Bringing Growth Theory "Down to Earth"

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WP 09-01

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Draft last revised: March 30, 2009

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Abstract: Explicitly accounting for certain basic physical laws governing the "earth" sector dramatically enriches our ability to explain a high degree of diversity in observed patterns of economic growth. We provide a theoretical explanation of why some countries have been able to sustain a more or less constant and positive rate of economic growth for many decades while so many others have failed to do so. The analysis predicts that countries that have an over abundance of physical capital (a concept that is precisely defined in the text) may be unable to sustain a positive rate of economic growth over the long run. Too much physical capital may affect the dynamics of the economy ultimately leading to stagnation. The plausibility of the growth model introduced here is demonstrated by its ability to predict some important stylized facts for which standard endogenous growth models generally cannot account.

Keywords: endogenous growth theory, unbalanced growth, structural change, stagnation

JEL: E22, Q01, O41

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I. Introduction

Economic growth does not take place in a vacuum but rather occurs in a natural world that is subject to its own physical laws. In this paper we explicitly consider this obvious fact by fully integrating the natural world into a growth model that allows for permanent and positive economic growth over the long run as a possible but not inevitable outcome. We show that explicitly accounting for basic physical laws governing the “earth’s” resources, even at a minimum level of complexity, results in some startling differences with the standard growth models, dramatically enriching our ability to explain the high degree of diversity in observed patterns of economic growth. The standard endogenous growth models are either agnostic toward certain important aspects of new development patterns or they require ad-hoc modifications to explain these new patterns. Our focus on patterns of economic growth as affected by the earth sector stands in stark contrast to much of the environment/growth literature that focuses mainly on the implications of economic growth on the environment. As will be discussed, the fundamental role played by the environment in affecting economic growth has been alluded to by many authors, particularly in the so-called “resource curse” literature; however, this is the first time the environment has been so intimately linked to describe a wide array of growth patterns.

An important contribution of the model developed below is that it exhibits a state-dependent bifurcation that allows us to explain contrasting growth patterns among countries evident today. That is, depending on the state of asset endowments, the economy may endogenously achieve an economic growth rate that is constant and positive or decreasing toward ultimate economic stagnation. From the model we derive a well-defined interval within which the initial asset composition must fall for sustainable and constant economic growth to be feasible over the long run. If the asset composition is outside this critical interval, the economy will not be able to preserve a constant (positive) rate of economic growth and eventually may take a path that leads toward long run stagnation. This occurs because the endogenous dynamics of the system cause the asset composition to diverge further and further away from the asset boundaries that allow for permanent economic growth, and thus prevent any autonomous correction that could place the economy back into a steady growth path. Countries that have an over abundance of physical capital vis-à-vis human capital (a concept that is rigorously defined below) may be unable to sustain a positive rate of economic growth over the long run. They may fall into what we call a "capital curse".

Some literature has questioned the ability of factor accumulation to explain the deep income differentials observed across countries (e.g., Acemoglu and Angrist, 2001; Easterly and Levine, 2001; Pritchet, 2001; Acemoglu and Dell, 2009). Empirical estimates using years of schooling and related measures as proxies for human capital have found that differences in such indicators explain no more than 40% of cross-country variations (physical capital explains much less). In our analysis the nature of asset composition and the productivity of human capital play a key role in long run growth, which would contradict the said empirical evidence. However, recent advances in the measurement of human capital have shown that using years of schooling without accounting for quality differences may dramatically under-state the true value of human capital. Hanushek and Woessmann (2008) discuss several new empirical findings and convincingly show that when quality indicators based on international standardized cognitive tests are used to adjust for schooling years, human capital is a much more powerful measure for explaining cross-country differentials in individual earnings and income growth. We provide a hypothesis of why the accumulation of human capital can be smothered in economies where the asset composition is "wrong" or can fuel permanent positive economic growth in countries where such composition is within "right" levels.

This prediction of the model – that over abundance of physical capital with respect to human capital may lead to stagnation, i.e., the capital curse – cannot be easily tested with available empirical data. Consequently, we probe other predictions of the model against four well established empirical stylized facts to demonstrate the accuracy and relevance of the model. The fact that the model enhances our understanding of these stylized facts not only gives credibility to the model, but also supports its extension to the findings about the capital curse.

But before presenting the stylized facts, consider the following observation. For much of the 20th Century, persistent economic growth was mainly circumscribed to a handful of countries accounting for less than 10% of world population: Western Europe, the USA and more recently Japan (henceforth, the "North"). The rest of the world (the "South") was not able to maintain a persistent rate of growth for prolonged periods (Africa, Latin America, and most of Asia). At the same time the South was the receptacle of vast natural resources that appeared to provide an almost unbounded supply of primary commodities to satisfy the North's demand and for which the South itself demanded little. Recently, however, the growth club has expanded into parts of the South, particularly China, India and several other countries which constitute more than 40% of world population and consequently the demand for primary commodities from the South has begun to rapidly increase[1]. The popular press is ablaze in speculation of the growth implications resulting from this major shift in the growth club over the last two decades. The theoretical growth literature, however, has been largely silent on the implications of this growth club expansion for the North's continued growth success.

We now turn to four stylized facts that have been established in the recent empirical literature but which have not been comprehensively incorporated into existing growth theory. First, economic growth in the North has resulted in structural change away from capital-intensive and commodity-intensive industries toward service sectors, high technology sectors, as well as other knowledge-intensive activities (Chenery, 1960; Kongsamut et al., 2001; Acemoglu and Guierrieri, 2008)[2]. This structural change has been complemented in the North with the increasing importation of primary commodities and other capital-intensive industrial goods at a rate that outstrips growth (Ghertner and Fripp, 2007)[3].

Second, empirical evidence has shown that the conventional Kaldorian assertion regarding the constancy of labor share in GDP has not held over the past few decades (Poterba, 1998; Krueger, 1999; Acemoglu, 2003). In fact, a cross-country analysis of over 100 countries between 1972 and 1995 finds an increasing labor share in the North and a decreasing labor share in the South (Jayadev, 2007).

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[1] In the period 1960-90, growth in the United States and the EU countries represented 51% of world GDP growth while India and China with 35% of the world's population accounted for less than 1% of world GDP growth. By contrast, in the period 1991- 2006, the United States and EU contribution to world GDP growth dropped to 45% while the contribution of China and India rose to 16%. And since 2000, Brazil, Russia, India, and China growth has accounted for 22% of world GDP growth. (World Bank: World Development Indicators Database).

[2] Structural change has also been a concern in the literature on trade, growth and the environment (Copeland and Taylor, 2004). See also Antweiler et al. (2001).

[3] The other side of the coin is the South which has increasingly supplied the demand for commodities from the North. This has caused the South to reduce the size of its productive (non-subsistence) service sector vis-à-vis the rest of the economy. In China, for example, the service sector representing only 30% of GDP is considered to be grossly under-developed given its per capita GDP (Farrell and Grant, 2005).

Third, natural resource wealth and economic growth appear to be closely connected even if the natural resource sector is a declining part of the economy. This stylized fact is validated by the influential and primarily empirical "resource curse" literature (Sachs and Warner, 1995, 2001; Mehlum, Moene and Torvik, 2006; Humphreys, Sachs and Stiglitz, 2007), which highlights the importance of natural resource wealth as a factor affecting the rate of economic growth. A second component of this stylized fact is the nature of this relationship. While early works provided empirical evidence suggesting that resource wealth and the rate of economic growth were inversely related, more recent studies have shown that such a relationship may work in the opposite direction under certain conditions (Barbier, 2005; Lederman and Maloney, 2008; Peretto, 2008). Consequently, the empirical evidence concerning the relationship between natural resource wealth and economic growth remains elusive.

Fourth, despite rapid growth in the North which demanded increasingly larger volumes of raw materials, real commodity prices have not trended upwards. In fact, commodity prices have been non-increasing for most of the 20th century (Page and Hewitt 2001; Zanias, 2005; Kellard and Wohar 2006)[4]. Some indicators suggest, however, that over the last decade there may be a break of such stable price trends. As a result, there is increasing interest in the connection between commodity prices and economic growth.

Our contribution can be partly assessed with reference to the above stylized facts. First, with few exceptions, growth models consider only one final goods sector and thus are agnostic to structural change away from commodity-intensive industries to service industries. Kongsamut et al. (2001) is one such exception; however, this paper assumed a particular type of non-homothetic preferences (that the consumer iso-utility map shifts away from primary commodities) to explain structural change. While this assumption may be "realistic," it is almost equivalent to imposing structural change and deviates from the tradition in general equilibrium and growth literature to assume neutral preferences. Baumol et.al. (1985) explain structural change by imposing the assumption that exogenous technical change is biased against production of commodities and favors productivity growth in the rest of the economy. Acemoglu and Guerrieri (2008) also present a model of unbalanced growth with exogenous technical change

[4] A notable exception to these downward trending commodity prices is world timber prices which have consistently increased over the past 100 years. We will reconsider this important exception in light of the assessment of the growth model in Section IX.

where the shifting direction of growth is determined by the elasticity of substitution in consumption. López et.al. (2007) examine supply-induced structural change in the context of a bang-bang investment model where all assets are produced by the same production function; however, they do not integrate structural change as part of the broader framework implied by the four stylized facts discussed earlier and do not account for the role of the initial asset composition in affecting growth and structural change.

We show that structural change in both outputs and productive assets is an intrinsic consequence of economic growth even when preferences are entirely neutral and productivity growth is endogenous and sector neutral. The growth model that we develop is notably not proportional such that the physical capital to human capital ratio (and the consumption to capital ratio) is perpetually changing over the long run, even when the rate of economic growth may become constant. That is, we extend the concept of structural change to include changes in the composition of factors of production, not merely of outputs. While asset ratios change over time we show that they follow certain systematic patterns that replicate specific types of structural change observed in growing economies. Continuous structural change both in terms of outputs and factors of production is perfectly consistent with a constant rate of economic growth.

Second, resolving the issue of changing labor share requires a model that leads to differential rates of growth of human and physical capital in the long run. Yet most growth models predict constant factor ratios over the long run. The fact that we extend the concept of structural change to include changes in asset composition allows us to establish the conditions under which the labor and capital shares can change throughout the process of economic growth. This replicates the second stylized fact regarding factor shares discussed above.

Third, explanations of the resource curse abound despite the general absence in formal growth models of a satisfactory theory to delineate the conditions under which a resource endowment is associated with faster or slower economic growth. An exception is Peretto (2008) who provides the most rigorous theoretical treatment of the resource curse yet available; however, this work considers the welfare implications resulting from a once-and-for-all exogenous shock on resource availability and ignores the resource dynamics. We explicitly consider such resource dynamics and endogenous checks of resource wealth. We corroborate the first tenet of the resource "curse" findings; namely, that changes in resource wealth and economic growth are closely linked in the intermediate run. We describe conditions under which resource wealth is positively or negatively linked to economic growth over the intermediate run. The speed of economic growth may, however, be decoupled from natural resource wealth over the long run when the rate of economic growth is driven primarily by the economy's ability to create and disseminate knowledge. However, while natural resources do not affect the rate of economic growth over the long run, we show that long run resource wealth does affect the likelihood that an economy is able to sustain economic growth or alternatively fall into a long run stagnation trap. That is, under certain conditions resource wealth as well as over abundance of physical capital can be a curse for economic growth.

Fourth, with respect to the constancy of commodity prices, the model predicts stability of real commodity prices under certain conditions likely to have prevailed throughout most of the 20th Century. The model allows capital investment from the North to scour the globe searching for new commodity sources where the marginal product of capital is high (Caselli and Feyrer, 2007). Increasing demand for commodities from the North leads to the development of new commodity sources in the South, thus shifting the commodity supply curve to the right along with the shifting demand curve (Deaton and Laroque, 2003). Key components of this part of the model are the recognition of the vastness of unexploited commodity resources in the South, the relatively low demand for such resources in the South, and the almost complete lack of regulation in the South that allow resource extraction without paying for the environmental costs associated with such extraction. Inclusion of these mechanisms provides a framework to evaluate how recent changes in world growth patterns, in particular the expansion of the growth club, may affect the North in the future through potential new trends in commodity prices.

The earth or commodity sector considered in this paper is similar in nature to Brock and Taylor's (2004) treatment in the environment/growth literature. It includes production sectors that rely either on natural resources as sources of productive inputs (e.g., timber, agriculture, fisheries, hydroelectric power), as sinks of pollution (e.g., pipeline and others), or as both (e.g., coal mining, chemical processing). As defined, these sectors accounted for roughly 14% of 2005 GDP in the United States [5]. While we explicitly model the earth sector after renewable natural resources, non-renewable resource extraction also imposes heavy demands upon the renewable

[5] Resource or environment-dependent sectors may include agriculture, energy, mining, utilities, fisheries, wood and pulp, mineral, metals, and others (US Census: Statistical Abstract of the United States: 2008)

resource sector, and is thus considered part of such a sector. That is, we model non-renewable resource extraction through its effect on the surrounding renewable resource base[6]. This approach assumes that the limiting economic factor is the renewable resource more so than the nonrenewable under-ground reserves[7].

Our research strategy charts the following course. We first elucidate the intrinsic model mechanics using a "small open" economy paradigm (open to trade in final goods) which does not regulate the use of the natural resources (or has no property rights on them). Apart from allowing us to highlight the key qualitative nature of the model devoid of complications associated with endogenous commodity prices and resource regulation, we suggest that the small open economy approach gives the basic micro foundation of the theory much like the theory of the firm and household provide the micro foundations for static macroeconomic analyses. The use of a small open economy as the basic unit of analysis is infrequent in growth modeling because the overwhelming majority of the growth literature assumes a closed economy. The small open economy paradigm is simple enough to allow analytical tractability of the model, but robust enough to reproduce the conventional Kaldorian stylized facts (Kaldor, 1961) as well as some but not all of the stylized facts discussed above. We next show that the presence of property rights on the natural resource has little qualitative impact on the analysis. Finally, in the full model we explicitly incorporate endogenous world commodity prices with property rights in the North. This last modeling effort allows us to replicate the remaining stylized facts that cannot be analyzed in the context of a small open economy with immobile capital. However, we show that the inherent logic of the analysis of the small open economy remains intact in the case of a large economy.

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[6] For example, non-renewable resource extraction affects water quality (mining, oil extraction), soils and forests (mountain top removal for coal extraction), all of which are renewable resources and are thus included in our earth sector. For recent works considering nonrenewable resources in a growth model see for example, Bretschger (2008), Pittel and Bretschger (2008), and André and Smulders (2008).

