Search, Nash Bargaining and Rule of Thumb Consumers[*]

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Abstract

This paper analyses the effects of introducing typical Keynesian features, namely rule-of-thumb consumers and consumption habits, into a standard labour market search model. It is a well-known fact that labour market matching with Nash-wage bargaining improves the ability of the standard real business cycle model to replicate some of the cyclical properties featuring the labour market. However, when habits and rule-of-thumb consumers are taken into account, the labour market search model gains extra power to reproduce some of the stylised facts characterising the US labour market, as well as other business cycle facts concerning aggregate consumption and investment behaviour.

Keywords: general equilibrium, labour market search, habits, rule-of-tumb consumers.
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1. Introduction

Business cycle models with search frictions and wage bargaining have improved our understanding of US labour market stylized facts, as shown by the work of Merz (1995), Andolfatto (1996), Chéron and Langot (2004) and Yashiv (2006 and 2007), among many others. However, the basic search model fails to account for some important correlations among labour market variables, such as those relating labour share and output, hours and labour productivity and hours and real wages. Chéron and Langot (2004) partially

overcome some of the limitations of the basic search model by introducing a particular set of non-separable preferences as developed in Rogerson and Wright (1988). The idea behind this change in preferences is straightforward. Given that in a standard bilateral bargaining environment individuals use insurance markets to equate consumption across states of employment and unemployment, a technological boom generates additional upward pressure on the real wage that makes this variable correlate positively with hours worked. The reason is that in a search setting, the real wage is given by labour productivity, but also by the worker's outside option (the reservation wage). With standard preferences, the outside option behaves procyclically in an expansion because unemployed workers enjoy more leisure, putting upward pressure on the reservation wage and, thus, on the bargained wage. Chéron and Langot (2004) reverse this mechanism by changing the preferences specification in a way that makes the outside option behave countercyclically. However, this approach has a cost in that balanced growth is not guaranteed.

In this paper we propose a different alternative to overcome this empirical shortcoming of the labour market search model, while maintaining standard preferences. To this end, we will introduce two typical Keynesian features, namely rule-of-thumb (*RoT*) consumers and consumption habits, into a standard labour market search model. For example, Galí, López-Salido and Vallés (2007), Andrés, Doménech and Fatás (2008), Coenen and Straub (2005), Erceg, Guerrieri and Gust (2005), Forni, Monteforte and Sesa (2006) or López-Salido and Rabanal (2006) are good examples of why it is important to take into account the presence of *RoT* consumers in explaining many features of advanced economies at business cycle frequencies. However, the merits of the existence of *RoT* consumers and/or consumption habits have not yet been explored in the context of the labour market, which is the main objective of this paper.

In our setting, two types of household will coexist in the economy. First, optimising consumers that maximise their utility along the life-cycle having access to the credit market. This type of worker owns firms and can accumulate wealth along the life-cycle. Second, *RoT* consumers that have no access to credit markets and are constrained to consuming their work earnings each period. The consumption of both types of consumers is subject to consumption habits. There is risk-sharing at household level, but not between households. Although optimising and *RoT* households have a different reservation wage, they pool together in the labour market and bargain with firms to distribute employment according to their shares in the working-age population. As we will see, the implication of this simplifying assumption is that all workers receive the same wages, work the same number of hours and suffer the same unemployment rates. The main feature of our model is that although *RoT* consumers are not allowed to use their wealth to smooth consumption over time, they take advantage of the fact that a matching today is to a certain extent

likely to continue in the future, yielding a labour income that in turn will be used to consume tomorrow. Therefore, they use the margin that hours and wage negotiation provides them to improve their lifetime utility, by narrowing the gap in utility with respect to Ricardian consumers. As we will show, as habits increase, for a given share of *RoT* consumers in the economy, the impact of a technology shock on total labour income is reduced due to pressure of *RoT* consumers to smooth consumption using the labour market negotiation in hours and wages as an intertemporal mechanism. This mechanism reduces the simulated correlation between the real wage and total hours to values close to those obtained empirically. In summary, when habits and rule-of-thumb consumers are taken into account, the labour market search model gains extra power to reproduce some of the stylised facts characterising the US labour market, as well as other business cycle facts concerning aggregate consumption and investment behaviour.

The paper is organised as follows. Section 2 provides a detailed description of the theoretical model. Section 3 deals with the empirical results. Section 4 presents the main conclusions.

2. Theoretical framework

We model a decentralised, closed economy where households and firms interact each period by trading one final good and two production factors. In order to produce output, firms employ physical capital and labour. While private physical capital is exchanged in a perfectly competitive standard market, the labour market is subject to search costs.

There are two types of households that possess the available production factors. On the one hand, optimising households own all the capital of the firms operating in the economy. They rent physical capital to firms, for which they are paid income in the form of interest. On the other hand, *RoT* consumers do not have access to capital markets. However, both types of households supply their labour services to competitive firms, which pay them wages. Also, both types of consumers will display similar internal habits in consumption spending.

Each household is made up of working-age agents who may be either employed or unemployed. New jobs are created after investing in searching activities. If unemployed, agents are actively searching for a job. Firms' investment in vacant posts is endogenously determined and so are job inflows. However, job destruction is exogenous. The fact that exchanges in the labour market are resource and time-consuming generates a monopoly rent associated with each job match. It is assumed that optimising and *RoT* workers are identical in terms of working capabilities, so households pool together in the labour market and bargain with firms over these monopoly rents in Nash fashion. Thus, wages and hours of work are simultaneously determined for the two types of households through an

efficient bargaining process.

2.1 Households

Following Galí et al. (2007), liquidity-constrained consumers are incorporated into the standard labour market search model. This extension is consistent with the large body of empirical research that finds consumption behaviour to deviate substantially from the permanent-income hypothesis. There are, hence, two types of representative households. One representative household, of size N_t^o, enjoys unlimited access to capital markets, so its members substitute consumption intertemporally in response to changes in interest rates. We will refer to these households as "Ricardian or optimising consumers". Another representative household, of size N_t^r, does not have access to capital markets, so its members can only consume out of current labour income. We will refer to these liquidity-constrained consumers as "rule-of-thumb (RoT) consumers". The size of the working-age population is given by $N_t = N_t^o + N_t^r$. Let $1 - \lambda^r$ and λ^r denote the fractions in the working-age population of Ricardian and RoT consumers, which are assumed to be constant over time. For the sake of simplicity, we assume no growth in the working-age population.

Both types of households maximise intertemporal utility by selecting streams of consumption and leisure. Inside each group of households, their members may be either employed or unemployed, but they internally ensure each other's consumption against fluctuations in employment, as in Andolfatto (1996) or Merz (1995). Due to labour search costs and the presence of unemployment, our specification for RoT consumers differs considerably from Galí et al. (2007).

Optimising households

Ricardian households face the following maximisation programme:

$$\max_{c_t^o, k_{t+1}^o} E_t \sum_{t=0}^{\infty} \beta^t \left[\ln \left(c_t^o - h^o c_{t-1}^o \right) + n_t^o \phi_1 \frac{(1 - l_{1t})^{1-\eta}}{1 - \eta} + (1 - n_t^o)\phi_2 \frac{(1 - l_2)^{1-\eta}}{1 - \eta} \right] \tag{1}$$

subject to

$$k_{t+1}^o - (1 - \delta)k_t^o + c_t^o = r_t k_t^o + n_t^o w_t l_{1t} \tag{2}$$

$$n_{t+1}^o = (1 - \sigma)n_t^o + \rho_t^w (1 - n_t^o) \tag{3}$$

All lower case variables in the maximisation problem above are normalised by the working-age population (N_t^o). In our notation, variables and parameters indexed by r and o respectively denote RoT and optimising households. Non-indexed variables apply indistinctly

to both types of households. Thus, c_t^o, n_t^o and $(1 - n_t^o)$ represent consumption, the employment rate and the unemployment rate of Ricardian households, respectively. The time endowment is normalised to one. l_{1t} and l_2 are hours worked per employee and hours devoted to job search by the unemployed. Note that while the household decides over l_{1t}, the same cannot be said of l_2: time devoted to job search is assumed to be exogenous so that individual households take it as given.

