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Signalling in a Dynamic Labour Market

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This paper analyzes a multiperiod version of the Spence Job Market Signalling Model in which workers cannot commit to an education choice and firms make wage offers at any point in time. The dynamic competition combined with the incomplete information yield a multiplicity of sequential equilibria, including ones that sustain implicit collusion, even though the length of the game is finite. Emphasis is placed on equilibria that satisfy the "independence of never weak best response" criterion of Kohlberg and Mertens (1986). It is shown that in the limit, as the time between offers tends to zero, any such equilibrium results (in expectation) in the unique stable outcome of the static Spence model.

1. INTRODUCTION

With his seminal paper "Job market signaling", Michael Spence was the first to study the important question of whether, in a situation where employers cannot directly observe the marginal product of workers prior to hiring, workers may be able to signal these productivities by means of their education choices. In a model where education costs were assumed to be negatively correlated with productivity (a pre-requisite for such signalling to occur at all), Spence (1973, 1974) found a multiplicity of equilibria. Some of these indeed had the property that the education choice revealed the productivity completely, but in others no information at all was revealed by this choice.

Spence's analysis was not explicitly game-theoretic but what he called "informational equilibrium" is what nowadays would be called "sequential equilibrium" (Kreps and Wilson (1982*a*)). The first complete game-theoretic analysis of the Spence model was performed in Cho and Kreps (1987). In the Cho/Kreps version of the model, workers, after having learned their type (i.e. productivity) move first by choosing an education level $t \in [0, \infty)$. Two risk-neutral firms observe this choice (and nothing more), they then bid (in the style of Bertrand) for the services of the worker, and the worker finally chooses whichever firm bids most. Cho and Kreps showed that only one of the sequential equilibria is "intuitive", viz. the Pareto-best separating equilibrium (also called the Riley outcome (after Riley (1979)). In this equilibrium, the least able type of worker chooses his first-best education level while the more able types just invest enough to separate themselves from their less able colleagues. Throughout this paper,[1] we will restrict ourselves to the most simple version of the Spence model in which education does not increase productivity

1. In Section 6 (and also at the end of Section 5) we point out how our results can be generalized to other specifications of the model.

and in which there are just 2 types of workers with education cost as in Table I. In this case, the Riley outcome has the type 1 (resp. type 2) worker choosing $t_1 = 0$ (resp. $t_2 = 1$) and receiving the wage $w_1 = 1$ (resp. $w_2 = 2$).

If we interpret the education level in Spence's model as an education duration, then the Cho/Kreps game is not entirely satisfactory since it assumes that the worker can *commit* himself to an education time thereby preventing the firms from hiring the worker immediately after the types have sorted themselves. In this paper we assume that the worker cannot commit himself and allow firms to make wage offers not only before or after, but also during the education process. In the modified model, the Riley perfect sorting outcome no longer seems an equilibrium. If the data are as in Table I and the type 1 worker plays according to the Riley outcome, then the first investment in education convinces the firms that the worker has high productivity and it seems that Bertrand competition forces them to offer the wage of 2 immediately. But if wages jump to 2 immediately after having enrolled in the education system, the type 1 worker will choose to invest in education as well, thereby upsetting the equilibrium.

The above-mentioned criticism of the Spence model was first formulated in Weiss (1983). Weiss tackles the problem by modifying the model; he assumes that firms not only care about a worker's productivity but also about his success or failure in education, i.e. about whether he passed the final exam or not. Recently, the criticism has also been advanced in Admati and Perry (1987), who conclude that it is impossible to have separation in a dynamic Spence model because "Once a high ability worker has gone to school long enough to distinguish himself from a worker of lower ability, the firms would offer wages appropriate to a high ability worker *before* enough time has elapsed to present an effective screen". (Admati and Perry (1987, footnote 7; also see p. 362).) Even though this argument sounds intuitive, there is clearly a need for a more formal analysis. Furthermore, if the Riley outcome is not an equilibrium, then what are the equilibrium outcomes if the worker cannot commit himself and how do they differ from the ones in the static game? Our aim in this paper is to solve these problems.

Ideally we would like to approach the problem in a continuous-time framework as this allows the best way to model the idea of not being able to commit oneself. Here we think of the firms as continuously making wage offers and workers responding instantaneously. However, as we are dealing with a delicate incomplete information problem, technically we do not feel ready for this approach. Still the continuous-time formulation may illustrate that matters are more subtle than the above intuition suggests. Namely, the Riley outcome *can* be sustained in equilibrium. Write $w_i(t)$ for the wage offered by firm i at time t and let $a_n(w, t)$ be the probability that the type n worker accepts the wage offer w at time t. The following strategy profile constitutes an equilibrium

$$w_i(t) = \begin{cases} 1 & \text{if } t = 0, \\ 0 & \text{if } \sup\{w_j(\tau), \tau \in (0, t), j = 1, 2\} \leqq 0 \\ & \text{and } t < 2 - \max\{w_j(0), j = 1, 2\}, \\ 2 & \text{otherwise,} \end{cases} \tag{1.1}$$

$$a_1(w, t) = \begin{cases} 1 & \text{if } t = 0 \text{ or } w \geqq 2, \\ 0 & \text{otherwise,} \end{cases} \tag{1.2}$$

$$a_2(w, t) = \begin{cases} 1 & \text{if } w \geqq 2, \\ 0 & \text{otherwise.} \end{cases} \tag{1.3}$$

TABLE I

Type	Productivity	Education cost	Probability
1	1	$t/1$	$1-\mu_0$
2	2	$t/2$	μ_0

It is as if firms agree to delay the wage offer of 2 until time 1; an agreement that is sustained by the threat to offer the wage 2 immediately after the opponent has deviated from the agreed upon plan. It is easily checked that workers respond optimally to the firms' wage offers. Furthermore, firms have zero expected profits and it is impossible for a firm to make positive profits by deviating unilaterally. Even though along the equilibrium path at any time $t \in (0, 1)$ firms know for sure that they face the productive worker, they make unacceptable offers. The reason is that this worker will reject any offer by means of which a firm could make profits (the worker receives 2 immediately after rejecting and, in continuous time, rejection is costless), so that one may as well make an unacceptable offer. Hence, we indeed have an equilibrium resulting in perfect sorting.

The continuous-time formulation has the drawback that it admits many equilibria, and that we do not have a criterion to judge how "plausible" these various equilibria are.[2] For example there also exists a perfect sorting equilibrium in which the type 1 worker accepts the wage 1 at $t=0$ while type 2 accepts the wage $\frac{3}{2}$ at $t=\frac{1}{2}$, hence, firms have positive profits. (Firms threaten to delay the offer 2 till time $t=\frac{3}{2}$ if the worker does not accept the offer at $t=\frac{1}{2}$, and each firm threatens (or promises) to offer 2 immediately after a deviation by the opponent; for details see Section 3.) As we already know from the theory of repeated games with complete information, such collusive equilibria may be expected if the game has infinite length or if it is played in continuous time. Intuitively, however, the reader may feel that the positive profit equilibria cannot be approximated by equilibria of discrete-time games provided that these have finite length. To sort out this issue, we will restrict ourselves to the finite, discrete specification. This restriction also has the advantage that we can make use of the "refinements" literature that deals with the question of how to define "plausible" beliefs. (Note that in the above, beliefs were implicit.)

Throughout the paper it is assumed that there exists a finite upper bound, L, on the time a worker is allowed to spend in the education system. L may be thought of as the length of a lifetime. In our model, this lifetime is split up into a large number of periods of equal length Δ. At the beginning of each period, firms (simultaneously) make offers valid for a worker who leaves school in this period, and the worker decides to accept a current offer or to stay in the school system. This formulation[3] affords the worker a slight possibility to precommit (if he doesn't accept a current offer, he commits to stay in school for Δ time units), but as Δ tends to zero, we naturally approach the no-commitment case.

Not surprisingly, our discrete dynamic model admits a plethora of sequential equilibria. A novel aspect, however, is that the usual multiplicity due to the incomplete information (which enables firms to threaten credibly by adopting incredible beliefs) is compounded by the repeated Bertrand competition. The latter enables firms to sustain implicit collusion by threatening to adopt incredible beliefs if the opponent deviates from the gentleman's agreement. Indeed there exist equilibria in which firms have positive profits. Moreover, as we show in Section 3, such equilibria cannot be eliminated by the

2. Defining strategies in continuous time is also a subtle issue. It is not completely clear that strategies as in (1.1)–(1.3) should be allowed.

3. Formally our model is what Vincent (1988) calls a dynamic auction. In Section 6 we compare our work to Vincent's.

"intuitive criterion" that worked so well in the static version of the model (Cho and Kreps (1987)). Still one would intuitively expect that "implicit collusion" should be eliminated by backwards induction arguments once the possibility of threatening through beliefs is excluded. This intuition is indeed supported by equilibria that satisfy the "independence of never weak best response" criterion (INWBR) from Kohlberg and Mertens (1986). We will show that all such equilibria do result in zero profits for the firms.

The same restriction on beliefs also lends support to the Weiss-Admati/Perry conclusion: for any positive period length Δ, it is indeed impossible to have full separation in any INWBR equilibrium. However, at the same time our analysis will show that the Weiss-Admati/Perry argument loses much of its force, since, as the period length tends to zero, the separation of types becomes complete in any sequence of INWBR equilibria, provided that the length of the games is long enough. (In particular, the equilibrium outcome implied by (1.1)-(1.3) is a limit of equilibrium outcomes of discrete time games, in fact, all *pure* plausible equilibria yield this outcome in the limit.) This conclusion is a special case of the general result we prove: for any given finite upper bound on the education duration, the expected outcomes induced by INWBR equilibria of the dynamic game coincide in the limit (as the period length tends to zero) with the INWBR outcomes of the static Spence game.

Interpreted on a slightly more abstract level, our main result establishes the "strategic equivalence" of the continuous-time dynamic auction model with a much simpler signalling game in which the informed party can commit to the time of trade. This result, at first, appears quite surprising. In our model the equivalence holds since the competition between the firms transforms the rejection of the first wage offer into an implicit commitment because of the fact that any INWBR equilibrium forces the firms to make unacceptable offers for some time following such a rejection. We conjecture that such equivalence holds for a broad class of dynamic auction models. (Some additional results pointing in this direction are described in Section 6). Besides being interesting in itself, this result also suggests the possibility of considerably simplifying the analysis of dynamic auctions.

