Contracting with Agents Seeking Status

Philippe Bontems
Toulouse School of Economics
(GREMAQ-INRA and IDEI)

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Philippe Bontems†

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Abstract

We explore in this paper the consequences of status seeking preferences among agents contracting with a private principal in the context of production. We examine in particular the case of envy and we show that in general envy entails augmented distortions due to asymmetric information in optimal contracts. Furthermore if the principal neglects the preferences of the agents with respect to status, then potentially there is under-participation to the contract. We also show that if the principal is free to choose who can participate to the contract, then under some conditions the principal may prefer to contract with only a subset of potentially "profitable" agents (that is where his utility is strictly positive). We then ask whether contracting with agents seeking status would yield to more incentives to exert unobservable effort. We actually show that the principal has incentives to discourage effort. In the last part of the paper, we consider the case of costly observation of private decisions so that we investigate whether envy encourages non compliance or not.

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* *Corresponding author* : Philippe Bontems, GREMAQ-INRA, Université des Sciences Sociales de Toulouse, 21 Allée de Brienne, 31000 Toulouse, France. Email: bontems@toulouse.inra.fr. Phone: (33) 561 128 522. Fax: (33) 561 128 520. This work has benefited from a grant under the S3E program (Ministère de l'Ecologie, du Développement et de l'Aménagement Durables).

†Toulouse School of Economics (GREMAQ, INRA, IDEI).

1 Introduction

It has long been recognized that individuals are probably motivated at least partly by a concern about their relative position in the population, in particular relative to income. For instance John Stuart Mills has observed that "men do not desire to be rich but richer than other men" (cited by Luttmer 2005). The effects of social comparisons on consumption has been analyzed in the classic work of Veblen (1899). Recently, Samuelson (2004) and Rayo and Becker (2004) have offered evolutionary explanations of relative consumption effects while Luttmer (2005) provides some empirical evidence on individual-level data.[1]

However, the standard modelling of preferences would rather state that individuals derive utility $U(C)$ from their own consumption level instead of a combination of own and relative consumption $U(C, C_{others})$ or $U(C, C/C_{others})$ where C_{others} is a measure of the consumption of relevant others (for instance the mean consumption in the population or the consumption of the richest people...). As suggested by Luttmer (2005), in general both formulations are isomorphic and hence unless an individual can affect C_{others}, they cannot be distinguished on the basis of individual behavior. This would explain why most economists would favor the standard formulation.

Nevertheless, policies will in general affect C_{others} and hence the formulation with relative concern will generate different conclusions compared to the standard formulation of utility. This problem has been analyzed by Boskin and Sheshinski (1978), Oswald (1983) and Ireland (1998) in the context of income taxation. Also, Dupor and Liu (2003) have shown that if the consumption of others affects marginal utility rather than the level of own utility, then the consumption of others will affect all kind of decisions a consumer can take, work effort, job search, risk takings and savings....

In this paper, we explore the consequences of retaining the idea of people taking decisions while being sensitive to relative income in the context of production and contracts. We model

[1] There is even some evidence that this pattern of behavior emerges among animals like capucin monkeys as shown by the work of Frans de Waal. Some studies have shown that these monkeys can exhibit some aversion to inequity in some experiments.

a principal-agent relationship where agents have preferences toward the allocations of others. The relationship is subject to adverse selection with respect to individual productivity. We first consider the case of a private principal that seeks to maximize her surplus net of the transfers to be paid to agents while taking care of participation and incentive compatibility constraints. We show that under perfect information in general the presence of preferences with relative consumption yields to a distortion in the optimal allocation in order to internalize the externality each individual exerts on the others. This distortion however disappears when the marginal rate of substitution between money and production is the same at the utility level and at the externality level. This assumption is satisfied for instance if we suppose that any agent has an utility function of the form $v(\pi, \overline{\pi})$ where π is his profit and $\overline{\pi}$ a weighted mean of profits in participating agents.

