^{\epsilon E}(l)\right)\right].$$

To see that this inequality is indeed satisfied, note that

$$\begin{aligned}
&\phi + \pi \left[b + \beta\left(\gamma \bar{V}^P(r) + (1-\gamma)\bar{V}^P(l)\right)\right] \\
&\quad > \quad (1-\epsilon)(b+\phi) + \epsilon\left[\phi + \pi\left[b + \beta\left(\gamma \bar{V}^P(r) + (1-\gamma)\bar{V}^P(l)\right)\right]\right] \\
&\quad > \quad (1-\epsilon)(b+\phi) + \epsilon\left[\phi + \pi\left[b + \beta\left(\gamma \bar{V}^{\epsilon E}(r) + (1-\gamma)\bar{V}^{\epsilon E}(l)\right)\right]\right]
\end{aligned}$$

where the first inequality comes from (2.15) and the second from the hypothesis $\bar{V}^P(s) > \bar{V}^{\epsilon E}(s)$.

Next, we show that whenever the parameters b, ϕ, β, π are such that there is no partisan equilibrium, then an ϵ-efficient equilibrium exists. We prove this by showing the converse, i.e. whenever there is no ϵ-efficient equilibrium, then there exists the partisan equilibrium. Whenever an ϵ-efficient equilibrium cannot be enforced, then by the one step deviation principle and the fact that enforceability in state r implies enforceability in state l, a single deviation for $\mu_t > 0.5$ and in state r must be profitable:

$$(1-\pi)b < \pi\left[b + \beta\left(\gamma \bar{V}^{\epsilon E}(r) + (1-\gamma)\bar{V}^{\epsilon E}(l)\right)\right] \tag{2.16}$$

We have to show that (2.16) implies that the partisan equilibrium can be enforced, i.e. that (2.15) holds (which implies that the second enforcement condition for state l is also satisfied). The same technique as above yields that (2.16) implies $\bar{V}^P(s) > \bar{V}^{\epsilon E}(s)$, $s \in \{l, r\}$. This together with (2.16) yields (2.15).

Proof of Proposition 2.5

Note that condition (2.14) is equivalent to $\overline{V}^R(r) \geq b + \phi$ where we use the same notation as in the base model, i.e. $\overline{V}^R(s_t) \equiv V^R(\mu_t, s_t)$ for $\mu_t > 0.5$. We have to show that this condition is satisfied if Assumption 2.2 holds.

At the same time, we show that Assumption 2.2 also implies $\overline{V}^R(l) \geq \frac{(1+\beta(1-\gamma))(b+\phi)}{1-\gamma\beta}$. Applying a contraction argument similar to Lemma A2.1, we have to show that whenever $\overline{V}^R(r) \geq b + \phi$, $\overline{V}^R(l) \geq \frac{(1+\beta(1-\gamma))(b+\phi)}{1-\gamma\beta}$ and Assumption 2.2, then the following two inequalities are satisfied:

$$b+\phi+\beta\left[\gamma\overline{V}^{R}(l)+(1-\gamma)\overline{V}^{R}(r)\right]\geq\frac{(1+\beta(1-\gamma))(b+\phi)}{1-\gamma\beta}$$
$$\pi b+\phi+q\beta\pi\left[(1-\gamma)\overline{V}^{R}(l)+\gamma\overline{V}^{R}(r)\right]+(1-q)(1-\pi)b\geq b+\phi$$

To see that the first inequality is true note that our hypothesis implies:

$$\begin{aligned} b+\phi+\beta\left[\gamma\overline{V}^{R}(l)+(1-\gamma)\overline{V}^{R}(r)\right] &\geq b+\phi+\beta\left[\gamma\frac{(1+\beta(1-\gamma))(b+\phi)}{1-\gamma\beta}+(1-\gamma)(b+\phi)\right] \\ &= \frac{(1+\beta(1-\gamma))(b+\phi)}{1-\gamma\beta} \end{aligned}$$

The second inequality is equivalent to $\pi b+\phi+\beta\pi\left[(1-\gamma)\overline{V}^{R}(l)+\gamma\overline{V}^{R}(r)\right]\geq b+\phi$. By our hypothesis, $\pi b+\phi+\beta\pi\left[(1-\gamma)\overline{V}^{R}(l)+\gamma\overline{V}^{R}(r)\right]\geq\pi b+\phi+\beta\pi\left[(1-\gamma)\frac{(1+\beta(1-\gamma))(b+\phi)}{1-\gamma\beta}+\gamma(b+\phi)\right]$. Simplifying Assumption 2.2 yields $\pi\beta\left[(1-\gamma)\frac{(1+\beta(1-\gamma))(b+\phi)}{1-\gamma\beta}+\gamma(b+\phi)\right]\geq(1-\pi)b$. Putting both observations together confirms the second inequality.

Chapter 3

Contracting with Specialized Agents

3.1 Introduction

Specialization and its positive effects on productivity is one of the main reasons of today's prosperity. As the variety of special products and services gets larger, it becomes more and more difficult for a customer to figure out which best fits his needs. In principle, specialized suppliers have a lot of expertise about their own and their competitors' products and therefore could help the customer in making the right choice. More generally, Darby and Karni (1973) speak of *credence goods* whenever an expert seller knows better than the customer which type of good or service suits the buyer's needs.[1] Credence goods comprise for instance medical treatments, repair services or the provision of complex goods. They enjoy increasing interest in the economic literature,[2] which emphasizes in particular that the opportunity to earn profits by selling their own service may induce sellers not to advise truthfully.

This chapter investigates optimal procurement schemes which account for the sellers' incentives to exploit the informational asymmetries in credence good markets. In contrast to the literature on credence goods, that has concentrated on goods where the

[1] Note that in contrast to experience goods, it is prohibitively costly or impossible for customers to evaluate credence qualities even after having purchased a credence good. See e.g. Darby and Karni (1973).

[2] See e.g. Dulleck and Kerschbamer (2006) for a survey.

seller typically sets the price, we focus on situations where the *buyer* can design and commit to a procurement mechanism. This chapter builds on the observation that the opportunity to earn profits by selling their own good may induce sellers to exaggerate its adequacy. Therefore, we are interested in how the buyer has to optimally design the procurement mechanism to extract the suitability information nevertheless.

A historic example serves to illustrate the issue.[3] In 1968, Atlantic Richfield (ARCO) successfully drilled for oil in the Prudhoe Bay area, Alaska. The problem soon became how to ship the product to U.S. and global markets. Several solutions were offered. For example, Boeing proposed tanker aircrafts, General Dynamics offered a line of tanker submarines for travel beneath the Arctic ice cap, and another group proposed extending the Alaska Railroad to Prudhoe Bay. Finally, the Alyeska Pipeline Service Company was instructed to build the famous Trans-Alaska Pipeline System (TAPS), which still is in use today.

This demonstrates that a single problem may be frequently solved by help of various alternatives which may differ considerably.[4] Sellers are often specialized only in a subset of these alternatives. In our example, it is conceivable that potential suppliers had already collected expertise with the products they offered, since these were also used for other oil fields. On the other side, not knowing the exact properties of the proposed alternatives made it difficult for ARCO to judge which of the products would suit the geographical challenges best. In order to choose the best alternative, ARCO had to extract this information from the potential sellers. Clearly, the importance of this infrastructure merits to design a sophisticated procurement scheme and to commit to its execution.

In our model, a principal has to decide from which of two specialized agents to buy a good or service. Each seller offers a different version of the good. Only one of the alternatives is well suited and generates a higher payoff to the principal than the remaining

[3] See Naske and Slotnick (1987) for further details.

[4] As another example, consider a house owner who seeks to install either a gas, oil, wood or electricity heating. Each heating has specific merits and drawbacks. For instance, electricity heatings are cheap to install but cause high electricity costs, while oil is presently cheaper but requires expensive ovens. Hence in terms of overall costs, electricity may be best suited for well isolated buildings while this might not be true for ancient buildings with high energy wastage. While expert sellers presumably know for which buildings their heating is especially well suited, a house owner typically does not know which characteristics of his house speak in favor of one or the other type of heating.

one. Each seller receives a private signal about the suitability of her alternative and has private information concerning the cost of her good, which is either high or low. Besides, each seller is protected by limited liability. The principal seeks to design a procurement scheme that maximizes his expected payoff and induces the sellers to reveal their two-dimensional private information truthfully.

Theoretically, contracts which condition on the customer's payoff could induce the sellers to give proper advice. However, there are practical reasons why the final payoff is often not contractible. For example, additional unobservable stochastic factors or hidden actions of the buyer may influence the value of the purchased good.[5] If the final payoff is realized long after the trade occurs it may also be difficult to write contracts depending on it. In addition, Pesendorfer and Wolinsky (2003) point out that the customer's payoff may be hard to verify. For these reasons, we assume that no contracts can be written on the principal's final payoff.

To isolate the effect of the sellers' private suitability information, we first consider a benchmark case where the principal, too, observes both sellers' signals. Endowed with this knowledge, it suffices to ensure that the sellers reveal their private cost information. Since the principal generally buys more often from sellers with a good signal than from those with a bad one, by standard results on adverse selection, higher rents have to be conceded to sellers with a low cost and a good signal.

Suppose now the principal does not know the sellers' signals, but still applies this benchmark scheme. The better prospects to sell their good would induce low cost sellers with a *bad* signal to exaggerate the suitability of their alternative. This suggests that the buyer needs to remunerate sellers for revealing a bad signal.

When the principal lacks the information on suitability, the principal faces a trade-off between maximizing the chances of buying the adequate good and economizing on rents. Intuitively, if the principal puts more emphasis on a good match, and therefore buys more often from sellers with a good signal, low cost sellers with a bad signal could generate higher rents by exaggerating their signal. Thus a higher remuneration

[5] In our introductory example, each of the alternatives to connect the oil field bears some risk of failure which depends on its adequacy. The event of a breakdown is then only a noisy signal of the product's suitability.

for them is required to assure truth-telling. Because of this trade-off, the optimal procurement scheme depends on the expected value of the sellers' suitability information as compared to the magnitude of cost uncertainty.

When choosing the superior alternative is of little importance, it may be optimal to commit to trading inefficiently often with agents who report a bad match. This makes it less tempting to exaggerate the suitability signal and therefore reduces the payments required to assure truth-telling. In this case, it may be even optimal to commit to trading sometimes with an agent whose product is as expensive as the competitor's but less likely to be suited. After reinterpreting the transaction probabilities as the proportion that each seller provides in a consortium of multiple sellers, this result may explain why sometimes inferior sellers are invited to participate in tendered projects. For example, we sometimes observe that in complex defense projects more sellers participate than would be necessary.

The optimal scheme vastly differs if the superior alternative is sufficiently important. Then the principal optimally gives priority to the adequacy of the good over the costs. However, this requires conceding extremely high rents to the sellers: Buying mainly from agents with a good signal requires to highly reward low cost sellers for reporting a bad signal. In addition, the principal rarely buys from these sellers. This tempts sellers with a bad signal to understate their costs in order to obtain the payments designed for the low cost sellers. We show that because of this problem, rents have to be conceded to all type of sellers. In order to implement a scheme where priority is given to the adequacy signal, the principal needs to exploit the correlated signal structure and to reward sellers whenever they agree on the adequate good.[6] This result, which relies on the correlation of the sellers' signals, seems remarkable, since in adverse selection problems the ex-ante participation constraint is usually binding at least for some type.[7]

A further result is that private information on adequacy makes it more likely that the buyer does not buy from high cost sellers at all. The rationale is similar as before: When the principal never trades with high cost sellers, then no concessions have to be made to extract the private cost information. But when the sellers have no chance of earning

[6]The sellers' limited liability prevents the principal from obtaining the information costlessly à la Cremer and McLean (1988).

[7]See Rochet and Stole (2003).

rents when being hired, then the incentives to exaggerate the suitability signal vanish. Since trading with high cost sellers is generally more costly than in the benchmark case mentioned before, it is optimal for a larger set of parameters to commit to barring high cost sellers.

Our results may be fruitfully applied in situations where the principal is endowed with commitment power. First, as already pointed out, we believe that in procurement decisions of valuable goods, buyers may design and credibly commit to sophisticated procurement schemes. Moreover, buyers may hire firms that are specialized on designing and enforcing optimal procurement schemes.[8] A second set of applications is the design of laws and provisions that apply for credence good transactions. For example, in the medical sector the process of identifying the optimal treatment is heavily regulated. Our results can thus be applied for specifying the optimal remunerations and the choice of the doctor. A further application is the design of rules for government procurement.[9]

Related Literature

Our theory brings together the literature on *credence goods* and on *optimal procurement* procedures.

Regarding the literature on *credence goods*, several authors have recently studied *vertically* differentiated markets:[10] Pitchik and Schotter (1987), Alger and Salanie (2004) and Wolinsky (1993, 1995) assume that a customer either needs a cheap or a costly treatment to solve a problem and only the expert seller can observe which one is needed. Emons (1997, 2001) investigate the role of capacity constraints on the incentives to give honest advice. As in this chapter, Pesendorfer and Wolinsky (2003) analyze a model with *horizontally* differentiated treatments and there is exactly one proper treatment which yields an extra payoff to the customer. However, they assume that finding out

[8]For example, the consultancy TWS-Partners (http://www.tws-system.com/index.html) is specialized in these services.

[9]For example, at the moment the question of how local government administrations should award contracts to build up network infrastructure to telecommunication sellers after having been advised by the these firms.

[10] For a more detailed survey of inefficiencies that arise in credence goods markets, see Dulleck and Kerschbamer (2006).

the customer's needs is costly and focus on the experts' incentives to actually exert investigative effort. Bolton, Freixas, and Shapiro (2007) assume that sellers of financial products have superior information about which of two horizontally differentiated products suits the customer's needs. Unlike our model, sellers suffer a reputational loss for selling a wrong product. While in our model each seller offers only one alternative, Bolton, Freixas, and Shapiro (2007) determine the number of products offered by each seller endogenously.

Our main contribution to the credence goods literature lies in allowing buyers to design more sophisticated procurement schemes. All the papers cited so far suppose that the *sellers* set prices and that buyers have little or no commitment power. While these assumptions seem appropriate for rather simple services, we are interested in complex and valuable goods that justify elaborate procurement methods. We also suppose that only a small number of sellers is capable to deliver these complex services.

The literature on *optimal procurement* is inter alia concerned with the question of how to take factors other than price into account in the procurement process. A seminal contribution that also analyzes a pure adverse selection problem is Che (1993), who investigates optimal auctions when sellers have private information regarding their costs to produce quality. He assumes that the quality level is verifiable, so that the allocation rule is contingent on the *actual* quality level provided by the winner. In contrast, Manelli and Vincent (1995) consider optimal procurement schemes when sellers have private information regarding their product's quality which is *not* verifiable. Asker and Cantillon (2010) analyze procurement mechanisms when quality is contractible, but when sellers have two-dimensional private information regarding their fixed costs and their marginal costs for quality. None of these papers investigates the trade-offs that arise when sellers have private cost information *and* non-verifiable private information regarding the suitability.[11]

This chapter also bears similarity to the literature on delegated expertise, where better informed agents have to be induced to gather and to honestly report the desired information. In Gromb and Martimort (2007), a principal faces two incompletely in-

[11] Jehiel, Moldovanu, and Stacchetti (1996) investigate the design of auctions when the winner inflicts a privately known externality on the remaining bidders. In our setup, the winning bidder inflicts a privately known externality on the principal who designs the auction.

formed agents. As in this paper, Gromb and Martimort (2007) use a mechanism design approach and allow payments that condition on both experts' reports. Similarly, in Dessein, Garicano, and Gertner (2007) local business managers have to be induced to combine their private information with that of a functional manager so that the right amount of standardization is implemented. In contrast to our model, both papers combine a one-dimensional adverse selection problem of revealing private information with one of moral hazard to induce agents to exert effort.

The methods employed to derive the optimal scheme draw on a further strand of literature. In our model sellers have two-dimensional private information (on costs and the suitability) which requires methods developed in the literature on multidimensional screening [e.g. Rochet and Stole (2003)] and multidimensional auctions [Armstrong (2000)]. While Armstrong (2000) allows the dimensions of an agent's type to be correlated, in our setup the suitability *across* agents is correlated.

The rest of the chapter is organized as follows. We introduce the model and formally define procurement schemes in Section 3.2. Section 3.3 derives the optimal mechanism when the principal knows the sellers' signals. Section 3.4 characterizes the distortions that are evoked by the optimal schemes of our main setup. In Section 3.5 we discuss which assumptions drive our results before Section 3.6 concludes.

3.2 The Model

A risk neutral principal (or buyer, "he") needs to procure one unit of an indivisible good or service. There are two risk neutral agents (or sellers, "she") indexed by $i \in \{1,2\}$ that may provide it. Each seller is specialized in a different version of the good. We assume that exactly one of both alternatives suits the buyer's needs.[12] Each seller is equally likely to offer the adequate alternative. The buyer does not know which version is better suited. The variable $\chi_i \in \{0,1\}$ captures the actual suitability with $\chi_i = 1$ indicating that seller i offers the superior alternative and $\chi_i = 0$ representing the inferior one. Buying a good yields the principal a base payoff of $S \in \mathbb{R}$. The

[12]This assumption is thoroughly discussed in section 3.5.

principal's payoff further increases by $\eta \in \mathbb{R}^+$ if he chooses the superior alternative. Thus, when seller i provides the good with probability q_i and the principal pays m_1 to seller 1 and m_2 to seller 2, his payoff is $u_p = q_1 (S + \chi_1 \eta) + q_2 (S + \chi_2 \eta) - m_1 - m_2$.[13]

Both agents possess two-dimensional private information: The first dimension concerns the superiority of a seller's alternative: Each agent receives a *noisy* signal $x_i \in \{0,1\}$ that indicates if her alternative is appropriate for the buyer. The *good* signal $x_i = 1$ means that the alternative is likely to be well suited while a *bad* signal of $x_i = 0$ indicates that the offered alternative is likely to be inappropriate. The probability of obtaining a correct signal, does not depend on a good's adequacy, that is $\Pr(x_i = \chi_i | \chi_i = 0) = \Pr(x_i = \chi_i | \chi_i = 1) \equiv \frac{1}{2}(1+\gamma)$ where $\gamma \in (0,1]$ measures the quality of the signal.[14] Hence, the unconditional probabilities of obtaining a good and a bad signal are $\alpha_1 = \alpha_0 = 1/2$, respectively. For $\gamma \to 0$, the sellers' signals are uninformative whereas $\gamma = 1$ captures perfect signals.

Since exactly one of the alternatives is appropriate, the sellers' signals are negatively correlated. The signals (x_1, x_2) are *consistent*, if they are compatible with a true match profile, that is if exactly one seller has a good signal: $x_1 \neq x_2$. They are *inconsistent* when both sellers have obtained the same signal: $x_1 = x_2$. Denote by α_{x_1,x_2} the joint probability that agents 1 and 2 receive the signals x_1 and x_2, respectively. A profile of consistent signals occurs more often than an inconsistent profile, since $\alpha_{1,0} = \alpha_{0,1} = \frac{1}{4}(1+\gamma^2)$ and $\alpha_{1,1} = \alpha_{0,0} = \frac{1}{4}(1-\gamma^2)$.

The second dimension of an agent's type refers to the cost of her alternative. A seller's alternative is either cheap with costs normalized to $\underline{c} = 0$ or expensive with costs $\overline{c} = \delta$: $c_i \in \{\underline{c}, \overline{c}\}$. With probability $\alpha_{\underline{c}} \in (0,1)$ a seller's cost is low and with probability $\alpha_{\overline{c}} = 1 - \alpha_{\underline{c}}$ it is high. Each seller's cost is drawn independently of her signal and of the competitor's cost. When agent i sells her good with probability q_i and receives the payment m_i, her payoff is $m_i - q_i c_i$.

In summary, a seller's type θ_i consists of her cost and her adequacy signal. Slightly abusing notation, the type space is $\Theta = \{\underline{c}0, \underline{c}1, \overline{c}0, \overline{c}1\}$. We will commonly use c_i and

[13]Note that the principal may pay both sellers even though buying only one version of the good.

[14]We use γ instead of $\Pr(x_i = \chi_i | \chi_i = 0)$ for convenience. It is easy to verify that γ equals the correlation between a seller's signal X_i and the true state χ_i.

x_i to refer to the cost and the signal of a seller with type θ_i. The ex-ante probability that the sellers are of type θ_1 and θ_2 is $\alpha_{\theta_1,\theta_2} \equiv \alpha_{c_1}\alpha_{c_2}\alpha_{x_1,x_2}$. Similarly, $\alpha_{\theta_i|x_j}$ refers to the probability that firm i has type θ_i, conditional on firm j having obtained the signal x_j.

An important assumption is that both sellers are protected by limited liability, which means that the payment from the principal must at least cover the *announced* costs of the good.[15] Indeed, often legal restrictions prevent the principal from forcing the agent to sell below the announced cost. Moreover, we require that payments do not depend on the buyer's final payoff.

Procurement schemes

The principal's objective is to design a procurement mechanism to which he then commits. This mechanism describes the available actions of the sellers and relates the outcomes to the sellers' actions. A version of the revelation principle ensures that we may restrict attention to *direct* mechanisms, in which the principal simply asks the sellers to announce their type.[16]

Without loss of generality, we focus on mechanisms that are *anonymous*, in the sense that the outcome does not depend on an agent's index.[17] Since ex-ante both sellers seem equally well suited to perform the task, it also seems reasonable that the buyer treats both sellers alike.

The outcome encompasses the winning probability q_i that agent i sells her good and

[15]This assumption rules out the application of the mechanism suggested by Cremer and McLean (1988), which would render the sellers' private information concerning the match worthless. It is closely related to the "no-slavery" assumption which is imposed for example by Strausz (1997) and allows the agent to walk away from any contract at a given time. The "no-slavery" assumption is stronger than our assumption, since it may be optimal to deviate twice, e.g. to step back from a contract after having reported too low costs. Both assumptions coincide if the adequacy is uncorrelated as analyzed in section 3.5.

[16]We are interested in allocations that are truthfully implementable in Bayesian Nash equilibrium (BNE). See e.g. Mas-Colell, Whinston, and Green (1995), Proposition 23.D.1.

[17]The notion of *anonymous* mechanisms is widely used in the auction and procurement literature. See e.g. Armstrong (2000). Formally, let $q_i(\theta_1, \theta_2)$ and $m_i(\theta_1, \theta_2)$ denote the transaction probability of seller $i \in \{1, 2\}$ and the payment to seller i, respectively. A mechanism is **anonymous** if it satisfies $q_1(\hat{\theta}, \tilde{\theta}) = q_2(\tilde{\theta}, \hat{\theta}) \equiv q(\hat{\theta}, \tilde{\theta})$ and $m_1(\hat{\theta}, \tilde{\theta}) = m_2(\tilde{\theta}, \hat{\theta}) \equiv m(\hat{\theta}, \tilde{\theta}) \quad \forall \hat{\theta}, \tilde{\theta} \in \Theta$. Optimal anonymous schemes remain optimal in the broader class of all schemes, since our setup is ex-ante symmetric. Suppose an asymmetric scheme S_1 were optimal. By symmetry, the scheme S_2, obtained from S_1 by reversing the roles of the sellers, is also optimal. But then, since the whole maximization problem described below is linear in (q, r), the symmetric scheme $S = \frac{1}{2}(S_1 + S_2)$ is also feasible and optimal.

the monetary transfer. Since all participants are risk neutral, it is convenient to express the outcome instead in terms of the agents' winning probability q_i and the expected payoff (or rent) $r_i \equiv m_i - q_i c_i$ after they have announced their private information and before the good is transferred.[18]

The procurement scheme (q, r) thus maps the agents' messages about their type, $(\hat{\theta}_1, \hat{\theta}_2)$, into the winning probabilities $q_i = q_{\hat{\theta}_i, \hat{\theta}_j}$ and the rents $r_i = r_{\hat{\theta}_i, \hat{\theta}_j}$ with $i, j \in \{1, 2\}$ and $i \neq j$.[19] It is *implementable* if it satisfies the resource, incentive and participation constraints, which are described below. Note that we allow seller i's rent and her transaction probability to be contingent on *both* sellers' announcements. Moreover, sellers may receive positive payments even when not selling their good.[20]

The *resource constraints* capture that the principal needs at most one good:

$$q_{\theta_i, \theta_j} + q_{\theta_j, \theta_i} \leq 1 \qquad \forall \theta_i, \theta_j \in \Theta \,. \tag{3.1}$$

In particular, if both sellers announce to be of the same type, then the restriction to anonymous schemes implies that both sellers win with equal probability: $q_{\theta_i, \theta_i} \leq 1/2 \qquad \forall \theta_i \in \Theta$. Note that the principal is allowed to abstain from buying a good.

The *incentive constraints* ensure that truth-telling is optimal for each seller, given that the competitor acts truthfully. This requires

$$\mathrm{E}\left[r_{\theta_i, \theta_j} | \theta_i\right] \geq \mathrm{E}\left[r_{\hat{\theta}_i, \theta_j} + (\hat{c}_i - c_i) q_{\hat{\theta}_i, \theta_j} | \theta_i\right] \qquad \forall \theta_i, \hat{\theta}_i \in \Theta \,, \tag{3.2}$$

where $\mathrm{E}\left[\cdot | \theta_i\right]$ refers to the expectation over θ_j conditional on θ_i.

As the type space of a seller contains four elements, there are three incentive constraints for every type and hence 12 incentive constraints in total. Figure 3.1 illustrates the

[18] As the sellers are risk neutral, defining that the rent does not depend on a seller *actually* selling the good is without loss of generality.

[19] The monetary transfer is thus $m_i = r_{\hat{\theta}_i, \hat{\theta}_j} + \hat{c}_i q_{\hat{\theta}_i, \hat{\theta}_j}$.

[20] This assumption seems realistic in procurement auctions for complex goods where only a small number of sellers participate. Similarly, Jehiel, Moldovanu, and Stacchetti (1996) consider a model with negative externalities where bidders sometimes pay even when not obtaining the good. However, in many common auction formats, such as the English or Dutch auction, only the winner of the object may obtain rents.

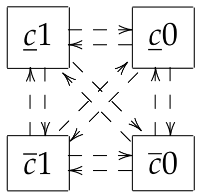


Figure 3.1: Incentive constraints

incentive constraints.[21] We refer to incentive constraints that prevent low (high) cost sellers from announcing high (low) cost as *downward (upward)* incentive constraints and those that prevent sellers from misreporting the fit-signal as *horizontal* incentive constraints.

The *limited liability* constraints ensure that after any truthful announcement each seller makes non-negative profits:[22]

$$r_{\theta_i,\theta_j} \geq 0, \qquad \forall \theta_i, \theta_j \in \Theta \tag{3.3}$$

The *limited liability* constraints also assure that the sellers are willing to participate in the procurement process independent of their own type.

Finally, we express the buyer's ex-ante expected payoff in terms of the variables introduced above. Due to the symmetry, from an ex-ante point of view, the buyer's expected payoff U_p is[23]

$$U_p = 2\mathrm{E}\left[q_{\theta_1,\theta_2}\pi_{\theta_1,x_2} - r_{\theta_1,\theta_2}\right] \tag{3.4}$$

where $\mathrm{E}[\cdot]$ refers to the expectation over θ_1 and θ_2, $\pi_{\theta_i,x_j} \equiv S + p_{x_i,x_j}\eta - c_i$ is the *gross surplu*s and $p_{x_i,x_j} \equiv Pr(\chi_i = 1|x_i, x_j)$ denotes the probability that a firm provides the superior approach. The gross surplus comprises the expected principal's payoff of the good minus the costs. The probability of providing the superior approach increases in the own signal and decreases in that of the rival. Applying Bayes rule yields $p_{1,0} = \frac{(1+\gamma)^2}{2(1+\gamma^2)} > p_{1,1} = 1/2 = p_{0,0} > \frac{(1-\gamma)^2}{2(1+\gamma^2)} = p_{0,1}$. Hence, the gross surplus depends on both competitors' signals and can be ordered as follows: $\pi_{c1,0} > \pi_{c1,1} = \pi_{c0,0} > \pi_{c0,1}$,

[21]Each dashed arrow represents one constraint that may or may not bind.

[22]Demougin and Garvie (1991) propose this formulation in a similar adverse selection setting as an alternative to requiring that the transfer m_i be non-negative. See also Laffont and Martimort (2002), Chapter 3.6.

[23]Symmetry implies that $\mathrm{E}\left[q_{\theta_2,\theta_1}\pi_{\theta_2,x_1} - r_{\theta_2,\theta_1}\right] = \mathrm{E}\left[q_{\theta_1,\theta_2}\pi_{\theta_1,x_2} - r_{\theta_1,\theta_2}\right]$.

$c \in \{\underline{c}, \bar{c}\}$. An implementable scheme is *optimal* if it maximizes the principal's expected payoff among all implementable schemes.

The *first best* allocation q^{FB} maximizes the expected surplus $2\mathrm{E}\left[q_{\theta_1,\theta_2}\pi_{\theta_1,x_2}\right]$ subject to the resource constraints (3.1). By inspection of equation (3.4), the first best allocation entails that the principal buys the good from the seller with the highest gross surplus provided it is positive.[24] If both signals are consistent, but the seller that is likely to provide the adequate good has higher costs than her rival, then the efficient choice depends on the ordering of $\pi_{\underline{c}0,1}$ and $\pi_{\bar{c}1,0}$: When the cost difference is small compared to the importance of the right alternative, i.e. $\frac{\delta}{\eta} < \frac{2\gamma}{(1+\gamma^2)}$, then it is efficient to give priority to the high cost firm which is more likely to suit the principal's needs and vice versa.

3.3 Benchmark: Buyer Knows Sellers' Signals

To isolate the effect of the sellers' private suitability information on the optimal procurement scheme, as a benchmark we assume in this section that the buyer observes the sellers' adequacy signals. Still, all players have noisy information regarding the match and each seller does not observe her rival's type.

In this benchmark situation, the buyer only needs to elicit the sellers' private cost information. Figure 3.2 depicts the following remaining incentive constraints:[25]

$$E\left[r_{\underline{c}x_i,\theta_j}|\theta_i\right] \geq E\left[r_{\bar{c}x_i,\theta_j} + \delta q_{\bar{c}x_i,\theta_j}|\theta_i\right] \quad \forall x_i \in \{0,1\}\ , \tag{3.5}$$

$$E\left[r_{\bar{c}x_i,\theta_j}|\theta_i\right] \geq E\left[r_{\underline{c}x_i,\theta_j} - \delta q_{\underline{c}x_i,\theta_j}|\theta_i\right] \quad \forall x_i \in \{0,1\}\ . \tag{3.6}$$

As the asymmetric information has been reduced to a single dimension, we can derive the benchmark scheme using standard techniques. For the moment, let us ignore the upward incentive constraints and assume that the downward constraints (3.5) bind.

[24]Formally, $q^{FB}_{\theta_i,\theta_j} = 1$ if $\pi_{\theta_i,x_j} > \max\left\{0, \pi_{\theta_j,x_i}\right\}$ and $q^{FB}_{\theta_i,\theta_j} = 0$ if $\pi_{\theta_i,x_j} < \max\left\{0, \pi_{\theta_j,x_i}\right\}$.

[25]Solid (dotted) lines represent constraints that are binding (slack).

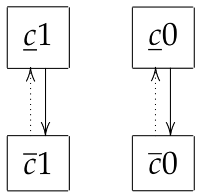


Figure 3.2: Benchmark incentive constraints

These constraints require that the expected minimum rent of a low cost seller with signal x_i be proportional to the probability that the principal buys from a high cost seller with the same match signal. In order to account for the induced rents, we define the virtual value

$$\psi^{BM}_{\theta_i,x_j} = \begin{cases} \pi_{\theta_i,x_j} & \text{if} \quad c_i = \underline{c}\,, \\ \pi_{\theta_i,x_j} - \frac{\delta\alpha_{\underline{c}}}{\alpha_{\bar{c}}} & \text{if} \quad c_i = \bar{c}\,. \end{cases}$$

Since the adjustment does not depend on the allocation of the task, the optimal procurement scheme obtains by maximizing "pointwise" as follows:

Lemma 3.1. *The optimal procurement scheme* q^{BM} *solves* $\max_q \mathrm{E}_{\theta_i,\theta_j}\left[q_{\theta_i,\theta_j}\psi^{BM}_{\theta_i,x_j}\right]$ *subject to the resource constraints* (3.1)*. High cost sellers do not obtain any rents and the expected payoff of low cost sellers is* $E[r_{\underline{c}x_i,\theta_j}|\theta_i = \underline{c}x_i] = \delta E[q^{BM}_{\bar{c}x_i,\theta_j}|\theta_i = \underline{c}x_i]$.

Proof. See Appendix A3.1. ■

Compared to the first best allocation, there arises no distortion at the top for $\underline{c}1$ sellers, since the principal buys from them whenever possible. If $\pi_{\bar{c}1,0} > \pi_{\underline{c}0,1} > 0$ and $\psi^{BM}_{\bar{c}1,0} < \psi^{BM}_{\underline{c}0,1}$, then the task is awarded to $\underline{c}0$ sellers too often compared to the first best. Similarly, there may be too little trade with high cost sellers compared to the first best.

The distortions only arise from the sellers' private information about their cost. In particular, if two sellers announce the same cost, the buyer never chooses a seller that is unlikely to match the principal's needs. This is because the buyer does not have to concede rents to acquire information regarding the suitability.

Let us now turn to some properties of optimal benchmark schemes that we later compare to those of the main setup. For future reference, we introduce the following partial order: Type $\theta \in \Theta$ *dominates* type $\tilde{\theta} \in \Theta$ whenever type θ is "better" than $\tilde{\theta}$ in one dimension and not worse in the remaining dimension.[26] Then, we may conclude:

[26]Formally, consider the linear order $\succeq_b$ to refer to "better" realizations: regarding costs $\underline{c} \succ_b \bar{c}$ and

Corollary 3.1. *Any optimal scheme of the benchmark setup has the following properties:*

i) If a seller's announced type is dominated by her competitor's report, then she never wins.

ii) The principal optimally buys from sellers with a good signal more often than from those with a bad signal: $\mathrm{E}\left[q_{c_i 1,\theta_j}|\theta_i = c_i 1\right] > \mathrm{E}\left[q_{c_i 0,\theta_j}|\theta_i = c_i 0\right]$, $c_i \in \{\underline{c}, \overline{c}\}$.

Since the buyer optimally chooses high cost sellers with a good match more often than those with a bad signal, $\underline{c}1$ sellers necessarily obtain higher rents than $\underline{c}0$ sellers. Precisely this rent differential creates problems when the buyer does not know which version is best. As we see in the next section, this may force the buyer to sometimes trade with high cost sellers even when they are unlikely to offer the right alternative.