The remainder of the paper is organised as follows. In Section 2 we briefly review the static Spence game and illustrate how the standard analysis is modified by the introduction of an upper bound on the education duration. In particular, it is shown that the "intuitive criterion" does not lead to a single oucome when the maximally allowed education time is too short. This section also motivates our use of the INWBR criterion. Section 3 introduces the dynamic model, gives examples of various unintuitive sequential equilibria (including ones in which firms implicitly collude) and formally describes the plausibility condition on beliefs that is implied by INWBR. In Section 4 we aim to provide the intuition for our main results by deriving them informally for the special (and usually studied) case in which the bound on education is not binding. In Section 5 we state and formally prove the main results. Section 6 concludes by indicating some limitations and possible extensions of our model.

2. REVIEW OF THE SPENCE MODEL

Given the data from Table I, we consider in this section the game $\Gamma(\mu_0, L)$ specified by the following rules:

(i) The worker learns his type and then takes an education choice $t \in [0, L]$,
(ii) the 2 firms observe t and then simultaneously make wage offers $w_i \in [0, \infty)$,
(iii) the worker chooses a firm.

Agents maximize expected payoffs; the payoff to a worker of type n is $w-t/n$ if he receives the wage w after an education choice of t; a firm has zero profits if it does not attract the worker while the profit is $n-w$ if it attracts the type n worker with a wage of w.

A sequential equilibrium of $\Gamma(\mu_0, L)$ specifies a (possibly random) education choice t_n for the type n worker and, for each education level t, a probability $\mu(t)$ that the firms assign to the worker having type 2 after having observed t as well as a wage offer $w_i(t)$ for each firm i. The following two conditions should be satisfied:

(i) The beliefs $\mu(t)$ should be derived from Bayes' rule whenever possible (i.e. when t occurs with positive probability),

(ii) Firms offer wages equal to the expected productivity i.e. $w_i(t)=1+\mu(t)$, and workers optimize their education choice given the wages.

If $L>0$, the game $\Gamma(\mu_0, L)$ admits infinitely many sequential equilibria, in fact it admits infinitely many equilibrium outcomes. This multiplicity is caused by the fact that the sequential equilibrium concept does not tie down firms' beliefs for education choices that do not occur in equilibrium. By imposing additional restrictions on beliefs, the multiplicity can be reduced. For the special case $L=\infty$, Cho and Kreps (1987) have shown[4] that only one outcome, viz. the separating equilibrium where the type n worker chooses $t_n=n-1$, survives application of the "intuitive criterion". This criterion requires that, for any out of equilibrium education choice, firms put zero weight on those types that are sure to lose (as compared to the equilibrium payoff) by taking this choice. Formally, if π_n is the equilibrium utility of the type n worker, then one requires that

$$\text{if } \pi_1>2-t \text{ and } \pi_2\leqq 2-t/2, \text{ then } \mu(t)=1. \tag{2.1}$$

It is easily seen that the Cho-Kreps arguments remain valid as long as $L\geqq 1$. However, if $L<1$, the separating outcome is no longer an equilibrium and in this case the intuitive criterion is not powerful enough to yield a unique outcome. Namely, assume $\mu_0<L<1$ and consider a pooling equilibrium s where both types of workers choose t^* with

$$\mu_0+L-1<t^*<\mu_0. \tag{2.2}$$

To break the pool, (2.1) requires the type 2 worker to take an education choice $t>2-\pi_1$ but such choices are not available in the game ($L<2-\pi_1=1+t^*-\mu_0$), hence, all such pooling equilibria survive application of the intuitive criterion. These equilibria are, however, eliminated by imposing criterion D_1 of Cho and Kreps (1987) or by requiring universal divinity (Banks and Sobel (1987)) or by insisting that the outcome survives "elimination of never weak best responses" (Kohlberg and Mertens (1986)).[5] These latter concepts basically require that for each t, firms put positive probability only on that type that is most likely to deviate to t. Specifically, if t^* is as in (2.2) and $t>t^*$ then the type 2 worker is most likely to deviate to t as this type definitely deviates to t for any wage $w(t)$ that would induce type 1 to deviate. (If $w(t)-t\geqq\pi_1$, then $w(t)-t/2>\pi_2$.) Hence, the criteria require $\mu(t)=1$ for $t>t^*$ and, therefore, $w(t)=2$ which upsets the equilibrium. Formally, independence of never weak best responses (INWBR) requires that, for each

4. Actually Cho-Kreps analyze a slightly different model, but their arguments apply as well to our specification. Also see Van Damme (1987, Section 10.6).

5. In general, criterion D_1 is weaker than universal divinity which in turn is weaker than "independence of never weak best responses". $\Gamma(\mu_0, L)$ satisfies "response monotonicity" (ceteris paribus, all types of workers prefer a higher wage) and Cho and Sobel (1987) show that for monotonic games the three criteria are equivalent.

t, firms only put positive weight on those types for which the constraint that it is not profitable to deviate to t is binding. Specifically

$$\text{if } \pi_2 + t/2 < \pi_1 + t, \text{ then } \mu(t) = 1. \tag{2.3}$$

(Note that $\pi_n + t/n$ is the wage that should be offered at t to make the type n worker indifferent between deviating to t or not.) It is straightforward to show that there exists a unique equilibrium outcome that can be supported by beliefs as in (2.3).

Proposition 2.1. *The game $\Gamma(\mu_0, L)$ has a unique equilibrium outcome satisfying INWBR. This outcome is given by*

$$t_1 = 0,\ t_2 = 1,\ w(0) = 1,\ w(1) = 2 \qquad \textit{if } L \geqq 1, \tag{2.4a}$$

$$t_1 = \begin{cases} 0 & \textit{with } \text{prob. } (L - \mu_0)/L(1-\mu_0) \\ L & \textit{with } \text{prob. } \mu_0(1-L)/L(1-\mu_0)' \end{cases}$$

$$t_2 = L,\ w(0) = 1,\ w(L) = 1 + L \qquad \textit{if } \mu_0 < L < 1, \tag{2.4b}$$

$$t_1 = t_2 = L,\ w(L) = 1 + \mu_0 \qquad \textit{if } L \leqq \mu_0. \tag{2.4c}$$

Proof. Condition (2.3) implies that, if type 2 chooses t with positive probability, then $w(t) = 2$ or $t = L$. In particular it follows that the type 2 worker does not randomize. If $w(t_2) = 2$ the equilibrium must involve complete separation, hence $t_1 = 0$ and $w(t_1) = 1$. But separation is possible in equilibrium only if $L \geqq 1$ and (2.3) implies $t_2 = 1$ in this case. If $L < 1$, then $w(t_2) < 2$ and $t_2 = L$. Consequently, the type 1 worker also should choose L with positive probability. If $\mu_0 < L$, this probability cannot be one (pure pooling at L is not an equilibrium), hence the type 1 worker is separated with positive probability. Given that type 1 receives the wage 1 if he reveals himself, he will randomize between 0 and L and the probabilities follow directly from Bayes rule. If $L \leqq \mu_0$ there does not exist an equilibrium where type 1 randomizes between 0 and L, hence, he must choose L for sure. ‖

3. THE MULTIPERIOD MODEL: PRELIMINARY RESULTS

In this section we start analyzing the model in which the worker cannot commit himself to an education duration. Again the basic data are as in Table I. After having specified the game, sequential equilibria are defined and it is indicated that there exist a plethora of them. In particular, it is shown that the freedom in beliefs at zero probability events allows firms to sustain implicit collusion in equilibrium. At the end of the section the restriction on beliefs implied by INWBR is specified. The remainder of the paper will then be devoted to showing that INWBR generates an essentially unique outcome and that this outcome converges to the INWBR outcome of the static game when the time between offers tends to zero.

Specifically, in the remainder of the paper we study the game $\Gamma(\mu_0, \Delta, L)$ defined by the following rules. The players are one risk neutral worker and two identical risk neutral firms. Before time 0 the worker learns his type. Decisions are made at time points $t\Delta$ with $t \in \{0, \dots, T\}$ and $T\Delta = L$.[6] At time $t\Delta$ firms, knowing the entire history of rejected wage offers, simultaneously offer wages $w \in [0, \infty)$ and the worker decides to accept a current offer or to stay in the education system. The game terminates if the worker accepts

6. In what follows we will refer to a time point $s = t\Delta$ either as "period t" or as "time s". To ease comparison with Section 2, we will sometimes write "time t" which, by the above convention, is *not* the same as "period t".

or if $t = T$, otherwise the game moves to period $t+1$. Payoffs are defined as in the static game, i.e. a worker of type n gets $w_i - t\Delta/n$ if he accepts the wage w_i of firm i at time $t\Delta$; firm i receives $n - w_i$ in this case, whereas the payoff to firm j is zero.

A history at stage t is a sequence $h_t = (w_1(\tau), w_2(\tau))_{\tau=0}^{t-1}$ of rejected wage pairs, hence, $h_t \in H_t := (\mathbb{R}_+^2)^t$. For convenience, write $H_0 = \{0\}$ and let $H = \bigcup_{t=0}^{T} H_t$ be the set of all histories relevant to the firms. A strategy for firm i is a function $w_i(\cdot)$ that specifies a (possibly random) wage $w_i(h)$ for each history h. A system of beliefs μ is a function $\mu : H \to [0, 1]$ where $\mu(h)$ represents the probability firms attach to the worker having type 2 after having observed the history h. Note that the sequential equilibrium concept (Kreps and Wilson (1982*a*)) requires firms to have the same beliefs as they have identical information. A strategy for the type n worker specifies for each $h \in H$ and each $w = (w_1, w_2) \in \mathbb{R}_+^2$ a triple $(a_n^1(h, w), a_n^2(h, w), r_n(h, w))$ where $a_n^i(h, w)$ is the probability the worker accepts w_i after h and where $r_n(h, w)$ is the probability that he rejects both offers. Given strategies for firms and workers we write $\pi_n(h)$ for the expected payoff of the type n worker resulting from these strategies when the game is started with history h.