Several parameters are present in the utility function of Ricardian households. Future utility is discounted at a rate of $\beta \in (0, 1)$. The parameter $-\frac{1}{\eta}$ measures the negative of the Frisch elasticity of labour supply. As consumption is subject to habits, the parameter h^o takes a positive value. In general $\phi_1 \neq \phi_2$, i.e., the subjective value of leisure imputed by workers may vary with employment status[1].

Maximisation of (1) is constrained as follows. First, the budget constraint (2) describes the various sources and uses of income. The term $w_t n_t^o l_{1t}$ captures net labour income earned by the fraction of employed workers, where w_t stands for hourly real wages. There is one asset in the economy, namely private physical capital (k_t^o). Return on capital is captured by $r_t k_t^o$, where r_t represents the gross return on physical capital. Total revenues can either be invested in private capital or spent on consumption. The household's consumption and investment are respectively given by c_t^o and $j_t^o = k_{t+1}^o - (1 - \delta) k_t^o$, where δ is the exogenous depreciation rate.

The remaining constraint faced by Ricardian households concerns the law of motion employment. Employment obeys the law of motion (3), where n_t^o and $(1 - n_t^o)$ respectively denote the fraction of employed and unemployed optimising workers in the economy at the beginning of period t. Each period employment is destroyed at the exogenous rate σ. Likewise, new employment opportunities come at the rate ρ_t^w, which represents the probability that one unemployed worker will find a job. Although the job-finding rate ρ_t^w is taken as exogenous by individual workers, it is endogenously determined at aggregate level according to the following Cobb-Douglas matching function[2]:

$$\rho_t^w (1 - n_t) = \vartheta_t (v_t, n_t) = \chi_1 v_t^{\chi_2} [(1 - n_t) l_2]^{1 - \chi_2} \tag{4}$$

Given the recursive structure of the above problem, it may be equivalently rewritten in terms of a dynamic programme. Thus, the value function $W(\Omega_t^o)$ satisfies the following Bellman equation:

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[1] Notice, that the only difference in the utility function with respect to Andolfatto (1996) and Cheron and Langot (2004) is the presence of habits in consumption.

[2] Note that this specification presumes that all workers are identical to the firm. This assumption will be commented further when we explain the bargaining process.

$$W(\Omega_t^o) = \max_{c_t^o, k_{t+1}^o} \left\{ \ln\left(c_t^o - hc_{t-1}^o\right) + n_t^o \phi_1 \frac{(1 - l_{1t})^{1-\eta}}{1 - \eta} + (1 - n_t^o)\phi_2 \frac{(1 - l_2)^{1-\eta}}{1 - \eta} + \beta E_t W(\Omega_{t+1}^o) \right\}$$

(5)

where maximisation is subject to constraints (2) and (3).

The solution to the optimisation programme above generates the following standard first-order conditions for consumption and the capital stock:

$$\lambda_{1t}^o = \left(\frac{1}{c_t^o - h^o c_{t-1}^o} - \beta \frac{h^o}{c_{t+1}^o - h^o c_t^o} \right)$$

(6)

$$1 = \beta E_t \frac{\lambda_{1t+1}^o}{\lambda_{1t}^o} \left\{ r_{t+1} + (1 - \delta) \right\}$$

(7)

According to condition (6) the current marginal utility of consumption depends on both past and expected future consumption due to the presence of habits. Expression (7) ensures that the intertemporal reallocation of capital cannot improve the household's utility.

Now it is convenient to derive the marginal value of employment for a worker (that is, the derivative of the value function with respect to employment, $\frac{\partial W_t^o}{\partial n_t^o} \equiv \lambda_{ht}^o$), as it will be used later to obtain the wage and hours equation in the bargaining process:

$$\lambda_{ht}^o = \lambda_{1t}^o w_t l_{1t} + \left(\phi_1 \frac{(1 - l_{1t})^{1-\eta}}{1 - \eta} - \phi_2 \frac{(1 - l_2)^{1-\eta}}{1 - \eta} \right) + (1 - \sigma - \rho_t^w)\beta E_t \frac{\partial W_{t+1}^o}{\partial n_{t+1}}$$

(8)

where λ_{ht}^o measures the marginal contribution of a newly created job to the household's utility. The first term captures the value of the cash-flow generated by the new job in t, *i.e.*, the labour income measured according to its utility value in terms of consumption (λ_{1t}^o). The second term on the right hand side of (8) represents the net utility arising from the newly created job. Finally, the third term represents the discounted present value of an additional employed worker, given that employment status will persist into the future, conditional to the probability that the new job will not be destroyed.

Rule-of-thumb households

RoT households do not benefit from access to capital markets, so that they face the following maximisation programme:

$$\max_{c_t^r} E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln\left(c_t^r - h^r c_{t-1}^r\right) + n_t^r \phi_1 \frac{(1 - l_{1t})^{1-\eta}}{1 - \eta} + (1 - n_t^r)\phi_2 \frac{(1 - l_2)^{1-\eta}}{1 - \eta} \right]$$

subject to the law of motion of employment and the specific liquidity constraint whereby each period's consumption expenditure must be equal to current labour income, as reflected in:

$$c_t^r = w_t n_t^r l_{1t} \tag{9}$$

$$n_{t+1}^r = (1 - \sigma) n_t^r + \rho_t^w s (1 - n_t^r) \tag{10}$$

where n_0^r represents the initial aggregate employment rate (in terms of the working-age population of *RoT* individuals), which is the sole stock variable in the above programme. Note that *RoT* consumers do not save and, as a result, they do not hold physical capital.

In this case, the value function $W(\Omega_t^r)$ satisfies the following Bellman equation:

$$W(\Omega_t^r) = \max_{c_t^r} \left\{ \ln \left(c_t^r - h c_{t-1}^r \right) + n_t^r \phi_1 \frac{(1 - l_{1t})^{1-\eta}}{1 - \eta} + (1 - n_t^r) \phi_2 \frac{(1 - l_2)^{1-\eta}}{1 - \eta} + \beta E_t W(\Omega_{t+1}^r) \right\} \tag{11}$$

where the maximisation is subject to constraints (9) and (10).

The solution to the optimisation programme is characterized by the following first-order condition:

$$\lambda_{1t}^r = \left(\frac{1}{c_t^r - h^r c_{t-1}^r} - \beta \frac{h^r}{c_{t+1}^r - h^r c_t^r} \right) \tag{12}$$

The marginal value of employment for a consumption-restricted worker ($\frac{\partial W_t^r}{\partial n_t^r} \equiv \lambda_{ht}^r$) can be obtained as,

$$\lambda_{ht}^r = \lambda_{1t}^r w_t l_{1t} + \left(\phi_1 \frac{(1 - l_{1t})^{1-\eta}}{1 - \eta} - \phi_2 \frac{(1 - l_2)^{1-\eta}}{1 - \eta} \right) + (1 - \sigma - \rho_t^w) \beta E_t \frac{\partial W_{t+1}^r}{\partial n_{t+1}} \tag{13}$$

which can be interpreted analogously to that of optimising households.

