Looking for Cattle and Hog Cycles through a Bayesian Window

Jeffrey H. Dorfman
Department of Agricultural and Applied Economics
The University of Georgia
Athens, GA 30602-7509
email: jdorfman@uga.edu

and

Myung D. Park
Department of Agricultural and Applied Economics
The University of Georgia
Athens, GA 30602-7509
email: mdpark@uga.edu

Selected Paper prepared for presentation at the Agricultural & Applied Economics Associations 2009 AAEA & ACCI Joint Annual Meeting, Milwaukee, WI, July 26-28, 2009

Abstract

The agricultural economics literature, both academic and trade, has discussed the assumed presence of cycles in livestock markets such as cattle and hogs for a very long time. Since Jarvis (1974), there has been considerable discussion over how these cycles impact optimal economic decision making. Subsequent studies such as Rucker, Burt, and LaFrance (1984), Hayes and Schmitz (1987), Foster and Burt (1992), Rosen, Murphy, and Scheinkman (1994), and Hamilton and Kastens (2000) have all investigated some aspect of how biological factors, economic events, or economic actions could be causes of and/or responses to cycles in hog and cattle inventories. There has also been debate, again both in the academic and trade literature, over the length of the cycle(s) present in hog and cattle stocks. To provide both academics and producers with accurate information on the number and periods of cycles that might be present in hog and cattle inventories, this paper provides a purely statistical view of the matter.

Using over 140 years of annual data on cattle and hog inventory levels, we estimate Bayesian autoregressive, trend-stationary models on cattle inventories, hog inventories, and the growth rate of cattle inventories. We then use those models to find the posterior distributions of both the number of cycles present in each series and the period lengths of those cycles. We find multiple cycles present in all three series. Cattle inventory results show clear evidence in favor of 4.5, 6, and 11 year cycles with other cycles present but not as clearly identified. Hog inventory results identify five cycles with periods of approximatley 4.5, 5.4, 6.8, 10 and 13 years. The data on the growth rate in cattle stocks has similar cycles to the series on the stock levels.

1. Introduction

For the last one hundred years, U.S. beef cattle stocks have cycled periodically between periods of high and low inventory numbers. Biological lags have been suspected as one of the big factors behind cattle cycles with those lags leading to rigidities in the accumulation of breeding stock and limiting the ability of producers to respond to changes in market prices. The seminal study of Jarvis (1974) examined how cattle investment decisions interact with biological production lags in the cattle cycle using data from Argentina. After Jarvis' work, more recent empirical studies have provided further understanding of the biological nature of the cattle cycle (cf., Rucker, Burt, and Lafrance (1984), Foster and Burt (1992) and Rosen, Murphy, and Scheinkman (1994)). Hayes and Schmitz(1987) and Hamilton and Kastens (2000) considered market timing effects which could cause a countercyclical response of providers as another factor of cattle and hog cycle.

Previous studies all provide either empirical evidence or theoretical reasons for agricultural commodities like cattle and hogs having cycles in their production and inventories. However, precisely estimating the number of cycles and the length of those cycles has been empirically difficult (or at least such estimates are still a matter of some dispute). Interestingly, none of past literature has really looked just at cycles without economic reasons. In this study, because our approach is from a purely time series point of view, it is in some sense neutral or agnostic in that we are not postulating any cause or mechanism for the cycles or their length. Instead, we are only examining the data-based evidence from the actual inventory levels themselves and investigate the cycle of cattle, hog, and growth rate of cattle inventories.

Our model follows Geweke's (1988) classic study that was at the beginning of the numerical Bayesian econometric literature to investigate the posterior densities of the number and length of cycles. Posterior densities of the cycles are estimated and the number and length of the cycles are found.

The remainder of this paper is organized as follows. In section 2 we present our cycle model, which is a simple time series model that follows Geweke's (1988) classical study and Bayesian estimation method. Section 3 describes the data. Section 4 provides the econometric results, and discusses those results. Conclusions follow in section 5.

