

## Fishing Capacity and Harvest Rules

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**Abstract** *In this note we consider the choice of optimum fishing capacity for fish stocks that vary at random. In models with stochastic variations of fish stocks, optimum fishing capacity is normally a decision variable separate from fishing effort. It is shown how the optimum fishing capacity depends on the price of fish, the cost of capacity, and the "harvest rule" linking the permitted catch to the size of the fish stock. Operating costs may also influence the optimum capacity through the effect of stock "thinning" on the cost per unit of fish caught and the level at which further depletion becomes unprofitable.*

**Keywords** Fisheries economics, fishing capacity, fish stock fluctuations.

### Introduction

In deterministic fishery models there is only one, basic decision variable; the rate of exploitation. Once this has been decided, optimum fishing effort follows. As a stationary pattern of fishing is obtained, there will be no reason to vary fishing effort and thus no difference between effort capacity and actual effort. Alternatively, one may take effort as a decision variable. The optimum effort implies an optimum rate of exploitation, and vice versa.

In stochastic fishery models the catch capacity of the fleet becomes a decision variable in its own right. Random variations in the abundance of fish will cause random variations in the permitted harvest, provided there is some "rule" linking the size of the fish stock at the beginning of a fishing season and the harvest it is permitted to take during the season. Therefore, the utilization of the fishing fleet will most likely have to be varied according to the status of the fish stock at a given time. Since the average degree of utilization of the fleet will probably be lower the larger it is, the optimum fishing capacity for any given probability distribution of the permitted catch is likely to depend on the cost of fishing capacity.

Questions related to the choice of optimum capacity for harvesting fluctuating stocks have surprisingly seldom been addressed in the literature, given the importance of the issue. Most stocks fluctuate considerably over time, and some so severely that the fishery is entirely closed down from time to time, while alternative stocks are often not available, or available only to a limited extent. Exceptions are Huppert (1981) and Charles and Munro (1985). Both of these papers discuss the trade-off between the cost of idle capacity and the ability to take large

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but infrequent catches. Wilson (1982) refers to the "peak load problem" for patchily distributed stocks.

In this note we explore the relation between optimum fishing capacity and two simple harvest rules linking the permitted harvest and the status (size) of the fish stock. Ideally such harvest rules ought to be based on an economic optimization, taking into account the biology of the stock. What often seems to happen in practice, however, is that the permitted harvest is based on some simple biological "rule of thumb", such as a target escapement or fixed fishing mortality. But even in this case we are left with a decidedly nontrivial economic optimization problem, namely to find the optimum fishing capacity, given the prospects with regard to permitted catches in future periods.

### Two Harvest Rules

We ignore deterministic population dynamics and regard the stock of fish,  $X$ , as a stochastic variable with a time-independent probability distribution.<sup>1</sup> Let the probability density function be denoted by

$$f(X), \quad (1)$$

and the cumulative distribution function by

$$F(X) = \int_0^X f(s) ds. \quad (2)$$

Suppose there is a harvest rule, specifying how much can be caught in each period, depending on how large the stock is at the beginning of the period.<sup>2</sup> Let the harvest rule be denoted by

$$Q = G(X), \quad (3)$$

where  $Q$  is the permitted catch. It will be assumed that the harvest rule is linear:

$$G(X) = \max[0, k(X - X^*)], \quad k \leq 1, \quad X^* \geq 0,$$

where  $X^*$  is a target minimum stock to be left behind after harvesting.

The linearity assumption is made for simplicity, but it is not unreasonable. Certainly one would expect the harvest rule to be monotone; that is,  $dG/dX \geq 0$  for all  $X$ .  $d^2G/dX^2 > 0$  for all  $X$  is unlikely or impossible, since eventually  $G(X) > X$  if  $d^2G/dX^2$  is large enough.  $d^2G/dX^2 < 0$  for all  $X$  makes little sense, since the harvest rule would then prescribe that a progressively smaller portion be caught of the stock as it becomes larger.

<sup>1</sup> This ignores the relationship between harvest in the current period and the available stock in later periods (i.e., the fish live longer than one period, recruitment depends on the spawning stock, etc.). This is considered in Hannesson (1987).