A sequential equilibrium specifies strategies for firms and workers together with a system of beliefs consistent with these strategies, having the property that no player can ever, i.e. no matter which history has realised, deviate to a more profitable strategy, given the beliefs associated with this history. There is no need to formally write down all these conditions. Let us just note that consistency requires that the initial beliefs be as in Table 1,

$$\mu(0) = \mu_0, \tag{3.1}$$

and that updating be done via Bayes' rule whenever possible, i.e.

$$\mu(h, w) = \frac{\mu(h) r_2(h, w)}{\mu(h) r_2(h, w) + (1 - \mu(h)) r_1(h, w)} \tag{3.2}$$

whenever the right-hand-side is well-defined. Furthermore, optimal behaviour on the part of the worker means that he should accept (reject) the maximal current offer if this yields more (less) than the expected payoff from rejecting adjusted by the cost of waiting, in particular (with $w_+ = \max(w_1, w_2)$)

$$\text{if } w_+ > \pi_n(h, w) - \Delta/n, \text{ then } r_n(h, w) = 0, \tag{3.3}$$

$$\text{if } w_+ < \pi_n(h, w) - \Delta/n, \text{ then } r_n(h, w) = 1. \tag{3.4}$$

Finally, let us remark that a straightforward dynamic programming argument shows that to have optimal behaviour on the part of the firms it is sufficient to check that no single-period deviation is profitable.

Next, let us indicate that the game admits many sequential equilibrium outcomes and, moreover, that there exist outcomes with qualitatively very different properties. In each case, we will just sketch the essential part of the argument, a detailed description of the strategies may be found in the Appendix. We restrict ourselves to the case $\Delta < \mu_0 < 1 - \Delta/2$, hence, also $\Delta < \frac{2}{3}$. First of all, there exists *a pooling equilibrium* in which both firms offer $w_i = 1 + \mu_0$ at $t = 0$, an offer that is accepted by both workers. Basically, this outcome is sustained by "passive" updating by the firms, i.e. $\mu(h, w) = \mu(h)$ whenever possible; firms refuse to draw inferences from an unexpected rejection. If firms follow this rule, the type n worker will accept (reject) any wage offer above (below) $1 + \mu_0 - \Delta/n$ at $t = 0$ and in this case it is indeed an equilibrium for firms to offer $1 + \mu_0$. Given such behaviour of workers, passive updating is consistent if $w_+ \geqq 1 + \mu_0 - \Delta/2$ or if $w_+ \leqq 1 + \mu_0 - \Delta$, but in the intermediate interval it violates (3.2). To restore consistency one puts $\mu(h, w) = w_+ - 1 + \Delta$ in this interval so that the type 1 worker becomes indifferent between accepting and rejecting, and this allows him to randomize in such a way that the posterior becomes $\mu(h, w)$. Since $\mu_0 > \Delta$ firms make losses by offering wages in the intermediate interval (type 2 surely rejects them), hence, they do not offer such wages, and the pooling outcome with immediate acceptance can be sustained.

In the current model one has the usual multiplicity of sequential equilibria due to the incomplete information which allows firms to threaten credibly against the worker by adopting incredible beliefs. However, this multiplicity is compounded by the dynamic competition: firms can threaten against each other to adopt incredible beliefs whenever the opponent "misbehaves" and indeed in this way one may sustain equilibria involving implicit collusion. Our next example, *a pooling equilibrium with positive profits*, illustrates this possibility. Assume that from time Δ on, the pooling equilibrium described above will be played and consider the following gentleman's agreement: both firms offer a wage $1 + \mu_0 - \Delta/2$ at $t = 0$ and if there are no deviations at $t = 0$ there is passive updating (i.e. firms put $\mu = \mu_0$ at time Δ). Given this agreement among firms, the worker optimizes by accepting the wage $1 + \mu_0 - \Delta/2$ if this is offered by both firms, and each firm can expect a tiny positive profit of $\Delta/4$. Why isn't it profitable for firm i to offer w_i slightly above $1 + \mu_0 - \Delta/2$? The answer is that in this case firm j retaliates by offering $w_j = w_i + \Delta$ in the next period. Given this response, it is optimal for the type 2 worker to reject, while the type 1 worker is indifferent so that he may randomize in such a way to make firm j's threat credible (i.e. to bring the posterior to $w_j - 1$). This mixing also has the consequence that deviating results in an expected loss to firm i (since $\mu_0 \geqq \Delta/2$). To put it differently, it is as if the type 1 worker participates in the agreement by promising to accept wages above the equilibrium level with the desired probability. Hence, no firm will deviate since by overbidding it adversely effects its pool of applicants, and *implicit collusion* can be an equilibrium outcome. Of course, profits are only tiny in the equilibrium described but by repeating the argument it is easily seen that profits can be substantial if the length of the game is long enough. Specifically, if L is large there exist equilibria where both firms offer wages below 1 at $t = 0$ and which are accepted by both types of workers.

As the reader may verify by inspecting the Tables A0–A2 from the Appendix, the paths described above can be sustained by equilibria which involve monotonic beliefs, $\mu(h, w) \geqq \mu(h)$ for all h and w. Intuitively, this monotonicity requirement is justified by the consideration that the higher type has a greater incentive to continue education (he has lower cost) so that after a rejection one should not decrease the probability that one faces this type. This requirement amounts to an intertemporal variant of divinity (Banks and Sobel (1987)) and it successfully reduced the multiplicity of equilibria in the Kreps and Wilson (1982b) chain store game with incomplete information. The above shows that in our game, the requirement does not produce a unique outcome.

The intuitive argument from the introduction that one cannot have full separation was based on the assumption that in equilibrium firms always offer a wage equal to the expected productivity but the above construction shows that this need not be the case. In fact, it is possible to have *full separation*; however, one needs non-monotonic beliefs to establish this. The key insight in getting full separation is that sequential equilibrium does not force the firms to offer the wage 2 in the case where they are sure that the worker is of type 2. Namely, take a history $h \in H_t$ with $\mu(h) = 1$ and $t < T$ and, as a preliminary step, consider the following firms' agreement at h: both of us offer $w = 1$, if nobody deviates we put $\mu(h, w) = 0$ and continue offering $w = 1$, otherwise we put $\mu(h, w) = 1$ and offer $w = 2$ for the remainder of the game. If the firms abide by the agreement it is optimal for the worker to accept the wage 1, hence at h each firm will have an expected profit of $\frac{1}{2}$. If a firm deviates, it has to offer at least $2-\Delta/2$ to attract the worker so that deviating yields lower profits. Hence, no firm will deviate and we have an equilibrium.

Next, one may use the above constructed equilibrium as a threat to establish separation. Let $1-\Delta < w^* < 1-\Delta/2$ and consider the following agreement among firms: at $t=0$ both of us offer w^*; if there are no deviations at $t=0$, then at time δ we put $\mu=1$ and continue as described in the last paragraph. Otherwise we continue with the pooling equilibrium with zero profits associated with the uniquely determined beliefs that are consistent with this continuation. In this case, the type 1 worker accepts w^* at $t=0$ while the type 2 worker waits for the wage 1 at $t=\Delta$, and firms have no incentive to deviate so that we have an equilibrium. Full separation does not result from the type 2 worker investing much more in education, rather it results from the fact that the wage offered to educated workers is so low that it does not pay for the type 1 worker to invest even a little bit.

We hope the reader agrees that not all equilibria described above are "reasonable". In fact, in our view none of them is, and we will formulate a refinement (an additional restriction on beliefs) to eliminate them all. We have already seen that requiring monotonicity of beliefs ($\mu(h, w) \geqq \mu(h)$) is not powerful enough to eliminate the first two. Also the "intuitive criterion" will not do the trick. Basically this criterion just requires $\mu(h, w) = 1$ if $w_+ > 2-\Delta$ (it is optimal for the type 1 worker to reject any lower wage if he (very optimistically) believes that the next period wage will be 2) and all equilibria described above survive application of this criterion. The refinement we will use is based on the ideas already discussed in the previous section, i.e. we will require independence of never weak best responses (INWBR). Hence, we require that at a zero probability event (i.e. after an unexpected rejection) firms put all weight on that type for which the constraint that it not be optimal to reject is binding. The following simple lemma implies that this will always be type 2.

Lemma 3.1. *If s is a sequential equilibrium of $\Gamma(\mu_0, \Delta, L)$, then $\pi_2^s(h) \geqq \pi_1^s(h)$ for all histories h.*

Proof. The type 2 worker has the possibility to mimic type 1, i.e. to play α_1. By doing so his expected payoff after h is at least $\pi_1^s(h)$ (since his education costs are lower than those of 1) hence this is a lower bound for his equilibrium payoff. ‖

This lemma implies that if it is weakly optimal for the type 2 worker to accept, then it is strictly optimal for type 1 to accept

$$\text{if } w_+ \geqq \pi_2(h, w) - \Delta/2, \text{ then } w_+ > \pi_1(h, w) - \Delta, \tag{3.5}$$

and, conversely, if it is weakly optimal for type 1 to reject then type 2 will surely reject the offer. Hence, (3.3) and (3.4) show that in any sequential equilibrium

$$\text{if } r_1(h, w) > 0, \text{ then } r_2(h, w) = 1, \tag{3.6}$$

and

$$\text{if } r_2(h, w) < 1, \text{ then } r_1(h, w) = 0. \tag{3.7}$$

These conditions in turn imply that, as long as beliefs are determined by (3.2) (i.e. as long as $r_2(h, w) > 0$, or equivalently $r(h, w) > 0$), it becomes increasingly likely over time that the worker has high ability

$$\mu(h, w) \geqq \mu(h) \quad \text{if } r(h, w) > 0. \tag{3.8}$$

Finally, equation (3.5) motivates our additional restriction on beliefs. Consider an equilibrium outcome in which after history h (in period t) the game terminates with the acceptance of the wage offer w (hence $r(h, w) = 0$). Equation (3.5) implies that if from period $t+1$ on an equilibrium is played that supports this outcome, then staying in the market till $t+1$ is suboptimal for type 1, but (at least if $w_+ \leqq 2 - \Delta/2$) there exist equilibrium continuations for which such behaviour is optimal for type 2. Hence, staying in the market is an inferior response for type 2 and INWBR requires that[7,8]

$$\mu(h, w) = 1 \quad \text{if } r(h, w) = 0. \tag{3.9}$$

Note that all sequential equilibria discussed in this section violate this requirement. From now on, whenever we speak of *equilibrium* we will mean a sequential equilibrium satisfying condition (3.9). In the next two sections we will show that with this refinement we obtain an essentially unique outcome.