3.4 Optimal Procurement Schemes

If the buyer does not know the match signal, the sellers may misreport their type not only with respect to costs but also concerning the suitability of their good.[27] Disposing of less information than in the benchmark setup, the buyer will be able to extract weakly less surplus than in the benchmark setup. As argued in the introduction, rents that are paid in order to elicit costs make it more difficult to induce sellers to reveal their suitability information truthfully. Thus, in order to exclude uninteresting cases where a buyer would never hire high cost sellers even if she knew the sellers' match signals, we assume for the rest of the chapter that $\pi_{\overline{c}1,0} > \delta\frac{\alpha_{\underline{c}}}{\alpha_{\overline{c}}}$.

Since both the surplus and the set of transaction probabilities is compact, we know that profit maximizing schemes always exist.[28] According to the following lemma, the most effective way to elicit the information on suitability is to concede rents only if both sellers report consistent signals.[29]

Lemma 3.2. *There always exists an optimal scheme that entails zero rents whenever the re-*

regarding signals, $1 \succ_b 0$. Then, θ dominates $\tilde{\theta}$ if $c \succeq_b \tilde{c}$ and $x \succeq_b \tilde{x}$ with at least one relation being strict. For example, $\underline{c}0$ dominates $\overline{c}0$ but not $\overline{c}1$.

[27] A seller can either misrepresent her cost or her match signal or both.

[28] Clearly, the buyer can guarantee an expected profit of at least 0. The set of (non-negative) rent payments that result in non-negative expected profits is also compact.

[29] Conceding rents only in case of consistent reports is strictly optimal in case horizontal or diagonal incentive constraints are binding.

ported signals are inconsistent: $r_{\theta_i,\theta_j} = 0$ *if* $x_i + x_j \neq 1$.

Proof. See Appendix A3.1. ■

Intuitively, if a seller misreports her suitability signal while the rival reports truthfully, then it is likely that the announced signals will be inconsistent due to their correlation. Paying no rents in this case thus reduces the incentives to misrepresent the suitability signal. Following Lemma 3.2 we can restrict attention to schemes that involve positive rents only when consistent suitability signals are announced. Since all players are risk neutral, we may summarize any optimal scheme by the expected rents $R_{\theta_i} \equiv E[r_{\theta_i,\tilde{\theta}_j}|\theta_i]$ that a seller of type θ_i obtains.[30]

The key difficulty of multidimensional adverse selection models lies in identifying the set of constraints that are binding at the optimum together with the associated partition of the parameter space.[31] While our discrete type space allows us to explicitly solve for optimal allocations, there are numerous cases that may distract from the economically relevant insights. Thus, we defer a full description to Appendix A3.2 and focus here instead on important properties. The following Proposition identifies three classes of solutions:

Proposition 3.1. *Optimal procurement schemes can be grouped into three classes according to the binding incentive constraints as shown in Figure 3.3.*[32] *The sellers' expected rents are determined by equations* (3.7)-(3.10).

[30]Due to risk neutrality, the expected rent $R_{\theta_i} \equiv E[r_{\theta_i,\tilde{\theta}_j}|\theta_i]$ is a sufficient statistic for the payoff of all players with respect to the rent payments $r(\cdot)$ obtained for *consistent* reports.

[31]See e.g. Armstrong and Rochet (1999), Armstrong 2000, Rochet and Stole 2003.

[32]A *binding* constraint means that the associated Lagrange multiplier is *positive*. In some cases the multipliers are not unique. However, for all parameter values, there is a set of multipliers that corresponds to one of the classes shown in Figure 3.3. Additional incentive constraints may hold with equality, but they are not binding in the sense that ignoring them does not change the solution.

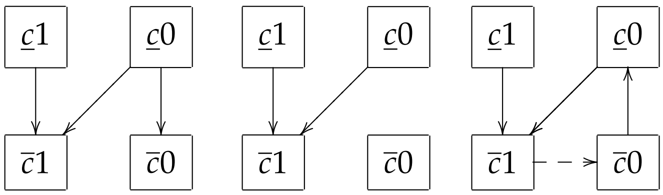
$$R^*_{\bar{c}0} = \max\left\{0, \frac{\delta(1+\gamma^2)}{\gamma^2} \sum_{\theta_j \in \Theta_j} \alpha_{\bar{c}0,\theta_j} \left[q^*_{\bar{c}1,\theta_j} - q^*_{\underline{c}0,\theta_j}\right]\right\} \tag{3.7}$$

$$R^*_{\bar{c}1} = \frac{1-\gamma^2}{1+\gamma^2} R^*_{\bar{c}0} \tag{3.8}$$

$$R^*_{\underline{c}0} = \sum_{\theta_j \in \Theta_j} \delta\alpha_{\theta_j|0} q^*_{\bar{c}1,\theta_j} + \frac{1-\gamma^2}{1+\gamma^2} R^*_{\bar{c}1} \tag{3.9}$$

$$R^*_{\underline{c}1} = \sum_{\theta_j \in \Theta_j} \delta\alpha_{\theta_j|1} q^*_{\bar{c}1,\theta_j} + R^*_{\bar{c}1} \tag{3.10}$$

Proof. See Appendix A3.1. ■



(a) Class I (b) Class II (c) Class III

Figure 3.3: Binding incentive constraints of optimal procurement schemes

Not knowing any information about adequacy forces the buyer to ensure that the sellers have no incentives to exaggerate their suitability signal.[33] In the benchmark setup, the buyer procures from high cost sellers with a good signal more often than from those with a bad signal since a seller's gross profit π increases in the own signal and decreases in that of the rival. As a higher probability of winning also implies higher rents, low cost sellers with a bad signal strictly prefer to announce type $\bar{c}1$ than $\bar{c}0$. Thus, in order to maintain the same transaction rule as in an optimal scheme of the benchmark setup, the principal would need to concede higher rents especially to $\underline{c}0$ sellers. Starting from an optimal scheme of the benchmark setup, the principal may save rents when committing to buying more often from $\bar{c}0$ sellers and less often from $\bar{c}1$ sellers. Indeed, since low cost sellers *strictly* prefer to imitate a $\bar{c}1$ agent, slightly increasing the payoffs of low cost sellers when pretending to be of type $\bar{c}0$ does not

[33]It is easy to verify that optimal procurement schemes of the benchmark setup violate the $(\underline{c}0 \to \bar{c}1)$ constraint for all admissible parameter values.

necessitate higher rents. This illustrates the trade-off between buying more often the adequate good and economizing on rents.

This trade-off is formally embodied in the result that the diagonal incentive constraint $(\underline{c}0 \to \bar{c}1)$ binds in any optimal scheme.[34] At first sight, it seems surprising that the horizontal constraints $(\underline{c}0 \to \underline{c}1)$ and $(\bar{c}0 \to \bar{c}1)$ do not bind. Intuitively, $(\bar{c}0 \to \bar{c}1)$ does not bind since a high cost seller cannot earn more rents by exaggerating the adequacy of the own good in order to trade more often. The reason why a $\underline{c}0$ seller prefers to pretend to be $\bar{c}1$ rather than $\underline{c}1$ at the optimum is less obvious: Clearly, the principal has to concede rents to $\underline{c}1$ sellers because of their private cost information. However, the principal optimally grants these rents only in case of consistent reports according to Lemma 3.2, which makes it is unlikely that seller $\underline{c}0$ will earn rents after misreporting her signal. In contrast, if the principal buys the good after inconsistent reports, then it is likely that seller $\underline{c}0$ obtains rents from selling the good after announcing to be of type $\bar{c}1$. Therefore, at the optimum, if the diagonal incentive constraint $(\underline{c}0 \to \bar{c}1)$ holds, sellers have no incentive to exaggerate their signal.

According to Proposition 3.1, optimal schemes can be grouped into three classes of solutions that differ in the set of binding incentive constraints. The solution classes summarize how much weight the principal optimally puts on the suitability signals. Figure 3.4 qualitatively summarizes for which parameter values these classes arise.

If the incremental value of the superior alternative or the base payoff is low (i.e. $\alpha_{\bar{c}1|0}\left(\pi_{\bar{c}1,0} - \max\left\{0, \pi_{\bar{c}0,1}\right\}\right) - \delta\alpha_{\underline{c}} < 0$), then optimal schemes belong to class I.[35] In this class, $\bar{c}0$ sellers are hired relatively often so that $\underline{c}0$ sellers find it equally tempting to either announce type $\bar{c}0$ or $\bar{c}1$. Intuitively, since the buyer has to concede enough rents to sellers so as to deter them from reporting *any* other type, this balanced allocation allows him to save on rents. Indeed, procuring more often from $\bar{c}1$ sellers at the detriment of $\bar{c}0$ sellers would break the indifference and increase the induced rent payments to *all* low cost sellers. No upward constraints are binding and no rents have to be conceded to high cost sellers, since schemes of this class entail $\underline{c}0$ sellers to be hired often enough. In particular, the buyer prefers to trade with a low cost seller if

[34]In addition, the vertical incentive constraint $(\underline{c}1 \to \bar{c}1)$ binds in any optimal scheme, for the same reason as in the benchmark case.

[35]The boundaries of the solution classes are proved in Appendix A3.1.

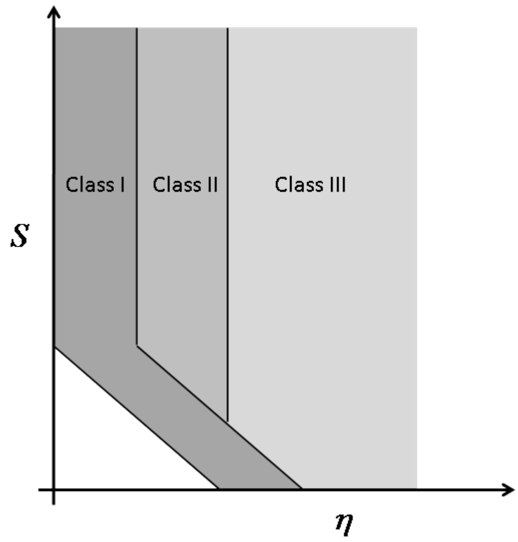


Figure 3.4: Solution classes

one agent announces $\underline{c}0$ and the second agent reports $\overline{c}1$. Indeed, any solution of this class satisfies $\sum_{\theta_j \in \Theta_j} \alpha_{\theta_j|0} \left[q_{\overline{c}1,\theta_j} - q_{\underline{c}0,\theta_j} \right] < 0$, which implies that high cost sellers obtain zero rents according to equations (3.7) and (3.8).

Solutions of class II obtain if the suitability is of intermediate importance, that is whenever $\alpha_{\overline{c}1|0} \left(\pi_{\overline{c}1,0} - \max \left\{ \pi_{\overline{c}0,1}, 0 \right\} \right) - \delta \alpha_{\underline{c}} > 0$, and $\alpha_{\overline{c}1|0} \left(\pi_{\overline{c}1,0} - \pi_{\underline{c}0,1} \right) - \delta \alpha_{\underline{c}} < 0$. In schemes of this class, high cost agents that announce a good signal sell their good more often than those with a bad signal. Low cost sellers therefore find it more tempting to pretend to be of type $\overline{c}1$ rather than $\overline{c}0$ as illustrated in Figure 3.3(b). Accordingly, buying from $\underline{c}1$ sellers is expensive at the margin, since this necessitates further rent payments both to $\underline{c}1$ and to $\underline{c}0$ sellers. By the same reasoning as above no rents have to be conceded to high cost sellers.

The third class obtains if the appropriate alternative is important, i.e. whenever $\alpha_{\overline{c}1|0} \left(\pi_{\overline{c}1,0} - \max \left\{ \pi_{\underline{c}0,1}, 0 \right\} \right) - \delta \alpha_{\underline{c}} > 0$. In this case, the principal would like to put a high weight on the suitability signals. However, as the buyer further increases the chances that $\overline{c}1$ agents win at the detriment of $\underline{c}0$ sellers, the upward incentive constraints ($\overline{c}0 \rightarrow \underline{c}0$) and ($\overline{c}1 \rightarrow \overline{c}0$) start to bind as shown in Figure 3.3(c). There are two subclasses: If the right alternative is not extremely important, or the base surplus is relatively low,[36] then the buyer commits to an allocation that assures that $\underline{c}0$ agents

[36]Formally, this subclass obtains if $\pi_{\overline{c}1,0} - \delta \left(\left(\frac{\alpha_{0|0}}{\alpha_{\overline{c}1|0} - \alpha_{\overline{c}1|1}} \right) + \frac{\alpha_{\underline{c}}}{\alpha_{\overline{c}}} \right) - \max \left\{ \pi_{\underline{c}0,1} + \delta \left(\frac{\alpha_{\underline{c}1|1} + \alpha_{\overline{c}1|0}}{\alpha_{\underline{c}1|0} - \alpha_{\underline{c}1|1}} \right) ; 0 \right\} < 0$.

win often enough, that is $\sum_{\theta_j \in \Theta_j} \alpha_{\theta_j|0} \left[q_{\bar{c}1,\theta_j} - q_{\underline{c}0,\theta_j} \right] = 0$. According to equations (3.7) and (3.8) such an allocation avoids that rents have have to be conceded to high cost sellers. If the adequacy is extremely important and the base surplus is high enough,[37] the principal optimally buys from the seller that is likely to offer the superior alternative whenever two consistent signals are announced. Putting so much weight on the suitability signal means that $\underline{c}0$ agents do not often sell their good but still have to obtain high rents to ensure truth-telling. This makes it attractive for high cost sellers to pretend being of type $\underline{c}0$. Thus, positive rents also have to be conceded to high cost sellers, which necessitates even higher rents for low cost sellers. This circular character of binding incentive constraints as shown in Figure 3.3(c) means that buying from $\bar{c}1$ sellers is extremely costly at the margin. Optimal schemes of this kind heavily rely on the correlation between the signals: Since misreporting the suitability signal is likely to produce a profile of inconsistent signals and because the sellers obtain positive rents only when reporting consistent signals, truthful revelation may be ensured if the rents are high enough.[38] Therefore, as the signals' precision and thus their correlation increase, implementing a given optimal transaction scheme q^* requires to concede less rents to high cost sellers. Compared to the literature on adverse selection, where optimal contracts usually entail that at least some agents cannot improve on their outside option, our result that all types may obtain a strictly positive rent seems remarkable.

The discussion thus far suggests that adverse selection with respect to the adequacy induces the principal to trade less often with sellers that announce a good signal and instead to buy more often from sellers that have reported a bad signal. Indeed, the following proposition confirms this important result.

Proposition 3.2. *Compared to the benchmark setup, optimal transaction probabilities are generically*

i) weakly lower *for high cost sellers that report a good suitability signal:* $q^*_{\bar{c}1,\theta_j} \leq q^{BM}_{\bar{c}1,\theta_j} \quad \forall \theta_j$,
ii) weakly higher *for high cost sellers that report a bad suitability signal:* $q^*_{\bar{c}0,\theta_j} \geq q^{BM}_{\bar{c}0,\theta_j} \quad \forall \theta_j$,
iii) weakly higher *for low cost sellers that report a bad suitability signal:* $q^*_{\underline{c}0,\theta_j} \geq q^{BM}_{\underline{c}0,\theta_j} \quad \forall \theta_j$
iv) and unchanged *for low cost sellers that report a good suitability signal* $q^*_{\underline{c}1,\theta_j} = q^{BM}_{\underline{c}1,\theta_j} \quad \forall \theta_j$.

[37] Formally, this subclass obtains if $\pi_{\bar{c}1,0} - \delta \left(\left(\frac{\alpha_{0|0}}{\alpha_{\bar{c}1|0} - \alpha_{\bar{c}1|1}} \right) + \frac{\alpha_{\underline{c}}}{\alpha_{\bar{c}}} \right) - \max \left\{ \pi_{\underline{c}0,1} + \delta \left(\frac{\alpha_{\underline{c}1|1} + \alpha_{\bar{c}1|0}}{\alpha_{\underline{c}1|0} - \alpha_{\underline{c}1|1}} \right); 0 \right\} > 0$.

[38] The correlation between signals makes it possible to implement allocations even though there are closed paths that violate condition (3.1) in Rochet and Stole (2003).

Proof. See Appendix A3.1. ■

Proposition 3.2 relies on the binding incentive constraints as shown in Figure 3.3. As already discussed, buying from $\bar{c}1$ sellers more often than from $\bar{c}0$ sellers necessitates higher rents compared to a more balanced allocation. Moreover, trading sufficiently often with low cost sellers with a bad signal helps to mitigate the circularity problem mentioned above. Hence, compared to the benchmark setup where these considerations are not relevant, the product is less often procured from high cost sellers with a good signal.

Regarding $\bar{c}1$ and $\underline{c}0$ sellers, the additional distortions caused by the desire to elicit the suitability signals have the same direction as those caused by the sellers' private cost information in the benchmark setup. Compared to the socially optimal allocation, the winning probabilities of $\bar{c}1$ sellers are therefore clearly distorted downwards and those of $\underline{c}0$ sellers are distorted upwards. The transaction probabilities of $\underline{c}1$ sellers are not distorted, since the buyer trades with these sellers as often as possible.

Since there is more trade with $\bar{c}0$ sellers than in the benchmark setup according to Proposition 3.2, is it possible that these sellers are hired inefficiently often? This possibility seems surprising at first glance since the private cost information induces the buyer to distort the transaction probabilities of high cost sellers downwards as discussed in section 3.3. The next corollary identifies conditions where this is indeed the case:

Corollary 3.2. *Suppose the base surplus is sufficiently large, namely* $S > \frac{\delta}{\alpha_{\bar{c}}} - \frac{(2\gamma^3+\gamma^2+1)\eta}{(\gamma^2+1)}$ *and the superior alternative is sufficiently unimportant, that is* $\frac{\eta}{\delta} < \frac{\alpha_{\underline{c}}}{\alpha_{\bar{c}}\gamma}$. *If both sellers announce a high cost and one seller is* less *likely to provide the superior alternative, the principal buys with a positive probability from the agent whose type is dominated by the rival's:* $q^*_{\bar{c}0,\bar{c}1} > 0$.

Proof. Follows directly from Solution I,a and Solution II,b of Appendix A3.2. ■

Corollary 3.2 examines a situation where the importance of the superior alternative is small compared to the cost uncertainty. In order to economize on rents, the buyer commits to trading sometimes with a $\bar{c}0$ seller even though the competitor is more likely to deliver the superior alternative at the same cost. Clearly this choice is not socially optimal, since the gross surplus of a dominated type lies always below that of her competitor. Moreover, this differs from the solution of the benchmark setup, where the principal never buys from dominated sellers even when η becomes very small. In Appendix A3.2 we show that if the base payoff is high enough, then it even becomes optimal for the buyer to commit to ignoring the suitability signals whenever both sellers announce high costs, that is $q^*_{\bar{c}1,\bar{c}0} = q^*_{\bar{c}0,\bar{c}1} = 1/2$. It is readily verified that in this case, the total expected rent conceded to the sellers is the same as in the benchmark setup.

Now we turn to another polar case where the importance of the superior alternative is relatively high compared to the cost uncertainty. In this situation, rents that the sellers could earn in the benchmark setup are relatively small compared to the incremental value of the superior alternative. Therefore, we expect that the principal gives priority to buy from the best suited seller, even though this necessitates high rents in order to elicit the private suitability information:

Corollary 3.3. *Suppose the base payoff and the incremental value of the superior alternative are sufficiently high:* $S \geq \frac{1}{2}\left[\frac{\left(1+2\alpha_{\underline{c}}\gamma^2-\gamma^2\right)\left(1+\gamma^2\right)\delta}{\gamma^2\alpha_{\bar{c}}(1-\gamma^2)} - \eta\right] + \delta$ *and* $\frac{\eta}{\delta} > \frac{(1+\gamma^2)^2}{4\gamma^3\alpha_{\underline{c}}\alpha_{\bar{c}}} - \frac{(1+\gamma^2)}{2\gamma}$. *Then the optimal winning probabilities coincide with the benchmark setup* ($q^*_{\theta_i,\theta_j} = q^{BM}_{\theta_i,\theta_j}$) *and whenever one seller is likely to offer the superior alternative, she is selected to deliver the good:* $q^*_{c_i1,c_j0} = 1\ \forall c_i, c_j$. *Positive rents have to be paid to all sellers:* $R^*_{\theta_i} > 0\ \forall \theta_i \in \Theta$.

Proof. Follows directly from Solution III,d of Appendix A3.2 and from Lemma 3.1. ■

While Corollary 3.3 presents a situation where asymmetric information regarding the adequacy does not affect the allocation of the task, the buyer has to concede much higher rents than in the benchmark setting in order to elicit the private information.

Corollary 3.3 is also useful to illustrate that the private information about suitability is only valuable as long as there is cost uncertainty. As the degree of cost uncertainty diminishes ($\delta \to 0$), the rents $R^*_{\theta_i}$ for all types θ_i also vanish since equations (3.10)-(3.7) are proportional to δ. Clearly, when the rent payments tend to zero, the principal optimally implements the efficient allocation. So in our model private suitability information may be socially harmful only to the degree that firms also possess private cost information.

Exclusion of high cost sellers

This section serves to analyze how private information concerning the right alternative may induce the buyer to exclude high cost sellers. The following proposition aims at situations where the surplus created by high cost sellers is low compared to the cost uncertainty.

Proposition 3.3. *The set of parameters that leads to a complete exclusion of high cost sellers (i.e.* $q^*_{\bar{c}x_i,\theta_j} = 0 \; \forall x_i \in \{0,1\}\,,\; \theta_j \in \Theta$*) is strictly larger than in the benchmark setup.*

Proof. See Appendix A3.1. ■

Completely barring high cost sellers allows the buyer to extract the sellers' information regarding the suitability at no cost. Indeed, if the principal never buys from high cost sellers, then low cost agents cannot earn rents by misrepresenting their cost. Therefore, sellers have no incentive to improve their chances of selling their product by announcing a good suitability signal. By committing to not buying from high cost sellers, the principal may thus extract the entire surplus, as in the benchmark setup. In contrast, any optimal allocation of the benchmark setup in which the principal sometimes buys from high cost sellers requires higher rents when the principal also has to elicit the adequacy signals. Hence, not buying from any high cost seller is optimal for a larger set of parameters. Excessive exclusion as pointed out by Proposition 3.3 is always socially inefficient, since the transaction probabilities for high cost sellers are already distorted downwards in the benchmark setup.

As an example, suppose that $\pi_{\bar{c}1,0} > \frac{\delta\alpha_{\underline{c}}}{\alpha_{\bar{c}}}$ and $\pi_{\bar{c}0,0} < 0$. Then the optimal scheme of the benchmark setup entails $q^{BM}_{\bar{c}1,\bar{c}0} = 1$ and $q^{BM}_{\bar{c}0,\bar{c}0} = 0$. Suppose now that the principal does not know the suitability signals. If additionally $\pi_{\bar{c}1,0} < \frac{\delta\alpha_{\underline{c}}}{\alpha_{\bar{c}}}\left(1 + \frac{1-\gamma^2}{1+\gamma^2}\right)$, then the gross surplus $\pi_{\bar{c}1,0}$ does not warrant the higher expected rents that have to be conceded in order to elicit the suitability information. Since $\pi_{\bar{c}0,0} < 0$ and therefore $\pi_{\bar{c}1,1} < 0$, any optimal scheme excludes high cost sellers.

Not buying any good after inconsistent reports

Another possibility to economize on rents is not to trade when both sellers claim to provide the superior alternative at high costs. In the benchmark setup, if the buyer *observes* that both sellers obtain a good signal, then the only consequence is that the belief regarding the sellers' adequacy remains unchanged. When the buyer has to elicit the suitability information, buying from high cost sellers when both sellers announce to be well suited creates an attractive opportunity for $\underline{c}0$ sellers to earn rents. Formally, we know from Proposition 3.1 that the diagonal incentive constraint $(\underline{c}0 \rightarrow \overline{c}1)$ always binds at the optimum. Similar to Section 3.3, we may define the virtual value that accounts for the expected rents needed to assure truthful revelation. The virtual value of a $\overline{c}1$ seller when both sellers make the same announcement is $\psi^*_{\overline{c}1,1} = \pi_{\overline{c}1,1} - \frac{\delta\alpha_{\underline{c}}}{\alpha_{\overline{c}}}\left(1 + \lambda^*_{\underline{c}0\rightarrow\overline{c}1}\frac{\alpha_{1|0}}{\alpha_{1|1}}\right)$ where $\lambda^*_{\underline{c}0\rightarrow\overline{c}1}$ is the Lagrange multiplier associated with the $(\underline{c}0 \rightarrow \overline{c}1)$ constraint.[39] The term $\frac{\alpha_{1|0}}{\alpha_{1|1}} = \frac{1+\gamma^2}{1-\gamma^2}$ reflects that the reports are more likely to be inconsistent if one seller exaggerates her signal than if both report sincerely. As the signals' precision γ augments, the afore-mentioned likelihood ratio increases, which suggests that trading with $\overline{c}1$ sellers after an inconsistent report becomes even more costly. The next proposition establishes that $\lambda^*_{\underline{c}0\rightarrow\overline{c}1}$ remains sufficiently high when the signals are very precise, so that it never pays to hire a high cost seller when both competitors claim to be well suited.[40]

Proposition 3.4. *If the suitability signals are precise enough, then the buyer optimally commits to not buying any product whenever both sellers announce high costs and a good signal: There exists a* $\overline{\gamma}(\eta, \alpha_{\underline{c}}, \delta) < 1$ *so that* $\gamma \geq \overline{\gamma}(\eta, \alpha_{\underline{c}}, \delta)$ *implies* $q^*_{\overline{c}1,\overline{c}1} = 0$.

Proof. See Appendix A3.1. ■

The virtual value $\psi^*_{\overline{c}1,1}$ of a $\overline{c}1$ seller when both competitors claim to sell well suited products is positive whenever the gross surplus $\pi_{\overline{c}1,1}$ outweighs the additional expected rents that have to be conceded. It can be shown that along the curve of parameters where this virtual profit is zero, the multiplier $\lambda^*_{\underline{c}0\rightarrow\overline{c}1}$ increases as choosing

[39] Note that in contrast to the benchmark case, the Lagrange multipliers now depend on the solution.

[40] Note however, that if the sellers' signals are not very precise, i.e. for $\gamma < 1/\sqrt{3}$, then $q^*_{\overline{c}1,\overline{c}1}$ may increase in γ. This may only occur if rents are paid to all sellers since in this case $\lambda^*_{\underline{c}0\rightarrow\overline{c}1} = \frac{1+\gamma^2}{2\beta\gamma^2}$ may be more elastic than $\frac{1+\gamma^2}{1-\gamma^2}$ with respect to γ.

the superior alternative becomes more important. Therefore, the required gross surplus $\pi_{\bar{c}1,1}$ to make it worthwhile to trade with an $\bar{c}1$ seller after an inconsistent report increases in η. Since the required gross surplus is constant in the benchmark setup, we can infer that concerning $q^*_{\bar{c}1,\bar{c}1}$, the distortion from having to elicit the suitability information increases as buying the appropriate alternative becomes more important.

3.5 Discussion

In this section we revisit some of the modeling assumptions and discuss alternatives.

Exactly one seller provides the appropriate alternative

Thus far, we have supposed that exactly one seller provides an adequate alternative which seems plausible in a number of situations and is commonly assumed in the literature.[41] Now we examine the optimal procurement scheme under the alternative assumption that the suitability of both alternatives is not correlated at all. We will see that the results are qualitatively robust up to the difference that the set of implementable allocations becomes constrained and that high cost sellers always obtain zero rents.

We assume that each sellers' alternative suits the principal's needs with probability α_1, independent of the competitor's adequacy.[42] Since there is no correlation between sellers any more, we can assume without loss of generality that the agents are perfectly informed about the suitability of their products. The gross surplus depends now solely on the seller's type so that $\pi_{\theta_i} \equiv S - c_i + x_i\eta$. The incentive constraints simplify to

[41]Our reasoning does not rely on perfect correlation and goes through if there is (negative) correlation at all. Assuming negative correlation seems plausible whenever a rare event triggers the need of some service. For example, falling ill is a rather rare event. Conditional on being sick, it is very unlikely to have two diseases at the same time. Suppose now there are only two diseases, which can be cured by different treatments. Conditional on being sick, the event of needing one treatment and the event of needing the other treatment are highly negatively correlated.

Regarding further literature, for example Bolton, Freixas, and Shapiro (2007) also assume that exactly one alternative matches each customer's needs. Spacial models of product differentiation (See e.g. Anderson, De Palma, and Thisse (1992), Chapter 4) are based on a similar idea.

[42]Thus, sometimes both alternatives may be adequate and sometimes none.

$$R_{\theta_i} \geq R_{\hat{\theta}_i} + (\hat{c}_i - c_i) Q_{\hat{\theta}_i} \qquad \forall \theta_i, \hat{\theta}_i \in \Theta \,, \tag{3.11}$$

where $Q_{\hat{\theta}_i} \equiv E\left[q_{\hat{\theta}_i,\theta_j}\right]$ is the probability of selling the item and $R_{\hat{\theta}_i} \equiv E\left[r_{\hat{\theta}_i,\theta_j}\right]$ is the expected rent conditional on reporting type $\hat{\theta}_i$. Due to the risk-neutrality of all players, the behavior of the sellers now depends exclusively on the expected probabilities $Q_{\hat{\theta}_i}$ and on the expected rents $R_{\hat{\theta}_i}$.[43] Therefore we may summarize any scheme by these two variables. Since the principal cannot exploit the correlation of the signals any more, the set of implementable schemes is now restricted as follows:

Lemma 3.3. *Any implementable scheme entails $Q_{\bar{c}x} \leq Q_{\underline{c}y}$ $\forall x, y \in \{0, 1\}$.*

Proof. Adding up the incentive constraints $(\bar{c}x \to \bar{c}y)$, $(\bar{c}y \to \underline{c}y)$ and $(\underline{c}y \to \bar{c}x)$ yields the result. ■

Lemma 3.3 points out a big difference compared to Section 3.4: The principal cannot implement schemes with $Q_{\bar{c}1} > Q_{\underline{c}0}$ any more. Clearly, as the adequacy becomes more important (η high), the difference $\pi_{\bar{c}1} - \pi_{\underline{c}0}$ increases, suggesting that the principal would prefer to buy from $\bar{c}1$ sellers rather than from $\underline{c}0$ ones. In this case, the restriction stated by Lemma 3.3 becomes relevant.

In order to concisely characterize the optimal procurement schemes, we introduce the linear order $\succeq$ defined as follows: If $\tilde{\theta} \succ \theta$, then whenever seller i reports $\tilde{\theta}$ while seller j announces θ and the principal buys at all, he buys from seller i, that is $q^{U*}_{\theta,\tilde{\theta}} = 0$.[44] Similarly, $\tilde{\theta} \sim \theta$ means that whenever seller i reports $\tilde{\theta}$ while seller j announces θ and the principal buys at all, then he buys from both sellers with equal probability. Note that once the types from which the principal does not buy are fixed, each order uniquely pins down the expected probabilities Q. Similar to Proposition 3.1, we get:

Proposition 3.5. *The optimal procurement schemes Q^{U*} can be grouped into three classes according to the binding incentive constraints as shown in Figure 3.3. In solution class I, II and III, the principal buys according to the priority $\underline{c}1 \succ \underline{c}0 \succ \bar{c}1 \sim \bar{c}0$, $\underline{c}1 \succ \underline{c}0 \succ \bar{c}1 \succ \bar{c}0$ and $\underline{c}1 \succ \underline{c}0 \sim \bar{c}1 \succ \bar{c}0$, respectively. The sellers' expected rents are*

[43] In particular, there is no point in rewarding consistent reports.

[44] The subscript $U*$ denotes an optimal value of this setup.

$$R^{U*}_{\underline{c}1} = R^{U*}_{\underline{c}0} = \delta Q^{U*}_{\bar{c}1}\,, \tag{3.12}$$

$$R^{U*}_{\bar{c}1} = R^{U*}_{\bar{c}0} = 0\,. \tag{3.13}$$

Proof. See Appendix A3.1. ■

In the proof of Proposition 3.5 we completely characterize the optimal procurement schemes and the boundaries of the solution classes.

The economic intuition of the optimal procurement schemes remains basically unchanged. When the importance of choosing an appropriate good is low (class I), the principal commits to ignore the adequacy signal of high cost sellers. For intermediate values of η (class II), he gives priority to the sellers' costs and buys from a seller with a good signal in case both sellers announce the same costs. If the principal values a well suited good much more than an inappropriate one (class III), he gives as much priority to the signal as is implementable.

There are two main differences compared to Proposition 3.1. First, the virtual value $\psi^{U*}_{\theta_i}$ that accounts for the induced rents of buying from a seller with type θ_i does not depend on the competitor's announcement any more. This reduces the number of different optimal schemes compared to section 3.4.[45] Second, as already pointed out by Lemma 3.3, allocations that give priority to the suitability signal are not implementable any more. High cost sellers never obtain rents, since precisely the allocations that would have required to concede strictly positive rents to high cost agents in the setup with correlated signals are not implementable any more.

Discrete type space

Assuming that the actual costs and the signals are drawn from a 2-by-2 discrete type space is a strong simplifying assumption and a concession to technical difficulties in multidimensional environments. Unfortunately, this assumption leads to optimal

[45]For example, if $q^{U*}_{\theta_i,\theta_j} > 0$ for $\theta_i \neq \theta_j$, then generically also $q^{U*}_{\theta_i,\theta_i} > 0$. Solution I,b of the main model, presented in Appendix A3.2, violates this property.

schemes that are discontinuous in the underlying parameters and therefore difficult to present. This problem was also encountered in related work by Armstrong and Rochet (1999), Armstrong (2000) or Asker and Cantillon (2010).

Armstrong (2000) points out that there is a lack of general solution techniques for general distributions of values. He presumes that beyond a discrete type space probably "numerical simulations (...) will provide the the most tractable method of generating further insights" [Armstrong (2000)]. As a robustness check, it would be certainly useful to apply numerical simulations on a version of our model with continuous types.[46] We thus see our model as a first step to highlight new trade-offs that may arise in a setting of multidimensional private information.

Collusion

One issue which we have not addressed so far is collusion among the sellers. An important element of the discussed optimal procurement schemes is that rents are only paid in case the sellers report consistent signals. If the sellers observe inconsistent signals and they can credibly communicate this information, then they can clearly improve on their payoff if one seller misrepresents her signal. In that case, the second seller does not have an incentive to further deviate with respect to the suitability signal, as this would again produce an inconsistent profile of signals. This suggests that the optimal schemes are likely to be vulnerable to collusion even if the sellers cannot commit to collusive reports among themselves. Therefore, this issue should be investigated in future research.

