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Modelling and Detecting Long Memory in Stock Returns

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1. Introduction

The existence of long memory in financial asset returns has been an important subject of both theoretical and empirical research. If asset returns display long memory, the series realizations are not independent over time, realizations from the remote past can help forecast future returns. Therefore the presence of long memory in asset returns contradicts the weak form of the market efficiency hypothesis, which states that, conditioned on historical returns, future asset returns are unpredictable. Mandelbrot (1971) suggests that in the presence of long memory, pricing derivative securities with martingale methods may not be appropriate.

A number of studies have tested the long memory hypothesis for stock market returns. The evidence is mixed. Using the classical rescaled-range method, Greene and Fielitz (1977) report evidence of persistence in daily U.S. stock returns. Lo and MacKinley (1988) and Poterba and Summers (1988), concluded that stock returns exhibit mean reversion. Fama and French (1988), who examined the autocorrelations of one-period returns, also found mean reversion. Lo (1991), using modified rescaled range statistic finds no evidence of long memory in a sample of U.S. stock returns. Mills (1993), using the modified statistic and the semi-parametric approach of Geweke and Porter-Hudak (1983), finds evidence of long memory in monthly U.K. stock returns. Cheung and Lai (1995), using the same methods, find no evidence of persistence in several international stock returns series. On the other hand, Henry (2000) finds long memory in the German, Japanese and Taiwanese markets. Lobato and Savin (1997) and Caporale and Gil-Alana (2001) find no evidence of long memory in daily Standard and Poor 500 returns.

In this paper we revisit this issue by using applying a range of parametric and semi-parametric techniques to daily, weekly and monthly index return data on nine countries, namely the USA, Japan, France, Great Britain, Taiwan, Singapore and Romania. We also discuss a continuous trading model based on the fractional Brownian motion (a stochastic process that exhibit long memory) and pricing derivative securities under this model.

This paper is divided into 5 sections. Section 2 outlines the methods used to model and detect long memory in time series. The third section describes the data and presents the empirical results. In section 4 we focus on pricing derivative securities under a continuous trading that exhibit long memory. The final section concludes.

2. Modelling and detecting long memory in time series

2.1 Modelling long memory

A popular method of capturing long memory is the fractionally differenced time series model of Granger (1980), Granger and Joyeaux (1981), and Hosking (1981).

A time series x_t follows a $ARFIMA(p,d,q)$ process if:

$$\Phi(L)(1-L)^d x_t = \theta(L)\varepsilon_t \ , \ \varepsilon_t \sim N(0,\sigma^2) \qquad (2.1)$$

where $\Phi(L),\theta(L)$ denote the autoregressive and moving average polynomials respectively. These polynomials are assumed to have no common roots.

The *ARFIMA* model generalizes the *ARIMA* model by allowing the differencing parameter d to take any real value.

Granger and Joyeaux (1981) show that

$$(1-L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)}{\Gamma(-d)\Gamma(k+1)} L^k \qquad (2.2)$$

where $\Gamma(\cdot)$ is the gamma function.

If $d \in (-0.5, 0.5)$ and the roots of the AR polynomial and of the MA polynomial are outside the unit circle the process is stationary and invertible.

Hosking (1981) shows that the autocorrelation function $\rho(\tau)$ displays a hyperbolic decay since for $d \neq 0$:

$$\rho(\tau) \propto \tau^{2d-1} \text{ as } \tau \to \infty$$

For positive d this implies that the sum of the absolute values of the autocovariance function is infinite. This is the case of long memory or long range dependence. The autocorrelations of such a process decline at a hyperbolic rate to zero, a much slower rate of decay than the exponential decay of the *ARMA* process.

The autocorrelation of such fractionally integrated processes remain significant at long lags.

Existing time series models of expected returns can be cast in terms of *ARFIMA* models. For example the common random walk model of asset prices is a special case of *ARFIMA* where p=0, d=1, and q=0. More generally expected returns can be modeled terms of particular cases of an *ARFIMA(p, d, q)*. Hence, finding a nonzero value of *d* implies the presence of long memory components in asset returns.

2.2 Testing for Long Memory

2.2.1 ADF, KPSS and FDF

Diebold and Rudebusch (1991) and Hassler and Wolter (1989) find that the standard Augmented Dickey-Fuller tests for the null hypothesis of unit root tend to have low power against the alternative hypothesis of fractional integration. Thus, in deciding whether economic data are fractionally integrated or not based on the Augmented Dickey-Fuller tests may be inadequate.

Lee and Schmidt (1996) propose the test of Kwiatkowski, Phillips, Schmidt, and Shin (1992), KPSS, as a test for the null of stationarity against the alternative hypothesis of fraction integration.

By testing both the unit root hypothesis (ADF) and the stationarity hypothesis (KPSS) we can distinguish economic series that appear to be unit root, series that appear to be stationary, and series that appear to be fractionally integrated.

KPSS's (1992) approach yields two types of statistics.

The η_τ statistic is based on the null hypothesis of trend stationary:

$$\eta_\tau = \frac{1}{T^2 S^2(l)} \sum_{t=1}^{T} S_t \qquad (2.3)$$

where $S_t = \sum_{i=1}^{T} \varepsilon_i$ and ε_i is the residual from regressing the series x_t against a constant and a trend.

The η_ρ statistic is based on the null hypothesis of level stationary and is computed as in (3) but we take the residual from regressing x_t against an intercept only.

$S^2(l)$ is a consistent estimator of the "long run variance" of $\varepsilon_i, \sigma_\varepsilon^2$, which is defined as:

$$\sigma_\varepsilon^2 = \lim_{T\to\infty} \frac{1}{T} E[S_T]$$

We employ estimators for the "long run variance" which are frequently used , the so-called heteroskedasticity and autocorrelation consistent (HAC) estimation. Recent studies, as for example Den Haan and Levin (1997), suggest that the accuracy of inference obtained using KPSS crucially depends on the actual choice of estimator for the "long run variance".

We consider estimators of the form:

$$S^2(l) = \gamma_0 + 2\sum_{j=1}^{T-1} k(l,j)\gamma_j \qquad (2.4)$$

where $\gamma_j = \frac{1}{T}\sum_{t=j+1}^{T} \varepsilon_t \varepsilon_{t-j}$ is used as the estimate of the j-th order autocovariance and $k(l,\cdot)$ is a kernel function depending on the bandwidth parameter l.

We consider two kernels: the Bartlett kernel and the Quadratic Spectral kernel.

KPSS (1992) considers only the integer valued bandwidth parameters and they use the Bartlett kernel :

$$k_B(l,j) = \begin{cases} 1 - \frac{1}{l+1}, j \le l \\ 0, j > l \end{cases} \qquad (2.5)$$

The values of the KPSS statistics are fairly sensitive to the choice of the bandwidth parameter. Therefore, the ability to reject the hypothesis of stationarity depends crucially upon the choice of l.

The Quadratic Spectral kernel gives a nonzero weight to all computable sample autocorrelations:

$$k_{QS}(l,j) = \frac{25}{12\pi^2\left(\frac{j}{i}\right)}\left[\frac{\sin\left(\frac{6\pi}{5}\frac{j}{l}\right)}{\frac{6\pi}{5}\frac{j}{l}} - \cos\left(\frac{6\pi}{5}\frac{j}{l}\right)\right] \qquad (2.6)$$

The QS kernel has been shown by Andrews (1991) to be more efficient. Newey and West (1994), indicate that it yields more accurate estimates of the "long run variance" than other kernels in finite samples.

We also have to choose the bandwidth *l*. We will use a data dependent procedure to estimate the optimal bandwidth parameter l . This approach was first explored by Andrews (1991) and later refined by Newey and West (1994). This procedure is as follows:

- choose an a priori bandwidth parameter l^0, $l^0{}_B = \left[T^{\frac{2}{9}}\right], l^0{}_{QS} = \left[T^{\frac{2}{25}}\right]$
- calculate $s_0 = \gamma_0 + 2\sum_{j=1}^{l^0} \gamma_j$, $s_k = 2\sum_{j=1}^{l^0} j^k \gamma_j$
- calculate the optimal bandwidth parameter

$$l_B = \left[1.1447\left(\left(\frac{s_1}{s_0}\right)^2 T\right)^{\frac{1}{3}}\right] \qquad l_{QS} = 1.3221\left(\left(\frac{s_2}{s_0}\right)^2 T\right)^{\frac{1}{5}}$$

Donaldo, Gonzalo and Mayoral (2002) proposed a Fractional Dickey-Fuller test (FDF) for testing the null hypothesis of $I(d_0)$ against the alternative hypothesis $I(d_1), d_1 < d_0$.

FDF is based on the *t*-statistic of the OLS estimator of ϕ in the regression:

$$\Delta^{d_o} x_t = \phi \Delta^{d_1} x_{t-1} + \varepsilon_t \qquad (2.7)$$

If $d_0 < 0.5$ or $d_0 = 1$ and $d_1 \geq 0.5$ we have that

$$t_{\hat{\phi}} \to N(0,1)$$

If $d_0 = 1$ and the DGP is given by $\Delta^{d_1^*} x_t = \varepsilon_t, \varepsilon_t \sim N(o, \sigma^2)$ then the FDF test is consistent for every $d_1 \in [0,1)$.

If $d_0 = 1$ and $d_1 < 0.5$ then the asymptotic distribution of the *t*-statistic is not standard and the critical values for the tests are obtained form the tables in Donaldo, Gonzalo and Mayoral (2002)

2.2.2. R/S Test

Let $\bar{\mu} = \frac{1}{T}(x_T - x_0)$ and let $\hat{\mu}$ be the OLS estimator of the coefficient of the trend from regressing the series x_t against a constant and a trend.

The classical R/S statistic is given by:

$$rsc = \frac{1}{\sqrt{\gamma_0 T}}\left(\max_{t \le T}\{x_t - \bar{\mu}t\} - \min_{t \le T}\{x_t - \bar{\mu}t\}\right) \qquad (2.8)$$

where γ_0 is the usual estimate of the variance of the series $\{\Delta x_t - \overline{\Delta x}\}$.

Lo (1991) shows that short-range dependence may compromise inferences about the presence of long-range dependence. Lo derives an adjustment to the classical R/S statistic that accounts for general forms of short-range dependence. The adjusted R/S statistic replaces the usual variance estimate with a consistent estimator of the "long run variance".

In this paper we will use Lo's generalized R/S statistic in the following form:

$$rslo = \frac{1}{\sqrt{S^2(l)T}}\left(\max_{t \le T}\{x_t - \bar{\mu}t\} - \min_{t \le T}\{x_t - \bar{\mu}t\}\right) \qquad (2.9)$$

where $S^2(l)$ is a consistent estimator of the "long run variance" of the series $\{\Delta x_t - \overline{\Delta x}\}$.

The correspondence between Lo's statistic and the one in (2.9) is proved in Cavaliere (2000). Therefore, Lo's range statistic is implicitly based on a detrendization of the time series under the unit root hypothesis.

Cavaliere (2001) introduces a new generalized R/S statistic:

$$rscav = \frac{1}{\sqrt{S^2(l)T}}\left(\max_{t \le T}\{x_t - \hat{\mu}t\} - \min_{t \le T}\{x_t - \hat{\mu}t\}\right) \qquad (2.10)$$

Range tests also seem to be unaffected by the so-called 'converse Perron effect' that is rejection of the unit root hypothesis (in favor of trend-stationarity) when the true generating process is $I(1)$ with a broken trend.

R/S statistics do not have standard asymptotic distributions, so the critical values for testing $I(1)$ against $I(1+d), d > 0$ are obtained form the tables in Cavaliere (2001).

2.2.3 Robinson LM Test

The null hypothesis of this test is $H_0 : d = d_0$.

Consider $z_t = (1,t)'$, and let u_t be the residual from regressing the series $\Delta^{d_0} x_t$ against $\Delta^{d_0} z_t$.

