

Volume 30, Issue 1**Insider patent holder licensing in an oligopoly market with different cost structures: Fixed-fee, royalty, and auction**

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The issue of the optimal licensing contract in firms having different cost structures is studied when the innovator is a producing patent holder who has three alternative licensing strategies, namely, the fixed-fee, royalty rate, and auction strategies. We conclude that the auction licensing strategy is not the best strategy when the innovator is a producing patent holder. This finding differs from that of Kabiraj (2004) where the auction licensing method is the optimal licensing strategy when the innovator is a non-producing patent holder. However, when we only compare two of the licensing methods, namely, the fixed-fee licensing method and the royalty licensing method, we conclude that if the inside innovator licenses to only some of the firms, then the royalty licensing method will be the best strategy. This result is different from that of Fosfuri and Roca (2004), who concluded that if only some of the licensees obtain a licensing contract, then the fixed-fee licensing method will be the best choice for a producing patent holder.

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1. Introduction

Technology licensing is an important business action from the perspective of the inventor and innovator. Rostoker (1984) surveyed the licensing mode of the firm and concluded that 13% of the sample used the fixed-fee licensing mode, 39% used the royalty licensing mode, and 46% used the two-part (fee plus royalty) licensing mode. Since technology transfer through auctions is less discussed than the other methods and the auction is also an important licensing mode, in this paper we focus on the licensing effect of an auction.

The seminal literature on technology transfer first started with Arrow (1962), who discussed the technology transfer effect for the royalty licensing method and found that the innovator has a larger licensing profit in a competitive market than in a monopolistic market. The contribution of licensing in a competitive industry based on comparing the fixed-fee licensing method with the royalty licensing method is shown in Kamien and Tauman (1984). However, the above two studies lack the firm's strategic interaction.

The licensing literature can be divided into where the innovator is either a non-producing patent holder or a producing patent holder. One important contribution of the former type is that by Kamien and Tauman (1986). The innovator uses a fixed-fee or a royalty rate to transfer technology to firms which produce homogeneous goods and engage in Cournot or Bertrand competition. The result shows that it is good for both the innovator's profit and the consumer's surplus to use the fixed-fee licensing method.

Some studies discuss the licensing effect of the auction licensing method. Kamien et al. (1992) considered three kinds of licensing contract – the fixed-fee, royalty, and auction – using a generalized demand function. They concluded that the auction licensing contract is better than the fixed-fee licensing contract, while the royalty licensing contract is inferior to each of the other two licensing contracts. Muto (1993) found that the royalty licensing mode dominates other licensing modes when the licensees have differentiated products and engage in Bertrand competition. Kabiraj (2004) set up the licensees as having the same cost structure and the licensor as a non-producing patent holder in order to choose the optimal licensing contract. His results show that the auction licensing mode is superior to other modes for the non-drastic case.

Articles in which the innovator is a producing patent holder include those by Katz and Shapiro (1985), Rockett (1990), Wang (1998), and Fosfuri and Roca (2004). Generally speaking, the royalty licensing method is the best licensing method when the innovator is a producing patent holder. However, Fosfuri and Roca (2004) concluded that the fixed-fee licensing method might be optimally chosen when a

producing patent holder licenses a new technology to only some of the licensees.

In our paper the innovator is a producing patent holder and each firm has different cost structures. The licensor chooses the optimal licensing strategy among the fixed-fee, royalty rate, and auction licensing strategies. Our results show that when a producing patent holder licenses to only certain of the firms, the royalty licensing contract is the best strategy. This conclusion differs from the finding of Fosfuri and Roca (2004). Besides, we also conclude that when three licensing contracts are available, providing royalty licensing to both firms is the optimal choice. In other words, the auction licensing contract is not the best licensing method. This result, however, contrasts with that of Kabiraj (2004), who concluded that of the three available licensing strategies, the optimal licensing contract for a non-producing patent holder is the auction.

