THE TECHNICAL EFFICIENCY OF ILLINOIS GRAIN FARMS: AN APPLICATION OF A RAY-HOMOTHETIC PRODUCTION FUNCTION

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Abstract

The purpose of this paper is to measure the extent of technical inefficiency among a sample of Illinois grain farms using the corrected ordinary least squares method. Instead of assuming a Cobb-Douglas production function, a linear form of the ray-homothetic is used. The results show a significant amount of technical inefficiency among all the farms in the sample, but with large farms being less technically inefficient than small farms.

Key words: technical efficiency, Illinois grain farms, ray-homothetic.

This paper analyzes the extent to which a sample of Illinois grain farmers have attained technical efficiency. The approach taken to measure the extent of technical inefficiency is the corrected ordinary least squares method (COLS). However, instead of assuming a Cobb-Douglas production form, a linear form of the ray-homothetic function will be used. This will not only allow for the measurement of the extent of technical inefficiency, but it will also allow the attribution of the inefficiency to operating off the isoquant (pure technical inefficiency) or operating at an inappropriate scale (non-constant returns to scale as opposed to constant returns to scale). In addition, the paper seeks to determine the relationship, if any, between farm size and technical efficiency.

Given the financial crisis facing many farmers in the United States and elsewhere, it is indeed important to determine to what degree farms are efficient. If significant inefficiencies are discovered and the causes identified, steps could be suggested to lower cost. Given the trend towards farm consolidation, it is also important to know if large farms are more technically efficient than smaller ones. Finally, it should be noted that the method developed here can be easily applied to farm data for other regions, different crops, etc.

The second section of this paper presents a brief review of the literature concerning the measurement of technical efficiency and discusses the methodology used. Section three discusses the data and the empirical results. Section four summarizes the paper.

METHODOLOGY

There are a variety of methods used for measuring and computing technical efficiency.[1] Most involve the construction of a best-practice frontier of one kind or another and the measurement of inefficiency relative to this frontier. In this paper, these various methods are divided into four basic approaches. These approaches differ in many ways, but two main differences involve the method used to determine the shape and placement of the frontier and the interpretation given to deviations from the frontier.

The beginning point for any discussion of frontiers and efficiency is the work of Farrell. This approach involves the construction of a deterministic, non-parametric frontier and is sometimes called the pure programming approach. Consider a firm using two inputs, x_1 and x_2, and producing one output, Y. If it is assumed that the production frontier is characterized by constant returns to scale, then it can be represented by a unit isoquant. Of course, the efficient unit isoquant is not observable and must be estimated. Farrell

[1] Much of this section is based on the work of Førsund, Lovell, and Schmidt.

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used linear programming techniques to construct the free disposal convex hull of the observed input-output ratios. This is supported by a subset of the sample with the rest of the points lying above it. Thus, each actual observation can be compared to the unit isoquant, and the extent to which the former lies above the latter measures the extent of technical inefficiency.

The Farrell approach has been extended so as to incorporate nonconstant returns to scale and to allow for the possibility of input congestion[2] (Färe, Grosskopf, and Lovell). Thus, not only can the extent of technical inefficiency be determined, but the source of the inefficiency can also be identified.

The principal advantage of this approach is that no functional form is imposed on the data. A major problem, however, is that the entire deviation of an observation from the unit isoquant is attributed to technical inefficiency. Since the frontier is non-stochastic, there is no allowance made for environmental heterogeneity, random external shocks, measurement error, etc. In addition, this approach is not amenable to statistical analysis.

A second approach involves the construction of a deterministic parametric frontier. The only difference between this approach and that discussed above is that the frontier is constructed using a specific functional form. This approach was first suggested by Farrell and has been extended by Aigner and Chu, Førsund and Jansen, and Førsund and Hjalmarsson. The principal advantages of this approach are the ability to characterize frontier technology in a simple mathematical form and the ability to accommodate non-constant returns to scale. There are two main drawbacks. First, the approach is deterministic and thus no allowance is made for noise, measurement error, etc. The second drawback is the inability to deal easily with multiple outputs.