[7] Several recent analyses suggest that with the exception of a few commodities, there are no signs of scarcity of non-renewable resources, but that scarcity mainly affects the renewable resources as a source of essential services (Simpson, Toman, and Ayres (2005). The US could, for example, dramatically increase its oil production by expanding off-shore or Alaskan production. The limiting factor is not the availability of under ground reserves but rather the steep environmental costs that such expansion would entail.

II. The Model

We consider three productive sectors: 1) a resource sector (referred to hereafter as the "commodity" sector) using natural resources and man-made inputs to produce a final good, 2) a second final good sector that does not use natural resources (referred to hereafter as the "service" sector), and 3) a knowledge sector that produces labor-augmenting human capital that benefits all three sectors including the commodity sector. There are three assets– human capital or knowledge, physical capital, and natural capital – and the economy invests in enhancing human and physical capital. Growth in human capital triggers labor-augmenting productivity growth that benefits all three sectors.

Consumption. The representative consumer has preferences defined over both the final service good (x_s) and the final commodity good (x_c) in the following indirect utility function:

$$U(c;p) = \frac{\varepsilon}{\varepsilon - 1}\left[\frac{c}{e(p,1)}\right]^{\frac{\varepsilon-1}{\varepsilon}} \tag{1}$$

where ε is the inverse of the elasticity of marginal utility and is fixed, $c = x_s + px_c$ is the level of total consumption expenditures in units of the service good, $e(p,1)$ is the consumer unit expenditure function, and p is the relative price of the commodity good relative to the service good[8]. The specification used in (1) imposes homothetic and strict concavity of consumer preferences (the latter condition due to the fact that $\varepsilon > 1$) but the fact that we do not impose any functional form on $e(p,1)$ means that the analysis is generally consistent with any type of homothetic strictly concave preferences, including Cobb-Douglas, CES, or other more general preference structures.

Production. Firms use raw labor (l_i) and physical capital (k_i) to produce each final good. Productivity growth is represented by human capital or knowledge (h). Production of the commodity good also requires a natural resource (n) as an additional factor of production. Firms produce the service and commodity goods according to standard neoclassical production functions:

[8] The levels of final good demands, x_s and x_c, can be determined through Roy's Identity from the indirect utility function *U(c;p)* once the optimal level of total real expenditures (*c*) is obtained from the solution to the ensuing dynamic optimization.

(2) $$y_s = Ak_s^{\alpha}\left(hl_s\right)^{1-\alpha}$$

(3) $$y_c = n^{\theta} Dk_c^{\beta}\left(hl_c\right)^{1-\beta}$$

where y_s and y_c are output levels of the service- and commodity-based goods, respectively, $0<\alpha<1$, $0<\beta<1$, $0\le\theta\le1$, and A and D are fixed parameters. We normalize human capital so that $h\ge1$. Increasing h is equivalent to increasing the effective size of the labor force.

The functional form for production of the commodity good adds human effort to the stock of natural resources to produce the final good. Human effort is represented by the composite of man-made assets, $Dk_c^{\beta}(hl_c)^{1-\beta}$, in equation (3). The parameter θ reflects the importance of the natural resource in production of the commodity good. This production function corresponds to the standard specification used for representing production of renewable natural resource-based commodities (Clark, 1990; Brander and Taylor, 1998).

For most of the analysis we will assume that $\beta>\alpha$ for developed countries where resource-based activities tend to be more physical capital intensive while the non-resource sector tends to be highly skill and labor intensive[9]. By contrast, developing countries may be characterized by $\alpha>\beta$, as the non-resource sector tends to consist of industrial production that is highly intensive in physical capital rather than knowledge. In developing countries, most of the population remains tied to a resource sector producing mainly primary commodities that use little capital but large amounts of labor[10].

Asset Accumulation and Market Equilibrium. The dynamics of the renewable natural resource stock are described by:

(4) $$\dot{n} = g\left(n\right) - \phi n^{\theta} Dk_c^{\beta}\left(hl_c\right)^{1-\beta}$$

[9] For example, in the USA for the period 2000-2006, the average employee compensation as a % of sector-specific GDP in the resource-based sectors is 27-38% (agriculture, forestry, fishing, and hunting), 22-35% (mining), and 22-25% (utilities); whereas, in non-resource sectors, labor accounts for 41-54% (information), 54-56% (finance and insurance), 69-74% (professional and business services), and 79-80% (education and health care services) of the respective sector-specific GDP level (US BEA: Gross Domestic Product by Industry Account, 1946-2007).

[10] Compare for example agriculture in the US which relies on a vast array of farm machinery and equipments with that in poor countries such as India depending mostly on human power.

The function $g(n)$ summarizes the physical laws governing intrinsic growth of the renewable natural resource[11]. The second term represents the reduction of the natural resource stock due to production of the commodity good. Producing one unit of the commodity good imposes demands upon the natural resource; the parameter $0<\phi<1$ represents the intensity of such demands. Thus, the parameters θ and ϕ represent the importance of the natural resource as a factor of production and the environmental impact due to commodity production, respectively. In fisheries or forest models (Gordon, 1954; Schaefer, 1957) the resource stock is assumed to be sufficiently scarce to influence the productivity of efforts and thus $\theta=1$ under most conditions[12]. In other sectors (mining, agriculture, energy) θ acquires intermediate values ($0<\theta<1$) showing a lesser dependence of production on the natural resource. Finally, $\theta=0$ implies that the resource is so abundant that output only depends on the level of effort, independent of the resource[13].

Human or knowledge capital growth is assumed to be subject to increasing returns as modeled by Lucas (1988) and others (Barro and Sala-i-Martin, 2004):

(5) $$\dot{h}=Bhl_r$$

where B is a fixed parameter. The stock of physical capital grows according to:

(6) $$\dot{k}=Ak_s^{\alpha}\left(hl_s\right)^{1-\alpha}+pn^{\theta}Dk_c^{\beta}\left(hl_c\right)^{1-\beta}-c$$

Equation (6) also represents the budget constraint of the economy, which in the context of an open economy reflects the equilibrium in the current account, i.e., the total value of domestic output (production of the service and commodity goods) is equal to the value of expenditures in consumption of the two goods plus investment. Discrepancies between production and consumption of each final good are filled by corresponding imports and exports.

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[11] When necessary for the analysis, we use a logistic specification for $g(n)=\gamma n[1-(n/\overline{n})]$ where $\gamma>0$ is a fixed parameter and $\overline{n}$ is the maximum carrying capacity of the natural system.

[12] For example, coal extraction often deforests large areas and even requires the complete destruction of the surrounding environment, but production of coal itself is unaffected by such environment, which implies a small value for θ and a large value for ϕ. Similarly, oil production often causes severe demands on renewable resources implying that $\theta=0$ and a large ϕ.

[13] For example, logging extraction in the Amazon is likely to depend only on logging effort, not on the forest stock due to the sheer immensity of the forest.

Labor and capital markets are perfectly competitive and thus full employment of labor and capital prevails. We assume that the total labor force L is fixed:

(7) $$l_c + l_s + l_r = L$$

(8) $$k_c + k_s = k$$

At this point we need to make certain assumptions about the parameter values:

Parameter Assumptions

A1. *The elasticity of marginal utility is fixed and less than one:* $1/\varepsilon < 1$.

A2. *Defining* $\rho > 0$ *as the pure time discount rate, the maximum rate of growth of human capital falls within the following range:* $\rho < BL < \rho/(1-1/\varepsilon)$.

With respect to assumption A1, Aghion and Howitt (1998) show that an elasticity of marginal utility greater than unity is implausible and implies odd behavior in the context of macroeconomic models. The first inequality in A2 implies that investment in human capital can be profitable. As we show below, this is a necessary condition for positive growth to be at all feasible. In fact, as will be clear below BL is not only the maximum rate of growth of human capital but is also equal to the rate of return to human capital in the long run. It will also be clear below that the second inequality effectively requires that the long run rate of return to human assets be not too large to crowd out investment in physical capital under any condition.

The Social Planner's Problem. Under the second welfare theorem, the central planner's conditions for maximizing social welfare are identical to the general equilibrium conditions arising from a decentralized model of perfectly competitive firms and households independently maximizing utility and profits, respectively. The social planner maximizes the discounted utility from consumption across all time:

(9) $$V \equiv \max_{c,k_c,k_s,l_c,l_s,l_r} \int_0^\infty U\left(c;p\right) e^{-\rho t} dt \,,$$

where t denotes time. Maximization of utility is subject to the constraints shown in equations (2) -(8) representing the production functions, asset growth equations and market clearing conditions for labor and capital. Consistent with the interpretation of (9) as that of a competitive market economy we assume that the planner takes p as given. In addition, the initial levels of human capital (h_0), natural capital (n_0), and physical capital (k_0), are assumed given and we have non-negativity constraints for consumption and production of the service and commodity sectors.

This problem can be solved by maximizing the current value Hamiltonian where λ, μ, and η are the co-state variables associated with physical, human, and natural capital, respectively.

(10) $H = U(c;p) + \lambda\left[Ak_s^{\alpha}(hl_s)^{1-\alpha} + pn^{\theta}Dk_c^{\beta}(hl_c)^{1-\beta} - c\right] + \mu Bhl_r + \eta\left[g(n) - \phi n^{\theta}Dk_c^{\beta}(hl_c)^{1-\beta}\right]$

We consider two polar cases regarding property rights on the natural resource: 1) complete open access which is equivalent to assuming that $\eta = 0$ and 2) perfect property rights on the resources which means that η is optimally chosen. We show below that the economy achieves a stationary level of the resource stock even if the resource is exploited under open access. For simplicity in presentation, we will assume initially that $\eta = 0$ but in Section VIII we show that key qualitative results do not change in the property rights scenario where $\eta > 0$ is endogenous. The resulting first order conditions for the maximization of the Hamiltonian under the assumption that $\eta = 0$ are derived below. The control variables are the allocation of labor across the three sectors, capital across the commodity and service sector, and consumption.

Interior vs. Corner solutions. We first assume interior solutions, e.g., that both final good sectors produce positive levels of output and that the knowledge sector is active ($l_r > 0$ and hence $\dot{h} > 0$). This allows us to present the first order conditions as equalities instead of their Kuhn-Tucker analogs. As discussed in the introduction, however, an interior solution with three productive sectors is not guaranteed and may not even be optimal to the social planner or to a competitive market solution. In Section V we will discuss the conditions required for diversification. We will be particularly concerned about the possibility that the non-negativity constraint be binding for l_r because permanent economic growth critically hinges on $\dot{h} > 0$ and hence on $l_r > 0$[14]. The other corner solution ($l_r = L$) can be ruled out as long as the stock of physical capital is greater than zero.

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[14] If $l_r = 0$ then the Kuhn-Tucker condition for (13) becomes $\tilde{w} > (\mu / \lambda)B$ and equation (16) becomes $\dot{\mu} = \mu\rho - \lambda\tilde{w}L$.

In addition to the equations of motion described in (4)-(6), the first order conditions assuming an interior solution are:

(11) $$U_c(c;p)=\lambda$$

(12) $$p(1-\beta)Dn^{\theta}\left(k_c/hl_c\right)^{\beta}=(1-\alpha)A\left(k_s/hl_s\right)^{\alpha}\equiv\tilde{w}$$

(13) $$p(1-\beta)n^{\theta}D\left(k_c/hl_c\right)^{\beta}=(\mu/\lambda)B$$

(14) $$\alpha A\left(k_s/hl_s\right)^{\alpha-1}=p\beta Dn^{\theta}\left(k_c/hl_c\right)^{\beta-1}\equiv r$$

The first condition states that the marginal value of consumption is equal to the shadow price of physical capital. The next two conditions represent the equalization of wages across the three sectors. And the fourth condition represents the equalization of the returns to capital between the service and commodity sector. We note that $\tilde{w}$ is the wage rate per unit of human capital (the wage per unit of effective labor time); then $w=\tilde{w}h$ (with $h\geq 1$) is the actual wage earnings per work time, both measured in terms of the service good. Also, r is the rental price of physical capital. The equations of motion for the co-state variables are:

(15) $$\dot{\lambda}=\lambda\left(\rho-\alpha A\left(k_s/hl_s\right)^{\alpha-1}\right)$$

(16) $$\dot{\mu}=\mu(\rho-BL)$$

Finally, the transversality conditions are:

(17) $$\lim_{t\to\infty}e^{-\rho t}\mu h=0,\qquad \lim_{t\to\infty}e^{-\rho t}\lambda k=0$$

Endogenous versus exogenous commodity prices. We first consider the "small" open economy case without property rights on the natural resource; that is, the case where the economy is open to trade in goods (but not in factors of production) so that the commodity price (*p*) is fixed and not affected by the production and demand conditions of the economy. In Section IX we present the full model which considers endogenous commodity prices and property rights. Whether or not p is endogenous, the nature of the first order conditions of the optimization problem above are not affected as long as the planner, like a competitive market economy, abstains from exercising market power. In both cases, the first order conditions are defined conditional on a particular level of p. The only difference is that if the economy is large enough to affect prices the evolution over time of the solution will trigger a price dynamic that will, in turn, feed back into the solution over time. By contrast, if the economy is small no price feedbacks exist.