The remainder of this paper is organized as follows. In Section 2 we set up the model and discuss the three licensing strategies. Section 3 analyzes the optimal licensing contract in term of maximizing the total profit of the patent holder. Section 4 provides the conclusion.

2. The model setup and analysis

We consider a three-firm game with one firm producing the patent and product, Firm L , and two manufacturing firms referred to as Firm M and Firm H . There are different costs among them, with firm L at c_L , firm M at c_M , and firm H at c_H , where $0 < c_L < c_M < c_H < a$. To simplify, let $c_L = c - 2\varepsilon$, $c_M = c - \varepsilon$, and $c_H = c$. If Firm L licenses to competitors, this will result in a reduction in each firm's unit cost to $c - 2\varepsilon$. The parameter ε can be interpreted as the innovation size. We further assume that ε is an exogenous parameter of the model.

The market demand is a linear form for a homogeneous product and is given by:

$$p = a - (q_L + q_M + q_H), \quad (1)$$

where p is the market price of the product and q_i is the quantity of the product supplied to firm i , where $i = L, M, H$. The benchmark model assumes there is no licensing action and the three firms engage in a Cournot competition. Hence, their initial payoffs are, respectively:

$$\pi_L^0 = [(a - c + 5\varepsilon) / 4]^2, \pi_M^0 = [(a - c + \varepsilon) / 4]^2, \pi_H^0 = [(a - c - 3\varepsilon) / 4]^2. \quad (2)$$

We focus our analysis on a non-drastic innovation case, and hence we have a non-drastic innovation condition, i.e., $\varepsilon \leq (a - c) / 3$, let $\pi_H^0 > 0$.

Given the innovation size, Firm L has three kinds of licensing strategy. Under the fixed-fee licensing method, the patent is transferred against a fixed fee. Under the royalty licensing method, the patent holder decides the optimal royalty rate of per unit output to transfer the patent. Under the auction licensing method, the innovator

transfers the technology using a first-price auction, and the highest bidder obtains the innovation.

This is a three-stage game. In the first stage, the patent holder decides the licensing strategy. In the second stage, it decides how many firms it will transfer the licensing to, i.e., Firm M , Firm H , or both. In the third stage, the three firms compete in quantities.

2.1 Fixed-fee licensing method

Under the fixed-fee licensing mode, the innovator decides to whom to license the patent by comparing the size of the total profit that is made up of the market competitive profit and the fixed-fee licensing revenue. We are thus able to derive:

$$\Pi_L^{FB} = \pi_L^{FB} + F_B = [(a - c + 2\varepsilon) / 4]^2 + (1/16)[-2\varepsilon^2 + 12(a - c)\varepsilon], \quad (3a)$$

$$\Pi_L^{FM} = \pi_L^{FM} + F_M = [(a - c + 4\varepsilon) / 4]^2 + (1/16)[15\varepsilon^2 + 6(a - c)\varepsilon], \quad (3b)$$

$$\Pi_L^{FH} = \pi_L^{FH} + F_H = [(a - c + 3\varepsilon) / 4]^2 + (1/16)[12(a - c)\varepsilon]. \quad (3c)$$

We explain the calculation process for the above results in Appendix A. The symbols Π_L^{FB} , Π_L^{FM} , and Π_L^{FH} represent the total profit of Firm L under the fixed-fee licensing mode when Firm L licenses to either or both of the firms, namely, Firm M and Firm H , respectively; the symbols π_L^{FB} , π_L^{FM} , and π_L^{FH} represent the competitive profit of Firm L in the product market under the fixed-fee licensing contract when Firm L licenses to either or both of the firms, Firm M and Firm H , respectively. Similarly, the symbols F_B , F_M , and F_H represent the licensing revenue of Firm L under the fixed-fee licensing contract when Firm L licenses to either or both of the firms, Firm M and Firm H , respectively.