4. HEURISTIC DERIVATION OF THE MAIN RESULTS

In this section we aim to provide the intuition for our main results. The argument contains some gaps which are filled in the next section. Throughout, whenever we speak of equilibrium, we will mean a sequential equilibrium of $\Gamma(\mu_0, \Delta, T)$ satisfying the INWBR requirement (3.9). We concentrate on the most interesting case where Δ is small (specifically $\mu_0 < 1 - \Delta/2$) and $L = T\Delta$ is large ($L \geqq 1 - \Delta/2$) so that in principle it is possible for the more able worker to separate himself.

Let us start with the behaviour of the firms. Since they are identical we may assume they follow the same strategy. In equilibrium they will have non-negative expected profits. Intuitively one expects, and indeed may formally show that non-negativity holds in any period of the game. Consequently, no firm will ever offer a wage above 2 as this would terminate the game immediately by attracting both types and would yield losses. More generally, if firms believe they face the type 2 worker with probability μ, then they will not offer $w > 1 + \mu$ unless they know for sure that this offer is rejected by both types. Finally, if it ever becomes common knowledge that the worker is of type 2, then we have an ordinary (finite-horizon) repeated Bertrand game in which both firms offer the wage 2 throughout and, hence, make zero profits. (As we know from the discussion in Section 3, we actually need a condition like (3.9) to derive this last result.)

Next note that the type 2 worker will reject for sure any wage offer that is strictly less than $2 - \Delta/2$. Namely, if he would accept $w < 2 - \Delta/2$ with positive probability, then (3.2), (3.7) and (3.9) force the belief μ to equal 1 in the next period in which case firms will offer 2, but then this worker is strictly better off by rejecting w, the desired contradiction. If $\mu_0 < 1 - \Delta/2$ firms are not willing to offer a wage $w \geqq 2 - \Delta/2$ at $t = 0$ (since the type 1

7. Formally, INWBR only requires that $\mu(h, w) = 1$ if $r(h, w) = 0$ *and* $w \leqq 2 - \Delta/2$, since, if $w > 2 - \Delta/2$, rejecting is inferior for both types. We could work equally well with this weaker condition. The referees preferred the formulation (3.9).

8. Cho/Kreps have given an example of what they claim is a counter-intuitive aspect of INWBR. Because of monotonicity (cf. footnote 5) such nasty things cannot happen in our model. The requirement (3.9) may also be justified in its own right as an independent refinement, it is similar to what Rubinstein (1985) calls "pessimistic conjectures".

worker will definitely accept, it results in losses) so that, in equilibrium, the able worker rejects the first period wage offers. Hence, he will invest in education.

Now turn to the type 1 worker. Let w^* be the maximal wage offer at $t=0$. Let us first exclude the possibility that this worker rejects w^* for sure. If this would be the case, the rejection contains no information and at Δ firms set beliefs again at μ_0. Assuming that equilibrium payoffs depend only on beliefs and not on the length of the game provided that this is long enough (properties which can indeed be shown to hold) we obtain a contradiction: the equilibrium payoffs starting from time 0 must be equal to those starting from time Δ, but the former are actually Δ smaller since there is one more education period. Hence, type 1 accepts w^* with positive probability and, therefore $w^* \leqq 1$ as otherwise firms would incur losses. Actually, a more or less standard Bertrand argument establishes that both firms will offer $w^*=1$. (Here one needs that the worker is willing to accept wages below 1 and that the acceptance probability is non-decreasing in the wage offered.) Finally, it is easy to see that the type 1 worker cannot accept $w^*=1$ for sure. If he would, the next period wage would be 2 and rejecting would have been strictly better. Consequently, the type 1 worker must randomize at $t=0$.

Since the type 1 workers is willing to incur an education cost, he must expect a wage $\tilde{w}>1$ when educated. Such a $\tilde{w}$ yields losses if it attracts only type 1, and this is impossible, so that we must have $\tilde{w} \geqq 2-\Delta/2$. Let $\tilde{t}$ be the first (possibly random) time at which such a $\tilde{w}$ is offered. Clearly, we must have that $E\tilde{t} \geqq 1-\Delta/2$ as otherwise type 1 is not willing to accept $w^*=1$ at $t=0$. On the other hand, at $\tilde{t}$ the firms must consider it sufficiently likely that they face type 2, specifically $\tilde{\mu} \geqq 1-\Delta/2$ at $\tilde{t}$, as otherwise they expect losses. Also note that at any time between 0 and $\tilde{t}$ firms do not learn anything (as both workers always reject) so that the posterior must be $\tilde{\mu}$ at these times. Now, if we would have $\tilde{\mu}>1-\Delta/2$, then firm i could make positive profits by offering $w_i \in (2-\Delta/2, 1+\tilde{\mu})$ at $t=\Delta$, and a standard Bertrand argument establishes that both firms offer $w=1+\tilde{\mu}$ already at $t=\Delta$ in this case. Hence, we would have $\tilde{t}=\Delta$, a contradiction. Consequently $\tilde{\mu}=1-\Delta/2$ and $\tilde{\omega}=2-\Delta/2$. Since the type 1 worker must be indifferent between $w^*=1$ at $t=0$ and $\tilde{w}=2-\Delta/2$ at $t=\tilde{t}$, it moreover follows that $E\tilde{t}=1-\Delta/2$. Finally, also the type 2 worker must accept $\tilde{w}$ with probability 1 since otherwise firms incur losses at $\tilde{t}$. Hence, the game terminates at $\tilde{t}$.

The above fully describes the set of all possible equilibrium paths of $\Gamma(\mu_0, \Delta, T)$ for the case $\mu_0<1-\Delta/2<\Delta T$. At $t=0$, the firms offer $w^*=1$ and the type 1 worker randomizes to bring the posterior to $\tilde{\mu}=1-\Delta/2$; at $t>0$, each type of worker rejects any wage until finally the wage $\tilde{w}=2-\Delta/2$ is offered; this occurs at a random time $\tilde{t}$ with $E\tilde{t}=1-\Delta/2$ and at $\tilde{t}$ any worker finishes his education. What still has to be verified is that such behaviour actually constitutes an equilibrium. It is clear that (at least along the path) workers cannot profit by deviating, what has to be established is that a firm cannot make positive profits by deviating at some time t in between 0 and $\tilde{t}$. Clearly, given the belief $\tilde{\mu}$ and optimal behaviour of the type 2 worker, positive profits can result only from the type 1 worker accepting a wage less than 1. However, since at $t=0$ this worker is indifferent between accepting and rejecting $w^*=1$, this worker prefers to reject $w<1$ at $t>0$ as long as the other firm follows the equilibrium strategy. Hence, it is impossible to make positive profits, consequently, one may as well make an unacceptable offer.

The analysis thus far supports a literal version of the Weiss-Admati-Perry argument: for any finite period length between the wage offers, there cannot be full separation; if the workers are not separated before starting the education process, they will remain pooled. At the same time, however, our analysis may show that that argument loses most of its force since the separation of types becomes perfect when the time between offers tends to zero. Namely, inverting Equation (3.2) gives the probability, r_1, that the type 1 worker rejects the equilibrium wage offer at $t=0$

$$r_1=\frac{\mu_0\Delta}{2(1-\Delta/2)(1-\mu_0)}$$

and we see that r_1 tends to zero as Δ tends to zero. In the limit, the type 1 worker almost surely does not go to school and different types of workers are separated with probability 1.

The foregoing shows that (for the special case $\mu_0<1-\Delta/2<T\Delta$), all expected equilibrium outcomes of our dynamic auction game converge, as Δ tends to zero, to the INWBR outcome of the static Spence game, i.e. to the Riley outcome. Hence, in a certain precise sense the two models are "strategically equivalent". In fact in the next section we show that this equivalence holds for any *finite* upper bound on the education duration that a worker is allowed. At first this result appears quite surprising since in the dynamic model there is no possibility for the worker to commit himself. The above discussion, however, clearly brings out the reason for this equivalence: in the dynamic model, the equilibrium forces the firms to make unacceptable offers for quite some time after the initial offer is rejected, hence, the result is as if the worker were committed to at least as long an education time.

The above established "equivalence" of the static and the dynamic versions of the Spence model could be strengthened if we would restrict ourselves to games $\Gamma(\mu_0, \Delta, L)$ with length $L=\Delta T=1$. In this case it follows easily that also the education duration of the type 2 worker in the dynamic model converges almost surely to this type's education choice in the INWBR outcome of the static model. However, this restriction is unwarranted and indeed if $L>1$ this stronger convergence property need not hold. Intuitively this will be clear as above we only derived a restriction on the expected education duration of the type 2 worker. To demonstrate the phenomenon formally, let $L=T\Delta=3/2$ and consider the path where at time $t\Delta$ each firm randomizes between $w=1$ and $w=2-\Delta/2$ choosing $w=1$ with the probability p_t given by

$$p_t=\begin{cases}1 & \text{if } t\neq T,\ T/2\\ (1/3-2\Delta/3)^{1/2} & \text{if } t=T/2, \text{ and}\\ 0 & \text{if } t=T.\end{cases} \tag{4.1}$$

(Describing the wages off the equilibrium path is somewhat cumbersome, hence, this will not be done, see Section 5. Consequently, optimality of behaviour will also only be verified along the path.) It is easily seen that, confronted with this path, the type 1 worker is indifferent at $t=0$ between accepting and rejecting, hence, he may randomize to bring the posterior to $\tilde{\mu}=1-\Delta/2$. One also readily verifies that at $t>0$ it is optimal for both types to reject $w=1$ so that the belief remains at $\tilde{\mu}$ as long as wages of 1 are offered. The type 2 worker will accept w only if $w\geqq 2-\Delta/2$, hence, given $\tilde{\mu}=1-\Delta/2$ firms cannot make a profit by offering acceptable wages. Therefore, they may as well make unacceptable offers; along the path their behaviour is optimal and the path (4.1) can be supported by an equilibrium satisfying INWBR. Now if $\tilde{t}(\Delta)$ is the education duration of the type 2 worker in $\Gamma(\mu_0, \Delta, L)$, then $\tilde{t}(\Delta)$ converges, as Δ tends to zero, to the random variable $\tilde{t}$

$$\tilde{t}=\begin{cases}3/4 & \text{with probability } 2/3\\ 3/2 & \text{with probability } 1/3.\end{cases}$$

We see that with positive probability it may happen that the type 2 worker invests strictly more than one time unit in education even in the limit. Hence, the equilibrium outcomes of the dynamic model only converge "in expectation" to the Riley outcome, they need not converge almost surely. (At the end of Section 5 it will be argued that this phenomenon is caused by a "degeneracy" in the model.)