<sup>2</sup> It is implicitly assumed that the size of the stock can be estimated with some confidence prior to each fishing season. The implications of uncertainty of these estimates will not be discussed here.

Two examples of the rule, to be discussed below, are

$$G(X) = kX, k < 1 \quad (X^* = 0); \quad (3a)$$

$$G(X) = X - X^*, X \geq X^* \quad (k = 1). \quad (3b)$$

The first of these is the well known constant fishing mortality rule. The second is the equally well known target escapement rule. These rules are illustrated in Figure 1.

### The Production Function, Harvest Rules, and Fishing Capacity

Imagine a sole owner who plans to invest in a fishing fleet as appropriate for utilizing a fluctuating stock described by the equations above. His catch rate ( $Y$ ) at any point in time may be written in general terms as

$$Y = \phi(\xi_1, \xi_2; X),$$

where  $\xi_1$  is a vector of ex-post fixed factors (boats and equipment) and  $\xi_2$  is a vector of variable factors. These factors interact with the fish stock ( $X$ ), resulting in a catch of fish ( $Y$ ).

The ex-ante optimal proportions of the fixed factors  $\xi_1$  are given by cost-minimization, as is the optimal size of the production units. For a maximization of

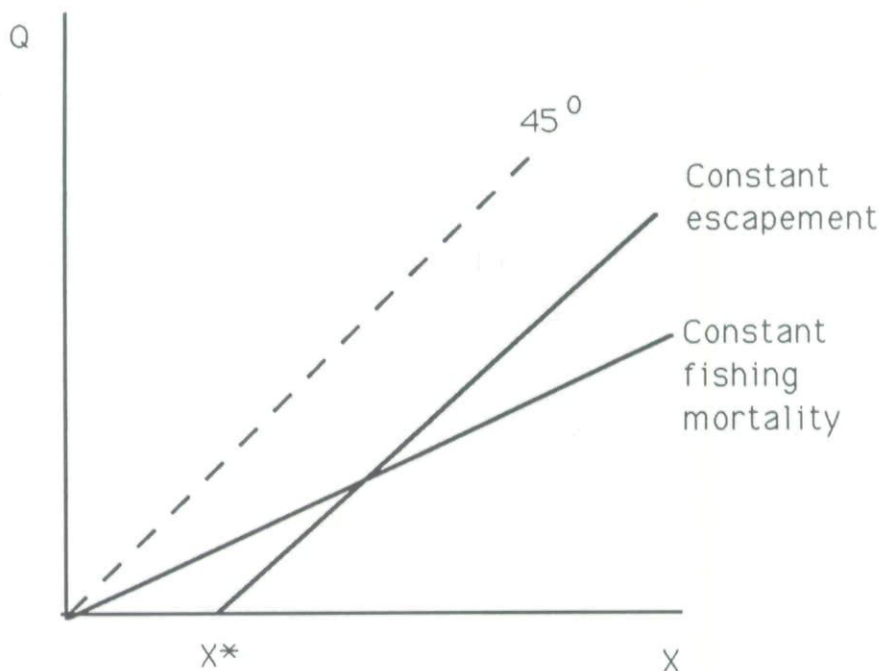


Figure 1. Two "harvest rules" linking the total allowable catch ( $Q$ ) to the size of the exploited stock ( $X$ ).

expected profits, the ex-ante optimal size of the production units will be given by the condition that the expected average cost be equal to the expected marginal cost.

Ex post the rate of utilization of the fixed factors  $\xi_1$  can be varied by varying the input of the variable factors  $\xi_2$ . We shall make the simplifying assumption that production increases proportionately with  $\xi_2$  for any given value of  $X$ . We further assume that the unit cost of each variable factor is constant up to a certain threshold value for some component in  $\xi_2$  (for example, so or so many hours per week for labor). This means that operating costs increase proportionately with  $\xi_2$  up to that threshold value. We finally assume that values of  $\xi_2$  beyond the said threshold are uneconomical.<sup>3</sup> This rules out the possibility of intensifying the use of the fixed factors beyond "normal" capacity when the permitted catch of fish is greater than the normal capacity. The purpose of this is to identify a unique value of full capacity utilization, which in this case is the combination  $\xi_1$  and the maximum economical  $\xi_2$ . According to the ordinary primal definition of capacity, the resulting value of  $Y$  would be the capacity limit. Here we shall use a different notion of capacity, namely the value of  $\xi_1$  combined with the maximum economical value of  $\xi_2$ . This would in ordinary language correspond to an intuitive notion such as so or so many fully utilized vessels. Due to the interaction with the fish stock ( $X$ ) this does not necessarily correspond to a unique value of  $Y$ , which is why we prefer this definition of capacity.<sup>4</sup>

