School of Economics and Management

TECHNICAL UNIVERSITY OF LISBON

Department of Economics

José Pedro Pontes

Microeconomics of Space – a Selective Survey

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Microeconomics of Space – a Selective Survey

by

José Pedro Pontes[1]

1. Introduction: why to study the microeconomics of space?

A representative firm takes two kinds of decisions concerning space: its location and the set of prices it quotes in each point in space.[2] Why is it important for an applied micro economist to examine these decisions?

Let us begin by defining the terminology. By "fob price" we mean the price set by the firm in its location, while "delivered price" labels the full price that the consumer pays at its living place, including the transport cost of the product between the locations of supply and demand.

Microeconomics has been traditionally dominated by the paradigms of "perfect market" and "perfect competition". A "perfect market" is a structure supporting transactions such that each consumer and each producer know the prices bid by all consumers and the prices asked by all producers. "Perfect competition" means that, among other assumptions, the products supplied by the firms are completely homogeneous, so that each consumer is indifferent among them when they are supplied at the same price. Moreover, "perfect competition" means that the number of producers competing in each market is high. Together, these two

[1] ISEG, Technical University of Lisbon and UECE. Contact address: ppontes@iseg.utl.pt.
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[2] A third type of spatial decision which will not be tackled here concerns the amount of land that will be used by the firm for its productive activity (see, e.g., PONTES, 2001, chapter 3)

assumptions (homogeneity and large number of producers) ensure that each firm is arbitrarily small in relation to the market, so that it cannot influence the price and that it faces an infinitely elastic demand curve.

"Perfect market" and "perfect competition" jointly determine that each product has a unique price at the market where the product is traded. For both conditions to hold, the market should be close to a "point" in geographical terms. The word "market" originally meant this physical "meeting point" (for instance, the stock exchange, or the commodities exchanges).

However, economic agents are spread over space rather than clustered in market "points". The model by ENKE (1942) tries to reconcile the dispersed pattern of agents with the assumptions of perfect competition (see Figure 1).

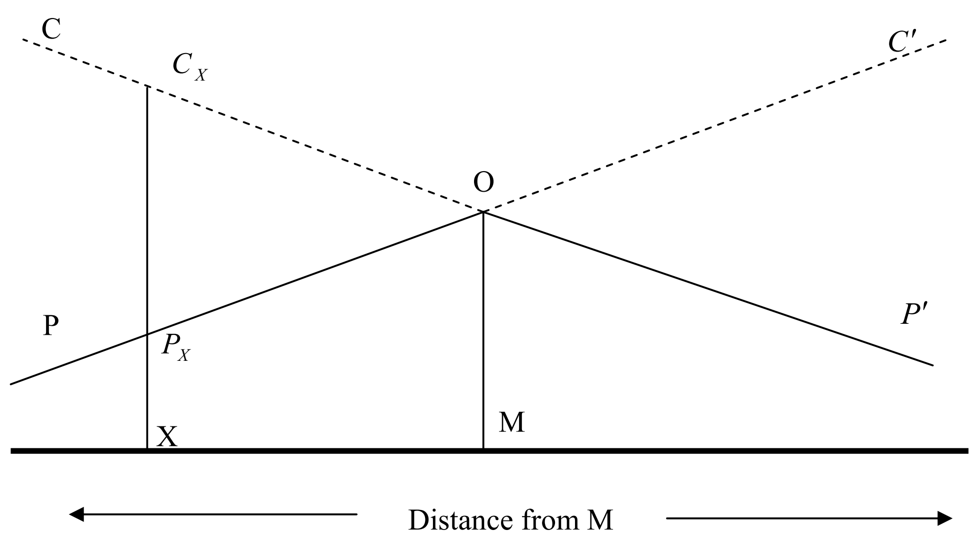


Figure 1: ENKE (1942)'s model of "central exchange"

In Figure 1, the consumers and producers are uniformly distributed along the line. All transactions are made at a "central exchange" in M, where a market price OM is formed through the meeting of all sellers and buyers. However, the real prices which are received by the producers (given by the curve POP') and paid by the consumers (given by the curve COC') differ from the market prices on account of freight charges. The slopes of line segments CO, PO and OC', OP' reflect the transport cost by unit of distance. Hence, a consumer located in X pays a delivered price C_X and a producer in the same location receives a fob price P_X. The difference between these prices and the market price corresponds to the transport cost between the "central exchange" M, the locations of the consumer and of the producer in X.

Why do the seller and the buyer placed in point X use the central exchange in M in order to transact instead of trading locally? After all, exchanging the product locally in X would allow saving transport costs for both groups of agents. Instead, locating the transactions in M brings two kinds of advantages:

1. In M, a large number of homogeneous buyers and sellers meet, so that none enjoys market power: each firm faces an infinitely elastic demand function, so that "perfect competition" prevails.
2. The localization of transactions yields "perfect information" of each side of the market about the prices bid and asked. Each agent has this kind of information without having to incur search costs: he must not travel between the sellers in order to inquire about prices.

However, the size of the region should be bounded from above. A too large region entails very high transport costs that do not allow the use of the "central exchange" as a transaction device.

Hence, it is usually assumed that the consumers and producers in an area are partitioned in a set of "regions" (PONTES, 1987). This partition is independent of the regional prices of the product. Within each "region", fixed numbers of consumers and producers transact through a "central exchange". Trade between "regions" takes place through the network of "central exchanges".

Demand by the consumers and supply by the producers are modeled by means of regional functions of demand and supply. Besides consumers and producers, a third category of agents (labeled as "traders") transports the product between regions. Let the term "excess supply" mean the difference between regional supply and regional demand at a given price. The equilibrium is a profile of regional prices such that:

1. The aggregate (across all regions) excess supply is zero, i.e. total exports equal total imports of the product in the spatial economy.
2. Each delivered price should not exceed the sum of the fob price and the transport cost between the origin and destination regions (i.e., the profit of the trader is non-positive).
3. If the export flow from an origin region to a destination is positive, then the delivered price equals the sum of the fob price and the transport cost between the regions (i.e., the profit of trader is zero).
4. If the delivered price is smaller than the sum of the fob price and the transport cost (i.e., the trader's profit is negative), the export flow is zero.

These conditions amount to the traditional conditions for a set of prices to be a competitive equilibrium, namely:

- Individual equilibrium: at these prices every agent maximizes either utility (consumers through the regional demand functions) or profit (producers through the regional supply functions; traders through the equilibrium conditions 2, 3 and 4).
- Market equilibrium: at these prices, total exports equal total imports, so that the interregional market of the product clears (equilibrium condition 1).

The equilibrium is depicted in Figure 2 for the case of two regions.

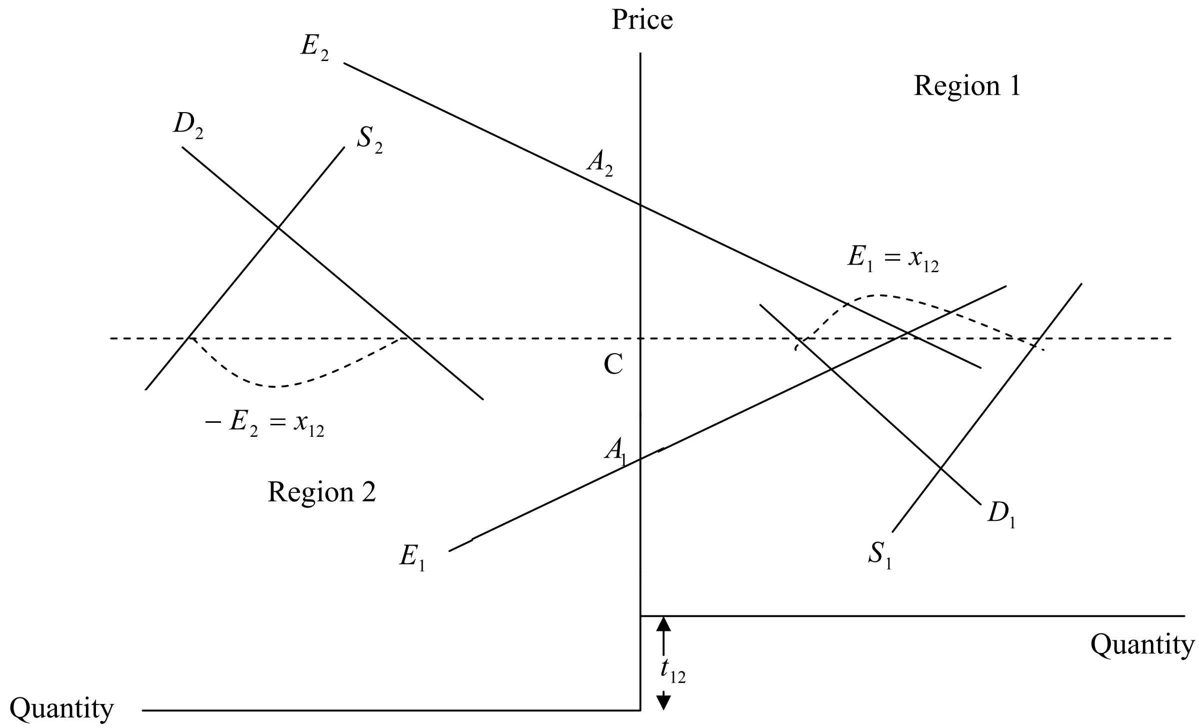


Figure 2: Competitive equilibrium between two regions

Where:

$D_1, D_2 \equiv$ Demand functions in regions 1,2

$S_1, S_2 \equiv$ Supply functions in regions 1,2

$E_1, E_2 \equiv$ Excess supply functions in regions 1,2

$A_1, A_2 \equiv$ Equilibrium prices in autarky in regions 1, 2

$C \equiv$ fob equilibrium price under interregional trade

$t_{12} \equiv$ Transport cost between the two regions

$C + t_{12} \equiv$ Delivered price under interregional trade

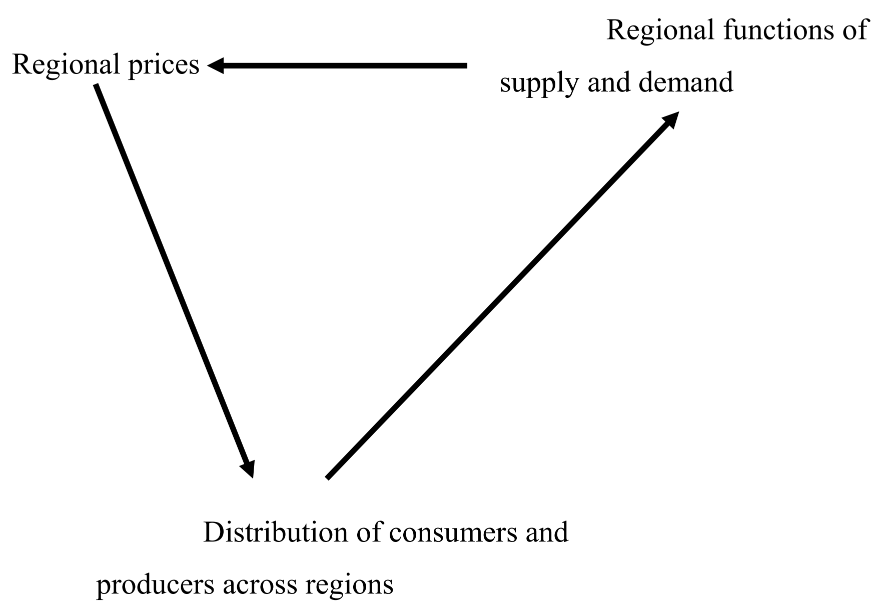
$x_{12} \equiv$ Exports from region 1 to region 2

With more than three regions it becomes difficult to find an analytical solution to the spatial price equilibrium model. This solution becomes feasible if we state the model as an optimization problem where either aggregate transport cost is minimized or social welfare is maximized subject to the following constraints (see PONTES, 1987):

1. Interregional product flows are nonnegative.
2. Aggregate imports of the product by a region should not be lower than regional demand.
3. Aggregate exports by a region should not exceed regional production.

