COTTON QUALITY, PRICE, AND USE VALUE: A STATISTICAL MODEL OF A TEXTILE PROCESSING PLANT

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THE PROBLEM

Cotton's position as the leading raw input supplier of the American textile industry is in grave danger. Recent years have seen various synthetic and man-made fibers make serious inroads into cotton's markets. In 1955, cotton enjoyed about 65 percent of the total domestic fiber market [7]. By 1960, with the introduction of relatively high priced substitutes such as nylon, cotton still commanded close to 65 percent of this market. Today, after the introduction of improved nylon, the polyesters, and the high-modulus rayons at more competitive prices, cotton comprises only about 50 percent of the textile market.

To some, this situation is simply a result of technological progress in which newer, better products have been and continue to be developed to supplant cotton in certain end uses. Although there is some merit in this view, those interested in the future of cotton see the market developments of the past few years as a challenge to more rapidly adapt cotton to the demands of a modern textile industry, to improve the marketing system for cotton so that it is efficiently and competitively priced, and to more aggressively disseminate knowledge of cotton's desirable characteristics to the consuming public.

Dr. Horne, of the National Cotton Council, has consistently admonished the cotton industry to shore up cotton's position on three fronts - - quality, price, and promotion [3, 4]. According to Dr. Horne, available resources should be used to improve cotton quality, to develop the marketing system to where cotton is priced both efficiently and competitively, and to aggressively promote the desirable features of cotton in the market place.

This study is a report of progress on a long-range study designed to (1) assist textile firms in using available technology to select and use cotton in the most profitable manner, and (2) as a result of firms' collective action in the market, improve the pricing and competitive position of cotton.

The specific objective of this paper is to present a statistical model of a textile processing plant through which the most profitable blending and processing of cotton can be determined. Effective use of the model by textile firms could improve their profit position and lead to more effective pricing of cotton in a competitive market. Use of the model would permit the industry to more systematically determine use values of various qualities of cotton, thus, price each different quality in accordance with its use value. This should lead simultaneously to a more efficient utilization of the nation's cotton production.

Although much work has been published showing relationships of fiber quality to mill processing performance and yarn quality, few studies have been published which attempt to include these relationships in a mathematical optimizing model of textile manufacturing. None were found which modeled both the blending of fibers and the selection of a processing organization as is done in this study.

This paper was made possible through the efforts of many people. Various personnel of the U.S. Department of Agriculture have worked on the overall project since its inception in the early 1960's. Dr. William A. Faught, John E. Ross, and F.W.S. Calkins, of ERS, were responsible for initiating the work. The ARS Pilot Spinning Laboratory has provided most of the basic information on which regression relationships are based.

Much of the work has been done under contract with outside research agencies. The primary theoretical framework and a nonlinear model were developed

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under contract by M.L. Bolinski and W.J. Baumol of Mathematica, Princeton, New Jersey. Various personnel of the Research Triangle Institute, Durham, North Carolina, have contributed to the development of the essential regression relationships.

More recently, Frank A. Tillman, Kansas State University, as a consultant to ERS, has developed, in cooperation with the author, the linear model to its present form [6]. At present, the linear model is more practical than the nonlinear model, since solution methods are more readily available.

THE THEORY

Briefly, the basic economic theory related to the model is as follows: on the demand side, the theory of marginal productivity is basic. This theory indicates that under competitive conditions (which are closely approximated in the American textile industry) and perfect knowledge, a productive input will be priced in equilibrium at the value of its marginal product. But supply, as well as demand, affects the equilibrium price. Thus, in determining the value of marginal product of an input in any time period, both supply of and demand for the input must be considered. Friedman discusses this point fully and summarizes the discussion as follows:

A complete theory (of value for price) requires a theory of both the demand of and the supply of factors of production [2].

A major obstacle to the accomplishment of the theoretical marginal pricing of cotton is imperfect knowledge. The relative marginal values of various cottons are difficult to know in a particular commercial setting. A prime purpose of the model under discussion is to make available a systematic means of estimating these marginal values.

There are many meaningful quality gradations of raw cotton. Because of its inherent qualities, one cotton is more useful than others in a given textile plant. Furthermore, various products in which cotton is used vary widely in their fiber quality demands and in their market values. The high quality cottons will command a substantially higher price in the market, ceteris paribus, because they are better fitted for use in high quality, high priced products. On the other hand, if these cottons are in abundant supply relative to poorer quality cotton, their price advantage will be diminished. The price advantage will never favor the poorer quality cotton because the higher quality will always substitute for the poorer quality.