The third approach, in contrast to the previous two, uses statistical techniques to estimate a deterministic statistical frontier. The technique was first proposed by Afriat and has been extended by Richmond and Greene. This approach involves assuming some sort of functional form for the frontier and estimating the frontier. The easiest way to estimate the frontier is by using corrected ordinary least squares (COLS). The functional form chosen (usually Cobb-Douglas) is first estimated using OLS, and then the constant term is corrected by shifting it up until no residual is positive and at least one is zero. Thus, the extent of a particular observation's inefficiency is measured by the ratio of actual output to potential output, with the latter given by the frontier itself. An example is provided by Russell and Young.

Another way of estimating the frontier is by maximum likelihood techniques. However, there are several difficulties involved. First, the estimated parameters depend on the particular distribution assumed for the error term. Second, not just any one-sided distribution for the error term will do. The usual desirable asymptotic properties of maximum likelihood estimators hold only if the density of the error term satisfies certain conditions. Greene has shown that the Gamma density satisfies these conditions. However, it is disturbing that the assumption about the distribution of technical inefficiency should be governed by statistical convenience.

Overall, the advantage of using the deterministic statistical approach to construct frontiers is the possibility of statistical inference based on the results. The disadvantages are that all deviations in the frontier are attributed to technical inefficiency and that a functional form must be specified.

The final approach involves the estimation of a stochastic frontier. This involves specification of a functional form and uses statistical techniques to estimate the frontier. However, in constrast to the deterministic statistical frontier approach, this approach allows the frontier to be stochastic. The essential idea is that the error term is composed of two parts. A symmetric component permits random variation of the frontier across observations and captures the effects of measurement error, random shocks, etc. A one-sided component of the error term captures the effects of inefficiency. This approach was first proposed by Aigner, Lovell, and Schmidt and Meeusen and van den Broeck and has been extended by Schmidt and Lovell, and Huang, among others.

There are a number of drawbacks to using this approach. First, considerable structure is usually imposed on the technology. In addition, the distribution of the one-sided error term must be specified when the model is estimated. Thus, additional structure is imposed on the distribution of technical inefficiency. Finally, this approach has difficulty dealing

[2] Congestion in the use of a particular input implies that the marginal product of that input is zero or negative.

with multiple outputs. The biggest advantage of the stochastic frontier approach is that, unlike the previous three approaches, it introduces a disturbance term representing noise, measurement error, and exogenous shocks beyond the control of the production unit.

In summary, there are a variety of methods which can be used to measure the extent of technical efficiency. There are advantages and disadvantages to each of these methods, and there is no obviously superior approach. Additional research involving a comparative evaluation of the strengths and weaknesses of the four alternative approaches is needed.

In this study, the deterministic statistical frontier is estimated using COLS. This method allows measurement of the technical inefficiency of each individual observation and statistical analysis of the results. It also does not require any special assumptions concerning the distribution of the error term. Finally, it is a tractable method of analysis that can be easily applied to relatively large samples. Obviously, additional research would involve using one or more of the alternative methods for measuring technical inefficiency in order to compare the results with those generated in this paper.

Instead of assuming a Cobb-Douglas form, a ray-homothetic structure will be used. The advantage of the ray-homothetic function is that it allows for the possibility that returns to scale will vary with output (the function is homothetic along a ray). It allows for the possibility that at low levels of output the firm will undergo increasing returns to scale, at some output level returns will be constant, and beyond this decreasing returns to scale will prevail. If this possibility exists, then one must also account for the fact that different firms are likely to have a different optimal scale (i.e., that level of output at which constant returns to scale prevail). Specifically, capital-intensive firms are likely to have a larger optimal scale than labor-intensive firms (the optimal scale varies with the input vector). The ray-homothetic function also allows for this possibility.

Letting x represent an input vector and $\phi(x)$ the maximum output attainable from the input vector x, the ray-homothetic production function can be written as [3]

$$(1)\ \phi(\lambda x) = F(\lambda^{H(x/\|x\|)} \cdot F^{-1}(\phi(x))),$$

where $\|x\|$ denotes the norm of x, $\lambda > 0$, and F is a monotonically increasing transformation of $(\lambda^{H(x/\|x\|)} \cdot F^{-1}(\phi(x)))$. If F is the identity function, then

$$(2)\ \phi(\lambda x) = \lambda^{H(x/\|x\|)}\phi(x),$$

with $H(x/\|x\|)$ greater than zero. The function given by equation (2) is a ray-homogeneous production function (Eichorn). Thus a ray-homothetic function is a monotonic transformation of a ray-homogeneous function. If the function $H(x/\|x\|)$ is a positive constant for all values of x, it can be seen that equation (2) becomes a homogeneous production function and equation (1), a homothetic production function (Shephard). Thus the homothetic, homogeneous, and ray-homogeneous functions are special cases of equation (1).