III. Basic Equilibrium Conditions

If the economy produces both final outputs we can combine equations (12) and (14) yielding:

Lemma 1: *The ratios of physical capital to human capital-augmented labor in the service and commodity sectors are proportional to each other:*

$$\frac{k_c}{hl_c} = \frac{\beta(1-\alpha)}{\alpha(1-\beta)}\frac{k_s}{hl_s} \equiv \Psi\frac{k_s}{hl_s} \tag{18}$$

Proof: *By inspection of equations (12) and (14).* $\otimes$

The capital to labor ratio in the commodity sector is always a constant multiple of that in the service sector. If $\beta > \alpha$ $(\alpha > \beta)$ we have that $\Psi > 1$ $(\Psi < 1)$ which means that the required physical capital to labor ratio in the commodity sector is greater (less) than that in the service sector at all times, regardless of the level of prices and other parameters. Using equation (18) in (14) allows us to solve for the equilibrium physical capital to human capital ratio which is exclusively a function of the natural resource stock:

$$k_c/hl_c = \left(\mathrm{X}\Psi^{\alpha}\right)^{1/\alpha-\beta}\left(pn(t)^{\theta}\right)^{1/\alpha-\beta} \tag{19}$$

where $\mathrm{X} \equiv \left(D(1-\beta)\right)/\left(A(1-\alpha)\right)$. Henceforth we define $Z_c(pn^{\theta}) \equiv k_c/hl_c$ and $Z_s(pn^{\theta}) \equiv k_s/hl_s$ where Z_s is solved using (19) in (18). Now we have the following Lemma:

Lemma 2. *(i) The physical capital to human capital-augmented labor ratios in both final goods sectors are decreasing (increasing) in the stock of natural resources (* n *) if the commodity sector is more (less) physical capital intensive than the service sector, that is if* $\beta > \alpha$ $(\alpha > \beta)$*. (ii) If* $\beta > \alpha$ $(\alpha > \beta)$ *then the wage rate per unit of human capital (* $\tilde{w}$ *) is falling (rising) and the rental price of physical capital (r) is rising (falling) in* n *.*

Proof: *By inspection of Equation (12), (14), and (19).* $\otimes$

Intuitively, increasing the natural resource stock expands the commodity sector and thus increases demand for physical capital and labor. If the commodity sector is physical capital intensive ($\beta > \alpha$), this expansion creates a greater demand for physical capital than for human capital which leads to incipient excess demand for physical capital and excess supply of labor. This, in turn, causes the price of physical capital to increase and the wage rate to fall. This factor price readjustment induces both final good sectors to reduce their capital-labor ratios, which is what Lemma 2 predicts. If $\beta > \alpha$ the lower wage rate induced by an increase in the natural resource stock results in an expansion of the knowledge sector, which is the most labor intensive sector in the economy. Knowledge and natural resources are complements in this case. By contrast, if $\alpha > \beta$ natural resources and knowledge are substitutes.

IV. The Nature of Convergence

Unlike most growth models in the literature (e.g., Acemoglu and Guerrieri, 2008), the present model does not allow one to distinguish between transitional dynamics and steady state. The system is in perpetual "transition" as it never reaches balanced growth often defined as the state where asset ratios and asset to consumption ratios stabilize. The dynamics of the system here, however, can still be separated into two stages: (i) **Stage 1** (the "intermediate run") in which consumption grows at varying rates over time; and (ii) **Stage 2** (the "long run") when the consumption growth rate becomes constant. In Stage 1 the natural resource is endogenously changing over time until it becomes constant in Stage 2. The constancy of the natural resource causes the consumption rate of growth to become constant as well. Below, we identify conditions under which Stage 2 with positive consumption growth is both feasible and sustainable.

Even in the long run (Stage 2) a growing economy is in perpetual evolution by adjusting asset ratios, consumption-to-asset ratios, as well as output composition. Despite the continual change of the k/h and c/k ratios, they are subject under certain initial asset endowment conditions to well-defined boundaries to which they may approach but never actually reach. We call these boundaries *infinite convergence points* (ICP). These boundaries are fundamentally different from the usual *asymptotic convergence points* (ACP) used in standard multi-asset endogenous growth models. More formally, we define these boundaries as follows:

Definition. *Consider two positive variables,* $N(t)$ *and* $M(t)$, *then we define the ICP as* $(N/M)^{\infty} \equiv \lim_{\substack{N\to\infty \\ M\to\infty}} [N(t)/M(t)]$, *and the ACP as the more traditional* $(N/M)^{*} \equiv \lim_{t\to\infty} [N(t)/M(t)]$.

Remark on Surrogate ICP*. A real variable* $x(t) \equiv f(N(t)/M(t))$ *is said to have a Surrogate ICP when the* N/M *ratio is at its ICP; that is,* $x^{\infty} = f[(N/M)^{\infty}]$.

One way of illustrating the difference between ICP and ACP is to perform the following experiment: assume that by chance the above ratios are initially at ICP. We show below that the system will necessarily move away from such condition even if no exogenous disturbance occurs. By contrast, in the standard growth model where the long run is characterized by ACP, an initial condition which by chance coincides with ACP will be permanent unless the system is perturbed. In fact, existence of ICP as natural boundaries provides the foundations for bifurcation and state dependence of the system. As we show below, diversified growth equilibrium is possible only if the initial asset ratios are on the "correct" side of their respective ICPs. If the ratios are at or on the "wrong" side of ICP, the economy converges to an ACP which is characterized by economic stagnation.

The two stages into which we have classified the dynamics of the system can be defined by their approach toward ACP (Stage 1) or ICP (Stage 2). Below we provide a description of each of these phenomena.

Stage 1: ACP convergence

There are two possible ACP equilibrium depending on whether an interior solution ($l_r > 0$) or a corner solution ($l_r = 0$) applies. We describe both cases below in sequence.

Natural resource: ACP convergence. Given the dependence of the physical capital-labor ratio on the resource stock, we now characterize the evolution of the natural resource based on any initial condition (n_0). With $l_r > 0$, we can use equation (13). Logarithmically differentiating it with respect to time and using (15), (16), and (19) we obtain the *equilibrium-required* rate of change of natural capital or "demand" side of natural capital (the "hat" denotes growth rate) which is the rate along the transition path necessary to satisfy the market wage equilibrium condition represented by equation (13).

(20) $$\hat{n}(t) = (\beta - \alpha)/(\theta\alpha)[BL - r(n)]$$

Since $r(n)$ is increasing (decreasing) in n for $\beta > \alpha$ ($\alpha > \beta$), we have that the rate of growth of natural capital is declining in $n(t)$ for both $\beta > \alpha$ and $\alpha > \beta$. This means that the natural resource converges to a stationary value (n^*) at which point $r(n^*) = BL$ regardless of the capital intensities of the final goods sectors and the initial level of n_0. Similarly, the wage rate per unit of human capital converges to a stationary value. This equilibrium is clearly of the ACP type.

The fact that the knowledge sector is competitive means that the marginal product of labor in the human capital sector is equal to the marginal product of labor elsewhere in the economy. That is, the relative (shadow) price of knowledge is set ($\tilde{w} = \mu B/\lambda$) so that equation (13) holds at all times. In a growing economy λ and μ are both falling, albeit at different rates. Therefore the wage rate has to adjust concomitantly to allow the equilibrium (13) to continuously hold and this adjustment must correspond to changes in the natural resource stock.

Finally, the adjustment of $n(t)$ fully determines the optimal size of the commodity sector and its corresponding allocation of labor. By equalizing the "demand" side (equation (20)) of the natural resource stock change with its "supply" side (equation (4)) a unique value for the commodity labor requirement can be determined at all points in time:

$$hl_c = \frac{1}{\phi D n^{\theta}[Z_c(pn^{\theta})]^{\beta}}\left[g(n) + \left(\frac{\beta-\alpha}{\theta\alpha}\right)n\left(r(n) - BL\right)\right] \tag{21}$$

Where Z_c is defined in (19). From (21) it is clear that when $n = n^* > 0$ (and thus $r = BL$) then $hl_c > 0$. Using (21) in (2) we determine the equilibrium production of the commodity output:

$$y_c = [1/\phi]\left[g(n) + \left(\frac{\beta-\alpha}{\theta\alpha}\right)n\left(r(n) - BL\right)\right] \tag{22}$$

Consumption Growth Rate: ACP Convergence. The dynamics of $n(t)$ also determines the rate of growth of the economy. Differentiating (11) with respect to time and using equations (15), (18), and (19) produces:

$$\hat{c}(t) = \varepsilon\left[r(n) - \rho\right] \tag{23}$$

Thus, an economy that is depleting its natural capital will experience a declining rate of growth over time if $\beta > \alpha$. As $n(t)$ converges toward its asymptotic stationary value, the rate of growth of the economy also converges to a constant value.

$$\hat{c}^*(t) = \varepsilon\left[r(n^*) - \rho\right] = \varepsilon\left[BL - \rho\right], \tag{24}$$

where the second equality in (24) follows from (20) when $\hat{n}(t) = 0$. Assumption A.2 guarantees that long run consumption growth in this diversified economy is positive (i.e., $\hat{c}^* > 0$). This convergence is also ACP. We formally define the ACP equilibrium in Proposition 1.

Proposition 1 [On ACP Characterization with Interior Solution]. *Assume* $l_r > 0$, *then*: *(i) The ACP equilibrium is described by the following constants:*

1) $n^* = \left(BL / A\alpha\right)^{(\beta-\alpha)/(\theta(1-\alpha))}\left(1/pX\Psi^{\beta}\right)^{1/\theta}$; $\hat{c}^* = \varepsilon[BL-\rho]$

2) $Z_c^* = \Psi\left(\dfrac{\alpha A}{BL}\right)^{\frac{1}{1-\alpha}}$; $Z_s^* = \left(\dfrac{\alpha A}{BL}\right)^{\frac{1}{1-\alpha}}$

3) $y_c^* = g(n^*)/\phi$; $(hl_c)^* = \dfrac{g(n^*)/n^{*\theta}}{\phi D Z_c^{*\beta}}$

4) $\tilde{w}^* = A^{\frac{1}{1-\alpha}}(1-\alpha)\left(\dfrac{\alpha}{BL}\right)^{\frac{\alpha}{1-\alpha}}$; $r^* = BL$

Achievement of these limits is contingent on a natural resource stock carrying capacity that is sufficiently large. (ii) The following additional conditions hold in the long run: $\hat{k}_c = \widehat{(hl_c)} = 0$; $\hat{k}_s = \hat{h} + \hat{l}_s$; $\widehat{(\tilde{w}h)} = \hat{h}$.

Proof: *See Appendix.* ⊗

We assume throughout the remaining analysis that $0 < n^* \le \bar{n}$. Thus, in ACP the economy achieves a constant rate of economic growth and a constant rental price of capital which under certain conditions can be supported over the long run. These results are consistent with stylized facts originally identified by Kaldor (1961). While $\tilde{w}$ reaches a stationary ACP value as well, the wage earnings rate ($\tilde{w}h$) will continue to grow at the rate of human capital growth.

Labor Allocation across sectors. Using the factor market clearing conditions (7) and (8) we can solve for actual levels of effort used in each of the three sectors in both Stages[15]:

(25) (i) $hl_s = \dfrac{1}{\Psi-1}\left[\Psi h\left(L-l_r\right)-\dfrac{k}{Z_s}\right]$; (ii) $hl_r = hL - \dfrac{k}{Z_s} + \left(\Psi-1\right)hl_c$.

In addition, hl_c is given by (21). From equation (25)(ii) it follows that an interior solution for l_r ($0 < l_r < L$) requires that

(26) $$Z_s[L + (\Psi-1)(hl_c)/h] > k/h > Z_s(\Psi-1)(hl_c)/h$$

[15] In deriving these conditions we have used the factor market clearing conditions $Z_c hl_c + Z_s hl_s = k$ and $hl_c + hl_s + hl_r = hL$ in conjunction with Lemma 2.

We note that in a diversified economy, effort in the commodity sector (hl_c) as defined in (21) is dependent only on n and hence, once $n = n^*$ then $hl_c = (hl_c)^*$ as defined in Proposition 1. From (26) it follows that diversification requires that the stock of capital be greater than a minimum level $k_{\min} \equiv Z_s^*(\Psi - 1)(hl_c)^*$ (constant in the long run once hl_c is at or in the neighborhood of ACP), and below a maximum level, $k_{\max} \equiv Z_s^* Lh + k_{\min}$. We note that $k_{\min} > 0$ if $\Psi > 1$ which happens if $\beta > \alpha$, but $k_{\min} < 0$ if $\Psi < 1$ which occurs when $\alpha > \beta$.

Stage 2: ICP Dynamics: Corner vs. Interior Solutions

Whether or not an economy can sustain an interior solution with $L > l_r > 0$ at all times depends on the dynamics of the k/h asset ratios and the bifurcation boundaries. We define $k' \equiv k - k_{\min}$ and, as shown in Proposition 2 below, we define the following ICP values for the respective ratios, $(c/k)^\infty = BL/\alpha - \varepsilon(BL - \rho)$ and $(k/h)^\infty = (Z_s^*/B)[BL - \varepsilon(BL - \rho)]$. Assumption A2 guarantees that both ICP values are positive. We can then use the two equations in (25) together with (5), (6), (21) and (23) to derive:

$$\widehat{c/k'} = \left[c/k' - (c/k)^\infty\right] - r(n)k_{\min}/k' - (\varepsilon - 1/\alpha)(BL - r(n)) \tag{27}$$

$$\begin{aligned} \widehat{k'/h} = B/Z_s \left[k'/h - (k/h)^\infty\right] - \widehat{c/k'} \\ - \varepsilon(BL - r(n)) + \left(1 - Z_s^*/Z_s\right)\left(BL(\varepsilon - 1) - \varepsilon\rho\right) \end{aligned} \tag{28}$$

In addition we have:

$$\widehat{c/k} = \left[c/k - (c/k)^\infty\right] + \left((1-\alpha)/\alpha\right) r(n)k_{\min}/k - (\varepsilon - 1/\alpha)(BL - r(n)) \tag{29}$$

$$\begin{aligned} \widehat{k/h} = B/Z_s \left[k/h - (k/h)^\infty\right] - \widehat{c/k} - (B/Z_s)k_{\min}/h \\ - \varepsilon(BL - r(n)) + \left(1 - Z_s^*/Z_s\right)\left(\varepsilon(BL - \rho) - BL\right) \end{aligned} \tag{30}$$

From Proposition 1, when $n \to n^*$, $Z_s \to Z_s^*$, $hl_c \to (hl_c)^*$ and $r(n) \to r(n^*) = BL$; similarly, $k_{\min}$ is fixed when $n \to n^*$. Thus, in ACP the last right-hand-side term in (27) and (29) and the last two right-hand-side terms in (28) and (30) all vanish. Proposition 2 provides a formal proof of the derivation of equations (27)-(30), of the ICP for the k/h and c/k ratios, and derives key implications.

