

A Two-Country Discontinuous General Equilibrium Model

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Abstract: The aim of this paper is to develop a continuous time general equilibrium model for a two country Lucas type economy. The model assumes that the output in the two countries follows a jump-diffusion stochastic process. We obtain the results concerning the evaluation of financial assets, the determination of the exchange rate, of the interest rate, and of the risk premium in this two-country economy.

Keywords: general equilibrium model, two-country Lucas economy, exchange rate, risk premium, jump-diffusion.

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1. INTRODUCTION

The theory of general equilibrium models established powerful results regarding asset pricing by relating interest rates and asset risk premium to underlying economic variables.

Merton (1973) developed an intertemporal capital asset pricing model deriving the relationship among the equilibrium expected rates of return on assets. He shows that when investment opportunities are changing stochastically over time the relationship includes effects which have no analogue in the classical one period CAPM.

Lucas (1978) considers an economy with homogeneous individuals and a single consumption good. The random output is exogenously given. Assets are defined as claims to all or a part of the output. The equilibrium asset prices are determined as a function of the physical state of the economy by solving a functional equation that is a generalization of the Martingale property of stochastic price sequences.

In the consumption-based CAPM of Breeden (1979), the asset risk premium is determined by the representative agent risk aversion and by the covariance between consumption and asset prices.

Lucas (1982) investigates the determination of prices, interest rates and currency exchange rates in an infinitely-lived two-country economy. The main assumption of the model is the perfect-pooling equilibrium (i.e. since agents are risk averse and since they have identical preferences, an equilibrium in which all agents hold the same portfolio will be indefinitely maintained) Bakshi and Chen (1997) extended the model in continuous time.

The economic structure of the model developed Cox, Ingersoll and Ross (1985) differs from Lucas (1978) in that the production is endogenously determined. The model is fully consistent with maximizing behavior of the economic agents. The model endogenously determines the relationship between the stochastic process followed by the equilibrium price of any financial asset and the underlying real variables.

In most general equilibrium models money often plays no role and all prices are denominated in units of the consumption good. Bakshi and Chen (1996) provide a way of integrating asset pricing theory with models from monetary economics. According to this model both production risk and monetary risk matter for the purpose of asset valuation. The expected risk premium on a risky asset is linear in its covariance with both production risk and monetary risk.

In this paper we develop a new continuous time general equilibrium model for a two country Lucas type economy. The model assumes that the output in the two countries follows a jump-diffusion stochastic process. We obtain the results concerning the determination of the exchange rate, of the interest rate, and of the risk premium.

This paper is organized as follows: in the second section we present the model hypotheses and obtain some basic results regarding asset pricing and the interest rates. In the third section we derive the relationship between the risk premium and the underlying macroeconomic variables. In the fourth section we study the dynamics of the exchange rate. The final section concludes.

2. THE MODEL

Consider a two-country Lucas type economy (Lucas 1978; Lucas 1982). The Home GDP equals $2y_t$, where y_t follows a dynamical equation consisting of a continuous part $\left(\frac{dy_t}{y_t}\right)_c$ given by a diffusion process driven by a Brownian Motion and a discontinuous part modeled using a compound Poisson process quantifying the jumps due to rare events. Hence, the GDP follows a jump-diffusion:

$$\frac{dy_t}{y_t} = \left(\frac{dy_t}{y_t}\right)_c + JdN_t \quad (1)$$

where the density function of the amplitude of the jumps J is given by φ_J , and N_t is a Poisson process with intensity λ (independent of W_t):

$$P(N_t = n) = p(n) = \frac{e^{-\lambda t} (\lambda t)^n}{n!} \quad (2)$$

Similarly, the Foreign GDP dynamics (the Foreign variables are denoted with $*$) is given by:

$$\frac{dy_t^*}{y_t^*} = \left(\frac{dy_t^*}{y_t^*}\right)_c + J^* dN_t^* \quad (3)$$

where N_t^* is a Poisson process with intensity λ^* . N_t and N_t^* are not correlated.

