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The Structural Estimation of Principal-Agent Models by Least Squares: Evidence from Land Tenancy in Madagascar

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Abstract

We develop a method to structurally estimate principal-agent models by ordinary least squares (OLS). We set up a general principal-agent model which explicitly incorporates the wealth levels of each party and the opportunity cost to the agent of entering the contract. This yields an optimal contract that is linearized by way of an Nth order Taylor approximation. This in turn imposes $\frac{N(3N-1)}{2}$ restrictions on the parameters and yields an empirical test of the canonical principal-agent model. In the application, we consider the case where $N = 2$ and apply our method to a sample of land tenancy contracts in rural Madagascar. Empirical tests lead to consistent failure to reject the hypotheses derived from our structural model, which lends support to our structural method as well as to the canonical principal-agent model.

Keywords: Principal-Agent Models, Structural Estimation.
JEL Classification: C12, C13, D86, O12, Q12.

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1 Introduction

In recent years, applied microeconomists have often struggled with reconciling the theoretical and empirical strands of the literature on contracts. Much of this struggle has stemmed from the inherent difficulty in measuring the variables necessary to properly test the canonical principal-agent model, let alone cutting-edge models that either seek to relax the axioms of expected utility theory or that seek to impose more structure on the data. Ackerberg and Botticini (2002), for instance, incorporate wealth levels into their contract choice equation in an effort to account for the respective risk preferences of the principal and the agent. Because wealth only proxies for an individual's coefficient of absolute risk aversion, however, and because principals and agents are very likely to match endogenously with one another along their risk preferences, then the estimated coefficients from a contract choice equation ignoring this endogenous matching problem are biased. In other words, because risk preferences are difficult if not impossible to derive from survey data, estimation of a simple contract choice equation is anything but straightforward.[1]

While solutions to such problems are important, it is also necessary to derive valid tests of the canonical model itself if we are to effectively use properly measured data and thus to understand the applicability of the principal-agent model. The contribution of this paper is thus to derive a structural method to estimate a principal-agent model by OLS. The obvious advantage of such a method is to free the econometrician from having to make strong functional form assumptions. Indeed, while the structural literature on contracts has generated important methodological contributions (see, for example, Ferrall and Shearer, 1999; Margiotta and Miller, 2000; Paarsch and Shearer, 2000; and Vera-Hernández, 2003), it is not always obvious how robust empirical results are to changes in functional forms. This paper thus presents some of the theoretical implications of including a set of observable variables in principal-agent models. As is shown below, these theoretical implications provide a simple empirical test of the validity of the canonical principal-agent model as well as a convenient means of accurately estimating the determinants of contract choice.

[1] Even if wealth were to accurately proxy for absolute or relative risk aversion, Bellemare and Brown (2008) show that tests of risk sharing relying on wealth as a proxy for risk aversion are almost always unidentified.

The goal of this paper is thus to concisely present the results of what amounts to a simple although hitherto neglected comparative statics exercise. In particular, we combine the first-order necessary conditions (FONCs) obtained from a general principal-agent model. This allows us to restrict the functional form of the optimal contract. We demonstrate the utility of these restrictions by estimating several specifications of a contract choice equation using data on the land tenancy contracts signed in rural Madagascar (Bellemare, 2008).

The remainder of this paper is organized as follows. In section 2, we present a general principal-agent model which explicitly accounts for wealth effects, we state the main results obtained from manipulating the FONCs, and discuss their empirical content. We then use Taylor expansions of the slope of the optimal contract in section 3, which yields a structural contract choice equation in which the slope of the contract (i.e., the level of risk sharing) is regressed on four observable variables (i.e., the wealth levels of the principal and the agent; the value of the agent's outside option; and output from the contract) and which yields a series of testable restrictions. In section 4, we present the data as well as some summary statistics. We then estimate several specifications of our structural contract choice equation by OLS, test the restrictions imposed by the theory, and discuss the limitations of our approach and our data in section 5. Section 6 concludes by providing directions for future research.

2 Theoretical Framework

Suppose a principal whose utility function is $V(\cdot)$ with $V' > 0$ and $V'' \leq 0$ contracts with an agent whose utility function is $U(\cdot)$ with $U' > 0$ and $U'' \leq 0$, and let the initial wealth levels of the principal and the agent respectively be z_p and z_a. The principal's final utility is defined over $z_p + q - w$, where q is the output from contract w, and conditional on θ_p, which embodies the principal's "type". Likewise, the agent's final utility is defined similarly over $z_a + w$ and conditional on θ_a.