Proposition 2 [On ICP Characterization]. A*ssume the following initial conditions:* $n = n^*$ *and* $k = k_o$*;* $h = h_o$*, where* k_o *and* h_o *are arbitrarily positive values. Define* $k_{\min} \equiv Z_s^*(\Psi - 1)(hl_c)^*$*, then full and permanent diversification is possible (i.e., all sectors' outputs including the human capital sector's are positive at all future times) if and only if the initial physical to human capital ratio satisfies the following conditions: (i) If* $\beta > \alpha$*:* $(k/h)^\infty < k_o/h_o < (k/h)^\infty + k_{\min}/h_o$*; (ii) If* $\alpha > \beta$*:* $(k/h)^\infty + k_{\min}/h_o < k_o/h_o < (k/h)^\infty$*. The ratios* k/h *and* c/k *converge toward but never reach non-negative and fixed constants* $(k/h)^\infty$ *and* $(c/k)^\infty$*, respectively as long as* k *and* h *are finite.*

Proof: *See Appendix* .⊗

Proposition 2 provides the conditions under which the economy can remain fully diversified indefinitely, i.e., $l_r > 0$ with positive production of both final goods. This Proposition combined with equation (26) also define the boundaries for the two corner solutions: 1) if $k \geq k_{\max}$, then there is no labor allocated to the production of human capital ($l_r = 0$); and 2) if $k \leq k_{\min}$, all of the labor would be allocated to the human capital sector ($l_r = L$) and the two final good sectors cease to be in operation. Note, however, that the condition for $l_r < L$ is extraordinary weak because if $l_r = L$ then $l_c = l_s = 0$, which imply that $k_{\min} = 0$. That is, as long as $k > 0$ we rule out this corner solution. Hence we only focus on the possibility that the lower bound constraint for l_r is binding.

If the conditions for $l_r > 0$ are not met then the solution changes dramatically. The economy invests only in physical capital (k) which means that the support for $r(n)$ is no longer set at BL. In fact, the economy continues investing in k causing n to change to a new ACP defined by $r(\tilde{n}) = \rho$, and $\tilde{n} = \left(\rho / A\alpha\right)^{(\beta-\alpha)/(\theta(1-\alpha))}\left(1/ p\mathrm{X}\Psi^{\beta}\right)^{1/\theta}$. From Lemma 2 it follows that $\tilde{n} < n^*$ ($\tilde{n} > n^*$) if $\beta > \alpha$ ($\alpha > \beta$). From (23) it is evident that at this point consumption growth becomes zero and the economy stagnates. Proposition 2 describes diversification boundaries relevant to the state variables (k, h, n) that will determine whether or not l_r can be positive and remain positive. We make the following additional remarks about the results in Proposition 2:

Remarks on Proposition 2. *(a)* ***Stable Diversification.*** *If $\beta > \alpha$ then $k_{\min} > 0$. Thus, if the initial k/h ratio is within the permanent diversification boundaries, it continuously falls but never reaches its lower bound, $(k/h)^{\infty}$, for finite k and h levels. When $\alpha > \beta$ then $k_{\min} < 0$. Thus, if the initial k / h ratio is within the permanent diversification boundaries, it continuously rises but never reaches its upper bound, $(k/h)^{\infty}$, for finite k and h levels.*

(b) ***Unstable Diversification.*** *If the initial k/h ratio is outside the diversification boundaries the economy cannot sustain diversification over time. If the k/h ratio is above its respective upper diversification boundary but below $k_{\max}$, the economy is able to remain diversified and grow for a period of time. However, physical capital in the economy inexorably moves toward $k_{\max}$, where the human capital sector is eventually crowded out, unable to compete for labor with the commodity sector. We refer to this situation as the "capital curse."*

(c) ***Physical Capital Intensive Corner Solution.*** *If the stock of physical capital k is initially above $k_{\max}$ or moves above it, the economy eventually stagnates as there is no longer investment in human capital ($l_r = \dot{h} = 0$).The economy specializes in the final goods, is unable to sustain the natural resource stock at n^* which eventually approaches a new equilibrium, $\tilde{n}$.*

The conditions stated by Proposition 2 concern the initial asset endowments k_o and h_o that reflect past historical conditions which cannot be endogenously changed in the short run. As stated in the Remarks to Proposition 2, whether or not these asset endowments satisfied the respective conditions has a dramatic effect on the long run growth potential of the economy.

Employment in Human Capital Sector: Surrogate ICP. Finally we consider the dynamics of labor in the human capital sector assuming that $n = n^*$ (i.e., the economy is in Stage 2). Using (5) and (23) we get that:

(31) $$\widehat{c/h} = \varepsilon\left[BL - \rho\right] - Bl_r$$

Using $c/h \equiv (c/k')(k'/h)$ it follows that $\hat{c/h} = \hat{k'/h} + \hat{c/k'}$. Hence, from Proposition 2 we have that $\hat{c/h} \to 0$ at ICP and we can solve (31) to obtain an expression for the surrogate ICP for the level of employment in the human capital sector:

(32) $$l_r^{\infty} = \varepsilon\left(BL - \rho\right)/B$$

which is obviously positive in a growing economy (i.e., if $BL > \rho$ by Assumption A1). The surrogate nature of the ICP for l_r is apparent by the fact that $l_r^{\infty} = L - (1/Z_s^*)(k/h)^{\infty}$. Combining (27) and (28) in (31) yields:

(33) $$\left[k'/h - \left(k/h\right)^{\infty}\right] = Z_s\left(l_r^{\infty} - l_r\right)$$

From Proposition 2 we have that if $\beta > \alpha$ ($\alpha > \beta$) the left-hand-side of equation (33) must be negative (positive) for permanent economic diversification and thus the level of employment in the human capital sector must be above (below) its respective ICP, i.e., $l_r^{\infty} < l_r$ ($l_r^{\infty} > l_r$). In addition from (33) it follows that if the conditions of Proposition 2 hold, l_r continuously falls (increases) over time becoming closer and closer to its lower (upper) boundary, l_r^{∞}, without ever reaching its boundary.

Under these diversified interior solution conditions, we can also characterize labor in the other two sectors. Given the constancy of $(hl_c)^*$ and positive allocation of labor to the human capital sector in a diversified economy, long run employment in the commodity sector (l_c) must continuously fall in a growing economy. The fact that l_r is also falling (rising) means that the service sector employment (l_s) must continuously expand (contract) over time towards its implicit surrogate ICP level when $\beta > \alpha$ ($\alpha > \beta$). This surrogate ICP is $l_s^{\infty} = L - l_r^{\infty}$ (note that the surrogate ICP for employment level in the commodity sector is zero; that is, $l_c^{\infty} = 0$).

V. ICP Dynamics and Bifurcation

In this section we discuss the Stage 2 conditions under which either a fully diversified interior solution or a specialized corner solution occurs. Figure 1 provides a graphical explanation of the possible Stage 2 asset endowments and the consequences for diversification and long run growth patterns of the economy in the $k-h$ space (for the case when $\beta > \alpha$). Figures 2A and 2B provide a different perspective in the $(k/h)-h$ space which permits a clear comparison of the case when $\beta > \alpha$ and $\alpha > \beta$.

Region (I) in Figure 1 ("Specialization and Stagnation") is defined by the $K\bar{K}$ boundary equivalent to the Physical Capital Intensive Corner Solution presented in the Remark to Proposition 2 ($hl_r = 0$). From (26) it follows that the $K\bar{K}$ line is defined in the long run by $k'/h = LZ_s^*$ or, equivalently by $k = k_{\max}$. Thus, if the initial endowment of human capital (h_0) is too low vis-à-vis the endowment of physical capital (k_0), the human capital sector may not operate causing human capital accumulation to be infeasible. Given that k/h must be increasing when the human capital sector is stagnant, it is clear that an economy that enters Region (I) will remain in Region (I). Intuitively, in this region human capital production is crowded out because it cannot compete with the final good sectors. The opportunity cost of labor in production of the final goods is too high relative to the human capital sector; equation (13) becomes an inequality ($\tilde{w} > (\mu/\lambda)B$) and the rate of return to human capital (equation (16)) ceases to have the level BL as support. Furthermore, the interior solution ACP for the natural resource level (n^*) cannot be sustained in Region (I) because the economy is growing by investing in physical capital only and instead converges to the corner solution ACP where $r(\tilde{n}) = \rho$, $\hat{c} = 0$ and the economy stagnates (when the stock of capital reaches a level $\tilde{k}$ in Figure 1). Thus, an economy that is too rich in physical capital and/or too poor in human capital may grow over the intermediate run on the basis of accumulating physical capital only but it will not be able to avoid a stagnation trap over the long run.

Region (II) in Figure 1 ("Temporary Diversification and Growth") allows for unstable diversification. This region is defined from below by the KK' line and above by the $K\bar{K}$ and represents an economy where the level of employment in the human capital sector is positive but below its ICP (i.e., $l_r < l_r^{\infty}$) for the case where $\beta > \alpha$. The KK' line is given by $k = (k/h)^{\infty} h + k_{\min}$. So in Region (II) we have that $k'/h \geq (k/h)^{\infty}$. Initially, the economy is fully diversified; the factor endowments allow all three sectors to be in operation. However, by Proposition 2, the k'/h ratio in this region is too high because it is above or at its ICP level

$(k/h)^{\infty}$. From equation (28) it is clear that k'/h is constantly increasing and continuously moving away from its ICP in this case. Thus, the $k'/h-(k/h)^{\infty}$ difference is permanently increasing which by (33) means that $l_r^{\infty}-l_r$ must also be constantly increasing until $l_r=0$. That is, positive and constant growth (Stage 2) is initially feasible but it cannot be sustained indefinitely. After enjoying a period of constant economic growth and natural resource sustainability, the economy eventually enters into Region (I) at which point the rate of economic growth gradually declines towards zero and the natural resource stock falls towards a new lower ACP equilibrium ($\tilde{n}$).

Thus, if at any point in time the economy finds itself in Regions I or II it will inexorably follow a path towards stagnation. Since eventual stagnation is due to a relative over abundance of physical capital we call this situation a (physical) capital curse.

By contrast, Region (III) ("Sustained Diversification and Permanent Growth") is defined by $k'/h<(k/h)^{\infty}$ which by (33) implies that $l_r>l_r^{\infty}$. This region is defined by Proposition 2 and (33) such that l_r must constantly approach but never actually reach its surrogate ICP. In this case the economy initially allocates a large level of employment to the human capital sector relative to its surrogate ICP level. As a result, in Region (III) a constant and positive rate of growth of consumption can be preserved *ad-infinitum*. From (27) and (28) one can see that for finite values of k and h the ICP boundaries are never reached, which means that positive economic growth is feasible and can be permanently sustained and that the natural resource stock can remain constant at its ACP. Or equivalently if an economy is in Region (III) it will never autonomously leave Region (III)[16].

Figures 2a and 2b allow us to compare the various growth and stagnation regions for the two relative capital-labor intensity cases: 1) the commodity sector is more capital intensive than the service sector ($\beta>\alpha$); and 2) the commodity sector is less capital intensive than the service sector ($\alpha>\beta$). The Figure clearly illustrates the growth boundaries described in Proposition 2. For sustained and permanent growth to exist, it is necessary that k/h be always above

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[16] In addition Figure 1 shows another region, Region IV which is characterized by a permanent reduction of the k/h ratio towards zero, which eventually would cause the human capital sector to use all the labor in the economy. This is a trivial and unlikely case which does not deserve further attention.

$(k/h)^{\infty}$ and decreasing if $\beta > \alpha$ (Figure 2a) and k/h must always be below $(k/h)^{\infty}$ and growing in the case when $\alpha > \beta$ (Figure 2b). This is also apparent from examination of equations (27)-(30). The other feature to note in Figure 2 is the potential for growth if there exists a risk that a country might have a high k/h ratio, either because of large natural resource wealth or an underdeveloped human capital sector. Comparing Figures 2a and 2b suggests that the risk of stagnation as a consequence of a high k/h ratio is greater when $\alpha > \beta$ than when $\beta > \alpha$. As can be seen in Figure 2, the maximum level of the k/h ratio consistent with permanent diversification and positive economic growth is higher, ceteris paribus, when $\beta > \alpha$ than when $\alpha > \beta$.

This finding is significant especially if our supposition that the $\beta > \alpha$ economy more closely reflects conditions in rich, developed countries while the $\alpha > \beta$ economy reflects better the conditions of a poor, developing economy. This would imply that poor countries have a higher risk of stagnating than richer countries. This conclusion is reinforced by the observation that poor countries appear to have historically encountered more difficulty expanding their human capital relative to their physical capital compared to rich countries. That is, in Figures 2 the initial point A for a poor country is likely to be higher and to the left of point A for a rich economy.

In summary, the long run rate of economic growth is *state dependent* and potentially affected by a drastic *bifurcation* process that is the difference between achieving a permanent and positive rate of economic growth and stagnation. Such bifurcation depends on the initial asset endowments for the economy. Ceteris paribus, the more physical capital rich and human capital poor an economy is, the more likely it is to not be able to sustain a permanent positive rate of economic growth.

Importantly, we do not make any normative statements about Region (I)-(IV). These results derive from the very conditions needed for welfare maximization. This analysis demonstrates that, depending on initial historical conditions, maximization of social welfare may not be consistent with permanent economic growth. The importance of these findings relies on the overwhelming focus of economists and the public in general on economic growth as a key indicator of economic success. Similarly, interventions to promote even faster growth in an economy that is in Region III – through for example subsidies to physical capital accumulation – may be counterproductive for both welfare and long term growth. After a few years of faster physical capital growth triggered by these interventions the economy's asset composition may shift from Region III to Region II. Thus policy makers are effectively trading faster growth over the intermediate run for long run economic stagnation.

VI. Natural Resource Wealth and Economic Growth

Intermediate Run Relationship. We denote the time that n takes to achieve ACP equilibrium the "intermediate run". Figure 3 shows the adjustment over the intermediate run under the assumption that the conditions for diversification in Proposition 2 are met. The top panel shows the dynamic path of the commodity sector as a function of the level of natural resources during the convergence towards its ACP equilibrium. At the ACP for natural resources the rate of growth of consumption also becomes constant (Figure 3, lower panel). From then on the economy will continuously grow by investing in physical and human capital over the long run.

The lower panel of Figure 3 shows that the rate of economic growth during ACP convergence is positively related to resource wealth if $\beta > \alpha$ and inversely related to resource wealth if $\alpha > \beta$. This relationship between resource wealth and economic growth conforms to the third stylized fact referenced in the Introduction on the resource curse. To examine the specific nature of the resource curse we first define a *Resource Poor* (*Resource Rich*) economy as one where the resource endowment is currently less (greater) than the ACP level of the resource (n^*). Using this definition, we summarize the relationship between economic growth and natural capital in Proposition 3.