We compare the magnitudes of Π_L^{FB} , Π_L^{FM} , and Π_L^{FH} and we obtain:

$$\begin{aligned} \Pi_L^{FH} &> \Pi_L^{FB} \quad \forall \varepsilon, \\ \Pi_L^{FM} &< \Pi_L^{FH} \quad \text{for } 0 < \varepsilon < (2/11)(a - c), \\ \Pi_L^{FM} &> \Pi_L^{FH} \quad \text{for } \varepsilon > (2/11)(a - c). \end{aligned} \quad (4)$$

Hence, we conclude that, under the fixed-fee licensing mode, the patent holder will license to a high cost firm when the innovation size is small; the innovator will license to a low cost firm when the innovation size is large.

2.2 Royalty licensing method

Under the royalty licensing mode, the total profit of the patent holder is derived from the profit obtained by selling the product and the licensing revenue is obtained by means of the royalty licensing mode. The total profits of the innovator under the different situations are shown as:

$$\Pi_L^{RB} = \pi_L^{RB} + R_B = [(a - c + 2\varepsilon) / 2]^2, \quad (5a)$$

$$\Pi_L^{RM} = \pi_L^{RM} + R_M$$

$$= [(7a - 7c + 28\varepsilon) / 22]^2 + [(3a - 3c + 12\varepsilon) / 11][(a - c + 4\varepsilon) / 22], \quad (5b)$$

$$\Pi_L^{RH} = \pi_L^{RH} + R_H$$

$$= [(7a - 7c + 21\varepsilon) / 22]^2 + [(3a - 3c + 9\varepsilon) / 11][(a - c + 3\varepsilon) / 22]. \quad (5c)$$

We explain the calculation process for the above result in Appendix B. The symbols Π_L^{RB} , Π_L^{RM} , and Π_L^{RH} represent the total profit of Firm L under the royalty licensing mode when Firm L licenses to either or both of the firms, Firm M and Firm H , respectively; the symbols π_L^{RB} , π_L^{RM} , and π_L^{RH} represent the competitive profit of Firm L in the product market under the royalty licensing contract when Firm L licenses to either or both of the firms, Firm M and Firm H , respectively. Similarly, the symbols R_B , R_M , and R_H represent the licensing revenue of Firm L under the royalty licensing contract when Firm L licenses to either or both of the firms, Firm M and Firm H , respectively.

We compare the magnitudes of Π_L^{RB} , Π_L^{RM} , and Π_L^{RH} and we obtain:

$$\Pi_L^{RB} > \Pi_L^{RM} > \Pi_L^{RH} \quad \forall \varepsilon. \quad (6)$$

According to the result in Equation (6), we conclude that under the royalty licensing method, it is beneficial for the licensor to license to both firms. The reason is that the inside patent holder can use the royalty rate to influence the two rivals' marginal production costs.

2.3 Auction licensing method

Under the auction licensing method, the maximum that a producing firm can pay the patent holder to obtain a new technology is its payoff when it gets the patent, however, the rival will fail to get minus its payoff when the rival succeeds in getting the patent while it fails.

We obtain the total profits that arise as Firm L licenses to Firm M or Firm H by means of the auction licensing method as:

$$\Pi_L^{AM} = [(a - c + 4\varepsilon) / 4]^2 + 5\varepsilon(2a - 2c + 3\varepsilon) / 16, \quad (7a)$$

$$\Pi_L^{AH} = [(a - c + 3\varepsilon) / 4]^2 + 7\varepsilon(2a - 2c - \varepsilon) / 16. \quad (7b)$$

We explain the calculation process in Appendix C. The symbols Π_L^{AM} and Π_L^{AH} represent the total profit of Firm L under the auction licensing method when Firm M or Firm H obtains the patent by auction, respectively. By comparing these two equations above, we have:

$$\Pi_L^{AM} < \Pi_L^{AH} \text{ for } 0 < \varepsilon < 2(a - c) / 29,$$

$$\Pi_L^{AM} > \Pi_L^{AH} \text{ for } \varepsilon > 2(a - c) / 29. \quad (8)$$

According to the above result, we conclude that, under the auction licensing method, the patent holder will license to a high cost firm when the innovation size is small; on the contrary, the patent holder will license to a low cost firm when the innovation size is large.

