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Revisiting The Original Ghosh Model: Can It Be More Plausible?

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REVISITING THE ORIGINAL GHOSH MODEL:
CAN IT BE MADE MORE PLAUSIBLE ?

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Abstract

We reconsider in this paper the alleged implausibility of Ghosh's model and we do so reformulating the model to incorporate an alternative closure rule. Our proposed closure rule is in line with the original allocation rules defined by A. Ghosh. The closure solves, to some extent, the implausibility problem that was pointed out by Oosterhaven for then value–added is correctly computed and responsive to allocation changes resulting from supply shocks. Some numerical examples illustrate the sectoral and aggregate consistency of the allocation equilibrium.

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1. INTRODUCTION

The debate on the validity and plausibility of the so called 'supply-driven' input-output model of Ghosh (1958) seems to keep resurfacing every so often. The difficulty to interpret Ghosh's model within conventional production theory has led to numerous interpretations and assertions that, periodically, put into question the structure and meaning of the model. Giarratani (1980), for instance, discussed the lack of well understood economic behaviour behind it. Oosterhaven (1988, 1989), in turn, called the attention over the 'implausibility' of a model that allocates output in response to changes in value–added in a given sector without those changes in output translating into further changes in value–added. In whatever way output turns out to be produced and allocated among sectors it surely makes little sense that value–added is not responsive to a general system reallocation. Gruver (1989), however, argues in favour of the plausibility of the model provided only small changes are considered. In turn, Dietzenbacher (1997) 'vindicates' Ghosh by way or reinterpreting it as a price model, which then happens to be fully and formally equivalent to Leontief's price model and we are back to the well-known and standard interindustry model. More recently De Mesnard (2009a, 2009b) has claimed the model to be uninteresting since it is implausible as an output model, unnecessary as a price model and less informative than Leontief's dual quantity and price models. More in-depth discussion and details can be found in the references provided by these authors but the essence of the problematic issues about Ghosh's model has been sufficiently laid out.

Our aim in this paper is reviewing and addressing the 'plausibility' debate regarding unresponsive value-added that was pointed out by Oosterhaven (1988, 1989). His sharp criticism is valid since value–added being unresponsive to output changes is a hard to sell economic fact. Under Ghosh's model conditions, suppose that value–added in sector *j*, say, increases. When this shock is subsequently absorbed by the output allocation system, we observe an increase in the output of sector *i* $(i \neq j)$, as a result of the endogenous output reallocation, but at the same time value–added in sector *i* is surprisingly unaffected. Needless to say this seems to violate common sense as well as some version of Debreu's axiom on the impossibility of the Land of Cockaigne (Debreu, 1959, chapter 3).

Let us consider for a moment Leontief's open quantity model (Leontief, 1936). When autonomous final demand for good *j* increases, the system generates increases in output and value–added in all sectors. There is more value–added around but this does not have, however, any effect whatsoever in final demand for other goods $(i \neq j)$. This is also somewhat surprising as far as economic logic goes. How can it be that consumption behaves in an unresponsive way to the new additional income? There are at least two ways out of this situation. The first one is to close Leontief's open model and make consumption endogenous using linearity assumptions. The second one is to move up from the input-output model towards general equilibrium models where consumption is endogenous and price and income responsive. If Ghosh's model is not 'plausible' because value–added is unresponsive to output reallocations, then a similar case could be made for Leontief's model being somewhat 'implausible' too because of the fact that consumption is unresponsive to income generation, which is also a rather peculiar behavior.

This having been said, perhaps the road to endow Ghosh's model with a bit more plausibility is formally similar to the road taken with Leontief's model: close it with an additional layer of endogeneity. If for Leontief we make the 'driving demand' force (consumption) endogenous, then for Ghosh we may attempt to make the 'driving supply' force (value–added) endogenous. Since Ghosh's model shares the basic mathematical linearity of the standard interindustry model, closing it may follow the same formal logic. We first need a rule stipulating a relationship between value–added and some output measure which is allocation compatible and, secondly, we need an instrument that reflects and captures external supply shocks that are subsequently incorporated into the allocation system.