[46] Jehiel, Moldovanu, and Stacchetti (1996) examine a related model with buyers that may differ both in their valuation as well as in the externality they impose on others. They assume that both the valuation and the externality is continuously distributed but uncorrelated across bidders. In this setup, the expected winning probability conditional on a bidder's type must be constant in the externality. Both the adequacy in our model as well as the externality in Jehiel, Moldovanu, and Stacchetti (1996) does not enter in the bidder's utility. Hence, we reckon that in a version of our model with *uncorrelated* adequacy and continuously distributed costs, any feasible mechanism exhibits a constant probability of trade in the sellers' adequacy. Yet, we believe that if the costs are continuously distributed and the adequacy is negatively correlated as in our base model, the probability of trade need not be constant in the sellers' adequacy.

Value of commitment

The ability to commit is particularly valuable for the principal because of the sellers' private suitability information. If an outcome of the procurement mechanism is ex-post inefficient, the buyer has to resist the temptation to renegotiate.[47] To see that uncertainty concerning the adequacy increases the ex-post inefficiency of the optimal mechanism, consider first the case where both sellers provide goods that are equally suited and whose costs are drawn from the same distribution. In this case, the principal optimally buys from the seller with the highest positive virtual value as defined in section 3.3. Provided that buying from some seller is always optimal, the mechanism is ex-post efficient since the principal always procures from the seller with the lowest costs. If the goods differ in their adequacy, but the principal knows the sellers' signals as in our benchmark setup, then the principal may inefficiently commit to buying from low cost sellers with a bad signal rather than from high cost sellers with a good signal. If the principal does not know the sellers' signals, then there is a further tendency to commit to buying inefficiently often from sellers with a bad signal as discussed in section 3.4. As the allocation of the goods becomes less efficient, the private adequacy information makes commitment more valuable since there is more temptation to renegotiate.

Only potential sellers know the adequacy of their goods

So far, we have assumed that only potential sellers are informed about the principal's needs. If the costs to diagnose the customer's needs and to acquire the knowledge of the alternative products are low compared to the cost uncertainty, then it may be advantageous to consult a third party expert who does not benefit from selling the credence good on which he gives advice. Indeed, in practice, we often observe that experts that advise the buyer are not allowed to serve additionally as suppliers. Our model can be easily extended to account for the possibility to resort to third party experts. Suppose there is an expert that may incur a verifiable diagnosis cost of c_D in order to learn the adequacy signals of both sellers. The principal may make a take-it-

[47] See e.g. Laffont and Martimort (2002), sections 2.13 and 9.4 for a survey.

or-leave-it offer and propose a wage for finding out the adequacy signals (which will be c_D). Denote the principal's expected maximized payoff of the benchmark and the main setup by U_p^{BM*} and U_p^*, respectively. Then it is optimal to hire an expert if the diagnosis cost is sufficiently small, that is $c_D < U_p^{BM*} - U_p^*$.

3.6 Conclusion

This chapter shows that rents which arise due to cost uncertainty may induce specialized sellers to excessively recommend the alternative they are specialized in. Adverse incentives of sellers to exaggerate the adequacy of their services may be mitigated by rewarding them for admitting a bad signal or by committing to taking their recommendations less into account than would be efficient. If the principal's payoff of the alternative versions does not differ too much, the need to concede additional information rents has negative effects on the allocative efficiency.

We conclude our study with an example of how our results can be applied. Before hiring management consultancies, it is common practice that clients invite various competitors to present suggestions how to proceed. After the suggestions are made, the client awards one consultancy with the project. In hope to acquire a profitable project, consultancies often accept to carry out the introductory screening at low fares. But exactly this practice exacerbates the adverse incentives of the consultancies to advocate the own suggestion against better judgment. If the buyer deems the proper alternative to be crucial, our results suggest that he should offer a remuneration in case the project is finally *not* awarded to a consultancy.

In this chapter we have focused on optimal mechanisms that the buyer may design in order to extract the sellers' adequacy information in the presence of cost uncertainty. In further research, it might be promising to analyze cost uncertainty in a credence goods market equilibrium framework.

A3.1 Appendix - Proofs of Lemmas and Propositions

Proof of Lemma 3.1

Ignore the upward constraints (3.6) for the moment. In any optimal scheme the downward constraints (3.5) must hold with equality since otherwise the expected payoff of the principal could be increased by reducing the rents of low cost sellers accordingly. For the same reason, $r_{\bar{c}x_i,\theta_j} = 0$ for all x_i, θ_j. Combining this with the binding incentive constraints (3.5) yields the expected rents of low cost firms. Inserting (3.5) with equality into (3.4) and rearranging yields $U_p = 2E\left[q_{\theta_i,\theta_j}\psi^{BM}_{\theta_i,x_j}\right]$. From $\pi_{\underline{c}x_i,x_j} = \pi_{\bar{c}x_i,x_j} + \delta$ follows $q^{BM}_{\underline{c}x_i,\theta_j} \geq q^{BM}_{\bar{c}x_i,\theta_j}$. This inequality together with (3.5) holding with equality imply that the upward constraints (3.6) are indeed satisfied.

Proof of Lemma 3.2

It suffices to show that for any implementable scheme (q, r) that leads to the expected payoff U_p, there is an alternative scheme $(q, \tilde{r})$ with the same transaction probabilities q and $\tilde{r}_{\theta_i,\theta_j} = 0$ whenever $x_i + x_j \neq 1$ that yields the same payoff.

Suppose (q, r) is a feasible scheme with $r_{c_ix_i,c_jx_j} \neq 0$ and $x_i + x_j \neq 1$ for some (x_i, x_j) . Define the alternative rent scheme $\tilde{r}$ as follows:

$$\alpha_{c_i1,\underline{c}0}\tilde{r}_{c_i1,\underline{c}0} = \sum_{\theta_j \in \Theta_i} \left(\alpha_{c_i1,\theta_j} r_{c_i1,\theta_j}\right) \ \forall c_i \in \{\underline{c}, \bar{c}\},$$

$$\tilde{r}_{c_i1,\theta_j} = 0 \qquad \forall \quad \theta_j \neq \underline{c}0,$$

$$\alpha_{c_i0,\underline{c}1}\tilde{r}_{c_i0,\underline{c}1} = \sum_{\theta_j \in \Theta_i} \left(\alpha_{c_i0,\theta_j} r_{c_i0,\theta_j}\right) \ \forall c_i \in \{\underline{c}, \bar{c}\},$$

$$\tilde{r}_{c_i0,\theta_j} = 0 \qquad \forall \quad \theta_j \neq \underline{c}1 .$$

Since all incentive constraints are satisfied under the original scheme, we have

$$E\left[r_{\theta_i,\theta_j}|\theta_i\right] \geq E\left[r_{\hat{\theta}_i,\theta_j} + (\hat{c}_i - c_i)q_{\hat{\theta}_i,\theta_j}|\theta_i\right] \qquad \forall \theta_i, \hat{\theta}_i \in \Theta .$$

By construction, if the agent truthfully reports her signal ($\hat{x}_i = x_i$), then $E\left[\tilde{r}_{\hat{\theta}_i,\theta_j}|\theta_i\right] =$

$E\left[r_{\hat{\theta}_i,\theta_j}|\theta_i\right]$. If instead $\hat{c}_i \in \{\underline{c}, \overline{c}\}$, $x_i = 0$ and $\hat{x}_i = 1$, then

$$\sum_{\theta_j \in \Theta_i} \alpha_{\theta_j|0} r_{\hat{c}_i 1,\theta_j} \geq \frac{\alpha_{0|0}}{\alpha_{0|1}} \sum_{\theta_j \in \Theta_i} \alpha_{\theta_j|1} r_{\hat{c}_i 1,\theta_j} = \alpha_{\underline{c}0|0} \tilde{r}_{\hat{c}_i 1,\underline{c}0}$$

where the inequality comes from $\alpha_{\theta_j|0} \geq \frac{\alpha_{0|0}}{\alpha_{0|1}} \alpha_{\theta_j|1}$, $\forall \theta_j \in \Theta$. Therefore, $E\left[\tilde{r}_{\hat{c}_i 1_i,\theta_j}|\theta_i = c_i 0\right] \leq E\left[r_{\hat{c}_i 1,\theta_j}|\theta_i = c_i 0\right]$. By the same reasoning, $E\left[\tilde{r}_{\hat{c}_i 0_i,\theta_j}|\theta_i = c_i 1\right] \leq E\left[r_{\hat{c}_i 0,\theta_j}|\theta_i = c_i 1\right]$. Hence, the expected rents when misreporting the signal are weakly lower, which ensures that the incentive constraints are still satisfied.

Proof of Proposition 3.1

We first solve a relaxed problem where we ignore some incentive constraints and then show that these constraints are satisfied at the optimum. The claim of the proposition is directly implied by Lemma 3.4 and Lemma 3.7 which are established in what follows.

Consider the following relaxed problem $\mathcal{R}$ where we ignore all constraints except $(\underline{c}1 \to \overline{c}1)$, $(\underline{c}0 \to \overline{c}1)$, $(\underline{c}0 \to \overline{c}0)$, $(\overline{c}1 \to \overline{c}0)$ and $(\overline{c}0 \to \underline{c}0)$ and express the problem in terms of expected rents R_{θ_i} under the assumption that rents are only positive for consistent reports as implied by Lemma 3.2. The relaxed problem $\mathcal{R}$ is

$$\max_{q,R} \sum_{\theta_1} \alpha_{\theta_1} \left[\left[\sum_{\theta_2} \alpha_{\theta_2|\theta_1} \pi_{\theta_1,x_2} q_{\theta_1,\theta_2} \right] - R_{\theta_1} \right]$$

s.t.

$$\alpha_{\underline{c}1}\left[\sum_{\theta_j\in\Theta}\left(\delta\alpha_{\theta_j|1}q_{\bar{c}1,\theta_j}\right)+R_{\bar{c}1}-R_{\underline{c}1}\right]\leq 0 \qquad (\underline{c}1\to\bar{c}1)$$

$$\alpha_{\underline{c}0}\left[\sum_{\theta_j\in\Theta}\left(\delta\alpha_{\theta_j|0}q_{\bar{c}1,\theta_j}\right)+\varphi R_{\bar{c}1}-R_{\underline{c}0}\right]\leq 0 \qquad (\underline{c}0\to\bar{c}1)$$

$$\alpha_{\underline{c}0}\left[\sum_{\theta_j\in\Theta}\left(\delta\alpha_{\theta_j|0}q_{\bar{c}0,\theta_j}\right)+R_{\bar{c}0}-R_{\underline{c}0}\right]\leq 0 \qquad (\underline{c}0\to\bar{c}0)$$

$$\alpha_{\bar{c}1}\left[\varphi R_{\bar{c}0}-R_{\bar{c}1}\right]\leq 0 \qquad (\bar{c}1\to\bar{c}0)$$

$$\alpha_{\bar{c}0}\left[R_{\underline{c}0}-R_{\bar{c}0}-\sum_{\theta_j\in\Theta}\left(\delta\alpha_{\theta_j|0}q_{\underline{c}0,\theta_j}\right)\right]\leq 0 \qquad (\bar{c}0\to\underline{c}0)$$

where $\varphi\equiv\frac{\alpha_{0|0}}{\alpha_{1|0}}=\frac{\alpha_{1|1}}{\alpha_{1|0}}$.

Lemma 3.4. *The schemes given by Appendix A3.2 and* (3.7)-(3.10) *solve the problem $\mathcal{R}$ when $\pi_{\bar{c}1,0}-\delta\frac{\alpha_{\underline{c}}}{\alpha_{\bar{c}}}>0$. The solutions are generically unique.*

Proof. Define $\psi^*_{\bar{c}1,x_j}\equiv\pi_{\bar{c}1,x_j}-\delta\frac{\alpha_{\underline{c}}}{\alpha_{\bar{c}}}\lambda^*_{\underline{c}1\to\bar{c}1}-\delta\frac{\alpha_{\underline{c}0|x_j}}{\alpha_{\bar{c}1|x_j}}\lambda^*_{\underline{c}0\to\bar{c}1}$, $\psi^*_{\bar{c}0,x_j}\equiv\pi_{\bar{c}0,x_j}-\delta\frac{\alpha_{\underline{c}}}{\alpha_{\bar{c}}}\lambda^*_{\underline{c}0\to\bar{c}0}$ $\psi^*_{\underline{c}1,x_j}=\pi_{\underline{c}1,x_j}$ and $\psi^*_{\underline{c}0,x_j}=\pi_{\underline{c}0,x_j}+\delta\frac{\alpha_{\bar{c}}}{\alpha_{\underline{c}}}\lambda^*_{\bar{c}0\to\underline{c}0}$ for $x_j\in\{0,1\}$, where $\lambda^*_{\theta\to\hat{\theta}}$ is the Lagrange multiplier (or dual variable) of the incentive constraint ($\theta\to\hat{\theta}$). Fix $\lambda^*_{\underline{c}1\to\bar{c}1}=1$ which assures that the FOC for $R_{\bar{c}1}\geq 0$, is satisfied. After eliminating the dual variables for the resource constraints, the first order conditions for $q_{\theta_i,\theta_j}=0$, $q_{\theta_i,\theta_j}\in(0,1)$ and $q_{\theta_i,\theta_j}=1$ with $\theta_i\neq\theta_j$ can be written as $\psi^*_{\theta_i,x_j}-\max\left\{0,\psi^*_{\theta_j,x_i}\right\}\leq 0$, $\psi^*_{\theta_i,x_j}-\max\left\{0,\psi^*_{\theta_j,x_i}\right\}=0$ and $\psi^*_{\theta_i,x_j}-\max\left\{0,\psi^*_{\theta_j,x_i}\right\}\geq 0$, respectively. Similarly, the first order conditions for $q_{\theta_i,\theta_i}=0$, $q_{\theta_i,\theta_i}\in(0,1/2)$ and $q_{\theta_i,\theta_i}=1/2$ boil down to $\psi^*_{\theta_i,x_i}\leq 0$, $\psi^*_{\theta_i,x_i}=0$ and $\psi^*_{\theta_i,x_i}\geq 0$, respectively. Since both the constraints and the objective function are linear, the FOCs are sufficient for an global maximum by the Optimality Condition Theorem [Luenberger and Ye (2008, p. 45)]. Moreover, by the same Theorem, a solution entails a strict maximum in the interior of the boundaries given below.

By the definition above and the property of π, generally $\psi^*_{\bar{c}0,0}>\psi^*_{\bar{c}0,1}$, $\psi^*_{\bar{c}1,0}>\psi^*_{\bar{c}1,1}$, $\psi^*_{\underline{c}x,x}>\psi^*_{\bar{c}x,x}$, $x\in\{0,1\}$ and $\psi^*_{\underline{c}1,0}>\psi^*_{\bar{c},1}$. The first order conditions thus imply $q^*_{\bar{c}1,\underline{c}1}=q^*_{\bar{c}0,\underline{c}0}=q^*_{\bar{c}0,\underline{c}1}=0$. Moreover, the FOCs imply that $q^*_{\underline{c}1,\underline{c}1}=1/2$ and $q^*_{\underline{c}1,\bar{c}1}=1$ if $\pi_{\underline{c}1,1}>0$

and $q^*_{\underline{c}1,\underline{c}1} = q^*_{\underline{c}1,\overline{c}1} = 0$ if $\pi_{\underline{c}1,1} < 0$.

Case 1: Solve the problem for $\alpha_{\overline{c}1|0}\left(\pi_{\overline{c}1,0} - \max\left\{\pi_{\overline{c}0,1}, 0\right\}\right) - \delta\alpha_{\underline{c}} \leq 0$. Note that $\pi_{\overline{c}1,0} - \delta\frac{\alpha_{\underline{c}}}{\alpha_{\overline{c}}} > 0$ and $\alpha_{\overline{c}1|0}\left(\pi_{\overline{c}1,0} - \max\left\{\pi_{\overline{c}0,1}, 0\right\}\right) - \delta\alpha_{\underline{c}} \leq 0$ imply $\pi_{\overline{c}0,1} > 0$.

Define $\lambda^*_{\overline{c}1\to\overline{c}0} = 0$, $\lambda^*_{\overline{c}0\to\underline{c}0} = 0$, $\lambda^*_{\underline{c}0\to\overline{c}1} = \min\left\{\frac{\alpha_{\overline{c}1,0}\left(\pi_{\overline{c}1,0}-\delta\frac{\alpha_{\underline{c}}}{\alpha_{\overline{c}}}\right)}{\delta\alpha_{\underline{c}0,0}}; \left(\pi_{\overline{c}1,0} - \pi_{\overline{c}0,1}\right)\frac{\alpha_{\overline{c}1,0}}{\delta\alpha_{\underline{c}}}\right\}$ and $\lambda^*_{\underline{c}0\to\overline{c}0} = 1 - \lambda^*_{\underline{c}0\to\overline{c}1}$ which guarantees that $\psi^*_{\overline{c}1,0} \geq \psi^*_{\overline{c}0,1}$ with equality if $\psi^*_{\overline{c}1,0} \geq 0$. Note that $\lambda^*_{\underline{c}0\to\overline{c}1} \in [0,1]$ due to $\pi_{\overline{c}1,0} - \delta\frac{\alpha_{\underline{c}}}{\alpha_{\overline{c}}} > 0$ and $\alpha_{\overline{c}1|0}\left(\pi_{\overline{c}1,0} - \max\left\{\pi_{\overline{c}0,1}, 0\right\}\right) - \delta\alpha_{\underline{c}} \leq 0$. By the properties of π we have the following order: $\psi^*_{\underline{c}1,0} > \psi^*_{\underline{c}1,1} = \psi^*_{\underline{c}0,0} > \psi^*_{\underline{c}0,1}$. Besides, $\pi_{\overline{c}0,1} > 0$ implies $\psi^*_{\underline{c}0,1} > 0$. By construction $\psi^*_{\overline{c}1,0} > 0$ implies that $\psi^*_{\overline{c}1,0} = \psi^*_{\overline{c}0,1} < \pi_{\overline{c}0,1} < \psi^*_{\overline{c}1,0}$. So we have $\psi^*_{\underline{c}0,1} > \psi^*_{\overline{c}1,0} \geq \psi^*_{\overline{c}0,1}$. Thus necessarily $q^*_{\underline{c}0,\overline{c}1} = q^*_{\underline{c}1,\underline{c}0} = q^*_{\underline{c}1,\overline{c}0} = q^*_{\underline{c}x,\overline{c}x} = 1$, $q^*_{\overline{c}1,\underline{c}0} = q^*_{\underline{c}0,\underline{c}1} = 0$ and $q^*_{\underline{c}1,\underline{c}1} = q^*_{\underline{c}0,\underline{c}0} = 1/2$.

By complementary slackness, $\lambda^*_{\underline{c}0\to\overline{c}x_i} > 0$ for $x_i \in \{0,1\}$ requires that $(\underline{c}0 \to \overline{c}1)$ and $(\underline{c}0 \to \underline{c}1)$ hold with equality. Subtracting $(\underline{c}0 \to \overline{c}1)$ from $(\underline{c}0 \to \underline{c}1)$ and simplifying yields $\alpha_{0,0}\left[q^*_{\overline{c}1,\overline{c}0} - q^*_{\overline{c}0,\overline{c}0}\right] + \alpha_{1,0}\left[q^*_{\overline{c}1,\overline{c}1} - q^*_{\overline{c}0,\overline{c}1}\right] = 0$.

Consider first the sub-case $\psi^*_{\overline{c}0,\overline{c}1} \leq 0$: Then by the considerations above, $\psi^*_{\overline{c}1,\overline{c}0} = 0$ and $\psi^*_{\overline{c}1,\overline{c}1} \leq 0$ so that the FOCs for $q^*_{\overline{c}1,\overline{c}1} = q^*_{\overline{c}0,\overline{c}1} = 0$ are satisfied. If additionally $\psi^*_{\overline{c}0,\overline{c}0} < 0$, then also $q^*_{\overline{c}1,\overline{c}0} = q^*_{\overline{c}0,\overline{c}0} = 0$; if instead $\psi^*_{\overline{c}0,\overline{c}0} \geq 0$, then $q^*_{\overline{c}1,\overline{c}0} = q^*_{\overline{c}0,\overline{c}0} = 1/2$ satisfies the associated FOCs, which corresponds to Solution I, c in Appendix A3.2.

Consider now $\psi^*_{\overline{c}0,\overline{c}1} \geq 0$, which implies $\psi^*_{\overline{c}0,\overline{c}0} > \psi^*_{\overline{c}0,\overline{c}1} = \psi^*_{\overline{c}1,\overline{c}0} > \psi^*_{\overline{c}1,\overline{c}1}$. Together with $q^*_{\overline{c}1,\overline{c}0} + q^*_{\overline{c}0,\overline{c}1} \leq 1$ this pins down the transaction probabilities $q^*_{\overline{c}1,\overline{c}0}$, $q^*_{\overline{c}0,\overline{c}1}$, $q^*_{\overline{c}1,\overline{c}1}$ and $q^*_{\overline{c}0,\overline{c}0} = 1/2$ for Solutions I, a ($\psi^*_{\overline{c}1,\overline{c}1} > 0$ implies $q^*_{\overline{c}1,\overline{c}1} = 1/2$) and I, b ($\psi^*_{\overline{c}1,\overline{c}1} < 0$ implies $q^*_{\overline{c}1,\overline{c}1} = 0$).

It is readily verified that for these allocations and for the rents given by (3.7)-(3.10) the incentive constraints $(\underline{c}1 \to \overline{c}1)$, $(\underline{c}0 \to \overline{c}1)$ and $(\underline{c}0 \to \overline{c}0)$ hold with equality so that we may indeed use positive multipliers. Since $R^*_{\overline{c}1} = R^*_{\overline{c}0} = 0$, $(\overline{c}1 \to \overline{c}0)$ holds with equality (but does not bind since $\lambda^*_{\overline{c}1\to\overline{c}0} = 0$). Moreover, it is readily verified that the solution satisfies $(\overline{c}0 \to \underline{c}0)$.

Case 2: Solve the problem when $\alpha_{\overline{c}1|0}\left(\pi_{\overline{c}1,0} - \max\left\{\pi_{\overline{c}0,1}, 0\right\}\right) - \delta\alpha_{\underline{c}} \geq 0$ and $\alpha_{\overline{c}1|0}\left(\pi_{\overline{c}1,0} - \pi_{\underline{c}0,1}\right) - \delta\alpha_{\underline{c}} \leq 0$ holds. Note that subtracting the second condition from $\alpha_{\overline{c}1|0}\pi_{\overline{c}1,0} - \delta\alpha_{\underline{c}} \geq 0$ yields $\pi_{\underline{c}0,1} \geq 0$ which implies $\psi^*_{\underline{c}0,1} \geq 0$.

Define $\lambda^*_{\bar{c}0\to\underline{c}0}=0$, $\lambda^*_{\underline{c}0\to\bar{c}1}=1$, $\lambda^*_{\bar{c}1\to\bar{c}0}=0$ and $\lambda^*_{\underline{c}0\to\bar{c}0}=0$ which implies $\psi^*_{\bar{c}0,1}=\pi_{\bar{c}0,1}$ and $\psi^*_{\bar{c}1,0}=(\pi_{\bar{c}1,0})-\frac{\delta\alpha_{\underline{c}}}{\alpha_{\bar{c}1|0}}\geq\max\left\{\psi^*_{\bar{c}0,1},0\right\}$. By the properties of π and the definition of ψ^*, we also have the following order: $\psi^*_{\underline{c}1,0}>\psi^*_{\underline{c}1,1}=\psi^*_{\underline{c}0,0}>\psi^*_{\underline{c}0,1}\geq 0$. Thus the FOCs for $q^*_{\bar{c}1,\bar{c}0}=q^*_{\underline{c}0,\bar{c}1}=q^*_{\underline{c}1,\underline{c}0}=q^*_{\underline{c}1,\bar{c}0}=q^*_{\underline{c}x,\bar{c}x}=1$, $q^*_{\underline{c}1,\underline{c}1}=q^*_{\underline{c}0,\underline{c}0}=1/2$, $q^*_{\bar{c}0,\bar{c}1}=q^*_{\bar{c}1,\underline{c}0}=q^*_{\bar{c}0,\underline{c}1}=0$ are satisfied.

If $\psi^*_{\bar{c}1,1}=\alpha_{\bar{c}1|1}\pi_{\bar{c}1,1}-\delta\alpha_{\underline{c}}\geq 0$, then $\psi^*_{\bar{c}0,0}>\psi^*_{\bar{c}1,1}\geq 0$ so that the FOCs for $q^*_{\underline{c}1,\underline{c}1}=q^*_{\underline{c}0,\underline{c}0}=1/2$ hold. This corresponds to Solution II,a in Appendix A3.2.

If $\psi^*_{\bar{c}1,1}\leq 0$ and , then the FOC for $q^*_{\underline{c}1,\underline{c}1}=0$ holds. If $\pi_{\bar{c}0,0}\geq 0$ and $\pi_{\bar{c}0,0}\leq 0$, then the FOC for $q^*_{\underline{c}0,\underline{c}0}=1/2$ and $q^*_{\underline{c}0,\underline{c}0}=0$, respectively, holds. This is Solution II,b.

It is readily verified that for these allocations and for the rents uniquely given by (3.7)-(3.10) , the incentive constraints $(\underline{c}1\to\bar{c}1)$ and $(\underline{c}0\to\bar{c}1)$ hold with equality and that the remaining constraints $(\underline{c}0\to\bar{c}0)$, $(\bar{c}0\to\underline{c}0)$ and $(\bar{c}1\to\bar{c}0)$ are satisfied.

Case 3: Solve the problem for $\alpha_{\bar{c}1|0}\left(\pi_{\bar{c}1,0}-\pi_{\underline{c}0,1}\right)-\delta\alpha_{\underline{c}}\geq 0$, $\alpha_{\underline{c}0|0}\pi_{\underline{c}0,1}+\alpha_{\bar{c}1|0}\pi_{\bar{c}1,0}-\delta\alpha_{\underline{c}}\geq 0$ and $\pi_{\bar{c}1,0}-\pi_{\underline{c}0,1}-\delta\left(\left(\frac{\alpha_{0|0}}{\alpha_{\bar{c}1|0}-\alpha_{\bar{c}1|1}}\right)+\frac{\alpha_{\underline{c}}}{\alpha_{\bar{c}}}+\left(\frac{\alpha_{\underline{c}1|1}+\alpha_{\bar{c}1|0}}{\alpha_{\underline{c}1|0}-\alpha_{\underline{c}1|1}}\right)\right)\leq 0$. Define $\lambda^*_{\underline{c}0\to\bar{c}1}=1+\frac{\alpha_{\bar{c}1|0}\left(\pi_{\bar{c}1,0}-\pi_{\underline{c}0,1}\right)-\delta\alpha_{\underline{c}}}{\delta\left(\alpha_{\bar{c}1|0}+\alpha_{\underline{c}0|0}\right)}$, $\lambda^*_{\underline{c}0\to\bar{c}0}=0$, $\lambda^*_{\bar{c}0\to\underline{c}0}=\frac{\alpha_{\underline{c}}}{\alpha_{\bar{c}}}\left(\lambda^*_{\underline{c}0\to\bar{c}1}-1\right)\geq 0$ and $\lambda^*_{\bar{c}1\to\underline{c}0}=\max\left\{\left(\lambda^*_{\bar{c}0\to\underline{c}0}-1\right)\frac{\alpha_{0|1}}{\alpha_{1|1}},0\right\}$. These multipliers are constructed to assure that $\psi^*_{\underline{c}0,1}=\psi^*_{\bar{c}1,0}$ which implies $\psi^*_{\bar{c}1,0}>\psi^*_{\bar{c}0,1}$. Inserting these multipliers yields $\psi^*_{\underline{c}0,1}=\frac{\alpha_{\underline{c}0|0}\pi_{\underline{c}0,1}+\alpha_{\bar{c}1|0}\pi_{\bar{c}1,0}-\delta\alpha_{\underline{c}}}{\left(\alpha_{\bar{c}1|0}+\alpha_{\underline{c}0|0}\right)}$ so that $\psi^*_{\underline{c}1,0}=\pi_{\underline{c}1,0}>\psi^*_{\underline{c}0,1}$. Moreover, the condition $\alpha_{\underline{c}0|0}\pi_{\underline{c}0,1}+\alpha_{\bar{c}1|0}\pi_{\bar{c}1,0}-\delta\alpha_{\underline{c}}\geq 0$ implies $\psi^*_{\underline{c}0,1}\geq 0$ and therefore $\psi^*_{\underline{c}0,0}>0$. Thus, the FOCs for $q^*_{\underline{c}1,\underline{c}0}=q^*_{\bar{c}1,\bar{c}0}=q^*_{\underline{c}1,\bar{c}0}=1, =1/2$ and $q^*_{\bar{c}0,\bar{c}1}=q^*_{\underline{c}0,\underline{c}1}=0$ are satisfied.
If $\psi^*_{\bar{c}1,1}=\alpha_{\bar{c}1|1}\pi_{\bar{c}1,1}-\delta\alpha_{\underline{c}}-\frac{\alpha_{\underline{c}0|1}\left[\alpha_{\bar{c}1|0}\left(\pi_{\bar{c}1,0}-\pi_{\underline{c}0,1}\right)-\delta\alpha_{\underline{c}}\right]}{\alpha_{\bar{c}0|1}+\alpha_{\underline{c}0|0}}\geq 0$, then $\psi^*_{\bar{c}0,0}>0$ and $\psi^*_{\underline{c}1,1}>0$. Thus the FOCs for $q^*_{\underline{c}0,\bar{c}0}=q^*_{\underline{c}1,\bar{c}1}=1$ and $q^*_{\underline{c}0,\bar{c}1}=q^*_{\bar{c}1,\underline{c}0}=q^*_{\underline{c}0,\underline{c}0}=q^*_{\underline{c}1,\underline{c}1}=q^*_{\bar{c}1,\bar{c}1}=q^*_{\bar{c}0,\bar{c}0}=1/2$ hold. Using these values, it is readily verified that the incentive constraints $(\underline{c}1\to\bar{c}1)$, $(\underline{c}0\to\bar{c}1)$ and $(\bar{c}0\to\underline{c}0)$ are satisfied with equality (Solution III,a).
If $\psi^*_{\bar{c}1,1}\leq 0$, then the FOC for $q^*_{\bar{c}1,\bar{c}1}=0$, $q^*_{\underline{c}0,\bar{c}1}=\frac{0.5\alpha_{\underline{c}0|0}}{\alpha_{\bar{c}1|0}+\alpha_{\underline{c}0|0}}$ and $q^*_{\bar{c}1,\underline{c}0}=\frac{\alpha_{\bar{c}1|0}+0.5\alpha_{\underline{c}0|0}}{\alpha_{\bar{c}1|0}+\alpha_{\underline{c}0|0}}$ hold. Moreover, $q^*_{\bar{c}0,\bar{c}0}=1/2$ if $\pi_{\bar{c}0,\bar{c}0}>0$ and $q^*_{\bar{c}0,\bar{c}0}=0$ if $\pi_{\bar{c}0,\bar{c}0}<0$ satisfies the FOC for $q^*_{\bar{c}0,\bar{c}0}$. Using the values of $q^*_{\underline{c}0,\theta_j}$ and $q^*_{\bar{c}1,\theta_j}$, it is easy to confirm that the incentive constraints $(\underline{c}1\to\bar{c}1)$, $(\underline{c}0\to\bar{c}1)$ and $(\bar{c}0\to\underline{c}0)$ are satisfied with equality, independent of $q^*_{\bar{c}0,\bar{c}0}$, $q^*_{\underline{c}1,\underline{c}1}$ and $q^*_{\underline{c}1,\bar{c}1}$ (Solution III,b).

Turning to the rents, the FOC for $R_{\underline{c}0} \geq 0$, which is $\lambda^*_{\overline{c}0\to\underline{c}0} = \frac{\alpha_{\underline{c}}}{\alpha_{\overline{c}}}\left(\lambda^*_{\underline{c}0\to\overline{c}1} - 1\right)$, is satisfied by construction. The FOC for $R_{\overline{c}0} = 0$, $\left(\lambda^*_{\overline{c}0\to\underline{c}0} - 1\right)\frac{\alpha_{0|1}}{\alpha_{1|1}} \leq \lambda^*_{\overline{c}1\to\overline{c}0}$ holds by construction of $\lambda^*_{\overline{c}1\to\overline{c}0}$. Finally, the condition $\pi_{\overline{c}1,0} - \pi_{\underline{c}0,1} - \delta\left(\left(\frac{\alpha_{0|0}}{\alpha_{\overline{c}1|0}-\alpha_{\overline{c}1|1}}\right) + \frac{\alpha_{\underline{c}}}{\alpha_{\overline{c}}} + \left(\frac{\alpha_{\underline{c}1|1}+\alpha_{\overline{c}1|0}}{\alpha_{\underline{c}1|0}-\alpha_{\underline{c}1|1}}\right)\right) \leq 0$ assures that $\lambda^*_{\underline{c}0\to\overline{c}1} \leq \frac{\alpha_{1|0}}{\alpha_{\underline{c}1|0}-\alpha_{\underline{c}1|1}}$. Rewriting the FOC for $R_{\overline{c}1} = 0$, $\alpha_{\underline{c}0,\underline{c}0}\lambda^*_{\underline{c}0\to\overline{c}1} + \alpha_{\underline{c}1,\underline{c}0} + \alpha_{\overline{c}1,\underline{c}0} \geq \alpha_{\overline{c}1,\underline{c}0}\lambda^*_{\overline{c}1\to\overline{c}0}$, and using the definitions of $\lambda^*_{\underline{c}0\to\overline{c}1}$, $\lambda^*_{\overline{c}0\to\underline{c}0}$ shows that this inequality holds for $\lambda^*_{\underline{c}0\to\overline{c}1} \leq \frac{\alpha_{1|0}}{\alpha_{\underline{c}1|0}-\alpha_{\underline{c}1|1}}$.