The returns to scale for a particular value of x is measured by the scale function, which is also referred to as the function coefficient or the elasticity of output, and can be written as

$$(3)\ u(x) = \lim_{\lambda \to 1} \frac{\lambda}{\phi(\lambda x)} \cdot \frac{\partial \phi(\lambda x)}{\partial \lambda}.$$

It can be shown that for equation (1), the ray-homothetic function, the scale function can be written as

$$(4)\ u(x) = u(x/\|x\|, \phi(x)).$$

In other words, the ray-homothetic production function allows returns to scale to vary with relative input intensity $(x/\|x\|)$ and output.

The parametric specification of the ray-homothetic production function used in this paper is[4]

$$(5)\ Y = \ln\Theta + a_N N' \ln N + a_F F' \ln F + a_P P' \ln P + a_S S' \ln S + a_E E' \ln E + a_B B' \ln B + a_L L' \ln L,$$

where:

$$N' = \frac{N}{N+F+P+S+E+B+L},$$

$$F' = \frac{F}{N+F+P+S+E+B+L},$$

$$P' = \frac{P}{N+F+P+S+E+B+L},$$

[3] Much of this section draws upon the work of Färe, Jansson, and Lovell.

[4] This functional form was first derived in Färe and Yoon.

$$(6)\ S' = \frac{S}{N + F + P + S + E + B + L},$$

$$E' = \frac{E}{N + F + P + S + E + B + L},$$

$$B' = \frac{B}{N + F + P + S + E + B + L},$$

$$L' = \frac{L}{N + F + P + S + E + B + L},$$

and N, F, P, S, E, B, and L are respectively labor, fertilizer, pesticide, seed, equipment, buildings, and land. Y represents the gross revenue of farm production. The parameters to be estimated are Θ, a_N, a_F, a_P, a_S, a_E, a_B, and a_L.

The returns to scale function for the production function given in equation (5) can be written as

$$(7)\ u = \frac{a_N(N')}{Y} + \frac{a_F(F')}{Y} + \frac{a_P(P')}{Y} + \frac{a_S(S')}{Y} + \frac{a_E(E')}{Y} + \frac{a_B(B')}{Y} + \frac{a_L(L')}{Y}.$$

The optimal scale of output (constant returns to scale) can be found by setting equation (7) equal to one and can be written as

$$(8)\ OPTY = a_N N' + a_F F' + a_P P' + a_S S' + a_E E' + a_B B' + a_L L'.$$

As can be seen from examining equation (7), returns to scale depends upon the factor intensity of input usage (N′, F′, P′, S′, E′, B′, and L′) as well as gross revenue (Y). Equation (8) shows that the optimal scale is dependent on factor intensity.

In order to determine the extent to which a farm is technically efficient and the degree to which the inefficiency is due to pure technical inefficiency (operating off the isoquant) or scale inefficiency, the following procedure will be used. First, equation (5) will be estimated using ordinary least squares (OLS).[5] This will give the best linear unbiased estimates of the coefficients. The intercept will then be corrected by shifting the function until no residual is positive and at least one is zero. The pure technical efficiency score for each farm will be calculated by taking the ratio of the actual to the potential level of output. The potential level of output is calculated by substituting the quantity of each input actually used by the farmer into the estimated ray-homothetic production function whose intercept has been corrected (i.e., the frontier function). In addition to calculating the above ratio, one can also determine the total output lost as a result of pure technical inefficiency by subtracting actual output from potential output.

The analysis discussed above can be easily illustrated using Figure 1. The x_i axis measures the vector of inputs (where movements to the right represent equi-proportional increases in all inputs) and the Y axis, output. Production function A represents the estimated ray-homothetic function whose intercept has been corrected. Farm 1 uses x_1 of the inputs and produces an actual output level of Y_1. This farm's potential output is Y_1', and thus the percent by which actual output falls short of potential output due to pure technical inefficiency is Y_1/Y_1'. The total output lost is $(Y_1' - Y_1)$.