2. Econometric Methods

2.1. A Simple Cycle Model

Begin with a simple autoregressive model with 12 lags and a trend, allowing enough flexibility to have up to six cycles. This model is parsimonious enough to minimize empirical difficulties that could arise in the numerical approximations to the posterior distributions to come while allowing for adequate modeling of the feature of interest: the cattle and hog cycles. Denote the stock of a commodity such as cattle or hogs at time t by y_t. The model can be rewritten as

$$y_t = \mu + \sum_{i=1}^{12} \beta_i y_{t-i} + \epsilon_t, \tag{1}$$

where ϵ_t is assumed to follow a normal density. This is similar to the model used by Geweke (1988).

The dynamic behavior of a process such as $\{y_t, t = 1, ..., T\}$ is governed by the roots of the polynomial $\beta(L) \equiv (1 - \Sigma_i \beta_i L^i)$ where L is the lag operator. In particular, we are interested in the cases where the roots form one or more complex conjugate pairs (implying cycles). By examining the eigenvalues of the matrix composed of the autoregressive parameters in the first row and an (i -1) dimension identity matrix in the lower left corner, cycles can be identified. The matrix can be expressed as

$$\begin{bmatrix} \beta_1 & \beta_2 & \beta_3 & \cdots & \beta_{11} & \beta_{12} \\ 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \tag{2}$$

Each cycle in the time series process is linked to a complex conjugate pair of eigenvalues in the above matrix. The length of each cycle is given by the formula

$$\omega = 2\pi / tan^{-1}[Im(\lambda)/Re(\lambda)], \tag{3}$$

where Im(λ) and Re(λ) are the absolute value of the imaginary and real components of one of the eigenvalues in the complex conjugate pair.

Establishing the presence or lack of stationarity is one of the most important steps in time-series analysis. Stationarity can be checked in the model used here by examining the eigenvalues of the matrix in equation (2). If all eigenvalues lie inside the unit circle, the model is stationary; if not, it is nonstationary. In the numerical Bayesian analysis to follow, we can test for stationarity by examining the posterior probability of stationarity. This probability will simply be the percentage of draws from our posterior sampler that have all eigenvalues inside the unit circle.

2.2. The Bayesian Estimation Algorithm

In a Bayesian analysis of a statistical model, the prior density and the likelihood function are the two key components (Zellner, 1971). As we mentioned before, the error term is assumed to follow a normal density. The likelihood function can be expressed as

$$L(y_t|\mu, \beta, \sigma^2) = \frac{T}{\sqrt{2\pi\sigma^2}} \exp\Big[-\frac{1}{2\sigma^2}\Big(\sum_{t=1}^{T} \epsilon_t^2\Big)\Big]. \tag{4}$$

The prior density describes the researchers' subjective beliefs and information about model parameters before looking at the data. We choose independent normal priors for all model parameters with variances that relative to the magnitude of the expected posterior means are relatively diffuse. All prior means are set to zero. Jeffrey's prior is used for σ^2, so prior information plays little role in the posterior density. The prior can be expressed as

$$p(\mu, \beta) = N_\mu(0, 5) \cdot \Pi_{i=1}^{12} N_{\beta_i}(0, 5). \tag{5.1}$$

$$p(\sigma^2) \propto \frac{1}{C} \tag{5.2}$$

Although we could derive the posterior density for μ and β analytically, we cannot derive the posterior of the number and length of cycles without resorting to numerical methods. The change in variables involves an eigenvalue decomposition and the nonlinear function of equation (3), ensuring that the posterior density for the cycle numbers and period lengths will not be anything approaching a standard form. Instead, we use the posterior distribution for β to generate a large number of draws from that posterior and then use those draws to build up empirical posterior distributions for the number of cycles and the length of those cycles. In this study, posterior simulation, the Random Walk Chain Metroplis - Hastings algorithm (Koop, 2003), is employed to estimate the posterior density. The candidate draws are generated according to

$$\theta^* = \theta^{(s-1)} + z \tag{6}$$

where z is called the increment random variable. The intial value of θ is the OLS estimates of the model. Each draw is accepted with the acceptance probability,

$$\alpha(\theta^*|\theta^{(s-1)}) = \min\left[\frac{p(\theta = \theta^*|y)}{p(\theta = \theta^{(s-1)}|y)},\ 1\right] \tag{7}$$

where $p(\theta|y)$ is the posterior distribution. If a draw is rejected, the previous one is reused. The density of the increment random variable z determines the candidate generating density. Commonly, the multivariate normal density is a suitable choice for z. The candidate generating density then can be written as

$$q(\theta|\theta^{(s-1)}) = f_N(\theta^{(s-1)}, c \cdot \widehat{\Sigma}). \tag{8}$$

where $\widehat{\Sigma}$ is the covariance matrix from OLS estimation and c is a tuning constant to adjust the acceptance rate approximately 0.5. There is no general rule for the optimal acceptance rate but an acceptance rate of approximately 0.5 has been recommended by Koop (2003). In our estimation, c is around 0.15 to achieve the recommended acceptance rate.