Then, it is easy that the necessary conditions of these problems reproduce the conditions of the spatial equilibrium problems: Kuhn-Tucker multipliers define regional prices that clear supply and demand in each region; at these prices, all categories of agents (consumers, producers and "traders") maximize either utility or profits.

However, the spatial equilibrium model bears a contradiction that follows from the fact that the partition of consumers and producers across the regions is fixed. Nevertheless, it determines the formation of equilibrium prices. Or it is widely known that the matching between producers and consumers is influenced by regional prices. If p_1 is much higher than p_2, then consumers in region 1 will prefer to buy the product in region 2, and this specially if t_{12} is not too high. In sum, we have the following indetermination:



In order to avoid this indetermination, the trading units should be conceived as individual consumers and producers rather than “regions” or “central exchanges”. However this shift would undermine the assumptions of “perfect competition” (large number of homogeneous sellers in each point of space) and “perfect market” (zero search costs of information about asked and bid prices). The consideration of space makes perfect competition an unrealistic description of the operation of markets.

Another instance of breakdown of the competitive price system can be found in KOOPMANS and BECKMANN (1957) following from the combination of the indivisibility of the productive activity and the technological interdependence of the location of plants through the exchange of intermediate goods.

It is an empirical fact that productive plants are often indivisible. For many productive activities, there is a minimum efficient scale of production. Let us assume that there are n indivisible plants that must be located or assigned to n different locations. There are n^2 assignments which are each given by a pair plant-location. For each of these assignments a profit score is defined. We make now the crucial assumption that the plants are technologically independent, i.e. the profitability of a plant in a location does not vary with the location chosen by another plant.

For instance, if there are 4 plants and 4 locations, a possible set of profit scores is given by the matrix:

	Locations				
		1	2	3	4
Plants	1	25	20	5	19
	2	18	3	0	12
	3	22	4	2	12
	4	16	7	-2	10

Then, to find feasible locations for the plants amounts to selecting a unique plant –location pair in each row and column of the matrix. If we compare all the feasible assignments from the viewpoint of aggregate profitability, we obtain the optimal location pattern. In this example, the optimal assignment is given by

	Locations				
		1	2	3	4
Plants	1	25	<u>20</u>	5	19
	2	18	3	<u>0</u>	12
	3	<u>22</u>	4	2	12
	4	16	7	-2	<u>10</u>

In this table, the underlined cells express optimal locations. The overall profitability of the optimal assignment is 52 units. It should be remarked that the most profitable pair (plant 1 in location 1) does not occur in the optimal assignment.

Each feasible assignment can be expressed by a *permutation matrix*, i.e. a matrix that has exactly a 1 in each column and row and zeros elsewhere. The permutation matrix of the optimal assignment is

T	Locations				
h Plants		1	2	3	4
i	1	0	1	0	0
	2	0	0	1	0
s	3	1	0	0	0
	4	0	0	0	1

is a problem of centralized planning. Can the optimal allocation be sustained if instead the owners of plants take decentralized location decisions based on their knowledge of profit scores and on some kind of prices (namely rentals of the plants and sites)?

The answer is yes. This can be seen if we introduce the possibility of fractional assignments where a plant can be distributed by several locations and a location can be occupied by shares of different plants. We require that the sum of plant shares across locations and across plants sums 1 so that a location is occupied exactly by a plant. Then, the assignment that maximizes overall profit can be found by means of a linear programming problem.

The feasible set of a linear programming problem is a convex set with a finite number of extreme points (vertices). In such a problem, the optimum is reached either in a vertex or in two adjacent vertices. In the latter case, it is also reached in the face that connects the two adjacent extreme points (see Figure 3).

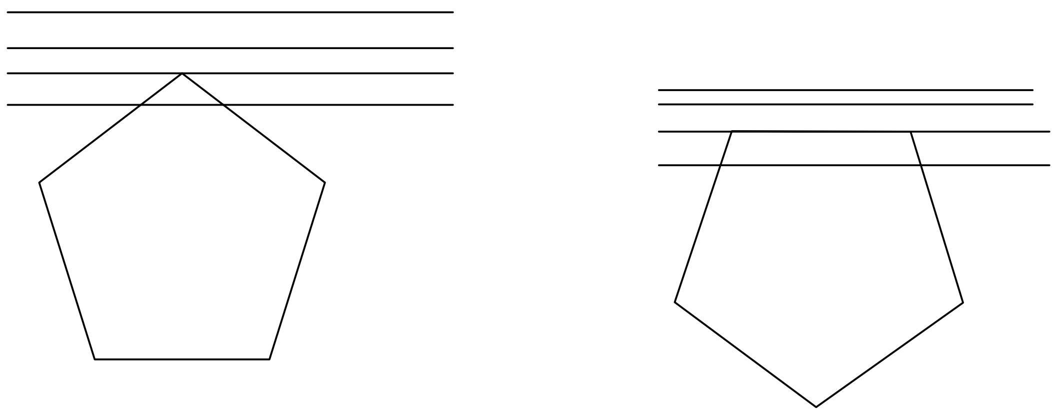


Figure 3: Optimum of a linear programming problem.

It can be established that the extreme points of the feasible set of the linear programming problem correspond to the permutation matrices in the problem of integral optimal assignment. Hence, when we exclude fractional assignments and pass to the optimal location of indivisible plants, we do not lose any optimal solutions.

Since a feasible solution of a linear programming location problem is optimal if and only if it has associated a set of prices (rentals of locations and of plants in this case), the same property holds for the optimal locations of indivisible plants.

Let us assume that there is a rental of each location and a rental of each plant and a score of profitability for the assignment of the plant to the location. Then, the sum of the costs (the rentals of location and plant) is higher than or equal to the profit of the assignment. Only in the case of an optimal plant /location assignment is the relation satisfied as equality. This means that, given the plant rentals, the owner of a location maximizes its rental in the optimal assignment. Conversely, given the location rentals, the owner of a plant maximizes its rental also in the optimal assignment. The set of prices (rentals) together with the knowledge of the profits related with each location-plant pair sustains the optimal set of locations of indivisible plants. Consequently, a competitive price system works in a spatial economy where plants are indivisible (may be, on account of increasing returns to scale in production).

Let us assume now instead that plants are technologically interdependent through the exchange of intermediate goods. Now the profit of a plant depends not only upon its location but also on the locations of the plants that supply its intermediate goods. The objective function of the integral assignment problem contains, besides the term connected with the pairing of plants and locations, a term that represents the transport cost of the traded inputs

Again, we can pass to a fractional assignment problem, where each plant is distributed across all locations and each location can be occupied by shares of all plants. This problem assumes that plants are divisible and it has the same constraints as before (namely the shares of a plant should sum 1 across locations and the shares of plants in a given location should sum 1 too). An additional constraint arises: the sum of the production of each intermediate good in a location with the imports of this good into that region should equal the use of the good in the region plus the amount of it that is exported.

It is clear that no integral solution with indivisible plants is an optimum of the fractional assignment problem. Assume that the gross profit score of each plant is invariant with relation to location. Then it is clear that profit maximization across locations by each plant is equivalent to minimization of the costs related with the movement of intermediate goods among plants. Consequently, if there are n plants and n locations, the optimum of the fractional assignment problem occurs when $\frac{1}{n}$ of each plant is placed in each location because then the transport costs of the intermediate goods are zero.

As no integral assignment is an optimum of linear programming (fractional assignment) problem, there is no competitive price system that sustains the optimal assignment plant/location as the outcome of decentralized decisions of the owners of plants and sites. Given any prices, there will always be a plant owner that has an incentive to shift location. As KOOPMANS and BECKMANN say:

> There will be always be an incentive for someone to seek a location other than the one he holds… there would be a continual game of musical chairs. (KOOPMANS and BECKMANN, 1957, p- 70)

2. Location of firms under oligopoly.

The same conclusion (that a perfectly competitive outcome is incompatible with the existence of a lengthy spatial market) can be reached from the viewpoint of oligopoly, i.e. an industry with few sellers. COURNOT (1838) devised the following framework. Two identically located firms (for instance, two springs of mineral water) sell homogeneous products to consumers agglomerated in a market that is a “point” (a market without length). Each firm competes through the choice of an output and faces the following trade-off: by selling one more unit it receives an additional price while depressing the prices at which all the other units are sold. Then a Cournot equilibrium (later generalized as “Nash equilibrium”) is achieved when each firm sells a quantity that maximizes its profit given the output chosen by its rival. In equilibrium, the output chosen by each firm is a “best reply” to the competitor’s output, so that no firm has an incentive to deviate unilaterally from the equilibrium output. When these outputs are sold, the equilibrium price of the homogenous good (the mineral water) is strictly higher than the unit cost of production, so that each firm makes a positive profit.

BERTRAND (1883) criticized Cournot’s results, saying that firms really compete through the quoting of prices rather than quantities. In this setting, two firms producing an homogenous good under constant and equal marginal costs will charge equilibrium prices equal to marginal costs. Consequently, both firms will have zero profits and the result is similar to perfect competition.

The rationale behind Bertrand’s claim is the discontinuity of the demand function addressed to each firm in the level where the prices of the sellers are equal.

In this level, if a firm charges a price slightly lower than the competitor's, it will get all the consumers. Since this applies to both firms, prices will fall successively until they reach the unit production costs. Demand discontinuity is crucially linked with the homogeneity of the two products.

HOTELLING (1929) restores the noncompetitive outcome under price competition by assuming that the market is lengthy (a line segment) rather than a point. Figure 4 depicts the spatial market.