Let $\lambda_j = \frac{2\pi j}{T}, \psi(\lambda) = \ln\left|2\sin\frac{\lambda}{2}\right|$ and $I(\lambda)$ be the peridogram of u_t.

In general the peridogram of a series $\{y_t\}_{t\le T}$ is given by:

$$I(\lambda) = \frac{1}{2\pi T}\left|\sum_{t=1}^{T} e^{it\lambda}(y_t - \bar{y})\right|^2 \qquad (2.11)$$

The statistic is defined as

$$r = \sqrt{\frac{T}{A}}\frac{a}{\hat{\sigma}^2} \qquad (2.12)$$

where $\hat{\sigma}^2$ is the usual estimator of the variance of u_t and

$$a = \frac{-2\pi}{T}\sum_{j=1}^{T-1}\psi(\lambda_j) \qquad A = \frac{2}{T}\sum_{j=1}^{T-1}\psi(\lambda_j)^2$$

Robinson (1994) showed that:

$$r \to N(0,1)$$

2.3 Estimating the degree of fractional differencing

2.3.1 Log Periodogram Estimator

A popular semiparametric estimate of d is the log peridogram estimate of Geweke, J. and S. Porter-Hudak, (1983) which is defined as the OLS estimator of b from the regression:

$$\ln(I(\lambda_j)) = a - b\ln\left(4\sin^2\left(\frac{\lambda_j}{2}\right)\right) + \varepsilon_j, \quad j = 1,...,m \qquad (2.13)$$

where $\lambda_j = \frac{2\pi j}{T}$ and $I(\lambda)$ is the peridogram of the analyzed time series defined in (11).

The bandwidth m is chosen such that for $T \to \infty, m \to \infty$ but $\frac{m}{T} \to 0$.

We will use three values for m : $\left[T^{0.45}\right],\left[T^{0.5}\right],\left[T^{0.55}\right]$.

Robinson (1995a) showed that:

$$\sqrt{m}\left(\hat{d}_{GPH}-d\right)\rightarrow N\left(0,\frac{\pi^2}{6}\right)$$

We will perform tests both using the result above and using the t ratios based on the standard deviations of the regression.

2.3.2 Gaussian semiparametric estimators

Robinson (1995b) has proposed a spectral maximum likelihood estimator for fractionally integrated models:

$$\hat{d}_R = \underset{b\in(-0.5,0.5)}{\arg\min} R(b) \qquad (2.14)$$

where

$$R(b)=\ln\left[\frac{1}{m}\sum_{j=1}^{m} j^b I(\lambda_j)\right]-\frac{b}{m}\sum_{j=1}^{m}\ln(j)$$

We have the following asymptotic distribution:

$$\sqrt{m}\left(\hat{d}_R-d\right)\rightarrow N(0,1)$$

We will have to choose the bandwidth m. Henry and Robinson (1996) proposed to set $m=\left[T^{\frac{4}{5}}\right]$.

A modified version of this estimator is (Giraitis and Robinson (2002)):

$$\hat{d}_{RM} = \underset{b\in(-0.5,0.5)}{\arg\min} R_h(b) \qquad (2.15)$$

where

$$R_h(b)=\ln\left[\frac{1}{m}\sum_{j=1}^{m} j^b I_h(\lambda_{3j})\right]-\frac{b}{m+1}\sum_{j=1}^{m}\ln(j)$$

and $I_h(\lambda)$ is the peridogram of the series $\left\{\frac{T}{\sum_{t=1}^{T} h_t^2} h_t x_t\right\}_{t\leq T}$

and $h_t=\frac{1}{2}\left(1-\cos 2\pi\frac{t}{T}\right)$

We have that $\sqrt{m}\left(\hat{d}_{RM}-d\right)\rightarrow N(0,1)$

2.3.3. Approximate wavelet MLE

We will consider that the time series x_t is $ARFIMA(p,d,q)$

The spectral density of x at frequency λ is:

$$f(\lambda) = \left|1 - e^{-i\lambda}\right|^{-2d} \frac{\left|\theta\left(e^{-i\lambda}\right)\right|^2}{\left|\Phi\left(e^{-i\lambda}\right)\right|^2} \qquad (2.16)$$

We first have to perform a wavelet transform of the time series x_t. In order to do so the sample size must be a power of 2 (i.e. $T = 2^{\max}$).

Let $w = \left(w_1', ..., w_{\max}'\right)'$ be the vector containing the wavelet coefficients, where $w_i = \left(w_{i,1}, ..., w_{i,2^{\max - i}}\right)$ contains $2^{\max - i}$ elements.

The wavelet coefficients can be computed recursively as follows:

- Let $s_{0,n} = x_n, n = 1,...,2^{\max}$
- $s_{m,n} = \sum_{k=0}^{2M-1} h_k s_{m-1,2n-k}, n = 1,...,2^{\max - m}$
- $w_{m,n} = \frac{1}{\sqrt{2}} \sum_{k=0}^{2M-1} g_k s_{m-1,2n+k}, n = 1,...,2^{\max - m}$

where $g_k = (-1)^k h_{2M-1-k}$ and $\{h_k\}_{k=0,...2M-1}$ are non-zero filter coefficients introduced by Daubechies (1988). We refer to this wavelet as the Daubechies wavelet of order M .

We have that

$$\sigma_m = Var\left[w_{m,n}\right] = \frac{2^{m+1}\sigma^2}{2\pi} \int_{2^{-m}\pi}^{2^{-m+1}\pi} f(\lambda) d\lambda, \forall n$$

$$Cov\left[w_{m^*,n^*}, w_{m,n}\right] = 0, m^* \neq m, n^* \neq n$$

The Daubechies wavelet may also formulated as a matrix operation. Consider W with the property that $w = Wx$.

It can be shown that the covariance matrix of the $ARFIMA$ process can be approximated by $W'\Omega W$ where $\Omega = diag\left(\underbrace{\sigma_1,..,\sigma_1}_{2^{\max}}, \underbrace{\sigma_2,...,\sigma_2}_{2^{\max-1}},..., \underbrace{\sigma_{2^{\max}}}_{2^{\max-\max}}\right)$

Jensen (2000) proposes the following approximate log-likelihood function:

$$Wmle\left(d,\theta,\phi,\sigma^2\right)=-\frac{2^{\max}-1}{2}\ln\left(2\pi\right)-\frac{1}{2}\sum_{m=1}^{\max}\left[2^{\max-m}\ln\left(\sigma^2\right)+\frac{w_m' w_m}{\sigma_m}\right]$$

where θ,ϕ represents the vectors of the AR and MA polynomial coefficients.

The estimator of σ^2 is given by:

$$\hat{\sigma}^2=\frac{1}{2^{\max}-1}\sum_{m=1}^{\max}\left[\frac{w_m' w_m}{\overline{\sigma}_m}\right], \overline{\sigma}_m=\frac{\sigma_m}{\sigma^2} \tag{2.17}$$

and the concentrated approximate log-likelihood function is:

$$Wmle\left(d,\theta,\phi\right)=-\frac{2^{\max}-1}{2}\ln\left(2\pi\right)-\frac{1}{2}\sum_{m=1}^{\max}\left[2^{\max-m}\ln\left(\hat{\sigma}^2\overline{\sigma}_m\right)+\frac{w_m' w_m}{\hat{\sigma}^2\overline{\sigma}_m}\right] \tag{2.18}$$

We will not perform tests for this estimate since the asymptotic distribution is not known.

2.3.3 Approximate Whittle estimator

Fox and Taqqu (1986) proposed a frequency domain method to estimate *ARFIMA* models by minimizing the implied white noise variance with respect to the parameters of the *ARFIMA* model:

$$\hat{\sigma}_T^2\left(d,\theta,\phi\right)=\sum_{k=1}^{T-1}\frac{I\left(\lambda_k\right)}{f\left(\lambda_k\right)} \tag{2.19}$$

where θ,ϕ represents the vectors of the AR and MA polynomial coefficients, $\lambda_k=\frac{2\pi k}{T}$, $I\left(\lambda\right)$ is the peridogram defined in (2.11) and $f\left(\lambda\right)$ is the spectral density defined in (15).

In the case of $ARFIMA\left(0,d,0\right)$ it can be shown that

$$\sqrt{T}\left(\hat{d}_W-d\right)\rightarrow N\left(0,\frac{2\hat{\sigma}_T^2(\hat{d}_W)}{\frac{\partial^2\hat{\sigma}_T^2}{\partial d^2}\left(d\right)}\right)$$

Schmidt and Tschernig (1994) discuss the identification of ARFIMA models using information criteria. They come to the conclusion that the Schwarz Criterion performs best in the detection of fractionally differenced noise and the Hannan Quinn Criterion displays the best performance when combinations of short and long memory components are considered. When computing this selection criteria we will use $\hat{\sigma}^2$ in (2.17) for the wavelet MLE and the minimum of the expression in (2.19) for the Whittle estimate.

Sowell (1992) derives the exact Maximum Likelihood Estimator of the ARFIMA(p, d, q) process. However the Sowell estimator is computationally burdensome and we will not discuss it in this paper.

3. Empirical Results

The study of long range dependence requires sufficiently long series to justify the application of large sample inference rules based on semiparametric models. The data used in this paper consist of daily, weekly and monthly observations of 7 international and three Romanian stock index returns over different periods of time up to June 2002. The data under consideration are: United States – S&P500 Index and NASDAQ Index; France - CAC40 Index; United Kingdom – FTSE100 Index, Japan - Nikkei 225 Index, Singapore- Straits Times Index, Taiwan - Weighted Index. In Romania we consider the following indices : the BET Index, that take into account the evolution of the ten most liquid companies on the Bucharest Stock Exchange; the BETC Index, which is the composite index of the Bucharest Stock Exchange and the RASDAQC Index, the composite index of the Romanian OTC market – RASDAQ.

We will consider different periods of time since the detection of long memory requires a large quantity of observations and a lot of observations would have been lost if we had trimmed all the data series to the size of the shortest one. In the case of the S&P500 Index we have also analyzed monthly stock returns since the large sample size for this index allowed it. On the other hand in the case of the Romanian indices we focus only on daily returns.

The tests and the estimation procedures described in the previous section were implemented in Mathcad 2000. As we have seen the estimation procedures requires either the peridogram or the wavelet coefficients. These are in fact the Fourier Transform and the Wavelet transform of the data series. Since the algorithms implemented in Mathcad to compute this two transforms (Fast Fourier Transform respectively Fast Wavelet Transform) requires that the number of inputs be a power of 2 , we have to reduce our samples to the largest power of two. The tests procedures proved to be a much bigger burden for the computer than the estimation procedures. So, due to lack of computing power, when conducting a test we reduced the sample size up to 2000 observations.