Regarding the money supply ($M_t = 2m_t, M_t^* = 2m_t^*$) the model assumes a dynamics given by a diffusion process generated by a Brownian Motion that can be correlated with the Brownian Motion influencing the GDP. Hence, there are no jumps in the evolution of the money supply:

$$\frac{dm_t}{m_t} = \left(\frac{dm_t}{m_t} \right)_c \quad (4)$$

Both Home and Foreign representative economic agents have the same utility function:

$$U(c, c^*) = u(c) + \bar{u}(c^*) \quad (5)$$

The model assumes that in each country there is only one equity share (i.e. a stock index), initially held by the country's representative agent, paying dividends given by $2y_t P_t$, where P_t is the Home price level. There are also traded zero-net-supply securities. The goods produced in each country can be purchased only in the producing country's currency. Goods and capital are perfectly mobile.

The optimization problem of the Home representative agent is given by:

$$\max_{c, c^*, \alpha, \alpha^*, \beta} E_t \int_t^{\infty} \exp\{-\phi(s-t)\} U(c_s, c_s^*) ds \quad (6)$$

where ϕ is the time preference parameter, α, α^* the number of Home, respectively Foreign stocks held in the portfolio, and β is the number of units held from a financial asset paying δ_t units of the domestic good and δ_t^* units of the foreign good.

Proposition 1. In equilibrium we have that:

a) the commodity price level at time t is:

$$P_t = \frac{m_t}{y_t}, \quad P_t^* = \frac{m_t^*}{y_t^*} \quad (7)$$

b) the nominal exchange rate at time t is:

$$e_t = \frac{y_t^* \bar{u}'(y_t^*) m_t^*}{y_t u'(y_t) m_t} \quad (8)$$

c) the real exchange rate at time t is:

$$\varepsilon_t = \frac{\bar{u}'(y_t^*)}{u'(y_t)} \quad (9)$$

d) the real price of the Home equity share is:

$$s_t = E_t \int_t^{\infty} \exp\{-\phi(\tau-t)\} \frac{u'(y_\tau)}{u'(y_t)} 2y_\tau d\tau \quad (10)$$

e) the real price of the Foreign equity share is:

$$s_t^* = E_t \int_t^\infty \exp\{-\phi(\tau-t)\} \frac{\bar{u}'(y_\tau^*)}{u'(y_t^*)} 2y_\tau^* d\tau \quad (11)$$

f) the real price (in Home goods) of a financial asset is:

$$q_t = E_t \int_t^\infty \exp\{-\phi(\tau-t)\} \frac{u'(y_\tau)}{u'(y_t)} [\delta_\tau + \varepsilon_\tau \delta_\tau^*] d\tau \quad (12)$$

Proof:

The methodology used to proof the results consists in solving a discrete time dynamic programming problem, by dividing the time horizon into discrete time interval of length dt and then in obtaining the continuous time limit (Bakshi and Chen 1996; Grossman and Shiller 1982).

The wealth of the Home representative agent denominated in Home currency is given by:

$$W_t = P_t c_t + e_t P_t^* c_t^* + \alpha_t S_t + \alpha_t^* e_t S_t^* + \beta_t Q_t \quad (13)$$

where S_t is the nominal price of the Home equity, S_t^* the nominal price of the Foreign equity (in Foreign currency), and Q_t is the nominal price of a financial asset paying δ_t units of the domestic good and δ_t^* units of the foreign good.

In order to buy goods the Home representative agent has n_t units of the Home currency and n_t^* units of the Foreign currency:

$$\begin{aligned} P_t c_t &\leq n_t \\ P_t^* c_t^* &\leq n_t^* \end{aligned} \quad (14)$$

Her wealth at time $t + dt$ is:

$$\begin{aligned} W_{t+dt} = & \left[n_t - P_t c_t + e_{t+dt} (n_t^* - P_t^* c_t^*) \right] + \\ & + \alpha_t (S_{t+dt} + P_{t+dt} 2y_t dt) + \alpha_t^* e_{t+dt} (S_{t+dt}^* + P_{t+dt}^* 2y_t^* dt) + \\ & + \beta_t [Q_{t+dt} + P_{t+dt} \delta_t dt + e_{t+dt} P_{t+dt}^* \delta_t^* dt] + \\ & + (M_{t+dt} - M_t) \end{aligned} \quad (15)$$