The agent produces output q by providing effort e, where output and effort are linked by the probability density function $f(q|e)$ and where the support of the distribution of output is $\mathbb{Q} = [\underline{q}, \overline{q}]$ The cost of effort to the agent is $\psi(e;\theta_a)$ with $\psi' > 0$ and $\psi'' > 0$. The agent must receive at least

$\overline{U} = U(z_a + c; \theta_a)$ in order to enter the contract, where c is defined here as the agent's opportunity cost from entering the contract.

The principal's problem is to design a contract $w(q)$ which solves

$$\max_{w(q)} \int_{\mathbb{Q}} V[z_p + q - w(q); \theta_p] f(q|e) dq \tag{1}$$

subject to

$$\int_{\mathbb{Q}} U[z_a + w(q); \theta_a] f(q|e) dq - \psi(e; \theta_a) \geq U(z_a + c; \theta_a), \text{ (IR)} \tag{2}$$

and

$$e = \arg\max_{\widetilde{e}} \left\{ \int_{\mathbb{Q}} U[z_a + w(q); \theta_a] f(q|\widetilde{e}) dq - \psi(\widetilde{e}; \theta_a) \right\}, \text{ (IC)} \tag{3}$$

where IR and IC denote the agent's individual rationality and incentive compatibility constraint, respectively. In most cases, the agent's IC constraint can be replaced by its first-order condition (Rogerson, 1985), which is such that

$$\int_{\mathbb{Q}} U[z_a + w(q); \theta_a] f_e(q|e) dq - \psi_e(e; \theta_a) = 0, \tag{4}$$

where $f_e(q|e) = \frac{\partial f(q|e)}{\partial e}$ and $\psi_e(e; \theta_a) = \frac{\partial \psi(e;\theta_a)}{\partial e}$. Suppressing the θ parameters for readability, we can now state the following results.

Proposition 1 *Let $w(q, z_p, z_a, c)$ denote the optimal contract. Then, there exist unique functions $\varpi : \mathbb{Q} \times \mathbb{R}^2 \to \mathbb{R}$ and $\rho : \mathbb{Q} \times \mathbb{R}^2 \to \mathbb{R}$ such that $w(q, z_p, z_a, c) = \varpi(q, z_p, z_a, c) - z_a = \rho(q, z_p, z_a, c) + z_p$ for all $q \in \mathbb{Q}$.*

Proof: See Appendix.

Corollary 2 *If the principal is risk-neutral, then there exists a unique function $\varpi : \mathbb{Q} \times \mathbb{R} \to \mathbb{R}$ such that $w(q, z_p, z_a, c) = \varpi(q, c) - z_a$ for all $q \in \mathbb{Q}$.*

Proof: See Appendix.

Corollary 3 *If the agent is risk-neutral, then there exists a unique function* $\varpi : \mathbb{Q} \times \mathbb{R} \rightarrow \mathbb{R}$ *such that* $w(q, z_p, z_a, c) = \varpi(q, \overline{U} - [z_a + z_p]) + \overline{U} - z_a$ *for all* $q \in \mathbb{Q}$ *and a unique function a unique function* $\rho : \mathbb{Q} \times \mathbb{R} \rightarrow \mathbb{R}$ *such that* $w(q, z_p, z_a, c) = \rho(q, z_a + z_p - \overline{U}) + z_p$ *for all* $q \in \mathbb{Q}$.

Proof: See Appendix.

3 Empirical Framework

Proposition 1 implies that the slope of the contract (i.e., the degree of risk sharing within the contract, or $w_q = \frac{\partial w}{\partial q}$) varies in precise way with respect to changes in the wealth levels of the principal and the agent, z_p and z_a. Specifically,

$$w_q = w_q(q, z_a + z_p, c + z_a; \theta_p, \theta_a), \tag{5}$$

and note that q, z_a, z_p, and c are in principle observable by the econometrician. A first-order Taylor expansion of w_q then yields

$$w_q = \beta_0 + \beta_z z_p + (\beta_z + \beta_c) z_a + \beta_c c + \beta_q q + \eta, \tag{6}$$

where the β parameters are to be estimated and η is a nonstochastic error term.

Likewise, a second-order Taylor expansion of η yields

$$\begin{aligned} \eta \;=\; & \beta_{qq} q^2 + \beta_{zz} z_p^2 + \beta_{cc} c^2 + (\beta_{zz} + 2\beta_{zc} + \beta_{cc}) z_a^2 \\ & + \beta_{zc} z_p c + \beta_{qc} q c + \beta_{qz} q z_p + (\beta_{qz} + \beta_{qc}) q z_a \\ & + (\beta_{zz} + \beta_{zc}) z_p z_a + (\beta_{zc} + \beta_{cc}) c z_a + r, \end{aligned} \tag{7}$$

where the β parameters are to be estimated and r is a nonstochastic error term.