Due to the minimum cost combination of the various elements in  $\xi_1$  there exists an aggregate expression of the vector  $\xi_1$ . This is a measure of capacity as defined above, and will be denoted by  $K$ . Similarly there exists, at given input prices, an aggregate expression for  $\xi_2$ . Reducing  $\xi_2$  below its maximum economical limit for one or more units of  $K$  reduces the use of capacity and variable costs proportionately. We refer to the capacity used as fishing effort and denote it by  $Z$ . The production function of the fishery can now be written as

$$Y = H(X,Z); Z \leq K. \quad (4)$$

Given the harvest rule (3) and the production function (4), the catch will be:

$$Y = \min[G(X), H(X,K)]. \quad (5)$$

If  $G(X)$  is linear in  $X$ ,  $H(X,K)$  is linear in  $X$  for a given  $K$ , and  $X^* = 0$ , we have one of the following:

$$G(X) > H(X,K) \text{ for all } X$$

$$G(X) = H(X,K) \text{ for all } X,$$

$$G(X) < H(X,K) \text{ for all } X.$$

<sup>3</sup> This implies limits to substitutability between the variable factors, such that it is not economical to increase the use of variable factors the prices of which remain constant to increase the degree of utilization of the fixed factors.

<sup>4</sup> In real life capacity is likely to be elastic, with the marginal cost curve rising as usually depicted in textbooks. The normal capacity of each production unit could then be defined as the minimum of the long run average cost curve. Smith and Hanna (1990) study capacity utilization in a fishing fleet.

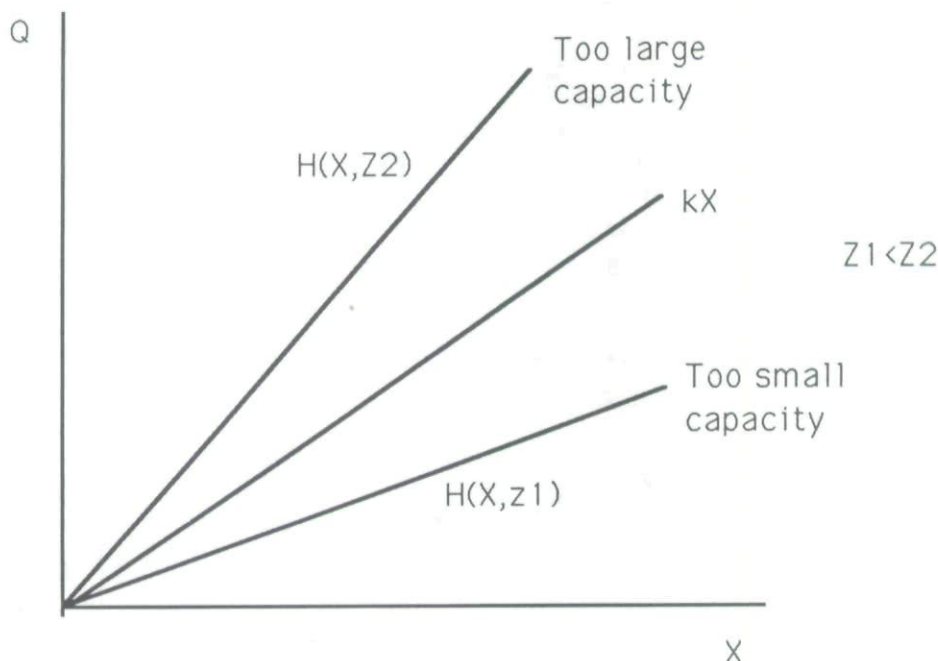
In the first case, the fishing capacity is always insufficient to take the permitted catch, while in the third case it is always too great. In the second case it is just sufficient, always. Hence, in this special case, there is no choice to be made with respect to optimal capacity, even if the fish stock and the permitted harvest fluctuate. Once the harvest rule (the value of  $k$ ) has been specified, the optimum capacity has been decided implicitly. This is illustrated in Figure 2. Operating costs affect this conclusion, however, as will be shown in Section 5 below, and imply a minimum exploitable stock level, which has a similar effect as a target escapement level  $X^* > 0$ .