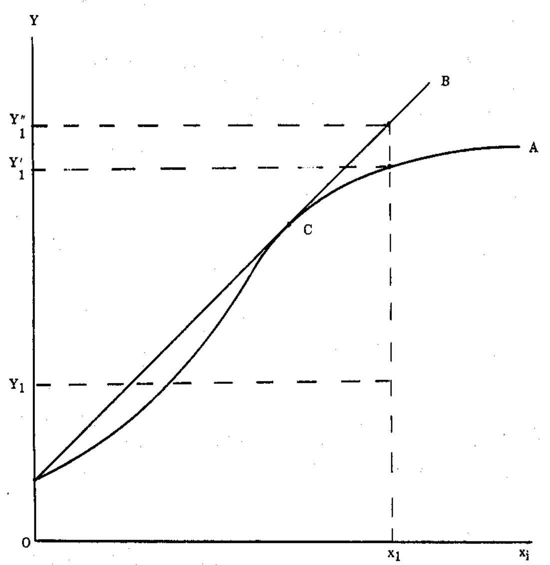

Figure 1. An Example of How Technical Efficiency is Measured.

In addition to pure technical inefficiency, the ray-homothetic function allows for the possibility of scale inefficiency. In order to

[5] Note that the equation (5) is linear.

determine the output lost as a result of this type of inefficiency, a simple procedure was developed (see appendix). An intuitive explanation involving Figure 1 will be presented here. Production function B represents a constant returns to scale function which is tangent to function A at point C, the optimal scale for function A. If farm 1 had used input vector x_1 and achieved constant returns to scale, output would have been Y_1''. Thus, if pure technical inefficiency was zero, actual output being Y_1', scale inefficiency could be measured as Y_1'/Y_1''. The total output lost due to scale inefficiency would be $(Y_1'' - Y_1')$.

Thus, for a firm using input combination x_1 and producing output Y_1, the total output lost as a result of technical inefficiency would be $(Y_1'' - Y_1)$. The lost output due to pure technical inefficiency would be $(Y_1' - Y_1)$, and the output lost due to scale inefficiency would be $(Y_1'' - Y_1')$. This procedure could be applied to all farms in the sample, and thus, one could find the total output lost due to technical inefficiency, that output lost as a result of pure technical inefficiency, and that output lost as a result of scale inefficiency.

Before proceeding further, several important points need to be made. Pure technical inefficiency occurs when, with the existing technology and input combination, a firm could produce more output with the inputs it employs (or the same level of output with fewer inputs). It represents an inability upon the part of the firm to solve certain technical problems in the production process and results in lost output for both the firm and society. A firm is scale inefficient when it operates at non-constant returns to scale. This notion corresponds to the Pareto efficiency conditions of a long-run (price-taking) competitive equilibrium (i. e., a zero profit long-run competitive equilibrium). This may not represent any inability to optimize on the part of the firm. The market may not be competitive, or price distortions may occur, and in these situations it may be optimal (in terms of maximizing profit, revenue, or some other activity) for the firm to operate at non-constant returns to scale. Thus, operating at non-constant returns to scale may be socially inefficient, but not necessarily inefficient from the individual firm's point of view.

EMPIRICAL ANALYSIS

The data used in this paper consist of information on production for a sample of Illinois grain farms operating in 1982. In order to control for variations in climate and soils, a sample of 88 farms from three contiguous counties in the south central portion of the Illinois grain belt: Christian, Montgomery, and Shelby, is used. A farm is considered a grain farm if the value of feed fed to livestock is less than 25 percent of the value of crop returns and if the value of feed fed to dairy or poultry is not more than one-sixth of the crop returns. The data source is the Illinois Farm Business/Farm Management Farm Business Analysis 1982 data tape "Annual Summary of Illinois Farm Business Records."