5. FORMAL EQUILIBRIUM ANALYSIS

In this section we formally prove the assertions made in the previous section. Throughout, Δ will be kept fixed and it will be assumed that $\Delta < \frac{2}{3}$. Again we are most interested in the case when $\mu < 1 - \Delta/2 < T\Delta = L$. However, as the proofs involve parametric backwards induction with respect to T and since μ increases as the remaining length becomes shorter, we are forced to treat the least interesting cases first. In each case we will fully specify the conditions that the equilibrium strategies have to satisfy (note that beliefs are fully determined by (3.2) and (3.9)) and, except for the case $\mu = 1 - \Delta/2$, we will leave to the reader to verify that any strategy-tuple satisfying these conditions is indeed an equilibrium. It will turn out that, except again when $\mu = 1 - \Delta/2$, the equilibrium payoffs $\pi_n(h)$ of the type n worker are a simple function of the belief $\mu(h)$ and the length of the game remaining after h. Consequently, if $h \in H_t$ it will be convenient to write $k = T - t$ and $\pi_n(h) = \pi_n(\mu(h), k)$.

The most simple case is when $h \in H_T$, i.e. we are in the final period. In this case one has a standard Bertrand game: workers accept any non-negative wage and firms offer wages equal to the expected productivity. We formally treat this case in Lemma 5.1.

Lemma 5.1. *If $h \in H_T$ and $\mu = \mu(h)$, then in subgame h the equilibrium is essentially unique and is described as follows*:

$$w_i(h) = 1 + \mu \qquad i = 1, 2, \tag{5.1}$$

$$r_n(h, w) = 0 \quad \textit{if } w_+ > 0, \textit{ and} \tag{5.2}$$

$$a_1^i(h, w) = a_2^i(h, w) \quad \textit{if } w = (1+\mu, 1+\mu) \textit{ for each firm } i. \tag{5.3}$$

Firms have zero expected profit and the workers' equilibrium payoffs are given by

$$\pi_n(h) = 1 + \mu. \tag{5.4}$$

Proof. Equation (5.2) follows trivially. Write M_i for the supremum of the support of w_i^* and m_i for the infimum. Assume $M_1 < 1 + \mu$. Then firm 2 can guarantee a positive profit by bidding slightly more than M_1. Hence, firm 2's equilibrium profits must be positive, therefore, $M_2 < 1 + \mu$, and both firms have positive profits. Consequently, bidding m_i for sure should result in positive profits and this is possible only if $m_1 = m_2$ and m_1 is an atom of both w_1^* and w_2^*. However, then firm i can strictly improve its payoffs by bidding $m_i + \varepsilon$ instead of m_i. Hence, in equilibrium we must have $M_i = 1 + \mu$. Therefore, all wages in the support of w_i^* must yield zero profit and this implies $m_i = 1 + \mu$. Consequently, both firms offer $1 + \mu$ for sure. This establishes (5.1). Finally to ensure that both firms are willing to offer $1 + \mu$, both types of workers must choose firm 1 with the same probability as otherwise some firm would make losses, hence, condition (5.3). Equation (5.4) follows trivially. Note that non-uniqueness is just caused by the probability in (5.3) not being completely determined. ∥

Before turning to the next case, let us note that a simple induction argument establishes that in no sequential equilibrium firms ever offer more than 2 so that rationality requires the type n worker to accept a wage above $2-\Delta/n$ for sure

$$r_n(h, w)=0 \quad \text{if } w_+>2-\Delta/n, \quad \text{for all } h, w, n. \tag{5.5}$$

Next, we turn to the case $h\in H_t$ with $t<T$ and $\mu=\mu(h)>1-\Delta/2$. In this case, (5.5) shows that the type 2 worker is willing to accept wages below the maximal wage (i.e. $1+\mu$) that firms are willing to offer and standard Bertrand-type arguments can again be used. As we show in Lemma 5.2, the equilibrium path has firm i offering $w_i=1+\mu$ and both workers accepting immediately.

Lemma 5.2. *If $h\in H_t$, $t<T$ and $\mu=\mu(h)>1-\Delta/2$, then subgame h only allows pooling equilibria with zero profits and immediate acceptance. Specifically, equilibrium is characterized by* (5.1), (5.3), (5.5) *and*

$$r_2(h, w)=1 \quad \text{if } w_+<2-\Delta/2, \tag{5.6}$$

$$r_1(h, w)=1 \quad \text{if } w_+\leqq 1+\mu-\Delta, \tag{5.7a}$$

$$r_1(h, w)=\frac{\mu(1-\nu)}{\nu(1-\mu)} \quad \text{if } w_+\in(1+\mu-\Delta, 2-\Delta), \tag{5.7b}$$

where

$$\nu=w_+-1+\Delta. \tag{5.8}$$

Firms have zero expected profits and the workers' payoffs are again given by (5.4).

Proof. The proof is by induction with respect to $k=T-t$. Note that this is possible since equilibrium beliefs are monotonic ((3.8) and (3.9)) so that we will remain in the case covered by Lemma 5.2. Assume the assertions have already been proved for $k-1$. We have already argued that (5.5) must hold. Equation (5.6) immediately follows from the INWBR requirement (3.9) and the induction step. If type 2 accepts w with positive probability, then the wage jumps to 2 immediately thereafter, so that type 2 must reject wages below $2-\Delta/2$. Now turn to the type 1 worker. If $w_+\leqq 1+\mu-\Delta$ and this worker would accept or randomize, the next period belief would be strictly higher, hence, the next period wage would be above $1+\mu$, implying that accepting with positive probability cannot be optimal. This establishes (5.7a). If $w_+\in(1+\mu-\Delta, 2-\Delta)$, the induction step together with consistency of beliefs imply that the worker must randomize, hence, he should be indifferent. This implies that the next period belief ν must be as in (5.8) and by inverting (3.2) it follows that $r_1(h, w)$ must be as in (5.7b). If $w_+=1+\mu-\Delta$, then (5.7b) reduces to $r_1(h, w)=1$ and this follows from consistency of beliefs together with the fact that $\pi_1(\cdot, k-1)$ is strictly increasing in the relevant range. It remains to establish (5.1) and (5.3), but given the strategies of the workers, this follows from exactly the same argument as in the proof of Lemma 1. Consequently, (5.4) also holds. ||

Now that it has been established that for $\mu=1$ firms always offer 2, it follows from (3.9) that, in equilibrium, (5.6) must be satisfied for any subgame h. This result considerably simplifies the analysis that follows and for later reference we state it as a corollary.

Corollary 5.3. *An equilibrium strategy for the type 2 worker satisfies* (5.6) *for every pair* (h, w) *with* $h \notin H_T$.

Next let us turn to the case $h \in H_t$ with $t < T$ and $\mu = \mu^* = 1 - \Delta/2$. As we know from the previous section, this belief plays a crucial role. It is easy to see that the strategies described in Lemma 5.2 still specify an equilibrium for this case and this obviously is the best equilibrium for the workers. Note that (5.3) and (5.5) require that the type 2 worker accepts the wage $w_+ = 2 - \Delta/2$ for sure even though he is indifferent. In fact, any equilibrium forces this behaviour onto type 2 as otherwise firms would make losses which (by Lemma 5.2) cannot be recouped later. The same argument establishes that any alternative wage offer must be strictly less than $2 - \Delta/2$. As such a wage is rejected by the type 2 worker, the condition that firms profits be non-negative implies that also the type 1 worker rejects the offer unless it is at most 1. However, $\Delta \leqq 2/3$, Lemma 5.2 and (3.2) imply that for this worker it is optimal to reject wages not exceeding 1. Hence, any alternative equilibrium wage offer is rejected by both workers. We see by induction that all equilibria at μ^* completely pool the workers and that they result in zero profits for the firms. Also the equilibrium payoffs of the workers are easily determined. Recall that $k = T - t$ is the remaining number of periods and write $\Pi(\mu^*, k)$ for the set of equilibrium payoff pairs. We have $\Pi_1(\mu^*, k) \subset [1, 2 - \Delta/2]$. The upper boundary has already been established, the lower one follows from an argument as in Lemma 5.1: if there would exist an equilibrium with payoff $\pi_1 < 1$, then firms would see possibilities for profit and Bertrand competition would force them to offer 1, this type 1 would accept, but this is impossible as we have seen above. Furthermore, we have that

$$\min \Pi_1(\mu^*, k) \geqq \min \Pi_1(\mu^*, k-1) - \Delta,$$

as the worst that can happen is that there is an initial unacceptable offer and then there is a continuation with the worst equilibrium from the remaining game. Actually as long as the right-hand-side in this inequality is at least 1, the inequality will be an equality as the above behaviour specifies an equilibrium. Hence, we see

$$\Pi_1(\mu^*, k) = [\max(1, \min \Pi_1(\mu^*, k-1) - \Delta), 2 - \Delta/2],$$

or alternatively if, for $k > 0$, we define

$$\mu_k = \min(k\Delta, 1 - \Delta/2), \tag{5.9}$$

then $\pi_1 \in \Pi_1(\mu^*, k)$ if and only if there exists some $x \in [0, 1]$ such that $\pi_1 = 1 + \mu^* - x\mu_k$. (Note that x may be interpreted as the probability that the worst equilibrium is played.) Given that all equilibria are pooling ones it follows that $\pi_2 = 1 + \mu^* - x\mu_k/2$ whenever $\pi_1 = 1 + \mu^* - x\mu_k$, hence

$$\Pi(\mu^*, k) = \{(1 + \mu^* - x\mu_k, 1 + \mu^* - x\mu_k/2);\ x \in [0, 1]\}. \tag{5.10}$$

The following Lemma summarizes our findings:

Lemma 5.4. *If* $h \in H_t$ *with* $t < T$ *and* $\mu(h) = \mu^* = 1 - \Delta/2$, *the equilibria in subgame* h *are completely pooling. Firms randomize between making offers that are unacceptable to both types and offering* $w_+ = 2 - \Delta/2$. *The game terminates the first time some firm offers the wage* $2 - \Delta/2$ *with both types accepting this offer. Firms have zero profits and the equilibrium payoffs of the workers are as in* (5.10).