Proposition 3 [On the Resource Curse]. *(i) If $\beta > \alpha$ ($\alpha > \beta$), then the economy's rate of growth of consumption over the intermediate run will be increasing (decreasing) in the natural resource stock. (ii) An economy that is initially resource abundant with $\beta > \alpha$ ($\alpha > \beta$), will reduce its resource level along the adjustment path and thus decrease (increase) the economy's rate of growth. Conversely, an economy that is initially resource poor with $\beta > \alpha$ ($\alpha > \beta$), will increase its resource level along the adjustment path and thus increase (decrease) the economy's rate of growth over the intermediate run.*

Proof: *See Appendix.* ⊗

The Proposition predicts that the relative capital-labor intensities between the service and commodity sectors are the key determinants of the connection between resource endowment and economic growth. As can be seen in the lower panel of Figure 3, when $\beta > \alpha$ ($\alpha > \beta$) there is a positive (negative) correlation between the rate of economic growth of consumption and the stock of natural resources. Thus, continuing with our definition of $\beta > \alpha$ corresponding to a developed country and $\alpha > \beta$ corresponding to a developing country, the relationship between growth and resource wealth appears as a resource blessing in developed countries and as a resource curse in developing countries[17]. This insight directly responds to the second tenet of the resource curse literature establishing a simple condition under which resource abundance is associated with faster or slower economic growth.

Intuitively, if the commodity sector is more physical capital intensive than the services sector, natural capital and human capital are complements. That is, the natural resource wealth depresses the wage rate which drives labor into the knowledge producing-sector. This increased investment in knowledge fuels growth. Alternatively, if the commodity sector is more labor intensive than the services sector, a large stock of natural capital requires a large amount of labor to work in the commodity sector, decreasing investment in the knowledge sector which reduces growth.

Long Run Relationship. In Proposition 1 we show that the potential rate of economic growth over the long run is independent of the level of resource wealth. Proposition 2 and its corresponding Remarks show that achieving a positive long run growth rate is not guaranteed, depending on the initial asset endowments. This forces the question of how the level of resource wealth affects the likelihood of these asset ratios being at a level consistent with permanent growth. That is, are natural resource wealthier countries more or less likely to sustain a positive rate of economic growth over the long run? The level of natural resources in the long run is

[17] Between 1985 and 2006, the real per capita GDP for resource poor, developing countries (Southeast Asia) grew at an annual average rate of 3.9%; whereas, during that same period real per capita GDP for resource rich, developing countries (Southern and Western Africa and South America) grew at an annual average rate of 0.9%. During the same period, the real per capita GDP of resource rich developed countries (Australia, US, Canada) grew at an annual average rate of 2.0%; whereas the real per capita GDP for resource poor, developed countries (mainland Western Europe and Japan) grew at an annual average rate of 1.8% (UN Statistics Division: National Accounts Main Aggregates Database).

endogenous so we have to consider exogenous forces that trigger changes in natural resource wealth. We first consider changes in the commodity price (p) which leads to resource wealth changes over the long run.

A higher p as we see from Proposition 1 implies a lower level of n^* while Z_c^* and Z_s^* are not affected. In addition, the ACP level of human capital-augmented labor used by the commodity sector ($(hl_c)^*$) increases as n^* falls in the case when the function $g(n)$ is logistic as specified in Footnote 11 and when the parameter θ is not too low. This, in turn means that, given the constancy of Z_s^*, $k_{\min}$ must rise. In addition we note that $(k/h)^\infty$ is also independent of the commodity price.

Hence, in Figure 2a, a higher $k_{\min}$ is the equivalent to shifting the $(k/h)^\infty + k_{\min}/h$ schedule to the right, thus making sustained diversification with growth more likely. The intuition for this stems from the complementary nature of human capital and natural resource wealth; the higher the resource wealth, the more labor the economy shifts to human capital production. Existence in Region (III) for an economy where $\beta > \alpha$ requires that $l_r > l_r^\infty$. In Figure 2b a larger commodity sector equates to a lower $k_{\min}$ (as $k_{\min} < 0$ when $\alpha > \beta$) and is the equivalent of shifting the $(k/h)^\infty + k_{\min}/h$ schedule to the right making the area for sustained diversification larger and thus increasingly the likelihood of sustained diversification as well. The intuition for this stems from the fact that commodity sector is the labor intensive sector when $\alpha > \beta$; a higher resource wealth will require a shifting of labor from the human capital sector to the commodity sector. And existence in Region (III) for an economy where $\alpha > \beta$ requires that $l_r < l_r^\infty$. The implications of this are that in both cases a higher commodity price – while causing a lower long run level of the natural resource – increases the ability of the economies to sustain positive economic growth over the long run.

Note that the commodity price affects the potential for sustained economic growth exclusively through its effect on n^*. In particular, the fact that Z_s^* and Ψ remain constant is crucial to the previous analysis because it implies that all boundaries in Figure 2 with the exception of the $(k/h)^\infty + k_{\min}/h$ schedule remain unchanged. Thus, we conclude that a lower (higher) level of long run resource wealth caused by changes in exogenous conditions that do not affect Z_s^* or Ψ will cause an increase (decrease) in the likelihood that a positive and constant rate of economic growth can be sustained over the long run. Another exogenous change that does not affect Z_s^* nor Ψ is the advent of natural resource property rights, an issue that we consider in Section VIII.

The significance of this analysis is that it shows that while the potential rate of economic growth over the long run is independent of natural resource wealth, there may be a trade-off between the likelihood of sustaining long run economic growth and resource wealth. Resource-rich economies may, ceteris paribus, be less likely than resource-poor economies to sustain a positive rate of economic growth over the long run.

VII. Economic Growth and Structural Change

In this section we consider the case when the conditions of Proposition 2 are met and the economy is able to sustain a constant and positive rate of economic growth. Of course if this were not the case and there were no growth there would be no structural change to discuss. Unlike standard endogenous growth models, economic growth over the long run occurs concomitantly with structural change produced by an economy that shifts labor from a fixed-output commodity sector to the rest of the economy. The contribution made by the commodity sector to economic growth continuously declines in a growing economy; however, this contribution never becomes zero as the market will always require the sector to remain in operation as along as $n^* > 0$. The service sector is continuously growing, the commodity sector is stationary, and human capital and physical capital are growing at different and varying rates. This defines structural change within a constant growth economy with unbalanced asset and sector growth.

This result conforms to the first stylized fact referenced in the Introduction: as the North grows, the size of the commodity sector as a percentage of GDP continuously falls. The fact that preferences are homothetic, the prices of the final goods are fixed, and total consumption increases at a constant rate also implies that consumption of the service good (x_s) and of the commodity good (x_c) both increase at the same rate. Given that domestic production of the commodity good is constant in the long run, this means that the economy must continuously increase its imports of the commodity good and increase exports of the service good. This trade equilibrium is assured by the economy's budget constraint represented by equation (6). If the economy is not small in the world economy, this increasing demand for the commodity good must eventually affect world prices, an issue considered in Section IX.

Returning to Proposition 2 when $\beta > \alpha$ we show that the long run rate of growth of human capital is faster than that of physical capital ($\hat{h} > \hat{k}$); the economy relies primarily on the accumulation of knowledge as a source of growth. By contrast, countries where $\alpha > \beta$ will rely more on physical capital accumulation than knowledge as a source of economic growth ($\hat{h} < \hat{k}$). Thus, structural change and unbalanced factor growth take place even in the long run. This result corroborates the second stylized fact discussed in the Introduction regarding empirically observed variable factor shares in the long run.

Proposition 4 [On Labor and Capital Share Dynamics]. *In the long run, the share of labor in total income increases (falls) and the share of capital falls (increase) if* $\beta > \alpha$ $(\alpha > \beta)$.

Proof: *Define the share of labor as* $s \equiv \tilde{w}hL \big/ \left(\tilde{w}hL + rk\right) = \tilde{w}L \big/ \left(\tilde{w}L + r(k/h)\right)$. *First note that* $\tilde{w}$ *and r are fixed in the long run. Next from Proposition 2 if* $\beta > \alpha$ $(\alpha > \beta)$ *it follows that* $\hat{k} < \hat{h}$ $(\hat{k} > \hat{h})$ *and therefore the k/h ratio is decreasing (increasing) and thus the share of labor increases (falls). Similar reasoning shows the results for the share of capital*. ⊗

Proposition 4 shows that with *r* fixed and the earnings rate ($\tilde{w}h$) increasing during long run growth, the share of labor does not remain constant over time. These predictions of 1) increasing labor share and decreasing capital share for developed countries ($\beta > \alpha$) and 2) decreasing labor share and increasing capital share for developing countries ($\alpha > \beta$) even in the long run are consistent with the second stylized fact presented in the Introduction.

VIII. Full Property Rights

The following proposition shows how the findings for an economy without regulation or property rights generalize to the case of full property rights:

Proposition 5 [On Property Rights]. *The existence of full property rights (PR) for the natural resource produces an economy with (i) an adjustment path during Stage 1 that is qualitatively similar to the open access (OA) case in that economic growth is increasing in the resource* *stock. However, ceteris paribus, the PR economy exhibits higher levels of consumption at each point in time and a slower rate of growth of consumption than the OA economy; (ii) a Stage 2 that is qualitatively and quantitatively identical to the OA case including an identical rate of economic growth and identical asymptotic levels for the* k/h $(k'/h\,)$*and* c/k $(c/k'\,)$ *ratios, with the exception that the ACP level of the resource is greater under a property rights regime* $n_{PR}^{*} > n_{OA}^{*}$ *and the ACP level of effort (* $E_c \equiv DZ_c^{\beta} hl_c$ *) in the commodity sector is less under a property rights regime* $E_{PR}^{*} < E_{OA}^{*}$.

Proof: *See Appendix.* ⊗

The implications of Proposition 5 are that whether property rights exist or not is irrelevant to determining the rate of economic growth. At first glance this may seem counterintuitive as a property rights solution is first best and should produce an economy that is welfare superior to the economy without property rights. However, identical ACP economic growth rates do not imply identical absolute levels of consumption. That is, the consumption levels for the property rights regime will be greater than the consumption level in the open access regime ($c_{PR}(t) > c_{OA}(t)$ for all t). The reason for this is clear: a PR economy will have a higher level of resource wealth at all future points in time than an otherwise identical OA economy. This triggers a wealth effect which is translated into a lower level of $\lambda(t)$ over all time in the PR economy with a consequent higher level of consumption. The PR economy will allow for a lower level of effort (E_c) but a higher commodity output level in the long run compared to the open access economy. This lower effort in the resource sector will facilitate a larger services sector, thus supporting more consumption.

While property rights on the natural resource are of no consequence for the potential rate of economic growth, they do affect the likelihood that the economy will be able to sustain positive economic growth over the long run. In fact, as shown in Section VI, a greater ACP resource wealth leaves Z_s^{*} and $(k/h)^{\infty}$ unchanged but reduces the likelihood that such potential positive rate of economic growth be sustained over the long run. This conclusion remains valid because it can be easily seen that the introduction of property rights is of no consequence for Z_s^{*} and $(k/h)^{\infty}$. Hence, we can conclude that property rights on the natural resource may increase the likelihood of long run economic stagnation. This is a potential long run cost of natural resource regulation or property rights which to the best of our knowledge has not been recognized in the literature[18].

IX. The Full Model: Endogenous Commodity Prices with Property Rights in the North

The economy described above exerts a small demand on commodities relative to total world commodity availability and thus its increasing demand over time does not have a price effect. While the fact that during the 20th Century the North continuously increased imports of the commodity good with roughly constant commodity prices is consistent with the small open economy paradigm, one cannot reasonably argue that the North as a whole is a small open economy. We now relax the assumption of constant world commodity prices.

The full model includes property rights in the North, endogenous world commodity prices, and the assumption that there are no property rights on the resource in the South. In addition, we return to the assumption that the economy in the North is fully diversified and thus the North sustains positive economic growth. We show that under certain conditions commodity prices may reach stability and economic growth can continue at a constant positive rate even if the commodity demand from the North is large and continuously increasing. We now explicitly include the South as a provider of primary commodity goods. We assume that the North can invest in commodity-generating production enclaves in the South that are operated with small linkages with the rest of the South and that their prime objective is to export primary commodities to satisfy the increasing import demands from the North. These enclaves are discussed extensively in the literature (Prebisch, 1959; de Janvry, 1975) and combine capital investment from the North with a local labor supply to produce the commodity good that is produced mainly to be exported to the North[19].

The Commodity Enclaves. With a relatively abundant supply of natural resources, the South is an attractive target for investment of Northern capital aimed at producing the

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[18] Needless to say, the introduction of property rights is necessarily welfare increasing. The issue discussed here only concerns whether or not property rights facilitate growth over the long run.

[19] The colonies played a key role of suppliers of raw materials for the colonial powers during the 19th Century and early 20th Century. Once most colonies became independent the process continued through massive multinational enterprises investing in raw materials.

commodity good to be re-exported to the North. We assume the following after-tax profit function for the investment by Northern firms in the South:

$$\pi\left(p,\tilde{w}_D;k_D\right)=\left(1-\tau\right)\left\{\max_{l_D}\; pn_D^{\theta_D}k_D^{\delta}l_D^{1-\delta}-\tilde{w}_D l_D\right\} \tag{34}$$
$$\pi\left(p,\tilde{w}_D;k_D\right)=\left(1-\tau\right)k_D\Gamma(pn_D^{\theta_D})^{1+\Omega}p^{1+\Omega}\tilde{w}_D^{-\Omega}$$

where τ is the tax rate by the host South country, k_D is the capital from the North invested in the enclave, l_D is the labor force (from the South) used by the enclave, $\tilde{w}_D$ is the labor wage rate in the South, δ is a parameter reflecting the capital share for production in the enclave implied by a Cobb-Douglas production function, $\Omega \equiv (1-\delta)/\delta$, and Γ is a constant related to the parameter Ω. We note that Ω is the price elasticity of supply of commodity production in the South. This can be seen by applying Hotelling's lemma to equation (34) which implies that $y_c^D = \partial\pi/\partial p$, where y_c^D is the enclave's commodity supply.

To reflect conditions prevailing for the most part of the 20th Century and earlier the parameter indicating the importance of the resource in production (θ_D) in the South can be approximated at zero. This is because the South has an abundant supply of raw natural resource stocks and imposes no restrictions on their extraction. The lack of regulation means that even if the resources are scarce such scarcity does not factor into production decisions in a significant way[20]. Later we relax the resource abundance assumption to project 21st Century conditions. If $\theta \approx 0$, the second line of equation (34) can be rewritten as:

$$\pi\left(p,\tilde{w}_D;k_D\right)=\left(1-\tau\right)k_D\Gamma p^{1+\Omega}\tilde{w}_D^{-\Omega} \tag{35}$$

Investing in the South now provides a third option for Northern capital investment, such that equation (6) now becomes:

$$\dot{k}=Ak_s^{\alpha}\left(hl_s\right)^{1-\alpha}+pn^{\theta}Dk_c^{\beta}\left(hl_c\right)^{1-\beta}+\left(1-\tau\right)k_D\Gamma p^{1+\Omega}\tilde{w}_D^{-\Omega}-c\,, \tag{36}$$

[20] Examples include logging in the Brazilian Amazon which is a function of logging effort but is generally not a function of the stock of trees in the Amazon (annual logging corresponds to less than 1% of Amazon forest stock); ocean fish harvest of certain species which until recently were so abundant that the size of their stock levels did not affect capture. Harvest was primarily determined by effort. The limiting factor to extraction comes from demand and price conditions rather than from the degree of resource abundance. When the resource stock is many orders of magnitude greater than the level of extraction, stock levels do not affect production conditions.