Contrary to standard models with RoT consumers, it is worth mentioning that the optimising behaviour of *RoT* households preserves to some extent the intertemporal dynamic nature of the model, due to consumption habits and the dynamic law of motion of employment.

2.2 Firms

The profit maximisation problem faced by each competitive producer can be written as

$$\max_{k_t, v_t} E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_{1t+1}^o}{\lambda_{1t}^o} \left(y_t - r_t k_t - w_t n_t l_{1t} - \kappa_v v_t \right) \tag{14}$$

subject to

$$y_t = z_t k_t^{1-\alpha} (n_t l_{1t})^\alpha \tag{15}$$

$$n_{t+1} = (1 - \sigma)n_t + \rho_t^f v_t \tag{16}$$

where, in accordance with the ownership structure of the economy, future profits are discounted at the household relevant rate $\beta^t \frac{\lambda_{1t+1}^o}{\lambda_{1t}^o}$. The firm incurs in a cost of renting capital, hiring labour and posting vacancies $(\kappa_v v_t)$ where v_t stands for the number of vacancies posted and κ_v for the cost of one vacancy open. Producers use two inputs, namely private capital and labour, so that technological possibilities are given by a standard Cobb-Douglas production function with constant returns to scale where z_t is a stochastic term representing random technological progress. The variable ρ_t^f represents the probability that a vacancy will be filled in any given period t. It is worth noting that the probability of filling a vacant post ρ_t^f is exogenous from the firm's perspective. However, this probability is endogenously determined at aggregate level according to the following Cobb-Douglas matching function:

$$\rho_t^f v_t = \chi_1 v_t^{\chi_2} \left[(1 - n_t) \, l_2 \right]^{1-\chi_2} \tag{17}$$

Analogously to households, we can express the maximum expected value of the firm in state Ω_t^f as a function $V(\Omega_t^f)$ which satisfies the following Bellman equation

$$V(\Omega_t^f) = \max_{k_t, v_t} \left\{ y_t - r_t k_t - w_t n_t l_{1t} - \kappa_v v_t + \beta E_t \frac{\lambda_{1t+1}^o}{\lambda_{1t}^o} V(\Omega_{t+1}^f) \right\} \tag{18}$$

Under the assumption of symmetry, the solution to the optimisation programme above generates the following first-order conditions for private capital and the number of vacancies

$$r_t = (1 - \alpha) \frac{y_t}{k_t} \tag{19}$$

$$\frac{\kappa_v}{\rho_t^f} = \beta E_t \frac{\lambda_{1t+1}^o}{\lambda_{1t}^o} \frac{\partial V_{t+1}}{\partial n_{t+1}} \tag{20}$$

where the demand for private capital, determined by (19), is positively related to the marginal productivity of capital $(1 - \alpha) \frac{y_t}{k_t}$ which, in equilibrium, must equate to the gross return on physical capital. Expression (20) reflects that firms choose the number of vacancies in such a way that the marginal recruiting cost per vacancy, κ_v, is equal to the expected

present value of the job once the vacancy has been filled.

Using the Bellman equation, the marginal value of an additional employment in t for a firm ($\lambda_{ft} \equiv \frac{\partial V_t}{\partial n_t}$) is

$$\lambda_{ft} = \alpha \frac{y_t}{n_t} - w_t l_{1t} + (1-\sigma)\beta E_t \frac{\lambda^o_{1t+1}}{\lambda^o_{1t}} \frac{\partial V_{t+1}}{\partial n_{t+1}} \tag{21}$$

where the marginal contribution of a new job to profits equals the marginal product net of the wage rate, plus the capital value of the new job in t, corrected for the probability that the job will continue in the future. Now using (21) one period ahead, we can rewrite condition (20) as

$$\frac{\kappa_v}{\rho^f_t} = \beta E_t \left[\frac{\lambda^o_{1t+1}}{\lambda^o_{1t}} \left(\alpha \frac{y_{t+1}}{n_{t+1}} - w_{t+1} l_{1t+1} + (1-\sigma)\frac{\kappa_v}{\rho^f_{t+1}} \right) \right] \tag{22}$$

2.3 Trade in the labour market: the labour contract

The key departure of search models from the competitive paradigm is that trading in the labour market is subject to transaction costs. Each period, the unemployed engage in search activities in order to find vacant posts spread over the economy. Costly search in the labour market implies that there are simultaneous flows into and out of the state of employment, such that an increase (reduction) in the stock of unemployment results from the predominance of job destruction (creation) over job creation (destruction). Stable unemployment occurs whenever inflows and outflows cancel each other out, i.e.,

$$\rho^f_t v_t = \rho^w_t (1-n_t) = \chi_1 v^{\chi_2}_t \left[(1-n_t) l_2 \right]^{1-\chi_2} = (1-\sigma)n_t \tag{23}$$

Because it takes time (for households) and real resources (for firms) to make profitable contacts, pure economic rent emerges with each new job, which is equal to the sum of the expected transaction (search) costs the firm and the worker will further incur if they refuse to match. The emergence of such rent gives rise to a bilateral monopoly framework.

Once a representative job-seeking worker and vacancy-offering firm match, they negotiate a labour contract in hours and wages. Risk-sharing exists at household level but not between households. Although optimising and *RoT* households have a different reservation wage, they pool together in the labour market and bargain with firms to distribute employment according to their shares in the working-age population. The implication of this assumption is that all workers receive the same wages, work the same number of hours, and suffer the same unemployment rates.

Following standard practice, the Nash bargain process maximises the weighted product of stakeholder surpluses from employment. Defining the weighted worker sur-

plus as

$$\lambda_{ht} \equiv (1 - \lambda^r) \frac{\lambda_{ht}^o}{\lambda_{1t}^o} + \lambda^r \frac{\lambda_{ht}^r}{\lambda_{1t}^r}$$

the objective function can be expressed as

$$\max_{w_t, l_{1t}} \left((1 - \lambda^r) \frac{\lambda_{ht}^o}{\lambda_{1t}^o} + \lambda^r \frac{\lambda_{ht}^r}{\lambda_{1t}^r} \right)^{\lambda^w} \left(\lambda_{ft} \right)^{1-\lambda^w} = \max_{w_t, l_{1t}} \left(\lambda_{ht} \right)^{\lambda^w} \left(\lambda_{ft} \right)^{1-\lambda^w} \tag{24}$$

where $\lambda^w \in [0, 1]$ reflects workers' bargaining power. The first term in brackets represents the worker surplus (as a weighted average of *RoT* and Ricardian workers' surpluses), while the second is the firm surplus. More specifically, $\lambda_{ht}^o / \lambda_{1t}^o$ and $\lambda_{ht}^r / \lambda_{1t}^r$ respectively denote the earning premium (in terms of consumption) of employment over unemployment for a Ricardian and a *RoT* worker. Notice that earning premia of workers are weighted according to the share of *RoT* consumers in the population (λ^r).