The paper is organized as follows. In Section 2 we extend Ghosh's model by formulating an alternative closure rule for solving the aforementioned lack of value-added responsiveness. In our view, this closure rule follows the original allocation rules more properly than previous work. Davis and Salkin (1984), for instance, make value-added endogenous using the Leontief perspective of input coefficient rather than Ghosh's output coefficient idea. We verify the consequences of the proposed closure rule for a correct accounting of output changes as well as value–added changes. We then illustrate the results with some numerical examples in Section 3 which show that the allocation system is consistent both at the sectoral and aggregate levels. Section 4 concludes.

2. CLOSING GHOSH'S MODEL: AN ALTERNATIVE APPROACH

Ghosh himself (1958) formulated his model as a vision of two quite different economic scenarios, one referring to capitalist economies with producers having monopolistic power, and therefore control of the supply, another referring to non-market economies with a central planner whose mission is to allocate output among the intervening agents. In both cases, the connecting formal aspect is the fact that supply may be subject to restrictions and under the control of some exogenous agent. Furthermore, in both interpretative cases the implausibility pointed out by Oosterhaven still stands. Because of the recent fundamental advances of industrial organization theory, we believe Ghosh's approach is way too simplistic and restrictive as a sensible tale for a monopolistic approach even in a disaggregated setting[1]. We will therefore approach the lack of plausibility issue using the context of a non-market economy as the basic storyline. In this setting we will assume that decisions on output allocation are taken by a benevolent central planner whose assigned task is to enhance the collective good and guarantee a viable distribution of goods. This alleged economy comprises *n* productive units and distinguishes a private agent (citizens) and a public one (the planner). The private agent provides labour services to all sectors and in exchange receives income (value–added) that is used to finance his consumption needs and his contribution to the sustainment of the collective. From this contribution the planner provides infrastructure services that are used in the allocation process. These services also provide value to the collective, which is in turn used by the public agent to facilitate goods to society in the form of public goods. The aggregate level of these public goods is of course constrained by the overall contributions to the collective[2].

Let us begin considering a reference or benchmark allocation table for this *n* good-*n* sector economy. The reference data in value flows for such an economy is represented in Table 1. Data in this Table represent an economic arrangement that is allocation feasible in the aggregate as well as budget feasible for all agents involved. All magnitudes are value magnitudes.

[1] See Nikaido (1975) for a disaggregated approach to monopolistic power and Tirole (1988) for a state of the art in industrial organization.
[2] In a market economy these citizens' contributions would take the form of taxes and the return to society would of course be labelled as public consumption and investment.

Table 1: Benchmark allocation data

	Sector 1	Sector 2	...	Sector n	Private Agent	Collective	Total
Sector 1	z_{11}	z_{12}	...	z_{1n}	f_1	c_1	x_1
Sector 2	z_{21}	z_{22}	...	z_{2n}	f_2	c_2	x_2
....	...	...	...	...	...	...	...
Sector n	z_{n1}	z_{n2}	...	z_{nn}	f_n	c_n	x_n
Value-added	v_1	v_2	...	v_n			
Collective	t_1	t_2	...	t_n			
Total	x_1	x_2	...	x_n			

Because of viability the following accounting identities hold true:

$$\sum_{j=1}^{n} z_{ij} + f_i + c_i = x_i \quad (i = 1, 2, ..., n) \tag{1}$$

$$\sum_{i=1}^{n} z_{ij} + v_j + t_j = x_j \quad (j = 1, 2, ..., n) \tag{2}$$

In Expressions (1) and (2) we have that z_{ij} is the amount of good i flowing to sector j, f_i is the consumption of good i by the private agent, c_i is collective consumption of good i, v_j is income accruing to the private agent in sector j whereas t_j is the materialization of the contribution to the collective. The identity in expression (1) shows, by rows, the 'output' distribution for each of the goods in terms of gross output x_i. Using columns, identity (2) shows the 'input' repercussions of the said output allocations that are budget feasible. Because of a 'Walras-like' aggregate feasibility constraint (1) and (2) imply:

$$\sum_{i=1}^{n} f_i + \sum_{i=1}^{n} c_i = \sum_{j=1}^{n} v_j + \sum_{j=1}^{n} t_j \tag{3}$$

The left-hand side of (3) can be interpreted as national output calculated from the expenditure side. The right-hand side, in turn, is national output as obtained from the income side. Alternatively, if the private and public agents behave so as to satisfy some sort of disciplined budget constraint, such as:

$$\begin{aligned} \sum_{i=1}^{n} f_i &= \sum_{j=1}^{n} v_j \\ \sum_{i=1}^{n} c_i &= \sum_{j=1}^{n} t_j \end{aligned} \tag{4}$$

then the national output accounting identity (3) follows from aggregation of the budget constraints in (4).