Under asymmetric information, the usual distortion due to the rent-extraction-efficiency trade-of depends on the presence of the externality generated by the assumption of relative consumption preferences. Suppose that there is envy (or jealousy) then individual would gain from having a profit larger than say the average profit in the population but they would experience an additional disutility if they earn less than the average profit. We then show that it is optimal for the principal to impose an augmented downward distortion to production in general. The intuition goes as follows: here leaving informational rents to the agents will have the additional effect of tightening the participation constraint of the least efficient type because this agent will earn less than the others and hence is jealous. It is therefore necessary to extract rents more than in the absence of jealousy, by decreasing production. To sum up, jealousy amounts to a more discriminating production schedule, although one can show that the gap between the highest and the lowest profits decreases (less inequality in terms of monetary payoffs). In a sense, the introduction of envy yields to some implicit redistribution between agents towards a more equal income distribution.[2]

[2] It is interesting to note that Cornéo and Grüner (2002) in an empirical analysis suggest the "social rivalry effect" as one of the possible explanations that drive people's support of governmental reduction of income inequality.

Furthermore, if the principal neglects the preferences of the agent with respect to relative income or consumption (hereafter the so-called "naive" principal), then a subset of agents among the less efficient might be reluctant to participate. There is thus potentially under-participation to the contract. We also show that if the principal is free to choose who can participate to the contract, then under some conditions the principal may prefer to contract with only a subset of potentially "profitable" agents (that is where his utility is strictly positive). This implies that there is under-participation because expanding the set of participating agents amounts to increase downward distortion on production levels which is inefficient.

We then ask whether contracting with agents seeking status would yield to more incentives to exert unobservable effort. We actually show that for a given production level, the presence of status seeking preferences induces more effort as the marginal benefit of production is expanded. But at the same time it also contributes positively to the size of the negative externality when there is envy. The principal hence designs a contract that reduces the production level in order to internalize the impact of envy over effort.

In the last part of the paper, we consider the case of costly observation of private decisions so that we investigate whether envy encourages non compliance or not.

The paper is organized as follows. The next section is devoted to a general model of contracting between a principal and agents with status seeking preferences. In section 3, we develop a particular specification with more details. Section 4 is devoted to the model with unobservable effort while section 5 is devoted to the model of public regulation. Section 6 concludes.

2 The model

2.1 Assumptions and notations

Consider an economy populated by a continuum of individuals indexed by θ. Each individual takes a decision q (e.g. production) and receives a transfer t from the principal. The utility of the principal when contracting with an agent is $V(q,t)$. The utility of the type-θ individual is $u(q,t,\theta,\alpha)$ where α is a simple sum of functions $H(q(\theta),t(\theta),\theta)$ of q and t accross the population

$$\alpha = \int_{\Theta} H(q(\theta),t(\theta),\theta)dF(\theta)$$

where F is the distribution function of θ. We normalize the set of types such that $u_\theta < 0$.[3] We also assume that the Spence-Mirrlees property holds:

$$\frac{\partial}{\partial\theta}\left(\frac{u_q}{u_t}\right) < 0.$$

This model can be viewed as an extension of Oswald (1983) to the private principal case. We also slightly generalizes his analysis by considering a general formulation for α. The role of α is precisely to incorporate any externality from an aggregate value of decisions and transfers in the individual utility. For instance, if we denote $\pi(\theta) = t(\theta) - c(q(\theta),\theta)$ the profit get by the individual by taking the decision $q(\theta)$ which costs $c(q(\theta),\theta)$, then one possible specification for H is simply

$$H(q(\theta),t(\theta),\theta) = \omega(\theta)\pi(\theta).$$

where $\omega(.)$ is a weight function. Then if $u_\alpha > 0$ then the individual is altruistic in the sense that an increase in the average profit in the population raises the utility. Conversely, if $u_\alpha < 0$ then there is envy or jealousy as an increase in the average profit yields to decrease utility ceteris paribus. In that case, a more specific model of interest could be written such that $u(q,t,\theta,\alpha) = v(\pi(\theta),\alpha) = \pi(\theta) + \rho(\pi(\theta) - \alpha)$ where $\alpha = \int_\Theta \omega(\theta)\pi(\theta)dF(\theta)$ and $\rho \geq 0$ is a parameter that represents the (common) intensity of envy. If the individual earns more than α then the utility is increased. Conversely, if the individual earns less than α then the utility is decreased. There are many interpretations of α: it could be an exogenous poverty line for instance, or it could be simply the non weighted mean of profits.

[3]Hereafter, we denote f_x the partial derivative of f with respect to x.