The results of the tests and estimation procedures are presented in Appendix 1. The information presented in the tables is as follows. For the ADF test we present the number of lags (determined according to Schwartz's information criterion), the ADF statistic value, the level of significance, the Schwartz criterion and the Durbin-Watson statistic. For the KPSS there are presented the statistics values calculated both using the Bartlett kernel and the Quadratic Spectral kernel. In the case of the FDF test we test the null hypothesis $I(d_0)$ versus $I(d_1), 0 \le d_1 < d_0$ for $d_0 = 1, 0.4, 0.3, 0.2, 0.1$. For every test performed we report the statistic's value, the p-value, the Schwartz criterion and the Durbin-Watson statistic. Following the author's recommendations we compute the fractional difference up to the lag $\lfloor \sqrt{T} \rfloor$. In the case of the R/S test we present the classical R/S statistic and Lo (1991) and Cavaliere (2000) modified R/S statistics calculated using the Bartlett kernel and the Quadratic Spectral kernel. In this case the input series is not the return series but the series of the index values taken in logarithm. We test the null hypothesis $I(1)$ versus $I(1+d), d > 0$. In the case of the Robinson LM test we report the value of the statistic and the p-value for the null hypothesis $d = d_0$ when $d_0 = -0.2, -0.1, 0, 0.1, 0.2$. We performed the GPH estimation for four values of the bandwidth $m = \lfloor T^k \rfloor$, where $k = 0.45, 0.5, 0.55, 0.8$. We choose to use in the GPH case the bandwidth recommended for the Gaussian semiparametric estimators of Robinson because the difference of the two estimators is too big when we take $k = 0.45, 0.5, 0.55$. For every value of k we report the estimate for d, the statistic value and the p-value for the null hypothesis $d = 0$, both when we use the t-ratio based on the standard deviations of the regressions and when we use the asymptotic distribution. In the case of the classical and modified Gaussian semiparametric estimators of Robinson we present estimate for d, the statistic value and the p-value for the null hypothesis $d = 0$ based on the asymptotic distribution. In the case of the wavelet MLE and Whittle we present the estimates for d in four cases $ARFIMA(0,d,0)$, $ARFIMA(1,d,0)$, $ARFIMA(0,d,1)$ and $ARFIMA(1,d,1)$. A test of the hypothesis $d = 0$ is performed and the value of the statistic and the p-value are presented only for the Whittle estimator in the case $ARFIMA(0,d,0)$ since in the other cases Mathcad failed to compute the asymptotic variance needed for the test and for the wavelet MLE the asymptotic distribution is not known. We also present the Schwarz Criterion and the Hannan Quinn Criterion in order to choose between the four models. In Appendix 1 are also presented the critical values for the KPSS test and for the R/S test.

The ADF test rejects the null hypothesis of unit root for all the returns series.

The evidence of long memory in international stock indices is mixed. In the case of S&P500 daily, NASDAQ daily, FTSE100 daily, Nikkei 225 weekly the KPSS test reject the null hypothesis of level stationarity at 5% indicating that the series might be $I(d), 0 < d < 1$. In the case of the NASDAQ daily and Nikkey 225 weekly the R/S test reject at 1% the null hypothesis that the series of the logarithm of index level is $I(1)$ against $I(1+d), d > 0$. In the case of S&P500 daily the modified R/S test proposed by Cavaliere calculated with the Barlett kernel rejects the null hypothesis at 5%. But for all three series the LM test can not reject the null $d = 0$. The FDF test rejects $I(0.1)$ showing that if the three series are fractionally integrated then $d < 0.1$. The R/S test and the LM test for FTSE100 daily reject the possibility of the existence of long memory. For S&P500 daily, NASDAQ daily and FTSE100 daily only the Whittle estimator for $ARFIMA(0, d, 0)$ is significantly different from zero. For S&P500 monthly, FTSE100 weekly , Taiwan WI weekly and CAC40 weekly return series all the test indicate that there is no evidence of long memory. The LM test for CAC40 daily rejects the hypothesis $d = 0.1$, but does not reject $d = 0.2$ indicating that the order of integration may be between 0.1 and 0.2 . But FDF rejects $I(0.1)$. For Nikkey 225 daily and NASDAQ weekly the LM test rejects the null hypothesis $d = 0$ but all the other tests reject the existence of long memory. For Taiwan WI daily and Singapore ST daily and weekly the majority of estimates are significantly different from zero.

In the case of Romania the KPSS test rejects at 5% the null hypothesis of level stationarity for all the three series of indices returns. The LM test for BET and BETC rejects the null hypothesis of $d = 0$ and $d = 0.1$ but does not reject $d = 0.2$. For both series the FDF rejects $I(0.4)$, but can not reject $I(0.3)$. We also observe that the GPH, Robinson and Whittle estimates are significantly different from zero. For RASDAQ the tests indicate that $d < 0$.

4. A continuous trading model with long memory

4.1 Fractional Brownian Motion

If $0 < H < 1$ the fractional Brownian motion (fBm) with Hurst parameter H is the continuous Gaussian process $\{B_H(t), t \in \mathbf{R}\}$, $B_H(t) = 0$ with mean $E[B_H(t)] = 0$ and whose covariance is given by:

$$C_H(t,s) = E[B_H(t)B_H(t)] = \frac{1}{2}\left\{ |t|^{2H} + |s|^{2H} - |t-s|^{2H} \right\} \qquad (4.1)$$

If $H = \frac{1}{2}$ then $B_H(t)$ coincides with the standard Brownian motion $B(t)$.

The fractional Brownian motion is a self-similar process meaning that for any $\alpha > 0$ $B_H(\alpha t)$ has the same law as $\alpha^H B_H(t)$.

The constant H determines the sign of the covariance of the future and past increments. This covariance is positive when $H > \frac{1}{2}$, zero when $H = \frac{1}{2}$ and negative when $H < \frac{1}{2}$.

Another property of the fractional Brownian motion is that for $H > \frac{1}{2}$ it has long range dependence in the sense that if we put

$$r(n) = Cov(B_H(1), B_H(n+1) - B_H(n))$$

then

$$\sum_{n=1}^{\infty} r(n) = \infty \qquad (4.2)$$

The self-similarity and long-range dependence properties make the fractional Brownian motion a suitable tool in different applications like mathematical finance. Since for $H \neq \frac{1}{2}$ the fractional Brownian motion is neither a Markov process, nor a semimartingale, we can not use the usual stochastic calculus to analyze it. Worse still after a pathwise integration theory for fractional Brownian motion was developed (Lin (1995), Decreusefond and Ustunel (1999)) it was proven that the market mathematical models driven by $B_H(t)$ could have arbitrage (Rogers (1997)). The fractional Brownian motion was no longer considered fit for mathematical modeling in finance. However after the development of a new kind of integral based on the Wick product (Duncan, Hu and Pasik-Duncan (2000), Hu and Oksendal (2000)) called fractional Ito integral, it was proved (Hu and Oksendal (2000)) that the corresponding Ito type fractional Black-Scholes market has no arbitrage.

We will present some result regarding the fractional Ito integral. For more aspects you may consult Duncan, Hu and Pasik-Duncan (2000) and Hu and Oksendal (2000).

We will consider for the rest of the paper that $H > 0.5$ although the results can be extended for the case $H < 0.5$ following Elliot and van der Hoek (2000) and Hu, Oksendal and Zhang (2002).

Consider the fractional differential equation:

$$dX(t) = \mu X(t)dt + \sigma X(t)dB_H(t), \quad X(0) = x$$

It can be shown that:

$$X(t) = x\exp\left(\sigma B_H(t) + \mu t - \frac{1}{2}\sigma^2 t^{2H}\right) \tag{4.3}$$

On the other hand, using the fractional Ito lemma, we have:

$$d\ln X(t) = \left(\mu - H\sigma^2 t^{2H-1}\right)dt + \sigma dB_H(t) \tag{4.4}$$

Another important concepts is that of *quasi-conditional expectation and quasi-martingale* which are important for the evaluation of derivatives, but we will not present their definitions in this paper.

4.2. A fractional Black-Scholes market

Consider a *fractional Black-Scholes market* that has two investment possibilities:

1. a money market account:

$$dM(t) = rM(t)dt, \quad M(0) = 1, \ \ 0 \le t \le T$$

where r represent the constant riskless interest rate.

2. a stock whose price satisfies the equation:

$$dS(t) = \mu\, S(t)dt + \sigma S(t)dB_H(t), \quad S(0) = S > 0, \ 0 \le t \le T \tag{4.5}$$

where $\mu, \sigma \neq 0$ are constants.

Using (4.3) we have that:

$$S(t) = S(0)\exp\left(\sigma B_H(t) + \mu t - \frac{1}{2}\sigma^2 t^{2H}\right) \tag{4.6}$$

In figure 4.1 are given some examples of sample paths of the stock price over a period of 1 year for $\mu = 0.1, \sigma = 0.2$ and $H = 0.5, 0.55, 0.6$.

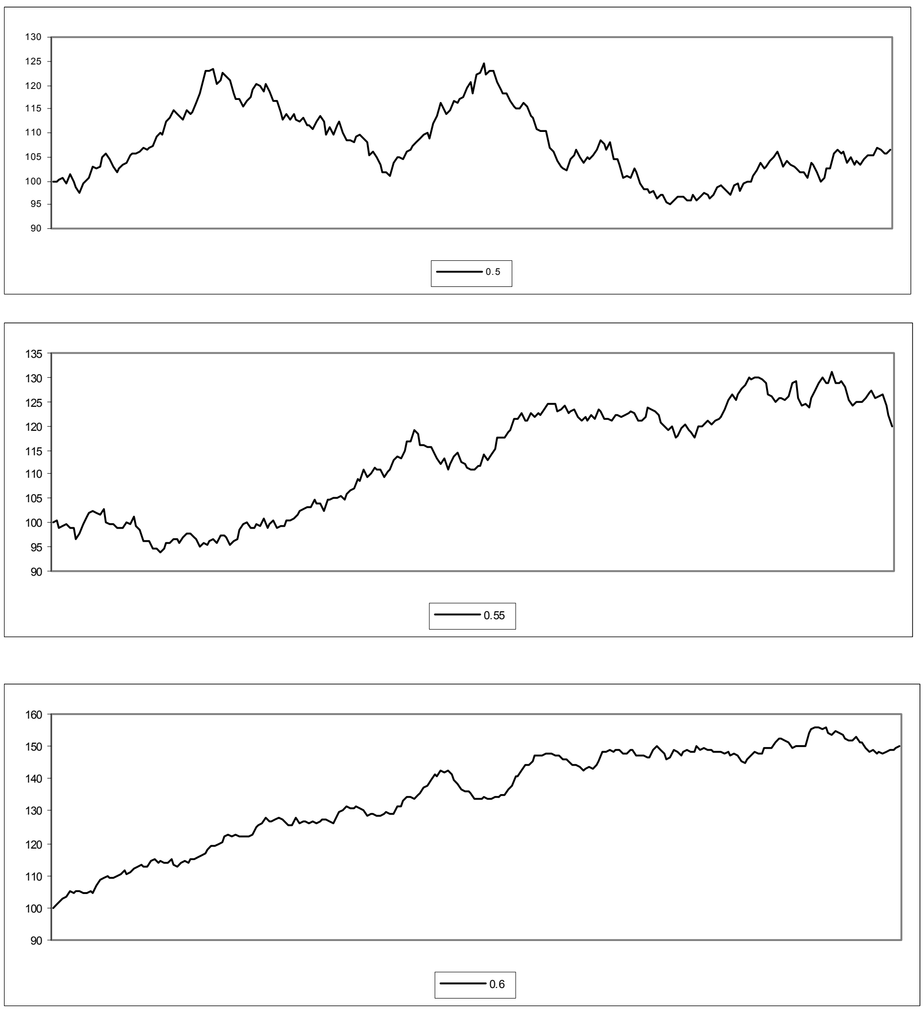

Figure 4.1 Sample path of the stock price

When H is close to 1 the sample paths become relatively smooth, due to the positive correlation of the future and past increments of the fBm.

From (4.6) results that

$$\ln S(t) = \ln S(0) + \left(\mu - \frac{1}{2}\sigma^2 t^{2H-1}\right)t + \sigma B_H(t) \tag{4.7}$$

One can see that the trend is not linear and it depends on H as in the classical Black-Scholes model.

A computationally efficient method of estimating the three parameters in equation (4.7) does not yet exist. An exact MLE method can be obtained but is computationally burdensome.

An approximate MLE method exist (Vidács and Virtamo (1999)) for the following model:

$$x(t) = mt + aB_H(t) \tag{4.8}$$

But as seen from (4.8) we can not use this method.

Hu and Oksendal (2000) have shown that this market does not have arbitrage and is complete.

They used in the definition of self-financing the Wick product instead of using the ordinary product as in the pathwise model, which leads to arbitrage.

