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The Relationship between the Hybrid New Keynesian Phillips Curve and the NAIRU over Time

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Abstract

New Keynesian models of the Phillips curve in the spirit of Galí and Gertler (1999) generally assume a short-run trade-off between inflation and a measure of excess demand due to nominal rigidities, while in the long run inflation is constant at the Non-Accelerating Inflation Rate of Unemployment (NAIRU). By contrast, Gordon (1997) in his 'triangle model' of inflation models a time-varying NAIRU. We combine both approaches and estimate state-space models of the hybrid New Keynesian Phillips curve (NKPC), where excess demand is measured by the unemployment gap and the NAIRU is allowed to vary over time as in Gordon (1997). Moreover, inflation expectations are measured directly from surveys on household's inflation expectations and not instrumented for. Our model is estimated for the US, the UK, Italy and Spain and we find considerable variation in the NAIRU over time with NAIRU estimates significantly different from HP-filter derived measures such as usually employed in dynamic stochastic general equilibrium (DSGE) models. In contrast to GMM results for the hybrid NKPC, we find that backward looking behaviour generally seems to be quantitatively more important for inflation than forward looking behaviour.

JEL classification: C32, E31

Keywords: Hybrid New Keynesian Phillips curve, time-varying NAIRU, state-space models

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1. Introduction

The most commonly used model of the Phillips curve in modern macroeconomics is the hybrid New Keynesian Phillips curve (NKPC) as developed in Galí and Gertler (1999), relating the inflation rate to lagged inflation, inflation expectations and a measure of excess demand, stating a short-run trade-off between inflation and unemployment and long-run equilibrium with constant inflation at the NAIRU. However, the NAIRU may change over time if the market characteristics underlying the equilibrium relation between inflation and unemployment change (Friedman, 1968, and Phelps, 1968). Feedback effects between labour productivity and unemployment as in Phelps' (1994) structural slumps and, accordingly, hysteresis of unemployment (e.g. Stiglitz, 1997) may also cause the underlying 'natural' rate of unemployment to shift. With a time-varying NAIRU, the unemployment rate that will keep inflation constant changes so that knowledge of these movements is of great importance for efficient monetary policy targeting.

In this paper, we investigate the relationship between the hybrid NKPC and a time-varying NAIRU for the US, the UK, Italy and Spain. We estimate state-space models of the hybrid New Keynesian Phillips curve, where the time-varying NAIRU is estimated as an unobserved component. Thus, we can analyse changes in the NAIRU within the theory-based system of the hybrid NKPC, taking account of the interdependencies between inflation, inflation expectations and the unemployment gap when determining changes in the 'natural' unemployment rate. We contrast our estimates of the time-varying NAIRU from the state-space model with mechanically calculated steady-state unemployment from an HP-filter, such as usually employed in dynamic stochastic general equilibrium (DSGE) models (Dees *et al.,* 2008).

Most empirical studies of the hybrid NKPC make use of the Generalized Method of Moments (GMM), instrumenting for inflation expectations with the output gap, the interest rate and additional lags of inflation.[1] These models generally find that while backward looking behaviour with regard to inflation is statistically significant, forward looking behaviour is quantitatively more important. If excess demand is measured by the output gap, it is often found insignificant; therefore, Galí and Gertler (1999) propose

[1] For examples of GMM estimates of the hybrid NKPC for the US see Galí/ Gertler (1999) and Galí/ Gertler/Lopez-Salido (2001, 2003, 2005).

to use real marginal cost instead. Proxying this with unit labour cost, most studies find a significant and correctly signed coefficient.

However, the GMM approach may be biased due to identification problems and weak instrument bias with regard to inflation expectations that impede the recovery of unique structural coefficients (e.g. Lindé, 2005; Rudd and Whelan, 2005, and Dees *et al.,* 2008). We avoid this problem by using direct survey measures for households' inflation expectations from the University of Michigan's Surveys of Consumers for the US and the EC Consumer Survey for the European economies in our estimations.[2] Overall, our state-space model of the hybrid NKPC thus avoids the identification problems encountered in standard GMM models and obtains time-varying estimates of the NAIRU within the theory-based system, where the restrictions on coefficients of the model can be tested directly. We find significant changes in the NAIRU over time in all the countries under investigation, which seem to move closely with actual unemployment rates.

The paper is structured as follows: A short discussion of theories of the Phillips curve is given in section 2, while section 3 presents the model and methodology used for the econometric estimations. Section 4 presents the results from our estimations of the state-space models of the time-varying NAIRU in a hybrid NKPC setting. Finally, section 5 summarises and concludes.

2. Theories of the Phillips Curve

The New Keynesian Model of the Phillips Curve

Assuming Calvo (1983) pricing with sticky prices and rational firms, the New Keynesian Phillips curve (NKPC)[3] is a function of expected inflation $E_t\pi_{t+1}$ and a measure of excess demand y_t, which according to the theory of profit-maximising firms

[2] Other empiricial studies of the New Keynesian Phillips curve that employ survey measures of inflation expectations are, e.g., Roberts (1995, 1997), Adam/Padula (2003) and Paloviita (2008).
[3] An extensive summary of the literature on New Keynesian theories of monetary policy is given in Clarida/Galì/Gertler (1999). Roberts (1995, 1997) and Mankiw/Reis (2002a), inter alia, provide empirical estimates of the sticky-prices New Keynesian Phillips curve.

is represented by the percentage deviation of firms' real marginal cost from their steady-state value (Galí and Gertler, 1999):[4]

$$\pi_t = \lambda y_t + \beta E_t \pi_{t+1} + u_t \,, \tag{1}$$

where π_t denotes the inflation rate (p_t – p_{t-1}), u is an i.i.d. disturbance term and $\lambda \equiv (1-\theta)(1-\beta\theta)/\theta$ is a function of the probability of price adjustment (1 - θ) and the subjective discount factor β. With rational expectations, unexpected movements in inflation will only have short-run real effects, since inflation expectations will adjust and influence current inflation. Iterating equation (1) forward gives the following closed form of the NKPC:

$$\pi_t = \lambda \sum_{j=0}^{\infty} \beta^j E_t y_{t+j} \tag{2}$$

Inflation should thus equal future discounted expected marginal costs. More recently, New Keynesian models of the Phillips curve have incorporated a lagged inflation term to account for the strong persistence of inflation typically observed in empirical data. First introduced by Galí and Gertler (1999)[5], it is assumed that of the firms who are able to adjust prices in any period, only a fraction adjusts to their optimal prices, while the others update last period's optimal prices with lagged inflation as a 'rule of tumb'. This results in the so-called hybrid NKPC:

$$\pi_t = \phi \pi_{t-1} + (1-\phi) E_t(\pi_{t+1}) + \gamma y_t + \varepsilon_t \,, \tag{3}$$

with $0 \leq \phi \leq 1$ and $\varepsilon_t \sim \mathrm{IID}(0, \sigma^2_\varepsilon)$.

The hybrid NKPC presented in equation (3) thus incorporates sticky prices as well as inflation inertia and has become the workhorse of modern macroeconomics. The lagged inflation term might also be explained by sticky information as in Mankiw and Reis

[4] Studies previous to Galí/ Gertler (1999) usually employed the output gap as the measure of excess demand. However, Galí/ Gertler (1999) as well as Galí *et al.* (2005) stress the importance of using real marginal cost (which, assuming a Cobb-Douglas production function, can be proxied by the labour share) instead of the output gap for empirical estimation of the NKPC.

[5] Fuhrer/ Moore (1995) also observe the missing persistence in inflation in standard New Keynesian models of the Phillips curve with staggered contracts à la Taylor (1980) and present a model similar to the hybrid NKPC, the so-called 'relative contracting model', where agents negotiate wages relative to existing wage contracts during the time their wage contract will be in effect. This introduces persistence both in inflation and excess demand and the authors show that the dynamics of the model match actual dynamics in inflation quite closely.

(2001, 2002a, 2002b) which could be due to rational inattention (Sims, 2003, and Reis, 2006) related, for instance, to media coverage on inflation (Carroll, 2001, 2003).

Most empirical studies of the hybrid NKPC in the literature obtain estimates of the coefficients of the model using the Generalized Method of Moments (GMM). By assuming rational expectations and i.i.d. errors, the forecast error of inflation must be uncorrelated with variables dated t and earlier, providing the following orthogonality condition:

$$(4) \qquad E_t\{(\pi_t - \lambda y_t - \beta\pi_{t+1})z_t\} = 0 ,$$

where $\boldsymbol{z_t}$ is a vector of variables dated t and earlier. Galí and Gertler (1999) as well as Galí, Gertler and Lopez-Salido (2001, 2003, 2005) amongst many others present GMM estimates of the hybrid NKPC for the US. While they find a significant impact of inflation inertia on current inflation, the effect of forward-looking behaviour, i.e. inflation expectations, on inflation seems to be quantitatively more important. The coefficient on excess demand is usually found significant and correctly signed. However, Lindé (2005), Rudd and Whelan (2005) and Dees, Pesaran, Smith and Smith (2008) argue that the GMM approach to the hybrid NKPC often suffers from identification problems and weak instrument bias: As is common practice in most papers in the literature, apart from the output gap and the interest rate, additional lags on inflation are used to instrument for inflation expectations. However, Dees *et al.* (2008) show that this is only appropriate if the output gap depends on past values of inflation, either directly or indirectly. If this is not the case, instruments do not fulfil the rank condition and results may be seriously biased due to the weak instruments.

Lindé (2005) proposes the use of full information maximum likelihood estimators (FIML) to avoid the possible bias in GMM single equation estimations. Nason and Smith (2005) also acknowledge the identification problems of GMM methods. They present an alternative identification method where a structural vector autoregressive (SVAR) system of the hybrid NKPC is estimated, introducing an additional error-covariance restriction between the two equations in the system: The output gap is assumed to follow a first-order autoregressive process and does not depend on current inflation, which is described by the hybrid NKPC. However, past lags of inflation are allowed to affect the output gap, so that the identification problem mentioned by Dees *et al*. (2008) is not solved.

Batini, Jackson and Nickell (2005) furthermore address the problem of a possible omitted variable bias of the standard NKPC for the case of an open economy such as the UK, including proxies for material input prices, foreign competition and employment adjustment costs. They find that marginal cost is inaccurately proxied by the labour share if employment adjustment costs are not accounted for and that inflation in the UK is significantly explained by shifts in real import prices and foreign competition. Bjørnstad and Nymoen (2008) also discuss a possible omitted variable bias for NKPC estimations, namely a linear combination of unit labour costs and the real exchange rate, since the NKPC is encompassed by imperfect competition models of inflation but not *vice versa*. They estimate the NKPC with a panel model for OECD countries and find that expected inflation and marginal cost in the model provide replacements for equilibrium correction terms in the imperfect competition model.