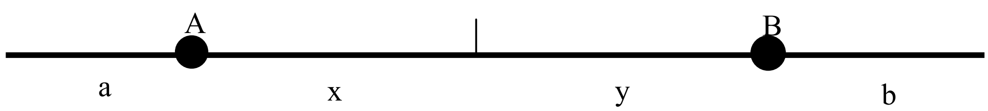


Figure 4: HOTELLING's spatial market

The consumers are uniformly spread in a line segment of length *l* (which may represent Main Street). Two firms selling homogenous products are located in points A and B. The distances between each firm and the closest extreme point of the market are given by a and by b, respectively.

The firms have identical constant unit production costs, which we assume to be zero w.l.g. The firms sell fob mill prices p_1 and p_2 and each consumer carries the product in the distance between the firm's location and his address, the transport cost per unit of distance being given by t. The delivered price of the product for a given consumer is the sum of the fob mill price and the transport cost. It is further assumed that each consumer purchases a unit of the product per unit of time irrespective of its price.

This model deals with two different problems: the setting of fob mill prices and the choice of locations by the firms. He implicitly assumes that the firms firstly select locations and then set prices. This is a two stage game that is, as usual solved by backward induction. Thus we first tackle the formation of prices and then the firms' locations.

The main point stressed by HOTELLING (1929) is that the demand addressed to each firm becomes continuous if the firms agree somehow to share the market, i. e. if the difference of the fob mill prices does not exceed the transport cost in the distance between the firms:

$$|p_1 - p_2| \leq t(x+y) \tag{1}$$

If this condition is met, the consumers placed between the firms will be split in two market areas whose boundary is given by the condition of indifference for the marginal consumer to purchase the product to either firm. This means that the delivered price of each firm to that consumer is the same. The market area of each firm comprehends its hinterland (*a* or *b*) and the consumers in the intermediate region for whom it is cheaper in terms of delivered price to purchase to the firm (segments *x* or *y*). If a firm lowers its fob mill price, some consumers will be transferred from the competitor to the firm. However, *as long as the previous condition is met*, the competitor will retain a share of its customers, who prefer to buy from it at a somehow higher price, so that the demand functions addressed to the firms are continuous.

As the outputs of the firms are proportional to the size of market areas, it is easy to write the profit functions of the firms and derive equilibrium fob mill prices: the price set by each firm maximizes its profit given the price set by the rival firm. Each price is a best reply to the price set by the rival firm. It is easy to show that these prices are strictly above the unit production costs and firms have positive profits. Consequently, the substitution of a lengthy market for a market with an exact point leads the economy away from the perfectly competitive outcome of Bertrand and restores the oligopoly result of Cournot which is now casted in price competition.

This model gave birth to a huge strand of literature. A share of this literature has devoted itself to generalize the assumptions of the model, but it is more interesting to recall the papers that have addressed the consistency and validity of the result.

D'ASPREMONT et AL. (1979) have shown that HOTELLING´s prices are not (unlike Cournot prices) Nash equilibrium prices, since for each firm, its price is a best reply only if it accepts to share the market with the competitor. The firms in HOTELLING (1929) choose profit maximizing prices constrained to the price set defined by condition (1). Were the prices of a Nash equilibrium type, they would have to be profit maximizing in the whole price set $[0, \infty)$ and not only in the price set defined by inequality (1).

The proof by D'ASPREMONT et AL (1979) proceeds as follows. If there is a Nash price equilibrium, then the prices (p_1, p_2) should belong to the price set defined by condition (1), because otherwise one firm would have zero sales and profit and it would have an incentive to change its price. However, if prices belong to this set, they form a Nash equilibrium if and only if no firm has incentive to deviate to a price outside the set in order to undercut the rival out of business. The authors prove that this is the case if the firms locate far apart (with symmetric locations, outside the quartiles), but not if they locate close. In the latter case, each firm can drive the competitor out of business with a modest price cut and a price war follows.

Figures 5-a and 5-b illustrate the cases of existence and absence of a Nash price equilibrium in HOTELLIG's (1929) oligopoly. We represent the profit function of firm 1 for a given value of firm 2' mill price $\overline{p_2}$.

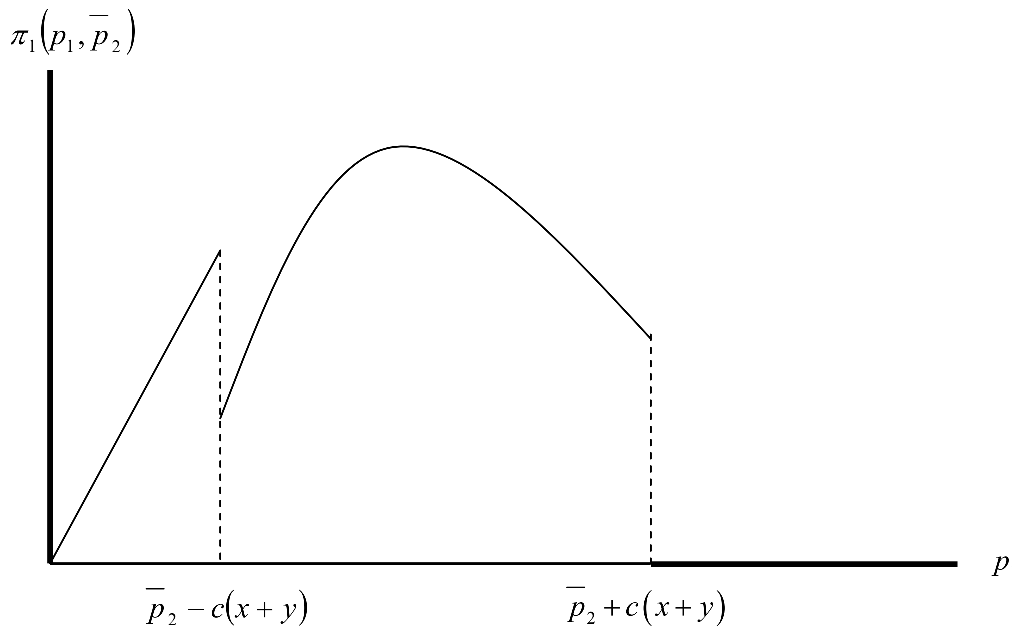


Figure 5-a: Existence of price equilibrium in HOTELLING's oligopoly

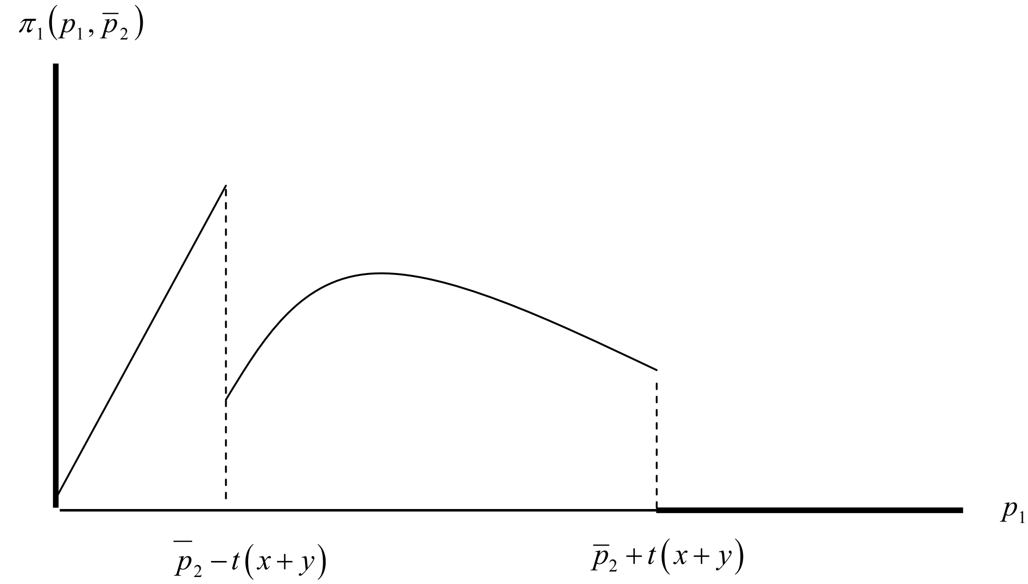


Figure 5-b: Nonexistence of price equilibrium in HOTELLING's oligopoly

In Figure 5, it is plotted the profit of firm1 as a function of its mill price p_1, given the price of its competitor $\bar{p}_2$. The profit function exhibits three regions separated by two discontinuities. If $p_1 < \bar{p}_2 - t(x+y)$, firm 1 sells to all customers, so that its profit is a linear function of the price. If $\bar{p}_2 - t(x+y) < p_1 < \bar{p}_2 + t(x+y)$, they share the market and the profit function is quadratic in price. For $p_1 > \bar{p}_2 + t(x+y)$, demand addressed to firm 1 and profit are zero.

It follows from the previous discussion that a Nash equilibrium in prices exists if and only if the profit of sharing the market with the rival firm exceeds the profit of undercutting it, i.e. if the local maximum of the profit function in the quadratic region exceeds the local maximum in the linear region. There is a Nash equilibrium in Figure 5-a but not in Figure 5-b. Equilibrium occurs whenever the firms are distant apart so that the cost of undercutting the rival is high for each firm.

However, this criticism seems unjustified since HOTELLING (1929) did not claim that his prices were analogous to Cournot's. Instead, he defended that his prices are **(locally) stable**.

The concept of stability of a dynamic variable, whose law of motion is described by a differential or difference equation, can be shortly described as follows. Let the equilibrium be a **stationary point**: if the variable reaches that value, it stays there indefinitely. This equilibrium is stable in a given set if, when it starts from any value within that set, it converges to the equilibrium as time tends to infinity. The degree of stability is measured by the size of the reference set. The variable is said to be locally stable if it is stable in a small neighborhood around equilibrium.

HOTELLING (1929) argues that his prices have that property of (local) stability. The possibility of undercutting by the firms is ruled out by some degree of collusion. Each firm realizes that to launch a price war will be detrimental for both firms and this will be avoided:

> It is of course, possible that A feeling stronger than his opponent and desiring to get rid of him once for all, may reduce his price so far that B will give up the struggle and retire from business. But during the continuance of this sort of price war A's income will be curtailed more than B's. In any case its possibility does not affect the argument that there is stability, since stability is by definition merely the tendency to return after *small* displacements. A box standing on end is in stable equilibrium, even though it can be tipped over. (HOTELLING, 1929:50).

This author is aware that if the firms get close, the mass of intermediate consumers shrinks and the degree of price stability decreases:

> But the danger that the system will be overturned by the elimination of one competitor is increased. The intermediate segment of the market acts as a cushion as well as a bone of contention; when it disappears we have Cournot's case and Bertrands's objection applies. Or, *returning to the analogy of the box in stable equilibrium though standing on end*, *the approach of B to A corresponds to a diminution in size of the end of the box.* (HOTELLING, 1929:52)

In this case, equilibrium prices fit into the definition of "local Nash equilibrium prices": prices which are mutually best replies in a neighborhood of the equilibrium. Small deviations from equilibrium are ruled out by considerations of private profitability of the firm. Larger deviations entailing the undercutting of the rival are excluded by the consideration that the rival will retaliate and launch a mutually destructive price war.