They compute the risk-neutral measure and under this measure we have that:

$$dS(t) = rS(t)dt + \sigma S(t)d\overline{B}_H(t), \quad S(0) = S > 0, \ 0 \le t \le T$$

In the same paper (Hu and Oksendal (2000)) a formula for the price of a European option at $t = 0$ is derived.

4.3 Pricing derivative securities

We will denote by $\widetilde{E}_t[\cdot]$ the quasi-conditional expectation with respect to the risk-neutral measure.

Necula(2002) have shown that in a *fractional Black-Scholes market* we have the following results.

Theorem 1 (fractional risk-neutral evaluation)

The price at every $t \in [0, T]$ of a bounded $\mathcal{F}_T^H$ - measurable claim F is given by

$$F(t) = e^{-r(T-t)}\widetilde{E}_t[F] \tag{4.9}$$

Theorem 2 (fractional Black-Scholes equation)

The price of a derivative on the stock price with a bounded payoff $f(S(T))$ is given by $D(t, S(t))$, where $D(t, S)$ is the solution of the PDE:

$$\frac{\partial D}{\partial t} + H\sigma^2 t^{2H-1} S^2 \frac{\partial^2 D}{\partial S^2} + rS\frac{\partial D}{\partial S} - rD = 0 \qquad (4.10)$$

$$D(T, S) = f(S)$$

Theorem 3 (fractional Black-Scholes formula)

The price at every $t \in [0, T]$ of an European call option with strike price K and maturity T is given by

$$C(t, S(t)) = S(t)N(d_1) - Ke^{-r\cdot(T-t)}N(d_2)$$

where $$d_1 = \frac{\ln\left(\frac{S(t)}{K}\right) + r(T-t) + \frac{\sigma^2}{2}\left(T^{2H} - t^{2H}\right)}{\sigma\sqrt{T^{2H} - t^{2H}}}$$ and

$$d_2 = \frac{\ln\left(\frac{S(t)}{K}\right) + r(T-t) - \frac{\sigma^2}{2}\left(T^{2H} - t^{2H}\right)}{\sigma\sqrt{T^{2H} - t^{2H}}} \qquad (4.11)$$

and $N(\cdot)$ is the cumulative probability of the standard normal distribution.

Theorem 4 (The Greeks)

The Greeks are given by:

$$\Delta = \frac{\partial C}{\partial S} = N(d_1)$$

$$\nabla = \frac{\partial C}{\partial K} = -e^{-r(T-t)}N(d_2)$$

$$\vartheta = \frac{\partial C}{\partial \sigma} = Sf(d_1)\sqrt{T^{2H} - t^{2H}} \qquad (4.12)$$

$$\rho = \frac{\partial C}{\partial r} = (T-t)Ke^{-r(T-t)}N(d_2)$$

$$\Gamma = \frac{\partial^2 C}{\partial S^2} = \frac{1}{S\sigma\sqrt{T^{2H} - t^{2H}}} f(d_1)$$

$$\Theta = \frac{\partial C}{\partial t} = -rKe^{-r(T-t)}N(d_2) - Ht^{2H-1}\frac{S\sigma}{\sqrt{T^{2H} - t^{2H}}}f(d_1)$$

where

$$f(z) = \frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}$$

The fractional Black-Scholes price of a European call option no longer depends only on $T - t$. A reason may be the fact that the fractional Brownian motion has long memory. The price of an option at a moment $t \in [0, T]$ will depend on the stock price $S(t)$, but despite the classical Black-Scholes model, will take into consideration the evolution of the stock price in the period $[0, t]$. This influence is reflected in the fractional Black-Scholes formula by the Hurst parameter H.

4.4 A comparison between the classical and the fractional Black-Scholes formulae

Consider a European call option with strike price $K = 100$. Also we suppose that $r = 0.1$ and $\sigma = 0.2$. For the fractional model we consider $H = 0.55$

First, we consider that the option has a maturity $T = 1$ and at a fixed moment of time $t = 0.5$ we will analyze the difference between the classical and the fractional Black-Scholes formulae for the price and for the sensitivity indicators of the call option if the stock price varies between $0.5K$ and $1.5K$.

Figure 4.2 shows the results. If we use the classical Black-Scholes model in the case of a market that exhibits long memory the call options will be underevaluated no matter they are in the money or out of the money. Depending on the moneyness of the option we could be overhedged or underhedged. The hedge ratio of near at the money options is more sensitive to changes of the stock price in the fractional case than in the classical model. In a fractional market the rate of decay in time of near at the money is lower while for the rest of the options is bigger and options are more sensitive to changes in volatility.

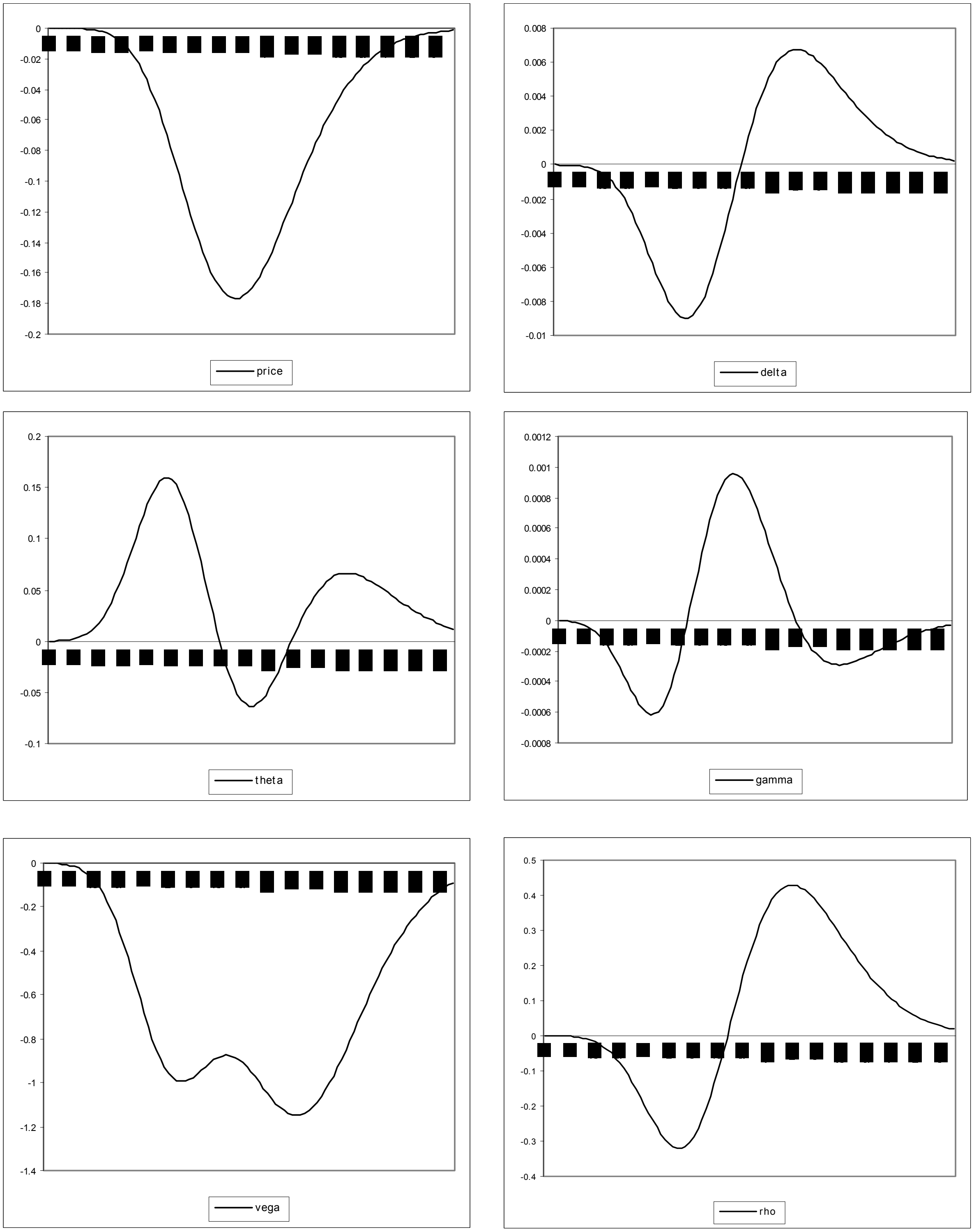

Figure 4.2 The difference between the indicators of the classical and fractional models for different values of the stock price

Second, we consider that the option has different maturities $T = 0.5$, $T = 1$ and $T = 2$ and for a fixed value of the stock price $S = 100$ we will analyze the evolution of the difference of indicators when time varies between 0 and T.

Figure 4.3 present the results regarding the difference between the prices of the two models. The graphs for the rest of the indicators are presented in Appendix 2.

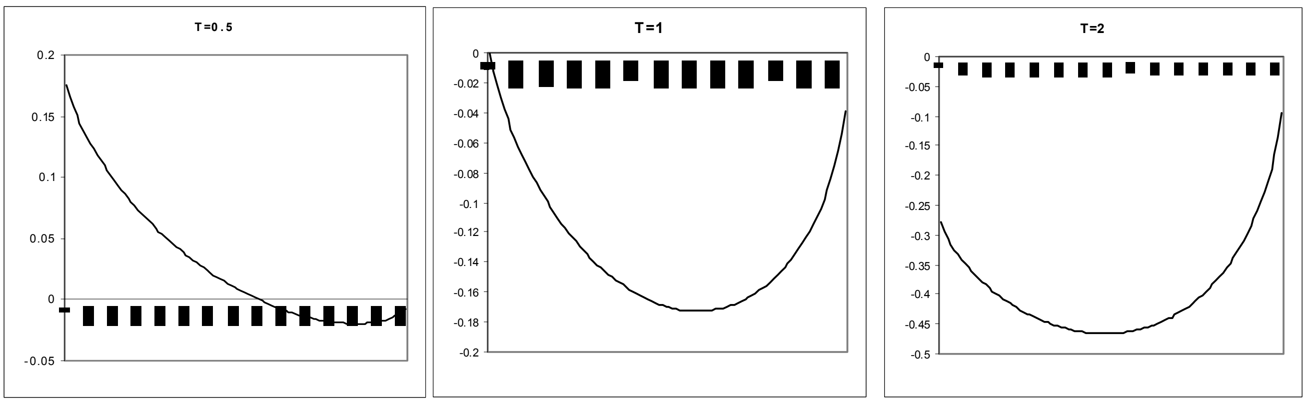

Figure 4.3 The difference in time between the prices of the two models

As it was expected the prices of options given by the two models are different for every t during the life of the option.

5. Conclusions

Using a wide range of test and estimation procedures we have investigated whether stock returns exhibit long memory. Some evidence of long range dependence was found in daily returns of S&P500, NASDAQ, FTSE100, Singapore ST and Taiwan WI indices and in weekly returns of Nikkei 225 and Singapore ST indices. Strong evidence of long memory was found in daily returns or Romanian BET and BETC indices.

We analyzed the properties of a continuous trading model that is a generalization of the classical Black-Scholes market and we made a comparison between the classical Black-Scholes formula and the option pricing formula obtained in this new model.