If  $X^* > 0$ , there will sometimes be redundant capacity, even if (4) is linear in  $X$ . Depending on the level of  $K$ , the capacity will become binding at some level of  $X(X^{**})$ , as is illustrated in Figure 3b.

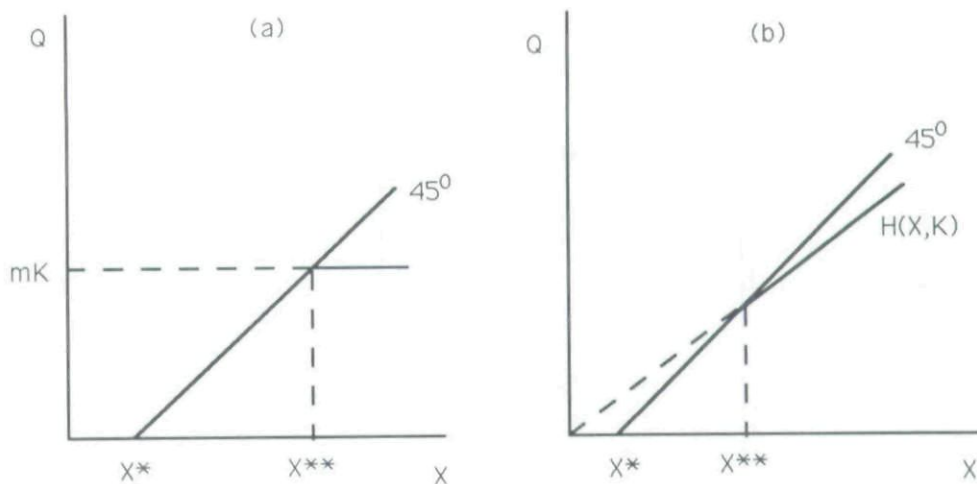
One special case of (4) is where it depends only on  $Z$ , such as

$$H(Z) = mZ, m \text{ constant.}$$

This is shown in Figure 3a. Here the capacity of the fleet is well defined in terms of output ( $mK$ ). Otherwise the capacity in terms of output will depend on the size of the exploited stock. With  $H(X,Z)$ , the catch of a fully utilized fleet of a given size  $K$  will increase with  $X$  as  $X$  increases beyond the critical level  $X^{**}$ . This is shown in Figure 3b for the case where  $H(\cdot)$  is linear in  $X$  for a given  $Z = K$ .



**Figure 2.** A case of optimum fishing capacity that is always fully used despite fluctuating allowable catch. The catch per unit of capacity used ( $H(X,Z)/Z$ ) is linear and the constant fishing mortality harvest rule ( $Q = kX$ ) applies.



**Figure 3.** Actual catch under the target escapement rule when (a) the catch per unit of capacity used is constant versus (b) proportional to the exploited stock.  $X^*$  = the target escapement stock level,  $X^{**}$  = the stock level at which the capacity constraint becomes binding.

### The Criterion for Optimum Capacity

Consider the choice of optimal fishing capacity, with a given investment cost,  $c$ , per unit of capacity. Assume that the price of fish,  $p$ , is constant and ignore operating costs for the time being. Since the probability distribution of the stock (and, by the harvest rule, the permitted catch) is the same in all time periods, the optimum capacity will remain constant in all future periods. Given risk neutrality, the conditions for optimum fishing capacity may therefore be derived by maximizing the expected present value of future profits ( $E\pi$ ) obtained by a fleet that is built from scratch and maintained at a given level ever after. We assume the fleet can be built in one period at a constant cost,  $c$ , per unit of fishing capacity,  $K$ . Hence we have the following problem:

$$\max E\pi = -cK + \sum_1^{\infty} \left[ \int_0^{X_{\max}} f(s)pY(X,K)ds - \delta cK \right] (1+r)^{-t}, \quad (6)$$

where  $X_{\max}$  is the maximum size of the stock,  $Y$  is given by (5),  $\delta$  is the rate of depreciation of the fleet, and  $r$  is the discount rate.