Equation (36) shows the closed economy analog of the current account equilibrium represented by equation (6). Here the North exports a fraction of its capital and imports commodities produced in the South by its capital. Maintaining our assumption of perfectly competitive capital markets, equation (8) is modified accordingly:

(37) $$k_c + k_s + k_D = k$$

The first order conditions are augmented with two additional conditions: one reflecting the optimal evolution of the resource co-state variable (η) as shown in Proposition 5 and a second describing an equalization of the (after tax) marginal return to capital in the Southern enclave with that in the North. This latter condition yields equilibrium in the capital market,

(38) $$\alpha A(k_s / hl_s)^{\alpha-1} = (1-\tau)\Gamma p^{1+\Omega}\tilde{w}_D{}^{-\Omega} \equiv r$$

Combining equations (38) with (18) and (19) (replacing *p* for *q* in the latter) yields:

(39) $$\alpha A\left[\mathrm{X}\Psi^{\beta}\right]^{\frac{1-\alpha}{\beta-\alpha}}\left(qn(t)^{\theta}\right)^{\frac{1-\alpha}{\beta-\alpha}} = (1-\tau)\Gamma p^{1+\Omega}\tilde{w}_D{}^{-\Omega}$$

Equation (39) governs the North-South flow of capital and the production of commodities in the South. Assuming a positive level of investment by the North in the South, the return to physical capital in the South should be equal to that in the North. If, for example, returns to capital in the South were above those in the North, capital would rapidly flow from the North to the South increasing the world supply of commodities which, in turn, would cause the commodity price to rapidly fall. The fall of *p* reduces the returns to capital in the South which is a change toward the re-establishment of the equilibrium condition (39). However, the returns to capital in the North also fall when *p* decreases (since *q* also decreases), which is a change in the direction of deepening the gap in the capital returns.

If the effect of the commodity price on the returns to capital in the North is stronger than its effect on the returns to capital in the South the above equilibrium may not be re-established when the system is perturbed. Thus, equation (39) motivates the implicit market stability condition whereby the price effect is stronger in the returns to capital of the South than in the North. The parameters in (39) must satisfy certain conditions to guarantee that the capital market equilibrium is stable. Proposition 6 describes the relationship between the world commodity price and the resource level in the North and provides the stability condition.

Proposition 6 [On Dynamic Equilibrium with Endogenous Prices]. *Assume that the wage rate and the tax rates in the South are fixed, that* $\theta_D = 0$*, and that the international capital market equilibrium is stable, then:* (i) *Along the Stage 1 intermediate run dynamics, the world commodity price increases (decreases) if the natural resource stock level in the North increases (decreases), i.e.,* $sign(\hat{p}) = sign(\hat{n})$. *(ii) The world commodity price and the stock of natural resources in the North achieve a constant ACP. (iii)* T*he required capital market stability condition is:* $(1+\Omega)(\beta-\alpha) > 1-\alpha$

Proof: *See Appendix.* ⊗

Part (i) of Proposition 6 describes the relationship between the dynamics of the world commodity price and the stock of natural resources in the North during the intermediate run (Stage 1) assuming that the wage rate and the tax rate in the South are constant [21]. The rate of return to capital in the North can only follow an increasing path if $n(t)$ is rising as well.

This co-evolution of the paths of $p(t)$ and $n(t)$ along the intermediate run (Stage 1) is not to be confused with the effects of exogenous shocks which may cause the world commodity price to change in opposite direction to the stock of natural resources in the North. For example, if investment conditions in the South improve (for example if $\tilde{w}_D$ falls) more capital will shift from the North to the South causing world commodity prices to fall while at the same time inducing an increase in the natural resource stock in the North over the long run due to the fact that the domestic after tax price in the North must also fall.

Instead, the co-evolution refers to endogenous changes in the intermediate run. As the resource level of the North falls (increases) toward its stationary value the commodity sector contracts (expands). This causes capital and labor to be released to the rest of the economy. If the commodity sector is more capital intensive than the rest of the economy this will cause a lower rate of return to capital in the North thus temporarily causing disequilibrium between the rates of return of the North and South. This causes a reallocation of capital away from the North

[21] One characteristic of these enclaves in the South is that they tend to employ only a small fraction of the labor force so that they do not affect the South's wage rate. For example, mining enclaves formed in the mid-20th Century in Chile represented 0.3% of the Chilean labor force and thus were not of a magnitude to affect the market wage.

toward the South with the consequent increase of the supply of commodities in the South and further declines of the commodity price.

Part (*ii*) of Proposition 6 demonstrates that an endogenous commodity price does not prevent the system from reaching a constant price level in ACP and consequently to reach a constant level of natural resources. This stems from the fact that the rate of return to capital in the North is entirely dependent on the stock of natural resources and the commodity price. If the resource reaches its ACP, the capital market equilibrium depicted by equation (39) remains unaffected and, therefore, the commodity price achieves a constant ACP as long as the wage and tax rate in the South do not change. This also means that the rate of growth of the economy reaches its ACP and the evolution of the other variables is qualitatively identical to the intermediate run dynamics of the fixed price case discussed in Section IV.

This result may seem paradoxical; as the North grows in the long run it demands increasing amounts of the commodity good while its production of such good is fixed. The resulting increase of commodity imports is perfectly matched by rising supply of commodity production in the South. This causes the commodity price to continuously suffer upward pressures as the North grows but since growth of the North also means greater availability of physical capital in the North, equilibrium (39) can be satisfied by a continuous increase of Northern investment in the South consistent with the rise of total capital in the North. Thus, in the long run the world commodity price remains fixed. The stability condition in Part (iii) assures that the converging price effect in the South dominates the divergent one in the North.

The results of Proposition 6 are consistent with the empirical evidence. Foster and Rosenzweig (2003) provide data about the increase in forested stocks over the whole of the 20th Century in the Northern and Southern US which was accompanied by increasing world timber prices through the same period (Kellard and Wohar, 2006). Timber represent one of the few exceptions to the general observation of non-increasing commodity prices during most of the 20th Century. The stocks of most other natural resources in the US declined during the 20th Century prior to the 1980's which is consistent with the predictions from Proposition 6 when paired with the fouth stylized fact in the introduction: non-increasing commodity price trends for most commodities (with the prominent exception of timber prices). Since the late 1980's, the North has started to rebuild natural resource stocks (including a dramatic reduction of pollution) which occurs concurrent with some of the first increases in world commodity prices and would appear to be consistent with Proposition 6. However, it is difficult to disentangle the general equilibrium effects described above from regulatory institution effects which developed at the same time.

Proposition 6 describes the intermediate run equilibrium conditions for arriving at ACP. Proposition 7 below describes several comparative static results from increasing growth in the South that leads to either a) rising wages, b) a desire by governments in the South to capture more rents from commodity product exports through higher taxes, or c) the emergence of resource scarcity in the South.

Proposition 7 [On Comparative Static]. *Assuming that n is at or near its ACP, (i) if the South's wage rate experiences a discrete increase or the South imposes a higher tax rate, the ACP level of the commodity price and resource stock in the North will be disrupted and the intermediate dynamics described in Section IV will govern the return to ACP. The new ACP will be characterized by: 1) A lower resource stock level in the North; 2) A higher commodity price level; and 3) No change in the long run rate of economic growth for the North.*

(ii) However, once the resource becomes sufficiently scarce in the South such that θ_D *becomes positive as the North grows, the commodity price level will continuously increase which will depress economic growth in the North and may eventually lead to stagnation.*

Proof: *See Appendix.* ⊗

Economic growth in the North causes an ever increasing demand on the South for the commodity good. If resource scarcity does not affect returns to capital in the South, the South may meet this ever increasing demand at constant commodity prices. However, as demand from the North continues to increase and the set of growing countries continuously expands, resource scarcity eventually emerges in the South (i.e., the parameter θ_D becomes positive) which results in the continuous rise of commodity prices. At this point, the growth rate of real consumption falls due to the cost-of-living effect caused by the commodity price increases. Income or GDP growth in the North may continue but the price effect offsets the impact of GDP growth on real consumption. The consumption growth rate in the North may fall to zero depending on the nature of preferences and the elasticity of substitution across consumption goods.

The model presented provides predictions that are highly consistent with stylized facts prevalent mainly during the 20th century. In contrast with earlier results, Proposition 7 predicts conditions not yet empirically corroborated: continuous growth in the North may trigger sufficient resource scarcity in the South that eventually may lead to economic stagnation. However, given the fact that the model seems to predict well confirmed patterns of growth in the North combined with the recent significant expansion of the growth club, one might suggest that the predictions from Proposition 7 may be a reasonable projection of future developments unless some structural break occurs. This necessary structural break may consist of a reorientation of technological change from current efforts to develop labor augmenting technologies to the development of natural resource augmenting technologies as well. Clearly, the non-increasing level of commodity prices in the past has been a powerful disincentive for the development of natural resource augmenting technologies. If natural resource scarcity begins to be reflected in the commodity prices, however, such incentives may emerge. Should this occur, resource augmenting technologies may reduce commodity demands and eventually prevent ever-increasing commodity prices, helping the economy avoid the limits to growth.

X. Final Remarks

We have shown that incorporating an earth sector in growth theory is fundamental to understanding certain important patterns of economic growth. Inclusion of the earth sector produces a model that does not deviate from commonly accepted conclusions of standard growth models but is richer than them by being able to explain other stylized facts on which the standard growth model is mainly mute.

We have provided a hypothesis to explain why a handful of countries around the world have been able to sustain a positive and approximately constant rate of economic growth for many decades while so many others have not been able to do so. History matters; presumably random phenomenon such as wars and natural disaster shocks as well as policy episodes that have in the past biased incentives in favor of investment in particular assets could result in factor endowments more favorable to sustain growth or instead more prone to long run stagnation. Two countries with identical preferences, production technologies and natural resources may follow opposite growth paths, one stagnating and the other preserving a positive growth rate in the long run if their historically determined levels of human and physical capital are different. In particular our analysis highlights the risks of over abundance of physical capital relative to human capital: it may create autonomous forces which in the absence of exogenous correction can lead the economy to stagnate over the long run. We have called this situation a capital curse.

The analysis provides plausible conditions under which resource wealth may stimulate or depress the pace of economic growth over the intermediate run. It provides a theoretical foundation to empirical observations that suggest that sometimes resource wealth may hinder the long run economic growth potential of countries as emphasized by the resource curse literature. An important contribution of this paper has been to show that while it makes sense to think of economic growth as being affected by the natural resource wealth over the intermediate run, in the long run the rate of economic growth and the level of natural resource wealth are both endogenous and simultaneously determined. Hence the relationship between natural resources and growth rate depends on factors that trigger a change in one (or both) of the variables. We have shown that when resource wealth changes are originated in changes in resource regulation or property rights, resource wealth is correlated with a greater likelihood of long term stagnation. That is, a potential trade-off between natural resource wealth and sustained economic growth may arise.

Inclusion of an earth sector provides a necessary benchmark model from which one can evaluate changes in world growth patterns. For example, as new large countries increase their growth rates, they too are drawing on these worldwide sources of commodity products. At the same time the abundance of natural resources in the South appears to subside and the South may increasingly try to capture their resource loss through increasing taxes on commodity exports to the North. As a result, the recent increases of commodity prices over the last few years may be signaling the end of an era where part of the world was able to persistently grow with the luxury of stable commodity prices. This may result in both an increased drawdown of the resource stock in the North and slower economic growth or even stagnation.

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Figures

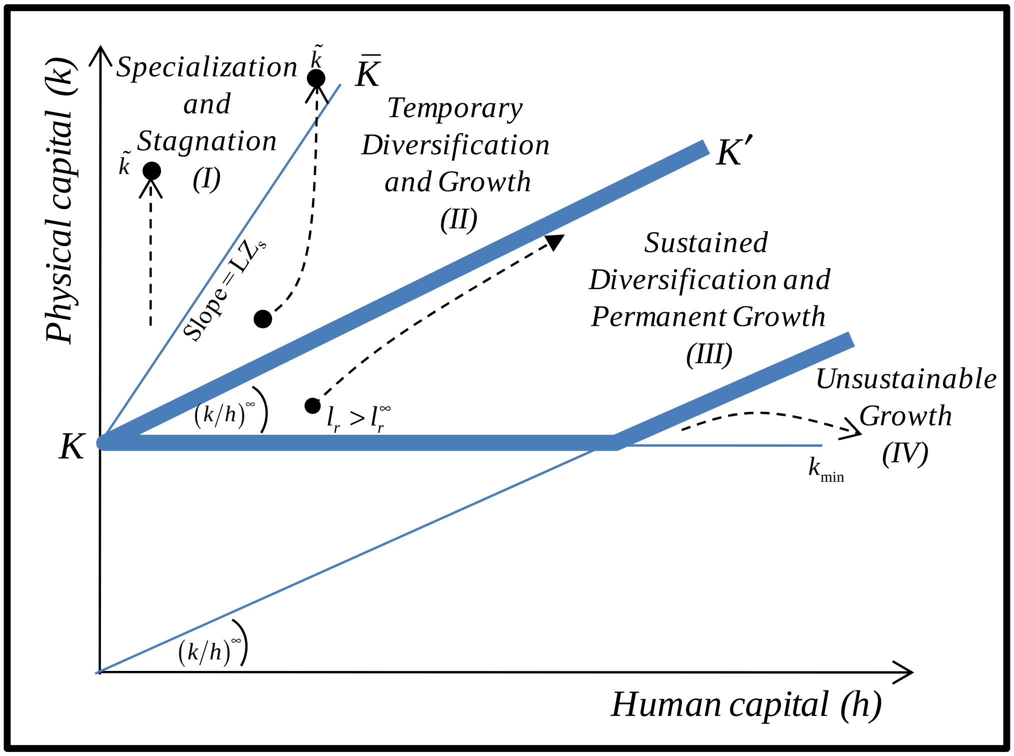

Figure 1. Shows the dynamics of the system in the four regions assuming that $\beta > \alpha$: I) Specialization and Stagnation delineated by the $K\bar{K}$ line above which $l_r = 0$. The resulting economic growth path is vertical causing the economy to specialize in only the commodity sector and eventually stagnating at $\tilde{k}$. II) Temporary diversification and growth where both final good sectors and the human capital sector are initially in operation; however, the economy eventually moves into Region I and consequently stagnates. III) Sustained Diversification and permanent growth where the economy gets closer but never reaches the KK' line. IV) The economy eventually specializes in only human capital production.