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Appendix 1 Tests and estimations results

USA SP500 daily

Tests

ADF	0		KPSS	level		trend	
	-44.5492			B	QS	B	QS
	0.01			0.67326	0.592527	0.079532	0.066981
	-6.17234						
	1.996661						

FDF	1	0.4	0.3	0.2	0.1
	-31.6636	-16.5623	-12.3267	-8.1272	-3.94297
	2.5E-220	6.53E-62	3.25E-35	2.2E-16	4.02E-05
	-5.88165	-6.11141	-6.11857	-6.13003	-6.14392
	2.163394	1.969368	1.974024	1.98309	1.994297

R/S	classical	Lo		Cavaliere	
		B	QS	B	QS
	1.411028	1.564693	1.473786	1.706952	1.60778

Robinson LM	-0.2	-0.1	0	0.1	0.2
	12.80773	4.235423	-1.54002	-5.42192	-8.22593
	1.48E-37	2.28E-05	0.123555	5.9E-08	1.94E-16

Estimations

GPH	0.45	0.5	0.55	0.8	Robinson	classical	modified
	0.161045	0.066315	0.053136	-0.02907		-0.04832	0.041141
	1.852118	0.914977	0.905625	-1.62411		-1.77656	1.512735
	0.064045	0.36023	0.365161	0.104391		0.075641	0.130347
	0.956287	0.49324	0.495428	-0.83329			
	0.338927	0.621843	0.620298	0.404684			

Wavelet MLE	0,d,0	1,d,0	0,d,1	1,d,1
	-0.01498	-0.02278	-0.02221	-0.04282
	-83052.1	-83043.6	-83043.5	-83036.1
	-83056.7	-83052.8	-83052.8	-83049.9

Whittle	0,d,0	1,d,0	0,d,1	1,d,1
	0.033668	-0.05058	-0.0432	-0.03582
	3.869149	-1	-1	-1
	0.000109	-1	-1	-1
	-75642.4	-75687	-75690.8	-75682.4
	-75647	-75696.2	-75700	-75696.3

USA SP500 weekly

Tests

ADF	0		KPSS	level		trend	
	-46.3889			B	QS	B	QS
	0.01			0.226664	0.227535	0.069074	0.069082
	-4.8889						
	1.997537						

FDF	1	0.4	0.3	0.2	0.1
	-35.5039	-18.9787	-14.6361	-10.2955	-5.93177
	2.1E-276	1.28E-80	8.27E-49	3.69E-25	1.5E-09
	-4.63949	-4.85547	-4.85863	-4.86394	-4.86919
	2.208396	1.986207	1.987417	1.992041	1.997687

R/S	classical	Lo		Cavaliere	
		B	QS	B	QS
	1.613597	1.639021	1.636311	1.428517	1.426155

Robinson LM	-0.2	-0.1	0	0.1	0.2
	16.56059	6.11513	-0.872	-5.51802	-8.75378
	1.34E-61	9.65E-10	0.383211	3.43E-08	2.06E-18

Estimations

GPH	0.45	0.5	0.55	0.8	Robinson	classical	modified
	0.030124	0.043886	0.053244	0.040631		0.008142	0.014063
	0.306414	0.55346	0.806433	1.305804		0.171946	0.297001
	0.759321	0.580009	0.420087	0.191766		0.86348	0.766466
	0.130772	0.232074	0.339808	0.669045			
	0.895956	0.81648	0.734001	0.503467			

Wavelet MLE	0,d,0	1,d,0	0,d,1	1,d,1
	0.0081	-0.04986	-0.03961	-0.12409
	-17739.6	-17737.4	-17737	-17732.9
	-17743.2	-17744.6	-17744.1	-17743.6

Whittle	0,d,0	1,d,0	0,d,1	1,d,1
	-0.01563	0.019439	0.019252	0.015527
	-0.894	-1	-1	-1
	0.371324	-1	-1	-1
	-15861.8	-15856.7	-15856.5	-15849.2
	-15865.3	-15863.8	-15863.7	-15859.9

USA SP500 monthly

Tests

ADF	0		KPSS	level		trend	
	-21.5339			B	QS	B	QS
	0.01			0.212969	0.216785	0.063964	0.065267
	-3.41311						
	1.996388						

FDF	1	0.4	0.3	0.2	0.1
	-15.41	-7.83504	-5.75831	-3.67739	-1.57947
	7.01E-54	2.34E-15	4.25E-09	0.000118	0.057114
	-3.16226	-3.3865	-3.39362	-3.40324	-3.41211
	2.184196	1.989756	1.993405	1.999525	2.003861

R/S	classical	Lo		Cavaliere	
		B	QS	B	QS
	1.524273	1.539558	1.553289	1.387194	1.399566

Robinson LM	-0.2	-0.1	0	0.1	0.2
	7.407008	2.963839	0.046084	-2.41337	-3.99261
	1.29E-13	0.003038	0.963243	0.015806	6.53E-05

Estimations

GPH	0.45	0.5	0.55	0.8	Robinson	classical	modified
	0.167383	0.257481	0.078563	0.024043		-0.01315	0.041141
	0.747054	1.086281	0.435288	0.28719		-0.1212	0.379301
	0.455722	0.278385	0.663723	0.774201		0.903531	0.704464
	0.470552	0.803029	0.287311	0.172829			
	0.637961	0.421958	0.773874	0.862786			

Wavelet MLE	0,d,0	1,d,0	0,d,1	1,d,1
	-0.01099	-0.14851	-0.11051	-0.20952
	-1886.42	-1884.14	-1883.72	-1879.11
	-1888.54	-1888.38	-1887.96	-1885.47

Whittle	0,d,0	1,d,0	0,d,1	1,d,1
	-0.01731	-0.04326	-0.04359	0.124519
	-0.33349	-1	-1	-1
	0.738766	-1	-1	-1
	-1591.24	-1585.85	-1585.86	-3593.86
	-1593.35	-1590.09	-1590.1	-3600.21

USA NASDAQ daily

Tests

ADF	0		KPSS	level		trend	
	-43.4256			B	QS	B	QS
	0.01			0.533588	0.516738	0.117237	0.111359
	-5.12547						
	1.995814						

FDF	1	0.4	0.3	0.2	0.1
	-30.5875	-15.1634	-10.9778	-6.8239	-2.68391
	9E-206	3.09E-52	2.44E-28	4.43E-12	0.003638
	-4.8342	-5.05873	-5.06894	-5.0829	-5.09786
	2.178904	1.988941	1.99377	2.001299	2.007627

R/S	classical	Lo		Cavaliere	
		B	QS	B	QS
	1.742987	1.783868	1.756378	1.958705	1.92852

Robinson LM	-0.2	-0.1	0	0.1	0.2
	18.3788	7.448223	0.297878	-4.39735	-7.6091
	1.94E-75	9.46E-14	0.765797	1.1E-05	2.76E-14

Estimations

GPH	0.45	0.5	0.55	0.8	Robinson	classical	modified
	0.093167	0.106415	0.138184	0.030081		0.040455	0.184567
	0.854541	1.275006	2.047246	1.284485		1.127665	5.144762
	0.392855	0.202379	0.040698	0.199045		0.259462	2.68E-07
	0.476346	0.663772	1.066584	0.653767			
	0.633828	0.506836	0.28616	0.513262			

Wavelet MLE	0,d,0	1,d,0	0,d,1	1,d,1
	0.080338	0.007949	0.01508	0.011592
	-42344.6	-42357.2	-42359	-42349.7
	-42348.7	-42365.3	-42367.2	-42362

Whittle	0,d,0	1,d,0	0,d,1	1,d,1
	0.043188	-0.0061	-0.01063	0.003347
	3.774799	-1	-1	-1
	0.00016	-1	-1	-1
	-34649.2	-34652	-34654.6	-34651.3
	-34653.2	-34660.2	-34662.8	-34663.5

USA NASDAQ weekly

Tests

ADF	0		KPSS	level		trend	
	-28.9025			B	QS	B	QS
	0.01			0.151179	0.168894	0.117102	0.130322
	-4.05552						
	2.003521						

FDF	1	0.4	0.3	0.2	0.1
	-22.4571	-10.5104	-7.58737	-4.65985	-1.71977
	5.5E-112	3.87E-26	1.63E-14	1.58E-06	0.042737
	-3.8371	-4.02172	-4.02684	-4.03343	-4.03808
	2.163769	1.982642	1.987127	1.994316	1.999437

R/S	classical	Lo		Cavaliere	
		B	QS	B	QS
	1.838239	1.618105	1.710288	1.395096	1.474573

Robinson LM	-0.2	-0.1	0	0.1	0.2
	15.15397	7.551807	2.171784	-1.73931	-4.44507
	7.13E-52	4.29E-14	0.029872	0.081981	8.79E-06

Estimations

GPH	0.45	0.5	0.55	0.8	Robinson	classical	modified
	0.288916	0.124045	0.143722	0.073439		0.104571	0.017877
	1.298585	0.737157	1.097664	1.310521		1.272159	0.217478
	0.194673	0.461366	0.272869	0.19061		0.203317	0.827836
	0.928798	0.463843	0.623923	0.696601			
	0.352994	0.642761	0.532678	0.486053			

Wavelet MLE	0,d,0	1,d,0	0,d,1	1,d,1
	0.072209	0.119384	0.113938	0.128866
	-4107.48	-4101.53	-4101.41	-4098.63
	-4110.06	-4106.68	-4106.56	-4106.35

Whittle	0,d,0	1,d,0	0,d,1	1,d,1
	0.014568	0.057785	0.058895	0.051905
	0.445893	-1	-1	-1
	0.655674	-1	-1	-1
	-3363.46	-3358.22	-3358.15	-3352.03
	-3366.04	-3363.37	-3363.31	-3359.76

UK FTSE100 daily

Tests

ADF	1		KPSS	level		trend	
	-33.414			B	QS	B	QS
	0.01			0.515203	0.446765	0.063407	0.053034
	-6.28924						
	2.006541						

FDF	1	0.4	0.3	0.2	0.1
	-28.8795	-14.4142	-10.1771	-5.96343	-1.75192
	1.1E-183	2.11E-47	1.25E-24	1.23E-09	0.039894
	-5.99133	-6.22063	-6.22954	-6.24365	-6.26115
	2.12132	1.943318	1.950565	1.963409	1.979693

R/S	classical	Lo		Cavaliere	
		B	QS	B	QS
	1.234109	1.361849	1.278485	1.555232	1.46003

Robinson LM	-0.2	-0.1	0	0.1	0.2
	12.68383	4.705447	-0.70349	-4.52956	-7.28154
	7.27E-37	2.53E-06	0.481753	5.91E-06	3.3E-13

Estimations

GPH	0.45	0.5	0.55	0.8	Robinson	classical	modified
	-0.08499	-0.09868	-0.09806	0.002709		0.020564	0.057924
	-0.68131	-0.96988	-1.34438	0.112093		0.573225	1.614605
	0.495717	0.332165	0.178901	0.910756		0.566492	0.106396
	-0.43454	-0.61553	-0.75689	0.058887			
	0.663896	0.538205	0.449115	0.953042			

Wavelet MLE	0,d,0	1,d,0	0,d,1	1,d,1
	0.03674	-0.02065	-0.01734	-0.00296
	-43367.7	-43372.3	-43373.7	-43389.5
	-43371.8	-43380.5	-43381.8	-43401.8

Whittle	0,d,0	1,d,0	0,d,1	1,d,1
	0.026444	-0.01833	-0.01528	-0.01528
	2.182967	-1	-1	-1
	0.029038	-1	-1	-1
	-37613	-37612.3	-37612.4	-37604.1
	-37617.1	-37620.4	-37620.6	-37616.4

UK FTSE100 weekly

Tests

ADF	1	KPSS	level		trend	
	-19.3276		B	QS	B	QS
	0.01		0.165668	0.156917	0.055414	0.051684
	-4.74537					
	1.989044					

FDF	1	0.4	0.3	0.2	0.1
	-24.7421	-11.594	-8.59582	-5.60489	-2.61488
	1.9E-135	2.21E-31	4.13E-18	1.04E-08	0.004463
	-4.58449	-4.74091	-4.74333	-4.7476	-4.75133
	2.10514	1.95261	1.958921	1.970463	1.984012

R/S	classical	Lo		Cavaliere	
		B	QS	B	QS
	1.025134	0.977849	0.951674	1.071591	1.042907

Robinson LM	-0.2	-0.1	0	0.1	0.2
	10.71233	4.890246	0.438975	-2.58421	-4.9692
	8.91E-27	1.01E-06	0.66068	0.00976	6.72E-07