Assuming r is negligible (e.g., assuming that third-order effects are negligible, or if w_q is approximately quadratic), then all of the β coefficients can be estimated consistently. Letting x be a vector of the variables q, z_a, z_p, and c as well as their squares and interaction terms, then the reduced form equation of interest is

$$w_q = \beta x + r. \tag{8}$$

The parameter vector β, however, depends on $\theta = (\theta_p, \theta_a)$, such that $\beta = \beta(\theta)$, and the problem is that θ is unobserved and varies across observations. One way to control for this is is to include principal as well as agent fixed effects, but we are not aware of any data set that includes multiple contracts both per principal and per agent. Thus, for cross-sectional analysis, we are interested in the moments of the distribution of β. Let $\overline{\beta} = E(\beta)$, and let $v = \beta - \overline{\beta}$. If $r = 0$ and $E(vx) = 0$, then $\overline{\beta}$ can be consistently estimated by ordinary least squares, the sufficient condition being the independence of v and x. This obviously means that z_p and z_a must be independent from θ_p and θ_a.

Although our framework does not obviate the endogenous matching problem (Ackerberg and Botticini, 2002), it provides useful testable implications that can help testing the internal consistency of the principal-agent problem. Specifically, any Nth order Taylor expansion of w_q will yield a set of $\frac{N(3N-1)}{2}$ testable implications.

4 Data and Descriptive Statistics

The data used in this paper were collected in Lac Alaotra, Madagascar, between March and August 2004. Lac Alaotra lies about 300 km northeast of Antananarivo, the country's capital, and is the country's most important rice-producing region. The survey methodology was as follows. First, the six communes[2] with the highest density of sharecropping around Lac Alaotra were selected from a 2001 commune census conducted by Cornell University in collaboration with Madagascar's Institut national de la statistique (INSTAT) and Centre national de la recherche appliquée au développement (FOFIFA) (Minten and Razafindraibe, 2003). Then, the two villages with the highest density of sharecropping were chosen in each commune after determining the density of sharecropping in each village by going through communal records. In an effort to oversample sharecropping so as to increase precision, five households known not to lease in or lease out land were selected, five households known to lease in or lease out under a fixed rent

[2]A commune is an administrative unit that is roughly equivalent to a district in the United States.

contract were selected, and 15 households known to lease in or lease out under a sharecropping contract were selected in each village. All households were from within the sampling frame in each village, and the end result is a sample of 300 selected households.[3]

For each selected household, plot-, household- and contract-level data were collected. Household- and (leased-in) plot-level data for the tenants of the 300 selected households as well as household-level and contract-level data for the landlords of the 300 selected households were then collected, which makes for a richer data set. A detailed discussion of the survey methodology is available upon request.

Table 1 presents contract-level summary statistics, and note that P denotes principals (landlords) and A denotes agents (tenants). As regards the structural variables, a little under 69 percent of the plots in our sample are sharecropped, with the remainder under a fixed rent contract. The sum of the values of the assets and the working capital owned is about \$137 for landlords, and about \$86 for tenants.[4,5] The opportunity cost of an hour for the tenant is equal to \$0.15, and we use this variable as a proxy for the value of the tenant's outside option, which we assume to be wage labor. Finally, the average plot is worth \$681 in these data, and we use this variable as a proxy for the output generated in each contract, which we assume to be a sufficient statistic for the plot's average output so as to obviate the endogeneity problem associated with regressing the tenant's incentives on his total output.

[3]The estimation results in this paper control for the oversampling of households that enter sharecropping agreements by incorporating sampling weights. Ideally, one would also control for the choice-based nature of the sample (Manski and Lerman, 1977). Unfortunately, population proportions at the contract-level have never been collected in Madagascar, making the choice-based sampling correction impossible to implement.

[4]The value of the household's working capital is then defined as the sum of the values of its hoe, harrow, cart, plow, tractor, and small tractor. Finally, the value of the household's assets is defined as the sum of the values of its non-productive assets, i.e., house, television, radio, car, and bank account balance. The value of landholdings is omitted because of the near impossibility of obtaining accurate values from respondents. The land sales market is extremely thin in Madagascar, given that sales tend to occur for the most part in distress situations (Randrianarisoa and Minten, 2001).

[5]US\$1 $\approx$ 2,000 ariary.

Turning to the control variables, 20 percent of the plots in our sample were previously owned by a relative of the landlord, and the average plot covers 1.1 hectares. Almost 40 percent of plots are titled, 7 percent are *tanety* (hillside plots), and 7.5 percent are *bas-fonds* (lowland plots). Well over three-quarters of the plots, as one would expect from a sample of rice plots, and the average distance between a landlord's house and her plot is about a 30-minute walk. The average landlord (tenant) household size is 5.5 (5.4) individuals, and the average landlord (tenant) household's dependency ratio is 0.45 (0.41).[6] The average landlord (tenant) household head is 53 (39) years of age and has about 5.4 (5.9) years of formal education. Moreover, approximately 20 percent of landlord household heads are female, and average landlord (tenant) household income was about $58 ($48) per capita. Finally, the average landlord-tenant pair has been contracting for over five years.