Denoting by $V(\cdot)$ the value function and with λ_t the Lagrange multiplier, the FOC are given by:

$$\frac{\partial U}{\partial c}(c, c^*) = u'(c_t) = \lambda_t P_t \quad (16)$$

$$\frac{\partial U}{\partial c^*}(c, c^*) = \bar{u}'(c_t^*) = \lambda_t e_t P_t^* \quad (17)$$

$$\lambda_t S_t = \exp(-\phi dt) E_t [V'(W_{t+dt}) (S_{t+dt} + P_{t+dt} Y_t dt)] \quad (18)$$

$$\lambda_t e_t S_t^* = \exp(-\phi dt) E_t [V'(W_{t+dt}) e_{t+dt} (S_{t+dt}^* + P_{t+dt}^* Y_t^* dt)] \quad (19)$$

$$\lambda_t Q_t = \exp(-\phi dt) E_t \{V'(W_{t+dt}) [(S_{t+dt} + P_{t+dt} \delta_t dt) + e_{t+dt} (S_{t+dt}^* + P_{t+dt}^* \delta_t^* dt)]\} \quad (20)$$

Using the envelope theorem one has that:

$$\lambda_t = V'(W_t) \quad (21)$$

The fact that there are financial assets with positive returns (risk free bonds) implies that the constraint (14) is binding:

$$\begin{aligned} P_t c_t &= n_t \\ P_t^* c_t^* &= n_t^* \end{aligned} \quad (22)$$

Since we assume a Lucas type economy (Lucas 1982) a perfect-pooling equilibrium is conjectured:

$$\begin{aligned} c_t &= y_t, \quad c_t^* = y_t^* \\ \alpha_t &= \frac{1}{2}, \quad \alpha_t^* = \frac{1}{2}, \quad \beta_t = 0 \end{aligned} \quad (23)$$

Using (22) and the similar equalities written for the Foreign representative agent one obtain the commodity price level (equation (7)).

Using (16), (17) and (23) the nominal exchange rate is:

$$e_t = \frac{\bar{u}'(y_t^*) P_t}{u'(y_t) P_t^*} \quad (24)$$

Using (21), the equations (18), (19) and (20) can be rewritten as:

$$u'(y_t) \frac{S_t}{P_t} = \exp(-\phi dt) E_t \left[u'(y_{t+dt}) \left(\frac{S_{t+dt}}{P_{t+dt}} + 2y_t dt \right) \right] \quad (25)$$

$$\bar{u}'(y_t^*) \frac{S_t}{P_t} = \exp(-\phi dt) E_t \left[\bar{u}'(y_{t+dt}^*) \left(\frac{S_{t+dt}^*}{P_{t+dt}^*} + 2y_t^* dt \right) \right] \quad (26)$$

respectively

$$u'(y_t) \frac{Q_t}{P_t} = \exp(-\phi dt) E_t \left[u'(y_{t+dt}) \left(\frac{Q_{t+dt}}{P_{t+dt}} + \delta_t dt + \varepsilon_{t+dt} \delta_t^* dt \right) \right] \quad (27)$$

By solving forward the equations (25), (26) and (27), and by obtaining the continuous limit one gets (10), (11), and (20).

q.e.d.

Corollary 1. In equilibrium we have that:

- a) the nominal price of the Home equity at time t is:

$$S_t = E_t \int_t^\infty \exp\{-\phi(\tau-t)\} \frac{y_\tau u'(y_\tau) m_t}{y_t u'(y_t) m_\tau} 2P_\tau y_\tau d\tau \quad (28)$$

- b) the nominal price of the Foreign equity at time t is:

$$S_t^* = E_t \int_t^\infty \exp\{-\phi(\tau-t)\} \frac{y_\tau^* u'(y_\tau^*) m_t^*}{y_t^* u'(y_t^*) m_\tau^*} 2P_\tau^* y_\tau^* d\tau \quad (29)$$

- c) the nominal price of a financial asset paying δ_t units of the domestic good and δ_t^* units of the foreign good:

$$Q_t = E_t \int_t^\infty \exp\{-\phi(\tau-t)\} \frac{y_\tau u'(y_\tau) m_t}{y_t u'(y_t) m_\tau} [P_\tau \delta_\tau + e_\tau P_\tau^* \delta_\tau^*] d\tau \quad (30)$$

- d) the nominal price of a financial asset with only one cash-flow, $\Pi_T = f(y_T, y_T^*, e_T, P_T, P_T^*)$, is:

$$Q(t, T) = \exp\{-\phi(T-t)\} E_t \left[\frac{y_T u'(y_T) m_t}{y_t u'(y_t) m_\tau} \Pi_T \right] \quad (31)$$

- e) the real price of a real zero-coupon bond with maturity T is:

$$b(t, T) = \exp\{-\phi(T-t)\} E_t \left[\frac{u'(y_T)}{u'(y_t)} \right] \quad (32)$$

- f) the nominal price of a nominal zero-coupon bond with maturity T is:

$$B(t, T) = \exp\{-\phi(T-t)\} E_t \left[\frac{y_T u'(y_T) m_T}{y_t u'(y_t) m_t} \right] \quad (33)$$

Proof:

The results are a straightforward application of Proposition 1.

q.e.d.