In matrix terms the input-output data information in Table 1 takes this shape:

$$\mathbf{Z} \cdot \mathbf{e} + \mathbf{f} + \mathbf{c} = \mathbf{x} \tag{1'}$$

$$\mathbf{e}' \cdot \mathbf{Z} + \mathbf{v}' + \mathbf{t}' = \mathbf{x}' \tag{2'}$$

where **e** is a summation vector. The rest of the notation with matrix **Z,** column vectors **f**, **c** and $\mathbf{x}$, and row vectors $\mathbf{v}'$, $\mathbf{t}'$ and $\mathbf{x}'$ is self-explanatory. Let us consider now the matrix $\mathbf{B}$ of 'allocation' coefficients, that is to say, the information on how output is sectorally distributed among productive agents:

$$\mathbf{B} = b_{ij} = \left(\frac{z_{ij}}{x_i} \right) = \hat{\mathbf{X}}^{-1} \cdot \mathbf{Z} \tag{5}$$

The notation $\hat{\mathbf{X}}$ stands for the diagonalised version of vector $\mathbf{x}$ while $\hat{\mathbf{X}}^{-1}$ is the inverse matrix of $\hat{\mathbf{X}}$. Solving for **Z** in Expression (5) and substituting in identity (2') we obtain now an equation in $\mathbf{x}'$:

$$\mathbf{e}' \cdot \mathbf{Z} + \mathbf{v}' + \mathbf{t}' = \mathbf{e}' \cdot \hat{\mathbf{X}} \cdot \mathbf{B} + \mathbf{v}' + \mathbf{t}' = \mathbf{x}' \cdot \mathbf{B} + \mathbf{v}' + \mathbf{t}' = \mathbf{x}' \tag{6}$$

This equation corresponds to the familiar 'supply-driven' equation of Ghosh and allocated output can be meaningfully solved provided matrix **B** satisfies the usual viability condition[3]:

$$\mathbf{x}' = (\mathbf{v}' + \mathbf{t}') \cdot (\mathbf{I} - \mathbf{B})^{-1} \quad (7)$$

We will now postulate a possible closing for value-added. Define the coefficient λ_i as value-added per unit of aggregate consumption. This coefficient expresses, in normalized terms, the value-added contribution in each sector *i* required for a unit of private consumption to be available[4]. Note that this coefficient might also be considered as an allocation coefficient for value-added. However, λ_i follows allocation rules in terms of final private consumption rather that overall output levels. Define too d_j as the allocation coefficient for the consumption of good *j* by the private agent:

$$\lambda_i = \frac{v_i}{\sum_{j=1}^{n} f_j} \qquad d_j = \frac{f_j}{x_j} \quad (8)$$

From (8) we easily find:

$$v_i = \lambda_i \cdot \sum_{j=1}^{n} d_j \cdot x_j \quad (9)$$

so that in compact matrix terms (9) becomes:

[3] **B** is non-negative, productive and (**I-B**) is singular. See Waugh (1950). Also, $(\mathbf{I\text{-}B})^{-1}$ can be expressed as a convergent matrix series.
[4] Recall that in Leontief's standard closed model consumption is made endogenous in terms of labor requirements. The closure rule in (8) can therefore be seen as reciprocal to the one used in Leontief's case.

$$\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_n \end{pmatrix} \cdot (d_1, d_2, ..., d_n) \cdot \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \boldsymbol{\lambda} \cdot \mathbf{d}' \cdot \mathbf{x} \qquad (10)$$

where the matrix $\boldsymbol{\lambda} \cdot \mathbf{d}'$ reflects the value-added allocation coefficients that derive from the private agent consumption. Observe too that the coefficients $(\lambda_i \cdot d_j)$ in this matrix are fully consistent with the allocation coefficient idea proposed by Ghosh and they are the basis for making sectoral value-added endogenous and quantity responsive. When matrix $\boldsymbol{\lambda} \cdot \mathbf{d}'$ is post-multiplied by vector **x**, we obtain scalable value-added in response to changes in output levels, which in turn are driven by the exogenous changes in the value contributed to the collective, i.e. **t**.