Finally we consider the case with $h \in H_t$, $t < T$ and $\mu < 1 - \Delta/2$. We will show that the equilibrium path depends essentially on μ and the "strategic length" μ_k of the game as defined in (5.9). (Roughly μ_k gives the maximal number of times firms can still make an unacceptable offer.) If this length is short, specifically $\mu \geqq \mu_k$ (hence $\mu_k = k\Delta$), the worker prefers the pooling wage $1+\mu$ at the end to being separated at the beginning and in this case the equilibrium involves pooling with delay: firms make unacceptable offers till the end of the game. If the remaining length is long enough, i.e. if $\mu < \mu_k$, the equilibrium yields partial separation: firms offer the wage 1 at $t = 0$ and the type 1 worker randomizes to bring the posterior to μ_k. If $\mu_k = 1-\Delta/2$, the remaining behaviour is as in Lemma 5.4 whereas, if $\mu_k = k\Delta$, firms continue with unacceptable offers till the end. Our final lemma proves these assertions formally.

Lemma 5.5. *If $h \in H_t$, $t < T$ and $\mu = \mu(h) < 1-\Delta/2$ then, after h, the workers' equilibrium strategy is given by* (5.5), (5.6) *and*

$$r_1(h, w) = 1 \quad \text{if } w_+ < \pi_1(\mu, k-1) - \Delta, \tag{5.11a}$$

$$r_1(h, w) = \frac{\mu(1-\nu)}{\nu(1-\mu)} \quad \text{if } w_+ \in [\pi_1(\mu, k-1) - \Delta, 2-\Delta), \tag{5.11b}$$

where ν is implicitly determined by

$$w_+ = \pi_1(\nu, k-1) - \Delta. \tag{5.12}$$

If $\mu < \mu_k$ (where μ_k is defined in (5.9)) both firms offer the wage 1, whereas they may pick arbitrary wages from the interval $[0, \pi_1(\mu, k-1)-\Delta]$ if $\mu \geqq \mu_k$. Consequently, along the equilibrium path, the updated belief ν is given by

$$\nu = \max(\mu, \mu_k). \tag{5.13}$$

Firms have zero expected profits and the workers' payoffs are given by

$$\pi_1(\mu, k) = 1 + \mu - \min(\mu, \mu_k), \tag{5.14a}$$

$$\pi_2(\mu, k) = \pi_1(\mu, k) + \mu_k/2. \tag{5.14b}$$

Proof. The proof is by induction with respect to k, so assume that Equation (5.14) has already been shown to hold for $k-1$. Then, given that $\pi_1(\cdot, k-1)$ is non-decreasing, (5.11a) follows from an argument that by now should be standard. Similarly, it follows that type 1 must randomize if w_+ lies strictly in between $\pi_1(\mu, k-1)-\Delta$ and $2-\Delta$. Equation (5.12) expresses the indifference of this worker and (5.11b) follows from Bayes' rule. Also note that (5.14a) implies that ν and hence $r_1(h, w)$ is uniquely determined unless $w_+ = 1-\Delta$ and $\mu < \mu_{k-1}$. If $\mu < \mu_{k-1}$ the type 1 worker may accept $w_+ = 1-\Delta$ with some positive probability, but if $\mu \geqq \mu_{k-1}$, then $\pi_1(\cdot, k-1)$ is increasing at μ, so that he should reject for sure and this is indeed what equation (5.11b) says that he should do. Next turn to the firms. To understand their behaviour, note first that the condition $\pi_1(\mu, k-1) - \Delta \geqq 1$ is equivalent to $\mu \geqq \mu_k$ since $\mu < 1-\Delta/2$. If $\mu \geqq \mu_k$, the above shows that the type 1 worker surely rejects any wage that is at most $\pi_1(\mu, k-1)-\Delta$. No firm is actually willing to offer a higher wage as it would yield losses, hence the equilibrium must involve rejection. Consequently, the firms can just offer any wage pair w with $w_+ \leqq \pi_1(\mu, k-1)-\Delta$. Equation (5.13) follows from (3.2) in this case. If $\mu < \mu_k$, the type 1 worker is willing to accept wages below 1 so that firms see possibilities for positive profits. However, noting that $r_1(h, w)$ is non-decreasing in w_+ we have a more or less standard Bertrand game and an argument as in Lemma 5.1 shows that firms compete away profits. Hence, both firms offer $w_i = 1$ and Equation (5.13) follows from solving

(5.12) with $w_+ = 1$, incorporating (5.4) and (5.14a). What remains is to derive Equation (5.14), but this just involves a series of elementary substitutions. If $\mu \geqq \mu_k$ both types of worker reject the first period wage, hence, $\pi_n(\mu, k) = \pi_n(\mu, k-1) - \Delta/n$ and (5.14) follows by induction. If $\mu < \mu_k$ firms offer 1 at h and the type 1 worker accepts with positive probability, so that his equilibrium payoff is 1 and (5.14a) holds. This worker randomizes to bring the posterior to μ_k, therefore the type 2 equilibrium payoff is equal to $\pi_2(\mu_k, k-1) - \Delta/2$. By substituting this expression into (5.14b) if $\mu_k < 1 - \Delta/2$ or into (5.10) if $\mu_k = 1 - \Delta/2$ one obtains that (5.14b) holds for k. (In the latter case one also has to use that $\pi_1(\mu_k, k-1) = 1 + \Delta$ since this determines the value of x.) This completes the induction step, hence, the proof of the lemma. ||

The above lemmata establish existence, as well as a complete characterization, of equilibria satisfying condition (3.9). Note that there do not exist equilibria in Markov strategies, i.e. the belief μ cannot be used as a state variable. The reason is that in a subgame h with $\mu(h) = 1 - \Delta/2$, the equilibrium continuation must depend on the previous period's wage offer in order to ensure that the type 1 worker is willing to randomize. For example, if $\mu_0 < 1 - \Delta/2 < (T-1)\Delta$, then (5.10)-(5.14) imply that for any first period wage offer with $w_+ \subset (1 - \Delta, 2 - \Delta)$ we must have

$$\mu(0, w) = 1 - \Delta/2 \quad \text{and} \quad \pi_1(0, w) = w_+ + \Delta,$$

i.e. the updated belief does not depend on the wage offered at $t = 0$ but the equilibrium continuation does. It follows that there does not exist a Markov equilibrium.

Next, let us turn to the paths implied by equilibrium play. Of course, they are completely determined by the previous lemmata, but their description is somewhat hidden in there. Not surprisingly, the paths depend crucially on how the initial belief μ_0 relates to the length of the game $L = T\Delta$. A simple description is possible using the (implicit) notion of unacceptable offer, i.e. an offer that both workers reject for sure in equilibrium. The explicit definition is contained in Lemma 5.5; offers below $1 - \Delta$ are always unacceptable, but even a wage slightly below $2 - 3\Delta/2$ may have this property. The following proposition is the main result of the paper.

Proposition 5.6. *The equilibrium paths of the game $\Gamma(\mu_0, \Delta, L)$ with $\mu_0 < 1 - \Delta/2$ are given by*

(i) If $L \geqq 1 - \Delta/2$, firms offer the wage 1 at $t = 0$ which the type 1 worker rejects with probability

$$r_1 = \frac{\mu_0 \Delta}{2(1 - \Delta/2)(1 - \mu_0)} \tag{5.15}$$

and which the type 2 worker rejects for sure. If $t > 0$, firms make unacceptable offers until a random time $\tilde{t}$ with $E\tilde{t} = 1 - \Delta/2$ at which a firm offers the wage $2 - \Delta/2$ which is accepted by both workers.

(ii) If $\mu_0 < L < 1 - \Delta/2$, firms offer the wage 1 at $t = 0$ which the type 1 worker rejects with the probability

$$r_1 = \frac{\mu_0(1 - L)}{L(1 - \mu_0)} \tag{5.16}$$

and which the type 2 worker rejects for sure. At $0 < t < L$, firms make unacceptable offers, while in the final period they offer $w = 1 + L$ which both workers accept.

(iii) If $L \leqq \mu_0$ firms make unacceptable offers till the end of the game, they then offer $w = \mu_0$ which is accepted.