Proposition 2. In equilibrium we have that:

a) **the real interest rate** is:

$$r_t = \phi - \frac{y_t u''(y_t)}{u'(y_t)} \frac{1}{dt} E_t \left(\frac{dy_t}{y_t} \right)_c - \frac{1}{2} \frac{y_t^2 u'''(y_t)}{u'(y_t)} \frac{1}{dt} E_t \left(\frac{dy_t}{y_t} \right)_c^2 - \lambda \int_{\mathbf{R}} \frac{u'(y_t(1+x)) - u'(y_t)}{u'(y_t)} \phi_j(x) dx \quad (34)$$

b) **the nominal interest rate** is:

$$R_t = \phi - \left[1 + \frac{y_t u''(y_t)}{u'(y_t)} \right] \frac{1}{dt} E_t \left(\frac{dy_t}{y_t} \right)_c - \left[\frac{y_t u''(y_t)}{u'(y_t)} + \frac{1}{2} \frac{y_t^2 u'''(y_t)}{u'(y_t)} \right] \frac{1}{dt} E_t \left(\frac{dy_t}{y_t} \right)_c^2 + \frac{1}{dt} E_t \left(\frac{dm_t}{m_t} \right) - \frac{1}{dt} E_t \left(\frac{dm_t}{m_t} \right)^2 + \left[1 + \frac{y_t u''(y_t)}{u'(y_t)} \right] \frac{1}{dt} E_t \left(\frac{dy_t}{y_t} \right)_c \left(\frac{dm_t}{m_t} \right) - \lambda \int_{\mathbf{R}} \frac{u'(y_t(1+x))(1+x) - u'(y_t)}{u'(y_t)} \phi_j(x) dx \quad (35)$$

Proof:

The relation between the real interest rate and the price of the real zero-coupon bond is:

$$r_t dt = b(t, t+dt)^{-1} - 1 \quad (36)$$

Using the approximation $\exp(\phi dt) \approx 1 + \phi dt$, equation (32) can be written as:

$$1 + \phi dt = E_t \left[\frac{u'(y_{t+dt})}{u'(y_t)} (1 + r_t dt) \right] \quad (37)$$

By applying the Ito Lemma for Levy processes to the function $u'(y)$ one has that:

$$du'(y_t) = y_t u''(y_t) \left(\frac{dy_t}{y_t} \right)_c + \frac{1}{2} y_t^2 u'''(y_t) \left(\frac{dy_t}{y_t} \right)_c^2 + [u'(y_t(1+J)) - u'(y_t)] dN_t \quad (38)$$

It follows that:

$$\frac{u'(y_{t+dt})}{u'(y_t)} = 1 + \frac{y_t u''(y_t)}{u'(y_t)} \left(\frac{dy_t}{y_t} \right)_c + \frac{1}{2} \frac{y_t^2 u'''(y_t)}{u'(y_t)} \left(\frac{dy_t}{y_t} \right)_c^2 + \left[\frac{u'(y_t(1+J)) - u'(y_t)}{u'(y_t)} \right] dN_t \quad (39)$$

Using some the stochastic calculus for Levy processes, respectively

$$\left(\frac{dy_t}{y_t} \right)_c dt = 0, \quad \left(\frac{dy_t}{y_t} \right)_c^2 dt = 0, \quad dN_t dt = 0 \quad (40)$$

one obtains equation (34).