Thus, a system for pricing cottons in the market according to their marginal value products must take into consideration the supply of, demand for, and various marginal value products of the different qualities of raw cotton. Provided quantitative measurements of supply, demand, quality, and technical processing relationships are available, a model of the industry which can relate these factors systematically can provide guidelines as to the most efficient industrial uses of various qualities of cotton and their relative values (prices) under various supply-demand conditions.

Such a model of the entire industry is some years away. However, a similar model for one firm is within reach at the present time. If each firm in the industry selected the appropriate blend (mix) of cottons and combination of machinery settings for maximum profit in the manufacture of its product(s) then, theoretically, market prices for the various qualities of raw cotton would gravitate toward levels of their respective marginal value products. Thus, general use of the firm model presented in this paper should result in improved utilization efficiency and more equitable pricing of cotton as well as improved profit positions of firms utilizing cotton.

For simplicity, a model of a yarn manufacturing plant utilizing n cottons in the production of one type of yarn will be presented. The principles involved in the simple model can be used to extend the model to include fibers other than cotton, multiple products, and additional stages of manufacturing.

THE VARIABLES

Variables which must be considered in any meaningful model of a textile firm may be broadly classified as follows: cotton types (and perhaps other fibers) and availability of each type, quality and price of each cotton type, quantity of each cotton type used in a mix, processing (machinery) organizations and related cost of each, yarn turnout, yarn quality and price, and profit per unit of time.

A cotton type is a distinct quality of cotton. The most important cotton quality variables which constitute a cotton type are fiber length, fiber fineness, fiber strength, grade, and price per pound.

Important yarn qualities, defined to include processing performance, are ends down per unit of time, yarn strength, yarn irregularity, and sales price per pound.

Processing organizations are discrete combinations of various machinery settings available in the spinning plant. In the model presented here, an organization consists of a specified spindle speed, a specified yarn twist multiplier, and a specified roving size. All other settings are constant across all organizations. To keep the model simple, only three possible levels of each

setting are considered. Thus, three spindle speeds x three twist multipliers x three roving sizes generate 27 combinations (or separate possible organizations) available. A specified manufacturing cost per pound is associated with each.

Since several cotton types may be blended and since waste is removed in the processing of cotton into yarns, quantity variables must be introduced to represent physical quantities of the various cotton types at each processing stage. Yarn turnout is the quantity of yarn made per unit of raw cotton. Finally, profit is measured as a function of total revenue, total input costs, and variable and fixed manufacturing costs.

THE MODEL

The model of a spinning plant is essentially a combination of multiple linear regression and linear programming techniques. Regression equations are used to estimate the effect on yarn quality of variations in raw cotton or blend quality and in processing organizations. Linear programming is used to determine the blend of cotton types (or cotton types and other fibers) and processing organization which produce the greatest profit while meeting all necessary specifications on yarn quality.

Although some of the relationships involved in the problem are not linear, close approximations are obtained with the linear methods. The overall project, under which this work is being accomplished, also, involves experimentation with nonlinear models of textile processing [1, 5].

Error

Unfortunately, the textile modeling process is plagued with both measurement and specification errors. Although considerable progress has been made recently in the measurement of fiber and yarn properties, considerable measurement error is still prevalent. Hopefully, the art of measurement is sufficiently developed to provide workable measurements of quality. Also, remarkable improvements currently are being made in the areas of quality measurement and equation specification.

Equations

The following equations provide the essential frame work for the model.

Regression equations are used to estimate the yarn qualities resulting from use of each cotton type (i), at a standard or base processing organization.

$$\text{Ends down}_i = a + b_1(\text{fiber length}_i) + b_2(\text{fiber fineness}_i) + b_3(\text{fiber strength}_i) + b_4(\text{grade}_i) = ED_i \quad (1)$$

$$\text{Yarn strength}_i = a + b_1(\text{fiber length}_i) + b_2(\text{fiber fineness}_i) + b_3(\text{fiber strength}_i) + b_4(\text{grade}_i) = YS_i \quad (2)$$

$$\text{Yarn irregularity}_i = a + b_1(\text{fiber length}_i) + b_2(\text{fiber fineness}_i) + b_3(\text{fiber strength}_i) + b_4(\text{grade}_i) = YI_i \quad (3)$$

where $i = 1, \ldots, n$

It is assumed that any blend of various cotton types will process exactly like any pure type, so long as it has the same average length, fineness, strength, and grade as the pure type.