Since all the terms in the sum, except the discount factor, are independent of time, the sum in (6) is easily calculated. After multiplying by  $r$  we get

$$\max E\pi_a = \int_0^{X_{\max}} f(s)pY(X,K)ds - (r + \delta)cK, \quad (7)$$

which represents the annual expected profit (expected annual revenue less the annualized capital cost). Using (5), we can write this as

$$\max E\pi_a = p \int_0^{X^{**}} f(s)G(s)ds + p \int_{X^{**}}^{X_{\max}} f(s)H(s,K)ds - (r + \delta)cK, \quad (8)$$

where  $X^{**}$  is the level of stock at which all the available fishing capacity is needed (i.e.,  $G(X^{**}) = H(X^{**},K)$ ). The necessary condition for maximum is

$$dE\pi_a/dK = p \int_{X^{**}}^{X_{\max}} f(s)[\partial H(s,K)/\partial K]ds - (r + \delta)c = 0. \quad (9)$$

This has a straightforward interpretation; the expected annual revenue resulting from a marginal increase in fishing capacity should be equal to the annualized cost of that capacity.

In the special case of  $H(Z) = mZ$ , (9) is particularly simple:

$$pm[1 - F(K)] = (r + \delta)c,$$

while in the special case of  $H(.) = nZX$  we get

$$p \int_{X^{**}}^{X_{\max}} f(s)nsds = (r + \delta)c.$$

### The Importance of Operating Costs

So far we have ignored operating costs. How would these affect the optimum capacity? If the operating costs are proportional to the quantity caught they can simply be incorporated in the price in expression (6). If the catch depends on  $X$  as well as  $Z$  (or  $K$ ) the operating costs per unit of fish caught will be inversely related to the abundance of fish. The operating costs will then affect the optimum level of fishing capacity through a "thinning effect"; i.e., as a stock that replenishes itself periodically is fished down, the operating cost per unit of fish increases and the fishery will be abandoned when the operating cost per unit of fish becomes equal to the price. The higher the operating cost the sooner the stock will be abandoned (if at all), and the less profitable an expansion of fishing capacity will be. We shall use a simple numerical example and the production function  $H(X,Z) = nZX$  to demonstrate this.

Suppose the probability density of the stock is constant and equal to one. For a zero minimum stock, this implies  $X_{\max} = 1$ . Suppose further that the harvest rule is a "target escapement policy", i.e.,  $G(X) = \max[0, X - X^*]$ , where  $X^* = 0.1$ . Above it was assumed that the operating cost per unit of capacity used is constant. Let this cost be denoted by  $q$ . The operating cost per unit of fish caught therefore is  $qZ/nZX = q/nX$ . The stock will be abandoned when the operating cost per unit of fish caught has become equal to the price. The abandonment level ( $X\ddagger$ ) thus is

$$X\ddagger = q/pn. \quad (10)$$

If  $X\ddagger > X^*$  the target escapement rule is redundant, as the stock will be abandoned before that rule becomes binding. Hence we may define  $X\ddagger$ , the minimum stock to be left behind (provided the unfished stock exceeds this level):

$$X^\dagger = \max(X^\dagger, X^*). \quad (11)$$

Note that with Rule (3a) the smallest stock to be left behind after harvesting would be  $X^\dagger > 0$ , with the same implications for optimum capacity choice as  $X^* > 0$ .

As the total catch is

$$\begin{aligned} X - X^\dagger &\text{ when } X \leq X^{**}, \\ nKX &\text{ when } X \geq X^{**}, \end{aligned}$$

we get the following expression for the total operating cost (O):

$$O(X) = \int_{X^\dagger}^X (q/nx) dx = (q/n)[\ln X - \ln X^\dagger] \text{ when } X \leq X^{**}, \quad (12a)$$

$$O(K) = \int_{X-nKX}^X (q/nx) dx = -(q/n)\ln(1 - nK) \text{ when } X \geq X^{**}. \quad (12b)$$

It may be noted that the total operating cost is in fact independent of the size of the stock in the latter case, but since the total catch increases with the stock, the cost per unit caught will decrease as the stock increases.