3. The optimal licensing strategy

In this section, we use a geometric figure to find the optimal licensing method and which firm is to be licensed. Figure 1 shows that the patent holder will obtain the lowest profit if it does not license to any firm, and the optimal licensing method for the producing patent holder will be to license to both firms by means of the royalty licensing method. In other words, the auction licensing method is not the best licensing method when the licensor is a producing patent holder. Our result differs in this respect from that of Kabiraj (2004), who claimed that the optimal licensing method is the auction licensing method when the licensor is a non-producing patent holder. We shall now provide an explanation for this different result. Our model setup differs in three respects from that of Kabiraj (2004). The first is that the licensor is the producing patent holder in our model, but the licensor is the non-producing patent holder in the Kabiraj model. The second is that the market structure in our model is the Cournot market structure, while the market structure in the Kabiraj model is the Stackelberg market structure. The third is that the firms are asymmetric producers in our model, while the firms are symmetric producers in the Kabiraj model. Hence, the differences in the results between our model and the Kabiraj model are caused by these three factors. However, the main factor that makes the results different is the first one. Since the auction licensing method does not change the rival's production behavior, the patent producing licensor in our model does not adopt the auction licensing method. In order to change the rival's production behavior, the producing licensor in our model will adopt the royalty licensing method to license to both firms.

Besides, our result can also be compared with the finding of Fosfuri and Roca (2004). Our model setup only differs in one respect from Fosfuri and Roca (2004). It is that there are asymmetric producers in our model, but symmetric producers in the Fosfuri and Roca model. Except for the difference in the model setup referred to above, the model setup for both our model and for the Fosfuri and Roca model are similar in that they both have a producing patent and product licensor and a Cournot market structure. In their article, Fosfuri and Roca only compare the fixed-fee licensing method with the royalty licensing method, and find that when the producing patent holder licenses to all firms, the best licensing method is the royalty rate. However, if the patent holder licenses to only certain of the firms, then a fixed-fee contract will replace a royalty contract as the optimal choice. However, we reach a different conclusion to that of Fosfuri and Roca (2004). In this study, we can show that if the producing patent holder only licenses to one firm, then the royalty licensing method always dominates the fixed-fee licensing method, i.e., $\text{Max} \{ \Pi_L^{RM}, \Pi_L^{RH} \} >$

$\text{Max} \{\Pi_L^{FH}, \Pi_L^{FM}\}$. The proof of this process appears in Appendix D. We provide the economic intuition of this result as follows. There are two differences between our study and Fosfuri and Roca (2004). First, our model features asymmetric producers, while there are symmetric producers in the Fosfuri and Roca model setup. Second, our model implicitly assumes that the producing patent holder can endogenously choose which firm it licenses to; however, which firm is licensed to is an exogenous decision in the Fosfuri and Roca model setup. Hence, the parameter space where $\Pi_L^{FH} > \Pi_L^{RH}$ in the Fosfuri and Roca model holds despite there being asymmetric producers. In other words, in our model, if the producing patent holder only licenses to one firm, then it will license to the low cost firm by means of the royalty licensing method. In the Fosfuri and Roca model, both licensees have the same production cost, and so the producing licensor will only license to one of the licensees by means of the fixed-fee licensing method.