Since the above equations relate to a given processing organization, the following equations are required to estimate the effect on yarn quality of changes in processing organization:

$$\Delta\ \text{Ends down}_j = b_1(\Delta\ \text{roving size}_j) + b_2(\Delta\ \text{spindle speed}_j) + b_3(\Delta\ \text{yarn twist}_j) \quad (4)$$

$$\Delta\ \text{Yarn strength}_j = b_1(\Delta\ \text{roving size}_j) + b_2(\Delta\ \text{spindle speed}_j) + b_3(\Delta\ \text{yarn twist}_j) \quad (5)$$

$$\Delta\ \text{Yarn irregularity}_j = b_1(\Delta\ \text{roving size}_j) + b_2(\Delta\ \text{spindle speed}_j) + b_3(\Delta\ \text{yarn twist}_j) \quad (6)$$

where Δ is the change of each yarn quality and organization setting from that of the standard or base processing organization, and $j = 1, \ldots, 27$.

The above constitute the regression components of the model. The object of the linear programming phase is to maximize the following profit function:

$$\text{Profit} = rt - p_1x_1 - p_2x_2 - \ldots - p_nx_n - c_1y_1 - c_2y_2 - \ldots - c_{27}y_{27} - f \quad (7)$$

where

r = price of yarn per pound, dollars
t = total yarn, pounds per hour
$p_1, \ldots, p_n$ = prices of cottons $1, \ldots, n$ per pound, dollars
$x_1, \ldots, x_n$ = quantities of cottons $1, \ldots, n$ used lbs. per hr.
$c_1, \ldots, c_{27}$ = variable processing cost per pound for organizations $1, \ldots, 27$
$y_1, \ldots, y_{27}$ = quantities of yarn processed through organizations $1, \ldots, 27$, pounds per hour
f = fixed processing cost per unit of time.

The above profit function is to be maximized subject to the following restrictions:

$$ED_1x_1 + \ldots + ED_nx_n = q_1 \quad (8)$$

$$YS_1x_1 + \ldots + YS_nx_n = q_2 \quad (9)$$

$$YI_1x_1 + \ldots + YI_nx_n = q_3 \quad (10)$$

and

$$(\text{turnout})q_1 + \Delta\, ED_1(y_1) + \ldots + \Delta\, ED_{27}(y_{27}) \leq ED\ \text{spec(t)} \quad (11)$$

$$(\text{turnout})q_2 + \Delta\, YS_1(y_1) + \ldots + \Delta\, YS_{27}(y_{27}) \geq YS\ \text{spec(t)} \quad (12)$$

$$(\text{turnout})q_3 + \Delta\, YI_1(y_1) + \ldots + \Delta\, YI_{27}(y_{27}) \leq YI\ \text{spec(t)} \quad (13)$$

where

q_1 = Ends down, in pounds - units per hour

q_2 = Yarn strength, in pounds-units per hour

q_3 = Yarn irregularity, in pounds-units per hour.

Equations (8) - (10) relate raw cotton quality to the yarn qualities at the standard processing organization. Equations (11) - (13) capture this effect and add the effect of organizational changes on the yarn qualities. Units are products of the quantity of stock (raw cotton in equations 8-10 and yarn in equations 11-13) and the weighted average yarn quality in each case.

Material balance constraints must also be met:

$$x_1 + \ldots + x_n = X \quad (14)$$

$$y_1 + \ldots + y_{27} = (\text{Turnout})\,(X) = t \quad (15)$$

$$C_1y_1 + C_2y_2 + \ldots + C_{27}y_{27} = 100 \quad (16)$$

where X = total quantity of raw material used, pounds per hr.

$C_1, C_2, \ldots, C_{27}$ = capacity factors which equate yarn production to frame capacity at the 27 processing organizations.

By setting C_j for the base organization equal to 1.0, those for faster frame speeds less than 1.0, and those for slower frame speeds greater than 1.0, each organization is forced in the model to generate the correct amount of production per hour.

Solution of the system to maximize the profit function describes the nature of the most profitable blend for a plant, prescribes the one best (maximum profit) processing organization, and through estimated "reduced costs" indicates the relative use values of different cottons considered.[1]

An Example

Consider six alternative cotton types for possible use by a plant manufacturing 40s carded yarn worth 95 cents per pound. It is assumed that availability of each cotton type is unlimited and that all yarn produced can be sold at 95 cents.

Table 1 shows the quality and price of each cotton type and the regression estimates of the yarn quality which would be produced by each cotton type if processed at the standard organization. Table 2 shows, for selected processing organizations, the definition of the organization, the associated manufacturing cost per pound, the change in each yarn quality arising from changing the organization from standard, and the associated spinning capacity.

Specifications on yarn quality are:

Ends down $\leq$ 30

Yarn strength $\geq$ 2000

Yarn irregularity $\leq$ 23.5.