Another empirical approach to the hybrid NKPC is developed by Sbordone (2002, 2005) who estimates the closed form of the NKPC in equation (2). The NKPC is estimated in a two-step procedure: First, unit labour costs (*ulc*) as a proxy for *nominal* marginal cost are forecasted in an unrestricted VAR. Second, taking the forecast as given, the distance between the path of the *price/ulc* ratio implied by the model and that of the real dynamic data is minimised in order to gain estimates of the structural parameters of the model. Similar to Galí and Gertler (1999) and subsequent papers, Sbordone (2002, 2005) finds that while backward looking behaviour with regard to inflation is significant, forward looking behaviour is relatively more important. Her approach has been criticised by Kurmann (2005): Kurmann's (2005) paper analyses the fit of the inflation path derived from the closed form NKPC with respect to actual inflation and concludes that while the fit of the model seems impressive, the confidence interval around the point estimates is relatively large so that it remains uncertain whether backward looking or forward looking behaviour dominates. In that sense, his critique applies also to Galí and Gertler (1999).

There exist several other studies of the New Keynesian Phillips curve that employ direct survey measures of inflation expectations instead of instruments: Roberts (1995, 1997) uses the Michigan survey of households' inflation expectations and the Livingston survey of professional forecasters' inflation expectations for the US in his study of the NKPC. He finds that expectations are not perfectly rational and there is evidence of a role for lagged inflation in explaining current inflation. Similarly, Adam and Padula (2003) analyse the NKPC for the US with data from the Survey of Professional Forecasters (SPF). Like Roberts (1995, 1997), they find that survey data of inflation expectations do not confirm the rationality hypothesis needed for the orthogonality assumption of forecast errors with respect to output, so that estimations instrumenting for expectations may be severely distorted. Furthermore, they find that lagged inflation enters the hybrid NKPC significantly. Finally, Paloviita (2008) estimates different models of the Phillips curve for European economies using survey data from Consensus Economics for inflation expectations. While she finds that the NKPC fits the data adequately, the New Classical and Hybrid NKPC model perform better and even when allowing for possible non-rationality of expectations, the lagged inflation term still enters significantly. Thus overall, there seems to be a strong case for including lagged inflation in the hybrid NKPC and using survey data to account for possible distortions due to non-rationality of expectations.

Modelling the NAIRU over Time

Gordon (1997) proposes a different model of the Phillips curve in his 'Triangle Model', where inflation depends on inflation inertia in the form of lagged values of inflation, present and past measures of excess demand (*D*) as well as present and past supply shocks (*z*) (Gordon, 1997):

$$\pi_t = \alpha(L)\pi_{t-1} + \beta(L)D_t + \gamma(L)z_t + \varepsilon_t \, , \qquad (5)$$

where (*L*) stands for the lag operator. Excess demand *D* is normalised to zero and can be represented by the output gap or the unemployment gap, which is defined as the gap between the current unemployment rate and its 'natural' value ($U - U^N$). If the sum of the α-coefficients equals exactly unity, it can be shown that there exists a 'natural' rate of unemployment consistent with constant inflation, hence a NAIRU. Long-run steady-state unemployment is thus explicitly modelled in equation (5).

The notion of changes in the NAIRU attributable to changes in the microeconomic relations governing the product and labour markets was acknowledged by Friedman and

Phelps already in 1968 and later ascribed for example to 'structural slumps' (Phelps, 1994) or hysteresis of unemployment (e.g. Stiglitz, 1997). Nevertheless, most empirical approaches to the Phillips curve test the performance of an assumed fixed value for the NAIRU. Gordon (1997) resigns from this approach and instead estimates a time-varying NAIRU in equation (5), specifying it as an unobserved component following a simple random walk (Gordon, 1997, p. 20):

(6) $$\pi_t = a(L)\pi_{t-1} + b(L)(U_t - U_t^N) + c(L)z_t + e_t \qquad \text{with } e_t \sim \text{IID } (0, \sigma^2_e)$$

(7) $$U_t^N = U_{t-1}^N + \nu_t \qquad \text{with } \nu_t \sim \text{IID } (0, \sigma^2_\nu)$$

The NAIRU is allowed to vary over time according to the state-equation in (7) and exists if the sum of the *a*-coefficients equals one and the sum of the *b*-coefficients is significantly negative. Thus, by using the unobserved components approach in a state-space model of the Phillips curve, Gordon (1997) employs a specific econometric technique to estimate changes in the NAIRU over time within the system set out by the triangle model, thereby providing testable estimates of those changes.

Gordon (1997) finds for the US in the time-period 1955(q2) – 1996(q2) that the NAIRU or long-run Phillips curve has varied significantly between 5.3% and 6.5%, contrary to the 'textbook' assumption of a constant NAIRU at 6% for the US after 1978.[6]

In a recent paper, Harvey (2007) uses the unobserved component approach to model a hybrid NKPC, where lagged inflation π_{t-1} is substituted for a random walk μ^*:

(8) $$\pi_t = (1-\gamma)\mu_t^* + \gamma E_t(\pi_{t+1}) + \beta^* x_t + \varepsilon_t^* \qquad \text{with } \varepsilon^*_t \sim \text{IID } (0, \sigma^2_{\varepsilon^*}),$$

(9) $$\mu_t^* = \mu_{t-1}^* + \eta_t^* \qquad \text{with } \eta^*_t \sim \text{IID } (0, \sigma^2_{\eta^*}),$$

where $0 \le \gamma \le 1$ and x represents the output gap in period t. Since inflation π is most commonly found to be integrated of order one, but the output gap x is stationary by construction, the unobserved component μ^* captures the long-run forecast of π and can thus be regarded as a measure of core inflation. Harvey (2007) then shows that in steady-state, a reduced form of (8) can be derived as

[6] Staiger/Stock/Watson (1997) use a similar model to estimate a time-varying NAIRU for the US over the time period 1961(q1) – 1996(q4). However, they solve the model to include the NAIRU in the constant term, which is then estimated with a flexible polynomial ('spline'). The authors find estimates of the NAIRU or long-run Phillips curve in a 95% confidence interval between 5% and 8.5%.

$$(10) \qquad \pi_t = \tilde{\mu}_t + \gamma\beta^* \sum_{j=0}^{\infty} \gamma^j E_t(x_{t+1+j}) + \beta^* x_t + \tilde{\varepsilon}_t \qquad \text{with } \tilde{\varepsilon}_t \sim \text{IID }(0,\ \sigma_{\tilde{\varepsilon}}^2),$$

$$(11) \qquad \tilde{\mu}_t = \tilde{\mu}_{t-1} + \tilde{\eta}_t \qquad \text{with } \tilde{\eta}_t \sim \text{IID }(0, \sigma_{\tilde{\eta}}^2).$$

However, assuming that x is driven by an AR(1) process with root $|\phi| < 1$, equation (10) becomes:

$$(12) \qquad \pi_t = \tilde{\mu}_t + \frac{\beta^*}{1-\phi\gamma} x_t + \tilde{\varepsilon}_t.$$

The model of the hybrid NKPC thus reverts back to a simple Phillips curve without expectations or dynamics and identification of γ is not possible unless the output gap follows a higher order AR(p) process with p $\geq$ 2. The unobserved component $\tilde{\mu}$ captures both core inflation and inflation expectations, making a direct interpretation difficult.

3. Model and Methodology

The model used in this paper combines the hybrid NKPC as developed by Galí and Gertler (1999) and the unobserved components approaches by Gordon (1997) and Harvey (2007). The hybrid NKPC is chosen as the baseline model because it has become the most widely used model of the Phillips curve in recent years and incorporates both nominal rigidities in the form of sticky prices and inflation inertia which might be due to some form of sticky information. Nevertheless, as in the original model developed by Friedman (1968) and Phelps (1968), the assumption of a vertical long-run Phillips curve at the NAIRU, hence no long-run trade-off, is retained, but the NAIRU may vary over time if structural characteristics of the labour and commodity markets change. It thus seems to be a good starting point for the analysis of the relationship between the short-run New Keynesian Phillips curve and the NAIRU over time. As in Gordon (1997), the NAIRU is modelled directly by substituting the output gap for the unemployment gap and modelling the time-varying NAIRU as an unobserved component in a state-space representation. In order to ensure that the unobserved component measures the time-varying NAIRU and to avoid the identification problem in Harvey (2007), we include survey measures of inflation expectations directly in the model. This gives the following model of the time-varying

NAIRU in a hybrid NKPC setting, taking full account of sticky prices and inflation inertia:

(13) $$\pi_t = \alpha\pi_{t-1} + (1-\alpha)E_t^{survey}(\pi_{t+1}) + \beta(U_t - U_t^N) + \varepsilon_t$$ with $\varepsilon_t \sim$ IID $(0, \sigma^2_\varepsilon)$

(14) $$U_t^N = U_{t-1}^N + \nu_t$$ with $\nu_t \sim$ IID $(0, \sigma^2_\nu)$,

where $0 < \alpha < 1$.

By allowing the 'natural' rate of unemployment, or NAIRU, to vary over time according to the state equation in (14), we can estimate changes in equilibrium unemployment within the system of the hybrid NKPC, controlling for the interdependencies between inflation, inflation expectations and unemployment. Thus, rather than assuming a fixed value of the NAIRU and testing its empirical performance, this approach provides econometrically testable estimates of structural changes in the NAIRU over time.

The state-space model of the hybrid NKPC presented in equation (13) has a number of advantages over other specifications and estimation methods found in the literature: Our model in equations (13) and (14) avoids the possible weak identification bias of GMM estimations of the hybrid NKPC described above by using independent survey measures of inflation expectations instead of IV procedures using further lags of inflation as instruments. Thus, survey measures of household's inflation expectations provide raw data that does not depend on any underlying econometric methodology.

A further advantage of the model given in equations (13) and (14) is that it allows the time-varying NAIRU to be estimated within the system set out by the hybrid NKPC. The interdependencies between inflation, inflation expectations and unemployment are used to determine steady-state unemployment over time as given by the state-variable U^N. The systems' approach thus provides estimates of the time-varying NAIRU that are grounded in macroeconomic theory rather than mechanically obtained as HP-filtered steady-state measures, such as usually applied in DSGE models (Dees *et al.,* 2008).

Finally, the estimates of the unobserved component of the time-varying NAIRU can be compared to mechanically derived steady-states measures of unemployment, such as HP-filtered trend unemployment. Furthermore, the significance of the restriction imposed on the coefficients of lagged and expected inflation in (13) ($\alpha + \beta = \alpha + (1-\alpha) = 1$) can be tested within the model. Overall, the state-space representation of the hybrid NKPC avoids identification problems of GMM approaches and provides a flexible and testable estimation method both for the standard short-run hybrid NKPC and the time-varying NAIRU.