Estimations

GPH	0.45	0.5	0.55	0.8	Robinson	classical	modified
	0.140469	0.086685	0.059817	0.00931		-0.03266	-0.09631
	1.040935	0.823179	0.490702	0.170166		-0.3973	-1.17166
	0.298399	0.410791	0.623849	0.864947		0.691148	0.241333
	0.451577	0.32414	0.259676	0.088309			
	0.651574	0.745832	0.795114	0.929631			

Wavelet MLE	0,d,0	1,d,0	0,d,1	1,d,1
	-0.09767	-0.07097	-0.07177	-0.07716
	-4444.8	-4438.71	-4438.67	-4433.88
	-4447.38	-4443.86	-4443.82	-4441.61

Whittle	0,d,0	1,d,0	0,d,1	1,d,1
	-0.05552	-0.04098	-0.04392	-0.03787
	-1.43742	-1	-1	-1
	0.150599	-1	-1	-1
	-3926.48	-3920.34	-3920.32	-3914.57
	-3929.06	-3925.49	-3925.47	-3922.3

France CAC40 daily

Tests

ADF	0		KPSS	level		trend	
	-43.8182			B	QS	B	QS
	0.01			0.31046	0.289298	0.180045	0.167191
	-5.78757						
	1.995065						

FDF	1	0.4	0.3	0.2	0.1
	-31.692	-15.9685	-11.7282	-7.51716	-3.3155
	1E-220	1.06E-57	4.57E-32	2.8E-14	0.000457
	-5.52385	-5.74196	-5.74958	-5.76108	-5.77415
	2.15311	1.970431	1.976094	1.98594	1.996965

R/S	classical	Lo		Cavaliere	
		B	QS	B	QS
	1.553511	1.63262	1.560068	1.541335	1.472839

Robinson LM	-0.2	-0.1	0	0.1	0.2
	28.02967	17.64668	10.24652	4.095692	0.139045
	7.1E-173	1.08E-69	1.23E-24	4.21E-05	0.889415

Estimations

GPH	0.45	0.5	0.55	0.8	Robinson	classical	modified
	0.123257	0.043824	0.072018	-0.01714		-0.05907	0.037197
	0.896988	0.376886	0.808853	-0.49547		-1.24739	0.785554
	0.369831	0.706297	0.418694	0.620319		0.212254	0.432129
	0.535078	0.231749	0.459623	-0.28221			
	0.592596	0.816733	0.645787	0.777786			

Wavelet MLE	0,d,0	1,d,0	0,d,1	1,d,1
	-0.00251	-0.05657	-0.05102	-0.06667
	-19424.8	-19422.9	-19423	-19415.2
	-19428.4	-19430	-19430.1	-19425.8

Whittle	0,d,0	1,d,0	0,d,1	1,d,1
	-0.00086	-0.03023	-0.03024	-0.02662
	-0.04902	-1	-1	-1
	0.960901	-1	-1	-1
	-17686.3	-17680.8	-17680.9	-17673.5
	-17689.9	-17687.9	-17688.1	-17684.1

France CAC40 weekly

Tests

ADF	0	KPSS	level		trend	
	-25.4235		B	QS	B	QS
	0.01		0.143923	0.14586	0.144805	0.146761
	-4.29286					
	2.001686					

FDF	1	0.4	0.3	0.2	0.1
	-18.9365	-9.95913	-7.57513	-5.20757	-2.84424
	2.85E-80	1.15E-23	1.79E-14	9.57E-08	0.002226
	-4.04961	-4.27319	-4.27973	-4.2892	-4.29909
	2.206323	1.995686	1.997819	2.002544	2.006705

R/S	classical	Lo		Cavaliere	
		B	QS	B	QS
	1.495712	1.500258	1.510319	1.176087	1.183974

Robinson LM	-0.2	-0.1	0	0.1	0.2
	9.019189	3.402423	-0.10284	-3.05974	-4.88187
	1.89E-19	0.000668	0.918088	0.002215	1.05E-06

Estimations

GPH	0.45	0.5	0.55	0.8	Robinson	classical	modified
	0.123257	0.043824	0.072018	0.037807		-0.05907	0.037197
	0.896988	0.376886	0.808853	0.665531		-1.24739	0.785554
	0.369831	0.706297	0.418694	0.506012		0.212254	0.432129
	0.535078	0.231749	0.459623	0.358613			
	0.592596	0.816733	0.645787	0.719885			

Wavelet MLE	0,d,0	1,d,0	0,d,1	1,d,1
	-0.04166	0.079885	0.171751	0.218759
	-4132.94	-4136.92	-4136.69	-4129.99
	-4135.52	-4142.07	-4141.84	-4137.72

Whittle	0,d,0	1,d,0	0,d,1	1,d,1
	-0.02563	-0.01006	-0.00668	0.035912
	-0.72457	-1	-1	-1
	0.468715	-1	-1	-1
	-3626.26	-3620.17	-3620.2	-3614.28
	-3628.84	-3625.33	-3625.35	-3622.01

Japan Nikkei 225 daily

Tests

ADF	1		KPSS	level		trend	
	-34.4968			B	QS	B	QS
	0.01			0.094648	0.099131	0.042186	0.044135
	-5.59348						
	1.994962						
FDF	1	0.4	0.3	0.2	0.1		
	-33.191	-18.2143	-14.0282	-9.89845	-5.80381		
	7.3E-242	1.99E-74	5.24E-45	2.11E-23	3.24E-09		
	-5.29387	-5.53769	-5.54579	-5.55771	-5.57139		
	2.224793	2.005782	2.007537	2.011521	2.015031		
R/S	classical	Lo		Cavaliere			
		B	QS	B	QS		
	1.055868	1.15348	1.17979	1.150935	1.177187		
Robinson LM	-0.2	-0.1	0	0.1	0.2		
	12.15783	3.241186	-2.7235	-6.55481	-9.30366		
	5.21E-34	0.00119	0.006459	5.57E-11	1.36E-20		

Estimations

GPH	0.45	0.5	0.55	0.8	Robinson	classical	modified
	0.092078	-0.00473	-0.03398	-0.00208		-0.01362	-0.01719
	0.617285	-0.04607	-0.46156	-0.08452		-0.37964	-0.4792
	0.537081	0.963256	0.644424	0.932644		0.704216	0.631793
	0.470778	-0.0295	-0.26224	-0.04521			
	0.6378	0.976466	0.793134	0.963943			
Wavelet MLE	0,d,0	1,d,0	0,d,1	1,d,1			
	-0.02236	-0.03117	-0.03167	-0.03294			
	-38730.4	-38722.5	-38722.6	-38724.3			
	-38734.5	-38730.7	-38730.7	-38736.6			
Whittle	0,d,0	1,d,0	0,d,1	1,d,1			
	-0.02697	-0.03571	-0.03811	-0.04212			
	-2.12804	-1	-1	-1			
	0.033334	-1	-1	-1			
	-34684.5	-34676.6	-34676.7	-63050.6			
	-34688.5	-34684.7	-34684.9	-63062.8			

Japan Nikkei 225 weekly

Tests

ADF	0		KPSS	level		trend	
	-31.5065			B	QS	B	QS
	0.01			0.536792	0.539105	0.072107	0.07209
	-4.31932						
	1.992699						

FDF	1	0.4	0.3	0.2	0.1
	-25.3722	-12.7533	-9.77464	-6.80518	-3.83364
	2.6E-142	1.49E-37	7.23E-23	5.05E-12	6.31E-05
	-4.13024	-4.3111	-4.314	-4.31816	-4.32077
	2.18088	1.977307	1.979479	1.984406	1.988438

R/S	classical	Lo		Cavaliere	
		B	QS	B	QS
	1.708221	1.634369	1.637886	1.600566	1.604011

Robinson LM	-0.2	-0.1	0	0.1	0.2
	14.50228	6.346252	0.715998	-3.04203	-5.62904
	1.17E-47	2.21E-10	0.473992	0.00235	1.81E-08

Estimations

GPH	0.45	0.5	0.55	0.8	Robinson	classical	modified
	-0.05084	0.049807	-0.12583	-0.03819		0.033798	0.030936
	-0.20799	0.241856	-0.75921	-0.61337		0.411168	0.376355
	0.83532	0.808989	0.448081	0.539904		0.680949	0.706653
	-0.16344	0.186244	-0.54624	-0.36223			
	0.870175	0.852254	0.5849	0.717183			

Wavelet MLE	0,d,0	1,d,0	0,d,1	1,d,1
	-0.03464	0.049359	0.038044	0.03636
	-4096.5	-4092.49	-4091.62	-4106.3
	-4099.08	-4097.64	-4096.77	-4114.03

Whittle	0,d,0	1,d,0	0,d,1	1,d,1
	-0.03275	0.016063	0.004269	0.008706
	-0.88163	-1	-1	-1
	0.377979	-1	-1	-1
	-3584.32	-3578.94	-3578.68	-3573.98
	-3586.89	-3584.09	-3583.84	-3581.71

Singapore Straits Times daily

Tests

ADF	0		KPSS	level		trend	
	-38.4229			B	QS	B	QS
	0.01			0.070898	0.068904	0.068214	0.066296
	-5.56765						
	1.997081						

FDF	1	0.4	0.3	0.2
	-27.8408	-10.4274	-6.15413	-1.87933
	7E-171	9.3E-26	3.77E-10	0.0301
	-5.33484	-5.5097	-5.52127	-5.5358
	2.104896	1.957377	1.969389	1.985289

R/S	classical	Lo		Cavaliere	
		B	QS	B	QS
	1.812642	1.605315	1.582857	1.597632	1.575281

Robinson LM	-0.2	-0.1	0	0.1	0.2
	26.14922	14.00325	5.707097	-0.04902	-4.05151
	1E-150	1.49E-44	1.15E-08	0.960904	5.09E-05

Estimations

GPH	0.45	0.5	0.55	0.8	Robinson	classical	modified
	-0.01656	0.062415	0.051656	0.067844		0.086969	0.277122
	-0.16488	0.68936	0.778041	2.218937		1.836667	5.852468
	0.869052	0.490675	0.436635	0.0266		0.066259	4.84E-09
	-0.0719	0.330062	0.329675	1.117129			
	0.942682	0.741353	0.741645	0.263939			

Wavelet MLE	0,d,0	1,d,0	0,d,1	1,d,1
	0.127531	0.091039	0.095457	0.07151
	-18434	-18429.6	-18429.5	-18422.7
	-18437.6	-18436.8	-18436.6	-18433.4

Whittle	0,d,0	1,d,0	0,d,1	1,d,1
	0.09845	-0.00161	0.014469	0.005947
	6.326595	-1	-1	-1
	2.51E-10	-1	-1	-1
	-17233.4	-17241.2	-17241.2	-17233.7
	-17236.9	-17248.3	-17248.3	-17244.3

Singapore Straits Times weekly

Tests

ADF	0	KPSS	level		trend	
	-26.9545		B	QS	B	QS
	0.01		0.210518	0.218095	0.04234	0.043832
	-4.04713					
	1.992582					

FDF	1	0.4	0.3	0.2
	-27.8408	-10.4274	-6.15413	-1.87933
	7E-171	9.3E-26	3.77E-10	0.0301
	-5.33484	-5.5097	-5.52127	-5.5358
	2.104896	1.957377	1.969389	1.985289

R/S	classical	Lo		Cavaliere	
		B	QS	B	QS
	1.509627	1.390537	1.415342	1.317541	1.341044

Robinson LM	-0.2	-0.1	0	0.1	0.2
	13.22912	6.450783	1.461439	-1.86031	-4.285
	5.96E-40	1.11E-10	0.143895	0.062842	1.83E-05

Estimations

GPH	0.45	0.5	0.55	0.8	Robinson	classical	modified
	-0.0264	0.080666	0.127329	0.143006		0.243997	0.126724
	-0.18922	0.592977	0.819741	2.534074		2.968356	1.541659
	0.849997	0.553459	0.412747	0.011573		0.002994	0.123156
	-0.08487	0.301633	0.552756	1.356476			
	0.932367	0.762932	0.58043	0.174948			