The relation between the nominal interest rate and the price of the nominal zero-coupon bond is:

$$R_t dt = B(t, t+dt)^{-1} - 1 \quad (41)$$

Equation (33) can be rewritten as:

$$1 + \phi dt = E_t \left[\frac{y_{t+dt} u'(y_{t+dt})}{y_t u'(y_t)} \frac{m_t}{m_{t+dt}} (1 + R_t dt) \right] \quad (42)$$

By applying the Ito Lemma for Levy processes to the function $y u'(y)$ one has that:

$$\begin{aligned} \frac{y_{t+dt} u'(y_{t+dt})}{y_t u'(y_t)} &= 1 + \left[1 + \frac{y_t u''(y_t)}{u'(y_t)} \right] \left(\frac{dy_t}{y_t} \right)_c + \left[\frac{y_t u''(y_t)}{u'(y_t)} + \frac{1}{2} \frac{y_t^2 u'''(y_t)}{u'(y_t)} \right] \left(\frac{dy_t}{y_t} \right)_c^2 \\ &\quad + \left[\frac{u'(y_t(1+J))(1+J) - u'(y_t)}{u'(y_t)} \right] dN_t \end{aligned} \quad (43)$$

Using the classical Ito Lemma it follows that:

$$\frac{m_t}{m_{t+dt}} = 1 - \frac{dm_t}{m_t} + \left(\frac{dm_t}{m_t} \right)^2 \quad (44)$$

Using the stochastic calculus results in (40) one obtains the equation (35).
q.e.d.

3. THE RISK PREMIUM

The dynamical equations of the real price (q) and of the nominal price (Q) for a financial asset paying δ_t units of the domestic good and δ_t^* units of the foreign good consists of a continuous part, as well as a discontinuous part quantifying the jumps due to shocks in the two countries:

$$\frac{dq_t}{q_t} = \left(\frac{dq_t}{q_t} \right)_c + J_q dN_t + J_q^* dN_t^* \quad (45)$$

$$\frac{dQ_t}{Q_t} = \left(\frac{dQ_t}{Q_t} \right)_c + J_Q dN_t + J_Q^* dN_t^* \quad (46)$$

The expected real return is given by:

$$\mu_q = \frac{1}{dt} E_t \left(\frac{dq_t + \delta_t dt + \varepsilon_t \delta_t^* dt}{q_t} \right) \quad (47)$$

and the nominal one by:

$$\mu_Q = \frac{1}{dt} E_t \left(\frac{dQ_t + P_t \delta_t dt + e_t P_t^* \delta_t^* dt}{Q_t} \right) \quad (48)$$

Proposition 3. In equilibrium we have that:

a) **the real risk premium** is given by:

$$\mu_q - r_t = - \frac{y_t u''(y_t)}{u'(y_t)} \frac{1}{dt} E_t \left(\frac{dy_t}{y_t} \right)_c \left(\frac{dq_t}{q_t} \right)_c - \lambda E \left[J_q \frac{u'(y_t(1+J)) - u'(y_t)}{u'(y_t)} \right] \quad (49)$$

b) **the nominal risk premium** is given by:

$$\mu_Q - R_t = - \left[1 + \frac{y_t u''(y_t)}{u'(y_t)} \right] \frac{1}{dt} E_t \left(\frac{dy_t}{y_t} \right)_c \left(\frac{dQ_t}{Q_t} \right)_c + \frac{1}{dt} E_t \left(\frac{dm_t}{m_t} \right)_c \left(\frac{dQ_t}{Q_t} \right)_c - \lambda E \left[J_Q \frac{u'(y_t(1+J))(1+J) - u'(y_t)}{u'(y_t)} \right] \quad (50)$$

Proof:

From (27), (32) and (36) one has that:

$$E_t \left[\frac{u'(y_{t+dt})}{u'(y_t)} \left(\frac{dq_t + \delta_t dt + \varepsilon_t \delta_t^* dt}{q_t} - r_t dt \right) \right] = 0 \quad (51)$$

Using (39), and the following stochastic calculus results

$$\left(\frac{dy_t}{y_t} \right)_c dN_t = 0, \quad \left(\frac{dy_t}{y_t} \right)_c dN_t^* = 0, \quad dN_t dN_t^* = 0, \quad dN_t^2 = dN_t \quad (52)$$

one obtains the equation (49).

From (27), (33) and (41) it follows that:

$$E_t \left[\frac{y_{t+dt} u'(y_{t+dt})}{y_t u'(y_t)} \frac{m_t}{m_{t+dt}} \left(\frac{dQ_t + P_t \delta_t dt + e_t P_t^* \delta_t^* dt}{Q_t} - R_t dt \right) \right] = 0 \quad (53)$$

Equation (50) can be obtained using (43) și (44).

q.e.d.