4. Empirical Results

Description of the Data

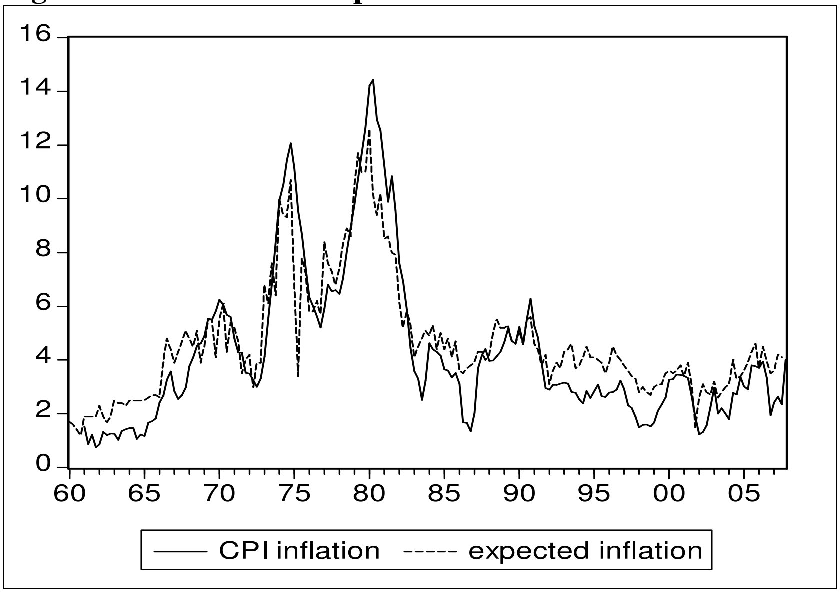
The model of the hybrid NKPC presented above was estimated for the US for the time period 1961(q1) to 2007(q3), for the UK and Italy for the time period 1985(q1) to 2007(q3) and for Spain for the period 1986(q3) to 2007(q3). The shorter estimation period for the European countries was due to shorter time series of survey data of household's inflation expectations.

We used quarterly data for consumer prices, the unemployment rate and inflation expectations. Data for the consumer price index (CPI) for all items and the standardised unemployment rate were taken from the OECD Main Economic Indicators (MEI) (OECD, 2008) database. The inflation rate was then calculated as the annual growth rate of the CPI. Survey measures of households' inflation expectations in the United States were provided by the University of Michigan's Surveys of Consumers (SCA), while for the European economies in our sample we employed survey data from the Consumer Survey of the 'Joint Harmonised EU Programme of Business and Consumer Surveys' directed by the European Commission.[7] While the Michigan Survey asks directly for a quantitative estimate of expected inflation, the EC Survey uses a qualitative measure of inflation expectations, asking interviewees about the direction of the expected price movement, rather than a specific point estimate. In order to derive a quantitative time series of inflation expectations, the qualitative answers were converted with the probability method of Carlson and Parkin (1975), scaling inflation expectations with one-period lagged inflation, recursive mean inflation until last period, recursively HP-

[7] Although the surveys are conducted by country-specific institutes, the questionnaire and timing of the survey are identical across European countries and sample sizes are similar, so that the data are consistent over time and across countries. Papers using the EC Consumer Survey data include Nielsen (2003) and Döpke *et al.* (2008).

filtered inflation and the recursively fitted values obtained from an ARMA(4,4)-model of inflation that were also filtered with an HP-filter as in Döpke *et al.* (2008).[8]

Figure 1: Inflation and expected inflation for the US



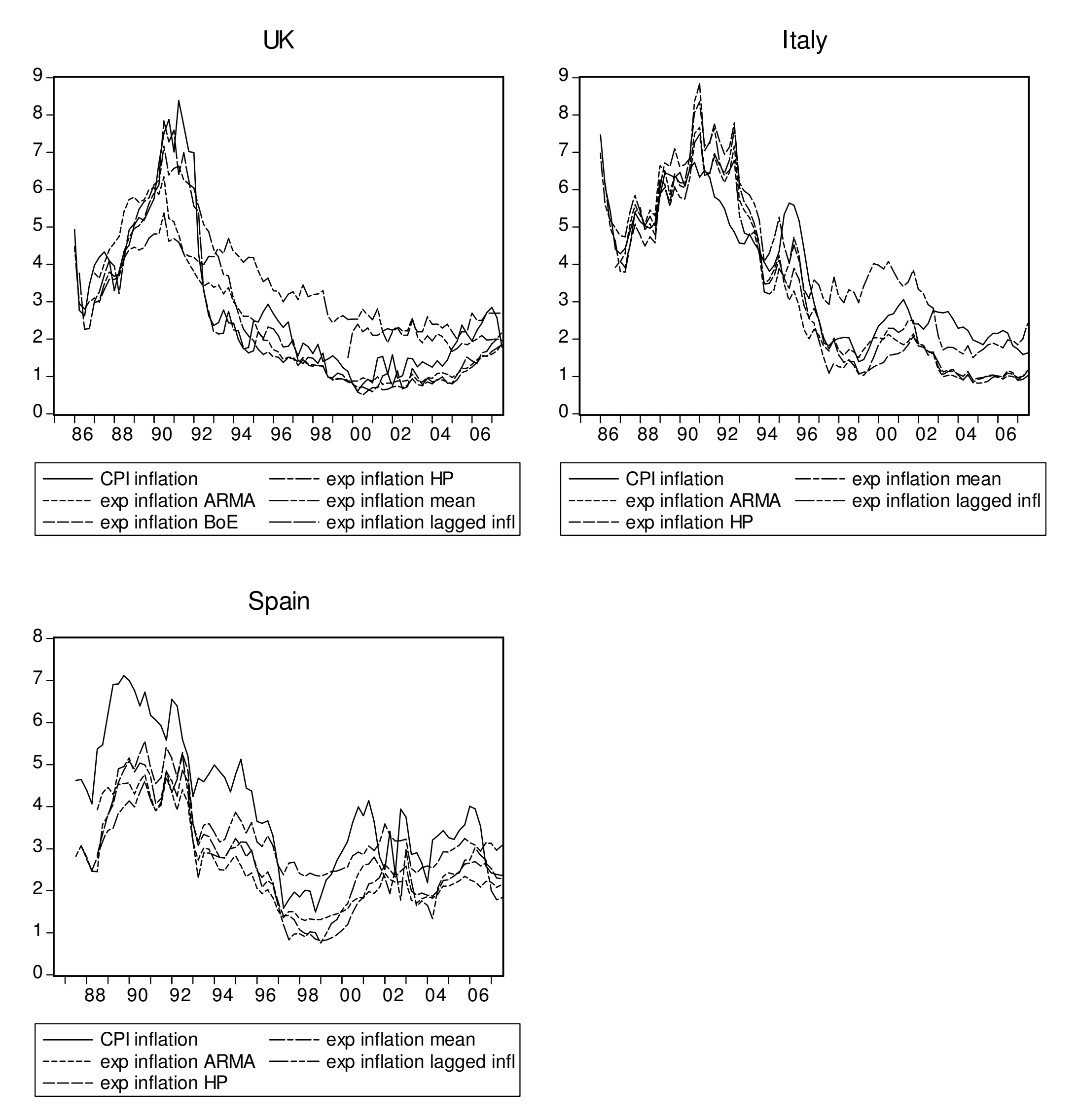
Source: OECD and SCA data, own calculations and graphs.

As can be seen from Figure 1, households' inflation expectations match actual inflation for the US relatively well, especially during the oil price shocks of the 70s and 80s. After a period of overshooting during the 90s, inflation expectations seem to have stabilised at around 3 – 4% since the beginning of the new millennium in line with actual inflation.

Figure 2 presents the resulting time series of expected inflation for the UK, Italy and Spain. The graph for the UK also shows the time series of expected inflation of the Inflation Attitudes Survey by the Bank of England for the time period 1999(q4) – 2007(q3). The time series' of expected inflation derived with the probability method are generally quite close for the three countries analysed here:

[8] Details of both surveys and on the probability method to extract a quantitative measure of inflation expectations from the qualitative survey of the EC are given in Appendix 1.

Figure 2: Inflation and Expected Inflation for the UK, Italy and Spain



Source: OECD, BoE and EC Consumer Survey data, own calculations and graphs.

Time series' of expected inflation for the UK fit actual inflation relatively closely, only expectations scaled with recursive mean inflation overshoot from 1992 onwards, but converge towards actual inflation rates towards the end of the sample period. Furthermore, they are found very close to the series of expected inflation published by the Bank of England. Inflation expectations in Italy match actual inflation rates quite closely until 1995; thereafter inflation expectations scaled with recursive mean inflation overshoot actual inflation rates until 2004. This matches the observation by several studies that inflation was severely overestimated during the time of the Euro introduction.[9] The remaining time series of expected inflation for Italy are below actual

[9] See Malgarini (2008) for a summary of studies on Italian inflation expectations.

inflation after 2002. In Spain, inflation expectations seem to have generally underestimated actual inflation up until the mid-90s. After a considerable drop in inflation rates, expected inflation rates approach actual rates in the second half of the sample period.

To discriminate more formally between the different series of inflation expectations derived from the probability method, we calculated the root mean squared error (RMSE) of the inflation expectations series with respect to actual inflation four quarter ahead:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(\pi_{t+4} -_t \pi_{t+4}^{e})}{n}} \qquad (15)$$

The RMSE thus gives a measure of forecasting accuracy of inflation expectations. Table 1 presents values of the RMSE for different scaling factors of expected inflation for the UK, Italy and Spain.

< *Table 1 here* >

The lowest forecasting error is achieved with the HP-filtered fitted values for inflation from the ARMA-model (*infl_exp_arma*) in all three countries under investigation here, although RMSEs of expected inflation with other scaling factors are quite close in the case of Italy and Spain. We thus decided to use *infl_exp_arma* in our model of the hybrid NKPC.

Testing for Unit Roots

Before we carried out any estimations, all time series in the model were tested for unit roots with the augmented Dickey-Fuller test (ADF test, Dickey and Fuller, 1981). Inflation and its expectations seem to be non-stationary in all the countries under investigation here (Table A1 in the Appendix). In the case of the US for the sample 1961(q1) – 2007(q4), this might be due to a structural break in inflation after the oil price shocks, when inflation rates in the US were stabilised substantially. Inflation rates of the European countries for the shorter sample from 1986(q1) – 2007(q3) seem to have stabilised after the turbulences of the ERM currency crisis 1991-1992. While the unemployment rate for the US was found to be stationary, the ADF tests could not reject the null of a unit root for the UK, Italy and Spain. This might be due to the significant fall in unemployment rates in the European countries from the mid-90s onwards.