5 Estimation Results

Before estimating the structural principal-agent model derived in sections 2 and 3 by OLS, we first impute (i) the likelihood that a plot is titled; and (ii) the value of the plot. Due to a mistake in survey design, these variables were only collected for a subset of the households in the data, and so they need to be imputed from observables.[7]

We present estimation and hypothesis test results for the structural principal-agent model derived above in table 2. We start with the most parsimonious specification, which only includes the variable specified by a first-order Taylor expansion of the theoretical model (model 1). We then augment this model by incorporating the variables from a second-order Taylor expansion of the theoretical model (model 2) and by successively including plot-level characteristics (model 3), landlord and tenant household-level characteristics (model 4), and the landlord's subjective perception of the likelihood that she will lose her plot as a consequence of the contract chosen (model 5). Each column of table 2 thus nests the specification found in the previous column. In every specification in table 2, the estimation results are probability-weighted

[6] The dependency ratio is the sum of the number of individuals under 15 and the number of individuals over 64 divided by the total number of individuals in the household.

[7] Estimation results for these imputations are omitted for brevity, but they are available from the authors upon request.

due to the sampling scheme, and the standard errors are both bootstrapped and robust.

Model 1

Because this model is the product of a first-order Taylor expansion, there is only one testable restriction, i.e., $(\beta_{z_p} + \beta_c) = \beta_{z_a}$. In this case, the $\chi^2(1)$ test statistic is equal to 0.07, with a p-value of 0.79. So in the simplest case of a first-order Taylor expansion, the data support the theoretical model of section 2.

Model 2

Because this model is the product of a second-order Taylor expansion, there are now five testable restrictions, i.e., $(\beta_{z_p} + \beta_c) = \beta_{z_a}$, $(\beta_{z_p z_p} + 2\beta_{z_p c} + \beta_{cc}) = \beta_{z_a^2}$, $(\beta_{q z_p} + \beta_{qc}) = \beta_{q z_a}$, $(\beta_{z_p z_p} + \beta_{z_p c}) = \beta_{z_p z_a}$, and $(\beta_{z_p c} + \beta_{cc}) = \beta_{c z_a}$. In this case, the $\chi^2(5)$ test statistic is equal to 2.06, with a p-value of 0.84. So when imposing more structure by applying a second-order Taylor expansion, the data again support the theoretical model of section 2.

Model 3

The results for models 1 and 2 were as parsimonious as possible in that they only included the variables of interest and market dummies. In this model, we reestimate model 2 and incorporate plot characteristics. The five testable restrictions are again $(\beta_{z_p} + \beta_c) = \beta_{z_a}$, $(\beta_{z_p z_p} + 2\beta_{z_p c} + \beta_{cc}) = \beta_{z_a^2}$, $(\beta_{q z_p} + \beta_{qc}) = \beta_{q z_a}$, $(\beta_{z_p z_p} + \beta_{z_p c}) = \beta_{z_p z_a}$, and $(\beta_{z_p c} + \beta_{cc}) = \beta_{c z_a}$. In this case, the $\chi^2(5)$ test statistic is equal to 2.05, with a p-value of 0.84. So when controlling for plot characteristics, the data support the theoretical model of section 2.

Model 4

In this case, we reestimate model 3 and incorporate characteristics of the landlord and tenant households. The five testable restrictions are again $(\beta_{z_p} + \beta_c) = \beta_{z_a}$, $(\beta_{z_p z_p} + 2\beta_{z_p c} + \beta_{cc}) = \beta_{z_a^2}$, $(\beta_{q z_p} + \beta_{qc}) = \beta_{q z_a}$, $(\beta_{z_p z_p} + \beta_{z_p c}) = \beta_{z_p z_a}$, and $(\beta_{z_p c} + \beta_{cc}) = \beta_{c z_a}$. In this case, the $\chi^2(5)$ test statistic is equal to 2.74, with a p-value of 0.74. So when controlling for plot characteristics and the characteristics of the landlord and the tenant, the data support the theoretical model of section 2.