HOTELLING (1929) contended that price equilibrium becomes "less stable" when firms choose close locations. Instead D'ASPREMONT el AL (1979) sustained that whenever firms get too close a Nash price equilibrium ceases to exist. These are two different ways to express the same basic idea. However, these two ways lead to different conclusions about the locations selected in equilibrium by the firms.

As we have said before, the economy is modeled by a game where firms select locations firstly and then prices. Each firm anticipates the impact of its location choice on the subsequent intensity of price competition. HOTELLING (1929) concludes that each firm has an incentive to move towards the location of the opponent in order to increase the mass of captive consumers in its hinterland. Consequently, the equilibrium of locations will entail agglomeration with both firms in the market center: it is the so called "Principle of Minimum Differentiation". This principle means that price competition is not strong enough to countervail the advantages for the firms to locate in a central position in relation to the mass of consumers.

D'ASPREMONT et AL. (1979) have a different point of view. They contend that the absence of a Nash price equilibrium when firms are close invalidates the "Principle of Minimum Differentiation". They rewrite the model using a quadratic function tx^2, where t is the transport cost parameter and x is distance. With this function, they argue that there exists a Nash price equilibrium for any firms' locations. Moreover, competition in the second stage will be strong enough to lead the firms to relax price competition by locating in the extreme points of the market in the context of a situation of maximal differentiation.

Our personal opinion is that HOTELLING's result is more consistent than those by their critics. Firstly, he did never contend that his prices are Nash equilibrium prices. He rather said only that they are prices endowed with "local stability". Secondly, real transport cost functions usually exhibit economies of scale, i.e. they are concave in distance rather than convex. From this it follows that agglomeration of competitors is a much more empirically common result than dispersion so that in the end HOTELLING (1929) was (approximately) right.

The fact that competition among firms does not countervail the advantages for the firms to agglomerate near the central point of the market becomes more evident if we consider that they compete in quantities rather than in prices (as in ANDERSON and NEVEN, 1991).

Let us assume instead a spatial oligopoly with two different assumptions. Firstly, firms A and B compete in each market point r selling quantities of output $q_1(r)$ and $q_2(r)$. Secondly, they carry themselves the product between their locations and the customers'. The behavior of the consumers in each market point r is expressed by a linear inverse demand function. The game has two stages: firstly, the firms choose locations; secondly, they select quantities of output in each market point. The assumption on the transport cost function $t(.)$ is more general than in HOTELLING (1929) and D'ASPREMONT et AL. (1979). It is assumed that $t(.)$ is:

- Continuous

- Increasing
- $t(0)=0$

Then, it can be proved that, with convex transport costs and assuming that each firm sells a positive output in each market point, there exists a unique equilibrium for the location-quantity duopoly. In this equilibrium, both firms locate in the market center.

The assumption on the convexity of the transport cost function is crucial. Let us assume that a firm is located in $\frac{l}{2}$ (the market center) and the other one is placed between $\frac{l}{2}$ and *l*. If it shifts to $\frac{l}{2}$, the second firm increases the profits that it makes with the consumers to whom it gets closer (in the segment $[0, x_2]$) and decreases the profits with the consumers from whom it gets far away (in the segment $[x_2, l]$). The former line segment is larger than the latter, this being the first reason behind the profitability of the move by the firm towards the center.

The second reason has to do with the convexity of the transport cost function. With this kind of function, profits of the firm moving towards the center increase faster in the segment $[0, x_2]$ than they will decrease in the segment $[x_2, l]$, because marginal transport costs (per unit of distance) increase with distance. Consequently, they increase more intensively in segment $[0, x_2]$ than in $[x_2, l]$.

It can also be easily concluded that, with a concave transport cost function, central agglomeration may not be an equilibrium set of locations.

Summing up, we can conclude that the centrifugal force of competition among firms is not strong enough to compensate the drive by each firm toward the market center in order to obtain accessibility to the whole set of consumers.

The location of oligopolistic firms has been renewed more recently by BELLEFLAMME et AL (2000). In their model, the economic space is made by two identical regions, A and B, each one endowed with the same number of consumers. Thus, by contrast with the previous oligopoly model, there is no natural “central region” of the market.

We assume first that there are two firms selling differentiated products. Each firm carries its product to the customers, so that it can price discriminate across markets. We make the assumption of a quadratic utility function for the customers given by:

$$U(q_1, q_2) = \alpha(q_1 + q_2) - \left(\beta/2\right)(q_1^2 + q_2^2) - \delta q_1 q_2 + q_0 \qquad (2)$$

Where the output q_i $(i = 1,2)$ is the quantity consumed of differentiated product *i* and q_0 is the quantity of an outside good. The following relations hold: $\alpha > 0$ and $0 \le \delta < \beta$. The maximization of this utility by the consumers subject to a budget constraint determines that each firm faces a linear demand function defined on the prices of both varieties:

$$q_i = a - bp_i + d(p_j - p_i) \qquad (3)$$

where $a \equiv \alpha/(\beta + \delta)$, $b \equiv 1/(\beta + \delta)$ and $d \equiv \delta/[(\beta - \delta)(\beta + \delta)]$ hold.

In (3), parameter *d* measures inversely the degree of differentiation of the products. These will be independent if $d = 0$. By contrast, they will be perfect substitutes when $d \to \infty$ holds.

In order to ship its product to another region each firm bears a unit transport cost *t*. The production costs of the firms depend on their relative locations. If the firms stay in different locations, each firm's unit production cost is $c > 0$. If they locate in the same region, there will be positive region-specific localization economies that decrease their unit production costs. These become expressed as $c - \theta_K$ $(K = A, B)$, the term θ_K measuring the regional intensity of agglomeration economies. In what follows, it is assumed that $\theta_B \le \theta_A < c$.

The concept of localization economies goes back to Alfred MARSHALL (1949) and expresses the fact that the production costs are reduced when firms belonging to the same industry locate in the same region. According to MARSHALL (1949) these cost savings can be modeled as non-market interactions and be classified in three categories:

1. Informal and unpredictable transfers of technological knowledge among firms (spillovers), that stem from the mere closeness of their employees. These kind of spillovers cannot be carried through electronic communication and they imply face-to-face contact. As MARSHALL says:

> The mysteries of the trade become no mysteries; but are as it were in the air, and children learn many of them unconsciously. Good work is rightly appreciated, invents and improvements in machinery, in processes and the general organization of business have their merits promptly discussed: if one man starts a new idea, it is taken up by others and combined with suggestions of their own; and thus it becomes the source of further new ideas. (MARSHALL, 1944: 225)

2. There is a difference between the skills that workers have and those that are required by the firms, so that the problem of assigning each worker to each firm in a "right" way is always present. If this matching is inefficient, each worker has to support training costs in order to compensate the gap between its skill and the one that is required by the firm. High training costs lead the workers to migrate to other regions and the scarcity of labor tends to reduce the agglomeration of firms. As MARSHALL says:

> Again, in all but the earliest stages of economic development a localized industry gains a great advantage from the fact that it offers a constant market for skill. Employers are apt to resort to any place where they are likely to find a good choice of workers with the special skill they require; while men seeking employment naturally go to places where there are many who need such skill as theirs and where therefore it is likely to find a good market. The owner of an isolated factory, even if he has access to a plentiful supply is often put to great shifts for want of some special skilled labor; and a skilled workman, when thrown out of employment in it, has no easy refuge …These difficulties are still a great obstacle to the success of any business in which special skill is needed, but which is not in the neighborhood of others like it: they are however being diminished by the railway, the printing press and the telegraph. (MARSHALL, 1949: 225/6)

3. A final type of economies of localization follows from the exchange of intermediate goods among firms. If several firms producing a consumption good cluster in space, the workers in each firm are a source of demand for the other firms. The resulting increase in market size allows a deepening of the division of labor, with the separation between firms producing final goods and firms producing intermediate goods. This specialization determines increasing returns to scale at two different

levels. On the one hand, the production of each input can be geographically concentrated in a single firm allowing the emergence of economies of scale. On the other hand, each firm producing the final good is no longer constrained to use a single intermediate good and it has a whole variety of available inputs. As MARSHALL says:

> Again, the economic use of expensive machinery can sometimes be attained in a very high degree in a district in which there is a large aggregate production of the same kind, even though no individual capital employed in the trade be very large. For subsidiary industries devoting themselves each to one small branch of the process of production, and working it for a great many of their neighbors, are able to keep in constant use machinery of the most highly specialized character, and make it to pay its expenses, though its original cost may have been high, and its rate of depreciation very rapid. (MARSHALL, 1949: 225)

In this survey, all these effects will be taken into account by means of the reduced form

$$\text{Unit production costs} = \begin{cases} c \text{ if firms locate in different regions} \\ c - \theta_K \text{ if firms co-locate in region K} \end{cases}$$

Then, it is possible to write a two-stage game, where the firms choose first to locate in regions A and B and then compete in delivered prices. The outcome of the game depends on the transport costs t and the intensity of localization economies θ_A and θ_B. Basically, if transport costs are very high in relation to agglomeration economies, the equilibrium will entail dispersion of firms. If transport costs are intermediate, the unique equilibrium will involve agglomeration in region A with larger economies of localization. Finally, if transport costs are very low, agglomeration in either region will become an equilibrium pattern. Then, there are two equilibrium patterns although the equilibrium in A Pareto dominates the agglomeration in B.

Consequently, if transport costs are high each firm locates in a different region in order to sell to local customers. If transport costs are low, it is easy for each firm to export to the other region. Hence, the firms agglomerate in order to make profit of economies of localization.

This framework allows an easy generalization to an imperfectly competitive industry with a large number of small differentiated producers. This large group works as in CHAMBERLIN (1948) according to the following principles:

1. Since each firm is very small in relation to the industry, it assumes that a price change has a negligible impact upon each individual competitor. Consequently, it has no meaningful impact upon the overall price index.
2. As each firm produces a differentiated good, it assumes that a decision about the amount of output influences its price.
3. Each firm takes into account the overall price index when quoting its price.

If we assume a continuum of firms $[0,1]$, the quadratic utility of the consumers becomes:

$$U(q_0; q(i), i \in [0,1]) = \alpha \int_0^1 q(i)di - \frac{\beta - \delta}{2} \int_0^1 [q(i)]^2 di - \frac{\delta}{2} \int_0^1 \int_0^1 q(i)q(j)didj + q_0 \qquad (4)$$

Where $q(i)$ is the quantity of variety $i \in [0,1]$, q_0 the quantity of numeraire and the parameters are such that $\alpha > 0$, $\beta > \delta > 0$. From the maximization of utility function (4) subject to a budget constraint we can derive linear demand functions addressed to each firm:

$$q(i) = a - bp(i) + d \int_0^1 [p(j) - p(i)]dj \qquad (5)$$

where $a \equiv \alpha/\beta, b \equiv 1/\beta$ and $d \equiv \delta/\beta(\beta - \delta)$ hold.