As mentioned by Fanelli (2007), most empirical studies on the hybrid NKPC fail to acknowledge the non-stationarity of inflation and inflation expectations. The author argues that non-stationarity may originate from the aggregation of sectoral and regional/national Phillips curves, with stationary variables at the firm level as assumed in theory. To rule out spurious results, we estimated simple OLS models of the hybrid NKPC with HP-filter derived output and unemployment gaps and tested the residuals for stationarity using special critical values from MacKinnon (1991). For all the models, residuals were stationary at the 1% level, suggesting cointegration of the variables.[10]

State-Space Models of the Time-Varying NAIRU

The state-space model of the hybrid NKPC presented in equations (13) and (14) was estimated in two different models: In the first specification, the coefficients of lagged inflation and expected inflation were estimated freely, while in the second specification they were restricted to sum to exactly one. We then extracted estimates of the time-varying NAIRU with the Kalman filter (Kalman, 1960). This enabled us to test for the significance of the restriction $\alpha + \beta = 1$ on the coefficients of lagged and expected inflation and compare the estimates of the time-varying NAIRU from the two models. In order to enable convergence, the variances of the observation equation and the state equation had to be restricted. Variances of the observation equation vary with each model, but the variance of the state equation was set uniformly to $\sigma^2_v = 0.20$ in accordance with Gordon (1997). To provide starting values for the iterations, the estimation periods were shortened, usually by 4 quarters.

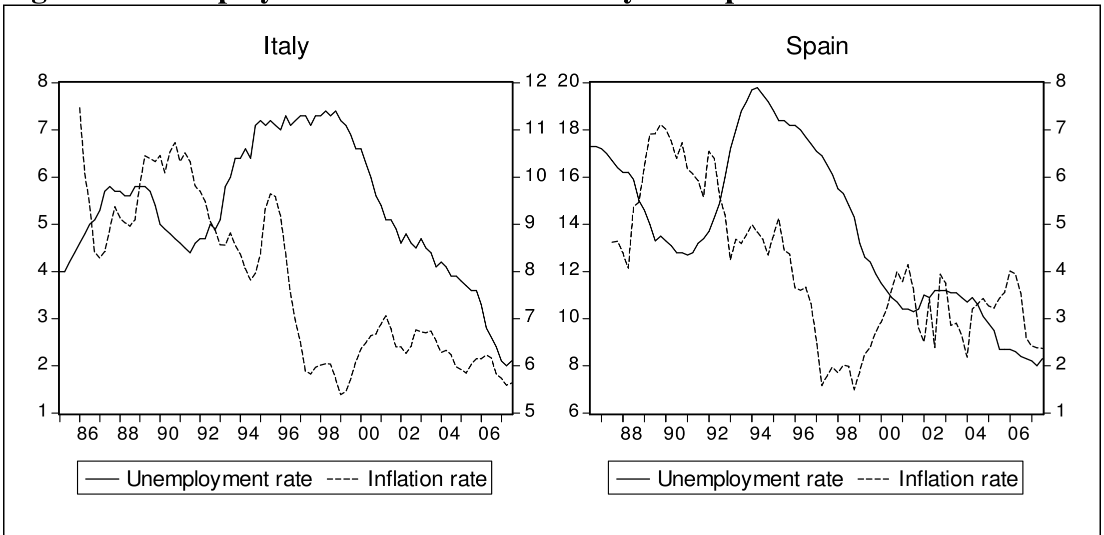
Fit of the Models

The estimated coefficients of the observation equation for both the restricted and the unrestricted model for the US, the UK, Italy and Spain are given in Tables A2 – A9 in the Appendix. Surprisingly, in contrast to the results of Galí and Gertler (1999), Galí *et al.* (2001, 2003, 2005) and Sbordone (2002, 2005), we find that the coefficient on

[10] We omit the results from the OLS models for reasons of space limitation, but they can be obtained from the author upon request.

lagged inflation is larger than that on expected inflation for all countries in our sample, with the notable exception of Spain. The reason for this finding might be the different estimation method employed here, where we use survey measures of inflation expectations instead of instruments and the different specification with the unemployment gap instead of real marginal cost. The unemployment gap generally enters the hybrid NKPC with a highly significant coefficient. For the US and the UK, the coefficient is negatively signed, as expected, but for Italy and Spain we find a significantly positive coefficient. This might be due to the estimation period used here, where a simultaneous drop in both inflation and unemployment occurred in the two countries in the latter half of the sample period. This was caused by monetary policies aimed at joining the EMU as well as labour market reforms and a boom that boosted employment in Italy and Spain.

Figure 3: Unemployment and Inflation in Italy and Spain

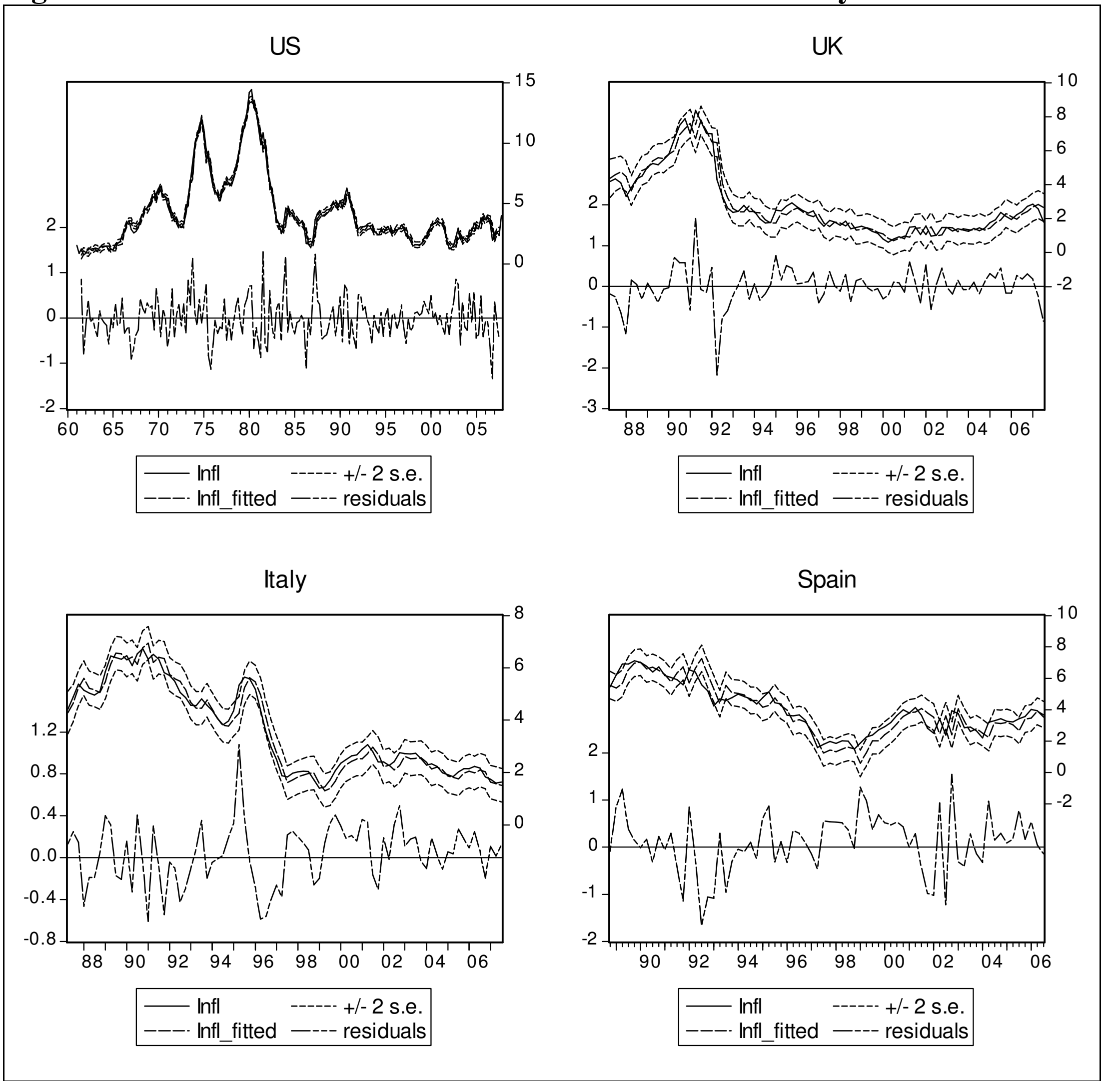


Source: OECD data, own calculations and graphs.

Nevertheless, a Phillips curve relation between inflation and unemployment is still visible at least in the first half of the sample period (Figure 3). In order to check for misspecification, we tested the residuals of all models for normality and stationarity. The ADF test rejected the null of a unit root for the residuals at the 1% level for all models, whereas the Anderson-Darling test for normality (Anderson and Darling, 1952, 1954) could not reject the null of a normal distribution for all models except those for the UK, where two large outliers (1991/1992) distorted the outcome.

Fitted values of the unrestricted and the restricted model (where coefficients on lagged and expected inflation were restricted to sum to one), as well as the residuals, are plotted in Figures 4 and 5, respectively.