The farms in the sample produce a variety of grains, including corn, soybeans, wheat, and double crop soybeans. The value of these and other outputs are included in the gross revenue measure of output used as the dependent variable in equation (5). In terms of inputs, land, labor, fertilizer, pesticides, seeds, equipment, and buildings are used. Labor is defined as annual paid and unpaid farm labor costs (wages are imputed for family labor). Fertilizer, pesticides, and seed are also defined in terms of annual costs. The equipment variable includes annual power and equipment fixed costs, and the building variable is annual building costs. The variable land is constructed based on the average of the beginning and ending land values for 1982 times an interest charge. The interest charge for 1982 is 2.8 percent reflecting net rents received by landlords from land used in agricultural production (see Wilkens et al.). Summary statistics concerning the above variables on a per acre basis are presented in Table 1.

TABLE 1. SUMMARY STATISTICS FOR A SAMPLE OF ILLINOIS FARMS: 1982

Variable($/Acre)	Mean	Standard Deviation	Minimum Value	Maximum Value
Gross Revenue	332.41	66.37	167.43	722.16
Labor	38.03	13.53	12.39	75.19
Fertilizer	37.61	13.54	5.11	72.81
Pesticide	16.80	8.19	0.00	50.62
Seed	13.95	4.91	0.28	29.60
Equipment	72.43	23.07	5.73	157.78
Buildings	11.19	7.36	0.76	39.63

In the analysis, output is measured in revenue terms rather than in physical terms. This limitation, caused by the data, might result in our analysis of technical inefficiency reflecting allocative inefficiencies as well. However, the observed farms are homogeneous in terms of output (i.e., grain farms) producing grain corn, full-season and second-crop soybeans, wheat and some milo). For these farms, all of the inputs are measured in terms of monetary cost to the farm. The location of the farms is such that spatial variation in either output or input markets should not result in significant price or cost differences among farmers. As with technology on the farms, differences among farmers would reflect managerial differences. However, given the dominant technology for the pervasive grain enterprises in the area, reasonably homogeneous inputs, and similar input and output markets throughout the area, we assume that differences among farmers reflect managerial abilities affecting the technical efficiency of production. Data in Table 1 indicate there is variance among the farms in the amount of the various inputs used on a per tillable acre basis. However, across the farms, the set of inputs is fairly homogeneous.

The first step in applying the corrected ordinary least squares approach is to estimate equation (5), the ray-homothetic function. The results are presented in Table 2. As can be seen, all of the coefficients are highly significant. A test for heteroscedasticity by Park and Glejser[6] indicates that heteroscedasticity is not present.

TABLE 2. REGRESSION RESULTS FROM ESTIMATING THE RAY-HOMOTHETIC FUNCTION

Coefficient	Estimate	Standard Error	t-Ratio[a]
$\ln \theta$	−1904270.2	208956.1	−9.11***
a_N	208234.6	34346.2	6.06***
a_F	199657.2	20654.8	9.67***
a_P	223213.6	30031.2	7.43***
a_S	248606.2	38230.7	6.50***
a_E	188374.2	21127.4	8.92***
a_B	239269.4	28248.4	8.47***
a_L	227861.2	19362.5	11.77***
R^2 = .78			
F Ratio = 40.23			

a. *** significant at 1% level

The next step is to adjust the intercept of the ray-homothetic function upwards until no residual is positive and at least one is zero.[7] Using the procedure outlined in the previous section, the output lost due to technical inefficiency can be calculated in terms of output lost as a result of pure technical inefficiency and output lost as a result of scale inefficiency (operating of non-constant returns). The results of applying this procedure are presented in Table 3 for all the farms.

As can be seen, the farms as a group are producing at about 58 percent of their potential, where potential output is that which would be obtained if there were neither scale nor pure technical inefficiencies. Of the total output lost as a result of technical inefficiency, about 60 percent is the result of pure technical inefficiency and 40 percent the result of scale inefficiency. Thus, operating off the isoquant is the main source of technical inefficiency.

As stated earlier, one of the objectives of this paper is to analyze the relationship between farm size and technical efficiency. Technical efficiency is measured by the ratio of actual output to potential output (i.e., the technical efficiency ratio). Two different size classifications are used. The first is based on tillable acres, where no adjustment is made for the quality of land, and classifies farms into those with fewer than 700 tillable acres and those with 700 or more. The second classification is based upon gross farm revenue. It divides farms into those with a gross farm revenue less than $100,000, greater than $100,000 but less than or equal to $200,000, greater than $200,000 but less than or equal to $300,000, and greater than $300,000. Summary statistics are presented in Table 3. As can be seen, the larger farms tend to be more scale inefficient while the smaller farms are subject to a greater degree of pure technical inefficiency.