Model 5

In this case, we reestimate model 4 and incorporate the slope of the asset risk function (i.e., the landlord's subjective perception that she will lose her plot as a result of the contract), which has been found to explain the emergence of sharecropping contracts in these data by Bellemare (2008). The five testable restrictions are again $(\beta_{z_p}+\beta_c) = \beta_{z_a}$, $(\beta_{z_p z_p}+2\beta_{z_p c}+\beta_{cc}) = \beta_{z_a^2}$, $(\beta_{q z_p}+\beta_{qc}) = \beta_{q z_a}$, $(\beta_{z_p z_p} + \beta_{z_p c}) = \beta_{z_p z_a}$, and $(\beta_{z_p c} + \beta_{cc}) = \beta_{c z_a}$. In this case, the $\chi^2(5)$ test statistic is equal to 2.92, with a p-value of 0.71. So when controlling for plot characteristics, the characteristics of the landlord and the tenant, and the landlord's perception of asset risk, the data still support the theoretical model of section 2.

Limitations

The estimation results in table 2 and the empirical tests of model structure we run above suffer from important limitations. First and foremost, none estimated coefficients for the structural variables z_p, z_a, q, and c are individually or jointly significant at any of the conventional levels in models 1 to 5. This lack of significance of the structural coefficient estimates could likely explain the structural test results we run for models 1 to 5. Yet these structural variables are derived from the theoretical model in section 2 and, as such, they must be included so as to accurately estimate a principal-agent model by OLS. Perhaps more importantly, the sample at hand is small, which could explain these results.

Moreover, we have had to rely on two proxies. First, since we do not observe the value of the agent's outside option, we had to assume that the value to the agent of refusing to enter the contract was proportional to his hourly wage, which implicitly assumes that the tenant's outside option is wage labor. Second, since output is most likely endogenous to contract choice due to Marshallian inefficiency, we had to rely on the value of the plot as a sufficient statistic for the value of the plot's output, which implicitly assumes that there are no fertility dynamics, i.e., that the plot is equally productive from year to year. A better way of incorporating output would be to use the previous period's output, but this variable was not available in these data.

Finally, given the cross-sectional nature of our data, it is impossible to control for θ_p and θ_a, i.e., the unobserved heterogeneity between principals and agents. Doing so would require longitudinal data which includes more than one observation for each principal and for each agent as well as enough variation in the slope of the contract they select into. Unfortunately, this is well-beyond the scope of these data, and we know of no data set that would allow to control for both these sources of unobserved heterogeneity.

6 Conclusion

This paper has developed a structural method to estimate a general principal-agent model by OLS. By combining the FONCs obtained from the canonical model, we restricted the functional form of the optimal contract, which we then linearized using a Taylor expansion. Doing so provided us with a structural contract choice equation estimable by OLS as well as with several testable restrictions.

The advantages of our method are twofold. First and foremost, it allows testing the canonical principal-agent model. In other words, provided one has the required variables (i.e., the slope of the contract; the wealth levels of the principal and the agent; the value of the agent's outside option; and the output from the contract), one can directly test the validity of the principal-agent model. Second, it allows accurately estimating the marginal effects of (i) the structural variables; (ii) any other variable of interest; and (iii) the control variables, provided there are no statistical endogeneity problems. Perhaps more importantly, our results are applicable to both linear and nonlinear contracts.

Applying our method to a sample of the land tenancy contracts signed in rural Madagascar, we found empirical support for the canonical principal-agent model in the data, but we were also careful to discuss the limitations of our empirical work. Indeed, our sample size was small, and one of our structural variables not only had to be proxied for, but the proxy itself had to be imputed due to a mistake in survey design. These data issues could explain our empirical results, so that we encourage future researchers to apply this method to larger, more carefully collected data so as to provide applied contract theorists with more solid evidence in favor of or against the canonical principal-agent model.

References

[1] Ackerberg, Daniel A., and Maristella Botticini (2002), "Endogenous Matching and the Empirical Determinants of Contract Choice," *Journal of Political Economy*, Volue 110, pages 564-591.

[2] Bellemare, Marc F. (2008), "Insecure Land Rights and Reverse Share Tenancy in Madagascar," Working Paper, Duke University.

[3] Bellemare, Marc F., and Zachary S. Brown (2008), "On the (Mis)Use of Wealth as a Proxy for Risk Aversion," Working Paper, Duke University.

[4] Caputo, Michael R. (2005), *Foundations of Dynamic Economic Analysis*, Cambridge: Cambridge University Press.

[5] Ferrall, Christopher, and Bruce Shearer (1999), "Incentives and Transactions Costs Within the Firm: Estimating an Agency Model Using Payroll Records," *Review of Economic Studies*, Volume 66, pages 309-338.

[6] Manski, Charles F., and Steven R. Lerman (1977), "The Estimation of Choice Probabilities from Choice-Based Samples", *Econometrica*, Volume 45, pages 1977-1988.

[7] Margiotta, Mary M., and Robert A. Miller (2000), "Managerial Compensation and the Cost of Moral Hazard," *International Economic Review*, Volume 41, pages 669-719.