Since we use an AR(12) model, 6 different complex conjugate pairs can be generated from the each draw at most, so the maximum number of cycles generated from parameters is 6 different cycles. We generate 30,000 draws to create an accurate approximation to the posterior density. The simple averages of draws and cycles from draws become the estimated posterior mean of parameters and cycles. We use the posterior mean for the point estimator in all empirical results.

3. Data Description

The data on annual cattle and hog inventory used to conduct this study are obtained from National Agricultural Statistic Service (NASS) in United States Department of Agricuture (USDA). The stock data of cattle and hog are measured in 1,000 head on 1 January for the period 1867 $\sim$ 2009 (T = 143) and on 1 December for the period 1866 $\sim$ 2008 (T = 143), respectively. To make the trend (μ) similar in magnitude to other parameters, the data on cattle and hog inventory are divided by 1,000 and by 10,000, respectively. We also analyze the growth rate in cattle inventories to see what cycles are present in that series. For the growth rate of cattle inventories, we use the first difference of the log of the stock data. Figure 1 displays annual cattle and hog inventory in U.S. for the period 1866$\sim$ 2009. Figure 2 provides the growth rate of annual cattle inventory. It clearly shows the growth rate of cattle inventory has a number of cycles.

4. Empirical Results

Table 1 provides the comparison between the OLS estimates of our model and the Bayesian posterior mean estimator for cattle inventories, hog inventories, and the change in cattle inventories. Since our prior distribution is relatively diffuse (non-informative), we expect the posterior mean to be close to the OLS estimator. Table 1 shows that, in fact, the two estimators are quite similar with some differences in both point estimates and standard deviations. The fact that the estimators are similar, but not identical is a good indication that the numerical Bayesian estimation approach worked correctly; that is, the prior had some influence on the posterior but not too much, and the numerical approximation appears accurate. From the 30,000 draws for each of the three series, the posterior probability of trend stationarity is over 99.5% for all three series; thus we can proceed under the assumption of trend stationarity.

4.1. Cycles in Cattle Inventories

Tables 2 and 3 provide the empirical results for the January 1 annual cattle inventory series for data from 1867 to 2009. Recall that by using an AR(12) model, we can find anywhere from zero to six cycles. What the data show, is that cattle inventories are indeed characterized by numerous cycles (table 2). The posterior distribution for the number of cycles present has a minimum of three cycles. The posterior probabilities of 0, 1, or 2 cycles are zero. With 30,000 draws from the posterior sampler and a total of 141,981 cycles found in those 30,000 draws, there are no cases of fewer than three cycles. This is powerful empirical evidence that researchers and industry members can stop arguing over the length of the cattle cycle and start talking about which of the many cycles they are talking about.

In terms of distribution of the number of cycles, we find posterior probabilities equal to 0.02% for three cycles, 37.19% for four cycles, 52.30% for five cycles, and 10.50% for six cycles. These results suggest that the cattle inventory data has a very complex dynamic process ongoing with multiple cycles that, as we shall discuss next, often have similar period lengths.

Table 2 also shows the posterior mean estimates of the period length for each cycle conditional on the total number of cycles in that draw. We find that regardless of the number of cycles present (3, 4, 5, or 6), the shortest cycle is always of 4.4 to 4.7 years in length (53 to 56 months). The next longest cycle tends to be 6 years long, again with great robustness across different cycle numbers in the model. When we get to the third cycle, we get more dispersion; here the number of cycles included in the model appears to matter. In fact, the more included cycles, the shorter this third cycle becomes. This suggest that in the 3- and 4-cycle models, this cycle is having to pick up dynamics that would be assigned to a longer cycle in the models with more included cycles. The 4-, 5-, and 6-cycle models regain their consensus on the fourth cycle, with clear agreement on a cycle of slightly over 11 years in length.