Using the acre size classification, an analysis of variance of the means of the two sets of farms is carried out. The results as well as the means for each group are presented in Table 4. As can be seen, those farms of 700 acres or more have a higher mean technical efficiency ratio than those farms of less than 700 acres. The F-statistic indicates that the variation between the two groups is significantly greater than the variation within each group (i.e., the means are statistically different). These results indicate that larger farms, in terms of acres, are more technically efficient than small farms.

Using the gross revenue size classification, an analysis of variance of the means of the four sets of farms is also carried out. The results as well as the means for each group are presented in Table 4. As can be seen, the

[6] See Pindyck and Rubinfeld.

[7] This approach is extremely sensitive to data outliers. In order to deal, to some extent, with this problem, a number of outlier farms (three) were deleted before the intercept was adjusted. The three were selected after visual inspection of the data revealed that they were producing large levels of output while using almost no inputs. No good method for dealing with the problem of outliers exists. However, an approach to this problem is indicated in the work of Schweder.

TABLE 3. EFFICIENCY RESULTS BY ACREAGE AND GROSS REVENUE FOR A SAMPLE OF ILLINOIS FARMS, 1982

Classification	Number of Farms	Total Actual Output ($) (mean)	Total Potential Output ($) (mean)	Technical Inefficiency ($) (mean)	Pure Technical Inefficiency (mean)	Scale Inefficiency ($) (mean)
All Farms	88	201238	346854	145616	87634	57982
< 700 acres	55	129821	233764	103943	91353	12590
≧ 700 acres	33	320265	535339	215074	81439	133635
< $100,000	17	76022	162403	86381	76505	9875
> $100,000 ≦ 200,000	32	136986	246522	109536	100816	8720
> 200,000 ≦ 300,000	24	250697	437369	186671	116566	75015
> 300,000	15	401083	625120	244037	33692	190345

TABLE 4. ANALYSIS OF VARIANCE FOR ACREAGE AND GROSS REVENUE FOR A SAMPLE OF ILLINOIS FARMS, 1982

Farm Size Classification	Mean Efficiency Ratio	Calculated F	Prob F
Acreage			
≧ 700 acres	.60		
< 700 acres	.55	4.18	.0438
Gross Revenue			
300,000 or more	.64		
200,001 to 300,000	.57		
100,001 to 200,000	.56		
100,000 or less	.47	7.98	.0001

TABLE 5. RESULTS OF MULTIPLE COMPARISONS BASED ON GROSS REVENUE

	Multiple t-Tests[c]		Tukey Tests[c]	
Mean Gross Revenue Comparisons[a]	Lower Confidence Limit[b]	Upper Confidence Limit[b]	Lower Confidence Interval[b]	Upper Confidence Interval[b]
A-B	.00491	.14751**	−.01777	.17019
A-C	.02769	.16326**	.00613	.18483**
A-D	.10940	.26288**	.08499	.18164**
B-A	−.14751	−.00491**	−.17019	.01777
B-C	−.03923	.07776	−.05724	.09637
B-D	.04126	.17860**	.01941	.20044**
C-A	−.16326	−.02769**	−.18483	−.00613**
C-B	−.07776	.03923	−.09637	.05784
C-D	.02565	.15567**	.00496	.17363**
D-A	−.26288	−.18614**	−.28729	−.08499**
D-B	−.17860	−.10993**	−.20044	−.01941**
D-C	−.15567	−.09066**	−.17636	−.00496**

a. A = farms with gross revenue greater than $300,000, B = farms with gross revenue greater than $200,000 but less than or equal to $300,000, C = farms with gross revenue greater than $100,000 but less than or equal to $200,000, and D = farms less than or equal to $100,000.
b. The confidence levels refer to differences in means.
c. **Five percent significance level.

larger the farm in terms of gross revenue, the higher the mean technical efficiency ratio. The analysis of variance indicates that the difference among the means is greater than the variation within each group.