[8] Minten, Bart and Roland Razafindraibe (2003), "Relations terres agricoles–pauvreté à Madagascar," Policy Brief Presented at the Projet Ilo/FOFIFA Conference on Agriculture and Poverty, Antananarivo, Madagascar.

[9] Paarsch, Harry J., and Bruce Shearer (2000), "Piece Rates, Fixed Wages, and Incentive Effects: Statistical Evidence from Payroll Records," *International Economic Review*, Volume 41, pages 59-92.

[10] Randrianarisoa, Jean Claude, and Bart Minten (2001), "Agricultural Production, Agricultural Land and Rural Poverty in Madagascar," Working Paper, Cornell University.

[11] Rogerson, William P. (1985), "The First-Order Approach to Principal-Agent Problems," *Econometrica*, Volume 53, pages 1357-1368.

[12] Vera-Hernández, Marcos (2003), "Structural Estimation of a Principal-Agent Model: Moral Hazard in Medical Insurance," *RAND Journal of Economics*, Volume 34, pages 670-693.

Appendix

Proof of Proposition 1: Fix $\overline{U}$ and suppress it as an argument. Let $e^* = e(z_a, z_p)$ denote the optimal effort level that solves the principal-agent problem along with $w(q, z_a, z_p)$. Further let $\partial_x w$ denote the derivative of $w(q, z_a, z_p)$ with respect to any argument x, keeping e^* fixed. Applying the Chain Rule to w, we thus have for $z \in \{z_a, z_p\}$,

$$w_z = \partial_z w + \partial_e w \cdot \partial_z e^*. \tag{9}$$

To establish the result, it suffices to show that the following two claims hold:

1. $\partial_{z_a} w = \partial_{z_p} w - 1$ for all $q \in \mathbb{Q}$; and

2. $\partial_{z_a} e^* = \partial_{z_p} e^*$.

In showing both claims, it is helpful to refer to the implicit function for the contract that is defined by the corresponding first-order condition. That is, define $\widetilde{w}(q, \lambda, \mu, z_a, z_p, e)$ implicitly by

$$\frac{V'[z_p + q - \widetilde{w}]}{U'[z_a + \widetilde{w}]} = \lambda + \mu \frac{f_e(q|e)}{f(q|e)}. \tag{10}$$

Also denote $\widetilde{\lambda}(z_a, z_p, e)$ and $\widetilde{\mu}(z_a, z_p, e)$ as the implicit functions defined through allowing $\widetilde{w}(q, \widetilde{\lambda}, \widetilde{\mu}, z_a, z_p, e)$ to satisfy both the IR and IC constraints. Applying the univariate Implicit Function Theorem to equation 10, one can first verify that

$$\partial_{z_a} \widetilde{w} = \partial_{z_p} \widetilde{w} - 1. \tag{11}$$

Subsequently, applying the bivariate Implicit Function Theorem to the system of equations defined by the IR and IC constraints and using the result in equation 11, one finds that

$$\partial_{z_a} \widetilde{\lambda} = \partial_{z_p} \widetilde{\lambda}, \text{ and} \tag{12}$$

$$\partial_{z_a} \widetilde{\mu} = \partial_{z_p} \widetilde{\mu}. \tag{13}$$

By noting that $w(q, z_a, z_p) \equiv \widetilde{w}(q, \widetilde{\lambda}(z_a, z_p, e^*), \widetilde{\mu}(z_a, z_p, e^*), z_a, z_p, e^*)$ and applying the Chain Rule to $\widetilde{w}$, we have, for $z \in \{z_a, z_p\}$,

$$\partial_z w = \partial_z \widetilde{w} + \partial_\lambda \widetilde{w} \cdot \partial_z \widetilde{\lambda} + \partial_\mu \widetilde{w} \cdot \partial_z \widetilde{\mu}. \tag{14}$$

Combining equations 11 to 14 establishes the first claim.

To establish the second claim, one must appeal to the First-Order Dynamic Envelope Theorem (Caputo, 2005), which implicitly defines $e^*(z_a, z_p)$ as follows

$$\widetilde{\mu}(z_a, z_p, e^*) = \tag{15}$$

$$\frac{\int_{\mathbb{Q}} V[z_p + q - \widetilde{w}(q, \widetilde{\lambda}(z_a, z_p, e^*), \widetilde{\mu}(z_a, z_p, e^*), z_a, z_p, e^*)] f_e(q|e^*) dq}{\psi''(e^*) - \int_{\mathbb{Q}} U[z_a + \widetilde{w}(q, \widetilde{\lambda}(z_a, z_p, e^*), \widetilde{\mu}(z_a, z_p, e^*), z_a, z_p, e^*)] f_{ee}(q|e^*) dq}.$$