The solution of the Nash maximisation problem gives the optimal real wage and hours worked (see Appendix 1 for further details)

$$w_t l_{1t} = \lambda^w \left(\alpha \frac{y_t}{n_t} + \frac{\kappa_v v_t}{(1 - n_t)} \right) \tag{25}$$
$$+ (1 - \lambda^w) \left(\frac{(1 - \lambda^r)}{\lambda_{1t}^o} + \frac{\lambda^r}{\lambda_{1t}^r} \right) \left(\phi_2 \frac{(1 - l_2)^{1-\eta}}{1 - \eta} - \phi_1 \frac{(1 - l_{1t})^{1-\eta}}{1 - \eta} \right)$$
$$+ (1 - \lambda^w)(1 - \sigma - \rho_t^w) \lambda^r E_t \beta \frac{\lambda_{ht+1}^r}{\lambda_{1t+1}^r} \left(\frac{\lambda_{1t+1}^o}{\lambda_{1t}^o} - \frac{\lambda_{1t+1}^r}{\lambda_{1t}^r} \right)$$

$$\alpha \frac{y_t}{n_t l_{1,t}} = \left[\frac{1 - \lambda^r}{\lambda_{1t}^o} + \frac{\lambda^r}{\lambda_{1t}^r} \right] \phi_1 (1 - l_{1t})^{-\eta} \tag{26}$$

Unlike the Walrasian outcome, the wage prevailing in the search equilibrium is related (although not equal) to the marginal rate of substitution of consumption for leisure and the marginal productivity of labour, depending on worker bargaining power λ^w. Putting aside the last term on the right hand side, the wage is a weighted average between the highest feasible wage (i.e., marginal productivity of labour plus hiring costs per unemployed worker) and the outside option (i.e., the reservation wage as given by the difference between the leisure utility of an unemployed and an employed worker). This reservation wage is, in turn, a weighted average of the lowest acceptable wage of Ricardian and *RoT* workers. They differ in the marginal utility of consumption (λ_{1t}^o and λ_{1t}^r). If the marginal utility of consumption is high, workers are prepared to accept a relatively low wage.

The third term on the right hand side of (25) is a part of the reservation wage that depends only on the existence of *RoT* workers (only if $\lambda^r > 0$ this term is different from zero). It can be interpreted as an inequality term in utility. The economic intuition is as follows. *RoT* consumers are not allowed to use their wealth to smooth consumption over time, but they can take advantage of the fact that a matching today is to some extent likely to continue (with probabilty $(1 - \sigma)$) in the future, yielding a labour income that in turn will be used to consume tomorrow. Therefore, they use the margin that hours and wage negotiations provide them to improve their lifetime utility by narrowing the gap in utility with respect to Ricardian consumers. In this sense, they compare the intertemporal marginal rate of substitution as if they were not constrained $\left(\frac{\lambda^o_{1t+1}}{\lambda^o_{1t}}\right)$ with the expected rate, given their present rationing situation $\left(\frac{\lambda^r_{1t+1}}{\lambda^r_{1t}}\right)$. For example if, *caeteris paribus*, $\frac{\lambda^o_{1t+1}}{\lambda^o_{1t}} > \frac{\lambda^r_{1t+1}}{\lambda^r_{1t}}$ the third term in (25) is positive, which indicates that *RoT* put additional pressure on the average reservation wage as a way to ease their period-by-period constraint in consumption. The importance of this inequality term is positively related to the earning premium of being matched in the next period $\left(\frac{\lambda^r_{ht+1}}{\lambda^r_{1t+1}}\right)$, because it increases the value of a matching to continue into the future, and negatively related to the job finding probability (ρ^w_t), that reduces the loss of breaking up the match. Finally, notice that when $\lambda^r = 0$, all consumers are Ricardian and, therefore, the solutions for the wage rate and hours simplify to standard values (see Andolfatto, 2004).

2.4   Aggregation and accounting identities in the economy

Aggregate consumption and employment can be defined as a weighted average of the corresponding variables for each household type:

$$c_t = (1 - \lambda^r) c^o_t + \lambda^r c^r_t \tag{27}$$

$$n_t = (1 - \lambda^r) n^o_t + \lambda^r n^r_t \tag{28}$$

For the variables that exclusively concern Ricardian households, aggregation is merely performed as:

$$k_t = (1 - \lambda^r) k^o_t \tag{29}$$

$$j_t = (1 - \lambda^r) j^o_t \tag{30}$$

Gross output is defined as the sum of consumption, investment and the cost of

vacancies:

$$y_t = c_t + j_t + \kappa_v v_t \tag{31}$$

whereas the value added generated in the economy is given by:

$$gdp_t = y_t - \kappa_v v_t \tag{32}$$

3. Empirical Results

3.1 Model parameterisation

Except for habits and the rate of RoT consumers, model parameters have been fixed as in Cheron and Langot (2004) in order to obtain the same steady state and dynamics when the share of RoT consumers (λ^r) is equal to zero. Thus, the Cobb-Douglas production function parameter $\alpha = 0.6$,the subjective discount rate, $\beta = 0.985$, the depreciation rate, $\delta = 0.012$, the exogenous job destruction rate and $\sigma = 0.15$ are taken from this paper. The scale parameter of the matching function, χ_1, and the elasticity of matchings to vacant posts, χ_2, have been calibrated at 1.007 and 0.6, respectively. Following Hosios (1990), workers' bargaining power, $\lambda^w = 0.4$, is set at $1 - \chi_2$. We choose a value of 4 for the intertemporal labour substitution, η, whereas the amount of time devoted to looking for a job, $l_2 = 1/6$, is estimated as half the quarterly average of working time and κ_v is calibrated to match an overall cost of vacant posts equal to 0.5 percentile points of GDP. As regards preference parameters in the household utility functions, ϕ_1 and ϕ_2, these parameters are computed to be consistent with steady-state restrictions as Cheron and Langot (2004) impose. The values for ϕ_1 and ϕ_2 imply that the imputed value for leisure by an employed worker is situated well above the imputed value for leisure by an unemployed worker.

The implied steady state values of the endogenous variables are given in Table 1. In the left panel we show the steady state from the basic labour market search model (BSM model), i. e. the model assuming no RoT consumers and no habits, which coincides with Cheron and Langot (2004). The right panel displays steady state values from our complete model including RoT and habits. The value for habit parameters ($h^o = h^r = 0.7$) is a little bit higher than that chosen by Smets and Wouters (2003), but lower than in many other studies. The fraction of RoT consumers in the economy ($\lambda^r = 0.5$) is also an upper bound standard value. In any case, we analyse the robustness of our results to changes in these parameters later.

Table 1 — Steady State

Basic search model (BSM)				BSM with RoT and habits			
c_t	0.819	y_t	1.00	c_t	0.819	n_t	0.57
j_t	0.176			c_t^o	1.043	n_t^o	0.57
k_t	14.69			c_t^r	0.594	n_t^r	0.57
l_{1t}	0.333			j_t	0.176	r_t	0.027
n_t	0.57			j_t^o	0.352	v_t	0.095
r_t	0.027			k_t	14.69	w_t	3.129
v_t	0.095			k_t^o	29.38	y_t	1.00
w_t	3.129			l_{1t}	0.333		

3.2 Comparing models

Assuming no habits ($h^o = h^r = h = 0$) and no *RoT* consumers ($\lambda^r = 0$), the model produces the impulse-response functions (Figure 1) of what we will call the Basic Search Model (*BSM*).[3] It is a well-known fact that a positive productivity shock triggers an increase in vacancies, a rise in labour productivity and a positive impact on the real wage and hours worked in a search model. The shock has also positive effects on both consumption and investment and, thus, output also grows on impact. As a result of the size of the impacts of the relevant variables, the labour share is reduced initially, to grow afterwards and gradually recover its long run level.