Case 4: Solve the problem for $\alpha_{\underline{c}0|0}\pi_{\underline{c}0,1} + \alpha_{\overline{c}1|0}\pi_{\overline{c}1,0} - \delta\alpha_{\underline{c}} \leq 0$, $\alpha_{\overline{c}1|0}\pi_{\overline{c}1,0} - \delta\alpha_{\underline{c}} \geq 0$ and $\alpha_{\overline{c}1|0}\pi_{\overline{c}1,0} - \delta\alpha_{\underline{c}1|0} - \delta\left(\frac{\alpha_{1|0}}{\alpha_{\underline{c}1|0}-\alpha_{\underline{c}1|1}}\right)\alpha_{\underline{c}0|0} \leq 0$. Define $\lambda^*_{\underline{c}0\to\overline{c}1} = 1 + \frac{\alpha_{\overline{c}1|0}\pi_{\overline{c}1,0}-\delta\alpha_{\underline{c}}}{\delta\alpha_{\underline{c}0|0}}$, $\lambda^*_{\underline{c}0\to\overline{c}0} = 0$, $\lambda^*_{\overline{c}0\to\underline{c}0} = \frac{\alpha_{\underline{c}}}{\alpha_{\overline{c}}}\left(\lambda^*_{\underline{c}0\to\overline{c}1} - 1\right) \geq 0$ and $\lambda^*_{\overline{c}1\to\underline{c}0} = \max\left\{\left(\lambda^*_{\overline{c}0\to\underline{c}0} - 1\right)\frac{\alpha_{0|1}}{\alpha_{1|1}}, 0\right\}$. $\lambda^*_{\underline{c}0\to\overline{c}1}$ is constructed such that $\psi^*_{\overline{c}1,0} = 0$ which implies $\psi^*_{\overline{c}1,1} < 0$. Moreover, these multipliers together with the condition $\alpha_{\underline{c}0|0}\pi_{\underline{c}0,1} + \alpha_{\overline{c}1|0}\pi_{\overline{c}1,0} - \delta\alpha_{\underline{c}} \leq 0$ imply $\psi^*_{\underline{c}0,1} \leq 0$.

Consider the sub-case $\alpha_{\underline{c}0|0}\pi_{\underline{c}0,0} + \alpha_{\overline{c}1|0}\pi_{\overline{c}1,0} - \delta\alpha_{\underline{c}} \geq 0$. Inserting the multipliers into $\psi^*_{\underline{c}0,0} = \pi_{\underline{c}0,0} + \delta\frac{\alpha_{\overline{c}}}{\alpha_{\underline{c}}}\lambda^*_{\overline{c}0\to\underline{c}0}$ and using the condition $\alpha_{\underline{c}0|0}\pi_{\underline{c}0,0} + \alpha_{\overline{c}1|0}\pi_{\overline{c}1,0} - \delta\alpha_{\underline{c}} \geq 0$ yields $\psi^*_{\underline{c}0,0} \geq 0$ which implies $\psi^*_{\underline{c}1,0} > 0$. Thus, the FOCs for $q^*_{\underline{c}1,\underline{c}0} = q^*_{\underline{c}1,\overline{c}0} = q^*_{\underline{c}0,\overline{c}0} = q^*_{\overline{c}1,\overline{c}0} = 1$, $q^*_{\underline{c}0,\underline{c}0} = q^*_{\overline{c}1,\underline{c}0} = 1/2$ and $q^*_{\overline{c}1,\overline{c}1} = q^*_{\underline{c}0,\overline{c}1} = q^*_{\overline{c}0,\overline{c}1} = q^*_{\underline{c}0,\underline{c}1} = 0$ are satisfied. Using the values of $q^*_{\underline{c}0,\theta_j}$ and $q^*_{\overline{c}1,\theta_j}$, it is easy to confirm that the incentive constraints ($\underline{c}1 \to \overline{c}1$), ($\underline{c}0 \to \overline{c}1$), ($\overline{c}1 \to \overline{c}0$) and ($\overline{c}0 \to \underline{c}0$) are satisfied with equality, independent of $q^*_{\underline{c}1,\theta_j}$ (This is the configuration of Solution III, c in Appendix A3.2).

Consider the sub-case $\alpha_{\underline{c}0|0}\pi_{\underline{c}0,0} + \alpha_{\overline{c}1|0}\pi_{\overline{c}1,0} - \delta\alpha_{\underline{c}} \leq 0$. Inserting the multipliers into $\psi^*_{\underline{c}0,0} = \pi_{\underline{c}0,0} + \delta\frac{\alpha_{\overline{c}}}{\alpha_{\underline{c}}}\lambda^*_{\overline{c}0\to\underline{c}0}$ and using the condition $\alpha_{\underline{c}0|0}\pi_{\underline{c}0,0} + \alpha_{\overline{c}1|0}\pi_{\overline{c}1,0} - \delta\alpha_{\underline{c}} \leq 0$ yields $\psi^*_{\underline{c}0,0} \leq 0$ The condition $\alpha_{\overline{c}1|0}\pi_{\overline{c}1,0} - \delta\alpha_{\underline{c}} \geq 0$ implies $\psi^*_{\underline{c}1,0} > 0$. Thus, the FOCs for $q^*_{\underline{c}1,\underline{c}0} = q^*_{\underline{c}1,\overline{c}0} = 1$, and $q^*_{\underline{c}0,\underline{c}0} = q^*_{\overline{c}1,\underline{c}0} = q^*_{\underline{c}0,\overline{c}0} = q^*_{\overline{c}1,\overline{c}0} = q^*_{\overline{c}1,\overline{c}1} = q^*_{\underline{c}0,\overline{c}1} = q^*_{\overline{c}0,\overline{c}1} = q^*_{\underline{c}0,\underline{c}1} = 0$ are satisfied. Since $q^*_{\overline{c}x_i,\theta_j} = 0\ \forall x_i, \theta_j$,the incentive constraints ($\underline{c}1 \to \overline{c}1$), ($\underline{c}0 \to \overline{c}1$), ($\overline{c}1 \to \overline{c}0$) and ($\overline{c}0 \to \underline{c}0$) are trivially satisfied with equality.

Turing to the rents, the FOC for $R_{\underline{c}0} \geq 0$, which is $\lambda^*_{\overline{c}0\to\underline{c}0} = \frac{\alpha_{\underline{c}}}{\alpha_{\overline{c}}}\left(\lambda^*_{\underline{c}0\to\overline{c}1} - 1\right)$, is satisfied by construction. The FOC for $R_{\overline{c}0} = 0$, $\left(\lambda^*_{\overline{c}0\to\underline{c}0} - 1\right)\frac{\alpha_{0|1}}{\alpha_{1|1}} \leq \lambda^*_{\overline{c}1\to\overline{c}0}$ holds by construction of $\lambda^*_{\overline{c}1\to\overline{c}0}$. Finally, the condition $\alpha_{\overline{c}1|0}\pi_{\overline{c}1,0} - \delta\alpha_{\underline{c}1|0} - \delta\left(\frac{\alpha_{1|0}}{\alpha_{\underline{c}1|0}-\alpha_{\underline{c}1|1}}\right)\alpha_{\underline{c}0|0} \leq 0$ assures that $\lambda^*_{\underline{c}0\to\overline{c}1} \leq \frac{\alpha_{1|0}}{\alpha_{\underline{c}1|0}-\alpha_{\underline{c}1|1}}$. Rewriting the FOC for $R_{\overline{c}1} = 0$, $\alpha_{\underline{c}0,\underline{c}0}\lambda^*_{\underline{c}0\to\overline{c}1} + \alpha_{\underline{c}1,\underline{c}0} + \alpha_{\overline{c}1,\underline{c}0} \geq \alpha_{\overline{c}1,\underline{c}0}\lambda^*_{\overline{c}1\to\overline{c}0}$, and using the definitions of $\lambda^*_{\underline{c}0\to\overline{c}1}$, $\lambda^*_{\overline{c}0\to\underline{c}0}$ shows that this inequality holds for $\lambda^*_{\underline{c}0\to\overline{c}1} \leq \frac{\alpha_{1|0}}{\alpha_{\underline{c}1|0}-\alpha_{\underline{c}1|1}}$.

Case 5: Solve the problem for $\pi_{\bar{c}1,0} - \delta\left(\left(\frac{\alpha_{0|0}}{\alpha_{\bar{c}1|0}-\alpha_{\bar{c}1|1}}\right)+\frac{\alpha_{\underline{c}}}{\alpha_{\bar{c}}}\right) - \max\left\{\pi_{\underline{c}0,1}+\delta\left(\frac{\alpha_{\underline{c}1|1}+\alpha_{\bar{c}1|0}}{\alpha_{\underline{c}1|0}-\alpha_{\underline{c}1|1}}\right);0\right\} > 0$. Define $\lambda^*_{\underline{c}0\to\bar{c}0} = 0$, $\lambda^*_{\bar{c}0\to\underline{c}0} = \frac{\alpha_{\underline{c}1|1}+\alpha_{\bar{c}1|0}}{\alpha_{\bar{c}1|0}-\alpha_{\bar{c}1|1}}$, $\lambda^*_{\underline{c}0\to\bar{c}1} = \frac{\alpha_{1|0}}{\alpha_{\underline{c}1|0}-\alpha_{\underline{c}1|1}}$ and $\lambda^*_{\bar{c}1\to\bar{c}0} = \frac{\alpha_{0|1}}{\alpha_{1|1}}\left(\lambda^*_{\bar{c}0\to\underline{c}0}-1\right)$. Note that these are the unique multipliers that solve the system of FOCs associated with $R_{\underline{c}0} > 0$, $R_{\bar{c}1} > 0$ and $R_{\bar{c}0} > 0$. Using these multipliers, the condition $\pi_{\bar{c}1,0} - \delta\left(\left(\frac{\alpha_{0|0}}{\alpha_{\bar{c}1|0}-\alpha_{\bar{c}1|1}}\right)+\frac{\alpha_{\underline{c}}}{\alpha_{\bar{c}}}\right) - \max\left\{\pi_{\underline{c}0,1}+\delta\left(\frac{\alpha_{\underline{c}1|1}+\alpha_{\bar{c}1|0}}{\alpha_{\underline{c}1|0}-\alpha_{\underline{c}1|1}}\right);0\right\} > 0$ is equivalent to $\psi^*_{\bar{c}1,0} \geq \psi^*_{\underline{c}0,1}$ and $\psi^*_{\bar{c}1,0} \geq 0$.

Consider the sub-case $\alpha_{\bar{c}1|1}\pi_{\bar{c}1,1} - \delta\alpha_{\underline{c}1|0} - \delta\alpha_{\underline{c}0|0}\lambda^*_{\underline{c}0\to\bar{c}1} \geq 0$. The condition of this sub-case is equivalent to $\psi^*_{\bar{c}1,1} \geq 0$ and implies $\psi^*_{\bar{c}0,0} > 0$, $\psi^*_{\underline{c}0,0} > 0$, $\psi^*_{\underline{c}1,1} > 0$, $\psi^*_{\underline{c}1,0} > 0$ and $\psi^*_{\bar{c}1,0} > 0$. Thus, the FOCs for $q^*_{\underline{c}1,\underline{c}0} = q^*_{\underline{c}1,\bar{c}0} = q^*_{\underline{c}1,\bar{c}1} = q^*_{\underline{c}0,\bar{c}0} = q^*_{\bar{c}1,\underline{c}0} = q^*_{\bar{c}1,\bar{c}0} = 1$, $q^*_{\underline{c}1,\underline{c}1} = q^*_{\underline{c}0,\underline{c}0} = q^*_{\bar{c}1,\bar{c}1} = q^*_{\bar{c}0,\bar{c}0} = 1/2$, $q^*_{\underline{c}0,\underline{c}1} = q^*_{\underline{c}0,\bar{c}1} = q^*_{\bar{c}0,\bar{c}1} = 0$ are satisfied. Moreover, using the rents (3.7)-(3.10), it is easy to verify that the incentive constraints $(\underline{c}1 \to \bar{c}1)$, $(\underline{c}0 \to \bar{c}1)$, $(\bar{c}0 \to \underline{c}0)$ and $(\bar{c}1 \to \bar{c}0)$ hold with equality (Solution III, d).

Consider the sub-case $\alpha_{\bar{c}1|1}\pi_{\bar{c}1,1} - \delta\alpha_{\underline{c}1|0} - \delta\alpha_{\underline{c}0|0}\lambda^*_{\underline{c}0\to\bar{c}1} \leq 0$ and $\alpha_{\underline{c}0|0}\pi_{\underline{c}0,0} + \alpha_{\bar{c}1|0}\pi_{\bar{c}1,0} - \delta\alpha_{\underline{c}} \geq 0$. The first, the second and the third condition implies $\psi^*_{\bar{c}1,1} \leq 0$, $\psi^*_{\bar{c}1,0} \geq 0$ and $\psi^*_{\underline{c}0,0} \geq 0$, respectively. Thus, the FOCs for $q^*_{\underline{c}1,\underline{c}0} = q^*_{\underline{c}1,\bar{c}0} = q^*_{\underline{c}0,\bar{c}0} = q^*_{\bar{c}1,\underline{c}0} = q^*_{\bar{c}1,\bar{c}0} = 1$, $q^*_{\underline{c}0,\underline{c}0} = 1/2$, $q^*_{\underline{c}0,\underline{c}1} = q^*_{\underline{c}0,\bar{c}1} = q^*_{\bar{c}0,\bar{c}1} = q^*_{\bar{c}1,\bar{c}1} = 0$ are satisfied. Moreover, $q^*_{\bar{c}0,\bar{c}0} = 1/2$ if $\pi_{\bar{c}0,\bar{c}0} > 0$ and $q^*_{\bar{c}0,\bar{c}0} = 0$ if $\pi_{\bar{c}0,\bar{c}0} < 0$ satisfies the FOC for $q^*_{\bar{c}0,\bar{c}0}$. Using the rents (3.7)-(3.10), it is easy to verify that the incentive constraints $(\underline{c}1 \to \bar{c}1)$, $(\underline{c}0 \to \bar{c}1)$, $(\bar{c}0 \to \underline{c}0)$ and $(\bar{c}1 \to \bar{c}0)$ hold with equality, independent of $q^*_{\bar{c}0,\bar{c}0}, q^*_{\underline{c}1,\bar{c}1}, q^*_{\underline{c}1,\underline{c}1}$ (Solution III, e).

Consider the sub-case $\alpha_{\underline{c}0|0}\pi_{\underline{c}0,0} + \alpha_{\bar{c}1|0}\pi_{\bar{c}1,0} - \delta\alpha_{\underline{c}} \leq 0$. The first and the second condition condition implies $\psi^*_{\bar{c}1,0} \geq 0$ and $\psi^*_{\underline{c}0,0} \leq 0$, respectively. This implies $\psi^*_{\underline{c}1,0} > 0$, $\psi^*_{\underline{c}1,1} < 0$, $\psi^*_{\bar{c}0,0} < 0$ and $\psi^*_{\bar{c}1,1}$. Thus, the FOCs for $q^*_{\underline{c}1,\underline{c}0} = q^*_{\underline{c}1,\bar{c}0} = q^*_{\bar{c}1,\underline{c}0} = q^*_{\bar{c}1,\bar{c}0} = 1$, $q^*_{\underline{c}1,\underline{c}1} = q^*_{\underline{c}1,\bar{c}1} = q^*_{\underline{c}0,\underline{c}1} = q^*_{\underline{c}0,\underline{c}0} = q^*_{\underline{c}0,\bar{c}1} = q^*_{\underline{c}0,\bar{c}0} = q^*_{\bar{c}0,\bar{c}1} = q^*_{\bar{c}1,\bar{c}1} = q^*_{\bar{c}0,\bar{c}0} = 0$ are satisfied. Using the rents (3.7)-(3.10), it is easy to verify that the incentive constraints $(\underline{c}1 \to \bar{c}1)$, $(\underline{c}0 \to \bar{c}1)$, $(\bar{c}0 \to \underline{c}0)$ and $(\bar{c}1 \to \bar{c}0)$ hold with equality (Solution III, f). ■

We further need the next two auxiliary Lemmas:

Lemma 3.5. *Consider the relaxed problem where all incentive constraints except* $(\underline{c}0 \to \bar{c}0)$, $(\underline{c}0 \to \bar{c}1)$ *and* $(\underline{c}1 \to \bar{c}1)$, $(\bar{c}1 \to \bar{c}0)$ *are ignored. If* $\pi_{\bar{c}1,0} - \delta\frac{\alpha_{\underline{c}}}{\alpha_{\bar{c}}} > 0$ *and* $(\bar{c}1 \to \bar{c}0)$ *holds with equality, then in any solution* $q^*_{\bar{c}1,\bar{c}0} + \frac{\alpha_{\underline{c}}}{\alpha_{\bar{c}}}q^*_{\bar{c}1,\underline{c}0} - q^*_{\bar{c}0,\bar{c}0} \geq 0$.

Proof. Subtracting the incentive constraints ($\underline{c}0 \to \bar{c}0$) from ($\underline{c}0 \to \bar{c}1$), using that ($\underline{c}0 \to \bar{c}1$) holds with equality due to Lemma 3.6, $\alpha_{\underline{c}1,\underline{c}0} = \frac{\alpha_{\underline{c}}}{\alpha_{\bar{c}}}\alpha_{\underline{c}1,\bar{c}0}$, and $q^*_{\bar{c}0,\underline{c}0} = q^*_{\bar{c}1,\underline{c}1} = q^*_{\bar{c}0,\underline{c}1} = 0$, yields $\alpha_{\bar{c}0|0}\left[q^*_{\bar{c}1,\bar{c}0} + \frac{\alpha_{\underline{c}}}{\alpha_{\bar{c}}}q^*_{\bar{c}1,\underline{c}0} - q^*_{\bar{c}0,\bar{c}0}\right] + \alpha_{\bar{c}1|0}\left[q^*_{\bar{c}1,\bar{c}1} - q^*_{\bar{c}0,\bar{c}1}\right] + \varphi R^*_{\bar{c}1} - R^*_{\bar{c}0} \geq 0$. Since ($\bar{c}1 \to \bar{c}0$) holds with equality, $\varphi R^*_{\bar{c}1} - R^*_{\bar{c}0} = \left(\varphi^2 - 1\right) R^*_{\bar{c}0} < 0$. Therefore,

$$\alpha_{\bar{c}0|0}\left[q^*_{\bar{c}1,\bar{c}0} + \frac{\alpha_{\underline{c}}}{\alpha_{\bar{c}}}q^*_{\bar{c}1,\underline{c}0} - q^*_{\bar{c}0,\bar{c}0}\right] + \alpha_{\bar{c}1|0}\left[q^*_{\bar{c}1,\bar{c}1} - q^*_{\bar{c}0,\bar{c}1}\right] \geq 0\,. \tag{3.14}$$

Suppose to the contrary that $q^*_{\bar{c}1,\bar{c}0} + \frac{\alpha_{\underline{c}}}{\alpha_{\bar{c}}}q^*_{\bar{c}1,\underline{c}0} - q^*_{\bar{c}0,\bar{c}0} < 0$. Then (3.14) implies $q^*_{\bar{c}1,\bar{c}1} - q^*_{\bar{c}0,\bar{c}1} > 0$, which in turn implies $q^*_{\bar{c}0,\bar{c}1} < q^*_{\bar{c}1,\bar{c}1} \leq 0.5$. Similarly, $q^*_{\bar{c}1,\bar{c}0} + \frac{\alpha_{\underline{c}}}{\alpha_{\bar{c}}}q^*_{\bar{c}1,\underline{c}0} - q^*_{\bar{c}0,\bar{c}0} < 0$ implies that $q^*_{\bar{c}1,\bar{c}0} < q^*_{\bar{c}0,\bar{c}0} \leq 0.5$. Hence, $q^*_{\bar{c}1,\bar{c}0} + q^*_{\bar{c}0,\bar{c}1} < 1$ so that the dual variable associated with the resource constraint of $q^*_{\bar{c}1,\bar{c}0}$ necessarily satisfies $\mu^*_{L1,L0} = 0$. Thus, the necessary FOC for $q^*_{\bar{c}1,\bar{c}0} < 1$ is $\pi_{L1,0} \leq \delta(\frac{\alpha_{\underline{c}}}{\alpha_{\bar{c}}} + \lambda^*_{H0\to L1}\frac{\alpha_{\underline{c}0|0}}{\alpha_{\bar{c}1|0}})$. This implies $\pi_{L1,1} < \delta(\frac{\alpha_{\underline{c}}}{\alpha_{\bar{c}}} + \lambda^*_{H0\to L1}\frac{\alpha_{\underline{c}0|1}}{\alpha_{\bar{c}1|1}})$ which implies $q^*_{\bar{c}1,\bar{c}1} = 0$, a contradiction. ∎

Lemma 3.6. *If the constraints* ($\underline{c}1 \to \bar{c}1$) *and* ($\underline{c}0 \to \bar{c}1$) *hold with equality, then the constraints* ($\underline{c}1 \to \underline{c}0$) *and* ($\underline{c}0 \to \underline{c}1$) *are necessarily satisfied.*

Proof. Since the constraints ($\underline{c}1 \to \bar{c}1$) and ($\underline{c}0 \to \bar{c}1$) hold with equality and by Lemma 3.2, we have

$$R_{\underline{c}1} = \sum_{\theta_j\in\Theta}\delta\alpha_{\theta_j|1}q_{\bar{c}1,\theta_j} + R_{\bar{c}1}$$

$$R_{\underline{c}0} = \sum_{\theta_j\in\Theta}\delta\alpha_{\theta_j|0}q_{\bar{c}1,\theta_j} + \varphi R_{\bar{c}1}$$

with $\varphi \equiv \frac{1-\gamma^2}{1+\gamma^2} < 1$. To see that ($\underline{c}0 \to \underline{c}1$) holds, note that $R_{\underline{c}0} \geq \varphi R_{\underline{c}1} = \sum_{\theta_j\in\Theta}\delta\varphi\alpha_{\theta_j|1}q_{\bar{c}1,\theta_j} + \varphi R_{\bar{c}1}$ since $\alpha_{\theta_j|0} \geq \varphi\alpha_{\theta_j|1}$. Similarly, for ($\underline{c}1 \to \underline{c}0$), note that $R_{\underline{c}1} \geq \varphi R_{\underline{c}0} = \sum_{\theta_j\in\Theta_j}\delta\varphi\alpha_{\theta_j|0}q_{\bar{c}1,\theta_j} + \varphi^2 R_{\bar{c}1}$ since $\alpha_{\theta_j|1} \geq \varphi\alpha_{\theta_j|0}$,$\forall\theta_j \in \Theta$. ∎

Lemma 3.7. *For* $\pi_{\bar{c}1,0} - \delta\frac{\alpha_{\underline{c}}}{\alpha_{\bar{c}}} > 0$, *any generic solution of the relaxed problem the problem* $\mathcal{R}$ *satisfies the remaining incentive constraints* (3.2).

Proof. We consider each of the remaining constraints in turn.

- Constraints ($\underline{c}1 \to \underline{c}0$) and ($\underline{c}0 \to \underline{c}1$) are satisfied due to Lemma 3.6.

- Constraint ($\underline{c}1 \to \overline{c}0$): By Lemma 3.4, in any solution condition ($\overline{c}1 \to \overline{c}0$) holds with equality which is equivalent to $R_{\overline{c}1} = \varphi R_{\overline{c}0}$. Hence, we have to show that $\sum_{\theta_j \in \Theta} \alpha_{\theta_j|1} \left(q_{\overline{c}1,\theta_j} - q_{\overline{c}0,\theta_j} \right) \geq 0$, which obtains after subtracting ($\underline{c}1 \to \overline{c}0$) from ($\underline{c}1 \to \overline{c}1$) and using that the latter constraint holds with equality as well as $R_{\overline{c}1} = \varphi R_{\overline{c}0}$. Subtracting ($\underline{c}0 \to \overline{c}0$) from ($\underline{c}0 \to \overline{c}1$) which holds with equality and using $R_{\overline{c}1} = \varphi R_{\overline{c}0}$ yields $\sum_{\theta_j \in \Theta_i} \alpha_{\theta_j|0} \left(q_{\overline{c}1,\theta_j} - q_{\overline{c}0,\theta_j} \right) + \left(\varphi^2 - 1 \right) R_{\overline{c}0} \geq 0$ which implies $\sum_{\theta_j \in \Theta} \alpha_{\theta_j|0} \left(q_{\overline{c}1,\theta_j} - q_{\overline{c}0,\theta_j} \right) \geq 0$. Since $\alpha_{0|1} > \alpha_{0|0}$, claim follows if $\alpha_{\underline{c}0|0} \left(q_{\overline{c}1,\underline{c}0} - q_{\overline{c}0,\underline{c}0} \right) + \alpha_{\overline{c}0|0} \left(q_{\overline{c}1,\overline{c}0} - q_{\overline{c}0,\overline{c}0} \right) \geq 0$. Since in any solution $q^*_{\overline{c}0,\underline{c}0} = 0$ by the associated FOC, this boils down to $\alpha_{\overline{c}0|0} \left(q_{\overline{c}1,\overline{c}0} + \frac{\alpha_{\underline{c}}}{\alpha_{\overline{c}}} q_{\overline{c}1,\underline{c}0} - q_{\overline{c}0,\overline{c}0} \right) \geq 0$ which is true according to Lemma 3.5.

- Constraint ($\overline{c}1 \to \underline{c}1$): From $\psi^*_{\underline{c}1,x_j} > \psi^*_{\overline{c}1,x_j}$ follows $q^*_{\underline{c}1,\theta_j} > q^*_{\overline{c}1,\theta_j}$ by the FOC. Since ($\underline{c}1 \to \overline{c}1$) holds with equality, the result follows.

- Constraint ($\overline{c}1 \to \underline{c}0$): We have

$$R_{\overline{c}1} \geq \varphi R_{\overline{c}0} \geq \varphi \left[R_{\underline{c}0} - \delta \sum_{\theta_j \in \Theta} \alpha_{\theta_j|0} q_{\underline{c}0,\theta_j} \right] \geq \varphi R_{\underline{c}0} - \delta \sum_{\theta_j \in \Theta} \alpha_{\theta_j|1} q_{\underline{c}0,\theta_j}$$

where the first inequality is constraint ($\overline{c}1 \to \overline{c}0$), the second is constraint ($\overline{c}0 \to \underline{c}0$) and the third inequality holds because $\frac{\alpha_{1|1}}{\alpha_{0|1}} \alpha_{\theta_j|0} \leq \alpha_{\theta_j|1} \quad \forall \theta_j \in \Theta$.

- Constraint ($\overline{c}0 \to \overline{c}1$): Since constraint ($\overline{c}1 \to \overline{c}0$) holds with equality $R_{\overline{c}1} = \varphi R_{\overline{c}0}$ which implies $\varphi R_{\overline{c}1} \leq R_{\overline{c}0}$.

- Constraint ($\overline{c}0 \to \underline{c}1$): We have

$$R_{\overline{c}0} \geq \varphi R_{\overline{c}1} \geq \varphi \left[R_{\underline{c}1} - \delta \sum_{\theta_j \in \Theta_i} \alpha_{\theta_j|1} q_{\underline{c}1,\theta_j} \right] \geq \varphi R_{\underline{c}1} - \delta \sum_{\theta_j \in \Theta_i} \alpha_{\theta_j|0} q_{\underline{c}1,\theta_j}$$

where the first inequality comes from ($\overline{c}0 \to \overline{c}1$), the second from ($\overline{c}1 \to \underline{c}1$) and the last inequality holds because $\frac{\alpha_{0|0}}{\alpha_{1|0}} \alpha_{\theta_j|1} \leq \alpha_{\theta_j|0} \quad \forall \theta_j \in \Theta$.

■

Proof of Proposition 3.2

Define $\psi^*_{\bar{c}x_i,x_j} \equiv \pi_{\bar{c}x_i,x_j} - \delta\frac{\alpha_{\underline{c}1,x_j}}{\alpha_{\bar{c}x_i,x_j}}\lambda^*_{\underline{c}1\to\bar{c}x_i} - \delta\frac{\alpha_{\underline{c}0,x_j}}{\alpha_{\bar{c}x_i,x_j}}\lambda^*_{\underline{c}0\to\bar{c}x_i}$, $x_i, x_j \in \{0,1\}$, $\psi^*_{\underline{c}1,x_j} = \pi_{\underline{c}1,x_j}$ and $\psi^*_{\underline{c}0,x_j} = \pi_{\underline{c}0,x_j} + \delta\frac{\alpha_{\bar{c}}}{\alpha_{\underline{c}}}\lambda^*_{\bar{c}0\to\underline{c}0}$. By Proposition 3.1, we can always set $\lambda^*_{\underline{c}1\to\bar{c}1} = 1$ and $\lambda^*_{\underline{c}1\to\bar{c}0} = 0$. Since $(\underline{c}0 \to \bar{c}1)$ binds in any optimal scheme, $\lambda^*_{\underline{c}0\to\bar{c}1} > 0$. This implies $\psi^*_{\bar{c}1,x_j} < \psi^{BM}_{\bar{c}1,x_j}$, $\psi^*_{\bar{c}0,x_j} > \psi^{BM}_{\bar{c}0,x_j}$, $\psi^*_{\underline{c}0,x_j} \geq \psi^{BM}_{\underline{c}0,x_j}$ and $\psi^*_{\underline{c}1,x_j} = \psi^{BM}_{\underline{c}1,x_j}$. The optimal scheme solves $\max_q \mathrm{E}\left[q_{\theta_i,\theta_j}\psi^*_{\theta_i,x_j}\right]$ subject to (3.1). Part i) therefore follows. Part ii) and iii) follow since generally $\psi^{BM}_{\bar{c}0,0} < \psi^{BM}_{\underline{c}0,0}$ and $\psi^*_{\bar{c}0,0} < \psi^*_{\underline{c}0,0}$. Part iv) follows since $\psi^{BM}_{\underline{c}1,x_j} > \psi^{BM}_{\theta_j,1}$ and $\psi^*_{\underline{c}1,x_j} > \psi^*_{\theta_j,1}$ $\forall\theta_j \in \{\bar{c}0, \bar{c}1, \underline{c}0\}$.

Proof of Proposition 3.3

From Lemma 3.1, $\psi^{BM}_{\bar{c}1,0} = \pi_{\bar{c}1,0} - \delta\frac{\alpha_{\underline{c}}}{\alpha_{\bar{c}}} > 0$ implies that $q^{BM}_{\bar{c}1,\underline{c}0} = 1$. By Lemma 3.4, $q^*_{\bar{c}x_i,\theta_j} = 0$ $\forall x_i \in \{0,1\}$, $\theta_j \in \Theta$ either if $\alpha_{\bar{c}0|0}\pi_{\bar{c}0,0} + \alpha_{\bar{c}1|0}\pi_{\bar{c}1,0} - \delta\alpha_{\underline{c}} < 0$ and $\alpha_{\bar{c}1|0}\pi_{\bar{c}1,0} - \delta\alpha_{\underline{c}} < 0$ or if $\alpha_{\underline{c}0|0}\pi_{\underline{c}0,0} + \alpha_{\bar{c}1|0}\pi_{\bar{c}1,0} - \delta\alpha_{\underline{c}} < 0$ and $\alpha_{\bar{c}1|0}\pi_{\bar{c}1,0} - \delta\alpha_{\underline{c}1|0} - \delta\left(\frac{\alpha_{1|0}}{\alpha_{\underline{c}1|0} - \alpha_{\underline{c}1|1}}\right)\alpha_{\underline{c}0|0} \leq 0$. Thus, for example when $\pi_{\bar{c}1,0} \in \left(\delta\frac{\alpha_{\underline{c}}}{\alpha_{\bar{c}}}, \delta\frac{\alpha_{\underline{c}}}{\alpha_{\bar{c}1|0}}\right)$ and $\pi_{\bar{c}0,0} < 0$, $q^{BM}_{\bar{c}1,\underline{c}0} = 1$ while $q^*_{\bar{c}x_i,\theta_j} = 0$ $\forall x_i \in \{0,1\}$, $\theta_j \in \Theta$.

Proof of Proposition 3.4

We show that for any class of solution $k \in \{I, II, III\}$, there is some finite threshold $\overline{\gamma}_k < 1$ such that $q^*_{\bar{c}1,\bar{c}1} = 0$ for $\gamma > \overline{\gamma}_k$. By Proposition 3.1, a sufficient condition for $q^*_{\bar{c}1,\bar{c}1} = 0$ is $\pi_{\bar{c}1,\bar{c}1} - \left(1 + \lambda^*_{\underline{c}0\to\bar{c}1}\frac{1+\gamma^2}{1-\gamma^2}\right)\frac{\delta\alpha_{\underline{c}}}{\alpha_{\bar{c}}} < 0$ where the Lagrange multiplier $\lambda^*_{\underline{c}0\to\bar{c}1}$ depends on the solution. Rewriting this condition yields $\lambda^*_{\underline{c}0\to\bar{c}1}\frac{1+\gamma^2}{1-\gamma^2} > \frac{\alpha_{\bar{c}}}{\delta\alpha_{\underline{c}}}\left(S + \frac{\eta}{2} - \delta\right) - 1$ where the right hand side is independent of γ. The first order conditions for $q^*_{\bar{c}1,\bar{c}1}$ and $q^*_{\bar{c}1,\bar{c}0}$ imply that whenever $q^*_{\bar{c}1,\bar{c}0} + q^*_{\bar{c}0,\bar{c}1} < 1$, then necessarily $q^*_{\bar{c}1,\bar{c}1} = 0$. Hence it suffices to consider solutions with $q^*_{\bar{c}1,\bar{c}0} + q^*_{\bar{c}0,\bar{c}1} = 1$ and show that the sufficient condition for $q^*_{\bar{c}1,\bar{c}1} = 0$ holds for $\gamma > \overline{\gamma}_k$. We consider each solution class in turn.

Class I: By Proposition 3.1, any solution with $q^*_{\bar{c}1,\bar{c}0} + q^*_{\bar{c}0,\bar{c}1} = 1$ requires that $\lambda^*_{\underline{c}0\to\bar{c}1} = \frac{\alpha_{\bar{c}}\gamma\eta}{2\delta\alpha_{\underline{c}}}$. Clearly, $\lambda^*_{\underline{c}0\to\bar{c}1}\left(\frac{1+\gamma^2}{1-\gamma^2}\right)$ increases in γ and $\lim_{\gamma\to1}\left(\lambda^*_{\underline{c}0\to\bar{c}1}\left(\frac{1+\gamma^2}{1-\gamma^2}\right)\right) = \infty$ so that a $\overline{\gamma}_I < 1$ exists as required.

Class II: Any solution of this class has $\lambda^*_{\underline{c}0\to\overline{c}1} = 1$ so that $\overline{\gamma}_{II} < 1$ exists.

Class III: By Proposition 3.1, in this class the constraint $(\overline{c}0 \to \underline{c}0)$ has to bind and $(\underline{c}0 \to \overline{c}0)$ does not so that $\lambda^*_{\underline{c}0\to\overline{c}1} > 1$. Since $\lim_{\gamma\to 1}\left(\frac{1+\gamma^2}{1-\gamma^2}\right) = \infty$, there exists $\overline{\gamma}_{III} < 1$.

Define $\overline{\gamma} = \max\{\overline{\gamma}_I, \overline{\gamma}_{II}, \overline{\gamma}_{III}\}$ and the claim is proved.

Proof of Proposition 3.5

We first solve for the solution of a reduced problem and then show that the remaining constraints are satisfied.