When comparing more than two means, an analysis of variance F-test indicates if the means are significantly different from each other, but it does not indicate which means differ from other means. Multiple-comparison methods give the most detailed information about the differences among means. The two multiple-comparison methods presented here are repeated t-tests and the Tukey method. The t-test approach involves doing a t-test on every pair of means. The results are presented in Table 5. The A category represents farms with gross revenue greater than $300,000, B greater than $200,000 but less than or equal to $300,000, C greater than $100,000 but less than or equal to $200,000, and D less than or equal to $100,000. As can be seen, the results indicate that the mean efficiency ratios for farms in groups B and C are not statistically different. However, the means for A and D are statistically different from each other and from B and C.

The Tukey method of multiple-comparison is a modification of the t-test approach. This modification controls the maximum experiment wise error rate. The results are presented in Table 5 with A, B, C, and D as previously defined. As can be seen, the means between groups A and B and groups B and C do not appear to be significantly different. However, all other comparisons of means do show statistically significant differences. Thus, there does seem to be a relationship between size and technical efficiency. The larger farms tend to have higher mean technical efficiency ratios.

SUMMARY AND CONCLUSIONS

In this paper, the corrected ordinary least squares method (COLS) is used to assess the extent of technical inefficiency among 88 grain farms in central Illinois in 1982. However, instead of assuming a Cobb-Douglas type production function, a ray-homothetic form is used. The advantage of this approach is that the ray-homothetic function allows the optimal scale to vary with both output and factor intensity. Thus it is possible, using COLS, to not only determine the extent of technical inefficiency, but also to determine whether the inefficiency stems from pure technical inefficiency or scale inefficiency.

On average, farms in this sample produce 58 percent of their potential output. Of the output lost due to technical inefficiency, 60 percent is due to pure technical inefficiency and 40 percent to scale inefficiency. Technical efficiency and the size of the farm are positively correlated. This holds whether one measures size in acres or in terms of gross farm revenue.

Among experts concerned with American agriculture, there has been much concern over the gradual disappearance of the small to medium-sized family-owned farm. Some argue that this will result in increased efficiency because the larger farms can take advantage of pecuniary economies of size. Others argue that there are no significant economies of scale and the movement to larger farms will reduce the efficiency of production (see Hall and LeVeen, and Bagi). The current analysis indicates that, from the perspective of technical efficiency, larger farms are indeed more efficient than smaller farms. However, neither the larger farms nor the smaller ones appear very efficient when actual production is measured against potential production. This large degree of inefficiency may be, to some extent, the result of the fact that 1982 was a year of recession in the economy at large. This could have dramatically reduced the ability of some farms to produce. However, many farm inputs cannot be easily or quickly liquidated. Thus, some farms may still appear to be utilizing significant quantities of inputs without producing much output.

An alternative explanation could involve differences among farmers in terms of the vintage of the technology used. It could be that significant inefficiencies exist because the vast majority of farmers are using older technologies while a few innovative farmers are using the most up to date technology. The results of this study would, from this perspective, indicate that large farmers tend to adopt the most recently developed technologies faster than smaller farmers. This may very well be due to large farmers having better access to credit, information, and other scarce inputs than small farmers. Large farmers may also have a better capacity for bearing risk (see Feder, Just, and Zilberman).

Given the importance of farm size (see Miller, and Lin et al.) in the policy debate over the effectiveness of past farm programs and the debate of the 1985 Farm Bill, it is significant that the data show that larger farms are more efficient in terms of resource utilization. Indeed, the data indicate at least one possible cause of the continued development of a dualistic farm structure (Carr) characterized by relatively few full-time farmers operating large farm businesses and a large number of farmers operating small farms on a full-time or part-time basis. The growth in larger farms might be partially accounted for by their relative technical efficiency.

Similarly, the results indicate that all farm operators potentially could either produce more given available resources or produce the same level of output using fewer resources. The first option, given excess supply and low commodity prices, would not benefit American farmers. The second option would have a direct impact on the financial conditions of individual farms. Given the financial crisis of American agriculture in the mid-1980s, farmers apparently can reduce input levels, hence direct and indirect cash costs, without reducing output. The effect should be an increase in farms profitability. In opposition to the myth of the efficient American farmer, these grain farmers in Illinois apparently could enhance their economic position and increase their probability for survival by improving their managerial skills and their level of economic efficiency.