Table 3 examines the posterior distribution of the cycle lengths from an unconditional view; that is, regardless of how many cycles are present in the model. Here we find a posterior probability of 88.6% in favor of a cycle of under five years in period length. Of all such cycles, the posterior mean cycle length is 4.59 years or 55 months. We also find overwhelming posterior support for cycles of between 5 and 7 years in length (at 93.2% support) and of between 9.5 and 12 years in length (with 88.1% posterior support). These three cycle lengths are clearly highly supported by the cattle inventory data. The other cycle length ranges shown in table 3 have much lower levels of posterior support, all under the 50% level.

The results of table 3 perhaps provide some rationale for simplifying a model of cattle inventory dynamics to one with three cycles with lengths of 4.5, 6 and 11 years, respectively. While the results displayed in table 2 show that there is a virtually 100% posterior probability of more than three cycles in the model, the length of those cycles is less clear. Table 3 makes clear that our posterior distribution can be clear on the periods of three of the cycles, but is less definitive on a fourth or fifth cycles length. This point is further amplified by figure 3. Figure 3 shows the posterior distributions of the four shortest cattle cycles (showing the longer cycles would cause us to lose detail on these four shorter cycles). From the figure we see well defined posterior distributions for the three cycles mentioned above (those with periods of 4.5, 6, and 11 years) but a bimodal distribution with a very large variance for the third cycle. Clearly the evidence on that cycle is more confused.

4.2. Cycles in Hog Inventories

Results for hog inventories are in tables 4 and 5. Table 4 shows that the posterior distribution for the number of cycles present in hog inventories is more informative than the same distribution for cattle inventories. With 82.4% posterior support for five cycles, there is a very clear choice for the most probable number of cycles to model. Further, the only other model with non-negligible posterior support is that with four cycles present (six cycles gets less than one percent posterior support and no other models ever occur in our 30,000 draws). Conditional on a model with five cycles, we find cycles with lengths of 4.4 years (53 months), 5.1 years, 6.7 years, 10.4 years, and 18.6 years. Except for the 18 year cycle, these are fairly similar in period to those in the cattle inventories.

Moving to the unconditional posterior distributions for cycle lengths (shown in table 5), we find near unanimous support for a cycle with a posterior mean estimated period of 4.6 years (55 months). There is also very strong support for cycles of between 9.5 and 12 years and between 5 and 7 years. Moderately strong support is evidenced for cycles of from 5 to 7 years and 12 to 18 years. The only noticeable difference from the conditional results is we now see that the longest (18 year) cycle from the five cycle model may be better represented by a cycle of just over 13 years in length. Unlike in the cattle inventories, here the unconditional distributions do not reduce the number of cycles worthy of inclusion in an accurate model of hog inventories. The difference is likely that in the hog inventories, a single model (five cycles) so dominates the posterior cycle probabilities that the conditional and unconditional distributions for cycle length do not differ greatly. Thus, these results suggest a model of hog inventory dynamics needs to allow for five cycles with approximate periods of periods of 4.5, 5.4, 6.8, 10.2, and 13.3 years.

Figure 4 shows the unconditional posterior distributions of the first five cycles. We see that the two shortest cycles have very informative posterior distributions, with very tight variances. The next three cycles have much larger variances, even relative to the posterior mean cycle length; however, these posteriors are still more definitive about the cycles' period lengths than we found for the third cycle for cattle inventories-none of them are bimodal.

4.3. Cycles in Cattle Inventory Growth Rates

Even though figure 2 shows such obvious cycling, the cycle of cattle inventory growth rates has not been oftenly investigated previously. Tables 6 and 7 contain results from the model for data on the growth rate in cattle inventories. Table 6 shows that the posterior support is evenly divided between the presence of five and six cycles, with minimal support for four cycles. The separate conditional distributions of cycles are consistent on the length of the cycles in cattle inventory growth rates, with little difference between the five and six cycle models. The posterior means of the first five cycle lengths are approximately 4.5, 5.75, 7.75, 11, and 22 years regardless of model examined. Not surprisingly, these are of similar periods to the cycles in the cattle inventory (level) data. In particular, the posterior mean lengths of the four shortest cycles for the growth rate series are all within a few months of the posterior means for the series in levels.