Proof. Assume $L \geqq 1-\Delta/2$. Then $\mu_0 < \mu_T$ and Lemma 5.5 shows that firms offer 1 in the first period. Equations (5.11b) and (5.14a) imply that the type 1 worker randomizes, hence, by (3.6) the type 2 worker rejects. By substituting $\nu = 1-\Delta/2$ (which follows from (5.13)) into (5.11b) we obtain (5.15). Given the updated belief $\nu = 1-\Delta/2$ at time Δ, we are in the case covered by Lemma 5.4, and imposing the additional restriction $\pi_1(\nu, T-1) = 1+\Delta$ (which expresses the indifference of the type 1 worker at $t=0$) on the equilibria of that Lemma, establishes (i) as the conclusion. The proof of (ii) proceeds similarly. Again $\mu_0 < \mu_T$ so that firms offer 1 and the type 1 worker randomizes. Substituting the posterior $\nu = \mu_T = T\Delta = L$ into (5.11b) yields (5.16). From time Δ on we have $\nu = L > \mu_k = k\Delta$ so that Lemma 5.5 implies that unacceptable offers are made till the last period. In this last period, equilibrium behaviour is as in Lemma 5.1. Finally, the proof of (iii) is just a repetition of the foregoing argument. ||

We are now in the position to compare the outcomes of the dynamic game $\Gamma(\mu_0, \Delta, L)$ with those of the static Spence game $\Gamma(\mu_0, L)$ from Section 2. Given an equilibrium s of $\Gamma(\mu_0, \Delta, L)$, define the *equilibrium outcome* as the 4-tuple of random variables $\langle w_n(\mu_0, \Delta L), t_n(\mu_0, \Delta, L)\rangle_{n=1}^2$ specifying the wages the workers receive in equilibrium together with the times at which they get these wages. Also write $\langle w_n(a_0, L), t_n(\mu_0, L)\rangle_{n=1}^2$ for the INWBR outcome of the static game. By combining the Propositions 2.1 and 5.6 we see that, if $L < 1-\Delta/2$, the outcomes of both games coincide, $\langle w_n(\mu_0, \Delta, L), t_n(\mu_0, \Delta, L)\rangle = \langle w_n(\mu_0, L), t_n(\mu_0, L)\rangle$ for all Δ. If $L \geqq 1-\Delta/2$, the correspondence is not perfect, but still the outcomes are very similar if Δ is small. In particular, we are interested in the limit as Δ tends to zero since this corresponds to vanishing commitment power on the part of the worker. Proposition 5.6 implies that, in any equilibrium, the equilibrium allocation of the type 1 worker converges almost surely to $\langle w_1(\mu_0, L), t_1(\mu_0, L)\rangle$. Furthermore, the wage the type 2 worker receives converges almost surely to this worker's productivity, and his expected education duration converges to $1 = t_2(\mu_0, L)$. In particular, in the limit there almost surely is perfect separation. Hence, we have

Corollary 5.7. *If $\langle(\mu_0, \Delta, L), t_n(\mu_0, \Delta, L)\rangle_{n=1}^2$ is an equilibrium outcome of $\Gamma(\mu_0, \Delta, L)$, then, as Δ tends to zero, $w_n(\mu_0, \Delta, L)$ converges almost surely to $w_n(\mu_0, L)$. Furthermore $t_1(\mu_0, \Delta, L)$ converges almost surely to $t_1(\mu_0, L)$ and $Et_2(\mu_0, \Delta, L)$ converges to $t_2(\mu_0, L)$.*

The above Corollary makes precise the sense in which the two models considered are equivalent. We have already seen at the end of Section 4 that, unless $L \geqq 1$, the education time of the type 2 worker need not converge almost surely.

This, however, is just an artifact of our model being in some sense degenerate: it is caused by the fact that the incremental education cost of the type 2 worker is constant. Changing this feature of the model, by allowing for non-linear cost functions (or non-evenly spaced decision points) yields a stronger convergence result.

To illustrate this claim, assume $L \geqq 1$ and that the type 2 worker faces strictly increasing marginal education costs with $c_2'(t) \leqq 1$ for $t \leqq 1$, while the other data remain as in Table 1. The type 1 worker again has to randomize at $t=0$. The crucial observation to make is that, if the offer at $t=0$ is rejected, the continuation equilibrium is one with complete, delayed pooling, i.e. $\mu(t\Delta)=\mu(\Delta)$ for $t>1$ along the equilibrium path.[9] The firms make zero profits, hence if the type 2 (and therefore also the type 1) worker accepts the wage w_2 at time $t\Delta$, then

$$w_2=1+\mu(t\Delta)=1+\mu(\Delta). \tag{5.17}$$

Note, therefore, that although the education duration of the type 2 worker may be random in equilibrium, the equilibrium wage is not; the latter is completely determined by the behaviour of the type 1 worker at $t=0$. Condition 3.9 implies that, for w_2 to be acceptable for type 2 at $t\Delta$, we should have

$$w_2 \geqq 2-[c_2((t+1)\Delta)-c_2(t\Delta)]. \tag{5.18}$$

On the other hand, firms should not find it profitable to offer w_2 already before $t\Delta$, hence

$$w_2 \leqq 2-[c_2(t\Delta)-c_2((t-1)\Delta)]. \tag{5.19}$$

The strict convexity of $c_2(\cdot)$ implies that, for fixed w_2, the equations (5.18)-(5.19) can be satisfied for at most two adjacent values of t, say t_2 and t_2+1. It follows that in the limit ($\Delta\to 0$) the education time of the type 2 worker becomes deterministic. Since the type 1 worker must be indifferent at $t=0$, we finally should have

$$w_2-(t_2+1)\Delta \leqq 1 \leqq w_2-t_2\Delta. \tag{5.20}$$

Combining the above inequalities yields that $w_2\to 2$ and $t_2\Delta\to 1$ as $\Delta\to 0$, hence, if the type 2 worker has increasing marginal education cost, the INWBR equilibrium outcomes of the dynamic game converge almost surely to the INWBR outcome of the static game for any finite upper bound on the education duration.

6. CONCLUDING REMARKS

In this section, we indicate some limitations and possible extensions of our analysis.

Throughout we have assumed that there is an exogeneously given finite upper bound, L, on the workers' education duration (L may be thought of as the workers' lifetime), and we have shown that this bound matters only if it is "small". The original analyses of the (static) Spence model (Spence (1974), Cho and Kreps (1987)) were, however, conducted for the case where $L=\infty$ and it will be clear that in this case our equivalence result (Corollary 5.7) does not hold: the game $\Gamma(\mu_0, \Delta, \infty)$ admits equilibria that do not produce the Riley outcome when Δ tends to zero. The reason is that, for $L=\infty$, the requirement (3.9) does not force firms to offer a wage of 2 when it is common knowledge that the worker has type 2; the infinitely repeated Bertrand game allows firms to implicitly collude in this case. Hence, if the game has infinite length, we will not be able to eliminate the folk theorem-type equilibria discussed in Section 3.

Recall that the motivation for this paper was to analyze what would happen in the Spence job market model if the worker could not commit himself to an education programme in advance. Note, however, that we have actually analyzed a model that does

9. This property clearly holds for the model formally analyzed in this section. The extension to the non-linear case considered here is not entirely straightforward. The main new complication is, to show that for no possible realization of firms' equilibrium strategies type 1 is willing to accept $w=1$ at $t>0$. Once this is established, it follows from the zero profits result and the fact that type 2 never randomizes in equilibrium that the continuation has to be characterized by complete pooling.

not even allow the worker to commit to quit the education system: in $\Gamma(\mu_0, \Delta, L)$ the worker is forced to stay in school if he does not accept a wage offer. It will be clear that our results remain valid for the modified game in which at each point the worker can (by quitting) obtain an outside option as long as the value of the latter is less than the value of our equilibrium. Nevertheless, the reader may argue that it would have been more interesting to analyze the game in which the order of the moves is reversed, i.e. first the worker chooses whether to continue education or not, the firms become active only if the worker quits, and they then play a one-shot Bertrand game for the worker's services. (Once he has dropped out of school, the worker is not allowed to re-enter.) Actually this game is somewhat easier to analyze as firms will always offer wages equal to the expected productivity. Furthermore, it can be shown that this game yields outcomes very similar to those obtained in Section 5 and that indeed a stronger version of Corollary 5.7 will continue to hold. These results can be derived by noting that INWBR, as in our analysis, implies that type 2 will never accept an offer below $2-\Delta/2$ before the very end of the game. In addition, quitting earlier than he is supposed to in equilibrium is an inferior response for type 2. Therefore, firms are forced to believe that the worker is of type 1 when quitting occurs too early. At this point it may be good to recall the assumption that each firm is always completely informed about the wage offers that the opponent has made in the past. Indeed, in the equilibrium constructed, firms do make essential use of this information. We do not know what the equilibria are in case each firm only knows its own rejected wage offers.

At the end of the previous section we already indicated that the assumption of constant marginal education cost is not necessary for our equivalence result to hold, and that Corollary 5.7 can be strengthened if the marginal cost is increasing. Similarly, the assumption that education is not directly productive is not essential (productivity should just not fall when education is increased), and we could introduce time preference as well. What drives the result is the "single crossing property" and the "response monotonicity" that give the type 2 worker an incentive to invest in education. Note that in our model (in contrast to the model considered in Gul and Sonnenschein (1988) the delay does not vanish when the time between offers become shorter. The cause is that our model involves "common values" whereas Gul/Sonnenschein consider independent values. (Also see Vincent (1988).)

The most important restriction of our analysis clearly is that it only covers the case of two types. We now wish to indicate that this assumption can be easily relaxed. Assume there is an additional third type of worker with productivity 3 and education cost $t/3$. Assume that Δ is small and L is large so that the static game allows a separating equilibrium. INWBR now requires firms to believe that they face the third type of worker in any zero probability event. Consequently, this type will only accept wages of at least $3-\Delta/3$. The Equations (3.6) and (3.7) continue to hold so that, as long as there is a positive probability that the type 1 worker is still in the market, the type 2 worker is in there for sure. In this case the wage offers will be strictly below $3-\Delta/3$, hence, they will actually be below 2. We see that the type 1 worker cannot benefit from the existence of type 3 and that the game naturally decomposes into one between the types 1 and 2 and one between the types 2 and 3. Therefore, the INWBR equilibria of the overall game consist of the equilibria of these respective games patched together: The type 1 worker randomizes at $t=0$ (accepting the wage 1 almost surely), at (an expected) time 1 firms offer (almost) 2 and type 2 randomizes (going out almost surely) and at (an expected) time 3 the remaining workers accept a wage of almost 3. Hence, the equivalence result from Corollary 5.7 still holds and it will continue to hold for any finite number of types.

Although we have not yet formally analyzed the game with a continuum of types, we are confident that our equivalence result continues to hold in this case.[10] Actually, this game should be easier to analyze since there will be no randomization: at each point in time a sub-interval of types drops out, with less productive workers dropping out earlier. If Δ tends to zero, the width of these intervals tends to zero and in the limit we obtain the Pareto-best separating outcome of the static game. Our confidence is strengthened by the results of Vincent (1988). Vincent analyzes a dynamic version of the Akerlof lemon problem. There are 2 identical buyers who make repeated offers to a seller with a car of unknown quality $q \in [0, 1]$. Vincent uses a modification of the Grossman/Perry concept of perfect sequential equilibrium to solve this game and he finds that, as the time between offers tends to zero, the equilibrium outcome converges to the Pareto-best separating equilibrium of the signalling game in which the seller can commit himself to a time at which he wants to trade. The reader may verify that in Vincent's model one obtains exactly the same solution if one imposes the refinement idea of our paper (which amounts to requiring that, whenever something unexpected happens, one believes one faces the highest-quality car for sure). The similarity of the models strongly suggests that Corollary 5.7 continues to hold when the type space is continuous.