Let N_A and N_B the numbers of firms locating in regions A and B, respectively. By definition, we have $N_A + N_B = 1$. Let us define $\Delta N = N_A - N_B$, so that ΔN fully characterizes the location equilibrium. The firms are subject to economies of localization that are directly linked with the number of firms within the region. The unit production cost in region *K* (*K=A,B*) is:

$$c_K(N_K) = c - \theta N_K$$

It is possible to conceive a two stage game, where the firms select firstly locations and then compete in delivered prices. This game can be solved by backward induction. Let *X* be a collection of parameters that is a decreasing function of the transport cost *t*. The equilibrium of locations is defined by the fact that there is a single stable equilibrium of locations. This one involves:

(i) Identical clusters $(\Delta N = 0)$ if and only if $X \leq 0$;

(ii) Asymmetric clusters $(\Delta N = \pm\sqrt{X})$ if and only if $0 < X \leq 1$;

(iii) A single cluster $(\Delta N = \pm 1)$ if and only if $1 < X$.

This equilibrium generalizes the case with only two firms and can be depicted in Figure 6.

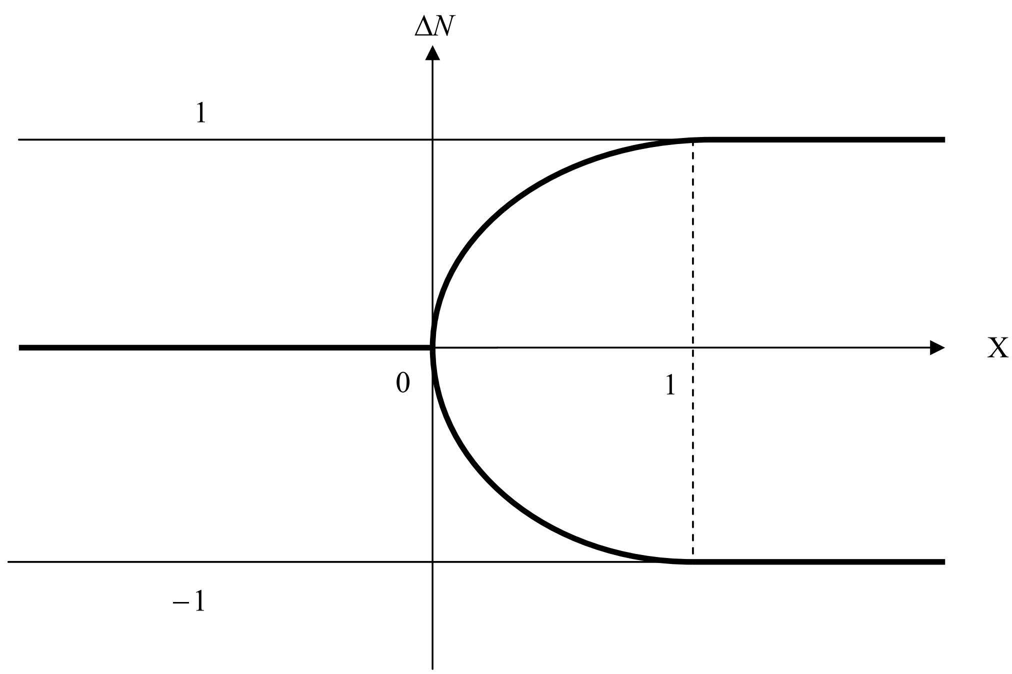


Figure 6: Stable location equilibria

The result plotted in Figure 6 is clear. If transport costs are high (X is low), the firms scatter in two equally sized groups each one being located in a different region in order to sell to nearby consumers. If transport costs are high (X is low), the firms export easily so that that they agglomerate in a region in order to exploit economies of localization. For intermediate levels of transport cost, the firms distribute themselves across the regions in an asymmetric way.

4. Location of firms under vertical monopoly.

Up to now we have dealt with the location of firms that interact on the basis that their products are substitutes (oligopoly models). However, firms also interact in the location choices when their products are complements. This is the case of vertical monopoly: an upstream firm U processes labor into an intermediate good that is sold to a downstream firm D. This latter firm combines the input with labor in order to manufacture a final product that is sold to consumers. The question that we pose here is similar to the question concerning spatial oligopoly: do vertically-related firms tend to choose separate locations in equilibrium or do they prefer to agglomerate?

In empirical terms, there are instances of both equilibrium strategies: in the textile industry, manufacturing is shifted to low wage countries, while design and marketing stay in developed countries; in engineering industries, such as the car industry, production of components co-locates with assembly even though factor intensities of both stages are very different.

The reference paper in this field is PAIS and PONTES (2008). The model has the following assumptions. There are two countries called Home (H) and Foreign (F). Country F has lower wages than H, so that $w_H > w_F \geq 0$. On the other hand, the purchasing power in country H exceeds the purchasing power in country F, this being expressed by the fact that the number of consumers in H, n_H, is higher than the number of consumers in F, n_F, i.e., $n_H > n_F$. Consumers in each country have identical demand functions $f(p)$, where p represents the delivered price, and satisfies the following assumptions:

1. f is continuous and differentiable;
2. f is decreasing;
3. The maximum price $\overline{p} = f^{-1}(0)$ is finite;
4. The total revenue function is strictly concave.

There are two vertically linked firms: the downstream firm (D), producing a consumer good to be sold in both countries, and the upstream firm (U), providing an intermediate good to the downstream firm.

U transforms $c_U\,(c_U \geq 0)$ units of labor into one unit of the intermediate good and D uses α units of the intermediate good together with $c_d\,(c_d) \geq 0$ units of labor to produce one unit of the consumer good. The parameter α, satisfying the condition $0 \leq \alpha < 1$, represents the intensity of vertical linkages.

Each firm carries its own product. The parameter t denotes the transport cost of both products (final and intermediate) between the two countries. Transport costs within each country are assumed to be zero. The assumption that transport costs are the same between goods rests on the fact that they usually vary in proportion.

When D locates in country X_D and U locates in country X_U, with the condition $X_D, X_U \in \{H, F\}$, firm D sets discriminatory prices $p_H^{X_D}$ and $p_F^{X_D}$ in each country, while firm U sets a delivery price k^{X_U} for the intermediate good.

With these assumptions, firm *D's* profit function is

$$\begin{aligned}\Pi_D^{(X_D,X_U)} &= n_H \cdot f\left(p_H^{X_D}\right)\left[p_H^{X_D} - \alpha \cdot k^{X_U} - c_D \cdot w_{X_D} - t \cdot d(X_D, H)\right] + \\ &+ n_F \cdot f\left(p_F^{X_D}\right)\left[p_F^{X_D} - \alpha \cdot k^{X_U} - c_D \cdot w_{X_D} - t \cdot d(X_D, F)\right] \qquad (6)\end{aligned}$$

And firm *U's* profit function is:

$$\Pi_U^{(X_D,X_U)} = \alpha\left[n_H \cdot f\left(p_H^{X_D}\right) + n_F \cdot f\left(p_F^{X_D}\right)\right]\left[k^{X_U} - c_U \cdot w_{X_U} - t \cdot d\left(X_D, X_U\right)\right] \quad (7)$$

where $d(X,Y)$ represents the distance between locations *X* and *Y*, with

$X, Y \in \{H, F\}$

Firm D and Firm U play a non-cooperative three-stage game. In the first stage, firms simultaneously choose their locations in the two-country economy. Given the adopted locations, X_D and X_U in the second stage, firm *U* sets k^{X_U}, the price of the intermediate good. Finally, in the third stage, firm *D* quotes $p_H^{X_D}$ and $p_F^{X_D}$, the prices for the final good in countries *H* and *F,* respectively.

The main results of the vertical monopoly can be described in Figure 7, where the equilibria of locations of firms (D,U) are plotted in the space of parameters (α, t).

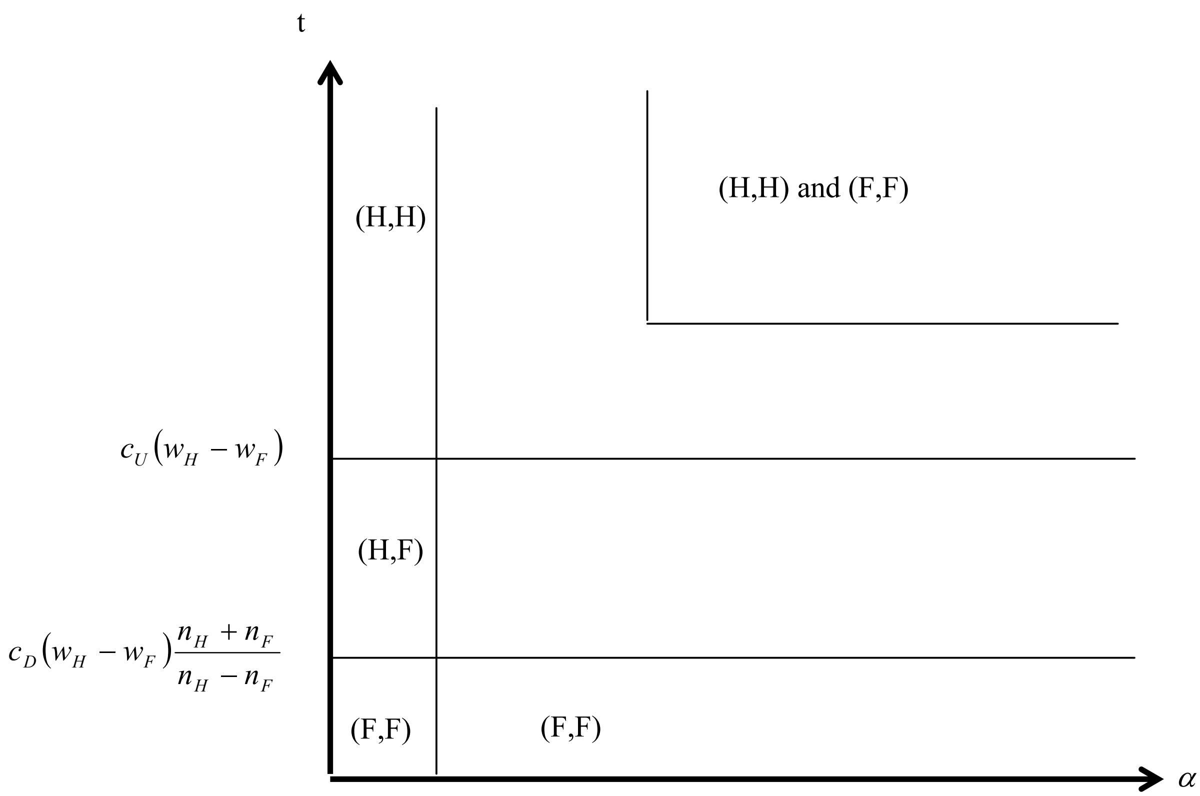


Figure 7: Location of firms in vertical monopoly

Figure 7 shows several aspects of the location of vertically related monopoly firms:

1. The fall of transport costs t to very low levels always leads to the agglomeration of the upstream and downstream firm in the low labor cost country, since the choice of locations is then driven by production costs only.
2. However, this process exhibits two distinct patterns depending on the intensity of vertical linkages α .
3. If α is low, the fall of trade costs may determine a transition from the agglomeration in the large, high labor cost *H* to spatial fragmentation, where the upstream firm locates in the small low labor cost country *F* and the downstream unit *D* stays in the large, high labor cost market *H*. Further reduction of t leads eventually to agglomeration in the small, low labor cost country *F*.
4. By contrast, if α is high, there are multiple agglomeration equilibria for high values of transport cost t, since in this case the transport cost of the intermediate good is high, and for each firm to cluster in either country is better than selecting an isolated location.
5. Fragmentation of production is more likely to arise if the countries are very asymmetric either in labor costs ($w_H - w_F$ is high) or in size ($n_H - n_F$ is high).