In the first columns of Table 2 we report statistics describing the cyclical properties of the US economy (for the 1964:1-2008:1 period), as well as the statistics stemming from a positive technological shock simulated in a standard real business cycle model (RBC) and in the basic search model (*BSM*). The shock is assumed to be highly persistent (a first-order autoregressive process with a parameter of 0.95), the standard deviation of the shock being 0.007 (as in Prescott, 1998). As expected, the RBC model is not capable of accounting for the stylized facts featuring the US labour market or the behaviour along the business cycle of both aggregate consumption and investment. In the standard labour market search model, however, there are clear gains in the explanation of some of the labour market facts. Although some relative volatilities improve, namely that of total hours worked, basically the *BSM* model reduces the correlations of certain variables to bring them in line with observed ones. Thus, correlations of output with the real wage (w_t) and with labour productivity ($\frac{y_t}{l_{1t}n_t}$) and of total hours ($l_{1t}n_t$) with productivity and the real wage come closer to US data. Nevertheless, the contemporaneous correlations of the real wage and of total hours worked with output are still clearly overestimated, as is the case with the

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[3] Given our calibration, this model coincides exactly with the *LMS*1 model in the terminology of Cheron and Langot (2004).
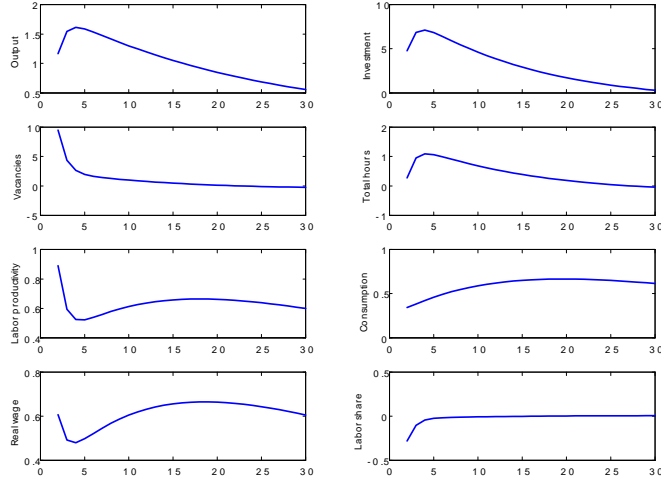
Figure 1: *IRF for the Basic Search Model ($\lambda^r = h^o = h^r = 0$)*

correlation between hours and labour productivity. Furthermore, the volatilities of both consumption and investment are the same as in the RBC model, and even now far from the real ones.

We have overcome the empirical shortcomings of the *BSM* model related to the volatilities of consumption and investment by incorporating *RoT* consumers (see column 4). This is a well-known result (see, for example, Galí, López-Salido and Vallés, 2007) in models with no search and matching frictions in the labour market. Moreover, the presence of rationed consumers exerts a positive impact on the explanation for the relative volatilities of labour productivity and wages with output. The demerits of this model are also noticeable and have to do, mainly, with the general worsening of cross correlations among labour market variables. For instance, the excessive procyclicality of labour productivity and real wages is increased with respect to the *BSM* model, along with the cross correlations between total hours worked and both wage and labour productivity are further increased.

In column 5 of Table 2 we present the results of incorporating habits in consumption ($h^o = h^r = h = 0.7$) into the *BSM* model. In this case we assume no presence of *RoT* consumers ($\lambda^r = 0$). As is readily apparent by comparing columns 5 and 3, introducing habits has little effect on the cyclical characterisation of the US economy simulated by the search model.

Table 2 − Cyclical Properties

Variable x	US (1)		RBC (2)		BSM (3) $\lambda^r = 0$ $h = 0$		SM (4) $\lambda^r = 0.5$ $h = 0$		SM (5) $\lambda^r = 0$ $h = 0.7$		SM (6) $\lambda^r = 0.5$ $h = 0.7$	
$\sigma(y)$	1.58		1.08		1.46		1.24		1.46		1.09	
ρ_y	0.91		0.72		0.84		0.80		0.84		0.87	
	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)
c_t	0.49	0.93	0.26	0.90	0.25	0.89	0.45	0.98	0.24	0.80	0.44	0.98
j_t	2.86	0.97	4.59	0.99	4.60	0.99	3.60	0.99	4.75	0.99	3.64	0.99
$l_{1t}n_t$	1.09	0.82	0.26	0.99	0.70	0.94	0.49	0.93	0.70	0.94	0.59	0.62
$\frac{y_t}{l_{1t}n_t}$	0.66	0.45	0.75	0.99	0.42	0.82	0.57	0.95	0.42	0.82	0.78	0.81
w_t	0.59	0.31	0.75	0.99	0.32	0.89	0.50	0.98	0.32	0.90	0.80	0.78
$\frac{w_t l_{1t}n_t}{y_t}$	0.60	0.14	0.0	0.0	0.13	-0.41	0.11	-0.46	0.13	-0.41	0.04	-0.13
$corr(l_{1t}n_t, \frac{y_t}{l_{1t}n_t})$	0.01		0.98		0.57		0.77		0.57		0.04	
$corr(l_{1t}n_t, w_t)$	-0.01		0.98		0.69		0.85		0.70		-0.01	

Columns (a): $\sigma(x)/\sigma(y)$, Columns (b): $corr(x,y)$.

Finally, in column 6 we present our main result, i.e., the joint introduction of *RoT* consumers and consumption habits. The interaction of habits and restricted consumers significantly affects many of the second moments displayed in the table. As can be observed, the search model augmented with restricted consumers and habits makes the best description of some of the business cycle facts of the US economy. This model produces no significant correlation between the labour input (total hours) and both labour productivity and real wages. In addition, the cross-correlation of the labour share with output is closest to that observed[4]. Also, labour productivity, consumption and investment display standard deviations with respect to output that are not far from observed those observed. This model is, however, unable to explain the high observed volatility of total hours and the labour share.

The main conclusion from the results in Table 2 is that the interaction of consumption habits and *RoT* individuals increases the explanatory power of the basic labour market search model, in terms of second-order moments. However, other alternatives in the literature have also arrived to a similar outcome. Cheron and Langot (2004) also improved the explanatory power of the search model by introducing non-separable preferences of the type assumed by Rogerson and Wright (1988). Comparing their results with ours, there are aspects of the business cycle that are captured well in both cases (i.e., the null correlations of total hours with both labour productivity and real wages, or the correlation of the labour share with output) and others where one alternative is superior to the

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[4] Notice that the observed correlation (0.14) and the simulated one (-0.13) are not significantly different from zero at conventional significance levels.

other. For example, Cheron and Langot's model better accounts for the low cross correlation of wages and output, whereas our model seems to better reflect the relative volatility of labour productivity and consumption.

3.3 Explanation of results

As shown previously, the null correlations of total hours with both labour productivity and real wages and the correlation of the labour share with output are well captured, including the interaction of consumption habits and *RoT* individuals in a standard search model. However, it is necessary to understand what is behind this result. To answer this question, we depict the impulse-response functions of the augmented model ($h^o = h^r = 0.7, \lambda^r = 0.5$) after a positive productivity shock in Figure 2 The differences with the BSM model impulse-response functions depicted in Figure 1 are very illustrative of what is going on. If we look at the time behaviour of wages, total hours worked and labour productivity, it is easy to understand why the implied cross-correlations turn to null values. In the basic model, the productivity shock generates a positive response from all three variables on impact. Although this continues to be true for labour productivity and real wages, in Figure 2 hours worked react negatively to the shock, which is behind the reduction in correlations. The negative response on behalf of hours on impact is in line with the results of Rotemberg, 2003, or Galí, 1999, or Andrés, Fatás and Domenech, 2008. Furthermore, the empirical null correlation of the labour share with output is related to the fact that the impact on the labour share turns from negative to positive when the basic search model is enriched.