2.2 Analysis

The problem of the principal is to choose an allocation $(q(\theta), t(\theta))$ for each individual subject to incentive compatibility and participation constraints.[4] The program of the principal writes as follows

$$\max_{q(.),t(.)} \int_{\Theta} V(q(\theta), t(\theta))dF(\theta)$$

$$\text{s.t.}$$

$$U(\theta) = u(q(\theta), t(\theta), \theta, \alpha) \geq 0$$

$$U(\theta) \geq U(\theta, \tilde{\theta}) \text{ for any } \theta, \tilde{\theta}$$

$$\alpha = \int_{\Theta} H(q(\theta), t(\theta), \theta)dF(\theta)$$

This corresponds to a standard principal-agent model except for the presence of an externality effect due to α. The following proposition establishes the properties of the optimal allocation of decisions. For this, we denote $\lambda(\theta)$ as the multiplier of the incentive compatibility constraint and ϕ as the multiplier of the externality constraint.

Proposition 1 *Assuming a separating equilibrium, the optimal decision for a type-θ individual is given by:*

$$V_q - \frac{u_q}{u_t}V_t = \frac{\lambda(\theta)}{f(\theta)}\left[u_{\theta q} - \frac{u_q}{u_t}u_{\theta t}\right] + \phi\left[H_q - \frac{u_q}{u_t}H_t\right] \quad (1)$$

Proof: See appendix A. ■

At the first best, the optimal decision for a type-θ individual is given by the equality between the marginal rates of substitution between the decision and money for the Principal and the agent, that is respectively $\frac{V_q}{V_t}$ and $\frac{u_q}{u_t}$. In the absence of externality ($\phi = 0$), asymmetric information imposes a distortion given by the first term of the RHS of equation (1). In the presence of the externality, not only the incentive distortion depends on ϕ through the value of $\lambda(\theta)$, but there is also a second term which appears independantly of the presence of

[4] We normalize the reservation utility level to 0 for any type-θ individual.

asymmetric information. Intuitively, the presence of the externality imposes a distortion to the optimal allocation of decision which depends on the specification of H. Indeed, the principal takes into account the marginal impact of the decision allocated to θ on his contribution H to the aggregate externality α. Note that in the presence of asymmetric information, the externality has also an impact on the cost of incentive compatibility.

Corollary 2 *If the marginal rate of substitution between the decision q and money is the same at the utility level and at the externality function level, namely $\frac{H_q}{H_t} = \frac{u_q}{u_t}$ then there is no reason for the principal to distort the allocation rule compared to the situation where the externality is absent.*

When $H_q - H_t \frac{u_q}{u_t} = 0$ then the principal has no incentives to distort the allocation rule (except that the value of $\frac{u_q}{u_t}$ depends itself on α). It suffices that there exists a function $\pi(q, t, \theta)$ such that $u(q, t, \theta, \alpha) = v(\pi, \alpha)$ and $H(q, t, \theta) = \omega(\theta)\tilde{H}(\pi)$ then we obtain that $H_q - H_t \frac{u_q}{u_t} = 0$ and there is no reason to control for the externality. Hence, the principal has to distort the allocation rule only if there is a difference between the marginal rate of substitution between q and t at the individual utility level and the individual contribution H to the externality level.

At the top, there is no incentive distortion as $\lambda(\underline{\theta}) = 0$, but obviously the correction due to the presence of externality subsists whenever $H_q - H_t \frac{u_q}{u_t} \neq 0$ and $\omega \neq 0$.

2.3 The optimal uniform allocation

Suppose that the principal restricts herself to the choice of an unique allocation whatever the type of individual. This may happen if the good under scrutiny or the environmental services are transferable between individuals. Alternatively, we obtain such a situation if for some institutional reasons the principal is forbidden to price discriminate between agents.

The program of the principal now reduces to

$$\max_{q,t} S(q) - t$$

s.t.

$$U(\theta) = u(q, t, \theta, \alpha) \geq 0$$

$$\alpha = \int_{\Theta} \omega(\theta) H(q, t, \theta) dF(\theta)$$

Computing the rate of growth of rents, we have that $U'(\theta) = u_\theta < 0$, so that the individual rationality constraints reduce to $U(\overline{\theta}) \geq 0$. The Lagrangean is thus given by

$$\mathcal{L} = S(q) - t + \tau u(q, t, \overline{\theta}, \alpha) + \phi(\alpha - \int_{\Theta} \omega(\theta) H(q, t, \theta) dF(\theta))$$