We now incorporate expression (10) into (7) via its transpose to obtain:

$$\mathbf{x}' = \mathbf{x}' \cdot \mathbf{B} + \mathbf{v}' + \mathbf{t}' = \mathbf{x}' \cdot \mathbf{B} + \mathbf{x}' \cdot \mathbf{d} \cdot \boldsymbol{\lambda}' + \mathbf{t}' \qquad (11)$$

We can now solve again for $\mathbf{x}'$ under this additional assumption to find:

$$\mathbf{x}' = \mathbf{t}' \cdot \left(\mathbf{I} - \mathbf{B} - \mathbf{d} \cdot \boldsymbol{\lambda}'\right)^{-1} \qquad (12)$$

The inverse matrix in Expression (12) can be interpreted as the ‘extended’ Ghosh inverse since it incorporates allocation coefficients for material flows, **B**, and value-added flows, $\mathbf{d} \cdot \boldsymbol{\lambda}'$. Supply shocks are caused by the exogenous actions of the central planner as represented by changes decreed in contributions to the collective, $\Delta \mathbf{t}'$, for example. The output vector $\mathbf{x}'$ that satisfies condition (12) can be interpreted as an ‘allocation equilibrium’ for this economy, and such an equilibrium turns out to be consistent with the allocation rules implicit in **B** and $\mathbf{d} \cdot \boldsymbol{\lambda}'$ and with the value of contributions to the collective, i.e. $\mathbf{t}'$. Allocated output is coherently distributed among sectors while at the same time it is value feasible. The new 'equilibrium' can be visualized in differential terms from:

$$\Delta \mathbf{x}' = \Delta \mathbf{t}' \cdot \left(\mathbf{I} - \mathbf{B} - \mathbf{d} \cdot \boldsymbol{\lambda}' \right)^{-1} \tag{13}$$

Provided the extended allocation matrix $\mathbf{B} + \mathbf{d} \cdot \boldsymbol{\lambda}'$ is also productive, in the sense that $(\mathbf{B} + \mathbf{d} \cdot \boldsymbol{\lambda}') \cdot \mathbf{x}' \leq \mathbf{x}'$ for all possible row vectors $\mathbf{t}' \geq 0$, then the new 'equilibrium' defined in (13) might be also rewritten as a power series of the form:

$$\Delta \mathbf{x}' = \Delta \mathbf{t}' + \Delta \mathbf{t}' \cdot \left(\mathbf{B} + \mathbf{d} \cdot \boldsymbol{\lambda}' \right) + \Delta \mathbf{t}' \cdot \left(\mathbf{B} + \mathbf{d} \cdot \boldsymbol{\lambda}' \right)^2 + \ldots + \Delta \mathbf{t}' \cdot \left(\mathbf{B} + \mathbf{d} \cdot \boldsymbol{\lambda}' \right)^k + \ldots \tag{14}$$

Expression (14) indicates that the endogenous effect on output levels can be decomposed into the following components: the "pure" impact of the contribution to the collective that adds value to production, i.e. $\Delta \mathbf{t}'$. This "pure" impact in output should be allocated in the system in the form of intermediate and private final demand, i.e. $\Delta \mathbf{t}' \cdot \left(\mathbf{B} + \mathbf{d} \cdot \boldsymbol{\lambda}' \right)$ generating additional multiplicative effects in output levels. This "second-round" impact further increases production in the remaining sectors round by round according the structure of the "allocation path" defined in (14), i.e. $\Delta \mathbf{t}' \cdot \left(\mathbf{B} + \mathbf{d} \cdot \boldsymbol{\lambda}' \right)^2 + \ldots + \Delta \mathbf{t}' \cdot \left(\mathbf{B} + \mathbf{d} \cdot \boldsymbol{\lambda}' \right)^k + \ldots$

Using the new output in the allocation equilibrium from expression (12) (or its differential version from (13)) it is straightforward to obtain a new and balanced Ghosh table using the technological information in matrices $\mathbf{B}$ and $\mathbf{d} \cdot \boldsymbol{\lambda}'$, along with the allocation coefficients for private consumption d_j from expression (8) and similarly constructed allocation coefficients for collective consumption (i.e. c_j / X_j using the benchmark data in Table 1). The new output level is fully consistent with the set of allocation rules and satisfies as well all the viability restrictions as described in expressions (1) and (2).