Step 1: Consider the problem of $\max_{q,r} U_p = \max_{q,r} 2E\left[q_{\theta_1,\theta_2}\pi_{\theta_1} - r_{\theta_1,\theta_2}\right]$ subject to the resource constraints (3.1), the incentive constraints $(\underline{c}1 \to \overline{c}1)$, $(\underline{c}0 \to \underline{c}1)$, $(\underline{c}0 \to \underline{c}0)$ and the implementability constraint $Q_{\underline{c}0} - Q_{\overline{c}1} \geq 0$. Since the objective function and the constraints are linear in the choice variables (q, r), the local optimality conditions are sufficient for global optimality. We may incorporate the incentive and the implementability constraints by defining $\psi_{\underline{c}1} \equiv \pi_{\underline{c}1}$, $\psi_{\underline{c}0} \equiv \pi_{\underline{c}0} + \frac{\tau}{\alpha_{\underline{c}0}}$, $\psi_{\overline{c}1} \equiv \pi_{\underline{c}1} - \delta\left(\frac{\alpha_{\underline{c}}}{\alpha_{\overline{c}}} + \frac{\alpha_{\underline{c}0}}{\alpha_{\overline{c}1}}\lambda_{\underline{c}0\to\overline{c}1}\right) - \frac{\tau}{\alpha_{\overline{c}1}}$ and $\pi_{\underline{c}0} - \delta\frac{\alpha_{\underline{c}0}}{\alpha_{\overline{c}0}}\lambda_{\underline{c}0\to\overline{c}0}$ where $\lambda_{\theta_i\to\hat{\theta}_i}$ is the Lagrange multiplier for the incentive constraint $(\theta_i \to \hat{\theta}_i)$ and τ is the multiplier for $Q_{\underline{c}0} - Q_{\overline{c}1} \geq 0$. The necessary and sufficient first order conditions for $q_{\theta_i,\theta_j} > 0$ are then

$$\psi_{\theta_i} \geq \max\left\{0, \psi_{\theta_j}\right\}$$

where the Lagrange multipliers have to satisfy $\tau \geq 0$ and $\lambda_{\underline{c}0\to\overline{c}1} \geq 0$, $\lambda_{\underline{c}0\to\overline{c}0} \geq 0$ with a strict inequality only if the respective constraint holds with equality, and additionally $\lambda_{\underline{c}0\to\overline{c}1} + \lambda_{\underline{c}0\to\overline{c}0} = 1$. We will now derive the solutions (where we treat all solutions $r, \tilde{r}$ with $E\left[r_{\theta_i,\theta_j}\right] = E\left[\tilde{r}_{\theta_i,\theta_j}\right] = R_{\theta_i}$ as one equivalence class) for different parameter values.

For $\alpha_{\overline{c}1}\eta \leq \delta\alpha_{\underline{c}}$ (class I), consider $\lambda_{\underline{c}0\to\overline{c}1} = \frac{\eta\alpha_{\overline{c}1}}{\delta\alpha_{\underline{c}}}$, $\lambda_{\underline{c}0\to\overline{c}0} = 1 - \lambda_{\underline{c}0\to\overline{c}1}$ and $\tau = 0$. Then it is readily verified that $\psi_{\overline{c}0} = \psi_{\overline{c}1} \leq S - \delta < \psi_{\underline{c}0} < \psi_{\underline{c}1}$. Therefore, choosing the seller according to the priority $\underline{c}1 \succ \underline{c}0 \succ \overline{c}1 \sim \overline{c}0$ and setting $q_{\theta_i,\theta_j} > 0$ only if $\psi_{\theta_i} \geq 0$ satisfies the first order conditions. Moreover, the necessary condition $Q_{\overline{c}1} = Q_{\overline{c}0}$ for $\lambda_{\underline{c}0\to\overline{c}1} > 0$

and $\lambda_{\underline{c}0\to\overline{c}1} > 0$ holds.

For $\delta\alpha_{\underline{c}} < \alpha_{\overline{c}1}\eta \leq \delta\left(\alpha_{\underline{c}} + \alpha_{\overline{c}1}\right)$ (class II), consider $\lambda_{\underline{c}0\to\overline{c}1} = 1$, $\lambda_{\underline{c}0\to\overline{c}0} = 0$ and $\tau = 0$ which implies $\psi_{\overline{c}0} < \psi_{\overline{c}1} \leq \psi_{\underline{c}0} < \psi_{\underline{c}1}$. Therefore, choosing the seller according to the priority $\underline{c}1 \succ \underline{c}0 \succ \overline{c}1 \succ \overline{c}0$ and setting $q_{\theta_i,\theta_j} > 0$ only if $\psi_{\theta_i} \geq 0$ satisfies the first order conditions.

For $\alpha_{\overline{c}1}\eta > \delta\left(\alpha_{\underline{c}} + \alpha_{\overline{c}1}\right)$ (class III), consider $\lambda_{\underline{c}0\to\overline{c}1} = 1$, $\lambda_{\underline{c}0\to\overline{c}0} = 0$ and $\tau = \left[\eta - \delta - \delta\frac{\alpha_{\underline{c}}}{\alpha_{\overline{c}1}}\right]\left(\alpha_{\overline{c}1} + \alpha_{\underline{c}0}\right)$ which implies $\psi_{\overline{c}0} < \psi_{\overline{c}1} = \psi_{\underline{c}0} < \psi_{\underline{c}1}$. Therefore, choosing the seller according to the priority $\underline{c}1 \succ \underline{c}0 \sim \overline{c}1 \succ \overline{c}0$ and setting $q_{\theta_i,\theta_j} > 0$ only if $\psi_{\theta_i} \geq 0$ satisfies the first order conditions. Moreover, the necessary condition $Q_{\overline{c}1} = Q_{\underline{c}0}$ for $\tau > 0$ holds.

Step 2: Verify that the remaining constraints are satisfied: Note that $R_{\overline{c}1} = R_{\overline{c}0} = 0$ and $R_{\underline{c}1} = R_{\underline{c}0} = \delta Q_{\overline{c}1}$. Therefore, all horizontal ICs are satisfied. In addition, $Q_{\underline{c}1} > Q_{\underline{c}0} \geq Q_{\overline{c}1} \geq Q_{\overline{c}0}$ so that all remaining incentive constraints are also satisfied.

A3.2 Appendix - Optimal Procurement Schemes

All optimal procurement schemes involve $q^*_{\overline{c}0,\underline{c}0} = q^*_{\overline{c}0,\underline{c}1} = q^*_{\overline{c}1,\underline{c}1} = q^*_{\underline{c}0,\underline{c}1} = 0$, $q^*_{\underline{c}1,\underline{c}0} = q^*_{\underline{c}1,\overline{c}0} = 1$, $q^*_{\underline{c}1,\underline{c}1} = 1/2$ and $q^*_{\underline{c}1,\overline{c}1} = 1$ if $\pi_{\underline{c}1,1} > 0$ and $q^*_{\underline{c}1,\underline{c}1} = q^*_{\underline{c}1,\overline{c}1} = 0$ if $\pi_{\underline{c}1,1} < 0$. Define $\lambda^{FR}_{\overline{c}0\to\underline{c}0} = \frac{\alpha_{\underline{c}1|1}+\alpha_{\overline{c}1|0}}{\alpha_{\overline{c}1|0}-\alpha_{\overline{c}1|1}}$ and $\lambda^{FR}_{\underline{c}0\to\overline{c}1} = \frac{\alpha_{1|0}}{\alpha_{\underline{c}1|0}-\alpha_{\underline{c}1|1}}$. The following table contains all solutions where high cost the principal sometimes buys from high cost sellers (i.e. $\exists x \in X,\ \theta_j \in \Theta$, s.t. $q^*_{\overline{c}x,\theta_j} > 0$) and presents the remaining 8 employment probabilities along with the necessary and sufficient conditions for the cases to obtain. If $\alpha_{\overline{c}0|0}\pi_{\overline{c}0,0} + \alpha_{\overline{c}1|0}\pi_{\overline{c}1,0} - \delta\alpha_{\underline{c}} < 0$ and $\alpha_{\overline{c}1|0}\pi_{\overline{c}1,0} - \delta\alpha_{\underline{c}} < 0$ or if $\alpha_{\underline{c}0|0}\pi_{\underline{c}0,0} + \alpha_{\overline{c}1|0}\pi_{\overline{c}1,0} - \delta\alpha_{\underline{c}} < 0$ and $\alpha_{\overline{c}1|0}\pi_{\overline{c}1,0} - \delta\alpha_{\underline{c}1|0} - \delta\left(\frac{\alpha_{1|0}}{\alpha_{\underline{c}1|0}-\alpha_{\underline{c}1|1}}\right)\alpha_{\underline{c}0|0} \leq 0$, then $q^*_{\overline{c}x_i,\theta_j} = 0\ \forall x_i \in \{0,1\},\ \theta_j \in \Theta$.

Solution	$q_{\underline{c}0,\underline{c}0}$	$q_{\underline{c}0,\bar{c}1}$	$q_{\underline{c}0,\bar{c}0}$	$q_{\bar{c}1,\underline{c}0}$	$q_{\bar{c}1,\bar{c}1}$	$q_{\bar{c}1,\bar{c}0}$	$q_{\bar{c}0,\bar{c}1}$	$q_{\bar{c}0,\bar{c}0}$
	Conditions							
I,a	0.5	1	1	0	0.5	0.5	0.5	0.5
	$\alpha_{\bar{c}1\mid 0}\left(\pi_{\bar{c}1,0}-\pi_{\bar{c}0,1}\right)-\delta\alpha_{\underline{c}}\leq 0,$ $\alpha_{\bar{c}1\mid 1}\pi_{\bar{c}1,1}-\delta\alpha_{\underline{c}}-\alpha_{0\mid 1}\left[\alpha_{\bar{c}1\mid 0}\left(\pi_{\bar{c}1,0}-\pi_{\bar{c}0,1}\right)-\delta\alpha_{\underline{c}}\right]\geq 0$							
I,b	0.5	1	1	0	0	$\frac{\alpha_{\underline{c}0,\bar{c}0}+2\alpha_{\underline{c}0,\bar{c}1}}{2\left(\alpha_{\underline{c}0,\bar{c}0}+\alpha_{\underline{c}0,\bar{c}1}\right)}$	$\frac{\alpha_{\underline{c}0,\bar{c}0}}{2\left(\alpha_{\underline{c}0,\bar{c}0}+\alpha_{\underline{c}0,\bar{c}1}\right)}$	0.5
	$\alpha_{\bar{c}1\mid 0}\left(\pi_{\bar{c}1,0}-\pi_{\bar{c}0,1}\right)-\delta\alpha_{\underline{c}}\leq 0,$ $\alpha_{\bar{c}1\mid 1}\pi_{\bar{c}1,1}-\delta\alpha_{\underline{c}}-\alpha_{0\mid 1}\left[\alpha_{\bar{c}1\mid 0}\left(\pi_{\bar{c}1,0}-\pi_{\bar{c}0,1}\right)-\delta\alpha_{\underline{c}}\right]\leq 0,$ $\alpha_{\bar{c}1\mid 0}\pi_{\bar{c}1,0}+\alpha_{\bar{c}0\mid 0}\pi_{\bar{c}0,1}-\delta\alpha_{\underline{c}}\geq 0$							
I,c	0.5	1	1	0	0	0.5	0	0.5
	$\alpha_{\bar{c}1\mid 0}\pi_{\bar{c}1,0}-\delta\alpha_{\underline{c}}\leq 0,$ $\alpha_{\bar{c}1\mid 0}\pi_{\bar{c}1,0}+\alpha_{\bar{c}0\mid 0}\pi_{\bar{c}0,1}-\delta\alpha_{\underline{c}}\leq 0,$ $\alpha_{\bar{c}0\mid 0}\pi_{\bar{c}0,0}+\alpha_{\bar{c}1\mid 0}\pi_{\bar{c}1,0}-\delta\alpha_{\underline{c}}\geq 0$							
II,a	0.5	1	1	0	0.5	1	0	0.5
	$\alpha_{\bar{c}1\mid 0}\left(\pi_{\bar{c}1,0}-\max\left\{0,\pi_{\bar{c}0,1}\right\}\right)-\delta\alpha_{\underline{c}}\geq 0,\ \alpha_{\bar{c}1\mid 1}\pi_{\bar{c}1,1}-\delta\alpha_{\underline{c}}\geq 0,$ $\alpha_{\bar{c}1\mid 0}\left(\pi_{\bar{c}1,0}-\pi_{\underline{c}0,1}\right)-\delta\alpha_{\underline{c}}\leq 0$							
II,b	0.5	1	1	0	0	1	0	0.5 or 0
	$\alpha_{\bar{c}1\mid 0}\left(\pi_{\bar{c}1,0}-\max\left\{0,\pi_{\bar{c}0,1}\right\}\right)-\delta\alpha_{\underline{c}}\geq 0,\ \alpha_{\bar{c}1\mid 1}\pi_{\bar{c}1\mid 1}-\delta\alpha_{\underline{c}}\leq 0,$ $\alpha_{\bar{c}1\mid 0}\left(\pi_{\bar{c}1,0}-\pi_{\underline{c}0,1}\right)-\delta\alpha_{\underline{c}}\leq 0$							
III,a	0.5	0.5	1	0.5	0.5	1	0	0.5
	$\alpha_{\bar{c}1\mid 0}\left(\pi_{\bar{c}1,0}-\pi_{\underline{c}0,1}\right)-\delta\alpha_{\underline{c}}\geq 0,$ $\alpha_{\bar{c}1\mid 0}\left(\pi_{\bar{c}1,0}-\pi_{\underline{c}0,1}\right)-\delta\left(\lambda^{FR}_{\underline{c}0\to\bar{c}1}\alpha_{\underline{c}0\mid 0}+\alpha_{\underline{c}1\mid 0}+\alpha_{\bar{c}1\mid 0}\left(\lambda^{FR}_{\underline{c}0\to\bar{c}1}-1\right)\right)\leq 0,$ $\alpha_{\bar{c}1\mid 1}\pi_{\bar{c}1,1}-\delta\alpha_{\underline{c}}-\frac{\alpha_{\underline{c}0\mid 1}\left[\alpha_{\bar{c}1\mid 0}\left(\pi_{\bar{c}1,0}-\pi_{\underline{c}0,1}\right)-\delta\alpha_{\underline{c}}\right]}{\alpha_{\bar{c}0\mid 1}+\alpha_{\underline{c}0\mid 0}}\geq 0$							
III,b	0.5	$\frac{0.5\alpha_{\underline{c}0\mid 0}}{\alpha_{\bar{c}1\mid 0}+\alpha_{\underline{c}0\mid 0}}$	1	$\frac{\alpha_{\bar{c}1\mid 0}+0.5\alpha_{\underline{c}0\mid 0}}{\alpha_{\bar{c}1\mid 0}+\alpha_{\underline{c}0\mid 0}}$	0	1	0	0.5 or 0
	$\alpha_{\bar{c}1\mid 0}\left(\pi_{\bar{c}1,0}-\pi_{\underline{c}0,1}\right)-\delta\alpha_{\underline{c}}\geq 0,$ $\alpha_{\bar{c}1\mid 0}\left(\pi_{\bar{c}1,0}-\pi_{\underline{c}0,1}\right)-\delta\left(\lambda^{FR}_{\underline{c}0\to\bar{c}1}\alpha_{\underline{c}0\mid 0}+\alpha_{\underline{c}1\mid 0}+\alpha_{\bar{c}1\mid 0}\left(\lambda^{FR}_{\underline{c}0\to\bar{c}1}-1\right)\right)\leq 0,$ $\alpha_{\bar{c}1\mid 1}\pi_{\bar{c}1,1}-\delta\alpha_{\underline{c}}-\frac{\alpha_{\underline{c}0\mid 1}\left[\alpha_{\bar{c}1\mid 0}\left(\pi_{\bar{c}1,0}-\pi_{\underline{c}0,1}\right)-\delta\alpha_{\underline{c}}\right]}{\alpha_{\bar{c}0\mid 1}+\alpha_{\underline{c}0\mid 0}}\leq 0,$ $\alpha_{\underline{c}0\mid 0}\pi_{\underline{c}0,1}+\alpha_{\bar{c}1\mid 0}\pi_{\bar{c}1,0}-\delta\alpha_{\underline{c}}\geq 0$							
III,c	0.5	0	1	0.5	0	1	0	0.5 or 0
	$\alpha_{\bar{c}1\mid 0}\pi_{\bar{c}1,0}-\delta\alpha_{\underline{c}}\geq 0,\ \alpha_{\underline{c}0\mid 0}\pi_{\underline{c}0,0}+\alpha_{\bar{c}1\mid 0}\pi_{\bar{c}1,0}-\delta\alpha_{\underline{c}}\geq 0,$ $\alpha_{\underline{c}0\mid 0}\pi_{\underline{c}0,1}+\alpha_{\bar{c}1\mid 0}\pi_{\bar{c}1,0}-\delta\alpha_{\underline{c}}\leq 0,$ $\alpha_{\bar{c}1\mid 0}\pi_{\bar{c}1,0}-\delta\alpha_{\underline{c}1\mid 0}-\delta\lambda^{FR}_{\underline{c}0\to\bar{c}1}\alpha_{\underline{c}0\mid 0}\leq 0$							
III,d	0.5	0	1	1	0.5	1	0	0.5
	$\alpha_{\bar{c}1\mid 0}\left(\pi_{\bar{c}1,0}-\pi_{\underline{c}0,1}\right)-\delta\left(\lambda^{FR}_{\underline{c}0\to\bar{c}1}\alpha_{\underline{c}0\mid 0}+\alpha_{\underline{c}1\mid 0}+\alpha_{\bar{c}1\mid 0}\left(\lambda^{FR}_{\underline{c}0\to\bar{c}1}-1\right)\right)\geq 0,$ $\alpha_{\bar{c}1\mid 0}\left(\pi_{\bar{c}1,0}-\pi_{\underline{c}0,1}\right)-\delta\left(\lambda^{FR}_{\underline{c}0\to\bar{c}1}\alpha_{\underline{c}0\mid 0}+\alpha_{\underline{c}1\mid 0}\right)\geq 0$							
III,e	0.5	0	1	1	0	1	0	0.5 or 0
	$\alpha_{\bar{c}1\mid 0}\left(\pi_{\bar{c}1,0}-\pi_{\underline{c}0,1}\right)-\delta\left(\lambda^{FR}_{\underline{c}0\to\bar{c}1}\alpha_{\underline{c}0\mid 0}+\alpha_{\underline{c}1\mid 0}+\alpha_{\bar{c}1\mid 0}\left(\lambda^{FR}_{\underline{c}0\to\bar{c}1}-1\right)\right)\geq 0,$ $\alpha_{\bar{c}1\mid 1}\pi_{\bar{c}1,1}-\delta\alpha_{\underline{c}1\mid 0}-\delta\alpha_{\underline{c}0\mid 0}\lambda^{FR}_{\underline{c}0\to\bar{c}1}\leq 0,$ $\alpha_{\bar{c}1\mid 0}\pi_{\bar{c}1,0}-\delta\alpha_{\underline{c}1\mid 0}-\delta\lambda^{FR}_{\underline{c}0\to\bar{c}1}\alpha_{\underline{c}0\mid 0}\geq 0,$ $\alpha_{\underline{c}0\mid 0}\pi_{\underline{c}0,0}+\alpha_{\bar{c}1\mid 0}\pi_{\bar{c}1,0}-\delta\alpha_{\underline{c}}\geq 0$							
III,f	0	0	0	1	0	1	0	0
	$\alpha_{\bar{c}1\mid 0}\left(\pi_{\bar{c}1,0}-\pi_{\underline{c}0,1}\right)-\delta\left(\lambda^{FR}_{\underline{c}0\to\bar{c}1}\alpha_{\underline{c}0\mid 0}+\alpha_{\underline{c}1\mid 0}+\alpha_{\bar{c}1\mid 0}\left(\lambda^{FR}_{\underline{c}0\to\bar{c}1}-1\right)\right)\geq 0,$ $\alpha_{\bar{c}1\mid 0}\pi_{\bar{c}1,0}-\delta\alpha_{\underline{c}1\mid 0}-\delta\lambda^{FR}_{\underline{c}0\to\bar{c}1}\alpha_{\underline{c}0\mid 0}\geq 0,$ $\alpha_{\underline{c}0\mid 0}\pi_{\underline{c}0,0}+\alpha_{\bar{c}1\mid 0}\pi_{\bar{c}1,0}-\delta\alpha_{\underline{c}}\leq 0$							

Table 3.2: Optimal schemes

Chapter 4

Certification and Minimum Quality Standards under Imperfect Competition

4.1 Introduction

Policy makers are frequently concerned about insufficient free-market qualities of goods and services. Recent examples are energy efficiency or safety properties of electrical devices, the quality of food, hazardous contents of play-toys, or the sustainability in the production of timber.

Previous research has identified various reasons for the asserted lack of quality. Externalities that are not fully taken into account either by consumers or by producers provide a convincing explanation. Furthermore, firms may under-provide quality if the marginal consumer has a lower taste for quality than the average customer [Spence (1975)]. Another reason which has recently gained more attention in public discussions is that customers often find it difficult or impossible to discern the products' quality prior to purchase. Firms respond to this problem in various ways like promising warranties, signaling quality with the price or developing a reputation for high quality goods. However, research on these market instruments has shown that they may alleviate the problem but rarely provide a fully satisfactory solution.[1]

[1] See e.g. Gal-Or (1989) for warranties, Daughety and Reinganum (2008b) for signaling and the liter-

Hence, there remains scope for governmental intervention. The adoption of minimum quality standards (MQS) and the certification of high grade goods are popular instruments to protect customers from poor quality products. A MQS restricts firms not to sell goods whose quality is below this standard. Examples are abundant and include safety standards for manufactured products, contents requirements for textiles or food, occupational licensing for professional services or environmentally related standards. Certification is a process where a third party verifies if a product fulfills certain criteria. Often, the government designs and enforces certification either directly, or promotes the creation of non-governmental organizations for these tasks.[2] Certification is used for similar goods as MQS: Among others, the New Car Assessment Programme (NCAP) inspects the security of cars, the CFA Institute certifies an additional qualification in accounting and finance and the Forest Stewardship Council (FSC) certifies timber from sustainable forestry.[3]

This chapter seeks to investigate the impact of MQS and of certification when adopted in oligopolistic markets. In particular, we are interested in the effects of these instruments on the firms' quality choices, on the equilibrium prices and on entry. Furthermore, we identify situations where we can rank these instruments in terms of social welfare. We focus on certification that indicates when a product exceeds some quality threshold but does not disclose the actual product's quality. Awarding a good with a certificate if it meets a publicly known criterion is very popular and for example used by the FSC and the CFA Institute. In contrast to the related literature surveyed below, we account for the observation that in practice, firms may build a reputation for producing goods above the lower quality bound even without any intervention.

To address these issues, we develop a model in which firms first decide on entry and on the quality of their products and then compete in prices with vertically differen-

ature reviewed below for reputation.

[2]In contrast to a MQS, certification does not require the power of coercion and can in principle be provided both by private and by public institutions. As discussed below, firms are often unable to build up a private certifying institution because of credibility issues.

[3]The NCAP has been founded in 1996 by several European national road administrations (See http://www.euroncap.com/home.aspx). The CFA Institute is a global not-for profit institution (See https://www.cfainstitute.org/about/history/Pages/index.aspx). FSC is an independent, non-governmental, not-for-profit organization established to promote the responsible management of the world's forests (See http://www.fsc.org).

Another example is the German label "Blauer Engel" which is directly awarded by the state (See http://www.blauer-engel.de).

tiated products. Some customers may discern the actual grade of the goods. These customers refrain from buying poor quality goods for an inadequate price. The remaining consumers may not observe the actual quality of the offered goods and base their purchase decision on the prices and on their belief concerning the qualities. The uninformed customers anticipate that the fear of losing revenues from informed customers incentivizes firms to produce goods above the lowest feasible quality level. Yet, since uninformed customers may not respond to the goods' actual quality, this reputational mechanism works imperfectly: In equilibrium, each firm's product quality is below the level it would provide if all customers were fully informed. As the fraction of informed customers shrinks, the quality which can be credibly produced and thus the equilibrium profits plummet.

We study an environment where social welfare could be improved by inducing firms to raise their quality, even if all consumers observed the goods' actual qualities. In our setup, the difference between marginal and average consumer valuation and the firms' desire to offer differentiated products so as to alleviate price competition induces them to provide goods of too low quality. The incentive to invest in quality is further reduced because some consumers cannot discern the actual quality as laid out above. The government may intervene before firms set up the production technology and either forbid to produce below some MQS or offer the opportunity that firms costlessly obtain a certificate if they produce goods with a quality not below some publicly announced threshold.

When the certification standard is set above the highest quality level that would be offered in an unregulated market, *all* firms may be induced to raise their quality. To see this, suppose two firms enter. In the unregulated market, each firm's profit would increase if it could commit to raise its quality level somewhat. With the certificate, a firm can demonstrate that the quality of its product meets at least the certification standard. Therefore, one firm will match this level if the standard is not too demanding. The remaining firm still produces a good below this level, because it would trigger a too intense price competition otherwise. Since the first firm produces a high quality in order to meet the certification threshold, the second firm may also raise its quality compared to the no-intervention level without compromising the degree of differentiation between the goods. As we show, this form of certification leads to more differentiated goods and to higher quality-weighted prices. Yet, we find that a suitable certification standard may increase both consumer surplus and industry profit.

When setting the certification standard, the government has to assure that obtaining the certificate is attractive for producers. A firm will only participate in the certification of its product if the profit from producing a good of sufficient quality to meet the threshold is not below the profit from selling a product without the certificate. Therefore, the certification threshold has to be set such that the required investments for quality are not unduly high compared to the expected revenues. In addition, the decision to sell a certified product crucially depends on the profit that a firm expects to earn from selling a good without the certificate. As argued above, the quality level and the resulting profit which can be sustained by help of reputational considerations without the certificate is low when the fraction of informed customers is sufficiently small. This leads to the surprising result that if the goods' quality is difficult to discern, a firm is more reliant on certification and tends to comply with a higher certification standard. The government may exploit the informational problems and achieve a higher maximum quality by help of certification than in case the quality was perfectly observable.

Importantly, by conferring a certificate only if a good's quality meets some fixed threshold instead of fully disclosing the product's quality, a firm may be induced to produce goods above the quality it would choose when it could credibly communicate its quality. Indeed, concerning the FSC certificate mentioned above, the timber industry's attempts to install another certification system with a relatively soft standard provide evidence that some firms would prefer less restrictive certification.[4] The observation that these attempts have been ineffective so far exemplifies that firms often lack the power to install certification in which consumers have confidence. This in turn allows the government - or institutions which may credibly enforce certification - to step in and manipulate the qualities in the market by setting suitable certification standards.[5]

[4]For a detailed report, see http://www.fern.org/sites/fern.org/files/media/documents/document_1890_1900.pdf.

[5]Our analysis is based on the assumption that policy interventions are adopted so as to maximize social welfare. Although we speak of governmental interventions for concreteness, our results also apply to non-governmental institutions whose objective is to maximize social welfare. For example, the FSC is a non-governmental, not-for-profit organization that promotes "environmentally appropriate, socially beneficial, and economically viable management of the world's forests" (See http://www.fsc.

This may also explain why simple threshold schemes are so popular.

The impact of a MQS differs vastly compared to that of certification. By restricting the admitted quality range and thus the firms' ability to differentiate their goods, a MQS intensifies price competition and requires higher minimum investments in quality. Therefore, adopting a MQS reduces the firms' profits and may deter them from entry. This is in line with Ronnen (1991) who analyzes a setup that essentially corresponds to ours in the limit when quality is observable to all customers. We show that this problem is particularly severe when the goods' quality is opaque so that the firms' ability to credibly produce a reasonable quality is further restricted. In contrast, our analysis indicates that suitable certification does not restrict entry. We thus conclude that suitable certification may improve welfare more than a MQS if only few customers are informed. If instead almost all consumers may observe the actual quality, firms' need not rely on certification so that the welfare gains from adopting a suitable MQS are higher.

Related Literature

Several authors have emphasized that for reputational reasons, firms may credibly undertake costly actions which are unobservable to consumers, like producing high quality products.[6] In their seminal work, Klein and Leffler (1981) have pointed out that consumers may expect firms to undertake costly actions if the fear of losing reputation exceeds the temporary advantage of cheating. This idea has been further refined and applied both on competitive and on monopolistic markets.[7] In our model, firms may rely on reputation since we assume that the production technology determines the quality of the goods and cannot be changed once it is in place. This allows consumers that buy the products at a later time to infer the quality from the experiences of

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`org/vision_mission.html`). This objective could be interpreted as maximizing social welfare.

[6] See Bar-Isaac and Tadelis (2008) for a survey.

[7] Klein and Leffler (1981) consider fully competitive markets with exogenously given equilibrium quality levels. Shapiro (1982) endogenizes the quality choice of a monopolist and Shapiro (1983) also endogenizes the customers' demand for different quality levels in a competitive environment. Shapiro (1983) also shows that introducing a MQS harms consumers with a low taste for quality and helps those that highly value quality. In a competitive model, Hoerner (2002) adds uncertainty about the firms' ability to provide high quality and presents an equilibrium in which it is indeed optimal for customers to abandon a firm after a negative experience. In all of these papers, a firm may decide on the quality repeatedly.

earlier customers. In this respect, our model is similar to Shapiro (1986), who studies the interaction between investment in human capital and the provision of high quality services in a competitive environment. Shapiro (1986) also analyzes the effect of certifying or imposing a minimum standard on the investment in human capital. He assumes that the quality of the services has a binary support (low/high) and that the policy interventions affect the quality only indirectly through the human capital. He is thus rather interested in the market share of the low and high quality segment instead of the endogenous quality levels. In contrast, our aim is to explore the effect of policy interventions on markups, quality choices and entry when firms have market power. Bar-Isaac (2005) and Dana and Fong (2008) also study reputation in an oligopolistic market. However, they assume that quality may be either low or high and concentrate on the non-monotonic effect of competition on the ability to sustain high quality.

Our model is closely related to the literature on oligopolistic competition and minimum quality standards in markets of vertically differentiated goods. Gabszewicz and Thisse (1979) and Shaked and Sutton (1982) point out that vertical product differentiation is a common strategy used by oligopolistic firms in order to relax competition and thus to raise profits.[8] Based on this insight, Ronnen (1991) demonstrates that regulatory authorities may increase welfare by adopting a MQS if improving a product's quality requires fixed investments but no variable costs.[9] Crampes and Hollander (1995) address the same question when firms incur variable costs for quality and obtain the same qualitative effect of introducing a MQS on total welfare if the costs of quality are convex enough.[10] Based on the commonly used setup of Ronnen (1991), we introduce consumers that may not discern the actual quality of the products in the market. Apart from adding a relevant aspect, this ingredient allows us to compare the efficiency of certification and MQS. Since the aim of certification is to reduce the amount of asymmetric information in the market, this policy instrument is void in an

[8]Wauthy (1996) endogenizes the market coverage.

[9]The same setup which was originally inspired by Tirole (1988) has been also analyzed by Choi and Shin (1992). Based on the same setup, Lehmann-Grube (1997) demonstrates that the high quality provider usually earns higher profits and shows that this result survives when firms chose their quality sequentially. In a slightly modified setup, Motta (1993) compares price and quantity competition.

[10]Kuhn (2007) shows that MQS may be detrimental in a setup with variable cost of quality if consumers derive some baseline benefits from the consumption of the good that is independent from its quality. Valletti (2000) demonstrates that a mildly restrictive MQS unambiguously reduces total welfare when firms compete in quantities instead of prices.

environment where all participants are perfectly informed.

Finally, our analysis investigates the effects of governmental certification as an instrument to reduce informational asymmetries. Assuming perfect commitment and treating the seller's quality as exogenously determined, Lizzeri (1999) studies the profit maximizing policy of a monopolistic and of oligopolistic certifiers. Based on a similar framework, Albano and Lizzeri (2001) endogenize the seller's quality choice. Strausz (2005) as well as Mathis, McAndrews, and Rochet (2009) concentrate on the incentives of certifiers to honestly rate the seller's quality. All of these models focus on the behavior of certifiers and consider a rather simple structure of the market for the rated goods.[11] In contrast, we abstract from problems associated with dishonest certification and focus on how costless certification affects the rated firms' behavior in a rather complex competitive environment. We are particularly interested in the interplay between certification and reputation which has not been investigated by any other paper to the best of our knowledge.[12] Daughety and Reinganum (2008a) examine how costly certification affects signaling via prices when the seller's quality is exogenously given. Since in our model the firms' quality choice is endogenous, we may also analyze the effect of certification on the traded quality. Moreover, while Daughety and Reinganum (2008a) consider disclosure of the actual quality level, we consider certification where a label is conferred whenever a product's quality does not lie below some threshold.

The remainder of this chapter is organized as follows. The basic framework is developed in Section 4.2. Section 4.1 derives the equilibrium prices for given quality choices. In Section 4.4, we solve for the equilibrium qualities and the number of active firms in the absence of any intervention. In Section 4.5 we carefully investigate the effects of certification on customers' beliefs and on the equilibrium quality choices. Moreover, we characterize the welfare maximizing certification standard. Section 4.6 compares the impact of a MQS to that of certification. Section 4.7 discusses an alternative modeling approach and Section 4.8 concludes.

[11]Similar to our model, Heyes and Maxwell (2004) consider the effect of MQS and certificates, but do not model the marked for the rated goods explicitly.

[12]Biglaiser and Friedman (1994) consider the impact of middlemen in a model of reputation but do not analyze how the power of reputation affects welfare-optimal certification.

4.2 The Model

There are two identical potential entrants to the market. Each firm i is constrained to offer products of a single quality $q_i \in Q \equiv [0, \infty)$. Both firms face the same costs $C(q)$ of installing the production technology to produce goods of quality q. The cost of the production technology is increasing and strictly convex for all feasible quality levels, that is $C^{'}(\cdot) > 0$ and $C^{''}(\cdot) > 0$. It also satisfies the regularity conditions $C(0) = C^{'}(0) = 0$ and $\lim_{q\to\infty} C^{'}(q) = \infty$. Once the production technology is in place, any quantity of products may be produced at zero unit production cost.

The demand comes from a continuum of consumers of measure one. They differ in their taste for quality and are indexed by their taste for quality θ which is distributed uniformly on the unit interval $[0, 1]$. Consumers buy up to one unit of the good. A consumer with taste θ derives a net surplus of $u(\theta, q, p) = \theta q - p$ when buying a product of quality q at price p.

Consumers also differ in their information about the product quality and the time they enter the market. The fraction $1 - \alpha$ of consumers enters the market early and cannot observe the actual quality of the products. Since all of these customers have the same level of information, they form a uniform belief $\hat{q}_i$ about the expected quality of each offered good i. The remaining fraction $\alpha \in (0, 1]$ of consumers enters the market later and observes the quality before deciding from which firm to buy or to abstain. This assumption captures that the available information about a product's quality increases in the time the good is on the market and simplifies the analysis.[13]

All consumers are short lived and have to make their decision in the same period when they enter the market. The information and the entry time of consumers is supposed to be independent of the taste θ.