Wavelet MLE	0,d,0	1,d,0	0,d,1	1,d,1
	0.007433	0.200232	0.219476	0.156095
	-3803.15	-3814.31	-3806.63	-3829.53
	-3805.73	-3819.46	-3811.79	-3837.26

Whittle	0,d,0	1,d,0	0,d,1	1,d,1
	0.04241	0.074342	0.080001	0.081063
	1.312266	-1	-1	-1
	0.18943	-1	-1	-1
	-3459.82	-3454.04	-3454.08	-3447.85
	-3462.4	-3459.19	-3459.24	-3455.58

Taiwan Weighted index daily

Tests

ADF	0		KPSS	level		trend	
	-33.1345			B	QS	B	QS
	0.01			0.064544	0.05998	0.06541	0.060787
	-5.06795						
	2.003198						

FDF	1	0.4	0.3	0.2	0.1
	-25.5592	-12.0961	-8.7279	-5.35457	-1.96402
	2.2E-144	5.54E-34	1.3E-18	4.29E-08	0.024764
	-4.85211	-5.03898	-5.04337	-5.05019	-5.05677
	2.165326	1.970652	1.975549	1.985225	1.996269

R/S	classical	Lo		Cavaliere	
		B	QS	B	QS
	1.413053	1.35443	1.305668	1.312111	1.264872

Robinson LM	-0.2	-0.1	0	0.1	0.2
	16.34629	7.779204	1.80793	-2.28051	-5.16802
	4.62E-60	7.3E-15	0.070617	0.022577	2.37E-07

Estimations

GPH	0.45	0.5	0.55	0.8	Robinson	classical	modified
	0.031958	0.097909	0.067245	0.077239		0.120972	0.075926
	0.192528	0.733993	0.662454	1.889172		1.935546	1.214816
	0.847367	0.463121	0.507829	0.059152		0.052923	0.224436
	0.119501	0.431842	0.355604	0.96357			
	0.904878	0.665856	0.722137	0.335261			

Wavelet MLE	0,d,0	1,d,0	0,d,1	1,d,1
	0.022791	-0.01903	-0.01493	-0.02832
	-8943.75	-8938.92	-8938.88	-8932.04
	-8946.81	-8945.04	-8945	-8941.21

Whittle	0,d,0	1,d,0	0,d,1	1,d,1
	0.044001	0.040821	0.041139	-0.04875
	1.904088	-1	-1	-1
	0.056899	-1	-1	-1
	-8081.07	-8074.14	-8074.14	-8068.56
	-8084.13	-8080.26	-8080.26	-8077.74

Taiwan Weighted index weekly

Tests

ADF	0		KPSS	level		trend	
	-15.9624			B	QS	B	QS
	0.01			0.059408	0.065796	0.059833	0.066238
	-3.34632						
	2.001517						

FDF	1	0.4	0.3	0.2	0.1
	-11.6483	-6.3465	-4.85264	-3.36034	-1.86097
	1.17E-31	1.1E-10	6.09E-07	0.000389	0.031374
	-3.07834	-3.31561	-3.32029	-3.32776	-3.33567
	2.223024	1.974445	1.974028	1.976375	1.979015

R/S	classical	Lo		Cavaliere	
		B	QS	B	QS
	1.238137	1.204891	1.268017	1.171541	1.232919

Robinson LM	-0.2	-0.1	0	0.1	0.2
	5.335129	2.090694	-0.11948	-1.95528	-3.14332
	9.55E-08	0.036555	0.904899	0.05055	0.00167

Estimations

GPH	0.45	0.5	0.55	0.8	Robinson	classical	modified
	0.125699	0.188504	0.085096	0.062099		0.070681	0.075926
	0.646473	1.27111	0.608044	0.611687		0.49477	0.531482
	0.519149	0.206031	0.544253	0.541847		0.620762	0.595085
	0.29402	0.509139	0.25697	0.338927			
	0.768742	0.610655	0.797202	0.734665			

Wavelet MLE	0,d,0	1,d,0	0,d,1	1,d,1
	-0.05236	-0.07335	-0.0785	-0.08448
	-895.467	-890.78	-890.827	-894.653
	-897.16	-894.166	-894.212	-899.732

Whittle	0,d,0	1,d,0	0,d,1	1,d,1
	0.017211	0.041267	0.05329	0.075677
	0.251821	-1	-1	-1
	0.80118	-1	-1	-1
	-773.6	-768.815	-768.842	-764.056
	-775.293	-772.201	-772.227	-769.135

Romania BET daily

Tests

ADF	0		KPSS	level		trend	
	-25.7844			B	QS	B	QS
	0.01			0.616224	0.637751	0.054558	0.054889
	-5.02044						
	2.000625						

FDF	1	0.4	0.3
	-18.5222	-3.86458	-0.56983
	6.84E-77	5.56E-05	0.284397
	-4.86661	-5.01102	-5.02764
	2.045159	1.955509	1.975536

R/S	classical	Lo		Cavaliere	
		B	QS	B	QS
	1.935119	1.4644	1.489759	1.484727	1.510438

Robinson LM	-0.2	-0.1	0	0.1	0.2
	26.94749	16.79774	9.375312	3.826367	-0.03631
	6.1E-160	2.54E-63	6.9E-21	0.00013	0.971031

Estimations

GPH	0.45	0.5	0.55	0.8	Robinson	classical	modified
	0.050982	0.01659	0.007108	0.195572		0.249901	0.345542
	0.444139	0.192866	0.105293	4.669356		3.998419	5.528668
	0.657036	0.847102	0.916164	3.42E-06		6.38E-05	3.23E-08
	0.190636	0.073171	0.037589	2.439785			
	0.848811	0.94167	0.970015	0.014696			

Wavelet MLE	0,d,0	1,d,0	0,d,1	1,d,1
	0.220784	-0.52469	0.026404	-0.45339
	-9039.88	-9092.67	-9064.88	-9084.82
	-9042.94	-9098.79	-9071	-9094

Whittle	0,d,0	1,d,0	0,d,1	1,d,1
	0.193001	0.037326	0.079587	0.035578
	10.03254	-1	-1	-1
	1.1E-23	-1	-1	-1
	-8062.45	-8069.72	-8068.79	-8062.79
	-8065.51	-8075.84	-8074.91	-8071.97

Romania BETC daily

Tests

ADF	0		KPSS	level		trend	
	-24.1307			B	QS	B	QS
	0.01			0.625873	0.728776	0.083495	0.092431
	-5.57191						
	2.001343						

FDF	1	0.4	0.3
	-17.3954	-3.82789	-0.7027
	4.47E-68	6.46E-05	0.24112
	-5.38662	-5.53575	-5.55136
	2.060288	1.956066	1.973778

R/S	classical	Lo		Cavaliere	
		B	QS	B	QS
	1.801999	1.266567	1.366728	1.275035	1.375866

Robinson LM	-0.2	-0.1	0	0.1	0.2
	28.02967	17.64668	10.24652	4.095692	0.139045
	7.1E-173	1.08E-69	1.23E-24	4.21E-05	0.889415

Estimations

GPH	0.45	0.5	0.55	0.8	Robinson	classical	modified
	0.088351	0.085298	0.12068	0.148007		0.292333	0.277366
	0.545883	0.717317	1.11177	3.673721		4.67733	4.437849
	0.585265	0.473342	0.266499	0.000251		2.91E-06	9.09E-06
	0.33037	0.376219	0.638177	1.846404			
	0.74112	0.706754	0.523358	0.064834			

Wavelet MLE	0,d,0	1,d,0	0,d,1	1,d,1
	0.187316	0.010785	0.07738	-0.7687
	-9875.88	-9883.75	-9881.75	-9895.63
	-9878.94	-9889.87	-9887.87	-9904.81

Whittle	0,d,0	1,d,0	0,d,1	1,d,1
	0.209532	0.071824	0.091056	0.113169
	11.70581	-1	-1	-1
	1.19E-31	-1	-1	-1
	-8594.67	-8601.09	-8602.67	-8596.11
	-8597.73	-8607.21	-8608.78	-8605.29

Romania RASDAQ daily

Tests

ADF	0		KPSS	level		trend	
	-40.0337			B	QS	B	QS
	0.01			0.324938	0.481132	0.067394	0.09872
	-5.85627						
	2.02315						

FDF	1	0.4	0.3	0.2	0.1
	-30.4091	-20.3003	-17.3931	-14.581	-11.8402
	2.1E-203	6.38E-92	4.66E-68	1.86E-48	1.21E-32
	-5.54924	-5.85611	-5.86392	-5.87459	-5.8862
	2.402077	2.123701	2.116916	2.107335	2.09235

R/S	classical	Lo		Cavaliere	
		B	QS	B	QS
	0.914573	0.976649	1.18842	1.109351	1.349896

Robinson LM	-0.2	-0.1	0	0.1	0.2
	1.685306	-3.51757	-5.11351	-8.77614	-10.1189
	0.091929	0.000436	3.16E-07	1.69E-18	4.55E-24

Estimations

GPH	0.45	0.5	0.55	0.8	Robinson	classical	modified
	0.143385	0.175931	-0.01728	-0.10716		-0.16671	-0.31997
	0.672634	1.010667	-0.12449	-1.82576		-2.02817	-3.89261
	0.501485	0.312655	0.900973	0.068471		0.042543	9.92E-05
	0.460951	0.657857	-0.07502	-1.01642			
	0.644834	0.51063	0.940199	0.309432			

Wavelet MLE	0,d,0	1,d,0	0,d,1	1,d,1
	-0.09287	0.035027	0.05585	0.002646
	-5795.02	-5799.48	-5796.67	-5824.43
	-5797.59	-5804.64	-5801.82	-5832.16

Whittle	0,d,0	1,d,0	0,d,1	1,d,1
	-0.11843	-0.07682	-0.07198	-0.04272
	-2.91684	-1	-1	-1
	0.003536	-1	-1	-1
	-5123.27	-5118.11	-5118.12	-5111.96
	-5125.85	-5123.26	-5123.27	-5119.69

Romania Leu/Usd exchange rate daily

Tests

ADF	0		KPSS	level		trend	
	-20.1743			B	QS	B	QS
	0.01			0.267725	0.253525	0.272444	0.257959
	-8.06356						
	1.953061						

FDF	1	0.4	0.3
	-12.9272	-1.5244	1.218639
	1.58E-38	0.063705	0.888509
	-7.84332	-8.00172	-8.02211
	2.006539	1.929734	1.953819

R/S	classical	Lo		Cavaliere	
		B	QS	B	QS
	2.616453	2.13332	2.075972	1.986022	1.932634

Robinson LM	-0.2	-0.1	0	0.1	0.2
	25.14012	15.04209	8.555493	2.988096	-0.16517
	1.8E-139	3.89E-51	1.17E-17	0.002807	0.868814

Estimations

GPH	0.45	0.5	0.55	0.8	Robinson	classical	modified
	0.303936	0.292666	0.386072	0.151869		0.197615	0.309455
	2.660289	3.467049	5.438559	3.383526		2.404091	3.764678
	0.008053	0.000571	8.35E-08	0.000771		0.016213	0.000167
	0.977084	1.094365	1.676001	1.440539			
	0.328528	0.273795	0.093738	0.149715			