To further understand the economic reasons behind the changes in the cross correlations of the labour market variables, it is convenient to recall the solutions for wages and hours obtained from the bargaining process:

$$
\begin{aligned}
w_t l_{1t} \approx{} & \lambda^w \left(\alpha \frac{y_t}{n_t} + \frac{\kappa_v v_t}{(1 - n_t)} \right) \\
& + (1 - \lambda^w) \left(\frac{(1 - \lambda^r)}{\lambda_{1t}^o} + \frac{\lambda^r}{\lambda_{1t}^r} \right) \left(\phi_2 \frac{(1 - l_2)^{1-\eta}}{1 - \eta} - \phi_1 \frac{(1 - l_{1t})^{1-\eta}}{1 - \eta} \right)
\end{aligned}
\tag{33}
$$

$$
\frac{(1 - l_{1t})}{l_{1,t}^{\frac{1}{\eta}}} = \left\{ \frac{n_t}{y_t} \left[\frac{1 - \lambda^r}{\lambda_{1t}^o} + \frac{\lambda^r}{\lambda_{1t}^r} \right] \frac{\phi_1}{\alpha} \right\}^{\frac{1}{\eta}}
\tag{34}
$$

Notice that for exposition purposes we have dropped the last term from the wage solution (33) and have slightly rewritten the hours equation (34).[5] Let us assume there is a

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[5] The reason is that this term is zero in the steady state solution and, therefore, has no impact on the bargained
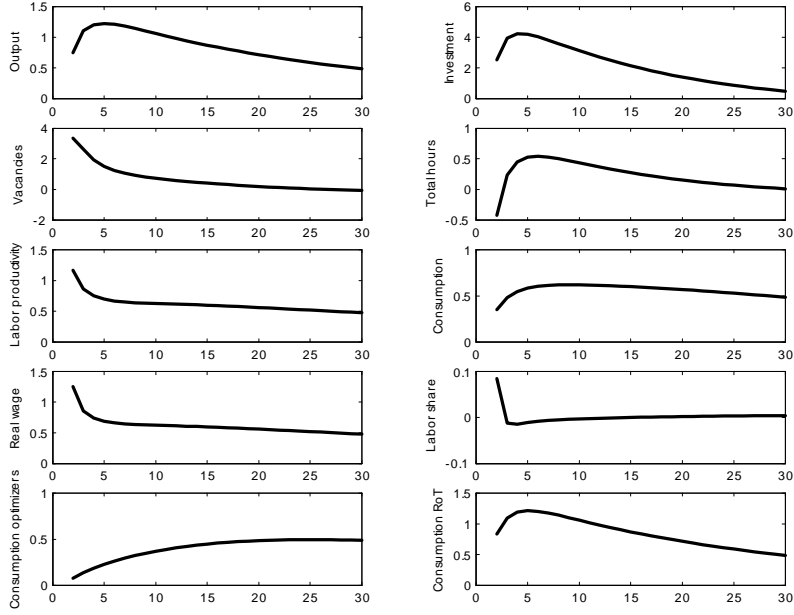
Figure 2: *IRF for the augmented search model ($\lambda^r = 0.5; h^o = h^r = 0.7$)*

positive technological shock in the economy. The shock boosts output, vacancies and consumption on impact (thus reducing the marginal utilities of consumption of both types of consumers, λ^r_{1t} and λ^o_{1t}). Looking at both terms in (33) this unambiguously produces, for given hours, an increase in the bargained wage. This is exactly what we observe in the impulse-response functions (Figures 1 and 2), irrespective of the degree of habits or the share of *RoT* consumers. However, the impact on wages is greater in the case of rationed consumers and consumption habits, while output is less affected. This means that the second term in the wage equation, i.e. the reservation wage, is responsible for wages suffering a heavier impact.

What happens with hours after the technology shock? In expression (34), we observe that bargained hours depend on two terms. First, the inverse of output per worker, $\frac{n_t}{y_t}$, which will be reduced on impact after the technology shock, given that y_t increases and n_t is a predetermined variable. Second, the weighted average of the inverse of the marginal utilities of consumption of both types of agents, $(\frac{1-\lambda^r}{\lambda^o_{1t}} + \frac{\lambda^r}{\lambda^r_{1t}})$, which will rise with the shock. Both effects go in opposite directions and hours can increase or decrease, depend-

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wage in the steady state. Although it may be relevant along the business cycle, its quantitative relevance is quite low: in our simulations it only accounts for less than one percentile point of the bargained wage along the steady state.

ing on which of them predominates. In the BSM model (see Figure 1), the total impact effect on hours is positive, given that the decrease in $\frac{n_t}{y_t}$ is larger than the effect produced by the increase in the inverse of the marginal utility. However, the augmented model generates exactly the opposite effect on bargained hours for two reasons. First, the ratio $\frac{n_t}{y_t}$ decreases less than in the standard model, because the impact on output is lower (see Figure 2). Second, and most importantly, the term $(\frac{1-\lambda^r}{\lambda_{1t}^o} + \frac{\lambda^r}{\lambda_{1t}^r})$ increases more markedly than is the case in a situation where there are no RoT consumers and no habits. To understand this result, according to equations (6) and (12), the marginal utility of consumption can be represented in terms of deviations with respect to the steady state as:

$$\widehat{\lambda}_{1t} = \frac{1}{(1-\beta h)(1-h)} \left[\beta h \left(\widehat{c}_{t+1} - h\widehat{c}_t \right) - \left(\widehat{c}_t - h\widehat{c}_{t-1} \right) \right] \equiv \frac{1}{(1-\beta h)(1-h)} \widehat{\Psi}_t \qquad (35)$$

As h tends to 1, $\widehat{\Psi}_t$ tends to the acceleration of consumption (corrected by β), but $\frac{1}{(1-\beta h)(1-h)}$ tends to infinite. Therefore, as h increases, the impact on $\widehat{\lambda}_{1t}$ also increases for a given volatility of consumption. Obviously, in a DGE model both consumption and $\widehat{\lambda}_{1t}$ are endogenous variables, but it seems clear from expression (35) that habits introduce a source of persistence as consumers are interested in smoothing not only their levels of consumption, but also its variations. And here the distinction between optimising and RoT consumers is crucial, as the former can use capital markets to smooth the levels and changes of consumption, whereas the latter cannot. As optimising consumers use their wealth to smooth the changes in consumption, habits introduce an extra incentive to accumulate wealth, increasing savings. This makes variations in Ψ_t smaller in absolute terms after a positive technology shock, explaining why $\widehat{\lambda}_{1t}$ does not vary much with habits for optimising consumers (see figure 3). However, RoT consumers have no access to financial markets and the only way they can smooth consumption is by smoothing variations in their current income. For this type of consumers, consumption is equal to current income

$$c_t^r = w_t l_{1t} n_t \qquad (36)$$

which in deviations with respect to the steady state can be written as

$$\widehat{c}_t^r = \widehat{w}_t + \widehat{l}_{1t} + \widehat{n}_t \qquad (37)$$

Therefore, equations (35) and (37) establish a relationship between $\widehat{\lambda}_{1t}^r$ and the deviations of wages, hours and employment for the whole economy. Given that current RoT consumption is tied up with current labour income, the negative impact of the technology shock on $\widehat{\Psi}_t$ is greater than in the case of optimising agents and, thus, λ_{1t}^r records a larger fall on impact. In addition, for a given share of RoT consumers in the economy, higher habits induce further falls in λ_{1t}^r according to the first term in expression (35). Thus, for a
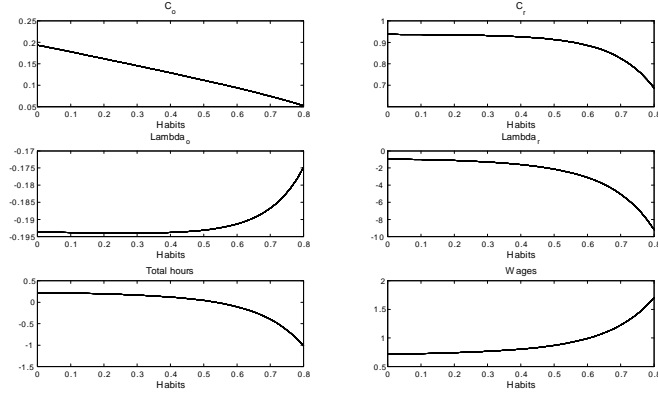
Figure 3: *Impacts of habits on key variables for RoT=0.5*

sufficiently large parameter of habits the initial positive impact on hours becomes zero or even negative (see figure 3), whereas pressure on wages increases (see figure 3), as shown by expressions (34) and (33).