Governmental intervention plays an important role in our model.[14] Before firms and consumers move, the government may either adopt a MQS or offer firms the opportunity to certify their production technology. Once the minimum standard q^{MQS} is

[13]In case of experience goods, some consumers have to buy and use the product before they can share their experiences with others.

[14]We use the term *governmental* intervention in order to highlight that these instruments are used to increase social welfare. We also abstract from problems like commitment, honesty etc.

enacted, no firm may sell products of a lower quality. In case the government provides certification, it determines a certification threshold q^C. Any firm that installs a quality level not below q^C obtains a certificate that all consumers and firms may observe. Note that the certificate only guarantees that a firm produces at least the quality level q^C; certification does not disclose the actual quality level of a particular firm.

The competition between firms takes place in three stages. At each stage, the firms make their choices simultaneously. In the first stage, each firm decides whether or not to enter the market. If a firm enters, it also chooses the quality level of the production technology. In case of certification, all firms that have installed at least q^C obtain the certificate. Before the second stage begins, each firm in the market learns whether a competitor has entered but does not observe the rival's quality level. Like consumers, each firm only observes if the competitor has obtained a certificate. Because firms have the same information about their rivals as the customers, we suppose that they form the same belief. In the second stage, each active firm sets the price of its product. Based on the prices and on their beliefs, early consumers decide which product to buy or whether to abstain.[15] Before the third stage begins, the firms learn their competitor's quality level. In the third stage, they may change their prices. Then late consumers enter and choose their preferred product. The sequential structure of the game reflects that changing the price can be done quickly while a change of the production technology is usually very time-consuming. For simplicity, firms do not discount their profits.[16]

Our equilibrium concept is Perfect Bayesian Equilibrium. Furthermore, we focus on pure strategy equilibria where beliefs are passive in the sense that consumers and rivals do not revise their beliefs about a firm's quality when the latter charges an unexpected price.[17] Since the production technology does not affect the marginal costs,

[15]For concreteness, we assume that consumers which are indifferent between several quality levels buy the highest one.

[16]If there is a discount factor $\delta < 1$ between the two selling stages, then the resulting game with the proportion of late consumers α yields the same equilibrium qualities as our setup without discounting and the fraction $\tilde{\alpha} \equiv \alpha\delta$ of late consumers.

[17]The restriction to equilibria with *passive beliefs* is commonly made in the literature on vertical contracts and discussed in Rey and Tirole (2007). In our setup, the restriction to passive beliefs does not affect the equilibrium qualities: The actual quality only affects the firms' revenues of the informed customers which do not depend on the uninformed customers' beliefs. Hence, both firms choose the quality so as to maximize the revenues of the informed customers minus the costs for quality. Yet, the beliefs affect the equilibrium prices and thus the profits in the second stage which has an impact on the firms'

firms cannot use the price as a signal for high quality.[18] If a single firm has entered, we use the index M. In case two firms have entered, we assign the firm that produces a weakly higher quality the index H and the remaining one the index L so that $q_H \geq q_L$. Accordingly, we will refer to firm H (L) as the high (low) quality producer. In equilibrium, the beliefs $(\hat{q}_L, \hat{q}_H)$ or $\hat{q}_M$ have to be correct, consumers make the utility maximizing choice and each firm's strategy maximizes its profits given the belief and the equilibrium strategy of the competitor and the consumers.[19]

4.3 Price Equilibrium

We first examine the equilibrium prices that obtain in the third stage. Ronnen (1991) analyzes a setup where *all* customers know the quality of the offered products, that is $\alpha = 1$. In what follows, we refer to this special case as *full information.* For convenience, we express the results in terms of the quality-deflated or hedonic price $x \equiv p/q$. If both firms have entered, $r \equiv q_H/q_L$ denotes the relative quality of the products. Ronnen (1991) provides a detailed derivation of the following results:

Lemma 4.1 (Ronnen (1991))**.** *Suppose that both firms have entered the market and that all consumers observe the actual quality level with $q_H \geq q_L$. In the unique price equilibrium, the hedonic prices of firm L and H are $x_L^* = \frac{r-1}{4r-1}$ and $x_H^* = \frac{2(r-1)}{4r-1}$, respectively. Defining $z^* = \frac{2r-1}{4r-1}$, consumers with $\theta \in [0, x_L^*)$ abstain from buying, those with $\theta \in [x_L^*, z^*)$ buy from firm L and those with $\theta \in [z^*, 1]$ buy from firm H in equilibrium. The equilibrium revenues of firm L and H are $R^L(q_L, q_H) \equiv q_L x_L^* (z^* - x_L^*) = q_L \frac{r(r-1)}{(4r-1)^2}$ and $R^H(q_L, q_H) \equiv q_H x_H^* (1 - z^*) = q_H \frac{4r(r-1)}{(4r-1)^2}$, respectively.*

decision to enter the market and on welfare.

[18]For example Daughety and Reinganum (2008b) examine a model of quality signaling in a market of imperfect competition. In this model, the firms' quality is exogenously given and high quality firms necessarily earn lower profits in any separating equilibrium. Thus, when endogenizing the quality choice, firms would not deliberately invest in high quality.

[19]Note that the customers' beliefs about the sellers quality may differ even though the firms are ex-ante identical. In particular, our restriction to passive beliefs implies that if firms produce vertically differentiated goods in equilibrium but the low quality producer deviates and charges the same price as its rival, the consumers maintain their negative beliefs about this firm. In this case, it may seem unorthodox that customers correctly identify which firm produces a lower quality even though they cannot distinguish between the prices. Yet, customers often have a rough understanding which of two products has a higher quality, even when not being able to evaluate the exact quality of the goods.

A firm's revenue increases when holding that firm's quality level fixed and raising the degree of disparity r, as more differentiated products give rise to higher equilibrium prices. If both firms produce the same quality, then the equilibrium prices are zero. The marginal revenues of the low quality and high quality firm with respect to the own quality are $R^L_{q_L} = \frac{r^2(4r-7)}{(4r-1)^3}$ and $R^H_{q_H} = \frac{4r(2-3r+4r^2)}{(4r-1)^3}$, respectively.[20] They depend on q_L and q_H only through the relative quality r.[21] It is easy to verify that for $q_H > q_L$, both firms' revenues are concave in the own quality level. Besides, the cross derivative $R^i_{q_L,q_H}$ for $i \in \{L, H\}$ is strictly positive on the relevant range, meaning that the quality levels are strategic complements.[22]

Now we turn to the equilibrium prices that obtain in the second stage. Observe that the prices in the second stage have no effect on the equilibrium revenues of the third stage. Since customers do not revise their belief $\hat{q}_i$ after they observe unexpected prices, we may conclude:

Corollary 4.1. *For given beliefs $(\hat{q}_L, \hat{q}_H)$, the equilibrium prices x_i^* in stage two are determined by the formulas given by Lemma 4.1 after replacing q_i by $\hat{q}_i$.*

Based on Lemma 4.1 and Corollary 4.1, the total revenues for the beliefs $(\hat{q}_L, \hat{q}_H)$ and the actual quality levels (q_L, q_H) are

$$\tilde{R}^i = (1-\alpha)\, R^i(\hat{q}_L, \hat{q}_H) + \alpha R^i(q_L, q_H) \quad , \; i \in \{L, H\}\,.$$

If the beliefs are correct, then $\tilde{R}^i = R^i$. Let $\Pi^i(q_L, q_H)$ denote the profits of firm i in case the consumers correctly anticipate that the provided qualities are q_L and q_H. Deducting the cost of installing the production technology from the equilibrium revenue yields $\Pi^i(q_L, q_H) = R^i(q_L, q_H) - C(q_i)$.

For later use, we define a standard welfare measure which is the difference between the aggregate value of consumption and the cost of supply: $W = q_L \int_{x_L^*}^{z^*} \theta \mathrm{d}\theta + q_H \int_{z^*}^{1} \theta \mathrm{d}\theta - C(q_L) - C(q_H)$. Suppose two firms have entered the market and provide goods of the quality levels (q_L, q_H) and the beliefs are correct. Then social welfare at the ensuing

[20] Subscripts refer to partial derivatives.

[21] Formally, $R^L_{q_L}$ and $R^H_{q_H}$ are homogeneous of degree zero.

[22] It is readily verified that $R^L_{q_L}$ increases in r while $R^H_{q_H}$ decreases in r.

equilibrium prices amounts to

$$W(q_L, q_H) = \frac{q_H \left(12r^2 - r - 2\right)}{2(4r-1)^2} - C(q_L) - C(q_H) \, . \tag{4.1}$$

In case a single firm has entered the market and thus becomes monopolist the revenue maximizing quality-deflated price is $x_M^* = \frac{1}{2}$. Assuming that consumers correctly anticipate the actual quality, the equilibrium revenue and the welfare are $R^M(q_M) \equiv q_M x_M^* \left(1 - x_M^*\right) = \frac{1}{4} q_M$ and $W^M(q_M) = \frac{3}{8} q_M - C(q_M)$, respectively.[23]

4.4 Quality Choice and Entry without Intervention

This section examines the equilibrium quality levels obtaining in the absence of any policy intervention. We are particularly interested in how the fraction of informed customers α affects the equilibrium quality investment decisions.

We first derive the quality choice that maximizes firm L's profit $(1-\alpha) R^L(\hat{q}_L, \hat{q}_H) + \alpha R^L(q_L, \hat{q}_H) - C(q_L)$ when the beliefs are $(\hat{q}_L, \hat{q}_H)$. As mentioned above, revenues from uninformed customers do not depend on the actual quality q_L. Therefore, the restricted best response $b^L(\hat{q}_H, \alpha) \equiv \arg\max_{0 \le q \le \hat{q}_H} \alpha R^L(q, \hat{q}_H) - C(q)$ of the low quality firm depends on the belief $\hat{q}_H$ but not on the belief $\hat{q}_L$. This observation will be important for our analysis below. Since any belief $\hat{q}_L$ that differs from $b^L(\hat{q}_H, \alpha)$ is inconsistent with the firm's desire to maximize its profit, we will also refer to $b^L(\hat{q}_H, \alpha)$ as the quality which firm L may *credibly* produce. Note that b^L is only a restricted best response since firm L is constrained to provide a quality below $\hat{q}_H$.[24] The properties of R^L and C ensure that $b^L(\hat{q}_H, \alpha) > 0$ and that the associated optimality condition

$$\alpha R^L_{q_L}(q_L, \hat{q}_H) = C'(q_L) \tag{4.2}$$

uniquely characterizes the best response whenever $b^L(\hat{q}_H, \alpha) < \hat{q}_H$. Observe that the best response b^L approaches 0 as $\alpha \to 0$ since $R^L_{q_L}$ is bounded above and $C'(0) = 0$.

[23] These formulae obtain from the ones already presented in the limit for $q_L \to 0$.

[24] We will later assure that the low quality firm has no incentives to provide a higher quality than its rival.

If $b^L(q_H,\alpha) < q_H$, then differentiating equation (4.2) and using the homogeneity of degree 0 of $R^L_{q_L}$ yields[25]

$$b^L_{q_H}(q_H,\alpha) = \frac{b^L(q_H,\alpha)}{q_H\left(1 - \frac{C''(b^L(q_H,\alpha))}{\alpha R^L_{q_L,q_L}(b^L(q_H,\alpha),q_H)}\right)} < \frac{b^L(q_H,\alpha)}{q_H}.$$

Similarly, let $b^H(\hat{q}_L,\alpha) \equiv \arg\max_{q\geq\hat{q}_L} \alpha R^H(\hat{q}_L,q) - C(q)$ denote the firms' restricted best response when having to provide a quality which is at least as high as the belief $\hat{q}_L$ about the competitor's quality. Since $R^H_{q_H}$ decreases in q_H and C is convex, b^H is uniquely defined by the optimality condition

$$\alpha R^H_{q_H}(\hat{q}_L,q_H) \quad = \quad C'(q_H) \tag{4.3}$$

whenever $b^H(\hat{q}_L,\alpha) > \hat{q}_L$.[26] In this case, after differentiating (4.3) we obtain

$$b^H_{q_L}(q_L,\alpha) = \frac{b^H(q_L,\alpha)}{q_L\left(1 - \frac{C''(b^H(q_L,\alpha))}{\alpha R^H_{q_H,q_H}(q_L,b^H(q_L,\alpha))}\right)} < \frac{b^H(q_L,\alpha)}{q_L}.$$

Since the marginal revenue is downward sloping in the own quality level, conditions (4.2) and (4.3) imply that the restricted best response increases in the fraction of informed customers when holding the rival's quality constant. Hence, $b^L_\alpha(q_H,\alpha) \geq 0$ and $b^H_\alpha(q_L,\alpha) \geq 0$ with a strict inequality if $b^L(q_H,\alpha) < q_H$ and $b^H(q_L,\alpha) > q_L$, respectively.

We can apply a similar reasoning as Ronnen (1991), who has examined the special case with $\alpha = 1$, to establish that at most one equilibrium exists.[27]

Proposition 4.1. *For any $\alpha \in (0,1]$, there is a unique pair of quality levels (q^*_L,q^*_H) that satisfies conditions* (4.2) *and* (4.3)*. When consumers correctly anticipate that the firms produce at these quality levels, the profits are $\Pi^H(q^*_L,q^*_H) > \Pi^L(q^*_L,q^*_H) > 0$. These quality levels are an equilibrium if $C''' \geq 0$.*

[25]This property implies $R^i_{q_L,q_L} + R^i_{q_L,q_H} r = 0$.

[26]In the proof of Proposition 4.1 we show that $b^H(\hat{q}_L,\alpha)$ always exists and is bounded above.

[27]It seems there is a minor mistake in the proof of Ronnen (1991), Theorem 1 which has been corrected by Lehmann-Grube (1997).

Proof. See Appendix A4.1. ■

Proposition 4.1 first stipulates that there is a unique candidate equilibrium where each firm's quality maximizes its profit when one firm is restricted to offer a quality below and the other to choose a quality above that of the rival. This result relies on the observation that the best responses of both firms are increasing and that their slope is bounded and satisfies $b^L_{q_H} b^H_{q_L} < 1$ on the relevant range. Proposition 4.1 further shows that the high quality firm earns higher profits than the low quality provider. This result relies on the functional form of the firms' revenue functions and on the convexity of the costs and is not intuitive. In contrast, after rewriting the equilibrium profit as $\Pi^{L*} = \int_0^{q_L^*} R^L_{q_L}(q, q_H^*) - C'(q)\,\mathrm{d}q$, it is easy to see that the low quality firm earns a positive profit. The optimality condition (4.2) implies that the marginal profit with respect to q_L, $R^L_{q_L}(q_L, q_H) - C'(q_L)$, must be non-negative at (q_L^*, q_H^*). The profit of firm L is thus positive, because each firm's marginal profit decreases in the own quality. Since both firms earn positive profits, we may conclude that it is optimal for them to enter the market in the first place. Moreover, the proposition verifies that firm H has no incentive to deviate to a quality level that lies below that of its rival. Conversely, assuming that the cost of quality exhibits non-decreasing convexity ($C''' \geq 0$) is grossly sufficient to guarantee that it is unprofitable for the low quality firm to deviate to a higher quality level than its competitor.

Importantly, when some consumers cannot discern the actual qualities, each firm would improve its profits if it could commit to producing higher quality goods. In equilibrium, the beliefs are correct, and the optimality conditions (4.2) and (4.3) imply that $\Pi^i_{q_i}(q_L^*, q_H^*) = (1-\alpha) R^i_{q_i}(q_L^*, q_H^*) > 0$. Thus, if a firm increased slightly its quality *and* consumers adapted their beliefs accordingly, the firm's profit would increase. However, the proportion $1-\alpha$ of consumers does not react to changes of the *actual* quality of a good, so that each firm has insufficient incentives to invest in quality. Therefore, it installs a production technology that maximizes $\alpha R^i(q_L, q_H) - C(q_i)$ instead of the whole profit $R^i(q_L, q_H) - C(q_i)$.[28]

For later reference, we note that a single monopolist sets the quality level q_M^* so as to maximize $\alpha R^M(q_M) - C(q_M)$. From $\lim_{q_L \to 0} R^H(q_L, q_M) = R^M(q_M)$ and $R^H_{q_H, q_L} > 0$

[28] A similar result has been discussed by Shapiro (1982) in a monopoly setup.

follows that for any $\alpha > 0$, the equilibrium quality level of the high quality firm exceeds the equilibrium quality level of a monopolist.

Our next result relies on the observations that each firm's best response increases in α and that the qualities are strategic complements.

Lemma 4.2. *The quality levels q_L^* and q_H^* increase in the fraction of informed customers α.*

Proof. In the proof of Proposition 4.1 we argue that q_L^* necessarily satisfies $B(q_L^*, \alpha) = 0$ with $B(q, \alpha) \equiv b^L\left(b^H(q, \alpha), \alpha\right) - q$ and $B_q(q, \alpha) < 0$ on the relevant range. We have $B_\alpha = \left(b_\alpha^L + b_{q_H}^L\right) b_\alpha^H > 0$, where the arguments are omitted for brevity. Therefore, q_L^* increases in α by the implicit function theorem. Since $q_H^* = b^H(q_L^*, \alpha)$ and $b_{q_L}^H > 0$, also q_H^* increases in α. ■

It is useful to relate the equilibrium qualities to those that maximize social welfare in order to assess the scope for governmental interventions. Our interest is centered on instruments that affect the quality choice, but do not directly intervene in the price stage.[29] Accordingly, a pair of quality levels is *second-best*, if the ensuing price equilibrium maximizes welfare: $(q_L^{SB}, q_H^{SB}) \equiv \arg\max_{q_L, q_H} W(q_L, q_H)$. Differentiating equation (4.1) and comparing to the marginal profits yields $W_{q_L} > \Pi_{q_L}^L$ and $W_{q_H} > \Pi_{q_H}^H$.[30] The firms' first order conditions (4.2) and (4.3) imply that $\Pi_{q_i}^i(q_L^*, q_H^*) = (1-\alpha) R_{q_i}^i(q_L^*, q_H^*) > 0$ for $i \in \{L, H\}$. Therefore, we may conclude that *locally* increasing each firm's quality raises welfare. The following Lemma asserts that indeed both second-best quality levels lie above the equilibrium values.

Lemma 4.3. *For any fraction of informed customers $\alpha \in (0, 1]$, both firms' equilibrium quality levels are socially too low: $q_L^{SB} > q_L^*$ and $q_H^{SB} > q_H^*$.*

Proof. See Appendix A4.1. ■

According to Lemma 4.3, a social planner that controls the quality choices but not the ensuing prices would implement a higher quality level than each firm does in equilibrium. The equilibrium quality levels do not correspond to those that maximize social

[29] Clearly, if the government could also determine the firms' prices, it would be optimal that a single firm serves the whole market at a price of zero. The first best quality level satisfies $C'(q^{FB}) = \int_0^1 \theta d\theta = \frac{1}{2}$.

[30] Formally, $W_{q_L}(q_L, q_H) = \frac{r^2(20r-17)}{2(4r-1)^3} - C'(q_L) > \frac{r^2(4r-7)}{(4r-1)^3} - C'(q_L) = \Pi_{q_L}^L(q_L, q_H)$ and $W_{q_H}(q_L, q_H) = \frac{24r^3-18r^2+5r+1}{(4r-1)^3} - C'(q_H) > \frac{4r(2-3r+4r^2)}{(4r-1)^3} - C'(q_H) = \Pi_{q_L}^H(q_L, q_H)$.

welfare for various reasons. First, we have already pointed out that the equilibrium qualities plummet as the proportion of informed customers shrinks. This is in contrast to the second best qualities which are not affected by the information level of customers. Second, firms usually chose the quality to cater the valuation of *marginal* customers, while a social planner cares for the *mean* value of quality. In addition, the quality choice usually affects how much of the total surplus can be optimally extracted by a firm as pointed out by Spence (1975) in a monopoly setup. In our duopoly setup, a third effect is present because of the strategic interaction between competitors. As discussed by Ronnen (1991), the total industry profit increases in the amount of product differentiation.

Importantly, according to Lemma 4.3, the socially optimal qualities even exceed the equilibrium levels (q_L^{FI}, q_H^{FI}) when customers are fully-informed ($\alpha = 1$). Put differently, our setup exhibits a general tendency that firms offer products of too-low quality that is even reinforced when there are further informational problems.

4.5 The Effect of Certification

This section explores the effects of a simple certification scheme on qualities in the market and on equilibrium prices. In contrast to most of the literature reviewed above, we concentrate on certification that aims at manipulating the offered qualities into a desired direction instead of maximizing some certifier's profit. Indeed, certification which is organized by the state or by many non-governmental institutions is frequently offered at a negligible price which is at most intended to cover the costs of certification.[31] Since we will later examine certification that is designed so as to maximize social welfare, we speak of *governmental* certification and keep in mind that the same insights apply to non-governmental organizations that pursue the same objective. More precisely, we assume that the government publicly announces a certification threshold $q^C \in Q$ before firms decide about entry and about their production technology. The government costlessly observes the quality of the products and awards a certificate to all firms that produce goods which are at least of quality q^C. We suppose that certifica-

[31] See the introductory examples.

tion is costless for the firms. Formally, the variable s_i takes on the value 1 if a certificate is awarded to firm $i \in \{L, H\}$ and the value 0 otherwise.

Consumers incorporate the additional information which is made available by the certification system. They know that a firm obtains the certificate precisely when its quality is at least q^C. To assure that consumers' beliefs are compatible with their observations, it is natural that they adapt their belief when they observe that a firm is unexpectedly (not) awarded with the certificate. Formally, $\hat{q}_i(s_i)$ captures the belief about the quality of firm i contingent on having obtained a certificate ($s_i = 1$) or not ($s_i = 0$).[32] Beliefs are in line with the certification outcome if $\hat{q}_i(1) \in [q^C, \infty)$ and $\hat{q}_i(0) \in [0, q^C)$ for $i \in \{L, H\}$. Although beliefs are adapted to the certification outcome, it remains plausible that beliefs are passive with respect to prices, since observing an unexpected price does not prove that the original belief regarding the quality is wrong.

In order to avoid implausible equilibria, we restrict attention to equilibria that survive the following natural refinement of out-of-equilibrium beliefs:[33] When a firm does unexpectedly not obtain the certificate, then the resulting belief has to coincide with the profit maximizing quality level within the interval $[0, q^C)$, holding fixed the belief about the rival's quality.[34] Likewise, if a firm is unexpectedly awarded the certificate, consumers believe that it produces the profit maximizing quality level within the interval $[q^C, \infty)$. In our setup this refinement is easy to apply, since the best response of firm i does not depend on the others' belief about this firm's quality $\hat{q}_i$. Moreover, it is appealing because the revised beliefs after an unexpected certification outcome about the deviant's quality coincide with the most profitable deviation.[35] Thus, this refinement

[32]Note that the belief may depend both on a firm's identity and on the certification outcome. This formulation contains the natural assumption that the belief about a firm's quality does not depend on whether the rival obtains a certificate.

[33]For example, the out-of equilibrium belief $\hat{q}_H(0) = 0$ implies a harsh punishment in case firm H is unexpectedly not awarded the certificate. As an implausible consequence, a firm would meet any certification threshold as long as it earns non-negative profits to avoid the stigma of not getting the certificate. We discuss an alternative approach in Section 4.7.

[34]Define $\varphi(q_1, q_2) \equiv \alpha R^L(q_1, q_2) - C(q_1)$ if $q_1 \leq q_2$ and $\varphi(q_1, q_2) \equiv \alpha R^H(q_2, q_1) - C(q_1)$ if $q_1 > q_2$. Formally, our refinement requires $\hat{q}_i(0) = \min\left\{\arg\max_{q \in [0, q^C)} \varphi(q, q_j^*)\right\}$ and $\hat{q}_i(1) = \min\left\{\arg\max_{q \in [q^C, \infty)} \varphi(q, q_j^*)\right\}$ for $i, j \in \{L, H\}, j \neq i$.

[35]This refinement can be thought of the reduced form of a more complex game, in which with a small probability a firm is forced to obtain/ abstain from the certificate but may still choose its production technology within the associated interval. This requirement resembles the concept of "wary" beliefs introduced by McAfee and Schwartz (1994).

leads to “more consistency” off the equilibrium path.

In any equilibrium, a firm either produces a quality that satisfies the first order condition $\alpha R^i_{q_i}(q_L, q_H) = C'(q_i)$ or installs a production technology that exactly matches the certification threshold q^C. To see that firms produce at no other quality level in equilibrium, suppose that consumers (and the rival) believed that firm i would install a quality level with $\alpha R^i_{q_i}(q_L, q_H) \neq C'(q_i)$ and $q_i \neq q^C$. Then, this firm may profitably deviate to some quality close to the anticipated level which would not change the outcome of the certification so that uninformed consumers would have no reason to revise their beliefs.

Moreover, there is no equilibrium in which both firms produce goods of the quality q^C, since they would both make zero revenues while having to incur the cost $C(q^C)$ otherwise. Therefore, at most one firm will exactly match the certification threshold.

We are interested in certification that raises the quality in the market since the qualities offered in the absence of any intervention are socially too low according to Lemma 4.3. Moreover, it will become clear in Section 4.6 that adopting a MQS is more efficient than certification when the government intends to manipulate primarily the lowest quality in the market. Therefore, we focus in what follows on situations where the government *targets* the high quality firm, that is, it chooses a certification threshold such that at most one firm offers the quality q^C or higher. Thus, in order to raise the offered qualities, it is necessary that the certification standard is set above the equilibrium level q^*_H that would obtain in the absence of any intervention.

Proposition 4.2. *Suppose that a certification threshold $q^C \geq q^*_H$ is in place. There exists a* certification equilibrium *in which two firms enter the market and produce the quality levels $q^{ce}_L = b^L(q^C, \alpha)$ and $q^{ce}_H = q^C$ if and only if the feasibility conditions*

$$\Pi^L\left(q^{ce}_L, q^C\right) \geq \Pi^H\left(q^C, b^H(q^C, \alpha)\right), \tag{4.4}$$

$$\Pi^H\left(q^{ce}_L, q^C\right) \geq \max\left\{\Pi^H\left(q^{ce}_L, b^H\left(q^{ce}_L, \alpha\right)\right), \Pi^L\left(b^L(q^{ce}_L, \alpha), q^{ce}_L\right)\right\} \tag{4.5}$$

hold. If additionally

$$\Pi^{H}\left(q_{L}^{*},q_{H}^{*}\right)<\Pi^{H}\left(q_{L}^{*},q^{C}\right)\ , \tag{4.6}$$

then this equilibrium is unique.

Proof. See Appendix A4.1. ■

Proposition 4.2 asserts that whenever the feasibility conditions (4.4) and (4.5) are satisfied, then there is a *certification equilibrium* (denoted by the superscript "ce") in which one firm exactly matches the certification standard, while a second firm produces goods of a quality below q^C and consequentially does not obtain the certificate. Condition (4.4) assures that the firm which is expected to produce low quality goods has no incentives to "leapfrog" the high quality producer, that is to install an even higher quality level than q^C.[36] If firm L unexpectedly obtains the certificate, then by our refinement consumers believe that this firm produces at the profit-maximizing level $b^H(q^C,\alpha)$ which belongs to the set $[q^C,\infty)$ and is thus compatible with the certification outcome. Since $q_L^{ce}=b^L(q^C,\alpha)$ is a restricted best response, firm L has no profitable deviation below q^C. Hence, there is no profitable deviation at all if and only if condition (4.4) holds. Similarly, condition (4.5) assures that the high quality producer has no incentive to deviate to a lower quality level than q^C. Condition (4.5) requires that the equilibrium profit be higher than the maximum this firm could earn either when producing a quality below q^C but still above q_L^{ce} or when deviating to a quality that is even below q_L^{ce}.

The second part of Proposition 4.2 states a condition which guarantees that the desired equilibrium is also unique. There is a second candidate equilibrium in which firms behave as if certification was not available and install the quality levels (q_L^*,q_H^*) characterized by Proposition 4.1. Condition (4.6) assures that (q_L^*,q_H^*) cannot be an equilibrium since H could increase its profit by producing goods of quality q^C and signal this deviation by help of the certificate. For simplicity, we will assume in what follows that the certification equilibrium is always selected whenever it exists.[37]

[36]Note that if $q^C=q_H^*$, then condition (4.4) is more restrictive than the related "no-leapfrogging" condition in the setup without certification. This is because uninformed customers will revise their beliefs upon observing that firm L unexpectedly obtains the certificate which makes deviating upwards more profitable now.

[37]Indeed, there is evidence that governments have some power in selecting equilibria, for example by

Compared to the equilibrium levels without any intervention, the certification equilibrium entails a higher quality of *both* firms when $q^C > q_H^*$. Clearly, the high quality firm produces a higher quality in order to match the required threshold. Anticipating this choice, firm L also raises its quality because the quality levels are strategic complements. Altogether, since the reaction of the low quality firm is relatively small ($b^L_{q_H} < \frac{1}{r}$), the degree of vertical differentiation in the certification equilibrium is larger than in the equilibrium without intervention: $r^{ce} = q^C/q_L^{ce} > q_H^*/q_L^*$. Thus, certification improves the possibility for firms to *credibly* differentiate their products.

Since certification typically leads to more differentiated products, it also raises the equilibrium hedonic prices by Lemma 4.1. Consumers thus benefit from higher quality products but suffer from higher prices. Specifically, if consumers correctly anticipate the qualities q_L and q_H, then consumer surplus is $CS(q_L, q_H) \equiv \int_{x_L^*}^{z^*} q_L\,(\theta - x_L^*)\,\mathrm{d}\theta + \int_{z^*}^{1} q_H\,(\theta - x_H^*)\,\mathrm{d}\theta$. Taking derivatives shows that $CS_{q_L} > 0$ is always true and that $CS_{q_H} > 0$ if and only if $r > 5/4$.[38] The optimality condition (4.2) of firm L implies that $R^L_{q_L}(q_L^*, q_H^*) > 0$ which requires $r^* > 7/4$. Hence, moving from the equilibrium without intervention to a certification equilibrium with $q_H^{ce} > q_H^*$ increases consumer surplus in our setup. Note however, that mainly customers with a high taste for quality benefit from the increase in the goods' grade and that the equilibrium utility of some consumers shrinks.[39]

It remains to discuss how certification affects the firms' profits. The low quality firm clearly benefits from less intense competition resulting from more differentiated products. Moreover, it optimally augments the quality of its products which further raises its profit.[40] In contrast, the impact of certification on the profit of the high quality firm is less clear. On the one hand, certification helps this firm to commit to producing higher quality products which tends to augment its profit according to inequality (4.5) when holding fixed the quality of the rival. On the other hand, the low quality provider anticipates that firm H meets the standard q^C and adapts its quality upwards by the

appealing to consumers' beliefs through public advertising. See also Section 4.7.

[38] Formally, $CS = \frac{q_H r(4r+5)}{2(4r-1)^2}$, $CS_{q_L} = \frac{r^2(28r+5)}{2(4r-1)^3}$ and $CS_{q_H} = \frac{r\left(8r^2-6r-5\right)}{(4r-1)^3}$.

[39] In contrast, the adoption of a small MQS benefits all consumers as shown by Ronnen (1991).

[40] Formally, rewriting the change in firm L's profit yields $\Pi^L(q_L^{ce}, q_H^{ce}) - \Pi^L(q_L^*, q_H^*) = \int_{q_H^*}^{q_H^{ce}} R^L_{q_H}(q_L^*, q)\mathrm{d}q + \int_{q_L^*}^{q_L^{ce}} \Pi^L_{q_L}(q, q_H^{ce})\mathrm{d}q > 0$ which is positive since the integrands of both terms are positive.

strategic complementarity. This in turn reduces the high quality producer's profit. Especially when the certification standard it set very high, which requires large investment costs to obtain the certificate, firm H may thus earn less profit in the certification equilibrium than without intervention.[41]

Since Proposition 4.2 states rather implicit conditions, we now turn to the question which thresholds admit a certification equilibrium. For simplicity, we assume $C''' \geq 0$ for the rest of the chapter. A threshold is *feasible* if it admits a certification equilibrium. In the light of Proposition 4.2, the set of feasible thresholds is $Q_H^C = \{q^C \geq q_H^* | \text{conditions (4.4) and (4.5) hold}\}$. For our next result, we implicitly define $q^\dagger$ as the unique solution to $q = b^H\left(b^L(q, \alpha), 1\right)$.[42] The quality level $q^\dagger$ can be interpreted as follows. Holding fixed the quality level of firm L at $q_L = b^L(q^\dagger, \alpha)$, firm H would deliberately produce goods of quality $q^\dagger$ if all consumers could observe the quality of its goods.

Lemma 4.4. *Condition* (4.4) *holds if the certification threshold q^C is set at least at the full information quality level of a monopolist $q_M^{FI} \equiv \arg\max_q \left(R^M(q) - C(q)\right)$. Condition* (4.5) *is satisfied for $q^C \in [q_H^*, q^\dagger]$.*

Proof. See Appendix A4.1. ■

According to Lemma 4.4, the set of thresholds that admit a certification equilibrium Q_H^C is non-empty and contains the interval $[\max\{q_H^*, q_M^{FI}\}, q^\dagger]$.[43] For thresholds above $\max\{q_H^*, q_M^{FI}\}$, leapfrogging the high quality firm would require high investments in quality and the ensuing revenues would not suffice to recover the initial costs. To see that it is indeed optimal for firm H to adopt the certificate for thresholds up to $q^\dagger$, suppose the threshold is set at $q^\dagger$ and that firm L chooses $q_L^{ce} = b^L(q^\dagger, \alpha)$. If early consumers observed the actual quality of firm H, it would deliberately produce goods of quality $q^\dagger$ since $q^\dagger = b^H(q_L^{ce}, 1)$. Since the early consumers' correctly anticipate the profit-maximizing quality level below the certification standard in case a firm is unexpectedly not awarded the certificate, deviating downwards is clearly unprofitable

[41]Of course, the highest profit when deviating downwards would be also below the equilibrium profit in the absence of certification.

[42]A similar reasoning as in the proof of Proposition 4.1 implies existence and uniqueness of $q^\dagger$.

[43]It is readily verified that $\max\{q_H^*, q_M^{FI}\} \leq q^\dagger$ with a strict inequality for $\alpha < 1$, by definition of $q^\dagger$.

when the certification standard is $q^\dagger$. Note that Q_H^C comprises *at least* this interval according to Lemma 4.4 and we will see below that it will usually be larger than that.[44]

The most important insight related to Lemma 4.4 is that for $\alpha < 1$ there are feasible certification thresholds that induce both firms to provide a higher quality relative to the no-intervention equilibrium. Since Lemma 4.3 points out that firms choose socially too low equilibrium quality levels in the absence of any intervention, certification may increase welfare.[45] We summarize the results of this section as follows.