The maximum-profit solution yields the following results:

Cotton Type	Pounds Used	Percent of Total	Implied Used Value[2]	Market Price
1	None	-	.2884	.3132
2	None	-	.2987	.3667
3	79.97	66	.2807	.2807
4	16.38	14	.2701	.2701
5	24.60	20	.3010	.3010
6	None	-	.2882	.2882
TOTAL	120.95	produced 108.11 pounds of yarn per hour at organization number nine.[3]		

[1] A "reduced cost" is an estimate of the amount by which the cost (price) of a cotton type would have to be reduced before the type would come into the blend.

[2] Implied use value = market price - "reduced cost".

[3] Fixed Cost - $.0891/lb.

TABLE 1. QUALITY AND PRICE OF SIX COTTON TYPES

Cotton Type	Length (32nd")	Fineness (Mike)	Strength (1000 psi)	Grade (index)	Price ($/lb.)	Estimated Yarn Quality at Standard Organization[a]		
						Ends Down (number)	Yarn Strength (break factor)	Yarn Irreg. (C.V.)
1	36	4.3	80	97	.3132	19	1,961	23.3
2	35	4.3	90	97	.3667	18	2,181	23.4
3	36	4.3	80	85	.2807	21	1,961	23.3
4	33	3.5	83	97	.2701	43	1,915	24.6
5	36	4.0	88	94	.3010	16	2,240	22.6
6	36	4.0	80	90	.2882	16	2,000	23.1

[a] Standard organization is: 11,100 rpm spindle speed
4.12 twist multiplier
1.15 roving size.

TABLE 2. PROCESSING ORGANIZATIONS AND THE PROCESSING COST, CHANGES IN YARN QUALITIES, SPINNING CAPACITIES FOR EACH

Org. No.	Roving Size (count)	Spindle Speed (rpm)	Yarn Twist (mult.)	Var. Cost ($/lb.)	Δ ED (no.)	Δ YS (units)	Δ YI (%)	Spinning Capacity Factor (C_j)
1	1.05	10,600	4.00	.1445	3	-38	0	1.015
2	1.15	10,600	4.00	.1460	0	-18	0	1.015
3	1.25	10,600	4.00	.1475	-3	2	0	1.015
4	1.05	11,100	4.00	.1425	8	-45	0	.970
5	1.15	11,100	4.00	.1440	5	-25	0	.970
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
9	1.25	11,600	4.00	.1435	7	-12	0	.925
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
14(std)	1.15	11,100	4.12	.1450	0	0	0	1.000
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
27	1.25	11,600	4.25	.1455	-3	38	0	.985

Profit = $52.91 per hour
Estimated ends down = 23
Estimated yarn strength = 2012
Estimated yarn irregularity = 23.3

THE ANTICIPATED RESULTS

As the model of a textile manufacturing firm is refined and used by indidivual firms, cotton's competitive position should be strengthened. Benefits should accrue to both individual textile manufacturing firms and the cotton industry.

Benefits to the Firm

By having reliable estimates of the relative use values of cottons in a particular plant, the firm will be able to select cottons more economically. In the shortrun, while considerable inequities exist in the pricing of cotton, the firm with the greater knowledge of use values of various cottons will be in position to take advantage of the inequities in the market. As a by-product, a firm will be able to obtain guidance from the model for planning processing research to obtain most needed information and for improving the way in which it utilizes cotton.

Benefits to the Industry

The additional knowledge of textile processing, particularly as it relates to use values of inputs, should lead to a more equitable cotton pricing system. Such pricing would provide additional guidance to plant breeders, cotton producers, cotton ginners, cotton merchants, and textile manufacturers in developing improved cottons, in handling and merchandising cotton, and in utilization of cotton.

Methodological Benefits

Such a model provides a basis for more properly selecting, blending, and utilizing cotton. However, the principles involved are applicable to other products used as inputs to various manufacturing processes. The modeling principles being used in the development of this model, for example, can be combined with an intimate knowledge of the wheat industry to develop a similar model for the wheat and milling industry. The successful completion of a model of any manufacturing firm or industry depends upon "workable knowledge" of critical relationships of input quality and processing variables to output quality (value). But it is precisely this kind of knowledge which, if combined systematically as in the model described, will lead to the anticipated firm and industry benefits.

SUMMARY

This paper presents a technique which could help textile firms and the industry to take increased advantage of the various qualities of cotton and to price cottons according to their relative use values. Such actions could lead to improvements in cotton quality as well as more efficient use of qualities now produced.

A simplified model of a textile manufacturing firm is presented which utilizes multiple regression and linear programming techniques. Solution of the model indicates the blend of fiber types and the processing organization which will meet specifications on processing performance and product quality and maximize profit. An example demonstrates the application of the model in a typical textile mill.

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