5. Location of firms under monopolistic competition: agglomeration of production under increasing returns.

KRUGMAN (1980) sets the foundations for the agglomeration of firms that operate under increasing returns. He assumes that there are a large number *n* of differentiated goods that have a constant elasticity of substitution. All consumers have the same utility function:

$$U = \sum_{i}^{n} c_i^{\theta} \qquad 0 < \theta < 1 \tag{8}$$

where c_i is the consumption of good i. Labor is the single production factor. All the goods have the same cost function:

$$l_i = \alpha + \beta x_i \qquad \alpha, \beta > 0,\ i = 1, \ldots, n \tag{9}$$

where l_i is labor used in the production of good i and x_i is the output of this good, so that there is a fixed cost and a constant marginal cost. Consequently, the average cost declines for all levels of output and the firm operates under increasing returns to scale.

The output of each good equals the sum of individual consumptions. We identify the consumers with workers. Hence the output of good *i* is equal to the consumption of a representative individual times the size of the labor force *L:*

$$x_i = Lc_i \qquad i = 1, 2, \ldots, n \tag{10}$$

It is assumed full employment, so that the labor force is equal to the total labor used in production:

$$L = \sum_{i=1}^{n} \left(\alpha + \beta x_i \right) \tag{11}$$

Finally, it is assumed that the firms maximize profits, but there is free entry by firms, so that in equilibrium profits are always zero.

Under this setting, firms operate under Chamberlinian monopolistic competition following the constant elasticity substitution version of DIXIT-STIGLITZ (1977). This market structure has the properties outlined in page 25.

KRUGMAN (1980) then proceeds considering two identical countries (regions) except in what concerns size (as measured by the labor force). He assumes in a first step, that transport costs are zero. Even in this case, openness to trade benefits the consumers in either country. Basically, trade determines that each variety is produced in a single plant in a single country thus allowing the exploitation of economies of scale. Increasing returns lead to more output, not in the form of a larger scale of production of each good but rather through the increase of the number of differentiated goods available to each consumer. It can also be shown that trade is always balanced in this case.

Then positive transport costs are introduced in the trade between the two countries. These costs have an "iceberg" form: if one unit of a good is exported from a country to the other, only $\tau < 1$ arrives to destination, a share $1 - \tau$ disappearing in transit.[3]

If it is assumed that the size of the Home country in terms of the number of workers/consumers is larger than the size of the Foreign country ($L > L^*$), it can be

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[3] This assumption is made for the sake of simplicity and it ensures that the spatial price policy is insesnsitive to the regional distribution of customers.

proved that there is a "Home Market effect": the former country becomes a net exporter of the differentiated consumer goods. This follows from two considerations:

1. Given the existence of economies of scale, it always pays off to concentrate the production of each variety in a single plant in a single country.
2. With positive transport costs, it always pays to locate this single plant in the larger market in order to avoid transport costs. The smaller market can be supplied through exports.

However, under the assumption of immobility of workers/consumers across countries, the trade flows between the two countries should be balanced. KRUGMAN (1980) considers two possible ways of achieving this goal.

If there is a single class of differentiated goods, the smaller country can produce and export an amount similar to its imports only if it has lower nominal wages, i.e. lower production costs, than the large country.

If there are two classes of differentiated goods and demand is symmetrically distributed, so that each country has a larger domestic demand in one class, trade balance can be achieved with each country becoming a net exporter of that class of goods. In this case, nominal wages are equal across the countries.

KRUGMAN (1991) goes further in this research line, by considering a two-region economy with two sectors: agriculture, operating under constant returns and perfect competition, and manufacturing, under increasing returns to scale and monopolistic competition. Each sector has a specific factor: farmers in agriculture, that are immobile; and workers in manufacturing, that are mobile across regions, according to relative real wages. The output of agriculture is transported without costs while the output of manufacturing bears an "iceberg" transport cost τ . In this context, the trade balance is not checked at an aggregate level but it rather follows from the fact that for each kind of consumers there is a balance between her income and the value of the output that it produces and sells in the global economy.

The "short run" equilibrium in this economy amounts to determining the nominal wages in both regions given a regional distribution of workers across the regions. If we assume that a region has more workers than the other one, there are two conflicting effects. On the one hand, the "Home Market effect" leads to higher nominal wages in the larger region. On the other hand, we have an "Extent of Competition" effect: more workers in a region mean a fiercer competition among manufactured goods producers for the local market made by the farmers. It follows a possible decrease of the nominal wages that the manufacturing firms can afford to pay. The interplay of these two factors leads to an uncertain outcome.

However, the main focus of KRUGMAN (1991) is the determination of the regional distribution of workers in the long run. Namely, it is sought whether in the long run the manufacturing workers are either evenly dispersed across regions or they instead agglomerate in one of the regions. It is assumed that workers move to the region where real wages are higher, so that migration is sensitive to the relative real wage ω_1/ω_2 .

Real wages are determined by nominal wages discounted by the price index of manufactured goods. The price of the agricultural goods is irrelevant since it is the same in both regions. The price index of manufactured goods is lower in the region that contains more manufacturing firms and workers, as manufacturing goods bear transport costs across regions. Hence, we have a "Price Index effect": if more workers and manufacturing firms enter a region, industrial goods become cheaper in that region, increasing workers' real wages and creating an incentive for the attraction of more workers.

Hence, considering the possibility of geographic concentration versus dispersion of manufacturing across the regions, the outcome is uncertain since we have two centripetal forces ("Home Market effect" and "Price Index effect") and a centripetal force ("Extent of Competition effect").

Basically, there will be regional convergence (dispersion of manufacturing) if ω_1/ω_2 decreases as a consequence of a movement of workers and firms from region 2 to region 1. And there will be regional divergence otherwise. This can be studied numerically and KRUGMAN (1991) concludes that regional divergence obtains for low transport costs (see Figure 8, where *f* stands for the share of workers in region 1).

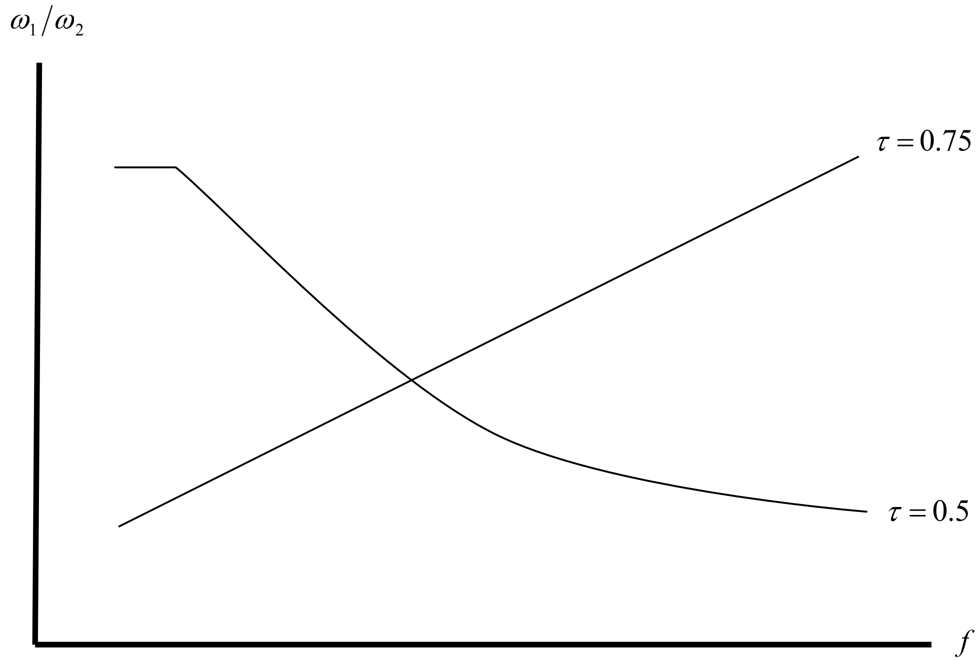

Figure 8: Regional convergence and divergence

It is possible to confirm this result in an analytic way, although with a slightly different meaning. Let us assume that all manufacturing firms and workers are agglomerated in region 1. Then agglomeration is sustainable provided that no firm finds profitable to "defect", i. e., to shift to region 2. KRUGMAN (1991) finds that three parameters matter for this decision:

1. The share of the manufacturing in expenditure and the allocation of labor, μ. It increases the likelihood of regional concentration by two reasons: it increases the relative size of region 1 under spatial concentration ("Home Market effect"); it decreases the relative cost of living in region 1 ("Price Index effect").

2. The transport cost of the manufactured good, inversely given by τ. A high value of τ (a low transport cost) has two contradicting effects: it decreases the strength of competition for the local market of farmers; it decreases the "Price Index effect". KRUGMAN (1991) proves that the first centrifugal effect predominates.
3. The strength of scale economies is inversely given by the parameter θ and it decreases the advantage of concentrating the increasing returns sector in region 1.

The combined influence of these parameters is depicted in Figure 9 in (μ, τ) space, for given values of σ $(\sigma = 4$ and $\sigma = 10)$.