Overall, as habits increase (for a given share of *RoT* consumers) the impact of the technology shock on total labour income is reduced due to the pressure of *RoT* consumers to smooth consumption by means of labour market negotiation in hours and wages. The gain in utility through *RoT* consumers accomplishing a smoother path of consumption, together with less hours of work, more than compensates the loss of utility due to the decrease in consumption on impact. Not surprisingly, the larger the share of *RoT* consumers in the economy, the larger the effect of the degree of habits of these consumers upon aggregate wages and hours. In summary, the result is that as habits of restricted consumers increase, total hours change by less on impact after the technology shock, while there is a larger increase in the real wage and, therefore, the correlation between the real wage and total hours is reduced. Notice that for similar reasons, increasing the degree of habits in consumption reduces the cross-correlation between total hours and labour productivity, in accordance with the reduction of the impact of the shock on total hours worked.

To finish our discussion of the results, Figures (4) and (5) depict three-dimensional plots showing the effects of habits and the share of *RoT* consumers on the cross-correlations between total hours worked and both wages and labour productivity. The aim of these graphs is to check the sensitivity of our results to different values of λ^r and h, given that the literature has used a wide range of these values in different models. Three results emerge from these figures. First, assuming no habits, a higher share of *RoT* consumers in the economy increases both correlations, further distancing them from empirical zero val-

ues, as shown in the bottom rows of column 4 in Table 2. Second, if there are no restricted consumers, more habits has little effect on these correlations (see column 5 in Table 2). Finally, for a wide range in the share of *RoT* consumers and in the degree of habits, cross-correlations record similar values to empirical results. Notice that correlations can even become negative and significant for high values of both parameters. Therefore, the main result is that when habits and rule-of-thumb consumers are taken into account, the labour market search model gains extra power to reproduce some of the stylised facts characterising the US labour market.

4. Conclusions

Although the basic search model improves our understanding of US labour market stylized facts, it fails to explain important correlations across prominent labour market variables. Chéron and Langot (2004) partially overcame some of these limitations of the basic search model by introducing a particular set of non-separable preferences. Our paper shares the same motivation, but explores a quite different route to accomplish this task. More specifically, we have analysed the effects of introducing typical Keynesian features, namely non optimising consumers and consumption habits, into the standard labour market search model. Without a doubt, these features are important nowadays to explain other business cycle facts in advanced economies like the US, as previous research has shown. For this reason, we confront the simulation results obtained with our model with a wide and sensible range of parameters that characterise the share of restricted consumers and the degree of habits in consumption.

Our results show that introducing habits alone into the search model does not improve the ability of the model to account for some labour market facts. The presence of *RoT* consumers alone even spoils the ability of the model to account for labour market correlations and volatilities. However, for a wide range in the share of *RoT* consumers and in the degree of habits, cross-correlations register values close to the empirical ones. The main feature of our model is that while *RoT* consumers are not allowed to use their wealth to smooth consumption over time, they will take advantage of the fact that they negotiate (in conjunction with optimising workers) wages and hours with the firms. Therefore, they use the margin that hours and wage negotiations provide them to improve their lifetime utility by narrowing the gap in utility with respect to Ricardian consumers. This, as we have shown in previous pages, weakens the correlations between the labour share and output, between hours and labour productivity and between hours and real wages.

Comparing our results with those of Chéron and Langot (2004), there are aspects of the business cycle that are well captured in both cases (e.g. the null correlations of total hours with both labour productivity and real wages, or the correlation of the labour
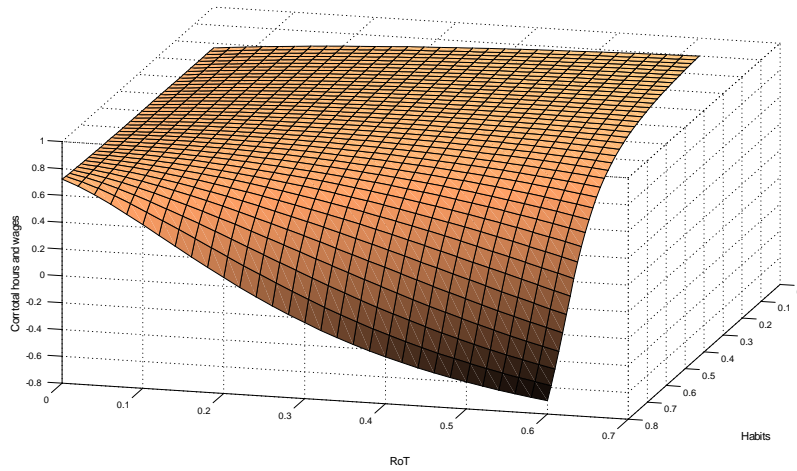
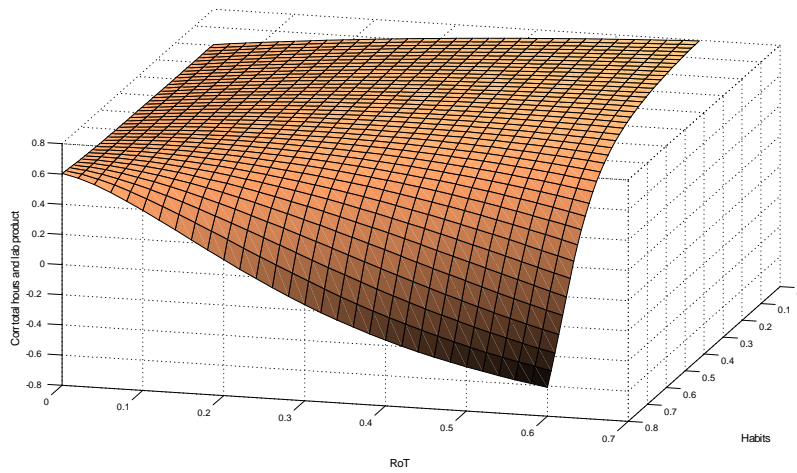**Figure 4:** *Correlation between total hours and wages*

Figure 5: *Correlation between total hours and labor productivity*

share with output). For some empirical moments, however, one alternative is superior to the other. For example, Cheron and Langot's model better accounts for the low cross correlation of wages and output, whereas our model seems to better reflect the relative volatility of labour productivity and consumption.

Appendix 1: Nash bargaining with RoT consumers

1. Maximisation problem

- The Nash bargaining process maximises the weighted product of stakeholder surpluses from employment.

$$\max_{w_t, l_{1t}} \left(\frac{\partial V_t}{\partial n_t} \right)^{1-\lambda^w} \left(\frac{1-\lambda^r}{\lambda_{1t}^o} \frac{\partial W_t^o}{\partial n_t} + \frac{\lambda^r}{\lambda_{1t}^r} \frac{\partial W_t^r}{\partial n_t} \right)^{\lambda^w} = \max_{w_t, l_{1t}} \left(\lambda_{ft} \right)^{1-\lambda^w} \left(\lambda_{ht} \right)^{\lambda^w} \tag{1.1}$$

where $\lambda_{ft} \equiv \frac{\partial V_t}{\partial n_t}$ and $\lambda_{ht} \equiv \frac{1-\lambda^r}{\lambda_{1t}^o} \frac{\partial W_t^0}{\partial n_t} + \frac{\lambda^r}{\lambda_{1t}^r} \frac{\partial W_t^r}{\partial n_t}$.