4. THE EXCHANGE RATE DYNAMICS

The dynamics for the real (ε) and the nominal (e) exchange rate is:

$$\frac{d\varepsilon_t}{\varepsilon_t} = \left(\frac{d\varepsilon_t}{\varepsilon_t} \right)_c + J_\varepsilon dN_t + J_\varepsilon^* dN_t^* \quad (54)$$

and respectively

$$\frac{de_t}{e_t} = \left(\frac{de_t}{e_t} \right)_c + J_e dN_t + J_e^* dN_t^* \quad (55)$$

The real and nominal expected depreciation is given by:

$$\mu_\varepsilon = \frac{1}{dt} E_t \left(\frac{d\varepsilon_t}{\varepsilon_t} \right), \quad \mu_e = \frac{1}{dt} E_t \left(\frac{de_t}{e_t} \right) \quad (56)$$

Proposition 4. In equilibrium we have that:

a) the jump amplitude of the real exchange rate is:

$$J_\varepsilon = \frac{u'(y_t)}{u'(y_t(1+J))} - 1, \quad J_\varepsilon^* = \frac{\bar{u}'(y_t^*(1+J^*))}{\bar{u}'(y_t^*)} - 1 \quad (57)$$

b) the real expected depreciation is:

$$\begin{aligned} \mu_\varepsilon - (r_t - r_t^*) = & \left[\frac{y_t u''(y_t)}{u'(y_t)} \right]^2 \frac{1}{dt} E_t \left(\frac{dy_t}{y_t} \right)_c - \frac{y_t u''(y_t) y_t^* \bar{u}''(y_t^*)}{u'(y_t) \bar{u}'(y_t^*)} \frac{1}{dt} E_t \left(\frac{dy_t}{y_t} \right)_c \left(\frac{dy_t^*}{y_t^*} \right) \\ & - \lambda E \left[J_\varepsilon \frac{u'(y_t(1+J)) - u'(y_t)}{u'(y_t)} \right] \end{aligned} \quad (58)$$

c) the jump amplitude of the nominal exchange rate is:

$$J_e = \frac{u'(y_t)}{u'(y_t(1+J))(1+J)} - 1, \quad J_e^* = \frac{\bar{u}'(y_t^*(1+J^*))(1+J^*)}{\bar{u}'(y_t^*)} - 1 \quad (59)$$

d) the nominal expected depreciation is:

$$\begin{aligned} \mu_e - (R_t - R_t^*) = & \frac{1}{dt} E_t \left\{ \left(\frac{dm_t}{m_t} \right)_c + \left[1 + \frac{y_t u''(y_t)}{u'(y_t)} \right] \left(\frac{dy_t}{y_t} \right)_c \right\}^2 \\ & - \left[1 + \frac{y_t u''(y_t)}{u'(y_t)} \right] \left[1 + \frac{y_t^* \bar{u}''(y_t^*)}{\bar{u}'(y_t^*)} \right] \frac{1}{dt} E_t \left(\frac{dy_t}{y_t} \right)_c \left(\frac{dy_t^*}{y_t^*} \right) \\ & - \frac{1}{dt} E_t \left(\frac{dm_t}{m_t} \right)_c \left(\frac{dm_t^*}{m_t^*} \right) - \lambda E \left[J_e \frac{u'(y_t(1+J))(1+J) - u'(y_t)}{u'(y_t)} \right] \end{aligned} \quad (60)$$

Proof:

By applying the Ito Lemma to the function given by (9) one can obtain:

$$\begin{aligned}
\frac{d\varepsilon_t}{\varepsilon_t} &= \frac{y_t^* \bar{u}''(y_t^*)}{\bar{u}'(y_t^*)} \left(\frac{dy_t^*}{y_t^*} \right)_c - \frac{y_t u''(y_t)}{u'(y_t)} \left(\frac{dy_t}{y_t} \right)_c + \frac{1}{2} \frac{y_t^{*2} \bar{u}'''(y_t^*)}{\bar{u}'(y_t^*)} \left(\frac{dy_t^*}{y_t^*} \right)_c^2 \\
&\quad - \frac{1}{2} \left\{ \frac{y_t^2 u'''(y_t)}{u'(y_t)} - 2 \left[\frac{y_t u''(y_t)}{u'(y_t)} \right]^2 \right\} \left(\frac{dy_t}{y_t} \right)_c^2 - \frac{y_t^* \bar{u}''(y_t^*)}{\bar{u}'(y_t^*)} \frac{y_t u''(y_t)}{u'(y_t)} \left(\frac{dy_t}{y_t} \right)_c \left(\frac{dy_t^*}{y_t^*} \right)_c \\
&\quad + \left[\frac{u'(y_t)}{u'(y_t(1+J))} - 1 \right] dN_t + \left[\frac{\bar{u}'(y_t^*(1+J^*))}{\bar{u}'(y_t^*)} - 1 \right] dN_t^*
\end{aligned} \tag{61}$$

Using the conditional expectation operator and the equation for the real interest rate (equation (34)) for the two countries one obtains equation (58).

In order to obtain the expected nominal depreciation one can use the Ito Lemma to the function given by (8) :

$$\begin{aligned}
\frac{de_t}{e_t} &= \left[1 + \frac{y_t^* \bar{u}''(y_t^*)}{\bar{u}'(y_t^*)} \right] \left(\frac{dy_t^*}{y_t^*} \right)_c - \left[1 + \frac{y_t u''(y_t)}{u'(y_t)} \right] \left(\frac{dy_t}{y_t} \right)_c + \left(\frac{dm_t}{m_t} \right)_c - \left(\frac{dm_t^*}{m_t^*} \right)_c \\
&\quad + \frac{1}{2} \left[\frac{y_t^{*2} \bar{u}'''(y_t^*)}{\bar{u}'(y_t^*)} + 2 \frac{y_t^* \bar{u}''(y_t^*)}{\bar{u}'(y_t^*)} \right] \left(\frac{dy_t^*}{y_t^*} \right)_c^2 \\
&\quad - \frac{1}{2} \left\{ \frac{y_t^2 u'''(y_t)}{u'(y_t)} - 2 \left[\frac{y_t u''(y_t)}{u'(y_t)} \right]^2 - 2 \frac{y_t u''(y_t)}{u'(y_t)} - 2 \right\} \left(\frac{dy_t}{y_t} \right)_c^2 \\
&\quad - \left[1 + \frac{y_t u''(y_t)}{u'(y_t)} \right] \left[1 + \frac{y_t^* \bar{u}''(y_t^*)}{\bar{u}'(y_t^*)} \right] \left(\frac{dy_t}{y_t} \right)_c \left(\frac{dy_t^*}{y_t^*} \right)_c \\
&\quad + \left(\frac{dm_t^*}{m_t^*} \right)_c^2 - \left(\frac{dm_t}{m_t} \right)_c \left(\frac{dm_t^*}{m_t^*} \right)_c \\
&\quad - \left[1 + \frac{y_t u''(y_t)}{u'(y_t)} \right] \left(\frac{dy_t}{y_t} \right)_c \left(\frac{dm_t}{m_t} \right)_c - \left[1 + \frac{y_t^* \bar{u}''(y_t^*)}{\bar{u}'(y_t^*)} \right] \left(\frac{dy_t^*}{y_t^*} \right)_c \left(\frac{dm_t^*}{m_t^*} \right)_c \\
&\quad + \left[\frac{u'(y_t)}{u'(y_t(1+J))(1+J)} - 1 \right] dN_t + \left[\frac{\bar{u}'(y_t^*(1+J^*))}{\bar{u}'(y_t^*)} (1+J^*) - 1 \right] dN_t^*
\end{aligned} \tag{62}$$

The equation (60) can be obtained by applying the conditional expectation operator and the equation for the nominal interest rate for the two countries (equation (35)).

q.e.d.

5. CONCLUSION

In this paper we developed a continuous time general equilibrium model for a two country Lucas type economy assuming that the output in the two countries follows a jump-diffusion stochastic process. We obtained results concerning the determination of the interest rate, and of the risk premium in this two-country world, as well as the dynamics equation of the real and of the nominal exchange rates.

The main result of the paper is the expression of the risk premium in this economy. The risk premium theorised by the model consists of two terms: the classical CCAPM term relating the premium to the risk aversion and the covariance between consumption and asset prices, and a term, specific to this model, quantifying the premium required by the investors due to jumps in output.

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