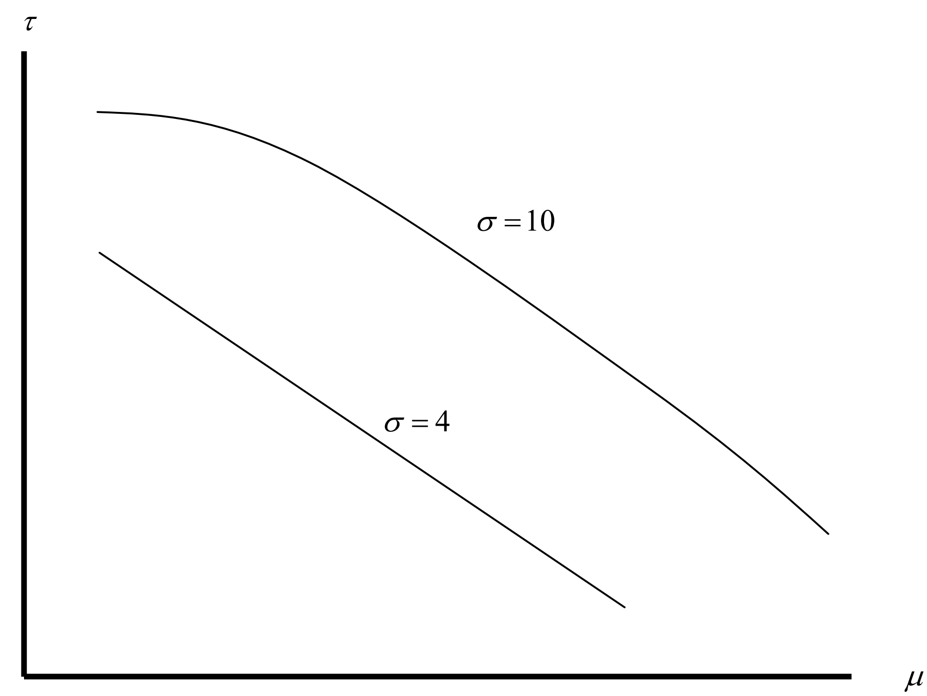

Figure 9: Economy parameters and geographic concentration

OTTAVIANO et AL. (2002) present a general equilibrium model of geographical agglomeration that is very similar to KRUGMAN's (1991). However, several different assumptions lead to more simple and clear results. They assume that the utility function of the consumers is given by (4) so that the direct demand functions addressed to the firms are described by (5). Transport costs are not "iceberg", but they are expressed in units of numeraire. Together these assumptions lead to a demand elasticity that varies with transport costs and according to the number of firms located in a market. Consequently, prices are not a fixed markup of costs but depend on the location of the firms. Prices are lower in the region where firms agglomerate reflecting the intensity of competition.

The "Price Index" and "Extent of Competition" effects do not stem only from the number of firms that locate in a region and thus avoid transport costs in supplying that region, but they follow also from lower prices in that region as a result of a higher intensity of competition.

This change of assumption gives the model a more realistic character and allows us to reach neater conclusions. KRUGMAN's (1991) model had implicitly a difference between the "sustain point", $T(S)$, i.e. the level of transport costs above which a full agglomeration of firms would be upset by a "defection" (a firm leaves the cluster and sets up in the other empty region), and the "break-point", $T(B)$, i.e. the level of transport costs below which a symmetric distribution of firms becomes unstable. Usually, the former point is higher than the latter (see Figure 10).[4]

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[4] Note that we are assuming now that $T = 1/\tau$, where τ is the amount of product that arrives to destination if one unit is exported. Hence *T* is the amount that must be sent in order that one unit arrives to destination.

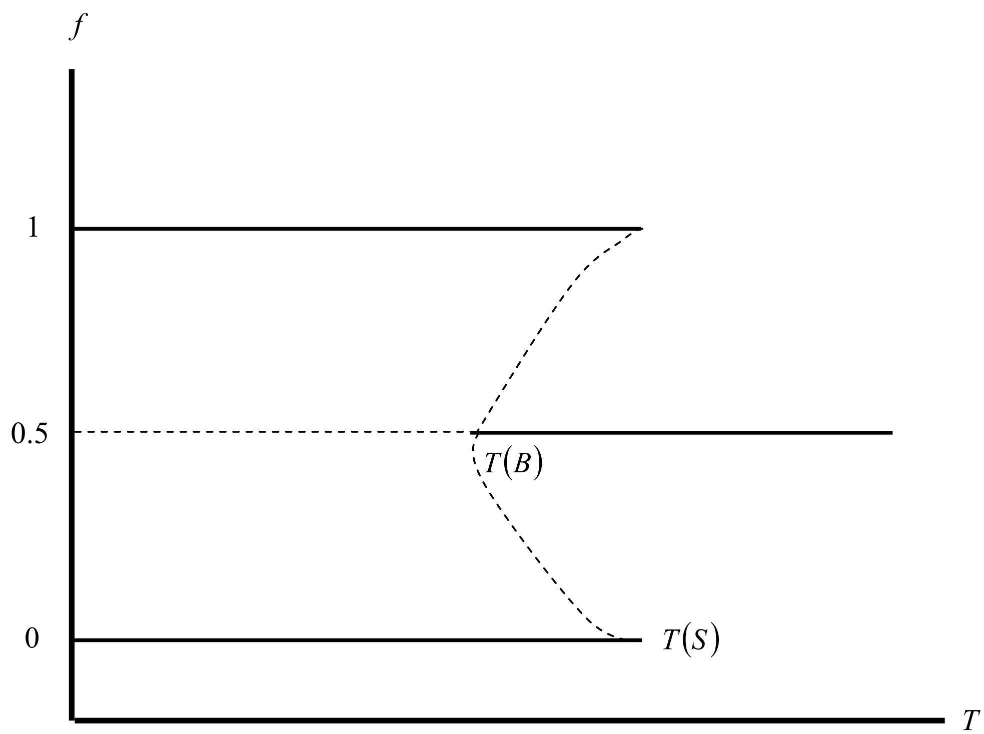

Figure 10: Long run spatial equilibria in (T, f) space.

Figure 10 depicts long run locational equilibria in (T, f) space, where $T = 1/\tau$ stands for the "iceberg" transport cost and f represents the share of workers that live in a region. Thick lines plot stable equilibria and dashed lines represent unstable equilibria, where stability means that a shift from the equilibrium is offset by labor movements that restore the initial location pattern. In the KRUGMAN (1991) economy, high transport costs $(T$ higher than $T(S))$ lead the economy to a symmetric division of manufacturing across the regions. By contrast, low transport costs $(T$ lower than $T(B))$ lead to a full agglomeration of increasing returns activities in one region. For intermediate transport costs $(T(B) < T < T(S))$, there are multiple equilibria, both dispersed and agglomerated, and this constitutes a weakness of the model.

In OTTAVIANO et AL (2002), there is a unique threshold value of transport cost T^* such that $T < T^*$ entails agglomeration and $T > T^*$ leads to symmetric dispersion of manufacturing. [5] The problem of existence of multiple patterns of location does not exist. Furthermore this analysis is not bounded to the definition of equilibrium spatial patterns, but allows us to make welfare considerations. Concerning welfare, it can be said that:

1. For extreme values of transport costs, the equilibrium is coincident with the socially optimum spatial pattern.
2. For intermediate values of *T*, the market discriminates against the dispersion of manufacturing, thus leading to an excessive geographical concentration.

6. Location of multi-plant (multinational) firms.

Up to now, we have assumed that each firm runs production activities in a single point in space. In this section, we consider the location choices by firms (multi-plant or multinational) that establish subsidiaries in regions (countries) different from their home location, in the context of Foreign Direct Investment (henceforth named as FDI).

The literature acknowledges two forms of FDI that differentiate according to their relationship with trade and transport costs. On the one hand, the firm sets up a plant in a foreign market in order to supply local consumers. By doing so, the firm substitutes local production for exports from the home country, trading off the

[5] Note that *T* means now a quantity of numeraire rather than an "iceberg" transport cost as in Figure 10.

benefits of proximity to final consumers (in the form of low transport costs) against the advantages of geographic concentration of production (in the form of higher economies of scale). This is clearly the type of FDI proposed by HORSTMANN and MARKUSEN (1992) where trade and FDI are substitutes.

Another form of FDI consists in splitting the production process into several vertically-related stages, each stage being intensive in a specific production factor. For instance, the activity of the firm can be split in two parts: headquarters (intensive in skilled labor) and plant (intensive in unskilled labor), that can be separated spatially, each one being placed in a country abundant in the factor that is used more intensively by that unit. This is clearly the form of FDI described by HELPMAN (1984). In this case, FDI implies the existence of trade in intermediate goods and is eased by low transport costs: trade and FDI complement each other.

The literature shows that the relationship between trade and FDI is not simple (see, for instance, PAIN and WAKELIN, 1998). In PONTES (2007), a non-monotonic relationship is proposed, inspired by BRAINARD (1993), of two vertically-linked firms with different degrees of divisibility. It is assumed that the upstream firm is indivisible and located in the home country. When the downstream firm invests abroad, it eliminates the transport costs of the final product, but it has to incur in the additional transport costs on the input that has to be imported from the home country. This yields the possibility of a non-monotonic pattern.

We consider a location decision by a monopolist industrial firm in a spatial economy made by two countries (regions): the home country, where the monopolist headquarters locate and the foreign country where all final demand is located. Final demand is described by $f(p)$, where p is delivered price. The demand function $f(p)$ exhibits the following properties:

1. It is continuous.
2. It is decreasing.
3. The associated revenue function $pf(p)$ is strictly concave.

The monopolist chooses among three strategies of supplying the foreign country:

- "0": Exit, the monopolist refrains from supplying the foreign country.
- "E": Export, the monopolist supplies the foreign city with exports.
- "D": FDI, the monopolist supplies the foreign city with a local plant.

It is assumed that production entails the successive manufacturing of an intermediate good and of a final good. The production of the intermediate good is indivisible and should be performed at the monopolist's headquarters in the home country. By contrast, the production of the consumer good can be either spatially divided across the two locations ("FDI") or it can concentrated at the monopolist's headquarters in the home country ("Export").

Both "Export" and "FDI" entail trade between the two locations, but its product composition differs: with "Export", consumer goods are sent to the foreign country, while "FDI" correspond to the export of intermediate goods only.

Let T be the unit transport cost between the two locations of a unit of any good (either intermediate or final) and $\alpha \in (0,1)$ the amount of input that is required to produce one unit of the consumer good. Hence "FDI" brings lower transport costs than "Export". Instead, "FDI" implies a fixed cost G related with the set-up of a foreign plant, while "Export" implies zero fixed costs. The intermediate good's price is parametric and given by w.