- Deriving w.r.t. w_t

$$(1-\lambda^w) \left(\frac{\lambda_{ht}}{\lambda_{ft}} \right)^{\lambda^w} (-l_{1t}) + \lambda^w \left(\frac{\lambda_{ht}}{\lambda_{ft}} \right)^{\lambda^w - 1} \left(\lambda_{1t}^o \frac{1-\lambda^r}{\lambda_{1t}^o} l_{1t} + \lambda_{1t}^r \frac{\lambda^r}{\lambda_{1t}^r} l_{1t} \right) = 0 \tag{1.2}$$

or

$$(1-\lambda^w)\lambda_{ht} = \lambda^w \lambda_{ft} \tag{1.3}$$

- Therefore, optimisation of this joint surplus w.r.t. wages implies that

$$\lambda^w \frac{\partial V_t}{\partial n_t} = (1-\lambda^w) \left(\frac{1-\lambda^r}{\lambda_{1t}^o} \frac{\partial W_t^o}{\partial n_t} + \frac{\lambda^r}{\lambda_{1t}^r} \frac{\partial W_t^r}{\partial n_t} \right) \tag{1.4}$$

2. Solution for hours

- Deriving equation (1.1) w.r.t. l_{1t}

$$(1-\lambda^w) \left(\frac{\lambda_{ht}}{\lambda_{ft}} \right)^{\lambda^w} (\alpha \frac{y_t}{n_t l_{it}} - w_t)$$

$$+\lambda^w \left(\frac{\lambda_{ht}}{\lambda_{ft}} \right)^{\lambda^w - 1} \left(w_t + \left[\frac{1-\lambda^r}{\lambda_{1t}^o} + \frac{\lambda^r}{\lambda_{1t}^r} \right] U_{lt} \right) = 0 \tag{5}$$

where U_{lt} is the marginal (des)utility of hours.

$$U_{lt} = -\phi_1 (1-l_{1t})^{-\eta} \tag{1.6}$$

- From equation (1.3)

$$\frac{\lambda_{ht}}{\lambda_{ft}} = \frac{\lambda^w}{(1 - \lambda^w)} \tag{1.7}$$

- Therefore, equation (1.5) can be written as

$$\frac{\alpha y_t}{n_t l_{i1}} = \phi_1 (1 - l_{1t})^{-\eta} \left[\frac{1 - \lambda^r}{\lambda_{1t}^o} + \frac{\lambda^r}{\lambda_{1t}^r} \right] \tag{1.8}$$

3. Solution for wages

- From the firm's side, we have the following FOC

$$\beta E_t \frac{\lambda_{1t+1}^o}{\lambda_{1t}^o} \frac{\partial V_{t+1}}{\partial n_{t+1}} = \frac{\kappa_v v_t}{\chi_1 v_t^{\chi_2} \left[(1 - n_{t-1}) l_2 \right]^{1 - \chi_2}} = \frac{\kappa_v}{\rho_t^f} \tag{1.9}$$

- Therefore

$$(1 - \lambda^w) E_t \frac{\beta}{\lambda_{1t}^o} \left((1 - \lambda^r) \frac{\partial W_{t+1}^0}{\partial n_{t+1}} + \lambda^r \frac{\lambda_{1t+1}^o}{\lambda_{1t+1}^r} \frac{\partial W_{t+1}^r}{\partial n_{t+1}} \right) = \lambda^w E_t \beta \frac{\lambda_{1t+1}^o}{\lambda_{1t}^o} \frac{\partial V_{t+1}}{\partial n_{t+1}} = \lambda^w \frac{\kappa_v}{\rho_t^f} \tag{1.10}$$

- From(1.3) and combining (8), (13), (21) and (1.4):

$$\frac{\lambda^w}{(1 - \lambda^w)} \left(\alpha \frac{y_t}{n_t} - w_t l_{1t} + (1 - \sigma) \frac{\kappa_v}{\rho_t^f} \right) = (1 - \lambda^r) w_t l_{1t} +$$

$$\frac{(1 - \lambda^r)}{\lambda_{1t}^o} \left(\phi_1 \frac{(1 - l_{1t})^{1 - \eta}}{1 - \eta} - \phi_2 \frac{(1 - l_2)^{1 - \eta}}{1 - \eta} \right) +$$

$$\lambda^r w l_{1t} + \frac{\lambda^r}{\lambda_{1t}^r} \left(\phi_1 \frac{(1 - l_{1t})^{1 - \eta}}{1 - \eta} - \phi_2 \frac{(1 - l_2)^{1 - \eta}}{1 - \eta} \right) +$$

$$(1 - \sigma - \rho_t^w s) \left(\frac{1 - \lambda^r}{\lambda_{1t}^o} \beta E_t \frac{\partial W_{t+1}^o}{\partial n_{t+1}} + \frac{\lambda^r}{\lambda_{1t}^r} \beta E_t \frac{\partial W_{t+1}^r}{\partial n_{t+1}} \right) \tag{1.11}$$

- Collecting terms:

$$
\frac{\lambda^w}{(1-\lambda^w)}\left(\alpha\frac{y_t}{n_t} - w_t l_{1t} + (1-\sigma)\frac{\kappa_v}{\rho_t^f}\right) = w_t l_{1t} +
$$

$$
\left(\frac{(1-\lambda^r)}{\lambda_{1t}^o} + \frac{\lambda^r}{\lambda_{1t}^r}\right)\left(\phi_1\frac{(1-l_{1t})^{1-\eta}}{1-\eta} - \phi_2\frac{(1-l_2)^{1-\eta}}{1-\eta}\right) +
$$

$$
(1-\sigma-\rho_t^w s)\frac{\beta E_t}{\lambda_{1t}^o}\left((1-\lambda^r)\frac{\partial W_{t+1}^o}{\partial n_{t+1}} + \lambda^r\frac{\lambda_{1t+1}^o}{\lambda_{1t+1}^r}\frac{\partial W_{t+1}^r}{\partial n_{t+1}} + \lambda^r\frac{\lambda_{1t}^o}{\lambda_{1t}^r}\frac{\partial W_{t+1}^r}{\partial n_{t+1}} - \lambda^r\frac{\lambda_{1t+1}^o}{\lambda_{1t+1}^r}\frac{\partial W_{t+1}^r}{\partial n_{t+1}}\right)
$$

$$(1.12)$$

or

$$
w_t l_{1t} = \lambda^w\left(\alpha\frac{y_t}{n_t} + \frac{\kappa_v v_t}{(1-n_t)}\right) +
$$

$$
(1-\lambda^w)\left(\frac{(1-\lambda^r)}{\lambda_{1t}^o} + \frac{\lambda^r}{\lambda_{1t}^r}\right)\left(\phi_2\frac{(1-l_2)^{1-\eta}}{1-\eta} - \phi_1\frac{(1-l_{1t})^{1-\eta}}{1-\eta}\right) +
$$

$$
(1-\lambda^w)(1-\sigma-\rho_t^w)\lambda^r E_t\beta\frac{\lambda_{ht+1}^r}{\lambda_{1t+1}^r}\left(\frac{\lambda_{1t+1}^o}{\lambda_{1t}^o} - \frac{\lambda_{1t+1}^r}{\lambda_{1t}^r}\right) \qquad (1.13)
$$

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