Corollary 4.2. *Introducing certification with $q^C \in \left(\max\{q_H^*, q_M^{FI}\}, q^\dagger\right]$ admits a certification equilibrium in which both firms' quality levels, prices, welfare and the profit of firm L increase relative to the no-intervention equilibrium.*

Optimal Certification

Having seen that certification which improves welfare is feasible, we now turn to *optimal* certification. A feasible certification threshold is optimal if the ensuing certification equilibrium leads to the highest welfare among all $q^C \in Q_H^C$.[46]

Let us ignore the feasibility constraints (4.4) and (4.5) for the moment and instead assume that the firms install the quality levels $q_H = q$ and $q_L = b^L(q,\alpha)$ with $q \geq q_H^*$. Denote by q^{TB} the *third best* quality level that maximizes welfare $\tilde{W}(q) \equiv W\left(b^L(q,\alpha), q\right)$ under this alternative assumption.[47] The marginal welfare when increasing the quality of firm H is $\tilde{W}_q(q) = b^L_{q_H}(q,\alpha) W_{q_L}\left(b^L(q,\alpha),q\right) + W_{q_H}\left(b^L(q,\alpha),q\right)$ where the first term

[44] Note however, that introducing certification may destroy all equilibria. If $\Pi^L\left(q_L^*, q_H^*\right) < \Pi^H\left(q_H^*, b^H(q_H^*,\alpha)\right)$ then after implementing the certification threshold $q^C = q_H^*$, the ensuing game has no equilibrium any more. In particular, the qualities (q_L^*, q_H^*) cease to form an equilibrium. This is because the certificate allows early customers to infer that the former low quality firm has leapfrogged its rival so that this deviation becomes more profitable.

[45] Formally, the change in welfare may be expressed as $W(q_L^{ce}, q^C) - W(q_L^*, q_H^*) = \int_{q_H^*}^{q^C} W_{q_H}\left(b^L(q,\alpha),q\right) + b^L_{q_H}(q_H,\alpha) W_{q_L}\left(b^L(q,\alpha),q\right) \mathrm{d}q$ where the integrand is necessarily positive for $q^C \leq q^\dagger$. For any $q \in [q_H^*, q^\dagger]$, $b^H(b^L(q,\alpha),1) \geq q$, so that optimality condition (4.3) and the concavity of R^H in q_H imply that $\Pi^H_{q_H}\left(b^L(q,\alpha),q\right) \geq 0$. Together with $W_{q_H} > \Pi^H_{q_H}$, this implies $W_{q_H}\left(b^L(q,\alpha),q\right) > 0$. The same reasoning also implies $W_{q_L}\left(b^L(q,\alpha),q\right) > 0$.

[46] An alternative objective function would be to maximize consumer surplus. This would even simplify our results, since consumer surplus always increases in the high grade firm's quality in any certification equilibrium. Therefore, the government would then set the certification standard as high as possible and condition (4.5) would always bind at the optimum.

[47] The subscript "third best" indicates that in contrast to the second best quality levels, only the quality of firm H may be chosen while firm L chooses a best response.

reflects that the low grade producer adapts its quality and the second term is the direct effect. Assuming that $\tilde{W}(q)$ is quasiconcave, we may conclude that $q^{TB} > q^{\dagger}$ because $\Pi^{H}_{q_H}\left(b^L(q^{\dagger},\alpha), q^{\dagger}\right) = 0$ and because the marginal welfare with respect to q_L and q_H exceeds the respective marginal profit everywhere.[48] Put differently, if the government could assure that one firm adopts the certificate and the second firm stays below the certification threshold, it would set a higher certification standard than $q^{\dagger}$.

Since the third best quality level lies above the feasible threshold $q^{\dagger}$, feasibility constraint (4.4) never binds at the optimum by Lemma 4.4 as a rather high certification standard makes it unattractive for the low quality firm to leapfrog its rival.

In contrast, feasibility constraint (4.5) potentially restricts the optimal certification threshold q^{C*}. According to the following proposition, implementing the certification threshold $q^C = q^{TB}$ is not feasible if the fraction of informed customers is sufficiently large.

Proposition 4.3. *i) If the fraction of informed customers is sufficiently small and $\Pi^M(q^{TB}) > 0$, then $q^{C*} = q^{TB}$. Moreover, the optimal certification level exceeds the full information equilibrium level of the high quality firm: $q^{C*} > q^{FI}_H$.*
ii) If the fraction of informed customers is sufficiently large, then $q^{C*} < q^{TB}$ and $\Pi^H\left(q^{ce}_L, q^{C*}\right) = \Pi^H\left(q^{ce}_L, b^H(q^{C*},\alpha)\right)$.

Proof. See Appendix A4.1. ■

The first part of Proposition 4.3 covers optimal certification when the fraction of informed customers is sufficiently small. In this case, implementing $q^{C*} = q^{TB}$ is feasible and therefore optimal. Intuitively, without the certificate, the quality level that firm H may credibly produce and hence the deviation profits are relatively low. To demonstrate high quality by means of the certificate, firm H will thus even install rather expensive production technologies. Indeed, Proposition 4.3 establishes that the optimal certification level is above the full information equilibrium quality level q^{FI}_H that would obtain if all customers could observe the goods' qualities. Hence, if sufficiently few customers observe the actual quality level, the government may exploit

[48] A grossly sufficient condition for concavity of $\tilde{W}(q)$ is $C'''(q) \geq 0$. Formally, $W_{q_H}\left(b^L(q^{\dagger},\alpha), q^{\dagger}\right) > \Pi^{H}_{q_H}\left(b^L(q^{\dagger},\alpha), q^{\dagger}\right) = 0$ and $W_{q_L}\left(b^L(q^{\dagger},\alpha), q^{\dagger}\right) > \Pi^{L}_{q_L}\left(b^L(q^{\dagger},\alpha), q^{\dagger}\right) > 0$ implies $q^{TB} > q^{\dagger}$.

the high grade firm's dependence on certification in order to raise the supplied quality towards the socially desired level. Proposition 4.3, part i) requires that a monopolist earns positive profits when selling goods of the third best quality level. Since $\lim_{q_L \to 0} \Pi^H(q_L, q) = \Pi^M(q)$, this condition assures that firm H earns positive profits when adopting the certificate. Otherwise, it would prefer to sell goods of a quality below q^{TB} without the certificate.

The second part of Proposition 4.3 asserts that when sufficiently many customers are informed, then the government is forced to set a certification threshold below q^{TB} such that investing in the technology in order to obtain the certificate is sufficiently attractive for firm H. If the proportion of informed customers is sufficiently large, then even without the certificate, the profit of each firm is close to the level that it could earn when all consumers were informed. Hence, in order to deter profitable deviations, the government has to set the certification threshold sufficiently close to $q^{\dagger}$. In the limit as $\alpha \to 1$, firms may credibly produce at their profit maximizing quality levels and certification loses its bite, that is the feasible set shrinks to the singleton q_H^{FI}.

The next proposition explores comparative statics of the optimal certification threshold with respect to the fraction of informed customers. These comparative statics are highly relevant, since for example technological developments change the customers' information level over time.

Proposition 4.4. *i) If the fraction of informed customers is sufficiently small and $\Pi^M(q^{TB}) > 0$, then the optimal certification level increases in the proportion of informed customers: $\frac{\mathrm{d}q^{C*}}{\mathrm{d}\alpha} > 0$.*

ii) If the fraction of informed customers is sufficiently large and the marginal cost has non-increasing elasticity, i.e. satisfies $\left(\frac{qC''(q)}{C'(q)}\right)' \leq 0 \ \forall q$, then the optimal certification level decreases in the proportion of informed consumers: $\frac{\mathrm{d}q^{C*}}{\mathrm{d}\alpha} < 0$.

Proof. See Appendix A4.1. ■

Proposition 4.4 highlights that the sign of the comparative statics crucially depends on whether the feasibility constraint (4.5) binds at the optimum. If the fraction of informed customers is sufficiently small and a monopolist would earn positive profits at the third best quality level ($\Pi^M(q^{TB}) > 0$), then setting the certification threshold at

q^{TB} is optimal according to Proposition 4.3 and the feasibility constraint (4.5) is slack. Remember that the socially optimal quality levels are complements, that is $W_{q_L,q_H} > 0$. Since a small increase of α raises the quality that the low quality firm can credibly produce given q_H, it also raises q^{TB} by the complementarity of the quality levels. Since the feasibility constraint (4.5) is slack, increasing the certification threshold is feasible, which implies the claim of Proposition 4.4, part i).

According to part ii) of Proposition 4.4, the optimal threshold q^{C*} decreases in the fraction of informed customers when sufficiently many consumers are informed. As discussed above, in this case the government is forced to set the threshold close to the quality level that maximizes the high grade firm's profit in order to deter a profitable deviation. A further increase in the fraction of informed customers has two effects. First, it allows the high grade firm to credibly produce higher quality goods even when not obtaining the certificate. Holding fixed the rival's quality, the profit of firm H thus increases after a deviation below q^C. Second, an increase in the fraction of informed customers also triggers an indirect effect. Holding fixed the quality level of firm H, an increase in α allows the low quality provider to credibly raise its quality which in turn fuels price competition. This indirect effect reduces the high grade firm's profit even more when deviating to a quality below the certification standard. The result of the proposition holds under the qualification that the first effect dominates the second. This is guaranteed by the grossly sufficient condition that the marginal cost has non-increasing elasticity or equivalently, $\left(\frac{qC''(q)}{C'(q)}\right)' \leq 0\ \forall q$. This condition is satisfied by a number of commonly used cost functions such as the power function $C(q) = \gamma q^n$ with $\gamma > 0$ and $n > 1$.

Figure 4.1 illustrates Proposition 4.4, part ii). It compares the optimal certification standards $\underline{q}^{C*}$ and $\overline{q}^{C*}$ that obtain when the proportions $\underline{\alpha}$ and $\overline{\alpha} > \underline{\alpha}$ of customers are informed, respectively. In the figure, the equilibrium quality of the low quality firm increases in α, that is $\overline{q}_L^{ce} > \underline{q}_L^{ce}$. Since a higher quality of firm L implies a more intense price competition, $\Pi^H(\overline{q}_L^{ce}, q_H) < \Pi^H(\underline{q}_L^{ce}, q_H)$ for any $q_H \geq \overline{q}_L^{ce}$. The dominant effect on the deviation profit comes from the quality level that firm H may credibly produce when not obtaining the certificate. This deviation quality level rises from $b^H\left(\underline{q}_L^{ce}, \underline{\alpha}\right)$ to $b^H\left(\overline{q}_L^{ce}, \overline{\alpha}\right)$ as α increases from $\underline{\alpha}$ to $\overline{\alpha}$. Since this increases the payoff from deviating downwards, the certification threshold has to be reduced from $\underline{q}^{C*}$ to $\overline{q}^{C*}$ so as to increase the payoff from obtaining the certificate in order to deter a deviation.

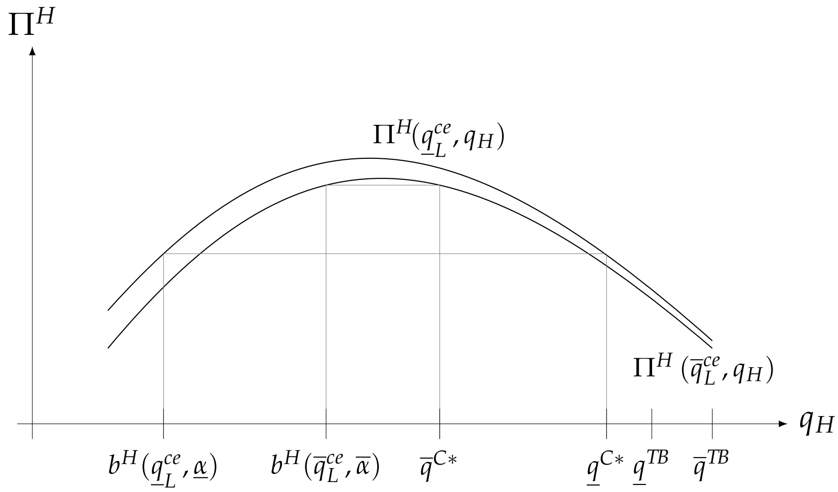

Figure 4.1: Comparative statics of the optimal certification standard

4.6 Certification versus Minimum Quality Standards

So far, we have examined the effect of awarding a certificate to firms if their quality exceeds the threshold q^C. Another way of increasing the quality of goods is to adopt a MQS that forces active firms to produce at least a minimum quality level q^{MQS}.[49] Following Ronnen (1991), we assume that the government publicly announces a minimum level q^{MQS} before firms decide about entry and about the production technology. Ronnen (1991) analyzes the effect of introducing a MQS in a model that essentially corresponds to ours when all customers observe the actual quality of the goods ($\alpha = 1$). He shows that introducing a suitable MQS is always welfare enhancing. If both firms enter, a MQS that is set above q_L^* forces firm L to install a higher quality compared to the unregulated equilibrium level. Anticipating this effect, firm H also increases its quality since $b_{q_L}^H > 0$. Ronnen (1991) shows that under a MQS the goods are less differentiated than in the unregulated market, which leads to lower quality weighted prices and thus to a higher participation.

[49]The government thus commits to oust firms that have installed a production technology below that level from the market.

Adopting a MQS may reduce the number of active firms. A MQS increases the minimum investment which is required to enter the market. Ronnen (1991) points out that a MQS which is set too restrictive does not allow both firms to recoup their initial investments in the production technology. This may result in only one or no firm entering the market. In order to maintain the competitive pressure on prices, the regulator may thus prefer to set a rather low MQS so as to assure that both firms enter.

The negative impact of a MQS on the firms' entry decision is particularly severe if only few customers observe the actual quality level. In this case, firm H has rather small incentives to invest in quality and therefore provides goods of a mediocre quality as discussed in Section 4.4. Adopting a MQS then quickly results in a small degree of differentiation that leads to low equilibrium revenues of both firms. Hence, if few customers are informed, then only a relatively low MQS still guarantees that both firms enter. In contrast, we have seen that certification never has a negative effect on entry. Enabling firm H to credibly produce a higher quality via certification increases the level of product differentiation and always raises firm L's profits. A small fraction of informed customers may even be helpful to implement a relatively high certification threshold.

On the other hand, we have seen that the set of feasible certification thresholds Q_H^C is tiny if almost all consumers are informed. The discussion so far suggests that either MQS or certification may be preferred, depending on the costumers' information level.[50] The next proposition refers to the welfare-maximizing quality in case a single firm is active, which is uniquely defined as $q_M^{SB} = \arg\max_q W^M(q)$.

Proposition 4.5. *Suppose the government has to decide between introducing a MQS or determining a certification threshold $q^C \in Q_H^C$ in order to maximize welfare. If the proportion of informed consumers α is sufficiently low and $\Pi^M\left(q_M^{SB}\right) > 0$, then optimal certification allows to attain higher welfare than adopting an optimal MQS. In contrast, if a sufficiently high fraction of consumers is informed, introducing an optimal MQS is preferred over certification.*

Proof. See Appendix A4.1. ■

[50]This result is important when the government decides between either MQS or certification. For example, this may be the case if introducing either policy intervention produces high fixed costs.

To see the intuition behind Proposition 4.5, assume first that almost all consumers do not observe the actual quality of the goods ($\alpha \to 0$). In this case, the highest q^{MQS} that allows two firms to earn non-negative profits is close to zero. The reason is that if the first firm offers goods of the quality q^{MQS}, the second firm cannot credibly sell goods of a much higher quality. This results in low revenues because of intense price competition and thus the firms cannot recoup their initial investments. Accepting that only a single firm enters and controlling the quality of this firm with a restrictive MQS therefore yields a higher social welfare. If $\Pi^M \left(q_M^{SB}\right) > 0$, it is optimal to require that the monopolist produces $q^{MQS*} = q_M^{SB}$. In this case, certification can do better, since setting $q^C = q^{MQS*}$ will generally induce two firms to enter, with firm H producing $q_H = q^{MQS*}$. Albeit firm L may credibly produce only at a very low quality level due to the small fraction of informed customers, this firms exerts some pressure on the ensuing prices and thus helps to improve welfare. Since any optimal certification level q^{C*} must lead to a weakly higher welfare than $q^C = q^{MQS*}$, optimal certification is preferred. The condition $\Pi^M \left(q_M^{SB}\right) > 0$ assures that firm H has no incentive to deviate to a quality below q^{C*} for α small enough.

In contrast, if almost all consumers observe the actual quality level, according to Proposition 4.3 the certification threshold has to be set close to the unregulated equilibrium quality level q_H^* in order to be accepted by firm H. Turning to a MQS, the drawback of deterring entry is smallest when many customers may observe the actual quality level as pointed out above. For $\alpha \to 1$, our model converges to that of Ronnen (1991), who has shown that there exist welfare improving $q^{MQS} > 0$. We thus conclude that MQS is preferred when almost all consumers are informed.

Two remarks are in order. First, the results of Proposition 4.5 depend in parts on our assumptions concerning the production costs. Since the marginal cost of quality converges to zero for low quality levels, both firms enter unless a MQS hinders them. In particular, this applies when only few customers are informed. Yet, if the marginal cost would be positive at $q = 0$ so that only one firm would enter in the absence of any intervention, certification may be preferred for a similar reason when α is small. In this case, certification may *raise* the number of entrants since it improves the firms' ability to differentiate their products. Second, we believe that our line of reasoning goes through if firms have to incur unit costs in addition to the fixed investments. In this case, equilibrium prices would be always above firms' unit costs. To what degree the revenues also cover the fixed investments would again depend on the level of differentiation. Hence, similar effects as discussed so far apply.

4.7 Discussion

In Section 4.2 we have introduced an refinement in order to rule out implausible equilibria. This refinement requires that uninformed players believe that a firm produces the profit-maximizing quality that is compatible with a certification outcome even when this outcome does not occur on the equilibrium path. Alternatively, we could rely on equilibrium selection instead of imposing restrictions on beliefs. One possibility is to treat all consumers as a single player and to argue that an equilibrium is selected if it payoff-dominates all other equilibria, that is if this equilibrium gives rise to the highest consumer surplus and to the highest profit for each firm. For $q^C > q_H^*$, by our reasoning above, only two pairs of qualities may be part of an equilibrium: the first pair is (q_L^*, q_H^*) defined by Proposition 4.1 and the second pair is (q_L^{ce}, q_H^{ce}) defined by Proposition 4.2. In case both equilibria exist, only the pair (q_L^{ce}, q_H^{ce}) possibly dominates (q_L^*, q_H^*) since $\Pi^L(q_L^*, q_H^*) < \Pi^L(q_L^{ce}, q_H^{ce})$ and $CS(q_L^*, q_H^*) < CS(q_L^{ce}, q_H^{ce})$. Thus, in case both equilibria coexist, the condition

$$\Pi^H(q_L^*, q_H^*) \leq \Pi^H(q_L^{ce}, q_H^{ce}) \tag{4.7}$$

assures that the equilibrium associated with (q_L^{ce}, q_H^{ce}) payoff-dominates (q_L^*, q_H^*). The out-of equilibrium belief $\hat{q}_i(s_i) = 0$ for any off-equilibrium certification outcome supports the candidate equilibrium quality levels (q_L^{ce}, q_H^{ce}) and (q_L^*, q_H^*) for the largest set of parameters. In particular, if (q_L^*, q_H^*) are equilibrium qualities without certification, then they remain equilibrium qualities if certification with $q^C \geq q_H^*$ is available. Likewise, it is readily verified that these out-of equilibrium beliefs support the certification equilibrium for a large set of certification thresholds. Roughly speaking, this indicates that condition (4.7) plays a similar role as condition (4.5) when computing the optimal certification standard in Section 4.5. Since $\Pi^H(q_L^{ce}, q_H^{ce})$ enters in condition (4.7) similarly as in condition (4.5), we believe that the results remain qualitatively unchanged when relying on equilibrium selection rather than on the refinement we propose.

4.8 Conclusion

This chapter has investigated the effects of certification and of a MQS when not all customers can discern the quality of traded goods. The presence of uninformed customers reduces firms' incentives to invest in quality. In equilibrium, quality is under-supplied which reduces both consumer surplus and firms' profits. By certifying their products, firms may demonstrate that their goods are of higher quality than consumers would expect otherwise. We have considered a simple form of governmental certification where firms are awarded with a certificate if they produce goods of a quality not below a publicly known threshold. The certification standard must be set low enough so that firms indeed raise their profits by investing in quality in order to obtain the certificate. Yet, a suitable certification standard may even induce some firms to raise their quality above the highest level that would be attained if all consumers were fully informed. We have pointed out that a large proportion of uninformed customers makes firms more reliant on certification which in turn allows the government to implement a high certification threshold. An increase in the proportion of informed customers typically allows firms to earn higher profits when selling goods that lack the certificate. This may force the regulator to lower the certification threshold so as to assure that firms still invest in high quality to obtain the certificate. Finally, we have compared certification to a MQS. When the proportion of informed customers is small, certification may be preferred over a MQS, since the latter potentially deters firms from entering the market. In contrast, when the proportion of informed customers is high, a MQS is typically more effective than certification.

Our results could be extended in several directions by further research. So far, we have analyzed the relative merits of certification and MQS, but have not considered adopting both instruments together. Introducing both instruments would improve the government's ability to manipulate the quality of active firms. In particular, if only few customers are informed and two firms have entered, then a suitable certification standard allows firm H to provide high quality goods and the increased disparity of goods results in higher revenues. Therefore, complementing a MQS with certification allows to alleviate the entry-deterring effect of a minimum standard. Certification may thus be particularly valuable when used in conjunction with a MQS.

The industry's ability to provide itself a certification system is also an interesting point that deserves further attention. We have pointed out that certification standards may increase the profits of all active firms. This suggests that firms may have an interest in building up an own certifying institution whose certification scheme is designed to maximize the industry profit rather than welfare. Why do we observe that firms rely on governmental certification, nevertheless? One important issue from which we have abstracted in our model is that a certifier must have correct incentives for designing and enforcing a certification scheme honestly.[51] The current experiences with private certifiers that have issued inflated ratings for financial products suggest that this requirement is more likely to be satisfied by governmental certification.[52] Nevertheless, the availability of privately run certification may restrict the leeway of the government to manipulate the firms' qualities by help of certification.

Another important aspect that could be incorporated in our framework is asymmetric information regarding production costs or consumer tastes. Concerning these two dimensions, firms are often better informed than the government. The government's lack of information may affect which policy intervention is preferred and how these instruments are implemented.[53]

[51] Gehrig and Jost (1995) study the incentives of self regulating organizations to conduct costly monitoring.

[52] See e.g. Mathis, McAndrews, and Rochet (2009).

[53] This assumes that the government cannot commit to more complicated mechanisms in order to extract this information.

A4.1 Appendix

Proof of Proposition 4.1

We first show that a unique candidate equilibrium that satisfies both conditions (4.2) and (4.3) exists. Let $\overline{q} > 0$ denote the unique solution to the equation $\alpha R^H_{q_H}(q,q) - C^{'}(q) = 0$. Define $B(q) \equiv b^L\left(b^H(q,\alpha),\alpha\right) - q$ on $[0,\overline{q}]$. By the properties of b^L and b^H, B is continuous, $B^{'}(q) = b^H_{q_L} b^L_{q_H} - 1 < 0$, $B(0) > 0$, and $B(\overline{q}) < 0$. Thus, there exists a unique q^*_L with $B(q^*_L) = q^*_L$. By construction, q^*_L and $q^*_H = b^H(q^*_L,\alpha))$ satisfy the equations (4.2) and (4.3).

Next, we show that $\alpha R^H(q^*_L,q^*_H) - C(q^*_H) > \alpha R^L(q^*_L,q^*_H) - C(q^*_L)$: Generally, $R^H(q_L,q_H) - R^L(q_L,q_H) = (q_H - q_L)\frac{r}{4r-1}$. Next, $\frac{r^*}{4r^*-1} \geq R^H_{q_H}(q^*_L,q^*_H) = \frac{4r^*\left(2-3r^*+4(r^*)^2\right)}{(4r^*-1)^3}$ is equivalent to $4r^* - 7 \geq 0$ which is true since $R^L_{q_L}(q^*_L,q^*_H) > 0$ implies $r^* > \frac{7}{4}$. Hence, $R^H(q^*_L,q^*_H) - R^L(q^*_L,q^*_H) \geq (q^*_H - q^*_L)\, R^H_{q_H}(q^*_L,q^*_H)$. Together with $\alpha R^H_{q_H}(q^*_L,q^*_H) = C^{'}(q^*_H)$, this implies $\alpha\left(R^H(q^*_L,q^*_H) - R^L(q^*_L,q^*_H)\right) \geq (q^*_H - q^*_L)\, C^{'}(q^*_H) > C(q^*_H) - C(q^*_L)$ where the last inequality is due to the convexity of C. The last inequality also means that $R^H(q^*_L,q^*_H) - R^L(q^*_L,q^*_H) > 0$, which implies that $\Pi^H(q^*_L,q^*_H) > \Pi^L(q^*_L,q^*_H)$.

Generally, $\alpha R^L(b^L(q,\alpha),q) - C(b^L(q,\alpha))$ increases in q by the envelope theorem and because $R^L_{q_H} > 0$. Similarly, for $q < \overline{q}$, $\alpha R^H(q, b^H(q,\alpha)) - C(b^H(q,\alpha))$ decreases in q by the envelope theorem and because $R^H_{q_L} < 0$.

Now we show that q^*_H is a global best response for $\hat{q}_L = q^*_L$ and $\hat{q}_H = q^*_H$. Clearly, deviating from q^*_H does not affect the revenues of the second stage, so it suffices to show that $\alpha R^H(q^*_L,q^*_H) - C(q^*_H) \geq \max_{q \leq q^*_L}\left[\alpha R^L(q,q^*_L) - C(q)\right] = \alpha R^L(b^L(q^*_L,\alpha),q^*_L) - C\left(b^L(q^*_L,\alpha)\right)$. By our results above, we have $\alpha R^H(q^*_L,q^*_H) - C(q^*_H) > \alpha R^L(q^*_L,q^*_H) - C(q^*_L) > \alpha R^L\left(b^L(q^*_L,\alpha),q^*_L\right) - C\left(b^L(q^*_L,\alpha)\right)$.

It remains to show that $C^{'''} \geq 0$ is sufficient for q^*_L to be a global best response for $\hat{q}_L = q^*_L$ and $\hat{q}_H = q^*_H$. Similarly as above, it suffices to show that $\alpha R^L(q^*_L,q^*_H) - C(q^*_L) \geq \alpha R^H\left(q^*_H, b^H(q^*_H,\alpha)\right) - C\left(b^H(q^*_H,\alpha)\right)$. Using, $C^{'''} > 0$ and $R^H_{q_H}(q,q) > R^H_{q_H}(q,\tilde{q})$ for $\tilde{q} > q$, we have $\alpha R^H\left(q, b^H(q,\alpha)\right) - C\left(b^H(q,\alpha)\right) = \int_q^{b^H(q,\alpha)} \alpha R^H_{q_H}(q,\tilde{q}) -$

$C^{'}(\tilde{q})\mathrm{d}\tilde{q} - C(q) < \frac{\left(\alpha R^{H}_{q_H}(q,q) - C^{'}(q)\right)^2}{2C^{''}(q)} - C(q)$. Moreover, using again $C^{'''} \geq 0$ and integrating by parts yields $C(q) > \frac{\left(C^{'}(q)\right)^2}{2C^{''}(q)}$ so that $\alpha R^{H}\left(q, b^{H}(q,\alpha)\right) - C\left(b^{H}(q,\alpha)\right) < \frac{R^{H}_{q_H}(q,q)}{2C^{''}(q)}\left(\alpha R^{H}_{q_H}(q,q) - 2C^{'}(q)\right)$. Clearly, $\alpha R^{H}\left(q, b^{H}(q,\alpha)\right) - C\left(b^{H}(q,\alpha)\right) < 0$ if $C^{'}(q) \geq \frac{\alpha}{2} R^{H}_{q_H}(q,q)$. Condition (4.3) implies that $C^{'}(q^{*}_{H}) = \alpha R^{H}_{q_H}(q^{*}_{L}, q^{*}_{H}) > \frac{\alpha}{4}$ where the last inequality comes from the fact that $R^{H}_{q_H}$ is bounded below by $\frac{1}{4}$. Hence, $C^{'}(q^{*}_{H}) > \frac{\alpha}{4} > \frac{2\alpha}{9} = \frac{\alpha}{2} R^{H}_{q_H}(q,q)$ and thus $\alpha R^{H}\left(q^{*}_{H}, b^{H}(q^{*}_{H},\alpha)\right) - C\left(b^{H}(q^{*}_{H},\alpha)\right) < 0$.

Proof of Lemma 4.3

First we check concavity of W: $U(q_L, q_H) \equiv W(q_L, q_H) + C(q_L) + C(q_H) = \frac{q_H\left(12r^2 - r - 2\right)}{2(4r-1)^2}$ is homogeneous of degree 1. Therefore, $\frac{1}{r}U_{q_L,q_L} + U_{q_L,q_H} = 0$ and $rU_{q_H,q_H} + U_{q_L,q_H} = 0$. Therefore, $U_{q_L,q_L}U_{q_H,q_H} - \left(U_{q_L,q_H}\right)^2 = U_{q_L,q_L}U_{q_H,q_H} - U_{q_L,q_L}U_{q_H,q_H} = 0$. Differentiating yields $U_{q_L,q_L} = -\frac{r^3(4r+17)}{q_H(4r-1)^4} < 0$ and $U_{q_H,q_H} = -\frac{r(4r+17)}{q_H(4r-1)^4} < 0$, so that U is concave and since C is strictly convex, W is strictly concave. Moreover, this implies $W_{q_L,q_H} = \frac{r^2(4r+17)}{q_H(4r-1)^4}$.

From the equilibrium conditions we have $\Pi^{i}_{q_i}(q^{*}_{L}, q^{*}_{H}) = (1-\alpha)R^{i}_{q_i}(q^{*}_{L}, q^{*}_{H}) > 0$ and therefore $W_{q_i}(q^{*}_{L}, q^{*}_{H}) > (1-\alpha)R^{i}_{q_i}(q^{*}_{L}, q^{*}_{H}) > 0$. Since W is strictly concave and $W_{q_i}(q^{*}_{L}, q^{*}_{H}) > 0$, as well as $W_{q_L,q_H} > 0$, we have $W_{q_L}(q^{*}_{L}, q_H) > 0$ for $q_H > q^{*}_{H}$ and $W_{q_H}(q_L, q^{*}_{H}) > 0$ for $q_L > q^{*}_{L}$ which implies $q^{SB}_{i} > q^{*}_{i}$ by the concavity of W.

Proof of Proposition 4.2

The following beliefs that support this equilibrium are generically uniquely pinned down by our refinement as follows: $\hat{q}_L(0) = b^{L}(q^{C}, \alpha)$, $\hat{q}_L(1) = b^{H}(q^{C}, \alpha)$ and $\hat{q}_H(1) = q^{C}$. In addition, $\hat{q}_H(0) = b^{H}(q^{ce}_{L}, \alpha)$ if $\alpha R^{H}\left(q^{ce}_{L}, b^{H}(q^{ce}_{L}, \alpha)\right) - C(b^{H}(q^{ce}_{L}, \alpha)) > \alpha R^{L}\left(b^{L}(q^{ce}_{L}, \alpha), q^{ce}_{L}\right) - C(b^{L}(q^{ce}_{L}, \alpha))$ and $\hat{q}_H(0) = b^{L}(q^{ce}_{L}, \alpha)$ otherwise.

First we show that $q_H = q^{ce}_{H}$ globally maximizes a firm's profit for the belief $\hat{q}_H$, when the competitor produces the quality $b^{L}(q^{C}, \alpha)$ and given the certification threshold q^{C}. Clearly, deviating upwards is unprofitable since for $q > q^{C} \geq q^{*}_{H}$, $\alpha R^{H}_{q_H}(q^{ce}_{L}, q) - C(q) < \alpha R^{H}_{q_H}(q^{ce}_{L}, q^{C}) - C(q^{C}) < \alpha R^{H}_{q_H}(b^{H}(q^{ce}_{L}, \alpha), q^{C}) - C\left(b^{H}(q^{ce}_{L}, \alpha)\right) =$

0 because $b^H(q_L^{ce},\alpha) = q^C$ and $R^H_{q_H q_H} < 0$. Suppose first that $\alpha R^H\left(q_L^{ce}, b^H\left(q_L^{ce},\alpha\right)\right) - C(b^H\left(q_L^{ce},\alpha\right)) > \alpha R^L\left(b^L(q_L^{ce},\alpha), q_L^{ce}\right) - C(b^L(q_L^{ce},\alpha))$. Then deviating downwards with $q^C > q > q_L^{ce}$ yields at most $R^H\left(q_L^{ce}, b^H(q_L^{ce},\alpha)\right) - C\left(b^H(q_L^{ce},\alpha)\right)$ and condition (4.5) guarantees that this deviation is unprofitable. Deviating downwards with $q < q_L^{ce}$ yields at most $(1-\alpha)\,R^H\left(q_L^{ce}, b^H(q_L^{ce},\alpha)\right) + \alpha R^L(b^L\left(q_L^{ce},\alpha\right), q_L^{ce}) - C\left(b^L\left(q_L^{ce},\alpha\right)\right) < R^H\left(q_L^{ce}, b^H(q_L^{ce},\alpha)\right) - C\left(b^H(q_L^{ce},\alpha)\right)$ so that this deviation is also unprofitable. If instead $\alpha R^H\left(q_L^{ce}, b^H\left(q_L^{ce},\alpha\right)\right) - C(b^H\left(q_L^{ce},\alpha\right)) \le \alpha R^L\left(b^L(q_L^{ce},\alpha), q_L^{ce}\right) - C(b^L(q_L^{ce},\alpha))$, then by the same reasoning, the highest deviation payoff is $R^L\left(b^L(q_L^{ce},\alpha), q_L^{ce}\right) - C(b^L(q_L^{ce},\alpha))$ which does not exceed the equilibrium payoff by condition (4.5).

Now we show that $q_L = q_L^{ce}$ globally maximizes a firm's profits when condition (4.4) holds, consumers have the belief $\hat{q}_L$, the competitor produces the quality $q_H = q^C$ and given the certification threshold q^C. Clearly, deviations with $q_L < q^C$ are unprofitable since $q_L^{ce} = b^L\left(q^C,\alpha\right)$. When deviating to some $q \ge q^C$, then we have $(1-\alpha)\,R^H\left(q^C, b^H(q^C,\alpha)\right) + \alpha R^H(q^C,q) - C\left(q\right) \le R^H\left(q^C, b^H(q^C,\alpha)\right) - C\left(b^H(q^C,\alpha)\right) \le \Pi^L\left(b^L(q^C,\alpha), q^C\right)$ where the first inequality is due to the definition of $b^H\left(\cdot,\alpha\right)$, and the second is satisfied by condition (4.4).