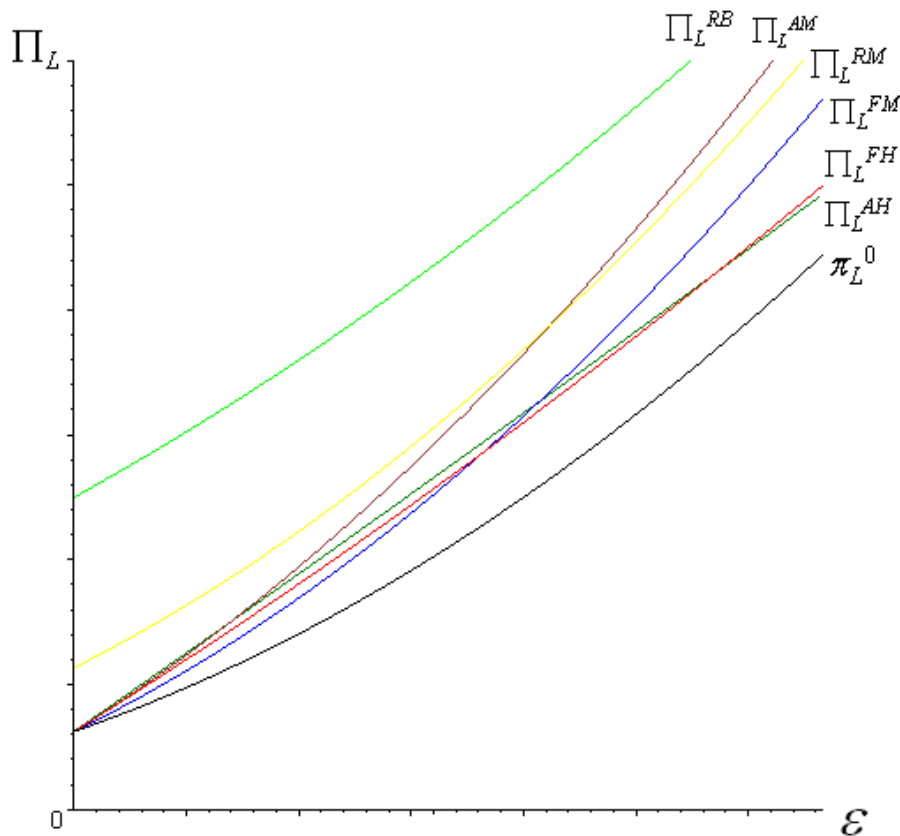


Figure 1. The Optimal Licensing Mode

4. Conclusion

In this paper we discuss the optimal licensing strategy in which the licensor is a producing patent holder. The producing patent holder has three alternative licensing

strategies: fixed-fee, royalty rate, and auction. There are also different cost structures between the licensor and the two licensees. We find that when the licensor is a producing patent holder and each firm has a different production cost, the auction licensing method is not the best licensing strategy. This result is different from the finding of Kabiraj (2004), which is that the auction licensing strategy is the optimal strategy among the three alternative licensing strategies when the licensor is a non-producing patent holder. Finally, we compare the licensing method between the fixed-fee licensing method and the royalty licensing method. We conclude that if the producing patent holder licenses to only certain firms, then the royalty licensing method is the best strategy. This result is different from that of Fosfuri and Roca (2004), who concluded that if only some of the licensees obtain a licensing contract, then the fixed-fee licensing method will be the best choice for a producing patent holder.

Appendix A

When Firm L licenses to both firms by means of the fixed-fee licensing method, the marginal costs of the three firms amount to $c - 2\varepsilon$, i.e., $c_L = c_M = c_H = c - 2\varepsilon$. The profit functions of three firms are each $\pi_i = (p - c_i)q_i$, where $p = a - \sum q_i$. We derive each firm's profit function with respect to its quantity and obtain three first-order conditions. By setting them equal to zero, we can solve three simultaneous equations. We obtain the optimal quantity $q_i^{FB} = (a - c + 2\varepsilon) / 4$, and the equilibrium profit is $\pi_i^{FB} = [(a - c + 2\varepsilon) / 4]^2$. The superscript FB represents the case where both firms are licensed by means of a fixed-fee licensing method. In the same way, if Firm L only licenses to Firm M , then the equilibrium quantities for the three firms are $q_L^{FM} = q_M^{FM} = (a - c + 4\varepsilon) / 4$, and $q_H^{FM} = (a - c - 4\varepsilon) / 4$. Furthermore, the equilibrium profits are $\pi_L^{FM} = \pi_M^{FM} = [(a - c + 4\varepsilon) / 4]^2$, and $\pi_H^{FM} = [(a - c - 4\varepsilon) / 4]^2$. The superscript FM represents the case where only Firm M is licensed by means of the fixed-fee licensing method. Finally, if Firm L only licenses to Firm H , then the equilibrium quantities we obtain for the three firms are $q_L^{FH} = q_H^{FH} = (a - c + 3\varepsilon) / 4$, and $q_M^{FH} = (a - c - \varepsilon) / 4$. In addition, the equilibrium profits are $\pi_L^{FH} = \pi_H^{FH} = [(a - c + 3\varepsilon) / 4]^2$, and $\pi_M^{FH} = [(a - c - \varepsilon) / 4]^2$. The superscript FH represents the case where only Firm H is licensed by means of the fixed-fee licensing method.