Defining $G(z_a, z_p, e^*)$ as the RHS of 15 and applying the univariate Implicit Function Theorem yields

$$\partial_z e^* = -\frac{\partial_z G - \partial_z \widetilde{\mu}}{\partial_e G - \partial_e \widetilde{\mu}} \text{ for } z \in \{z_a, z_p\}. \tag{16}$$

From equations 12, 13, and 16 it now suffices to show that $\partial_{z_p} G = \partial_{z_a} G$. Expanding $\partial_{z_p} G$ and $\partial_{z_a} G$ yields

$$\partial_{z_p} G = \frac{\int_{\mathbb{Q}} V' \cdot (1 - \partial_{z_p} w) \cdot f_e dq}{\psi''(e^*) - \int_{\mathbb{Q}} U f_{ee} dq} + \frac{\left(\int_{\mathbb{Q}} V f_e dq\right) \left(\int_{\mathbb{Q}} U' (\partial_{z_p} w) f_{ee} dq\right)}{\left(\psi''(e^*) - \int_{\mathbb{Q}} U f_{ee} dq\right)^2} \tag{17}$$

and

$$\partial_{z_a} G = \frac{\int_{\mathbb{Q}} V' \cdot (-\partial_{z_a} w) \cdot f_e dq}{\psi''(e^*) - \int_{\mathbb{Q}} U f_{ee} dq} + \frac{\left(\int_{\mathbb{Q}} V f_e dq\right) \left(\int_{\mathbb{Q}} U' (1 + \partial_{z_a} w) f_{ee} dq\right)}{\left(\psi''(e^*) - \int_{\mathbb{Q}} U f_{ee} dq\right)^2}. \tag{18}$$

Applying the first claim to equations 17 and 18 establishes the second claim, i.e., $\partial_{z_a} e^* = \partial_{z_p} e^*$. Thus, $w_{z_a} = w_{z_p} - 1$ for all $q \in \mathbb{Q}$. Solving this linear first-order partial differential equation yields the result. ■

Proof of Corollary 1: When the principal is risk-neutral, then changing z_p has no effect on the optimal contract, i.e., $\frac{\partial w(q)}{\partial z_p} = 0$. Applying Proposition 1 to this case yields $\frac{\partial w(q)}{\partial z_a} = -1$. Keeping $\overline{U}$ constant, integrating over z_a, and noting that the constant of integration depends on q and $\overline{U}$ yields the result. ■

Proof of Corollary 2: Given that $\int_{\mathbb{Q}} f(q|e)dq = 1$ and $\int_{\mathbb{Q}} f_e(q|e)dq = 0$ for all e, increasing z_a when the agent is risk-neutral has no effect on the IC constraint and has the same effect on the IR constraint as a decrease in $\overline{U}$ by the same amount. Applying Proposition 1 to this case, one can choose a function $\delta(\cdot)$ such that $w(q, z_a, \overline{U}) = \delta(q, z_a + z_p, \overline{U}) - z_a = \delta(q, z_p, \overline{U} - z_a)$. Totally differentiating both sides of the last equation with respect to $\overline{U}$ and z_a yields

$$\partial_{z_a} \delta + \partial_{\overline{U}} \delta = 1, \tag{19}$$

where ∂_{z_a} refers to the partial derivative of δ with respect to its second argument. Note that above, δ is evaluated at $(q, z_a + z_p, \overline{U})$, i.e., the same point for both partials. This is a linear first-order partial differential equation whose general solution yields the desired result. ■

Table 1: Descriptive Statistics

Variable	Mean	Std. Dev.	*N*
Sharecropping Dummy	0.687	(0.464)	387
z_p (1,000,000 Ariary)	0.273	(0.652)	380
z_a (1,000,000 Ariary)	0.171	(0.275)	384
c (1,000 Ariary)	0.294	(0.178)	383
q (1,000,000 Ariary)	1.361	(0.890)	387
Family-Owned Plot Dummy	0.199	(0.400)	387
Plot Size (Ares)	109.809	(84.164)	387
Titled Plot Dummy	0.384	(0.488)	125
Tanety Dummy	0.070	(0.255)	387
Bas-Fonds Dummy	0.075	(0.264)	387
Irrigated Plot Dummy	0.755	(0.431)	387
Distance from House (Walking Minutes)	33.013	(36.266)	387
P Household Size (Individuals)	5.475	(2.807)	387
P Household Dependency Ratio	0.450	(0.252)	387
P Household Head Age (Years)	53.359	(16.391)	387
P Household Head Female Dummy	0.196	(0.398)	387
P Household Head Education (Years)	5.413	(3.882)	387
P Household Income Per Capita (100,000 Ariary)	1.162	(2.349)	386
Relationship Length (Years)	5.373	(7.433)	386
Kin Contract Dummy	0.638	(0.481)	387
A Household Size (Individuals)	5.753	(2.566)	384
A Household Dependency Ratio	0.412	(0.216)	384
A Household Head Age (Years)	39.036	(11.098)	384
A Household Head Income (100,000 Ariary)	0.931	(1.484)	384
A Household Head Education (Years)	5.927	(3.427)	384
Slope of Asset Risk Function (r_a)	-0.258	(3.960)	387