The profit function of the monopolist for the different location strategies is:

$$\begin{aligned}
&\pi_0 = 0 \text{ for the "Exit" strategy} \\
&\pi_E(p,T) = (p - \alpha w - T) f(p) \text{ for the "Export" strategy} \\
&\pi_D(p,G,T) = (p - \alpha w - \alpha T) f(p) - G \text{ for the "FDI" strategy}
\end{aligned} \qquad (12)$$

Using the envelope theorem, we can eliminate the price and write the profit function in terms of costs only:

$$\begin{aligned}
&\pi_0 = 0 \text{ for the "Exit" strategy} \\
&\pi_{\mathrm{E}}(T) = \max_p \pi_{\mathrm{E}}(p,T) \text{ for the "Export" strategy} \\
&\pi_{\mathrm{D}}(T,G) = \max_p \pi_D(p,T,G) \text{ for the "FDI" strategy}
\end{aligned} \qquad (13)$$

It is possible to define the location decision by the monopolist in the space (T,G), by means of two thresholds:

- $\widetilde{T}$, such that $\pi_E(T) > \pi_0 = 0$ iff $T < \widetilde{T}$: the profitability of “Export” is positive if and only if the transport cost is lower than $\widetilde{T}$.
- $\widetilde{G}$, such that $\pi_D(\widetilde{G},\widetilde{T}) = \pi_0 = 0$. It has two properties:
    1. It is positive: $\widetilde{G} > 0$.
    2. It equalizes the profits of “Export “and “FDI” when the transport cost is $\widetilde{T} : \pi_E(\widetilde{T}) = \pi_D(\widetilde{G},\widetilde{T})$.

Figure 11 summarizes the mode of supply choice by the monopolist in space (T,G).

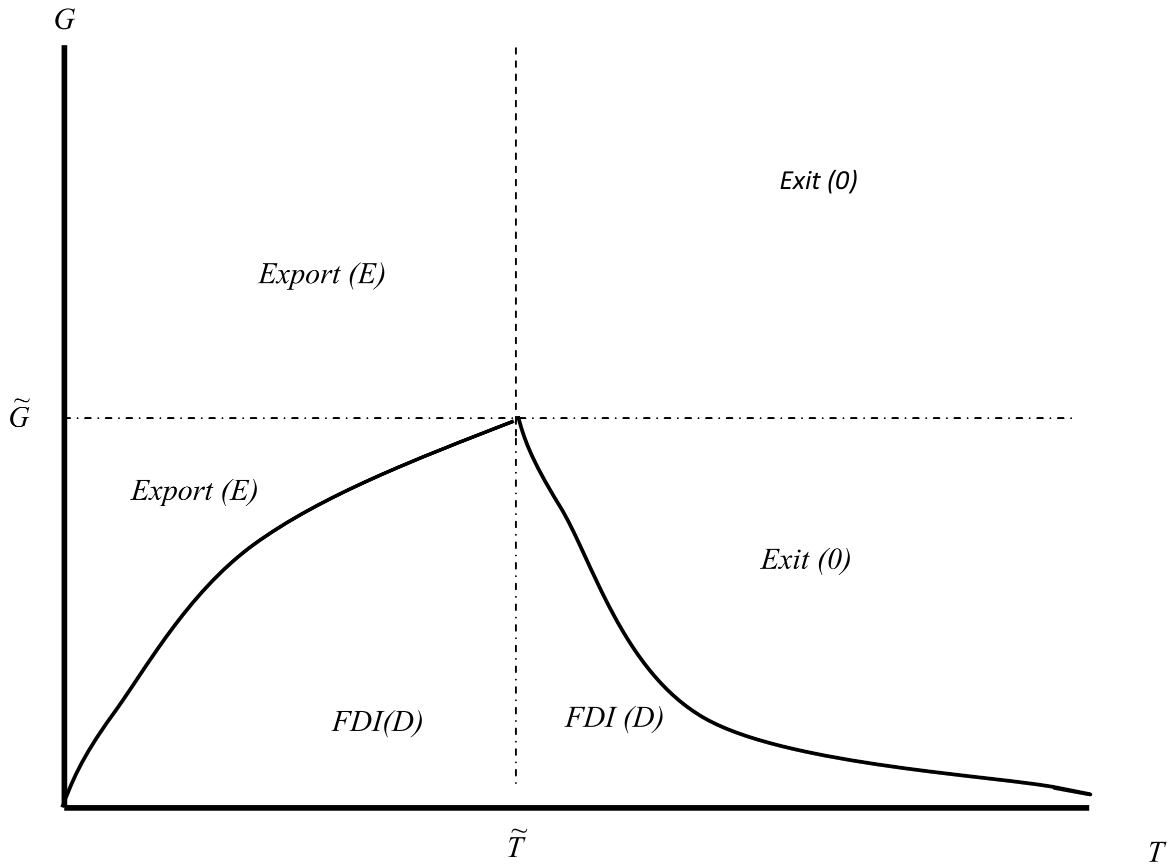

Figure 11: Location choice by a multinational firm.

7. Endogenous determination of transport technology.

Economic geography models as in KRUGMAN (1991) stress that agglomeration of the increasing returns productive activity follows from the fall of transport costs associated with the progress of the working of the transport sector. However the causality runs also in a reverse way: the feasibility of the adoption of modern transport technologies implies a spatially concentrated pattern of productive locations.

TAKAHASHI (2006) gives as an example the comparison between two cities, Los Angeles and Paris. In Los Angeles, a dispersed city, transport is almost exclusively made by cars, whereas in Paris, a spatially concentrated city, a large share of transportation is carried by mass transit systems such as the underground and the tramways.

This author inserts in a KRUGMAN-type economic geography model with two regions the possibility of choice between two transport technologies: a traditional T (i.e., the motorway) and a modern M (i.e., the railway) transport technology. The former has a constant "iceberg" transport cost $t_T > 1$ incurred by the consumers, who self-transport the product "on foot". By contrast, the modern technology implies that the firm must export $\gamma > 1$ units in order that one unit reaches the consumer. Note that this "iceberg" transport cost is smaller than the "iceberg" transport cost in the T technology cost: we have that $\gamma \in (1, t_T)$. However, under the modern technology transport services are supplied by a specialized transport sector whose operation is characterized by three features:

1. The transport sector has to set up and maintain an infrastructure (for instance, high-speed railway tracks) that leads to a fixed cost *F*.
2. For the sake of simplicity, it is assumed that the transport sector operates with a zero marginal cost.

3. The transport sector charges to the user a rate $u \in [0,1]$ which is a share of the amount transported. Hence, in order that one unit of output reaches the consumer, this must buy $\frac{\gamma}{1-u}$ units.

Consequently, the profit function of the transport sector under the modern technology is

$$\pi = uD - F \tag{14}$$

where D stands for the volume of interregional trade. In order to make the problem tractable, the author assumes that the profit of the transport sector is zero. We consider two kinds of economic landscape: a symmetric pattern, where all firms producing differentiated goods under increasing returns are evenly dispersed across the regions (pattern *S*); and a concentrated pattern where all varieties are produced in a single region (say, region) and the other region is a periphery (pattern *C*).

The traditional technology is always available to the consumers since they self-transport the product. By contrast, the modern technology may be or not be available. The consumers can use it only if there is a rate $u \in [0,1]$ such that the profit of transportation is zero. TAKAHASHI (2006) establishes the conditions under which the modern technology is available (see Assumption 1 and Proposition 1). If both technologies are available, the advantage of the modern technology with the concentrated location pattern is revealed mainly if both F and γ are high, i. e. if the modern transport technology is relatively less efficient. If the modern transport technology is very efficient it can be adopted for any location pattern, the existence of the concentrated pattern being no necessary condition.

Given the choice of transport technology, the locational outcome is similar to KRUGMAN's (1991): there is a "sustain point" $T(S)$ and a "break point" $T(B)$, with $T(B) < T(S)$ (see Figure 10).

We now deal with the simultaneous determination of the geographic structure and the technology choice. We say that a pair (i,k) is *maintainable* if the transport technology $i \in \{T,M\}$ is adopted in the economy given the geographic pattern $k \in \{S,C\}$ and, at the same time, the spatial distribution pattern $k \in \{S,C\}$ is an equilibrium given the transport technology $i \in \{T,M\}$.

In this context, there are “lock-in” effects. An economy is said to be “locked-in” if both pairs (i,k) and (j,l) are *maintainable* for $i \in \{T,M\}, k \in \{S,C\}, j \in \{T,M\}$ with $j \neq i$ and $l \in \{S,C\}$ with $l \neq k$.In this case, it is possible to conclude that there is a situation with two equilibria (T,S) and (M,C). This means that the economy with the symmetric locational pattern is “locked-in” in the traditional transport technology, while the economy with the core-periphery structure is “locked-in” in the situation with the modern technology. In this case, the transition to a modern transport technology seems dependent on the coordination among firms towards the selection of more concentrated productive locations.

8. Agglomeration and transport technology adoption in Portugal.

In Portugal, according to SOUSA and SILVA (2005), in the last four decades of the twentieth century a concentration of the population in the coastal areas took place together with a population loss in the hinterland. Population has been concentrating in three main corridors, namely:

1. A narrow coastal strip (with about 50 Km width) located between Viana do Castelo and Setúbal. This is the most important corridor in both demographic and economic terms: it contains about 7.5 millions of inhabitants ($\approx 80\%$ of the population of mainland Portugal).
2. The coastal area of the Algarve between Lagos and Vila Real de St. António, with a more dense occupation between Lagos and Faro, with almost 400 thousand inhabitants.
3. The axis Aveiro/Viseu/Guarda, the single area of horizontal passing through with a human density higher than 100 inhabitants per Km^2, and with a total population similar to the Algarve agglomeration.

As a whole, these three corridors contained about 90% of the population of mainland Portugal in 2001.

Outside these narrow areas of demographic concentration, the human occupation is very thin, with densities that are mostly lower than 20 inhabitants per Km^2.

The concentration of population is matched by a similar concentration of the productive activity. The two main metropolitan areas (Lisbon and Oporto) and Algarve exhibited values of per capita GDP much higher than the national average (between 25% and 72%, this latter value holding for the Lisbon metropolitan area).

This agglomeration process has been sustained by a steady decrease in transport costs. TEIXEIRA (2006) finds that these costs have fallen approximately by 45% in average between the provincial capitals (*capitais de distrito)* during the period 1985-1998. He further estimates a decline in transport costs in 42% for the period 1998-2010.

However, this progress of the transport system has relied mainly on the "traditional" technology (i.e., the motorway) rather than in "modern" transport technology (i.e., the railway). Hence, the evolution of the Portuguese transport network has been at odds with the European Transport Policy, which nowadays puts an emphasis upon the railway mode. It is estimated (PONTES, 2005) that the railway investments in the *Trans-European Transport Networks (TEN-T)* reached about 185 thousand millions of euros at 1993 prices (most of them dedicated to High-Speed Rail), which is twice the amount invested in roads and approximately six times the investment in airports.

While the extent of freeways and roads with separate traffic strips increased dramatically, the Portuguese railway network lagged far behind in relative terms in spite of some modest progress. This explains the difference in accessibility by road and by rail. While the area whose points can be reached by road in less than two hours, departing from Lisbon or Oporto (central cities) covers almost all territory of mainland Portugal up to the Spanish border, the railway correspondent area barely covers the coastal strip between Viana do Castelo and Aveiro.

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