4. CLOSING GHOSH'S MODEL: A NUMERICAL EXAMPLE

To illustrate the formal description and interpretation of the closed Ghosh model presented in Section 3, we use now a numerical example. We start using a 3 sector, 3 good economy whose reference data in value flows is shown in Table 2a.

TABLE 2a: Reference data: Numerical example

	Sector 1	Sector 2	Sector 3	Private Agent	Collective	Total
Sector 1	*30*	*20*	*10*	*35*	*5*	*100*
Sector 2	*20*	*10*	*40*	*5*	*25*	*100*
Sector 3	*10*	*20*	*5*	*30*	*35*	*100*
Value-added	*20*	*10*	*40*			*70*
Collective	*20*	*40*	*5*			*65*
Total	*100*	*100*	*100*	*70*	*65*	

Let us assume now that this economic system has a benevolent central planner that decides to allocate additional resources to sector 1 in such a way that its value contribution increases by 1 unit of value. As an example, these additional exogenous resources decided by the central planner would be materialized in new equipment whose services could be used in sector 1 and that increases production levels either in value or quantity terms. This refers to what we have named the "pure impact" in Expression (14), i.e. $\Delta \mathbf{t}'$. This impact additionally boosts output levels due to the multiplicative effects generated by this supply shock in the remaining sectors according to the structure of the allocation path in (14), i.e. if additional intermediate supply is allocated to the remaining sectors, there would be endogenous supply effects coming from these sectors that further affect the output values in sector 1 increasing overall value-added in the system. By repeating the same decision in sectors 2 and 3 we can envision and compare the overall results in terms of the new allocation equilibria in Table 3.

TABLE 3: Synthetic indicators after evaluating Δt_i=1 units sequentially in each sector.

	% Endogenous Changes in "key" variables				
Exogenous Shock	Δv	Δx_1	Δx_2	Δx_3	Δx
Δt_1 **=1 unit**	*2.10*	*2.60*	*1.18*	*1.65*	*1.81*
Δt_2 **=1 unit**	*1.29*	*1.08*	*1.81*	*1.41*	*1.43*
Δt_3 **=1 unit**	*1.46*	*0.96*	*0.84*	*2.12*	*1.31*

As it can be asserted from Table 3, the impact of an identical unitary exogenous increase in the contribution from the collective is distributed in an unequal way, reflecting the distinct values of the allocation coefficients in each sector. If the flow is contributed to Sector 1, for instance, total value-added in all three sectors increases by 2.10 percent and, on average, total output increases by 1.81 percent. Differently to the open version of the Ghoshian approach, value-added changes everywhere and does so simultaneously and homogeneously. The homogeneity of the endogenous change in value-added is due to the "allocation rules" as dictated by the matrix $\mathbf{d} \cdot \boldsymbol{\lambda}'$. The set of Tables 2b-2d in the Annex show the readjusted allocation flows in sectoral detail. Note that each of the additional exogenous units contributed to each sector is fully and endogenously redistributed over collective consumption according to allocation rules. This is because, following the disciplined budget constraints defined in expression (4) of this closed version of the Ghoshian approach, the exogenous unit contributed from the central planner cannot be withheld by the private agent but rather devoted to collective consumption.

According to the simulation results presented in Table 3, if the benevolent central planner wished to maximise economy-wide effects, the contribution to the collective should be decreed in Sector 1. This is so because the implied reallocation effects, both in value-added and in output, are higher here than those obtainable should the contribution be allotted in Sectors 2 or 3.