The necessary conditions are

$$\frac{\partial \mathcal{L}}{\partial q} = S' + \tau \overline{u}_q - \phi \int_{\Theta} \omega H_q dF(\theta) = 0 \quad (2)$$

$$\frac{\partial \mathcal{L}}{\partial t} = -1 + \tau \overline{u}_t - \phi \int_{\Theta} \omega H_t dF(\theta) = 0 \quad (3)$$

$$\frac{\partial \mathcal{L}}{\partial \alpha} = \tau \overline{u}_\alpha + \phi = 0 \quad (4)$$

Hence, we have $\tau = -\phi / \overline{u}_\alpha$. Hence, either we have $\overline{u}_\alpha < 0$ and $\phi > 0$ or $\overline{u}_\alpha > 0$ and $\phi < 0$. Eliminating ϕ, we obtain that

$$S'(q) = -\frac{\overline{u}_q + \overline{u}_\alpha \int_{\Theta} \omega H_q dF(\theta)}{\overline{u}_t + \overline{u}_\alpha \int_{\Theta} \omega H_t dF(\theta)} \quad (5)$$

This equation together with $u(q, t, \overline{\theta}, \alpha) = 0$ gives us the optimal allocation (q, t).Note that in the absence of externality ($u_\alpha = 0$), we would obtain the optimal decision as

$$S'(q) = -\frac{\overline{u}_q}{\overline{u}_t}$$

that is by equalizing the marginal rate of substitution between q and t for the highest type which is the marginal type from the principal's point of view. In the presence of the externality, the marginal individual utility of q should be corrected for its impact on the aggregate externality α. And the same for t.

In the special case where $H = \pi = t - c(q,\theta)$ and where $u(q,t,\theta,\alpha) = v(\pi,\alpha)$, then we obtain

$$S'(q) = \frac{\overline{v}_\pi c_q(q,\overline{\theta}) + \overline{v}_\alpha \int_\Theta \omega c_q dF(\theta)}{\overline{v}_\pi + \overline{v}_\alpha \int_\Theta \omega dF(\theta)}$$

If furthermore $\omega = 1$ and $v(\pi,\alpha) = (1+\rho)\pi - \rho\alpha$ then we get

$$S'(q) = c_q(q,\overline{\theta}) + \rho \left[c_q(q,\overline{\theta}) - \int_\Theta c_q(q,\theta)dF(\theta) \right]$$

As $c_{\theta q} > 0$, then the second term of the RHS is positive and there is a downward distortion to production due to the impact of ρ. At the first best, we would have $S'(q) = c_q(q,\overline{\theta})$, that is the optimal production level is the one which is optimal for the least efficient agent. The downward distortion comes from the fact that the participation constraint imposes that $u(q,t,\overline{\theta},\alpha) = (1+\rho)\overline{\pi} - \rho\alpha = 0$. Hence, the minimum level of profit devoted to the least efficient agent is equal to $\rho\alpha/(1+\rho) > 0$. Participation is hence more and more costly as ρ increases and this calls for an increasing downward distortion in order to decrease the rents left to agents. Here, the presence of ρ induces a lower production and thereby hurts welfare.

3 A special case with discriminating contracts

Back to the case of second best price discrimination, we adopt in this section the following specification.

Definition 3 (Specification) **(i)** *The utility $u(q,t,\theta,\alpha)$ of the agent is a function of his monetary payoff $\pi(q,t,\theta) = t - c(q,\theta)$ where c is the cost of producing q and θ an index of productivity. We assume that $c_\theta > 0$ and $c_{\theta q} > 0$ (Spence-Mirrlees property). We have*

$$u(q,t,\theta,\alpha) = v(\pi(q,t,\theta),\alpha)$$

where $v_\pi > 0$ and $v_{\pi\pi} \leq 0$.

(ii) *The aggregate externality α is defined as a weighted sum of monetary payoffs so that*

$H(q,t,\theta) = \pi(q,t,\theta)$:

$$\alpha = \int_{\Theta} \omega(\theta)\pi(\theta)dF(\theta).$$

This specification has the particularity that there is no reason for the principal to distort production allocations under perfect information even if the agent's utility depends on the aggregate externality α as shown by Corollary 2. Indeed, we have

$$H_q - H_t \frac{u_q}{u_t} = -c_q - \frac{v_\pi(-c_q)}{v_\pi} = 0.$$

Hence, distorting production allocations across types compared to first-best only becomes optimal under imperfect information.