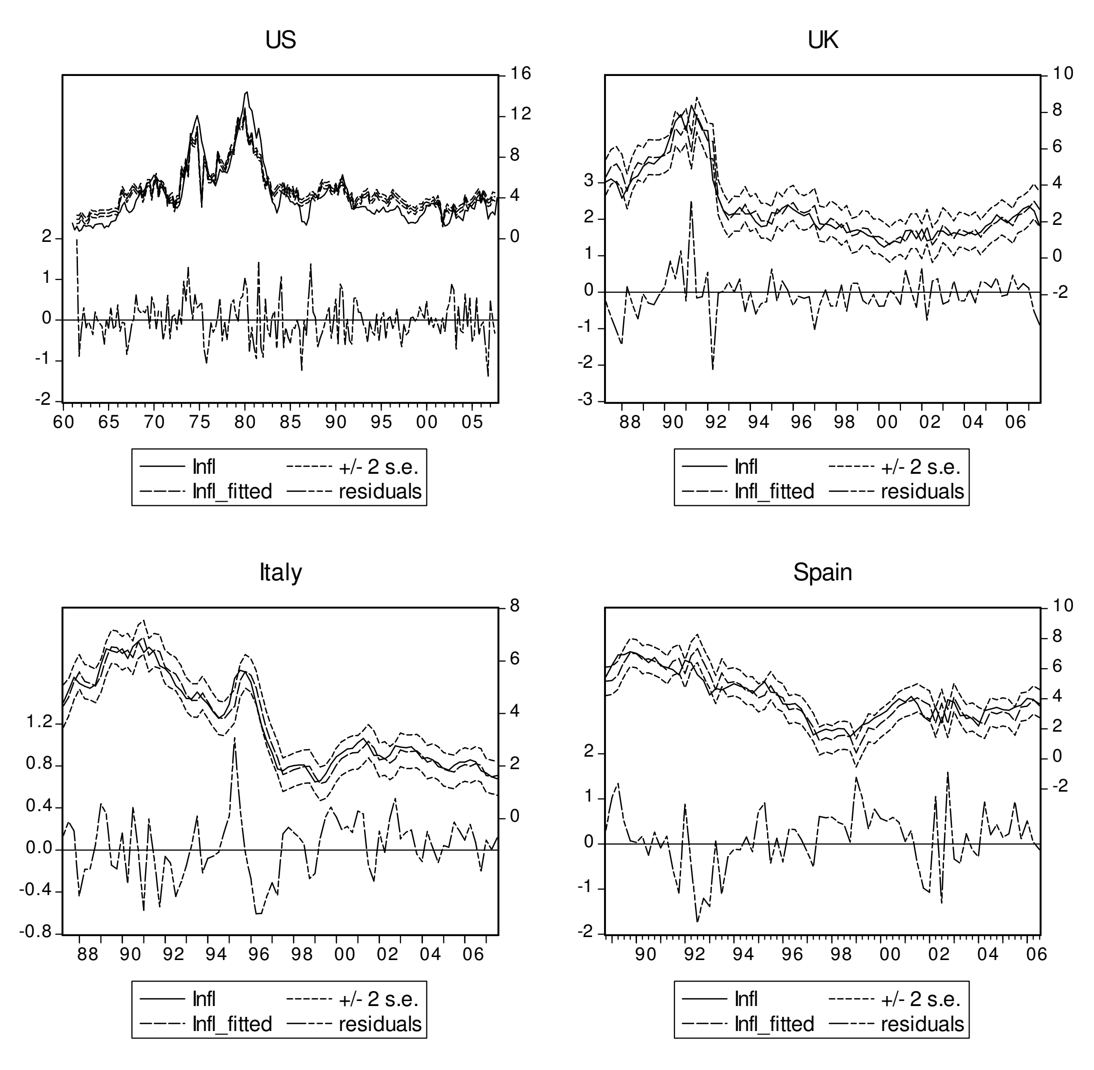
Figure 4: Fitted Values from the Unrestricted Model of the Hybrid NKPC



Source: OECD, EC Consumer Survey and SCA data, own estimations, own graphs.

In most of the countries under investigation here, fitted values from the unrestricted and the restricted model differ only marginally, and the fit of the model generally seems very close with respect to actual inflation rates. Only in the case of the US it seems that the fit from the unrestricted model is tighter, with exceptionally low standard errors. Nevertheless, fitted values from the restricted model for the US still fit actual inflation rates very closely. As indicated by the tests for stationarity and normality, the residuals plotted in Figures 4 and 5 generally seem to follow white noise processes around mean zero.

Figure 5: Fitted Values from the Restricted Model of the Hybrid NKPC

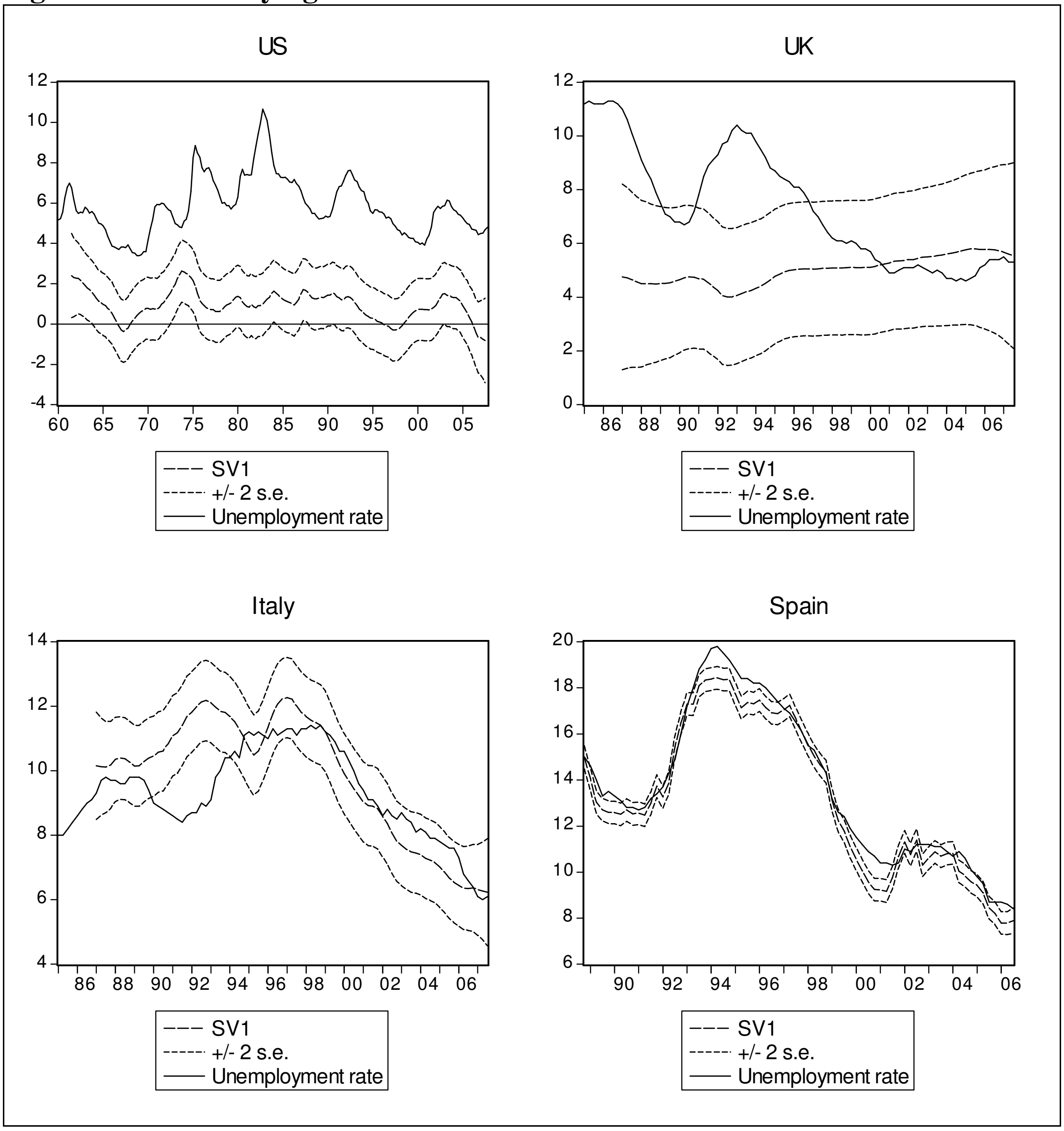


Source: OECD, EC Consumer Survey and SCA data, own estimations, own graphs.

Time-varying NAIRU Estimates

From the state-space model of the hybrid NKPC as in equations (13) and (14) we derived smoothed estimates of the time-varying NAIRU with the Kalman filter. Figure 6 presents the time-varying NAIRU estimates for the US, the UK, Italy and Spain from the unrestricted model.

Figure 6: Time-Varying NAIRUs from the Unrestricted Model

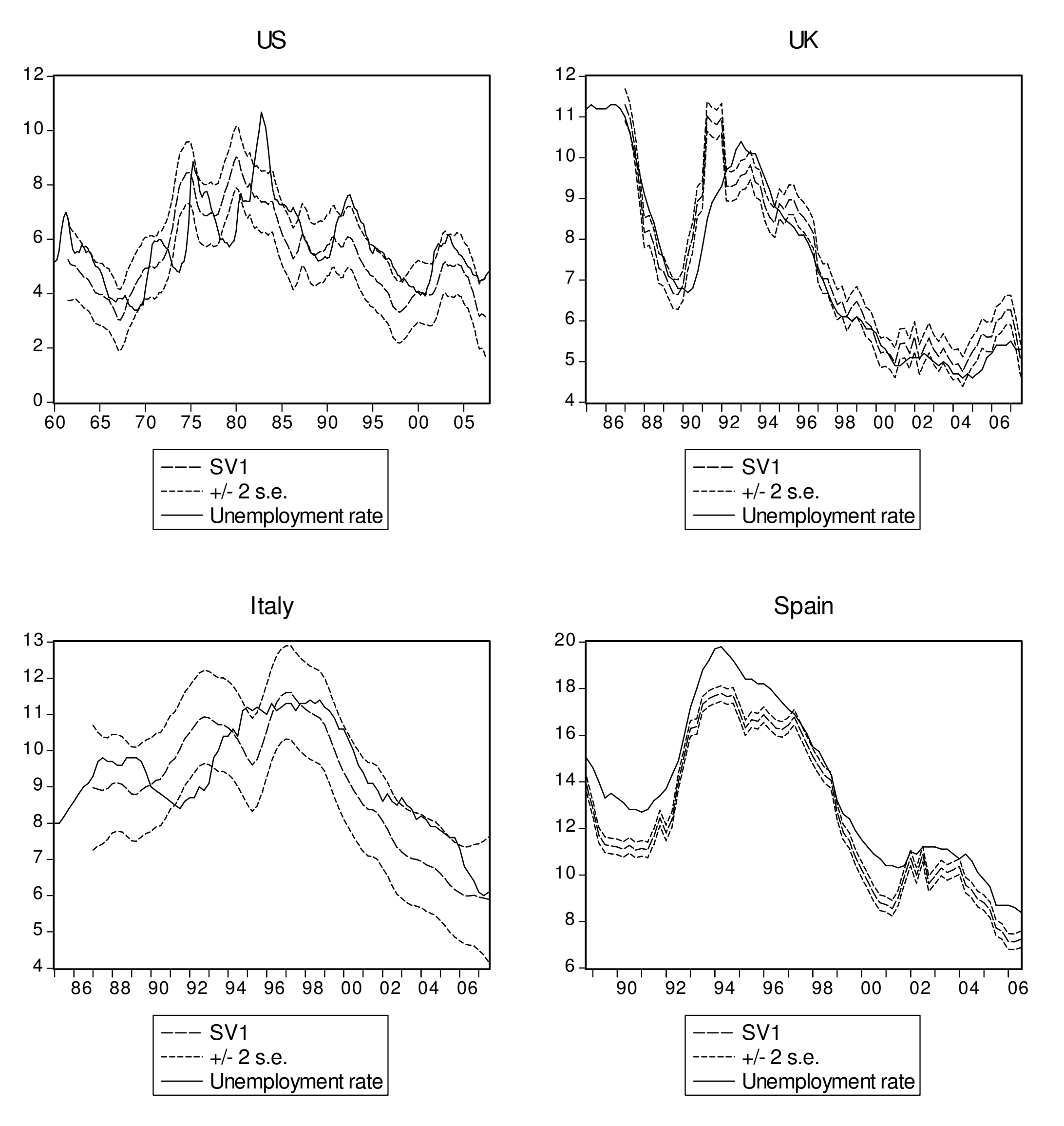


Source: OECD, EC Consumer Survey and SCA data, own estimations, own graphs.