Table 7 displays the unconditional (integrated over the number of cycles present) results for cycle length for the cattle inventory growth rate data. The results shown here are not particularly enlightening with regard to the five shorter cycles since the conditional results from table 6 were so consistent across the models with different numbers of cycles present. However, for the sixth cycle (the longest), we find considerable uncertainty over its period length. While table 6 shows a posterior mean of 35 years and virtually 50% of the models containing that sixth cycle, table 7 shows only 25% of the draws having a cycle with a length of over 30 years in length. Those draws have a posterior mean length of just over 50 years, so clearly the other draws that went into the table 6 posterior mean of 35 years are considerably shorter than 30 years. The diffusion in the posterior for this sixth cycle suggests maybe a five cycle model is a good choice for approximating this series since the length of the sixth cycle is so uncertain. Figure 5 shows the unconditional posterior distributions of the first five cycles. Posterior densities of the first four cycles are very informative with small variances; however, the posterior of the fifth cycle is scattered and relatively noninformative.

5. Conclusion

The topic of cycles in cattle and hog inventories has been discussed and investigated in both the academic and trade literatures for a least 35 years. Researchers have investigated both biological and economic causes and explanations. The topic is of interest to academics, producers, and processors because a better understanding of these cycles could lead to better risk management and higher profitability for producers and processors.

This article attempts to document the statistical evidence on the number and length of cycles in both hog and cattle inventories. We take an agnostic view of these cycles and stay neutral a priori on the number of cycles and their period lengths. The statistical evidence found in our posterior distributions is strongly in favor of multiple cycles in hog inventories, cattle inventories, and the growth rate of cattle inventories. These series should not be characterized as having a cycle present, but many cycles (4 or 5 according to our results). Both hog and cattle inventories have a short cycle of approximate length of 4.5 years and a longer cycle of around 11 years in period. Both series have multiple short series with period lengths that are quite similar, and then some longer cycles.

These results are certainly not the final word in hog and cattle cycles. However, they hopefully provide a new starting point for researchers to build theoretical models and offer explanations that could cause such complex dynamic behavior to arise.

References

Foster, K. A., and O. Burt. A Dynamic Model of Investment in the U.S. Beef-Cattle Industry, Journal of Business and Economic Statistics 10(1992):419-426.

Geweke, J. The Secular and Cyclical Behavior of Real GDP in 19 OECD Countries: 1957-1983, Journal of Business and Economic Statistics 6(1988):479-486.

Hamilton, S.F. and T.L. Kastens. Does Market Timing Contribute to the Cattle Cycle? American Journal of Agricultural Economics 82(2000):82-96.

Hayes, D. J., and A. Schmitz. Hog Cycles and Countercyclical Production Response, American Journal of Agricultural Economics 69(1987):762-770.

Jarvis, L. S. Cattle as Capital Goods and Ranchers as Portfolio Managers: An Application to the Argentine Cattle Sector, Journal of Political Economy 82(1974): 489-520.

Koop, G., 2003. Bayesian Econometrics. John Wiley & Sons, Chichester, England.

Rosen, S., K.M. Murphy, and J.A. Scheinkman. Cattle Cycles, Journal of Political Economy 102(1994):468-492.

Rucker, R. R., O. R. Burt, and J. T. LaFrance. An Econometric Model of Cattle Inventories. American Journal of Agricultural Economics 66(1984):131-144.

Zellner, A. 1971. Bayesian Inference in Econometrics. John Wiley & Sons, New York.