3.1 Analysis

Using this specification and the results contained in Proposition 1, we obtain that:

$$S'(q(\theta)) = c_q(q(\theta),\theta) + \frac{\lambda(\theta)}{f(\theta)} \left[v_\pi \left(-c_{\theta q}\right)\right] \tag{6}$$

with

$$\lambda(\theta) = \frac{\int_{\underline{\theta}}^{\theta}(1+\phi\omega(x))dF(x)}{v_\pi}$$

and

$$\phi = -\frac{v_\alpha(\pi(\overline{\theta}),\alpha)}{v_\pi(\pi(\overline{\theta}),\alpha) + v_\alpha(\pi(\overline{\theta}),\alpha)\int_{\underline{\theta}}^{\overline{\theta}} \omega(\theta)dF(\theta)}$$

where $\pi(\overline{\theta}) = \pi(q(\overline{\theta}), t(\overline{\theta}), \alpha)$ is such that $v(\pi(\overline{\theta}),\alpha) = 0$.

Replacing in (6), we have

$$S'(q(\theta)) = c_q(q(\theta),\theta) + \frac{F(\theta) + \phi\int_{\underline{\theta}}^{\theta}\omega(x)dF(x)}{f(\theta)} c_{\theta q}(q(\theta),\theta) \tag{7}$$

which shows that there is a distortion of production for any type except for the most efficient one ($\underline{\theta}$).

By contrast, note that the standard model of procurement is obtained when $v_\alpha = 0$ so that $\phi = 0$. Indeed, we get the familiar condition:

$$S'(q^s(\theta)) = c_q(q^s(\theta),\theta) + \frac{F(\theta)}{f(\theta)} c_{\theta q}(\theta, q^s(\theta)). \tag{8}$$

The sign of the distortion in (7) depends on the sign of ϕ and on the weight function $\omega(.)$. Let us assume that $\omega(\theta) \geq 0$ for any type. Suppose further that $v_\alpha > 0$ for the least efficient agent, then ϕ is clearly negative. This implies that if the least efficient agent is altruistic then the downward distortion due to the efficiency-rent extraction trade-off is reduced compared to the standard model and may even turn to an upward distortion for some types. The intuition goes as follows: it is less necessary to extract rents as giving rents will increase α and thereby will relax the participation constraint of the least efficient type. In the case where $\omega(\theta) = 1$ for any type, then $0 < \phi < 1$ and consequently the principal imposes a reduced downward distortion everywhere on production except at the top.

Suppose on the contrary that $u_\alpha < 0$ for the least efficient type, then ϕ is positive if and only if $v_\pi(\pi(\overline{\theta}), \alpha) > -v_\alpha(\pi(\overline{\theta}), \alpha) \int_{\underline{\theta}}^{\overline{\theta}} \omega(\theta) dF(\theta)$ which means that the direct effect of profit on utility must outweigh the impact of α sufficiently. In that case, the principal imposes an augmented downward distortion to production except at the top. The intuition goes as follows: here leaving informational rents to the agents will have the additional effect of tightening the participation constraint of the least efficient type because this agent is jealous. It is therefore necessary to extract rents more than in the absence of jealousy, by decreasing production. In a sense, jealousy amounts to a more discriminating production schedule, although one can show that the gap between the highest and the lowest profits decreases (less inequality in terms of monetary payoffs).

Suppose that the "naive" principal neglects the preferences of the agent with respect to α. In that case, it is optimal to offer the standard production schedule defined in (8). However, the least efficient agent will get $\pi^s(\overline{\theta}) = 0$ such that his utility is $v(0, \alpha^s)$ where $\alpha^s = \int_{\underline{\theta}}^{\overline{\theta}} \omega(\theta) \pi^s(\theta) dF(\theta)$. This utility level might be negative so that he is not willing to participate and this is also true for a subset of the less efficient types. There is thus potentially under-participation to the contract.

3.2 Optimal shutdown

In this section, we analyze the optimal shutdown policy for the principal, that is the identity of the marginal individual who is indifferent between partipating and not as part of the optimal policy. If the principal is free to choose who should enter into the mechanism, her expected utility is

$$\max_{\theta^*} W = \int_{\underline{\theta}}^{\theta^*} \{S(q(\theta)) - \pi(\theta) - c(q(\theta), \theta)\} \, dF(\theta)$$

where $\pi(\theta)$ and $q(\theta)$ depend on θ^*. Hence, the first-order condition is

$$\frac{dW}{d\theta^*} = [S(q(\theta^*)) - \pi(\theta^*) - c(q(\theta^*), \theta^*)] \, f(\theta^*) + \int_{\underline{\theta}}^{\theta^*} \left\{ [S'(q(\theta)) - c_q(q(\theta), \theta)] \, \frac{dq(\theta)}{d\theta^*} - \frac{d\pi(\theta)}{d\theta^*} \right\} dF(\theta)$$