Generally, unrestricted NAIRU estimates for the four countries under investigation here show considerable variation, usually in line with actual unemployment rates, with the notable exception of the UK, where NAIRU point estimates seem relatively stable. For the US and the UK, the unrestricted model yields rather implausible values of the NAIRU, suggesting that unemployment was significantly above its ‘natural’ rate in the US over the whole estimation period, albeit with very large confidence bands. By contrast, NAIRU estimates for Spain show a tight confidence band and are found close to actual unemployment rates, implying that unemployment was above its ‘natural’ rate only at the peak in 1994/95 and below in 2000. A similar result applies for Italy, with a

NAIRU close to actual unemployment from 1994 onwards, and unemployment below its ‘natural’ rate from 1990 to 1994.

Figure 7: Time-Varying NAIRUs from the Restricted Model



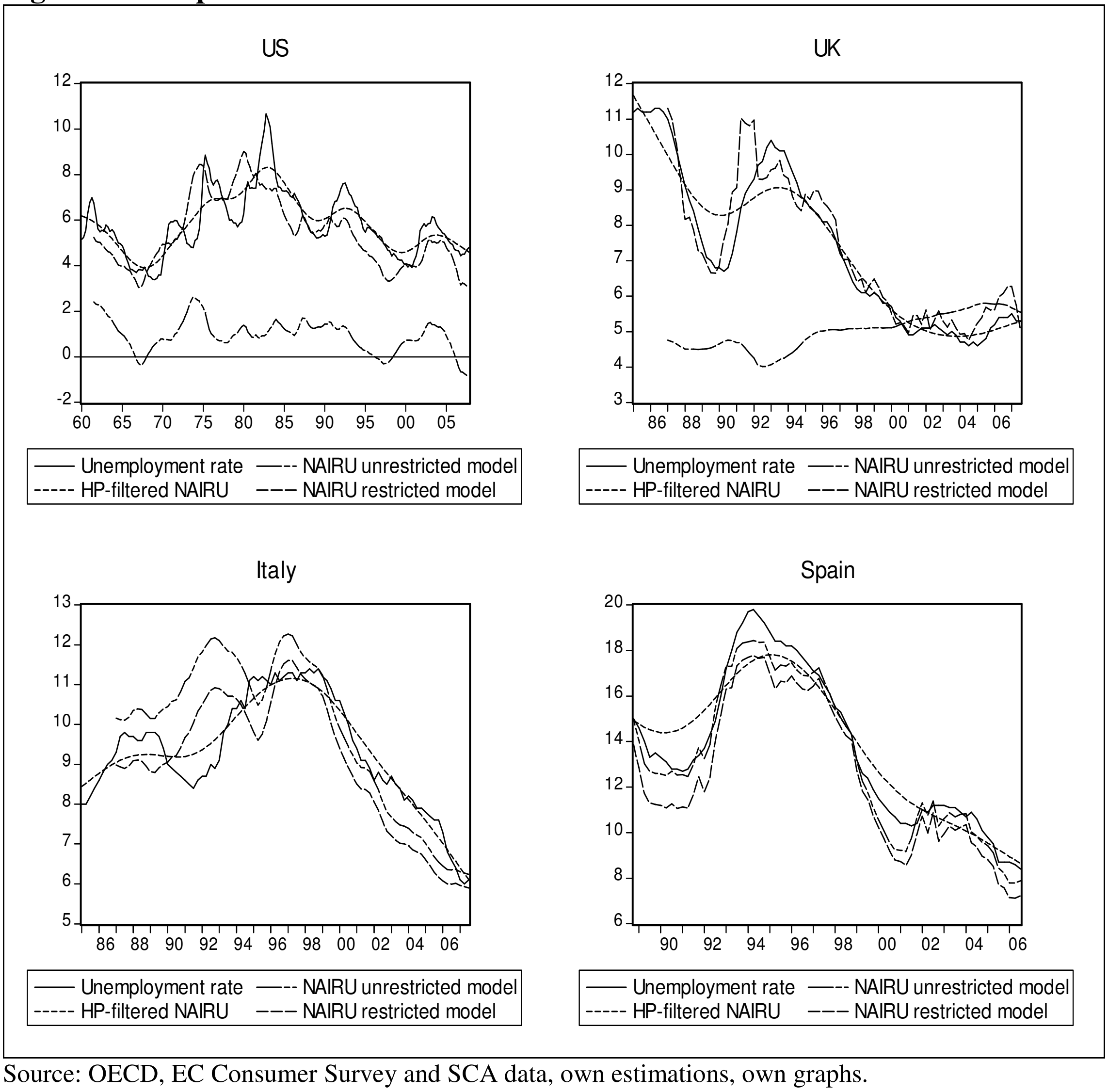
Source: OECD, EC Consumer Survey and SCA data, own estimations, own graphs.

Contrasting time-varying estimates of the NAIRU from the restricted models with those from the unrestricted model, the improvement in significance and variation of the NAIRU is remarkable. Especially for the US and the UK we now find highly significant NAIRU estimates with low standard errors close to actual unemployment rates. While we find for the UK that unemployment was below the ‘natural’ rate only in 1990 – 1992 and in the last years of the sample period, in the US unemployment seems to fluctuate around the NAIRU, with the NAIRU leading actual unemployment during the period of oil price shocks in the 70s and 80s. In the case of Italy, the picture seems mostly unchanged compared to the unrestricted model, with being 1992 – 1994 the only years where confidence bands of the NAIRU are above actual unemployment. For Spain, the fit of the time-varying NAIRU is again remarkable, but we now find that unemployment was above its 'natural' rate for most of the sample period.

Comparison of the Models

Finally, we compare the Kalman-filtered smoothed estimates of the time-varying NAIRU from the unrestricted and the restricted model of the hybrid NKPC to each other and to a mechanically calculated NAIRU derived with the HP-filter, shown in Figure 8.

Figure 8: Comparison of NAIRU Estimates



Source: OECD, EC Consumer Survey and SCA data, own estimations, own graphs.

Overall, we find three main results: First, as already noted above, NAIRU estimates from the restricted model of the hybrid NKPC generally seem more plausible in relation to actual unemployment and in the case of the US and the UK yield significantly different estimates. Second, all NAIRU estimates derived from the state-space models are significantly different from the mechanically derived HP-filtered NAIRUs, suggesting that estimating the NAIRU in a theoretically grounded macroeconomic model, taking account of the interaction of inflation, inflation expectations and the unemployment gap, yields significant new insights. Third, all NAIRU estimates from the state-space models of the hybrid NKPC imply a drop in 'natural' unemployment rates in the second half of the 90s, which in the case of Italy extends until the end of the estimation period.[11] For the US, Italy and Spain, the drop in 'natural' unemployment rates is even more pronounced than the fall in actual unemployment rates, suggesting that unemployment remained above its 'natural' value in this period.

As noted above, in the case of the US and the UK the unrestricted model gives implausibly low values of the NAIRU, suggesting that actual unemployment was always significantly above its 'natural' value. These results are not in line with those found in the literature for the US (e.g. Gordon, 1997, and Staiger *et al.*, 1997). By contrast, time-varying estimates of the NAIRU from the restricted model imply a mean 'natural' unemployment of 5.5% (7.25%) for the US (UK), close to actual mean unemployment of 5.8% (7.37%). Note that average 'natural' unemployment is still estimated to be lower than average actual unemployment. For Italy, estimates of the NAIRU from the state-space models differ mostly in the first half of the sample period, where the NAIRU implied by the restricted model is lower. Overall, the restricted NAIRU (9.23%) has a mean closer to average actual unemployment rates in Italy (9.39%) than the unrestricted NAIRU (10.0%). By contrast, in the case of Spain the time-varying NAIRU from the unrestricted model (mean: 13.23%) seems to be closer to actual unemployment (mean: 13.69%) than the time-varying NAIRU from the restricted model (mean: 12.44%).

In order to discriminate more formally between the unrestricted and the restricted model of the hybrid NKPC, we analysed the information criteria and conducted a Wald test on

[11] This result is in line with those in Gordon (1997) and Staiger *et al.* (1997) for the US.

the restriction $\alpha + \beta = 1$. Furthermore, since the unrestricted model encompasses the restricted one, we could run a likelihood ratio test according to the formula

$$(16) \qquad 2[\mathcal{L}(\theta) - \mathcal{L}(\theta^*)] \approx \chi^2(m),$$

where $\mathcal{L}(\theta)$ is the log likelihood of the unrestricted model, $\mathcal{L}(\theta^*)$ the log likelihood of the restricted model and m the number of restrictions, which here equals one.[12] The information criteria of the models are found in Tables A2 – A9 in the Appendix, and test values for the Wald test and the likelihood ratio test are shown in Table 2.

< *Table 2 here* >

Generally, coefficients on lagged and expected inflation summed closely to one in all the unrestricted models of the hybrid NKPC, so that the Wald test could not reject the null hypothesis of the restriction $\alpha + \beta = 1$ in all countries except for the US. However, the information criteria and the likelihood ratio test are less conclusive: While both also favour the restricted model in the case of Italy; for the US, the UK and Spain information criteria are smaller for the unrestricted model and the likelihood ratio test rejects the null of the validity of the restriction. In the case of the UK this might be due to the non-normality of the residuals which violate a condition for a valid likelihood ratio test. Judging from the very tight fit of the model for the US, it might be the case that the unrestricted state-space model assigns too much of the variability in the data to the coefficients of the model, leading to the implausible estimate of the NAIRU. Finally, in the case of Spain, estimates of the NAIRU from both models are very close so that the restriction might not be necessary.

5. Conclusion

Most models of the Phillips curve assume that there is no long-run trade-off between inflation and unemployment or output due to rational expectations of agents and that the long-run Phillips curve is hence vertical at the 'natural' rate of unemployment or NAIRU. Pioneered by Friedman (1968) and Phelps (1968), this concept is by now well accepted and embodied in the most commonly used model of the Phillips curve, the hybrid New Keynesian Phillips curve (NKPC) derived by Galí and Gertler (1999).

[12] See Hamilton (1994).

Introducing additional rigidities such as information stickiness (Mankiw and Reis, 2001, 2002a, 2002b) yields a much slower adjustment process of expectations, more inertia in inflation and, thus, a longer-lived trade-off between inflation and unemployment.