To summarize, we think it is fair to say that the results of this paper indicate that for a large class of models with one-sided incomplete information and common values, the dynamic Bertrand game in which uninformed buyers make offers is equivalent to the static signalling game in which the informed seller commits to the terms of trade. This insight should be of use in analyzing dynamic auctions in general.

APPENDIX. DESCRIPTION OF THE EQUILIBRIA FROM SECTION 3

In this appendix, we provide the complete description of the strategies discussed in Section 3 and briefly indicate why they constitute equilibria. Let us first specify the common elements of all equilibria: firms will follow the same strategy and workers will randomize equally among both firms when they make the equilibrium wage offers. Furthermore, firms will never offer wages above 2 in equilibrium, so that for high wages we may specify the updating rule and the worker's responses as given in Table A0.

Note that as long as beliefs are as in Table A0, the equilibrium survives application of the intuitive criterion (Cho and Kreps (1987)). If $w_+ < 2-\Delta$, it is optimal for both types of worker to reject if they expect the wage to jump to 2 immediately after w was offered, hence, equilibrium dominance does not impose any restrictions in that case. Hence, this criterion cannot eliminate any of the equilibria to be described below.

Next let us specialize to the pooling equilibrium with zero profits and immediate acceptance. We restrict ourselves to the case $\mu_0 \geqq \Delta$; details are slightly different in the other case. Firms offer $w_i(h) = 1+\mu$ for any history h with $\mu(h) = \mu$, and the posterior beliefs and the worker's responses are as in Table A1.

It is easily checked that updating as in Tables A0, 1 is consistent with (3.2). (Note that indeed the rejection probability of player 1 lies between 0 and 1.) Furthermore, given such updating and given the strategies of the firms, the worker has the choice between accepting w_+ now or receiving $1+\nu$ after one more education period, from which it follows that $r_n(\cdot)$ as in these tables is in agreement with (3.3)-(3.4), hence, the worker plays

TABLE A0

Case	$\nu = \mu(h, w)$	$r_1(h, w)$	$r_2(h, w)$
$w_+ \geqq 2-\Delta/2$	1	0	0
$2-\Delta \leqq w_+ < 2-\Delta/2$	1	0	1

10. By adapting the arguments from Gul, Sonnenschein and Wilson (1986), it is seen that our results also continue to hold for the case in which there are infinitely many workers with abilities distributed as in Table 1, provided we make the (restrictive) assumption that firms do not condition their strategies on the behaviour of subsets of workers with measure zero.

TABLE A1

Case	$\nu=\mu(h, w)$	$r_1(h, w)$	$r_2(h, w)$
$1+\mu-\Delta/2 \leqq w_+ < 2-\Delta$	μ	0	0
$1+\mu-\Delta < w_+ < \min(1+\mu-\Delta/2, 2-\Delta)$	$w_+-1+\Delta$	$\mu(1-\nu)/\nu(1-\mu)$	1
$w_+ \leqq 1+\mu-\Delta$	μ	1	1

optimally. Finally, given the strategies of the workers, firms play an ordinary Bertrand game in which bidding $w_i(h)=1+\mu(h)$ is an equilibrium. Hence, we have a sequential equilibrium.

Now we turn to the pooling equilibrium with positive profits. Assume $\Delta/2<\mu_0<1-\Delta/2$ and let players continue with the offers from Tables A0, 1 for any posterior ν at time Δ. Let firms offer $w_i(0)=1+\mu_0-\Delta/2$ at $t=0$, and let the responses of workers and the posterior ν at Δ be as in Table A2.

We claim that the firms' strategies together with the Tables A0, 2 describe a sequential equilibrium. Again it is easy to check that updating is consistent with (3.2). The type 1 worker randomizes exactly as to bring the posterior to the desired level. Furthermore, in the situations in which he randomizes, the condition (3.3) and (3.4) allow him to do so since he is indifferent. Hence, the worker behaves optimally. It remains to check that firms cannot gain by deviating at $t=0$. A firm has equilibrium profits of $\Delta/4$. If a firm bids below $1+\mu_0-\Delta$ it does not attract the worker and profits are zero. Any bid above $1+\mu_0-\Delta$ results in losses if it attracts only the type 1 worker (since $\mu_0 \geqq \Delta$). To attract the type 2 worker, the wage offer has to be at least $2-\Delta/2$, but any such offer results in losses since it also attracts the type 1 worker and since $\mu_0<1-\Delta/2$. Consequently, a firm cannot gain by deviating. The essential point is that by bidding above the equilibrium wage, the firm adversely changes the pool of workers it attracts, and this makes overbidding suboptimal.

Next, consider a subgame $h \in H_t$ with $\mu(h)=1$ and $t<T$. Suppose that after any history $(h, w) \in H_{t+1}$ play continues with the pooling equilibrium from the Tables A0, 1. Let firms offer $w_i(h)=1$ and let the updated belief and the response of the type 2 worker be as in Table A3.

We need not specify the response of the type 1 worker after (h, w), since this does not enter firms' calculations. Note that the sequential equilibrium concept allows such updating if $\mu_0<1$. It is easily checked that, given this updating, both firms and workers behave optimally, so that we indeed have an equilibrium for subgame h. (A firm does not attract the worker by bidding more than 1, unless it bids at least $2-\Delta/2$, but then expected profits are lower; the worker accepts the wage 1, since if he does not, firms will continue to offer this low wage for the remainder of the game.)

Finally, we specify a completely separating equilibrium. Let both firms offer w^* with $1-\Delta<w^*<1-\Delta/2$ at $t=0$, and let the responses and the equilibrium continuations at $t=\Delta$ be as in Table A4.

(A1 means continue with the pooling equilibrium described by the Tables A0, 1.) It is trivial to check that Tables A0, 4 specify a sequential equilibrium if $\mu_0 \geqq \Delta$. A firm cannot increase profits by bidding w_+ above w^*: If $w_+ \leqq 1$, the type 1 worker rejects, while the type 2 worker rejects any period 1 wage below $2-\Delta/2$. In equilibrium, the type 1 worker accepts w^* at $t=0$, type 2 worker accepts the wage 1 in period 1; there is full separation and firms have positive profits.

TABLE A2

Case	$\nu=\mu(0, w)$	$r_1(0, w)$	$r_2(0, w)$
$1+\mu_0-\Delta/2 < w_+ < 2-\Delta$	$w_+-1+\Delta$	$\mu(1-\nu)/\nu(1-\mu)$	1
$w_+=1+\mu_0-\Delta/2$	μ_0	0	0
$1+\mu_0-\Delta < w_+ < 1+\mu_0-\Delta/2$	$w_+-1+\Delta$	$\mu(1-\nu)/\nu(1-\mu)$	1
$w_+ < 1+\mu_0-\Delta$	μ_0	1	1

TABLE A3

Case	$\nu=\mu(h, w)$	$r_2(h, w)$
$w=(1, 1)$	0	0
$w \neq (1, 1), w_+ < 2-\Delta/2$	1	1

TABLE A4

Case	ν	Continuation	$r_1(h, w)$	$r_2(h, w)$
$1+\mu_0-\Delta < w_+ < 2-\Delta$	$w_+ - 1 + \Delta$	A1	$\mu(1-\nu)/\nu(1-\mu)$	1
$w^* < w_+ \leqq 1+\mu_0-\Delta$	μ_0	A1	1	1
$w_+ = w^*$	1	A3	0	1
$w_+ < w^*$	μ_0	A1	1	1

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REFERENCES

ADMATI, A. and PERRY, M. (1987), "Strategic Delay in Bargaining", *Review of Economic Studies*, **54**, 345-364.

BANKS, J. and SOBEL, J. (1987), "Equilibrium Selection in Signaling Games", *Econometrica*, **55**, 647-663.

CHO, I. K. and KREPS, D. (1987), "Signaling Games and Stable Equilibria", *Quarterly Journal of Economics*, **102**, 179-221.

CHO, I. K. and SOBEL, J. (1987), "Strategic stability and uniqueness in signaling games" (Discussion Paper 87-10, U.C. San Diego).

GUL, F. and SONNENSCHEIN, H. (1988), "On delay in bargaining with one-sided uncertainty", *Econometrica*, **56**, 601-612.

GUL, F., SONNENSCHEIN, H. and WILSON, R. (1986), "Foundations of dynamic monopoly and the Coase conjecture", *Journal of Economic Theory*, **39**, 155-190.

KOHLBERG, E. and MERTENS, J-F. (1986), "On the strategic stability of equilibria", *Econometrica*, **50**, 1003-1039.

KREPS, D. and WILSON, R. (1982a), "Sequential equilibria", *Econometrica*, **50**, 863-894.

KREPS, D. and WILSON, R. (1982b), "Reputation and imperfect information", *Journal of Economic Theory*, **27**, 253-279.

RILEY, J. (1979), "Informational Equilibrium", *Econometrica*, **47**, 331-359.

ROSENTHAL, R. W. and WEISS, A. (1984), "Mixed Strategy Equilibrium in a Market with Asymmetric Information", *Review of Economic Studies*, **51**, 333-342.

RUBINSTEIN, A. (1985), "The choice of conjectures in a bargaining game with incomplete information", in Roth, A. E. (ed.), *Game Theoretic Models of Bargaining* (Cambridge: Cambridge University Press).

SPENCE, A. M. (1973), "Job market signaling", *Quarterly Journal of Economics*, **90**, 225-243.

SPENCE, A. M. (1974) *Market Signaling, Information Transfer in Hiring and Related Processes* (Cambridge, Mass.: Harvard University Press).

VAN DAMME, E. (1987) *Stability and Perfection of Nash Equilibria* (Berlin: Springer Verlag).

VINCENT, D. (1990), "Dynamic Auctions", *Review of Economic Studies*, **57**, 49-61.

WEISS, A. (1983), "A sorting-cum-learning model of education", *Journal of Political Economy*, **91**, 420-442.