Table 2: Estimation Results for

	(1)		(2)		(3)		(4)		(5)	
	Coefficient	(Std. Err.)	Coefficient	(Std. Err.)	Coefficient	(Std. Err.)	Coefficient	(Std. Err.)	Coefficient	(Std. Err.)
z_p	-0.02	(0.09)	0.33	(0.34)	0.25	(0.34)	0.35	(0.36)	0.38	(0.36)
z_a	0.04	(0.12)	-0.03	(0.65)	0.00	(0.70)	0.35	(0.71)	0.30	(0.69)
c	0.00	(0.19)	0.19	(0.80)	0.43	(0.81)	0.21	(0.83)	0.10	(0.82)
q	0.03	(0.04)	0.01	(0.14)	-0.01	(0.20)	-0.16	(0.21)	-0.11	(0.20)
z_p^2			-0.03	(0.06)	-0.02	(0.06)	-0.02	(0.07)	-0.02	(0.07)
z_a^2			-0.14	(0.27)	-0.13	(0.28)	-0.25	(0.28)	-0.20	(0.26)
c^2			-0.21	(0.76)	-0.29	(0.72)	-0.17	(0.76)	-0.11	(0.78)
q^2			-0.01	(0.04)	-0.04	(0.05)	-0.01	(0.05)	-0.01	(0.05)
z_p*q			-0.06	(0.11)	-0.04	(0.10)	-0.05	(0.11)	-0.08	(0.11)
z_a*q			0.13	(0.19)	0.08	(0.20)	0.05	(0.21)	0.02	(0.20)
$c*q$			0.13	(0.26)	0.06	(0.27)	0.19	(0.28)	0.18	(0.28)
z_p*c			-0.85	(0.70)	-0.86	(0.68)	-1.03	(0.71)	-1.03	(0.72)
z_a*c			0.29	(1.86)	0.27	(2.01)	0.29	(2.01)	0.30	(1.93)
z_p*z_a			0.20	(0.55)	0.19	(0.54)	-0.03	(0.55)	0.02	(0.52)
Family-Owned Plot					-0.11	(0.08)	0.18	(0.16)	-0.12*	(0.07)
Plot Size					0.00*	(0.00)	0.21**	(0.13)	0.00*	(0.00)
Titled Plot					0.15	(0.27)	-0.34	(0.16)	0.24	(0.27)
Tanety					0.01	(0.17)	0.04	(0.17)	-0.01	(0.17)
Bas-Fonds					-0.04	(0.17)	0.06	(0.17)	-0.04	(0.16)
Irrigated Plot					-0.09	(0.10)	-0.24	(0.15)	-0.11	(0.10)
Distance from House					0.00	(0.00)	-0.12	(0.29)	0.00	(0.00)
P Household Size							0.05	(0.15)	0.01	(0.01)
P Household Dependency Ratio							0.05	(0.14)	0.00	(0.13)
P Household Head Age							-0.39*	(0.17)	0.00*	(0.00)
P Household Head Female							-0.48	(0.19)	-0.02	(0.07)
P Household Head Education							-0.12	(0.08)	-0.01	(0.01)
P Household Income Per Capita							0.00	(0.00)	0.01	(0.01)
Relationship Length							0.29	(0.28)	0.02	(0.01)
Kin Contract							0.00	(0.18)	0.01	(0.07)
A Household Size							-0.06	(0.17)	-0.02	(0.02)
A Household Dependency Ratio							-0.10*	(0.10)	0.26*	(0.15)
A Household Head Age							0.00	(0.00)	0.00	(0.00)
A Household Head Income							0.01*	(0.01)	-0.05*	(0.02)
A Household Head Education							0.00	(0.13)	-0.01	(0.01)
Slope of Asset Risk Function (r_a)									0.03***	(0.01)
Intercept	0.77***	(0.12)	0.73***	(0.23)	0.69**	(0.31)	1.06***	(0.39)	1.20***	(0.38)
N	376		376		376		376		376	
Village Dummies	Yes		Yes		Yes		Yes		Yes	
Replications	1000		1000		1000		1000		1000	
R^2	0.19		0.21		0.23		0.30		0.33	
p-value (All Coefficients)	0.00		0.00		0.00		0.00		0.00	
p-value (Structural Coefficients)	0.94		0.99		0.99		0.94		0.97	