When Firm L licenses to both firms by means of the fixed-fee licensing method, the licensing revenue for Firm L is $F_B = (\pi_M^{FB} - \pi_M^0) + (\pi_H^{FB} - \pi_H^0) = (1/16)[-2\varepsilon^2 + 12(a - c)\varepsilon]$. When Firm L licenses to Firm M by means of fixed-fee licensing, the licensing revenue for Firm L is $F_M = (\pi_M^{FM} - \pi_M^0) = (1/16)[15\varepsilon^2 + 6(a - c)\varepsilon]$. In addition, when Firm L licenses to Firm H by adopting the fixed-fee licensing

approach, the licensing revenue for Firm L is $F_H = (\pi_H^{FH} - \pi_H^0) = (1/16)[12(a - c)\varepsilon]$.

In summarizing the above analysis, the total profits for Firm L under the fixed-fee licensing mode when Firm L licenses to either or both of the firms, Firm M and Firm H are $\Pi_L^{FB} = \pi_L^{FB} + F_B$, $\Pi_L^{FM} = \pi_L^{FM} + F_M$, and $\Pi_L^{FH} = \pi_L^{FH} + F_H$. They are explicitly shown in Equations (3a), (3b), and (3c) in Section 2.

Appendix B

When Firm L licenses to both firms by means of the royalty licensing method, the marginal costs of the two licensees (Firm M and Firm H) are $c - 2\varepsilon + r$, i.e., $c_M = c_H = c - 2\varepsilon + r$. The profit functions of the two licensees are $\pi_M = (p - c + 2\varepsilon - r)q_M$, and $\pi_H = (p - c + 2\varepsilon - r)q_H$. However, the profit function of the licensor is $\pi_L = (p - c + 2\varepsilon)q_L$, where $p = a - \sum q_i$. We derive Firm i 's profit function with respect to q_i and obtain three first-order conditions. Let them to be zero and solve three simultaneous equations. We then obtain the optimal quantities $q_L^{RB} = (a - c + 2\varepsilon + 2r) / 4$, and $q_M^{RB} = q_H^{RB} = (a - c + 2\varepsilon - 2r) / 4$. The equilibrium profits are $\pi_L^{RB} = [(a - c + 2\varepsilon + 2r) / 4]^2$, and $\pi_M^{RB} = \pi_H^{RB} = [(a - c + 2\varepsilon - 2r) / 4]^2$. The superscript RB represents the case where both firms are licensed by means of the royalty licensing method. According to the same calculation process, if Firm L only licenses to Firm M , then the optimal quantities for the three firms are $q_L^{RM} = (a - c + 4\varepsilon + r) / 4$, $q_M^{RM} = (a - c + 4\varepsilon - 3r) / 4$, and $q_H^{RM} = (a - c - 4\varepsilon + r) / 4$. The equilibrium profits are $\pi_L^{RM} = [(a - c + 4\varepsilon + r) / 4]^2$, $\pi_M^{RM} = [(a - c + 4\varepsilon - 3r) / 4]^2$, and $\pi_H^{RM} = [(a - c - 4\varepsilon + r) / 4]^2$. The superscript RM represents the case where only Firm M is licensed by means of the royalty licensing method. Finally, if Firm L only licenses to Firm H , then the equilibrium quantities for the three firms are $q_L^{RH} = (a - c + 3\varepsilon + r) / 4$, $q_M^{RH} = (a - c - \varepsilon + r) / 4$, and $q_H^{RH} = (a - c + 3\varepsilon - 3r) / 4$. In addition, the equilibrium profits are $\pi_L^{RH} = [(a - c + 3\varepsilon + r) / 4]^2$, $\pi_M^{RH} = [(a - c - \varepsilon + r) / 4]^2$, and $\pi_H^{RH} = [(a - c + 3\varepsilon - 3r) / 4]^2$. The superscript RH represents the case where only Firm H is licensed by means of the royalty licensing method.