Table 1. OLS and Bayesian posterior mean estimates of AR(12) model

	Cattle		Hog		The growth rate	
	OLS	Posterior	OLS	Posterior	OLS	Posterior
μ	0.9864	1.0194	0.9249	0.9126	0.0027	0.0028
	(0.5381)	(0.5411)	(0.3738)	(0.3809)	(0.0021)	(0.0021)
β_1	1.8367	1.8380	0.8579	0.8559	0.8599	0.8589
	(0.0921)	(0.0870)	(0.0918)	(0.0925)	(0.0923)	(0.0931)
β_2	-1.0099	-1.0054	-0.3172	-0.3091	-0.0792	-0.0713
	(0.1925)	(0.1849)	(0.1215)	(0.1261)	(0.1213)	(0.1218)
β_3	0.1199	0.1078	0.1211	0.1141	-0.1013	-0.1070
	(0.2135)	(0.2142)	(0.1242)	(0.1391)	(0.1214)	(0.1219)
β_4	-0.0679	-0.0597	0.1581	0.1605	-0.0699	-0.0820
	(0.2139)	(0.2198)	(0.1247)	(0.1215)	(0.1218)	(0.1140)
β_5	-0.0097	-0.0114	-0.2594	-0.2547	-0.1453	-0.1344
	(0.2136)	(0.2022)	(0.1254)	(0.1221)	(0.1219)	(0.1212)
β_6	0.2175	0.2126	0.1926	0.1883	0.0411	0.0397
	(0.2146)	(0.1997)	(0.1274)	(0.1239)	(0.1227)	(0.1264)
β_7	-0.0369	-0.0360	-0.0968	-0.0941	0.0944	0.0947
	(0.2155)	(0.2115)	(0.1272)	(0.1302)	(0.1227)	(0.1206)
β_8	-0.1215	-0.1131	0.0559	0.0589	-0.0474	-0.0511
	(0.2156)	(0.2206)	(0.1255)	(0.1250)	(0.1223)	(0.1200)
β_9	0.0459	0.0338	-0.0106	-0.0175	-0.0550	-0.0615
	(0.2158)	(0.2194)	(0.1246)	(0.1234)	(0.1222)	(0.1230)
β_{10}	0.1100	0.1236	0.1416	0.1438	0.0531	0.0541
	(0.2156)	(0.2179)	(0.1241)	(0.1255)	(0.1219)	(0.1277)
β_{11}	-0.0762	-0.0903	-0.0685	-0.0677	0.0814	0.0838
	(0.1943)	(0.1884)	(0.1215)	(0.1164)	(0.1219)	(0.1148)
β_{12}	-0.0182	-0.0106	0.0658	0.0636	-0.0781	-0.0826
	(0.0922)	(0.0879)	(0.0897)	(0.0881)	(0.0920)	(0.0878)
R^2	0.9953	0.9953	0.6442	0.6442	0.6469	0.6468

Table 2. Conditional cattle inventory posterior mean cycle periods

Cycles	3	4	5	6
Shortest	4.6874	4.7144	4.6223	4.4428
	5.1601	6.0197	5.8580	6.1810
	10.1822	9.3153	8.3618	7.0740
		11.5711	11.4020	11.1030
			65.0033	24.1172
Longest				211.1379
Percentage	0.0002	0.3719	0.5230	0.1050

Table 3. Unconditional distribution of cattle inventory cycle periods

Period length	% of cycles	% of draws	Posterior Mean
Cycle < 5	0.1949	0.8862	4.5858
$5 \leq$ Cycle < 7	0.2638	0.9316	5.9456
$7 \leq$ Cycle < 9.5	0.1124	0.4814	8.3494
$9.5 \leq$ Cycle < 12	0.2361	0.8808	10.7393
$12 \leq$ Cycle < 18	0.0690	0.2951	14.5230
$18 \leq$ Cycle < 30	0.0575	0.2685	22.0607
$30 \leq$ Cycle	0.0663	0.2974	160.7447

* Total number of cycles : 141981

Table 4. Conditional hog inventory posterior mean cycle periods

Cycles	4	5	6
Shortest	4.5743	4.4489	4.5489
	5.2121	5.1197	5.1221
	8.0140	6.6773	6.9850
	10.9368	10.3992	9.7756
		18.6294	40.8773
Longest			102.1935
Percentage	0.1687	0.8238	0.0075

Table 5. Unconditional distribution of hog inventory cycle periods

Period length	% of cycles	% of draws	Posterior Mean
Cycle < 5	0.2767	0.9884	4.5575
$5 \leq$ Cycle < 7	0.1775	0.7347	5.3682
$7 \leq$ Cycle < 9.5	0.1342	0.6214	6.7641
$9.5 \leq$ Cycle < 12	0.2554	0.8772	10.2275
$12 \leq$ Cycle < 18	0.1211	0.5215	13.2599
$18 \leq$ Cycle < 30	0.0158	0.0762	24.1689
$30 \leq$ Cycle	0.0194	0.0890	64.5243

* Total number of cycles : 145165

Table 6. Conditional cattle inventory growth rate posterior mean cycle periods

Cycles	4	5	6
Shortest	4.5391	4.4560	4.4880
	6.0726	5.7662	5.7640
	8.8022	7.9544	7.6788
	11.4459	11.2365	10.6267
		22.4360	21.7486
Longest			35.1558
Percentage	0.0412	0.4643	0.4945