This is difficult to evaluate because both production and profit depend on θ^* in a complex way. In particular, $q(\theta)$ depends on θ^* through ϕ when $\omega(\theta)$ is not constant. Moreover

$$\pi(\theta) = \pi(\theta^*) + \int_{\theta}^{\theta^*} c_\theta(q(x), x) dx$$

and

$$\begin{aligned} \frac{d\pi(\theta)}{d\theta^*} &= \dot{\pi}(\theta^*) + c_\theta(q(\theta^*), \theta^*) + \int_{\theta}^{\theta^*} c_{\theta q}(q(x), x) \frac{dq(x)}{d\theta^*} dx \\ &= \int_{\theta}^{\theta^*} c_{\theta q}(q(x), x) \frac{dq(x)}{d\theta^*} dx \end{aligned}$$

The rule becomes

$$\begin{aligned} \frac{dW}{d\theta^*} &= [S(q(\theta^*)) - \pi(\theta^*) - c(q(\theta^*), \theta^*)] \, f(\theta^*) \\ &\quad + \int_{\underline{\theta}}^{\theta^*} \left\{ [S'(q(\theta)) - c_q(q(\theta), \theta)] \, \frac{dq(\theta)}{d\theta^*} - \int_{\theta}^{\theta^*} c_{\theta q}(q(x), x) \frac{dq(x)}{d\theta^*} dx \right\} dF(\theta) \end{aligned}$$

Note that

$$\begin{aligned} \int_{\underline{\theta}}^{\theta^*} \int_{\theta}^{\theta^*} c_{\theta q}(q(x), x) \frac{dq(x)}{d\theta^*} dx dF(\theta) &= \left[F(\theta) \int_{\theta}^{\theta^*} c_{\theta q}(q(x), x) \frac{dq(x)}{d\theta^*} dx \right]_{\underline{\theta}}^{\theta^*} \\ &\quad + \int_{\underline{\theta}}^{\theta^*} c_{\theta q}(q(\theta), \theta) \frac{dq(\theta)}{d\theta^*} F(\theta) d\theta \\ &= \int_{\underline{\theta}}^{\theta^*} c_{\theta q}(q(\theta), \theta) \frac{dq(\theta)}{d\theta^*} F(\theta) d\theta \end{aligned}$$

Hence, the rule becomes

$$\begin{aligned}\frac{dW}{d\theta^*} &= \left[S(q(\theta^*)) - \pi(\theta^*) - c(q(\theta^*),\theta^*)\right] f(\theta^*) \\ &\quad + \int_{\underline{\theta}}^{\theta^*} \left\{ \left[S'(q(\theta)) - c_q(q(\theta),\theta)\right] \frac{dq(\theta)}{d\theta^*} - c_{\theta q}(q(\theta),\theta) \frac{dq(\theta)}{d\theta^*} \frac{F(\theta)}{f(\theta)} \right\} dF(\theta) \\ \frac{dW}{d\theta^*} &= \left[S(q(\theta^*)) - \pi(\theta^*) - c(q(\theta^*),\theta^*)\right] f(\theta^*) + \int_{\underline{\theta}}^{\theta^*} \left\{ \frac{\phi \int_{\underline{\theta}}^{\theta} \omega(x) dF(x)}{f(\theta)} c_{\theta q}(q(\theta),\theta) \frac{dq(\theta)}{d\theta^*} \right\} dF(\theta)\end{aligned}$$

Note that the value of ϕ is

$$\phi = -\frac{v_\alpha(\pi(\theta^*),\alpha)}{v_\pi(\pi(\theta^*),\alpha) + v_\alpha(\pi(\theta^*),\alpha) \int_{\underline{\theta}}^{\theta^*} \omega(\theta) dF(\theta)}$$