When Firm L licenses to both firms by means of the royalty licensing method, the total profit for Firm L is $\Pi_L^{RB} = \pi_L^{RB} + R_B$, where $R_B = r(q_M^{RB} + q_H^{RB})$. Firm L maximizes the total profit with respect to r and we obtain the optimal royalty rate $r^{RB} = (a - c + 2\varepsilon) / 2$. Similarly, when Firm L licenses to Firm M , the total profit for Firm L is $\Pi_L^{RM} = \pi_L^{RM} + R_M$, where $R_M = r q_M^{RM}$. Firm L maximizes the total profit with respect to r and we obtain the optimal royalty rate $r^{RM} = (3a - 3c + 12\varepsilon) / 11$. Finally, when Firm L licenses to Firm H , the total profit for Firm L is $\Pi_L^{RH} = \pi_L^{RH} + R_H$, where $R_H = r q_H^{RH}$. Firm L maximizes its total profit with respect to r and we obtain the optimal royalty rate $r^{RH} = (3a - 3c + 9\varepsilon) / 11$. Furthermore, the optimal total licensing profits for Firm L Π_L^{RB} , Π_L^{RM} , and Π_L^{RH} are shown in Equations (5a),

(5b) and (5c), respectively, in Section 2.

Appendix C

If Firm M obtains the patent by means of the auction licensing method, then the marginal costs for Firm L and Firm M are $c - 2\varepsilon$. Furthermore, the marginal cost for Firm H is c . The profit functions for Firm L , Firm M , and Firm H are $\pi_L^{AM} = (p - c + 2\varepsilon)q_L$, $\pi_M^{AM} = (p - c + 2\varepsilon)q_M$, and $\pi_H^{AM} = (p - c)q_H$, respectively. The superscript AM indicates that the patent holder licenses to Firm M by means of the auction licensing method. We derive three profit functions with respect to their quantities, and obtain the three first-order conditions. Let the three first-order conditions be zero and solve three simultaneous equations. The optimal quantities for the three firms are $q_L^{AM} = q_M^{AM} = (a - c + 4\varepsilon) / 4$, $q_H^{AM} = (a - c - 4\varepsilon) / 4$. In addition, the equilibrium profits for the three firms are $\pi_L^{AM} = \pi_M^{AM} = [(a - c + 4\varepsilon) / 4]^2$, and $\pi_H^{AM} = [(a - c - 4\varepsilon) / 4]^2$. Based on the same calculation process, if Firm H obtains the patent by means of the auction licensing method, then the optimal quantities for the three firms are $q_L^{AH} = q_H^{AH} = (a - c + 3\varepsilon) / 4$, and $q_M^{AH} = (a - c - \varepsilon) / 4$. Furthermore, the equilibrium profits are $\pi_L^{AH} = \pi_H^{AH} = [(a - c + 3\varepsilon) / 4]^2$, and $\pi_M^{AH} = [(a - c - \varepsilon) / 4]^2$. The superscript AH indicates that the patent holder licenses to Firm H by means of the auction licensing method.