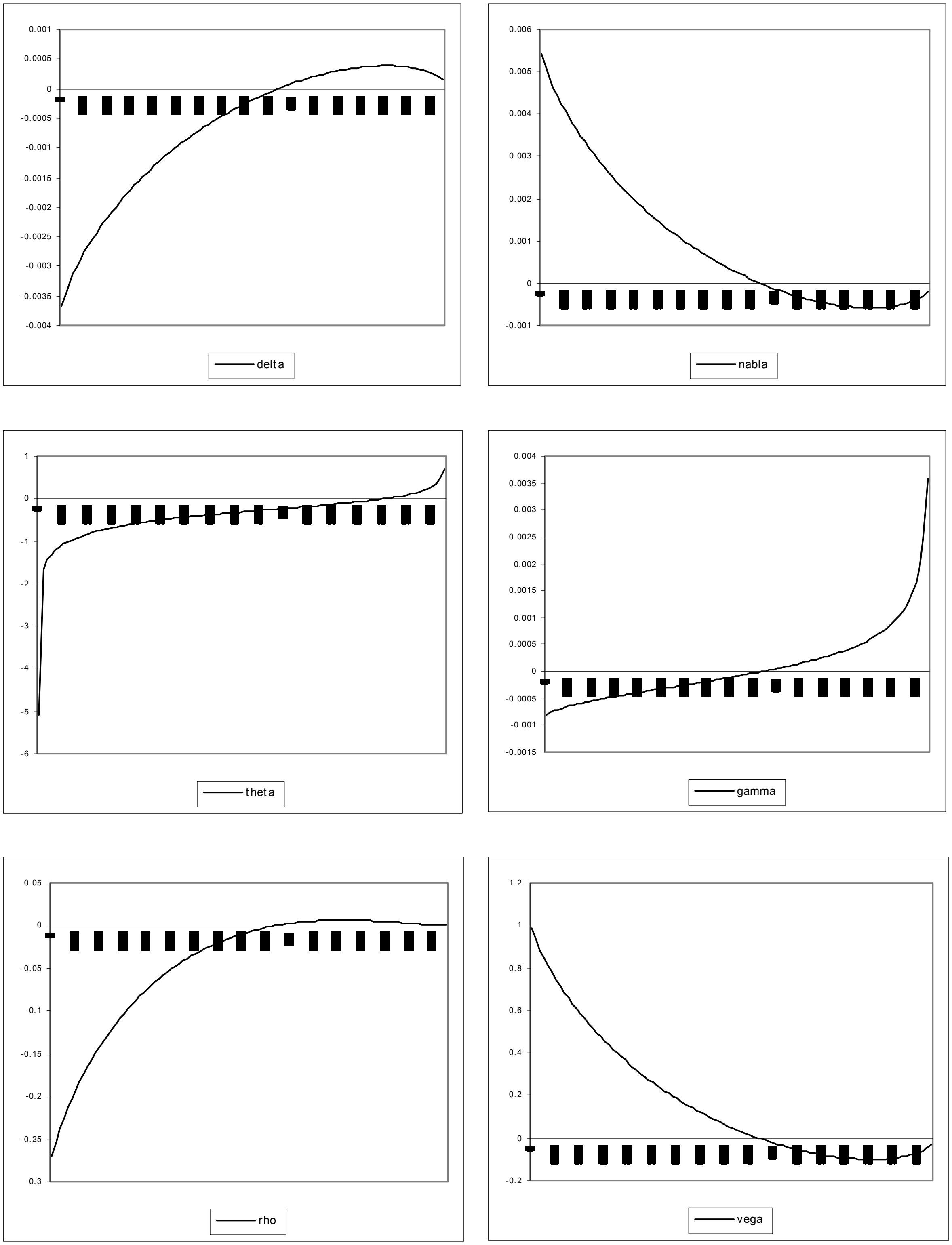
Wavelet MLE	0,d,0	1,d,0	0,d,1	1,d,1
	0.232588	0.164671	0.148242	0.164629
	-7088.04	-7085.21	-7087.47	-7153.63
	-7090.62	-7090.36	-7092.62	-7161.36

Whittle	0,d,0	1,d,0	0,d,1	1,d,1
	0.222211	-0.02145	-0.0081	0.022792
	8.310969	-1	-1	-1
	9.49E-17	-1	-1	-1
	-5544.82	-5566.59	-5581.37	-5575.89
	-5547.4	-5571.74	-5586.53	-5583.62

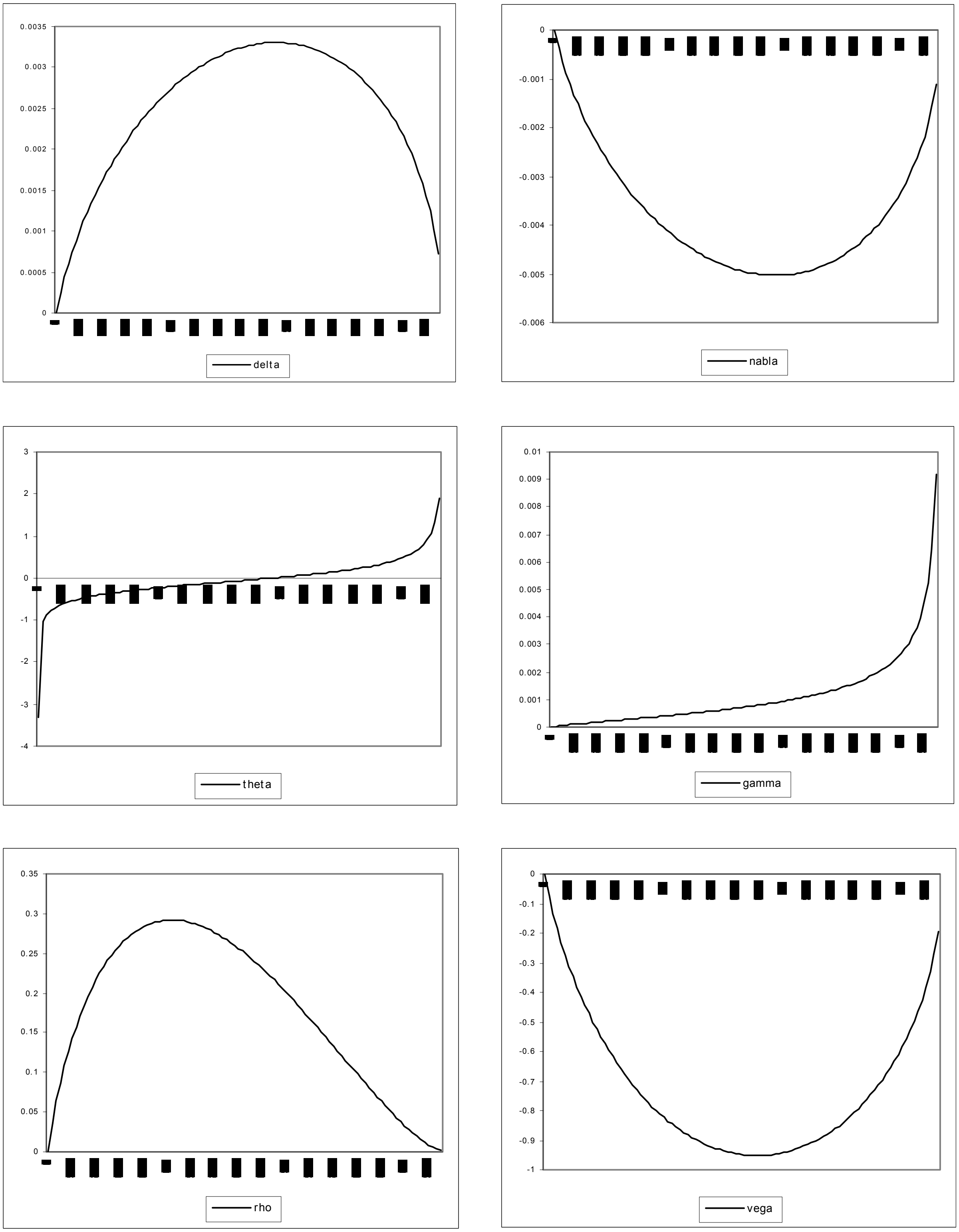
Critical values for the KPSS test			
	0.1	0.05	0.01
KPSS level	0.347	0.463	0.739
KPSS trend	0.119	0.146	0.216

Critical values for the R/S test			
	0.1	0.05	0.01
R/S Lo	1.62	1.747	2.001
R/S Cavaliere	1.55	1.682	1.956

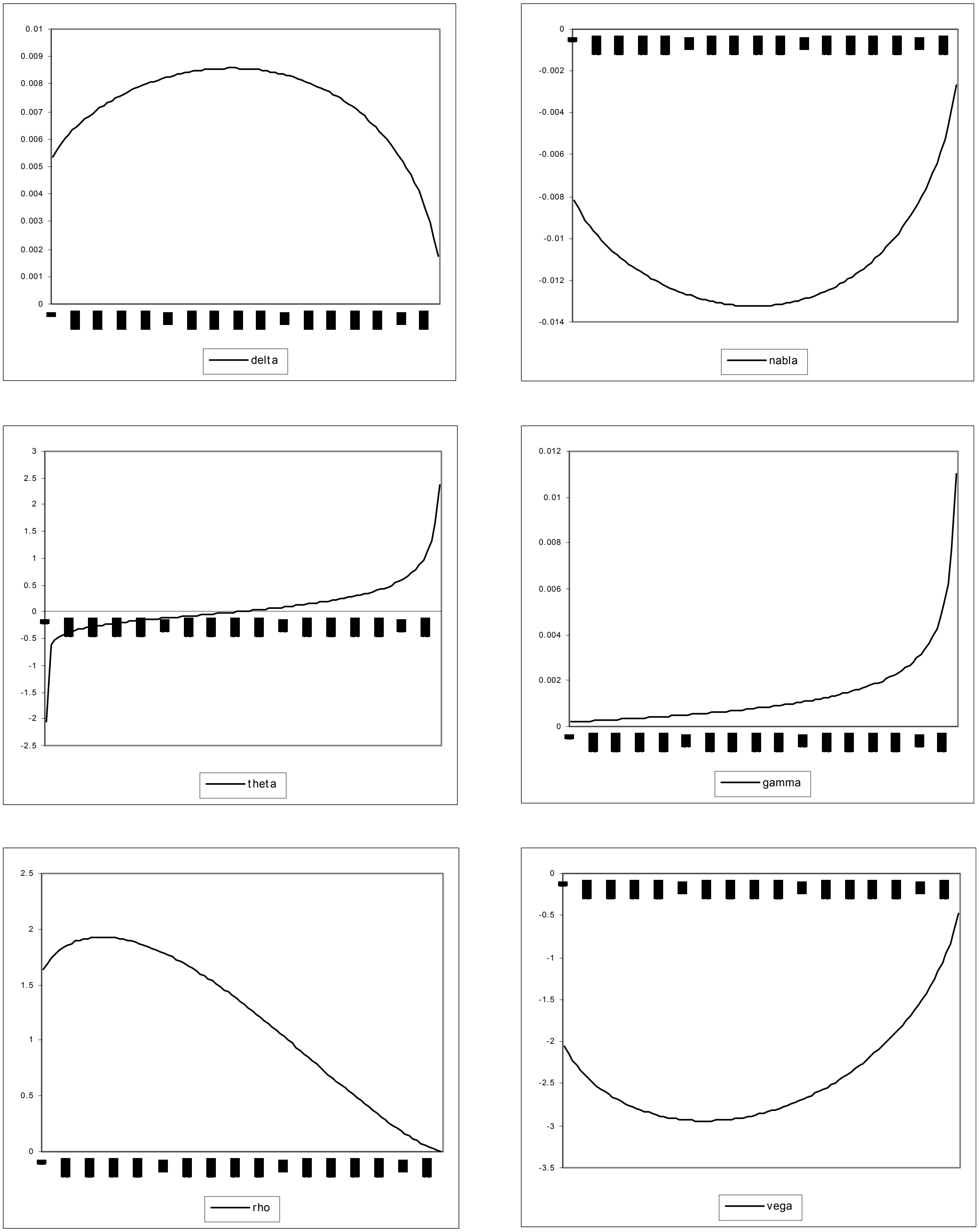
Appendix 2



The difference in time of the indicators for the two models for T=0.5



The difference in time of the indicators for the two models for T=1



The difference in time of the indicators for the two models for T=2