Table 7. Unconditional distribution of cattle inventory growth rate cycle periods

Period length	% of cycles	% of draws	Posterior Mean
Cycle < 5	0.1932	0.9862	4.4863
$5 \leq$ Cycle < 7.5	0.2503	0.9620	6.1923
$7.5 \leq$ Cycle < 10	0.1333	0.6162	8.6464
$10 \leq$ Cycle < 15	0.1796	0.8424	11.3855
$15 \leq$ Cycle < 30	0.1941	0.7581	22.1157
$30 \leq$ Cycle	0.0495	0.2484	50.3523

* Total number of cycles : 163601

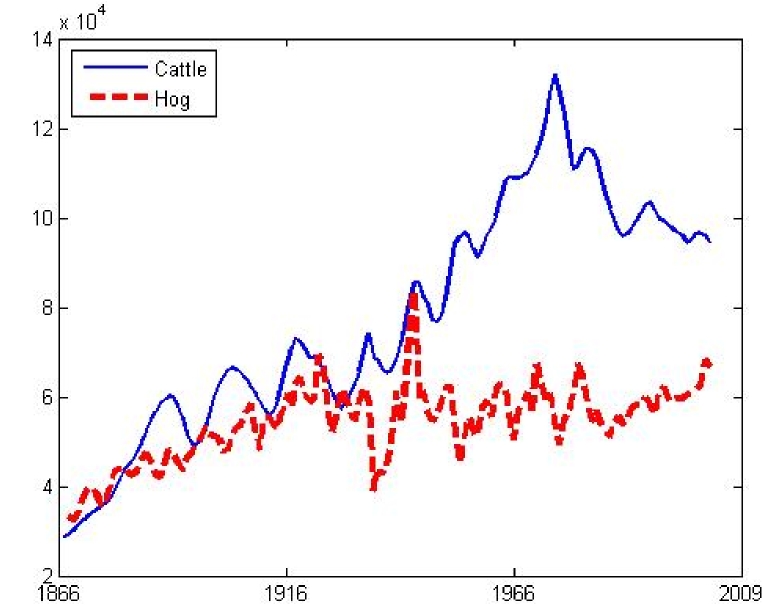