We estimated the shifts in the ‘natural’ rate of unemployment or NAIRU as an unobserved component in a state-space model of the hybrid NKPC, combining approaches of Gordon (1997) and Harvey (2007). Using direct survey data for inflation expectations from the University of Michigan’s Surveys of Consumers and the EC Consumer Survey to avoid the problems of weak instrument bias often encountered in standard GMM approaches, the model was estimated for the US, the UK, Italy and Spain. Both the models for the US and the UK showed a significant short-run trade-off between inflation and output or unemployment, whereas in the case of Italy and Spain, we found a significantly positive coefficient. Nevertheless, in the first part of the estimation period, a Phillips curve relation between inflation and unemployment is also visible in the latter two countries. As expected, coefficients on lagged and expected inflation summed closely to one in all the countries and the restriction $\alpha + \beta = 1$ could not be rejected except in the model for the US.

The Kalman-filtered smoothed estimates of the time-varying NAIRU all showed considerable variation over time, usually in line with variation in unemployment rates. Comparing estimates from an unrestricted and a restricted hybrid NKPC model, estimates from the restricted model generally gave more plausible values, although formal tests preferred the unrestricted model for the US, the UK and Spain. Furthermore, all estimates of the time-varying NAIRU differed significantly from steady-state measures of unemployment calculated from the HP-filter, implying that a theory-based systems’ approach yields important new information.

It is thus suggested for all countries investigated here that the NAIRU has shifted considerably with the business cycle and economic shocks during the estimation period, with actual unemployment rates fluctuating around their ‘natural’ rate. This has important implications for monetary policy, since inflation targeting and stabilisation will be the more accurate, the better the knowledge of the NAIRU at any given point in time. Still further questions remain for future research: What is the direction of causality between changes in unemployment and changes in the NAIRU – is it unemployment that continually adjusts to a changing 'natural' unemployment rate or is the opposite the case? And how do changes in the NAIRU feed back into unemployment and inflation?

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7. Appendix

Appendix 1:

Survey Data of Expected Inflation and the Probability Method

The SCA of the University of Michigan asks interviewees specifically about their inflation expectations over the next 12 months: "By about what percent do you expect prices to go (up/down), on the average, during the next 12 months?"[13] The Survey hence provides a direct quantitative measure of annual inflation expectations, which is published online on a quarterly basis from 1960(q1) – 2007(q3).

Survey measures of household's inflation expectations for the European countries in this paper are provided by the Consumer Survey of the European Commission (EC, 2008), which is integrated into the Joint Harmonised EU Programme of Business and Consumer Surveys. Data on households' inflation expectations are available from January 1985 onwards, in the case of Spain from June 1986 onwards. Unfortunately, the EC Consumer Survey only provides a qualitative measure of households' inflation expectations instead of asking for a quantitative estimate of expected inflation as in the Michigan Survey. Interviewees are asked in question 6 of the Consumer Survey: "By comparison with the past 12 months, how do you expect that consumer prices will develop in the next 12 months? They will...

++ increase more rapidly

\+ increase at the same rate

= increase at a slower rate

\- stay about the same

-- fall"[14]

The EC Consumer Survey on households' inflation expectations is thus a pentachotomous qualitative survey, which needs to be transformed in order to recover a quantitative time series of expected inflation.

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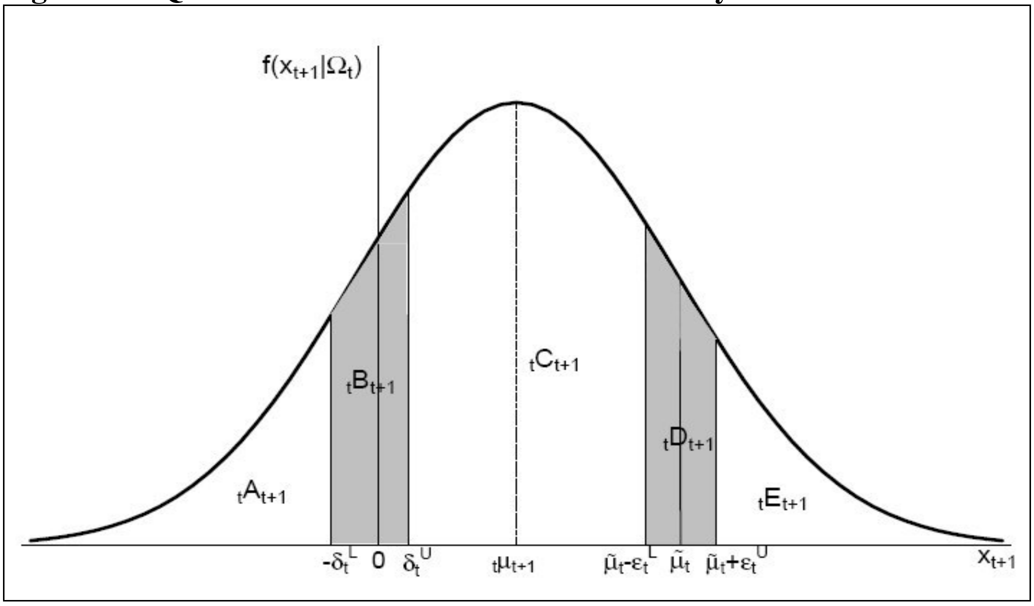
[13] See University of Michigan (2008b): Survey Description, p. 5, http://www.sca.isr.umich.edu/documents.php?c=i.

[14] See European Commission (2007), p. 48, http://ec.europa.eu/economy_finance/indicators/business_consumer_surveys/userguide_en.pdf .

A widely used method for obtaining quantitative estimates of expected inflation from qualitative surveys is the so-called probability method, which was derived by Carlson and Parkin (1975) for a trichotomous survey and extended for a pentachotomous survey by Batchelor and Orr (1988). It is assumed that individuals form their expectations on inflation based on a subjective probability distribution function, which can be aggregated across individuals in the joint probability distribution function (*pdf*) $f(\pi_{t+1}|\Omega_t)$, where π_{t+1} is expected inflation in period *t* based on the information set Ω available at *t* (Nielsen, 2003). Carlson and Parkin (1975) apply the Central Limit Theorem to argue that the joint *pdf* can be assumed to be normal, while Batchelor and Orr (1988) use the logistic *pdf* for computational convenience. Based on calculations with data from the EC Consumer Survey, Nielsen (2003) tests the properties of inflation expectations derived with the normal *pdf*, the logistic *pdf*, the central and non-central *t*-distribution and the χ^2-distribution to allow for peakedness and skewness. The author finds, however, that none of the alternative probability distribution functions significantly improves the forecasting abilities compared to estimates based on the normal distribution. We thus decided to base our estimates of inflation expectations for the European countries on the normal *pdf*, in line with Döpke *et al.* (2008) who also use the EC Consumer Survey data. A quantitative measure of expected inflation is then derived from the qualitative pentachotomous survey as follows (Batchelor and Orr, 1988, and Nielsen, 2003):

It is assumed that there exists a symmetric interval $(-\delta_t^L, \delta_t^U)$ around 0 such that interviewees will answer 'be stable' if the price change expected by them lies in this interval. Similarly, there exists a symmetric interval $(\tilde{\mu}_t - \varepsilon_t^L, \tilde{\mu}_t + \varepsilon_t^U)$ around the subjective mean perceived inflation rate $\tilde{\mu}_t$ such that interviewees will answer 'increase at the same rate' if the expected price change falls within this interval. Denoting the proportions of the total response of interviewees in each category described in section 4.2 as ${}_tA_{t+1}$ 'fall', ${}_tB_{t+1}$ 'stay about the same', ${}_tC_{t+1}$ 'increase at a slower rate', ${}_tD_{t+1}$ 'increase at the same rate' and ${}_tE_{t+1}$ 'increase more rapidly', the probability *P* of the expected price change x_{t+1} lying in one of the intervals can be expressed in terms of aggregated probability distribution functions (see Figure A1).

Figure A1: Quantification of Pentachotomous Survey Data



Source: Nielsen (2003), p. 5. See also Batchelor/Orr (1988), p. 320 for a similar graph.

As ${}_tA_{t+1} + {}_tB_{t+1} + {}_tC_{t+1} + {}_tD_{t+1} + {}_tE_{t+1} = 1$[15], ${}_tE_{t+1}$ can be dropped and the following quantiles of the distribution function with respect to expected inflation ${}_t\mu_{t+1}$ be specified, where ${}_ta_{t+1}$ returns the value of expected inflation below which random interviewees will answer that prices 'fall', i.e. the probability that expected prices lie in the range $x_{t+1} \le -\delta_t^L$. Consequently, ${}_tb_{t+1}$ returns the value where random interviewees will answer 'fall' or 'stay the same' and so on:

(33) $$\frac{-\delta_t^L - {}_t\mu_{t+1}}{{}_t\sigma_{t+1}} = F^{-1}({}_tA_{t+1}) = {}_ta_{t+1}$$

(34) $$\frac{\delta_t^U - {}_t\mu_{t+1}}{{}_t\sigma_{t+1}} = F^{-1}({}_tA_{t+1} + {}_tB_{t+1}) = {}_tb_{t+1}$$

(35) $$\frac{\tilde{\mu}_t - \varepsilon_t^L - {}_t\mu_{t+1}}{{}_t\sigma_{t+1}} = F^{-1}({}_tA_{t+1} + {}_tB_{t+1} + {}_tC_{t+1}) = {}_tc_{t+1}$$

(36) $$\frac{\tilde{\mu}_t + \varepsilon_t^U - {}_t\mu_{t+1}}{{}_t\sigma_{t+1}} = F^{-1}({}_tA_{t+1} + {}_tB_{t+1} + {}_tC_{t+1} + {}_tD_{t+1}) = {}_td_{t+1}$$

[15] We divided the proportions of answers 'don't know' proportionally among the five answers to ensure that ${}_tA_{t+1}+{}_tB_{t+1}+{}_tC_{t+1}+\mathrm{a}\,{}_tD_{t+1}+{}_tE_{t+1} = 1$ holds.

Finally, rearranging equations (38) – (41) gives the following expressions for expected inflation and its standard error:

$$(37) \qquad {}_t\mu_{t+1} = \tilde{\mu}_t({}_t a_{t+1} +{}_t b_{t+1})/({}_t a_{t+1} +{}_t b_{t+1_t} - c_{t+1} -{}_t d_{t+1})$$

$$(38) \qquad {}_t\sigma_{t+1} = -\tilde{\mu}_t * 2({}_t a_{t+1} +{}_t b_{t+1_t} - c_{t+1} -{}_t d_{t+1})$$

From equations (42) and (43), it can be seen that the quantitative time series of expected inflation derived with the probability method depends not only on the quantiles of answers from the survey and the distribution function used, but also crucially on the scaling factor $\tilde{\mu}_t$, i.e. the measure of inflation that agents base their expectations upon. In line with Nielsen (2003) and Döpke *et al.* (2008), we calculated expected inflation scaled with one-period lagged inflation, recursive mean inflation until last period, recursively HP-filtered inflation and the fitted values obtained from an ARMA(4,4)-model of inflation that were also recursively filtered with an HP-filter. The lag length of the both the AR- and the MA-terms was chosen according to the Akaike and the Schwarz info criterion.