In the proof of Proposition 4.1 we show that $\Pi^H\left(b^L(q^C,\alpha), b^H(b^L(q^C,\alpha))\right) > \Pi^L\left(b^L(q^C,\alpha), q^C\right)$. Therefore we have $\Pi^H\left(b^L(q^C,\alpha), q^C\right) > \Pi^L\left(b^L(q^C,\alpha), q^C\right) > 0$, so that both firms enter.

To prove uniqueness, recall that in any equilibrium necessarily either the quality locally maximizes a firm's profits, or the firm produces exactly the quality q^C. This means that there are only two other candidate equilibria, in which the firms either produce (q_L^*, q_H^*) or $\left(q^C, b^H(q^C,\alpha)\right)$. Condition (4.6) assures that (q_L^*, q_H^*) is no equilibrium, since the high quality firm has a profitable deviation when producing q^C instead of q_H^*. To see that $\left(q^C, b^H(q^C,\alpha)\right)$ cannot be an equilibrium, consider two cases. If $\Pi^L\left(q^C, b^H(q^C,\alpha)\right) \le \Pi^H\left(q^C, b^H(q^C,\alpha)\right)$, then $\Pi^L\left(b^L(q^C,\alpha), b^H(q^C,\alpha)\right) > \Pi^L\left(b^L(q^C,\alpha), q^C\right) \ge \Pi^H\left(q^C, b^H(q^C,\alpha)\right) \ge \Pi^L\left(q^C, b^H(q^C,\alpha)\right)$ where the first inequality is true since $R^L_{q_H} > 0$ and the second is true by condition (4.4). Thus the low quality firm has a profitable deviation. If instead $\Pi^L\left(q^C, b^H(q^C,\alpha)\right) > \Pi^H\left(q^C, b^H(q^C,\alpha)\right)$, then $\Pi^L\left(b^L(q^C,\alpha), q^C\right) \ge \Pi^H\left(q^C, b^H(q^C,\alpha)\right) > \Pi^L\left(q^C, b^H(q^C,\alpha)\right)$ where the first inequality holds by condition (4.4). Thus, in this case the former high quality firm profitably deviates to $b^L(q^C, \alpha)$.

Proof of Lemma 4.4

We first prove that condition (4.4) holds for $q^C \geq q_M^{FI}$. In Proposition 4.1, we have shown that $C''' \geq 0$ implies $\alpha R^H\left(q, b^H(q,\alpha)\right) - C\left(b^H(q,\alpha)\right) < 0$ if $\alpha R^H_{q_H}(q,q) \leq 2C'(q)$. Applying this result for $\alpha = 1$ we can infer from $C'(q_M^{FI}) = R^M_{q_M}\left(q_M^{FI}\right) = \frac{1}{4}$ and $R^H_{q_H}(q,q) = \frac{4}{9} \leq \frac{4}{8} = 2C'(q_M^{FI})$ that $R^H\left(q, b^H(q,\alpha)\right) - C\left(b^H(q,\alpha)\right) < 0$ for $q \geq q_M^{FI}$ since $C'(q)$ increases in q.

Next, we show that condition (4.5) is satisfied for $q^C \in [q_H^*, q^\dagger]$ with $q^\dagger$ being uniquely defined by $q^\dagger = b^H\left(b^L(q^\dagger, \alpha), 1\right)$. Note that $\Pi^H\left(q_L^{ce}, q^C\right) = R^H\left(q_L^{ce}, q^C\right) - C(q^C) \geq R^H\left(q_L^{ce}, b^H(q_L^{ce},\alpha)\right) - C(b^H(q_L^{ce},\alpha))$ since $R^H_{q_H}(q_L^{ce}, q) - C'(q) > 0$ for $q \in \left[b^H(q_L^{ce},\alpha), b^H(q_L^{ce},1)\right)$. In addition, for $q^C \in [q_H^*, q^\dagger]$, there is some $\tilde{\alpha} \leq 1$ s.t. $q^C = b^H(q_L^{ce}, \tilde{\alpha})$ by continuity of b^L in α. In Proposition 4.1 we have shown that if $\frac{q^C}{q_L^{ce}} \geq \frac{7}{4}$ (which is necessarily satisfied since $q_L^{ce} = b^L(q^C, \alpha)$), then $\tilde{\alpha} R^H(q_L^{ce}, q^C) - C\left(q^C\right) > \tilde{\alpha} R^L(q_L^{ce}, q^C) - C\left(q_L^{ce}\right)$ which implies $R^H(q_L^{ce}, q^C) - C\left(q^C\right) > R^L(q_L^{ce}, q^C) - C\left(q_L^{ce}\right)$. Since also $R^L(q_L^{ce}, q^C) - C\left(q_L^{ce}\right) - \left[R^L(b^L(q_L^{ce},\alpha), q_L^{ce}) - C\left(b^L(q_L^{ce},\alpha)\right)\right] = \int_{q_L^{ce}}^{q_H^{ce}} R^L_{q_H}(b^L(q_L^{ce},\alpha), q)\mathrm{d}q + \int_{b^L(q_L^{ce},\alpha)}^{q_L^{ce}} \Pi^L_{q_L}(q, q_H^{ce})\mathrm{d}q > 0$ because the integrands of both terms are positive, we have $R^H(q_L^{ce}, q^C) - C\left(q^C\right) > R^L(b^L(q_L^{ce},\alpha), q_L^{ce}) - C\left(b^L(q_L^{ce},\alpha)\right)$. Hence, for $q^C \in [q_H^*, q^\dagger]$, condition (4.5) is satisfied.

Proof of Proposition 4.3

Part i):

Since $\lim_{\alpha\to 0} b^L(q,\alpha) = 0$ and $\lim_{\alpha\to 0} b^H(q,\alpha) = q$ for any $q \in Q$, $\lim_{\alpha\to 0} \Pi^H\left(b^L(q,\alpha), b^H\left(b^L(q,\alpha),\alpha\right)\right) = \Pi^L\left(b^L\left(b^L(q,\alpha),\alpha\right), b^L(q,\alpha)\right) = 0$ for any $q \in Q$. By our hypothesis, $\Pi^M(q^{TB}) = \Pi^H(0, q^{TB}) > 0$. Therefore, there exists an $\tilde{\alpha} > 0$ s.t. condition (4.5) is satisfied for $q^C = q^{TB}$. Since $W_{q_H}\left(b^L(q,\alpha), q\right) > \Pi^H_{q_H}\left(b^L(q,\alpha), q\right) \geq 0$ and $W_{q_L}\left(b^L(q,\alpha), q\right) > \Pi^L_{q_L}\left(b^L(q,\alpha), q\right) > 0$ for all $q \in \left[q_H^*, q^\dagger\right]$, necessarily $q^{TB} > q^\dagger$. Hence, Lemma 4.4 implies that $q^{TB} \in Q_H^C$. Since $q^{TB} \in \arg\max_{q^C \geq q_H^*} \tilde{W}(q^C)$, it remains a maximizer subject to the condition $q^C \in Q_H^C$.

To see that $q^{C*} > q_H^{FI}$, note first that in any equilibrium with $\alpha = 1$ (full information), the equilibrium condition of the low quality firm implies $R_{q_L}^L\left(q_L^{FI}, q_H^{FI}\right) = \frac{\left(r^{FI}\right)^2 (4r^{FI}-7)}{(4r^{FI}-1)^3} > 0$ with $r^{FI} \equiv \frac{q_H^{FI}}{q_L^{FI}}$. This implies $r^{FI} > \frac{7}{4}$. Thus, $R_{q_H}^H\left(q_L^{FI}, q_H^{FI}\right) = \frac{4r^{FI}\left(2-3r^{FI}+4\left(r^{FI}\right)^2\right)}{(4r^{FI*}-1)^3} < R_{q_H}^H(q_L, \frac{7}{4}q_L) = \frac{7}{24}$ since $R_{q_H}^H$ decreases in r. Since $W_{q_H}(q_L, q_H) + C'(q_H) = \frac{24r^3-18r^2+5r+1}{(4r-1)^3}$ decreases in r and $\lim_{r\to\infty} \frac{24r^3-18r^2+5r+1}{(4r-1)^3} = \frac{9}{24}$, necessarily $W_{q_H}(b^L(q_H,\alpha), q_H) > R_{q_H}^H\left(b^L(q_H,\alpha), q_H\right) - C'(q_H) = 0$ for any $q_H \in \left[q_H^*, q_H^{FI}\right]$. Therefore $q^{TB} > q_H^{FI}$.

Part ii):

Define the highest feasible certification level as follows: $\overline{q}^C(\alpha) \equiv \max\left\{q|\Pi^H\left(b^L(q,\alpha), q\right) - \Pi^H\left(b^L(q,\alpha), b^H\left(b^L(q,\alpha),\alpha\right)\right) \geq 0\right\}$. Note that $\overline{q}^C(\alpha) \geq q^\dagger(\alpha)$ since $\Pi^H\left(b^L(q^\dagger,\alpha), q^\dagger\right) - \Pi^H\left(b^L(q^\dagger,\alpha), b^H\left(b^L(q^\dagger,\alpha),\alpha\right)\right) \geq 0$. By definition of $\overline{q}^C(\alpha)$, the lowest derivative of $\Pi^H\left(b^L(q,\alpha), q\right) - \Pi^H\left(b^L(q,\alpha), b^H\left(b^L(q,\alpha),\alpha\right)\right)$ w.r.t. q which is non-zero must be negative at $\overline{q}^C(\alpha)$. For simplicity of this proof we assume w.l.o.g. that $\frac{\mathrm{d}}{\mathrm{d}q}\left[\Pi^H\left(b^L(q,\alpha), q\right) - \Pi^H\left(b^L(q,\alpha), b^H\left(b^L(q,\alpha),\alpha\right)\right)\right]|_{q=\overline{q}^C} < 0$.

We first compute $\lim_{\alpha\to 1}\left[b^H(q_L,1) - b^H(q_L,\alpha)\right]$: By definition of b^H, we have $R_{q_H}^H\left(q_L, b^H(q_L,\alpha)\right) - C'\left(b^H(q_L,\alpha)\right) = (1-\alpha)\, R_{q_H}^H\left(q_L, b^H(q_L,\alpha)\right)$ and in particular $R_{q_H}^H\left(q_L, b^H(q_L,1)\right) - C'\left(b^H(q_L,1)\right) = 0$. These two equations imply $(1-\alpha)\, R_{q_H}^H\left(q_L, b^H(q_L,\alpha)\right) + \int_{b^H(q_L,\alpha)}^{b^H(q_L,1)}\left[R_{q_H,q_H}^H(q_L,q) - C''(q)\right]\mathrm{d}q = 0$ and thus $\left[b^H(q_L,1) - b^H(q_L,\alpha)\right]\min_{q\in[b^H(q_L,\alpha);b^H(q_L,1)]}\left\{-R_{q_H,q_H}^H(q_L,q) + C''(q)\right\} \leq (1-\alpha)\, R_{q_H}^H\left(q_L, b^H(q_L,\alpha)\right)$. Since $b^H(q_L,1)$ is bounded, $\min_{q\in[b^H(q_L,\alpha);b^H(q_L,1)]}\left\{-R_{q_H,q_H}^H(q_L,q)\right\}$ is bounded away from 0 and thus

$$\lim_{\alpha\to 1}\left[b^H(q_L,1) - b^H(q_L,\alpha)\right] \leq \lim_{\alpha\to 1}\left[\frac{(1-\alpha)\, R_{q_H}^H\left(q_L, b^H(q_L,\alpha)\right)}{\min_{q\in[b^H(q_L,\alpha);b^H(q_L,1)]}\left\{-R_{q_H,q_H}^H(q_L,q) + C''(q)\right\}}\right] = 0.$$

Since also $b^H(q_L,1) - b^H(q_L,\alpha) \geq 0$, the claim follows.

Next, we derive an upper bound for $\Pi^H\left(b^L(q^C,\alpha), q^C\right) - \Pi^H\left(b^L(q^C,\alpha), b^H\left(b^L(q^C,\alpha),\alpha\right)\right)$ and use $b^L = b^L(q^C,\alpha)$ and $b^H = b^H\left(b^L(q^C,\alpha),\alpha\right)$ for brevity.

$$\begin{aligned}
&\Pi^H\left(b^L,q^C\right)-\Pi^H\left(b^L,b^H\right)\\
&=\int_{b^H}^{b^H(b^L,1)}\Pi^H_{q_H}\left(b^L,q\right)\mathrm{d}q+\int_{b^H(b^L,1)}^{q^C}\Pi^H_{q_H}\left(b^L,q\right)\mathrm{d}q\\
&<(1-\alpha)\left(b^H(b^L,1)-b^H\right)R^H_{q_H}(b^L,b^H)+\int_{b^H(b^L,1)}^{q^C}R^H_{q_H}\left(b^L,q\right)-C^{'}(q)\mathrm{d}q\\
&<(1-\alpha)\left(b^H(b^L,1)-b^H\right)\overline{R}^H_{q_H}+\int_{b^H(b^L,1)}^{q^C}\left(q-b^H(b^L,1)\right)R^H_{q_H,q_H}\left(b^L,q^C\right)\mathrm{d}q\\
&=(1-\alpha)\left(b^H(b^L,1)-b^H\right)\overline{R}^H_{q_H}-\frac{1}{2}\left(q^C-b^H(b^L,1)\right)^2R^H_{q_H,q_H}\left(b^L,q^C\right)
\end{aligned}$$

where the first inequality comes from $R^H_{q_H,q_H}\left(b^L,q\right)-C^{''}(q)<0$ and $\Pi^H_{q_H}\left(b^L,b^H\right)=(1-\alpha)R^H_{q_H}(b^L,b^H)$ and the second inequality is due to $R^H_{q_H}(b^L,b^H)<\overline{R}^H_{q_H}\equiv\max_{q_L,q_H}R^H_{q_H}(q_L,q_H)=\frac{4}{9}$ and from $R^H_{q_H,q_H,q_H}>0$. Using our results from above we get

$$\lim_{\alpha\to1}\Pi^H\left(b^L,q^C\right)-\Pi^H\left(b^L,b^H\right)<\frac{1}{2}\left(q^C-b^H(b^L,1)\right)^2R^H_{q_H,q_H}\left(b^L,q^C\right)<0$$

for all $q^C>q^{\dagger}(1)$. Since $\overline{q}^C(\alpha)\geq q^{\dagger}(\alpha)$, we conclude that $\lim_{\alpha\to1}\overline{q}^C(\alpha)=\lim_{\alpha\to1}q^{\dagger}(\alpha)=q^{FI}_H$.

It remains to check that for $\alpha\to1$ and any $q^C\in\left[q^{\dagger}(\alpha),\overline{q}^C(\alpha)\right]$, $\Pi^H\left(b^L,b^C\right)>\Pi^L\left(b^L(b^L,\alpha),b^L\right)$. In the proof of Lemma 4.4 we have shown that, $\Pi^H\left(q^{ce}_L,q^{\dagger}(\alpha)\right)>R^L(b^L(q^{ce}_L,\alpha),q^{ce}_L)-C\left(b^L(q^{ce}_L,\alpha)\right)$. Since the last inequality is strict, by continuity it also holds for $q^C\in\left[q^{\dagger}(\alpha),\overline{q}^C(\alpha)\right]$ for $\alpha\to1$.

Since for all α, $q^*_H<q^{TB}$ and $\lim_{q\searrow q^*_H}\tilde{W}_q(q)>0$, the result follows.

Proof of Proposition 4.4

We maintain the notation used in the proof of Proposition 4.3 where we have derived that $\lim_{\alpha\to1}\overline{q}^C(\alpha)\to q^{\dagger}(\alpha)$ and use $b^L=b^L(q^C,\alpha)$ and $b^H=b^H\left(b^L(q^C,\alpha),\alpha\right)$ for brevity. We first prove the following auxiliary Lemma:

Lemma 4.5. *If the elasticity of the marginal cost decreases in q, then there exists some $\epsilon>0$ s.t.*

$\Pi^H\left(b^L, q^C\right) - \Pi^H\left(b^L, b^H\right)$ *is convex in* $\alpha \in [1-\epsilon, 1)$ *for all* $q^C \in \left[q^\dagger(1-\epsilon), \overline{q}^C(1-\epsilon)\right]$.

Proof. Taking derivatives yields

$$\frac{\mathrm{d}}{\mathrm{d}\alpha^2}\left[\Pi^H\left(b^L, q^C\right) - \Pi^H\left(b^L, b^H\right)\right] = b^L_{\alpha,\alpha}(q^C,\alpha)\left(R^H_{q_L}\left(b^L, q^C\right) - R^H_{q_L}\left(b^L, b^H\right)\right)$$
$$+ b^L_\alpha(q^C,\alpha)\left[b^L_\alpha(q^C,\alpha)\left(R^H_{q_L,q_L}\left(b^L, q^C\right) - R^H_{q_L,q_L}\left(b^L, b^H\right)\right) - \frac{\mathrm{d}b^H}{\mathrm{d}\alpha}R^H_{q_L,q_H}(b^L,b^H)\right]$$
$$-\frac{\mathrm{d}^2b^H}{\mathrm{d}\alpha^2}\Pi^H_{q_H}(b^L,b^H) - \frac{\mathrm{d}b^H}{\mathrm{d}\alpha}\left[\frac{\mathrm{d}b^H}{\mathrm{d}\alpha}\Pi^H_{q_H,q_H}(b^L,b^H) + b^L_\alpha(q^C,\alpha)\Pi^H_{q_H,q_L}(b^L,b^H)\right]$$

where $\frac{\mathrm{d}b^H}{\mathrm{d}\alpha} = b^H_\alpha(b^L,\alpha) + b^H_{q_L}(b^L,\alpha)b^L_\alpha(q^C,\alpha)$ and $\frac{\mathrm{d}^2b^H}{\mathrm{d}\alpha^2} = b^H_{\alpha,\alpha}(b^L,\alpha) + 2b^H_{\alpha,q_L}(b^L,\alpha)b^L_\alpha(q^C,\alpha) + b^H_{q_L}(b^L,\alpha)b^L_{\alpha,\alpha}(q^C,\alpha)$. Simple algebra yields

$$\lim_{\alpha\to 1}\left[\frac{\mathrm{d}b^H}{\mathrm{d}\alpha}\Pi^H_{q_H,q_H}(b^L,b^H) + b^L_\alpha(q^C,\alpha)\Pi^H_{q_H,q_L}(b^L,b^H)\right]$$
$$= \lim_{\alpha\to 1}\left[b^H_\alpha(b^L,\alpha)\Pi^H_{q_H,q_H}(b^L,b^H)\right.$$
$$\left.+b^L_\alpha(q^C,\alpha)\left(\frac{\alpha R^H_{q_H,q_L}(b^L,b^H)}{-\alpha R^H_{q_H,q_H}(b^L,b^H) + C''(b^H)}\Pi^H_{q_H,q_H}(b^L,b^H) + \Pi^H_{q_H,q_L}(b^L,b^H)\right)\right]$$
$$= b^H_\alpha(b^L,1)\Pi^H_{q_H,q_H}(b^L,b^H)\,.$$

Using this result and $\lim_{b^H\to q^C} R^H_{q_L,q_L}\left(b^L,b^H\right) - R^H_{q_L,q_L}\left(b^L,q^C\right) = 0$, $\Pi^H_{q_H}(b^L, b^H(b^L,1)) = 0$, $R^H_{q_L,q_H}(b^L,b^H) + \frac{b^H}{b^L}R^H_{q_H,q_H}(b^L,b^H) = 0$ and using that $\frac{\mathrm{d}^2b^H}{\mathrm{d}\alpha^2}$ as well as $b^L_{\alpha,\alpha}(q^C,\alpha)$ are bounded, we get

$$\lim_{\alpha\to 1}\left[\frac{\mathrm{d}}{\mathrm{d}\alpha^2}\left[\Pi^H\left(b^L, q^C\right) - \Pi^H\left(b^L, b^H\right)\right]\right]$$
$$= \frac{\mathrm{d}b^H}{\mathrm{d}\alpha}\left[R^H_{q_H,q_H}(b^L,b^H)\left(\frac{b^H}{b^L}b^L_\alpha(q^C,1) - b^H_\alpha(b^L,1)\right) + b^H_\alpha(b^L,1)C''(b^H)\right]\,.$$

A sufficient condition for $\lim_{\alpha\to 1}\left[\frac{\mathrm{d}}{\mathrm{d}\alpha^2}\left[\Pi^H\left(b^L, q^C\right) - \Pi^H\left(b^L, b^H\right)\right]\right] > 0$ is thus $\frac{b^H}{b^L}b^L_\alpha(q^C,\alpha) < b^H_\alpha(b^L,\alpha)$. Writing this inequality explicitly and rearranging yields

$$\frac{R^L_{q_L}(b^L,q^C)}{\left[-\alpha R^L_{q_L,q_L}(b^L,q^C)+C''(b^L)\right]}\frac{b^H}{b^L}<\frac{R^H_{q_H}(b^L,b^H)}{-\alpha R^H_{q_H,q_H}(b^L,b^H)+C''(b^H)}$$

$$\Leftrightarrow \alpha\left[\frac{-R^L_{q_L,q_L}(b^L,q^C)}{R^L_{q_L}(b^L,q^C)}-\left(\frac{b^H}{b^L}\right)\left(\frac{-R^H_{q_H,q_H}(b^L,b^H)}{R^H_{q_H}(b^L,b^H)}\right)\right]>\left(\frac{b^H}{b^L}\right)\frac{C''(b^H)}{R^H_{q_H}(b^L,b^H)}-\frac{C''(b^L)}{R^L_{q_L}(b^L,q^C)}$$

Using $R^L_{q_L,q_L}=-\frac{2r^3(8r+7)}{q_H(4r-1)^4}$ and $R^H_{q_H,q_H}=\frac{-8r(5r+1)}{q_H(4r-1)^4}$ we have

$$\begin{aligned}&\lim_{b^H\to q^C}\left(\frac{-R^L_{q_L,q_L}(b^L,q^C)}{R^L_{q_L}(b^L,q^C)}-\left(\frac{b^H}{b^L}\right)\frac{-R^H_{q_H,q_H}(b^L,b^H)}{R^H_{q_H}(b^L,b^H)}\right)\\&=\left(\frac{2r^C}{q_H(4r^C-1)}\right)\left[\frac{(8r^C+7)}{(4r^C-7)}-\frac{(1+5r^C)}{2-3r^C+4\left(r^C\right)^2}\right]>0\end{aligned}$$

since $\frac{(8r+7)}{(4r-7)}-\frac{(1+5r)}{(2-3r+4r^2)}>0$ holds iff $21-16r+16r^2>0$ which is always true for $r\geq\frac{7}{4}$. Therefore, a grossly sufficient condition is that $\left(\frac{b^H}{b^L}\right)\frac{C''(b^H)}{C'(b^H)}-\frac{C''(b^L)}{C'(b^L)}\leq 0$ or equivalently $\frac{b^HC''(b^H)}{C'(b^H)}\leq\frac{b^LC''(b^L)}{C'(b^L)}$ which is satisfied since the elasticity of C' is decreasing in q. ■

Part i):

Since $\lim_{\alpha\to 0}b^L(q,\alpha)$ and $\lim_{\alpha\to 0}b^H(q,\alpha)=q$ for any $q\in Q$, $\lim_{\alpha\to 0}\Pi^H\left(b^L(q,\alpha),b^H\left(b^L(q,\alpha),\alpha\right)\right)=\Pi^L\left(b^L\left(b^L(q,\alpha),\alpha\right),b^L(q,\alpha)\right)=0$ for any $q\in Q$. By our hypothesis, $\Pi^H(0,q^{TB})>0$. Therefore, there exists an $\tilde{\alpha}>0$ s.t. condition (4.5) is satisfied for $q^C=q^{TB}$.

Part ii):

By Lemma 4.5, there exists some $\epsilon>0$ s.t. $\Pi^H\left(b^L,q^C\right)-\Pi^H\left(b^L,b^H\right)$ is convex in $\alpha\in[1-\epsilon,1)$ for all $q^C\in\left[q^\dagger(1-\epsilon),\overline{q}^C(1-\epsilon)\right]$. This implies together with $\lim_{\alpha\to 1}\left[\Pi^H\left(b^L,q^{FI*}\right)-\Pi^H\left(b^L,b^H\right)\right]=0$ and $\lim_{\alpha\to 1}\left[\frac{\mathrm{d}}{\mathrm{d}\alpha}\left[\Pi^H\left(b^L,q^{FI*}\right)-\Pi^H\left(b^L,b^H\right)\right]\right]=0$ that $\Pi^H\left(b^L,q^{FI*}\right)-\Pi^H\left(b^L,b^H\right)>0$ for $\alpha\in(1-\epsilon,1)$. By continuity, for any $\alpha\in(1-\epsilon,1)$ there exists some $q^C>q^{FI*}_H$ s.t. $\Pi^H\left(b^L,q^C\right)-\Pi^H\left(b^L,b^H\right)>0$, implying that $\overline{q}^C(\alpha)>q^{FI*}_H$.

In the proof of Proposition 4.3 we have shown that for any $q^C > q_H^{FI*}$, $\Pi^H\left(b^L(q^C,1),q^C\right) - \Pi^H\left(b^L(q^C,1),b^H\left(b^L(q^C,1),1\right)\right) < 0$. By Lemma 4.5, there exists some $\underline{\alpha} < 1$ such that for any $q^C \in (q_H^{FI*},\overline{q}^C(\underline{\alpha})]$, $\Pi^H\left(b^L,q^C\right) - \Pi^H\left(b^L,b^H\right)$ is convex in $\alpha \in (\underline{\alpha},1)$. The convexity together with $\Pi^H\left(b^L(q^C,1),q^C\right) - \Pi^H\left(b^L(q^C,1),b^H\left(b^L(q^C,1),1\right)\right) < 0$ implies $\left[\frac{\mathrm{d}}{\mathrm{d}\alpha}\left(\Pi^H\left(b^L,q^C\right) - \Pi^H\left(b^L,b^H\right)\right)\right]|_{\Pi^H\left(b^L,q^C\right)=\Pi^H\left(b^L,b^H\right)} < 0$ for any $q^C \in (q_H^{FI*},\overline{q}^C(\underline{\alpha})]$.

Recall that $q^{TB}(\alpha) > q_H^{FI*}$ for all α by Proposition 4.3. By the proof of that proposition, there exists some $\tilde{\alpha} \in (\underline{\alpha},1)$ s.t $q^{TB}(\tilde{\alpha}) \geq \overline{q}^C(\tilde{\alpha})$ which implies $\Pi^H\left(b^L,q^{TB}(\alpha)\right) - \Pi^H\left(b^L,b^H\right) < 0$ and hence $\overline{q}^C(\alpha) < q^{TB}(\alpha)$ for all $\alpha \in (\tilde{\alpha},1]$ since $q^{TB}(\alpha)$ increases in α.

Fix any $\hat{\alpha} \in (\tilde{\alpha},1)$. Proposition 4.3 implies that $q^{C*} = \overline{q}^C(\hat{\alpha})$ and that the constraint (4.5) binds. Applying the implicit function theorem yields sign$\left(\frac{\mathrm{d}q^{C*}}{\mathrm{d}\alpha}\right) =$ sign$\frac{\mathrm{d}}{\mathrm{d}\alpha}\left[\Pi^H\left(b^L,q^{C*}\right) - \Pi^H\left(b^L,b^H\right)\right] < 0$ since $\frac{\mathrm{d}}{\mathrm{d}q^C}\left[\Pi^H\left(b^L,q^{C*}\right) - \Pi^H\left(b^L,b^H\right)\right] < 0$ as derived above.

Proof of Proposition 4.5

We first prove the result for $\alpha \to 0$. Define $\overline{q}(\alpha)$ as the unique solution to $\alpha R_{q_H}^H(q,q) - C'(q) = 0$.[54] Note that $\Pi^H\left(\overline{q}(\alpha),b^H(\overline{q}(\alpha))\right) = \Pi^H\left(\overline{q}(\alpha),\overline{q}(\alpha)\right) = -C(\overline{q}(\alpha)) < 0$ since the high quality firm cannot credibly produce any quality level higher than $\overline{q}(\alpha)$. From $R_{q_H}^H(q,q) = \frac{4}{9}$ follows that $\lim_{\alpha\to 0}\overline{q}(\alpha) = 0$.

Define the highest MQS that entails an equilibrium in which two firms enter by $\hat{q}(\alpha) \equiv \max\left\{q \geq q_L^* | \Pi^L\left(q,b^H(q,\alpha)\right) \geq 0 \wedge \Pi^H\left(q,b^H(q,\alpha)\right) \geq 0\right\}$. Since Π^L and Π^H are continuous, and since $\Pi^L(q_L^*,q_H^*) > 0$ and $\Pi^H(q_L^*,q_H^*) > 0$ as well as $\Pi^H\left(\overline{q}(\alpha),b^H\left(\overline{q}(\alpha),\alpha\right)\right) < 0$, we know that for $\alpha > 0$, $\hat{q}(\alpha)$ exists, is necessarily unique and satisfies $q_L^*(\alpha) < \hat{q}(\alpha) < \overline{q}(\alpha)$. Since $\lim_{\alpha\to 0}\overline{q}(\alpha) = 0$ and $\hat{q}(\alpha) \geq 0$, we also have $\lim_{\alpha\to 0}\hat{q}(\alpha) = 0$.

Next we show that if α is small enough, then it is optimal to set the MQS such that only one firm enters, i.e. $W^M(q_M^{SB}) > \max_{q^{MQS}\leq\hat{q}(\alpha)} W(q^{MQS},b^H(q^{MQS},\alpha))$. Define

[54] Clearly, $\alpha MR^H(q,q) - C'(q) = \frac{4}{9}\alpha - C'(q)$ decreases in q. Our assumptions on C assure that $\frac{4}{9}\alpha - C'(0) > 0$ for $\alpha > 0$ and that there exists a $\hat{q}$ with $\frac{4}{9}\alpha - C'(\hat{q}) < 0$, so that a unique solution exists.

$r^{MQS} \equiv \frac{b^H(q^{MQS})}{q^{MQS}}$ and $\psi(r) \equiv \frac{(12r^2-r-2)}{2(4r-1)^2}$ so that $W(q_L, q_H) = q_H\psi\left(\frac{q_H}{q_L}\right)$. Note that $\max_r \psi(r) = \psi(1) = \frac{1}{2}$. We have

$$\begin{aligned} & \max_{q^{MQS}\leq\hat{q}(\alpha)} W(q^{MQS}, b^H(q^{MQS}, \alpha)) \\ = & \max_{q^{MQS}\leq\hat{q}(\alpha)} b^H(q^{MQS}, \alpha)\psi(r^{MQS}) - C(q^{MQS}) - C(b^H(q^{MQS}, \alpha)) \\ \leq & \max_{q^{MQS}\leq\hat{q}(\alpha)} b^H(q^{MQS}, \alpha) \max_r \{\psi(r)\} - C(q^{MQS}) - C(b^H(q^{MQS}, \alpha)) \\ = & \max_{q^{MQS}\leq\hat{q}(\alpha)} b^H(q^{MQS}, \alpha)\frac{1}{2} - C(q^{MQS}) - C(b^H(q^{MQS}, \alpha)) \\ \leq & \frac{1}{2}\overline{q}(\alpha) \end{aligned}$$

where the last inequality holds because $b^H(q, \alpha) < \overline{q}(\alpha)$. Hence, $\lim_{\alpha\to 0} \max_{q^{MQS}\leq\hat{q}(\alpha)} W(q^{MQS}, b^H(q^{MQS})) = 0$ since $\overline{q}(\alpha) \to 0$. The optimal welfare if only one firm enters corresponds to $W^M(q_M^{SB}) > 0$ with q_M^{SB} being characterized by $\frac{3}{8}q_M^{SB} = C'(q_M^{SB})$. Thus, $W^M(q_M^{SB}) > \max_{q^{MQS}\leq\hat{q}(\alpha)} W(q^{MQS}, b^H(q^{MQS}))$ for α small enough. By our hypothesis, $\Pi^M(q_M^{SB}) > 0$, so that $q^{MQS*} = q_M^{SB}$ induces one firm to enter.

Now consider the certification level $q^C = q_M^{SB}$. Clearly, $W\left(b^L\left(q^{M*}\right), q_M^{SB}\right) > \lim_{q_L\to 0} W\left(b^L(q_M^{SB}, \alpha), q_M^{SB}\right) = W^M(q_M^{SB})$ since $W_{q_L}\left(q, q_M^{SB}\right) > \Pi_{q_L}^L\left(q, q_M^{SB}\right) \geq 0$ for $q \in \left(0, b^L(q_M^{SB}, \alpha)\right)$. To see that $q^C = q_M^{SB}$ is feasible, note that $\lim_{\alpha\to 0} \Pi^H\left(b^L(q_M^{SB}, \alpha), q_M^{SB}\right) = \Pi^M(q_M^{SB}) > 0$, $\lim_{\alpha\to 0} \Pi^H\left(b^L(q_M^{SB}, \alpha), b^H\left(b^L(q_M^{SB}, \alpha), \alpha\right)\right) = 0$ and $\lim_{\alpha\to 0} \Pi^L\left(b^L\left(b^L(q_M^{SB}, \alpha), \alpha\right), b^L(q_M^{SB}, \alpha)\right) = 0$, as well as $\lim_{\alpha\to 0} \Pi^L\left(b^L(q_M^{SB}, \alpha), q_M^{SB}\right) = 0$ and $\lim_{\alpha\to 0} \Pi^H\left(q_M^{SB}, b^H\left(q_M^{SB}, \alpha\right)\right) < 0$ so that conditions (4.4) and (4.5) are satisfied for α small enough.

Since $\max_{q^C} W\left(b^L\left(q^C, \alpha\right), q^C\right) \geq W\left(b^L(q_M^{SB}, \alpha), q_M^{SB}\right) > W^M\left(q_M^{SB}\right)$ the claim for a small α is proved.

Our result for $\alpha \to 1$ holds because when all consumers observe the actual quality level of the goods, certification becomes void. Moreover, for $\alpha \to 1$, our model converges to Ronnen (1991), who has shown that there exist welfare increasing $q^{MQS} > 0$.

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Eidesstattliche Versicherung

Ich versichere hiermit eidesstattlich, dass ich die vorliegende Arbeit selbständig und ohne fremde Hilfe verfasst habe. Die aus fremden Quellen direkt oder indirekt übernommenen Gedanken sowie mir gegebene Anregungen sind als solche kenntlich gemacht.

Die Arbeit wurde bisher keiner anderen Prüfungsbehörde vorgelegt und auch noch nicht veröffentlicht.

München, 9. September 2010

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