Figure 1. The Annual Inventories of Cattle and Hogs

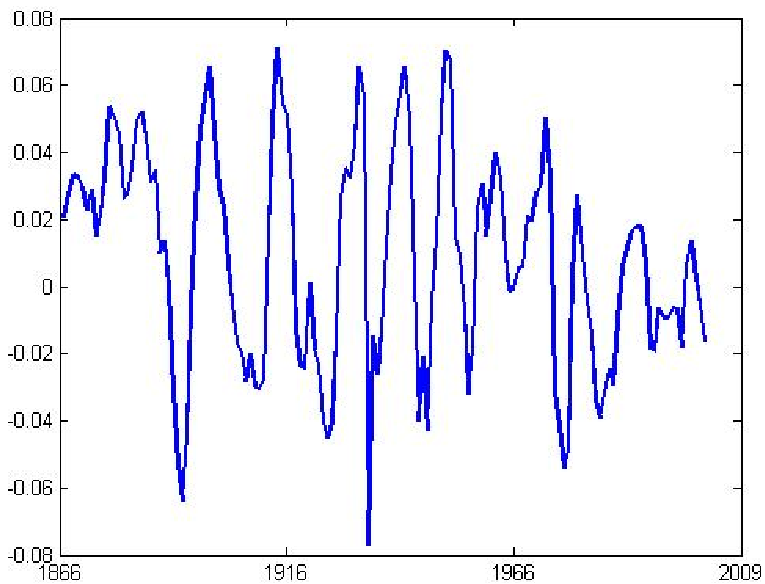

Figure 2. The Growth Rate of Cattle Inventories

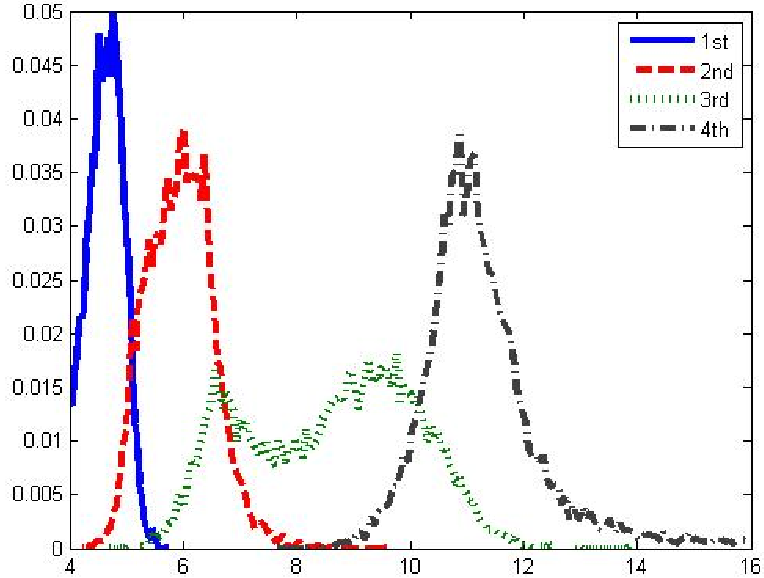

Figure 3. The Posterior Densities of Cattle Cycles

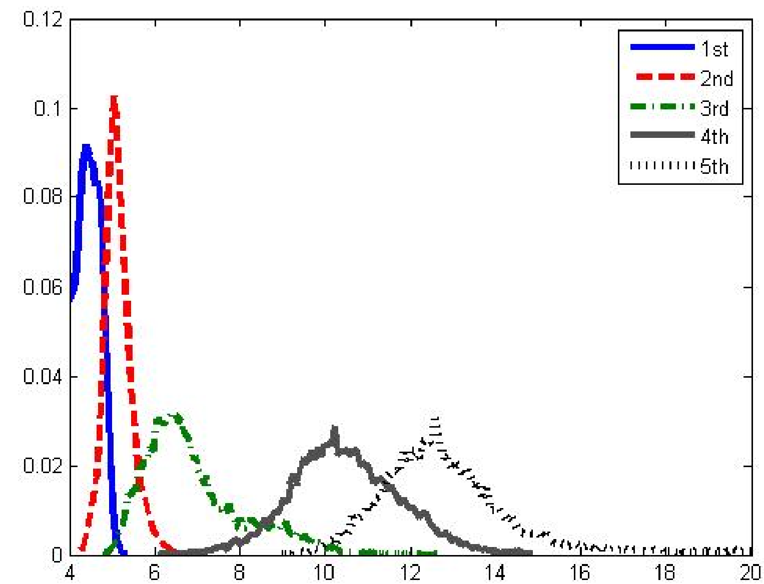

Figure 4. The Posterior Densities of Hog Cycles

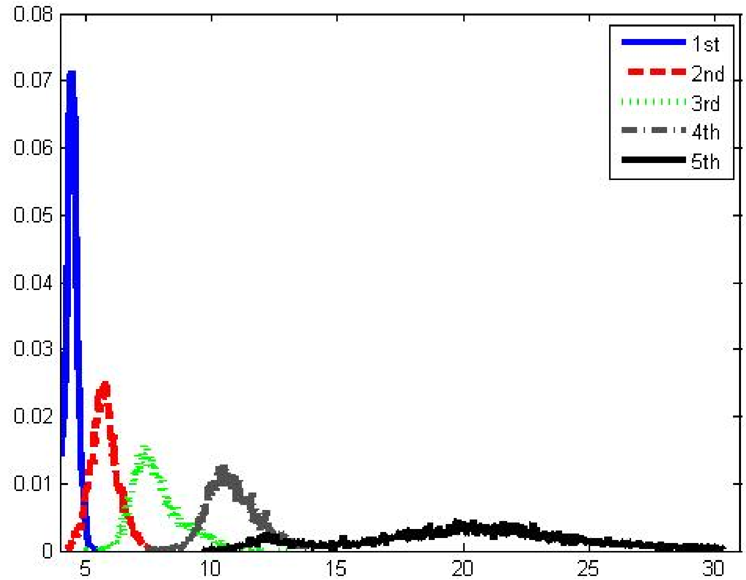

Figure 5. The Posterior Densities of Cattle Inventory Growth Rates