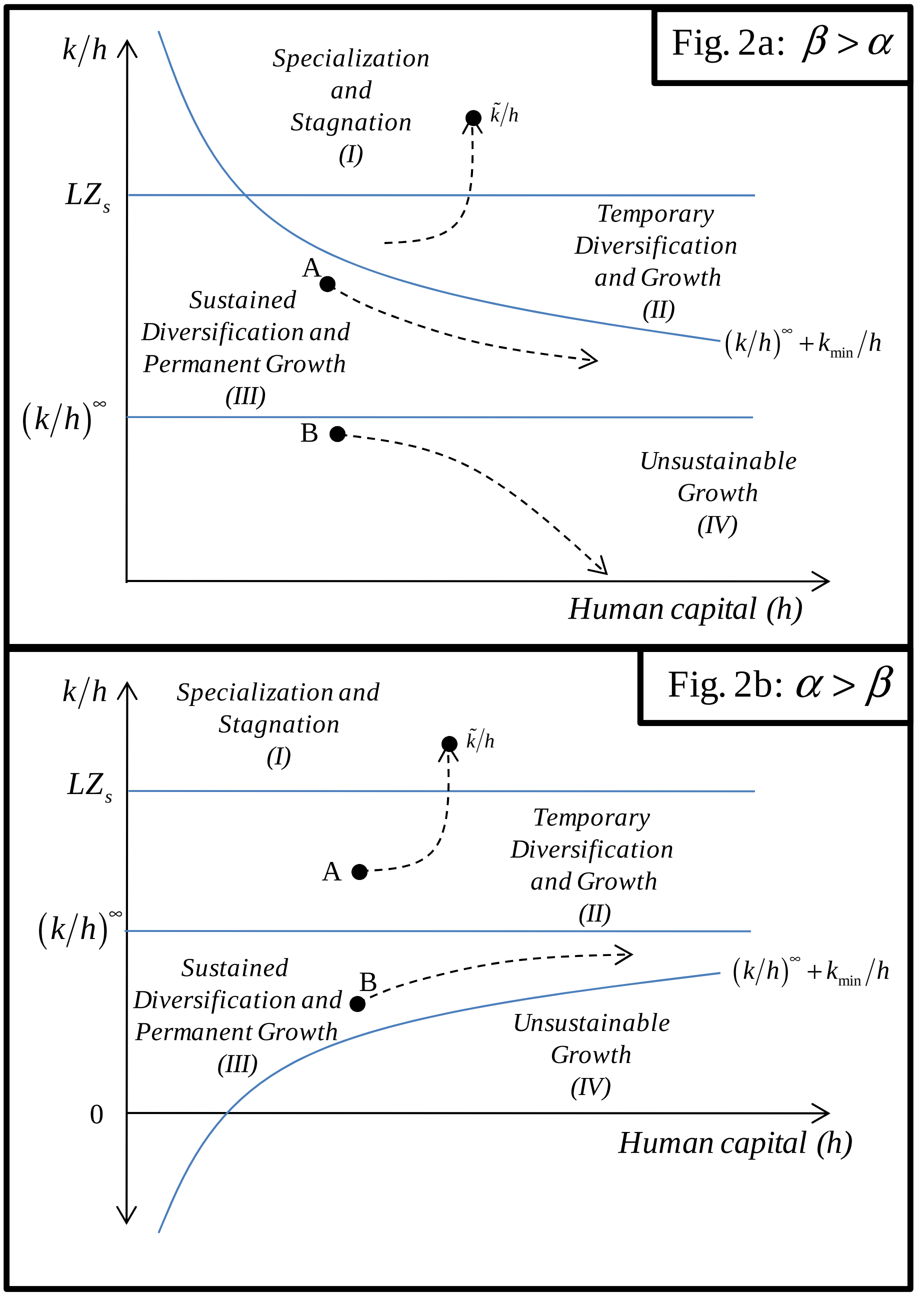

Figure 2: The dynamics of the economy in the k/h - h space where Figure 2a reflects an economy that has a commodity sector that is more physical capital intensive and Figure 2b reflects an economy that has a commodity sector that is more labor intensive.

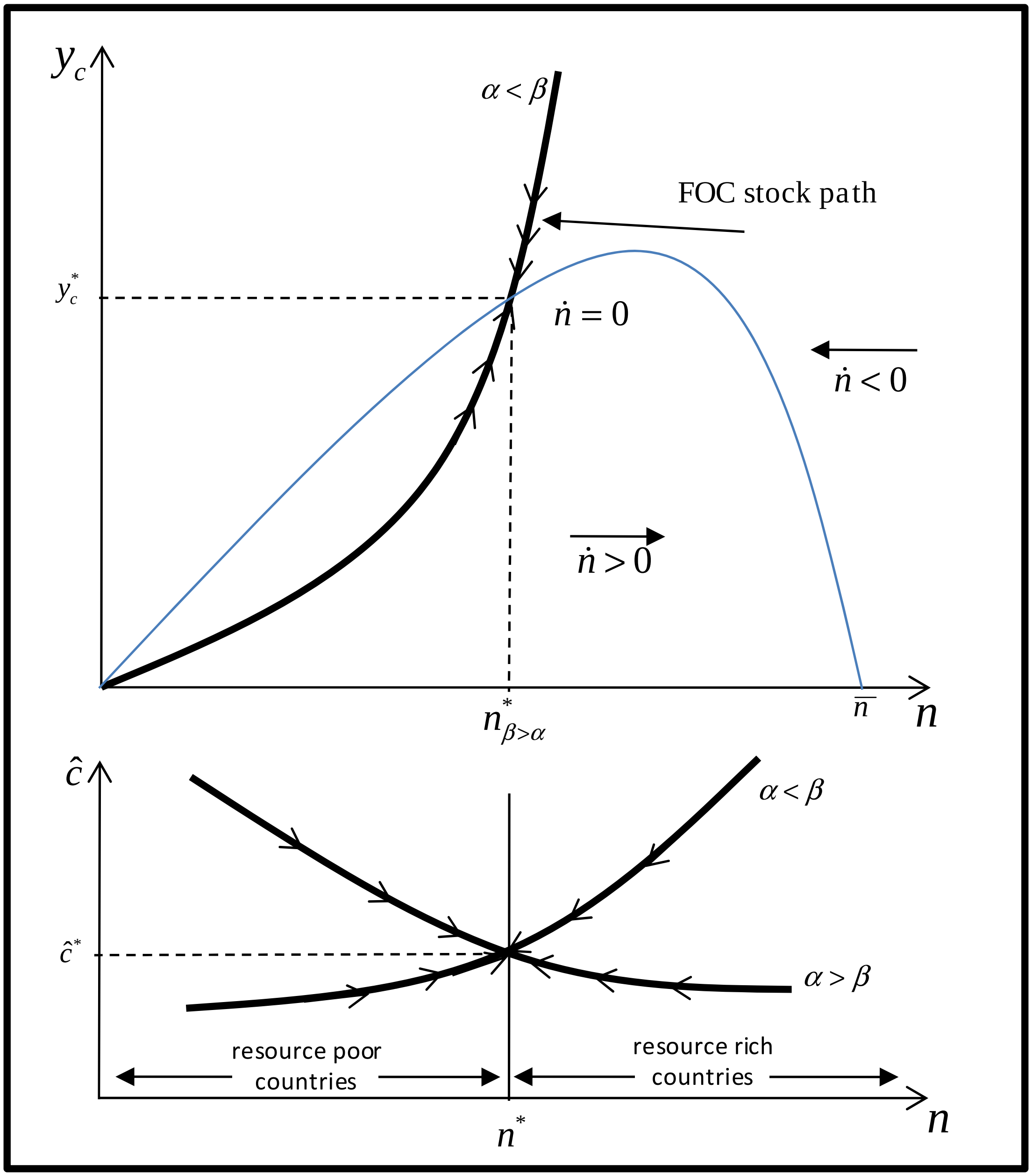

Figure 3. Transitional Dynamics. Assumes $\theta = 1$ and a logistic growth function for the renewable resource as shown in Footnote 11. The light inverted-U line in the top panel references the $\hat{n} = 0$ locus (equation (4)). For any point below (above) that line *n* is increasing (decreasing). Points along this line qualify as long-run equilibrium (y_c and n are stable). The thicker curves in the top panel describe the adjustment dynamics of y_c as a function of n (equation (22)). In the lower panel, the two lines represent equation (23).

Appendix

Proof of Proposition 1. (i) From (20) ACP equilibrium for n requires that $\hat{n}=0$ and hence that $r(n^*)=BL$. Thus, using (19) in (14) we have that $A\alpha\left(\mathrm{X}\Psi^{\beta}\right)^{1-\alpha/\beta-\alpha}\left(pn(t)^{\theta}\right)^{1-\alpha/\beta-\alpha}=BL$, which yields the expression for n^* as in 1). Using (24) we get the expression for $\hat{c}^*$ in 1). Using (18), (19) and the expression for n^* in 1) yields 2). The expression for y_c^* and $(hl_c)^*$ in 3) follow by setting (4) equal to zero and using $r(n^*)=BL$ in (21), respectively. Also 4) follows by using the definitions of $\tilde{w}$ and r as given in (12) and (14) and (18) and (19). Stability is guaranteed from equation (20). Existence requires that $0\le n^*\le\bar{n}$ where $\bar{n}$ is the carrying capacity of the resource. Expanding the second inequality yields:

$$\bar{n}\ge\left[\frac{A^{\frac{1-\beta}{1-\alpha}}(1-\alpha)}{D(1-\beta)\,p\Psi^{\beta}}\left(\frac{BL}{\alpha}\right)^{\frac{\beta-\alpha}{1-\alpha}}\right]^{1/\theta}\tag{A-1}$$

(ii) Following Part (i), constant $\left(k_c/hl_c\right)^*$ and $(hl_c)^*$ $\Rightarrow\hat{k}_c=0$. Logarithmically differentiating $\left(k_s/hl_s\right)^*$ yields $\hat{k}_s=\hat{h}+\hat{l}_s$. Finally, because the wage rate per unit of human capital $\tilde{w}^*$ is a constant in ACP, the earnings rate ($\tilde{w}h$) increases with h. ⊗

Proof to Proposition 2. We start by deriving equations (29) and (30). In the ACP we have the following equalities: 1) $\hat{c}^*=\varepsilon\left(BL-\rho\right)$; 2) $\hat{h}=BL+B\left(\Psi-1\right)\left(hl_c\right)^*/h-\left(B/Z_s^*\right)\left(k/h\right)$; and 3) $\hat{k}=BL\left[Z_s^*\left(L-l_r\right)\left(h/k\right)\left(1-\alpha\right)/\alpha+1\right]-c/k$. The expression for $\hat{k}$ is derived by combining equations (6) and (25). Noting that $\widehat{c/k}=\hat{c}-\hat{k}$ and using (25).ii produces:

$$\widehat{c/k}=\varepsilon\left(BL-\rho\right)-BL\left[1/\alpha-\left(k_{\min}/k\right)\left(1-\alpha\right)/\alpha\right]-c/k\tag{A-2}$$

Similarly, noting that $\widehat{k/h}=\hat{k}-\hat{h}$ yields:

$$\begin{aligned}\widehat{k/h}=&\,BL\left[1/\alpha-\left(k_{\min}/k\right)\left(1-\alpha\right)/\alpha\right]-c/k\\&-BL-B\left(\Psi-1\right)\left(hl_c\right)^*/h+\left(B/Z_s^*\right)\left(k/h\right)\end{aligned}\tag{A-3}$$

Set $\lim\limits_{\substack{c\to\infty\\k\to\infty}}\widehat{c/k}=0$ and $\lim\limits_{\substack{k\to\infty\\h\to\infty}}\widehat{k/h}=0$ and solve for c/k and k/h. Both ratios turn out to be constants and therefore correspond to $\left(c/k\right)^{\infty}\equiv\lim\limits_{\substack{c\to\infty\\k\to\infty}}c/k$ and $\left(k/h\right)^{\infty}\equiv\lim\limits_{\substack{k\to\infty\\h\to\infty}}k/h$. These limits in (A-2) and (A-3) are $\left(c/k\right)^{\infty}=\varepsilon\rho-\left(\varepsilon-1/\alpha\right)BL$ and $\left(k/h\right)^{\infty}=\left[BL-\varepsilon\left(BL-\rho\right)\right]Z_s^*/B$.

Using these boundaries in (A-2) and (A-3) yields:

$$\hat{c/k} = \left(c/k - (c/k)^{\infty}\right) + \left((1-\alpha)/\alpha\right) BL\, k_{\min}/k \tag{A-4}$$

$$\hat{k/h} = B/Z_s^* \left(k/h - (k/h)^{\infty}\right) - \hat{c/k} - B/Z_s^*\, k_{\min}/h \tag{A-5}$$

Where $k_{\min} = Z_s^* (\Psi - 1)(hl_c)^*$. When $k' = k - k_{\min}$ and following the same methodology but solving for $\hat{c/k'}$ yields:

$$\hat{c/k'} = \left(c/k' - (c/k)^{\infty}\right) - BL\, k_{\min}/k' \tag{A-6}$$

$$\hat{k'/h} = B/Z_s^* \left(k'/h - (k/h)^{\infty}\right) - \hat{c/k'} \tag{A-7}$$

When $\beta > \alpha$, $\Psi > 1 \Rightarrow k_{\min} > 0$. Thus we show (26) to (29).