These conclusions are independent from the benchmark value flows of the contribution to the collective. Consequently, there is no need to perform any normalization to appraise their robustness. Notice that the exogenous shock is carried out homogeneously in all three sectors and the impacts in Table 3 depict the percentage between benchmark and simulated allocations.

These numerical examples of the closed Ghosh model outlined in Section 2 approximate better, we believe, the initial idea posed by Ghosh. In his seminal work, this author highlighted that his approach could be used, in planned economies, for the assessment of economy-wide impacts of government employment programs. The main question that Ghosh wanted to address using his modelling proposal was the following: if the labour force is forcefully allocated in a given sector, what would the economy-wide impact be according to the allocation rules that are used in planned economies?. The economy-wide output impacts of the open version of this model turn out not to be value feasible when answering this question (Oosterhaven, 1988). Our closed version, however, not only makes it possible to answer this question in a more plausible way but also helps in understanding the initial purposes of A. Ghosh. Needless to say, we were not attempting to propose here a complete theory of planning for centralized economies.

5. CONCLUSIONS

The objective of this paper is simply to reconsider the initial purposes of A. Ghosh while trying to contribute to the extensive debate in the literature around his original model published in 1958. Since then, there has been an over-use of his model by researchers that, after some time, has given rise to an over-criticism.

To this end we describe a way of closing the Ghoshian approach that resolves, to some extent, the implausibility problem that afflicts its open version (Oosterhaven, 1988, 1989). Supply shocks in this modified version of the Ghosh model stem from the actions of a benevolent central planner. This central planner exogenously contributes to production which will generate value to the economic system, applying it in one or more sectors. This initial impact is spread through the economic system further boosting output levels that, differently to the open version, are accompanied by simultaneous endogenous and allocation compatible increases in value-added. Therefore, our proposal for closing Ghosh's model makes it more plausible.

Lastly, we would like to include here a comment about duality (Oosterhaven, 1996). In our view we coincide with the conclusion of De Mesnard (2009a) in the sense that duality is not fulfilled under the original Ghoshian approach. Consequently, under the original Ghosh model, we have to know the ex-post changes in quantities, i.e. after the change in value-added has taken place, to identify the ex-post changes in prices. Note that in our alternative closed version of the Ghosh model, all changes are expressed in value terms and thus, duality is not accomplished as it is the case under the original open version of the Ghosh model (1958).

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ANNEX OF TABLES

TABLE 2b: Simulated changes of the Reference data after evaluating an exogenous supply shock in Sector 1: $\Delta t_1 = 1$ units of value.

	Sector 1	Sector 2	Sector 3	Private Agent	Collective	Total
Sector 1	30.78	20.52	10.26	35.91	5.13	102.60
Sector 2	20.24	10.12	40.47	5.06	25.29	101.18
Sector 3	10.17	20.331	5.08	30.49	35.58	101.65

	Sector 1	Sector 2	Sector 3
Value-added	20.42	10.21	40.84
Collective	**21.00**	40.00	5.00
Total	102.60	101.18	101.65

TABLE 2c: Simulated changes of the Reference data after evaluating an exogenous supply shock in Sector 2: $\Delta t_2 = 1$ units of value

	Sector 1	Sector 2	Sector 3	Private Agent	Collective	Total
Sector 1	30.33	20.22	10.11	35.38	5.05	101.08
Sector 2	20.36	10.18	40.72	5.09	25.45	101.81
Sector 3	10.14	20.28	5.07	30.42	35.49	101.41

	Sector 1	Sector 2	Sector 3
Value-added	20.26	10.13	40.51
Collective	2.00	**41.00**	5.00
Total	101.08	101.81	101.41

TABLE 2.d: Simulated changes of the Reference data after evaluating an exogenous supply shock in Sector 3: $\Delta t_3 = 1$ units of value.

	Sector 1	Sector 2	Sector 3	Private Agent	Collective	Total
Sector 1	30.29	20.19	10.10	35.34	5.05	100.96
Sector 2	20.17	10.08	40.34	5.04	25.21	100.84
Sector 3	10.21	20.42	5.11	30.64	35.74	102.12

	Sector 1	Sector 2	Sector 3
Value-added	20.29	10.15	40.58
Collective	20.00	40.00	**6.00**
Total	100.96	100.84	102.12