APPENDIX

The analysis discussed in this section will be based on a simple two-input case. It can be easily generalized to the situation of n inputs. The ray-homothetic production function in this case can be written

$$(10)\ Y_i = \ln\Theta + a_K \frac{K_i}{K_i+L_i} \ln K_i + a_L \frac{L_i}{K_i+L_i} \ln L_i,$$

Where Y_i, K_i, and L_i are output, capital, and labor used by farm i respectively, and Θ, a_L, and a_K are parameters to be estimated. The optimal scale of output in this two-input case would be given by

$$(11)\ Y_i^o = a_K \frac{K_i}{K_i+L_i} + a_L \frac{L_i}{K_i+L_i}.$$

This is the two input version which corresponds to the multiple-input case given in equation (8) in the text.

Multiplying K_i and L_i in equation (10) by u, a constant, and setting equation (10) equal to the optimal level of output for firm i, Y_i^o, gives

$$(12)\ Y_i^o = \ln\Theta + a_K \frac{K_i}{K_i+L_i} \ln(K_i \cdot u) + a_L \frac{L_i}{K_i+L_i} \ln(L_i \cdot u).$$

Thus, u would be the number by which we would have to multiply L_i and K_i if the optimal level of output were to be produced by the ith farm. Note that the ratios $\frac{K_i}{K_i+L_i}$ and $\frac{L_i}{K_i+L_i}$ would not change. Solving for lnu gives

$$(13)\ \frac{Y_i^o - Y_i}{a_K \left(\frac{K_i}{L_i+K_i}\right) + a_L \left(\frac{L_i}{L_i+K_i}\right)} = \ln u.$$

Substituting equation (11) into the denominator of equation (13) gives

$$(14)\ \frac{Y_i^o - Y_i}{Y_i^o} = \ln u$$

or

$$1 - \frac{Y_i}{Y_i^o} = \ln u.$$

Taking the antilog will give the actual value for u. Finally, note that u will be positive and less than one for farms experiencing decreasing returns to scale and positive and greater than one for farms with increasing returns to scale.

Examining Figure 2, the x_i axis measures the vector of inputs, with movements to the right representing an equi-proportional increase in all inputs, and the Y_i axis output. Production function A represents the variable returns to scale production function estimated in this paper. Farm one uses x_1 of the inputs and produces Y_1 actual output. Production function B represents the constant returns to scale function. u-1 represents the percent by which all input usage would have to increase to allow the farm to produce at the optimal scale represented by point c, using x_0 inputs and producing Y_0 output. In addition $\frac{u-1}{u}$ and thus Y_1^o could be derived as

$$(15)\ Y_1^o = Y_0 - \left(\frac{u-1}{u}\right) Y_0.$$

For firms operating at decreasing returns to scale the same sort of logic prevails. Thus, the output that could be produced by the firm if it were operating at constant returns to scale would be

$$(16)\ Y_1^o = Y_0 + \left(\frac{1-u}{u}\right) Y_0.$$

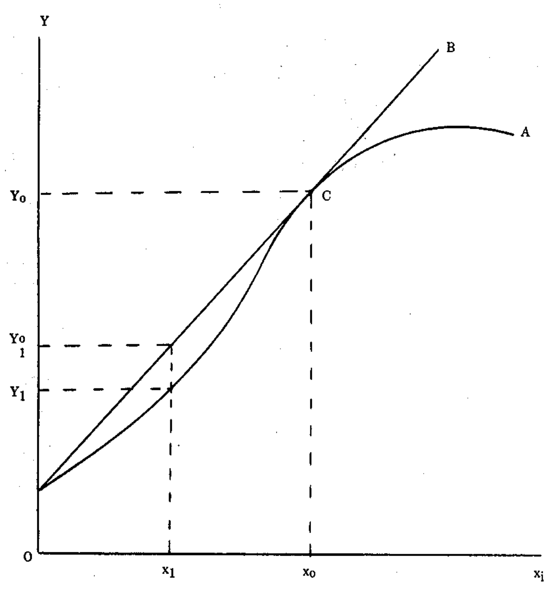

Figure 2. An Example of Measuring the Components of Technical Efficiency.

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