The maximum that Firm M is willing to pay for new technology is Firm M 's profit when it obtains the patent, however, Firm H fails to get minus Firm M 's profit when Firm H succeeds in getting the patent while Firm M fails. Thus, the maximum amount that Firm M can spend for getting the patent of new technology is $A_M = \pi_M^{AM} - \pi_M^{AH} = [(a - c + 4\varepsilon) / 4]^2 - [(a - c - \varepsilon) / 4]^2 = 5\varepsilon(2a - 2c + 3\varepsilon) / 16$. In the same way, the maximum amount that Firm H can spend to obtain the patent for the new technology is $A_H = \pi_H^{AH} - \pi_H^{AM} = [(a - c + 3\varepsilon) / 4]^2 - [(a - c - 4\varepsilon) / 4]^2 = 7\varepsilon(2a - 2c - \varepsilon) / 16$.

According to the above analysis, if Firm L licenses to Firm M , then the total profit of the licensor is $\Pi_L^{AM} = \pi_L^{AM} + A_M$; if Firm L licenses to Firm H , then the total profit of the licensor is $\Pi_L^{AH} = \pi_L^{AH} + A_H$. The reduced forms for Π_L^{AM} and Π_L^{AH} are shown in Equations (7a) and (7b) in Section 2.

Appendix D

We have the producing patent holder's profit functions under different licensing methods as follows:

$$\Pi_L^{RM} = \pi_L^{RM} + R_M = \left(\frac{7a - 7c + 28\varepsilon}{22}\right)^2 + \left(\frac{3a - 3c + 12\varepsilon}{11}\right)\left(\frac{a - c + 4\varepsilon}{22}\right), \quad (\text{A-1})$$

$$\Pi_L^{RH} = \pi_L^{RH} + R_H = \left(\frac{7a-7c+21\varepsilon}{22}\right)^2 + \left(\frac{3a-3c+9\varepsilon}{11}\right)\left(\frac{a-c+3\varepsilon}{22}\right), \quad (\text{A-2})$$

$$\Pi_L^{FH} = \pi_L^{FH} + F_H = 2\left(\frac{a-c+3\varepsilon}{4}\right)^2 - \left(\frac{a-c-3\varepsilon}{4}\right)^2, \quad (\text{A-3})$$

$$\Pi_L^{FM} = \pi_L^{FM} + F_M = 2\left(\frac{a-c+4\varepsilon}{4}\right)^2 - \left(\frac{a-c+\varepsilon}{4}\right)^2. \quad (\text{A-4})$$

It is obvious that Π_L^{RM} must be larger than Π_L^{RH} , i.e., $\text{Max}\{\Pi_L^{RM}, \Pi_L^{RH}\} = \Pi_L^{RM}$. We next compare Π_L^{RM} and Π_L^{FH} , and obtain

$$\Pi_L^{RM} - \Pi_L^{FH} = \left(\frac{1}{176}\right)[3(a-c)-11\varepsilon]^2 + \left(\frac{1}{176}\right)[100\varepsilon^2 + 28(a-c)\varepsilon] > 0. \quad (\text{A-5})$$

We finally compare Π_L^{RM} and Π_L^{FM} , and arrive at

$$\begin{aligned} \Pi_L^{RM} - \Pi_L^{FM} &= \left(\frac{9}{176}\right)(a-c)^2 + \left(\frac{6}{176}\right)(a-c)\varepsilon - \left(\frac{21}{176}\right)\varepsilon^2 > 0, \text{ if} \\ \left(\frac{1-\sqrt{22}}{7}\right)(a-c) &< \varepsilon < \left(\frac{1+\sqrt{22}}{7}\right)(a-c). \end{aligned} \quad (\text{A-6})$$

The non-drastic innovation case requires that $\varepsilon \in (0, (a-c)/3)$. Hence, $\Pi_L^{RM} - \Pi_L^{FM} > 0$ must hold. From (A-5) and (A-6), we get $\Pi_L^{RM} > \text{Max}\{\Pi_L^{FH}, \Pi_L^{FM}\}$.

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