The expected catch value net of operating costs (EV) will then be as follows (note that the probability density is 1):

$$\begin{aligned} EV &= \int_{X^\dagger}^{X^{**}} p(X - X^\dagger) dX - (q/n) \int_{X^\dagger}^{X^{**}} (\ln X - \ln X^\dagger) dX + \int_{X^{**}}^1 pnKX dX \\ &\quad + (q/n) \int_{X^{**}}^1 \ln(1 - nK) dX \\ &= \frac{1}{2} p(X^{**} - X^\dagger)^2 - (q/n)[X^{**}\ln X^{**} - X^{**} - X^{**}\ln X^\dagger + X^\dagger] \\ &\quad + \frac{1}{2} pnK[1 - (X^{**})^2] + (q/n)(1 - X^{**})\ln(1 - nK). \end{aligned} \quad (13)$$

The necessary condition for maximum is most easily derived from the integral terms, noting that  $X^{**}$  depends on  $K$ , since (note that  $X^\dagger$  replaces  $X^*$  in Figure 3)

$$(X^{**} - X^\dagger) = nKX^{**}. \quad (14)$$

Due to (14), the derivative of the upper limit of the first and third integral in (13) is equal to the derivative of the lower limit of the second and fourth integral, with sign changed. Hence we get

$$\begin{aligned} dEV/dK &= pn \int_{X^{**}}^1 X dX - (q/n) \int_{X^{**}}^1 [n/(1 - nK)] dX \\ &= \frac{1}{2} pn[1 - (X^{**})^2] - [q/(1 - nK)](1 - X^{**}). \end{aligned} \quad (15)$$



After substituting for  $X^{**}$  from (14) and setting (15) equal to  $(r + \delta)c$  (cf. 9) we get

$$K = -\sqrt{(\beta^2/4\alpha^2 + \gamma/\alpha)} - \beta/2\alpha, \quad (16)$$

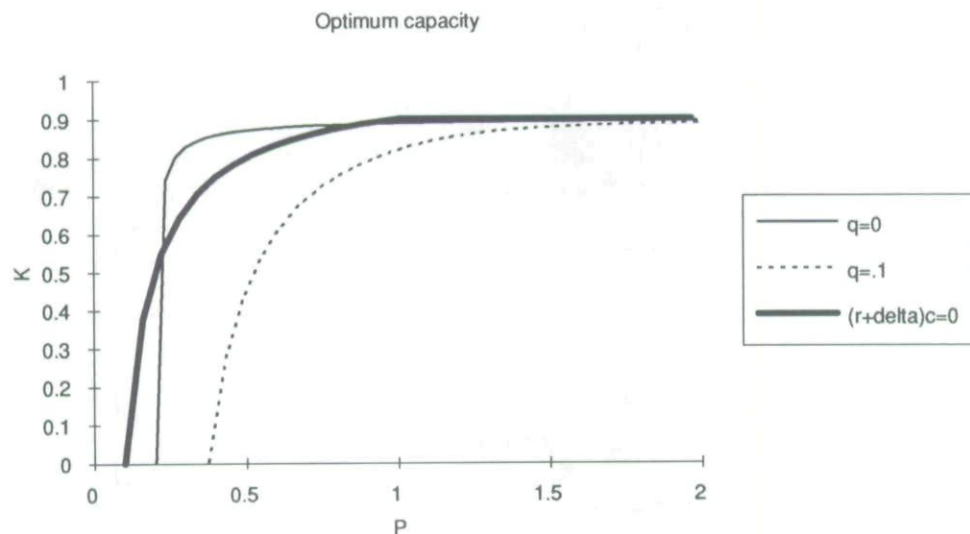
where

$$\alpha = \frac{1}{2}pn^3 - n^2(r + \delta)c,$$

$$\beta = -pn^2 + qn + 2n(r + \delta)c,$$

$$\gamma = (r + \delta)c - \frac{1}{2}pn[1 - (X^\dagger)^2] + q(1 - X^\dagger).$$