By inspection of equation (A-4), it is clear that for c/k to approach its ICP, $c/k < (c/k)^{\infty}$ such that $\hat{c/k} > 0$. Note that by definition the ICP can never be reached as long as k and h are finite. Similarly for c/k' to approach its ICP, $c/k' > (c/k)^{\infty}$ such that $\hat{c/k'} < 0$. Using this in equations (A-5) and (A-7) demonstrates that for k/h to approach its ICP, $(k/h)^{\infty} < k/h < (k/h)^{\infty} + k_{\min}/h$. Again, the asset ratio cannot reach its ICP as long as k and h are finite. Same reasoning produces the convergence results for the case where $\beta < \alpha$, $\Psi < 1 \Rightarrow k_{\min} < 0$. ⊗

Proof to Proposition 3. (i) Follows from equation (23) using (19) and the definition of $r(n)$; (ii) Follows from combining Prop 3.i with the definition of a resource poor (rich) country as one with a resource endowment less (more) than the ACP level of resource. ⊗

Proof of Proposition 5. Here $\eta > 0$ and optimally chosen. Maximizing (10) yields the following first order conditions:

$$\tilde{w}_{PR} \equiv A(1-\alpha) Z_s^{\alpha} = qn^{\theta} D(1-\beta) Z_c^{\beta} \tag{A-8}$$

$$qn^{\theta} D(1-\beta) Z_c^{\beta} = \frac{\mu}{\lambda} B \tag{A-9}$$

$$r_{PR} \equiv A\alpha Z_s^{\alpha-1} = n^{\theta} D\beta q Z_c^{\beta-1} \tag{A-10}$$

$$\hat{\eta} = \rho - g'(n) - D\theta n^{\theta-1} Z_c^{\beta} hl_c \left(\frac{q\phi}{p-q}\right) \tag{A-11}$$

Where $q \equiv p - (\eta/\lambda)\phi$ is the tax-adjusted commodity price ($0 < q < p$). In addition, equations (16), (17), and (18) still hold in identical form in the case of property rights. Combining equations (A-8) and (A-10) reproduces Lemma 1 (equation (19)). Following the methodology used in the open access case, Lemma 1 is used in the first order conditions above to obtain the property rights analog to the equations (19), the precursor and simplified equation (20), and (23), respectively:

$$Z_c = \left(\frac{D(1-\beta)\Psi^{\alpha}}{A(1-\alpha)} \right)^{1/(\alpha-\beta)} \left(qn^{\theta}\right)^{1/(\alpha-\beta)} \tag{A-12}$$

$$\hat{n}(t) = \left(BL - r_{PR}\right)\left(\frac{\beta-\alpha}{\theta\alpha} \right) + \left(\frac{1}{\theta} \frac{\left(\hat{\eta}-\hat{\lambda}\right)}{\left(q/\left(p-q\right)\right)} \right) \tag{A-13}$$

$$\widehat{qn^{\theta}} = \left(\frac{\beta-\alpha}{\alpha} \right) \left[BL - A\alpha\left(\mathrm{X}\Psi\right)^{1-\alpha/\beta-\alpha} \left(qn^{\theta}\right)^{1-\alpha/\beta-\alpha} \right] \tag{A-14}$$

$$\hat{c}(t) = \varepsilon \left[A\alpha\left(\mathrm{X}\Psi^{\beta}\right)^{1-\alpha/\beta-\alpha} \left(qn(t)^{\theta}\right)^{(1-\alpha)/\beta-\alpha} - \rho \right] \tag{A-15}$$

ACP Convergence: Equation (A-14) demonstrates that qn^{θ} converges to a stationary level identical to the stationary level of pn^{θ} in the open access case shown in equation (20):

$$\left(qn^{\theta}\right)^{*} = \frac{1}{\left(\mathrm{X}\Psi^{\beta}\right)} \left(\frac{BL}{\alpha A} \right)^{(\beta-\alpha)/(1-\alpha)} \tag{A-16}$$

Comparing (A-16) with Proposition 1, Part (*i*), we see that the production value of the resource is identical in the open access and property rights case, $\left(pn_{0A}^{\theta}\right)^{*} = \left(qn_{PR}^{\theta}\right)^{*}$; however, since $q < p \Rightarrow n_{PR}^{*} > n_{OA}^{*}$ (the ACP resource level under the property rights regime is greater than the ACP resource level under the open access regime). Substituting equation (A-16) in (A-15) demonstrates that the rate of economic growth in a property rights regime converges to a constant rate, and in fact, the same ACP as in the open access regime:

$$\hat{c}^{*} = \varepsilon[BL - \rho] \tag{A-17}$$

Given that *q* is bounded, we know that at ACP, $\hat{\eta} = \hat{\lambda}$. If this were not true and $\hat{\eta} > \hat{\lambda} \Rightarrow q \to -\infty$. If $\hat{\eta} < \hat{\lambda} \Rightarrow q \to p$ which can also not be the case as use of the resource incurs real costs (represented by ϕ) on society which must be accounted for in the commodity price (*q*). From this and equation (A-9) it follows that at a diversified ACP, $\hat{\eta} = \hat{\lambda} = \hat{\mu}$. Combining equations (A-16) with (A-12) yields:

$$Z_c^{*} = \Psi\left(\frac{\alpha A}{BL} \right)^{\frac{1}{1-\alpha}};\ Z_S^{*} = \left(\frac{\alpha A}{BL} \right)^{\frac{1}{1-\alpha}};\ \tilde{w}^{*} = A^{\frac{1}{1-\alpha}}(1-\alpha)\left(\frac{\alpha}{BL} \right)^{\frac{\alpha}{1-\alpha}};\ r^{*} = BL \tag{A-18}$$

These are identical to the comparative values in Proposition 1. Other than the ACP resource level and size of the effort applied to the commodity sector (E_c), the economy converges to an identical ACP. In addition, ICP convergence levels of the $\left(k/h\right)^{\infty}$ and $\left(c/k\right)^{\infty}$ ratios is unaffected by the property rights regime.

Intermediate Run Economic Growth: Convergence to the ACP resource level can be described in the *qn* - *n* space (see Figure A-1). For notational simplicity, we assume that $\theta = 1$. The two governing equilibrium loci are $\dot{n}=0$ and $\dot{\overline{qn}}=0$. The $\dot{n}=0$ locus is derived by starting with the ACP condition $\hat{\eta}=\hat{\mu}$ combined with equation (4) to arrive at *n* as only a function of *qn*:

$$BL-g'(n)-\frac{g(n)\theta}{n}\left(\frac{qn}{pn-qn}\right)=0 \tag{A-19}$$

Thus (A-19) yields the combinations of n and qn that are consistent with $\dot{n}=0$. From (A-19) it can be shown that $\dot{n}=0$ is upward sloping, $\partial(qn)/\partial n>0$ along the $\dot{n}=0$ schedule. Dynamics around this ACP locus can be characterized by noting that for a given level of *qn*, if *n* is below the corresponding value along the $\dot{n}=0$ schedule then $BL<g'(n)+\frac{g(n)\theta}{n}\left(\frac{qn}{pn-qn}\right)$ which implies that $\hat{\eta}<\hat{\mu}$ but that $\hat{\mu}=\hat{\lambda}$ because neither is a function of *n* for fixed *qn* (and consequently $\hat{\eta}<\hat{\lambda}$). Differentiating the function $q=p-(\eta/\lambda)\phi$ yields:

$$\hat{q}=\frac{\eta}{\lambda}\frac{\phi\left(\hat{\lambda}-\hat{\eta}\right)}{p-\phi(\eta/\lambda)} \tag{A-20}$$

Thus, $\hat{\eta}<\hat{\lambda}\Rightarrow\hat{q}>0\Rightarrow\hat{n}<0$ for a given level *qn*. The opposite occurs if for a given qn, *n* is above the $\dot{n}=0$ schedule. As indicated earlier, the other condition characterizing an ACP point is $\dot{\overline{qn}}=0$, defined be equation (A-14). For a given level of n, if qn is above (below) the $\dot{\overline{qn}}=0$ locus, *qn* falls (rises). This combined with the motion to the left or to the right of the $\dot{n}=0$ schedule discussed above gives the motion of the system outside the ACP as in Figure A-1. Thus, the system converges in the same way as in the open access case.

Suppose initially the economy is in open access equilibrium and is subject to a property rights reform. The economy initially at a point n_{OA} in Figure A-1 (Position 1) reduces the price of the resource from p to q which for the constant n_{OA} causes *qn* to fall to Position 2 and consequently increase along the line depicting the optimal adjustment under property rights. This causes n to start increasing towards the new ACP, n^*_{PR} . The reduction of the commodity price is caused by the fact that η becomes positive and there is also an instantaneous reduction of λ (reflecting a wealth effect). This causes an instantaneous jump in the consumption level but then the economy grows at a slower (but increasing) rate towards the new ACP level. The level of $\lambda(t)$ under property rights need to be less or equal to the level of $\lambda(t)$ under open access for all t. Hence, the consumption level under property rights is at or above than the consumption level under open access at all times.

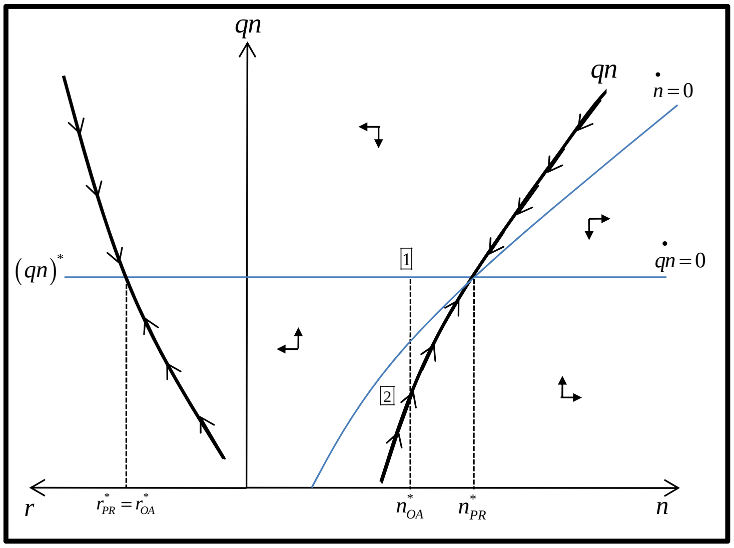

Figure A-1. Dynamics of the property rights solution. The figure shows the adjustment path and equilibrium under property rights. The arrows indicate the direction of changing *n* and *qn* over time. The upward sloping light line shows the combinations of *n* and *qn* values consistent with long-run equilibrium (equation (A-19)) and the horizontal light line shows the unique level of *qn* that satisfies equation (A-14) when $\dot{qn} = 0$. The left panel shows the intermediate run dynamics for the price of capital.

Proof to Proposition 6. For (*i*) and (*ii*) the results from the Proof to Proposition 5 (full property rights) remain valid when the commodity price is endogenous. The one additional first order condition (equation (38)) simply fixes the price of capital equal to the returns to capital in the South. Thus, Figure A-1 accurately describes the intermediate run dynamics of the full model. As described in Proposition 5, for a given level of *qn*, both *n* and *qn* will fall (rise) toward the ACP level of *qn* and thus, $sign\left(\hat{n}\right) = sign(\widehat{qn})$. Logarithmically differentiating equations (38) and (A-8) and re-arranging yields:

$$\left((1-\alpha)\big/\left[(\beta-\alpha)(1+\Omega)\right]\right)\widehat{qn} = \hat{p} \qquad \text{(A-21)}$$

Thus, $sign\left(\hat{p}\right) = sign(\widehat{qn}) = sign\left(\hat{n}\right)$. (*iii*) *Market Stability condition.* Assume initially that n is at or in the neighborhood of its ACP (n^*). Re-write equation (39) as follows:

$$\chi(p) \equiv \alpha A\left[\mathrm{X}\Psi^{\beta}\right]^{\frac{1-\alpha}{\beta-\alpha}} \left(pn^*_{OA}(t)^{\theta}\right)^{\frac{1-\alpha}{\beta-\alpha}} - (1-\tau)\Gamma p^{1+\Omega}\tilde{w}_D^{\ -\Omega} = 0$$

where we have used the equilibrium condition from Proposition 5 $\left(pn^{\theta}_{0A}\right)^* = \left(qn^{\theta}_{PR}\right)^*$. Thus, it follows that the ACP will be re-established after a shock only if the price adjusts in the direction of closing the gap caused by the shock. This requires that $\partial\chi/\partial p < 0$, which, in turn, requires that $(1+\Omega)(\beta-\alpha) > 1-\alpha$. ⊗

Proof to Proposition 7. (*i*)Initially the economy is at an ACP as depicted in Proposition 6. By (A-16) changes in τ or w_D do not affect $(qn_{PR}^{\theta})^*$. Hence, from (39) and Proposition 6 it follows that an increase in τ or w_D must cause p to rise in the long run. Since (A-19) is still valid as an ACP condition, implicit differentiating shows that $\partial p/\partial n < 0$. Logarithmic differentiation of equation (11) allowing endogenous commodity prices combined with equations (A-8), (A-10), and (A-21) yields:

$$\hat{c} = \varepsilon\left(A\alpha\left(\mathrm{X}\Psi^{\beta}\right)^{1-\alpha/\beta-\alpha}\left(qn(t)^{\theta}\right)^{(1-\alpha)/\beta-\alpha} - \rho\right) + \frac{\xi(1-\varepsilon)(\beta-\alpha)(1+\Omega)}{(1-\alpha)}\left(\widehat{qn}\right) \qquad \text{(A-22)}$$

At ACP when $\widehat{qn} = 0$ and (A-16) applies, equation (A-22) collapses to $\hat{c}^* = \varepsilon\left(BL - \rho\right)$ and the long run economic growth is unchanged.

*(ii) Case when $\theta_D > 0$ in the South***.** Rewriting the first order condition shown in equation (39) to incorporate resource scarcity in the South, $\theta_D > 0$, yields:

$$\alpha A Z_S^{\alpha-1} = (1-\tau)\Gamma(pn_D^{\theta_D})^{1+\Omega}\tilde{w}_D^{-\Omega} \qquad \text{(A-23)}$$

Logarithmic differentiation of conditions equations (A-8), (A-9), and (18) combined with equations (15) and (16) yield:

$$\alpha\hat{Z}_s = \alpha A Z_s^{\alpha-1} - BL \qquad \text{(A-24)}$$

With $\alpha < 1$, Z_s converges to $Z_s^* = (\alpha A / BL)^{1/(1-\alpha)}$. Thus by (A-23), $pn_D^{\theta_D}$ must also converge to a constant value at the same rate as Z_s. Upon convergence, $\theta_D\hat{n}_D = -\hat{p}$ and thus the continuously falling resource stock in the South causes the commodity price level to continuously rise at the same rate the resource level is falling. $\otimes$