Table A1: ADF tests for unit roots

Country	Variable	t-adf stat.	Prob. value[1]	Lag length
H_0: The variable has a unit root. Exogenous: constant				
US	π	-1.930127	0.3179	8
	$\Delta(\pi)$	-6.965019	0.0000	7
	$E_t(\pi_{t+1})$	-2.334283	0.1624	5
	$\Delta(E_t(\pi_{t+1}))$	-5.049304	0.0000	5
	u	-3.272	0.0176	1
UK	π	-1.485	0.536	5
	$\Delta(\pi)$	-8.599	0.000	0
	$E_t(\pi_{t+1})$_arma	-2.025	0.276	5
	$\Delta(E_t(\pi_{t+1})$_arma)	-3.037	0.036	2
	u	-2.270	0.184	5
	Δu	-3.523	0.010	0
Italy	π	-0.795	0.815	5
	$\Delta(\pi)$	-6.103	0.000	3
	$E_t(\pi_{t+1})$_arma	-1.364	0.595	7
	$\Delta(E_t(\pi_{t+1})$_arma)	-3.285	0.019	5
	u	-0.920	0.777	5
	Δu	-2.856	0.055	2
Spain	π	-1.207	0.668	5
	$\Delta(\pi)$	-9.215	0.000	0
	$E_t(\pi_{t+1})$_arma	-2.065	0.259	5
	$\Delta(E_t(\pi_{t+1})$_arma)	-4.621	0.000	2
	u	-1.477	0.540	5
	Δu	-3.594	0.008	0

[1] MacKinnon (1996) one-sided p-values.

Source: OECD, EC Consumer Survey and SCA data, own estimations.

Table A2: Results of the unrestricted state-space model for the US

Observation equation: $infl_t = c(1)*infl_{t-1} + c(2)*infl\_exp_t + c(3)*(u_t - sv1_t) + e_t$ State equation: $sv1_t = sv1_{t-1} + v_t$				
	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.745630	0.019850	37.56404	0.0000
C(2)	0.387542	0.031542	12.28658	0.0000
C(3)	-0.150547	0.029268	-5.143814	0.0000
	Final State	Root MSE	z-Statistic	Prob.
SV1	-0.811555	1.136853	-0.713861	0.4753

No. of observations	185	Akaike info criterion	1.363914
Log likelihood	-123.1620	Schwarz criterion	1.416136
No. of iterations	23	Hannan-Quinn criterion	1.385078

Anderson-Darling test for normality of the residuals	0.604	Prob.	0.115
ADF test on residuals	-7.825	Prob.	0.000

Source: OECD and SCA data, own estimations.

Table A3: Results of the restricted state-space model for the US

Observation equation: $infl_t = c(1)*infl_{t-1} + (1-c(1))*infl\_exp_t + c(3)*(u_t - sv1_t) + e_t$ State equation: $sv1_t = sv1_{t-1} + v_t$				
	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.729131	0.027256	26.75116	0.0000
C(3)	-0.281417	0.037613	-7.482014	0.0000
	Final State	Root MSE	z-Statistic	Prob.
SV1	3.118526	0.870759	3.581388	0.0003

No. of observations	185	Akaike info criterion	1.415126
Log likelihood	-128.8991	Schwarz criterion	1.449940
No. of iterations	17	Hannan-Quinn criterion	1.429235

Anderson-Darling test for normality of the residuals	0.706	Prob.	0.065
ADF test on residuals	-5.333	Prob.	0.000

Source: OECD and SCA data, own estimations.

Table A4: Results of the unrestricted state-space model for the UK

Observation equation: $infl_t = c(1)*infl_{t-1} + c(2)*infl\_exp\_arma_t + c(3)*(u_t - sv1_t) + e_t$
State equation: $sv1_t = sv1_{t-1} + v_t$

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.806362	0.027987	28.81187	0.0000
C(2)	0.264229	0.040499	6.524410	0.0000
C(3)	-0.058089	0.029610	-1.961797	0.0498
	Final State	Root MSE	z-Statistic	Prob.
SV1	5.542817	1.783607	3.107646	0.0019

No. of observations	83	Akaike info criterion	1.507086
Log likelihood	-59.54409	Schwarz criterion	1.594514
No. of iterations	48	Hannan-Quinn criterion	1.542210

Anderson-Darling test for normality of the residuals	1.457	Prob.	0.001
ADF test on residuals	-8.427	Prob.	0.000

Source: OECD and EC Consumer Survey data, own estimations.

Table A5: Results of the restricted state-space model for the UK

Observation equation: $infl_t = c(1)*infl_{t-1} + (1-c(1))*infl\_exp\_arma_t + c(3)*(u_t - sv1_t) + e_t$
State equation: $sv1_t = sv1_{t-1} + v_t$

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.407533	0.081924	4.974512	0.0000
C(3)	-0.897442	0.042933	-20.90341	0.0000
	Final State	Root MSE	z-Statistic	Prob.
SV1	5.036696	0.488386	10.31295	0.0000

No. of observations	83	Akaike info criterion	1.966783
Log likelihood	-79.62151	Schwarz criterion	2.025069
No. of iterations	27	Hannan-Quinn criterion	1.990199

Anderson-Darling test for normality of the residuals	1.766	Prob.	0.000
ADF test on residuals	-20.387	Prob.	0.000

Source: OECD and EC Consumer Survey data, own estimations.

Table A6: Results of the unrestricted state-space model for Italy

Observation equation: $infl_t = c(1)*infl_{t-1} + c(2)*infl\_exp\_arma_t + c(3)*(u_t - sv1_t) + e_t$				
State equation: $sv1_t = sv1_{t-1} + v_t$				
	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.828536	0.085636	9.675086	0.0000
C(2)	0.208977	0.078227	2.671404	0.0076
C(3)	0.161354	0.056595	2.851042	0.0044
	Final State	Root MSE	z-Statistic	Prob.
SV1	6.233446	0.943550	6.606376	0.0000
No. of observations	83	Akaike info criterion	0.584316	
Log likelihood	-21.24911	Schwarz criterion	0.671744	
No. of iterations	47	Hannan-Quinn criterion	0.619440	
Anderson-Darling test for normality of the residuals		0.618	Prob.	0.104
ADF test on residuals		-8.385	Prob.	0.000

Source: OECD and EC Consumer Survey data, own estimations.

Table A7: Results of the restricted state-space model for Italy

Observation equation: $infl_t = c(1)*infl_{t-1} + (1-c(1))*infl\_exp\_arma_t + c(3)*(u_t - sv1_t) + e_t$				
State equation: $sv1_t = sv1_{t-1} + v_t$				
	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.814319	0.071115	11.45080	0.0000
C(3)	0.151379	0.057554	2.630211	0.0085
	Final State	Root MSE	z-Statistic	Prob.
SV1	5.897647	0.970336	6.077943	0.0000
No. of observations	83	Akaike info criterion	0.564167	
Log likelihood	-21.41292	Schwarz criterion	0.622452	
No. of iterations	30	Hannan-Quinn criterion	0.587582	
Anderson-Darling test for normality of the residuals		0.544	Prob.	0.157
ADF test on residuals		-4.471	Prob.	0.000

Source: OECD and EC Consumer Survey data, own estimations.

Table A8: Results of the unrestricted state-space model for Spain

Observation equation: $infl_t = c(1)*infl_{t-1} + c(2)*infl\_exp\_arma_t + c(3)*(u_t - sv1_t) + e_t$
State equation: $sv1_t = sv1_{t-1} + v_t$

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.334502	0.060608	5.519084	0.0000
C(2)	0.910038	0.121401	7.496107	0.0000
C(3)	0.683568	0.078028	8.760543	0.0000
	Final State	Root MSE	z-Statistic	Prob.
SV1	7.889068	0.526501	14.98395	0.0000

No. of observations	73	Akaike info criterion	2.541410
Log likelihood	-89.76147	Schwarz criterion	2.635538
No. of iterations	19	Hannan-Quinn criterion	2.578922

Anderson-Darling test for normality of the residuals	0.655	Prob.	0.084
ADF test on residuals	-17.953	Prob.	0.000

Source: OECD and EC Consumer Survey data, own estimations.

Table A9: Results of the restricted state-space model for Spain

Observation equation: $infl_t = c(1)*infl_{t-1} + (1-c(1))*infl\_exp\_arma_t + c(3)*(u_t - sv1_t) + e_t$
State equation: $sv1_t = sv1_{t-1} + v_t$

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.217899	0.061100	3.566259	0.0004
C(3)	0.903304	0.074232	12.16867	0.0000
	Final State	Root MSE	z-Statistic	Prob.
SV1	7.231519	0.481385	15.02230	0.0000

No. of observations	73	Akaike info criterion	2.632641
Log likelihood	-94.09141	Schwarz criterion	2.695394
No. of iterations	21	Hannan-Quinn criterion	2.657649

Anderson-Darling test for normality of the residuals	0.643	Prob.	0.090
ADF test on residuals	-6.354	Prob.	0.000

Source: OECD and EC Consumer Survey data, own estimations.

Tables:

Table 1: Root Mean Squared Errors of Time Series' of Inflation Expectation

Country	Scaling Factor	RMSE
UK	HP-trend from ARMA model of inflation	0.868
	Recursive HP-trend	1.300
	Recursive mean	1.545
	Last period's inflation	1.145
Italy	HP-trend from ARMA model of inflation	1.006
	Recursive HP-trend	1.310
	Recursive mean	1.181
	Last period's inflation	1.223
Spain	HP-trend from ARMA model of inflation	1.516
	Recursive HP-trend	1.543
	Recursive mean	1.549
	Last period's inflation	1.676

Source: OECD and EC Consumer Survey data, own calculations.

Table 2: Comparing Unrestricted and Restricted Models of the Hybrid NKPC

Country	Wald Test		Likelihood Ratio Test	
	$\chi^2(1)$	Prob.	$\chi^2(1)$	Prob.
US	23.393	0.000	11.4742	0.001
UK	2.053	0.152	40.155	0.000
Italy	0.287	0.592	0.328	0.567
Spain	3.563	0.059	8.660	0.003

Source: OECD, EC Consumer Survey and SCA data, own calculations.