Figure 4 shows how the optimum fishing capacity depends on the price of fish ( $p$ ), the operating cost of fishing per unit of effort ( $q$ ), and the capacity cost  $(r + \delta)c$ . The optimum fishing capacity increases with the price of fish, for a given cost of capacity and operating cost per unit of effort. The reason is straightforward; if the capacity is high, it will often not be fully needed. Investing in a large fishing capacity will therefore be worth while only if the price of the fish is high. Note that the price has to exceed a certain level to make it worth while to invest in fishing capacity. The optimum capacity increases with the price of fish towards the asymptotic value  $K = 0.9$ . This value implies that the largest permitted catch can be taken, as  $(X_{\max} - X^*) = 0.9$  for  $n = 1$  and  $X_{\max} = 1$ , the values used in the example. Comparison between the curves for  $q = 0$  and  $q = 0.1$  in Figure 4, both of which are drawn for  $r + \delta = 0.1$  and  $c = 1$ , shows that the minimum price



**Figure 4.** Optimum capacity ( $K$ ) as a function of the price of fish ( $p$ ), the operating cost per unit of capacity used ( $q$ ), and the annualized cost per unit of capacity  $(r + \delta)c$ , when a fish stock is "thinned" over a fishing season.

necessary to make fishing worth while increases with the operating cost per unit of effort and that the optimum capacity rises less steeply with the price of fish the higher is the operating cost per unit of effort.

The third curve in Figure 4 shows how the optimum fishing capacity varies with the price of fish when the capital cost is zero but the operating cost per unit of effort is 0.1. The optimum capacity is seen to increase with the price of fish over some interval instead of being constant at the level that permits the maximum allowable catch ( $X_{\max} - X^*$ ) to be taken. This may appear surprising, because it might be expected that a zero cost of capacity would imply a constant optimum capacity high enough to take the maximum allowable catch. The reason why the optimum capacity increases with the price of fish over some interval is simple, however. It is not profitable to deplete the stock below  $X_{\ddagger}$  (see Equation 10). For a sufficiently low price,  $X_{\ddagger} > X^*$ , and the capacity  $(X_{\max} - X_{\ddagger})/n$  will be sufficient for taking the maximum worthwhile catch ( $X_{\max} - X_{\ddagger}$ ). As the price rises,  $X_{\ddagger}$  decreases. The target escapement rule is in fact redundant in this case, as it is not profitable to deplete the stock below the level  $X_{\ddagger} > X^*$ .

If on the other hand the operating cost and the capacity cost are both zero, the optimum fishing capacity is  $K = (X_{\max} - X^*)/n$  irrespective of price. This may be seen by setting  $(\delta + r)c$  and  $q$  equal to zero in Equation (16).

## Conclusion

In this note the distinction between fishing effort, in the sense of fishing capacity used, and fishing capacity has been emphasized. When the total permitted catch from a stock varies in part due to the influence of random factors that are unrelated to the pattern of exploitation, the management of fisheries will, at the most general level, consist of two parts: (i) applying rules for optimum use of existing fishing capacity, and (ii) deciding the optimum fleet capacity. Here it has been shown how the optimum capacity depends qualitatively on the cost of capacity, operating cost, and the price of fish.

It can be argued that optimum fleet capacity to take uncertain harvests is a more topical subject for the economics of fisheries management than setting the right harvest rate. As mentioned in the introduction, the permitted harvest from fish stocks often appears to be determined by simple biological rules of thumb, such as a target escapement or a constant fishing mortality, sometimes modified by administrators under political pressure. There nevertheless remains the important economic issue to determine optimum fishing capacity for stocks with randomly fluctuating allowable catch. This will not happen automatically. To the extent the optimum solutions derived above imply a positive rent, this would under free access attract additional effort until all rent is dissipated. Overcapacity and rent dissipation in free access fisheries does not depend, therefore, on dynamic growth and stock effects; this can also happen in cases like here where the future catch possibilities are entirely random (but with a known probability distribution). Overcapacity would in that case be the result of a competition for the largest possible share of a given total catch, much like in fisheries managed by closed seasons or a total catch quota for which all can compete.

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