Here, implicitely, we assume that the agent values only the profits get by participating agents, i.e. $\alpha = \int_{\underline{\theta}}^{\theta^*} \omega(\theta)\pi(\theta) dF(\theta)$.[5] Recall that $v(\pi(\theta^*),\alpha) = 0$ which implies that $\pi(\theta^*) = h(\alpha)$. Hence,

$$\phi = -\frac{v_\alpha(h(\alpha),\alpha)}{v_\pi(h(\alpha),\alpha) + v_\alpha(h(\alpha),\alpha) \int_{\underline{\theta}}^{\theta^*} \omega(\theta) dF(\theta)} = \frac{h'(\alpha)}{1 - h'(\alpha) \int_{\underline{\theta}}^{\theta^*} \omega(\theta) dF(\theta)}$$

$$\begin{aligned}\frac{d\phi}{d\theta^*} &= \frac{\partial \phi}{\partial \alpha} \frac{d\alpha}{d\theta^*} + \frac{\partial \phi}{\partial \theta^*} \\ &= \frac{h''(\alpha)}{\left[1 - h'(\alpha) \int_{\underline{\theta}}^{\theta^*} \omega(\theta) dF(\theta)\right]^2} \frac{d\alpha}{d\theta^*} + \frac{\left[h'(\alpha)\right]^2 \omega(\theta^*) f(\theta^*)}{\left[1 - h'(\alpha) \int_{\underline{\theta}}^{\theta^*} \omega(\theta) dF(\theta)\right]^2}\end{aligned}$$

and hence

$$sign(\frac{d\phi}{d\theta^*}) = sign(h''(\alpha)\pi(\theta^*) + \left[h'(\alpha)\right]^2)$$

But we have from the definition of h',

$$h'' = -\frac{1}{v_\pi}\left[v_{\pi\pi} h'^2 + 2 v_{\pi\alpha} h' + v_{\alpha\alpha}\right]$$

The sign of $\frac{d\phi}{d\theta^*}$ explains the sign of $\frac{dq(\theta)}{d\theta^*}$. Suppose that $\phi > 0$ and suppose that $\frac{d\phi}{d\theta^*} > 0$ then $\frac{dq(\theta)}{d\theta^*} < 0$. Then the principal prefers to contract with only a subset of potentially "profitable" agents (that is where his utility $(S - \pi - c)$ is strictly positive) : there is under-participation

[5] Otherwise, the analysis should be conducted with $\alpha = \int_{\underline{\theta}}^{\theta^*} \omega(\theta)\pi(\theta) dF(\theta) + k \int_{\theta^*}^{\overline{\theta}} \omega(\theta)\pi(\theta^*) dF(\theta)$. Parameter k with value 1 represents the situation where the agent includes the non participating agents when computing α. When parameter k is zero, then the agent computes α only on the subset of participating agents.

because expanding the set of participating agents amounts to increase downward distortion on production levels.

On the contrary, if $\phi > 0$ but suppose that $\frac{d\phi}{d\theta^*} < 0$ then $\frac{dq(\theta)}{d\theta^*} > 0$ then there is over-participation. Here, expanding the set of contracting agents allows to reduce the intensity of distortion.

3.3 The impact of envy

To pursue further the analysis, let us suppose that $v = \pi + \rho(\pi - \alpha)$. There is thus jealousy or envy whenever the agent earns less than α. Then, $\pi(\theta^*) = \frac{\rho}{1+\rho}\alpha$ and

$$\phi = \frac{\rho}{1 + \rho - \rho \int_{\underline{\theta}}^{\theta^*} \omega(\theta) dF(\theta)}$$

Note that ϕ is positive if and only if $\int_{\underline{\theta}}^{\theta^*} \omega(\theta) dF(\theta) < \frac{1+\rho}{\rho}$. When $\omega(\theta) > 0$, we also get

$$\frac{d\phi}{d\theta^*} = \frac{\rho^2 \omega(\theta^*) f(\theta^*)}{\left[1 + \rho - \rho \int_{\underline{\theta}}^{\theta^*} \omega(\theta) dF(\theta)\right]^2} > 0$$

which means that for a given θ the downward distortion is higher when the set of contracting types increases. Intuitively, the participation constraint is more and more stringent when θ^* increases. This means in turn that $\frac{dq(\theta)}{d\theta^*} < 0$ (and consequently $\frac{d\pi(\theta)}{d\theta^*} < 0$). In that case, there is under-participation. The principal would prefer to restrict participation to the contract more compared to a standard model without envy.

In the particular case where $\omega(\theta) = 1$ for any θ then

$$\phi = \frac{\rho}{1 + \rho - \rho F(\theta^*)} < \rho$$

and

$$S'(q(\theta)) = c_q(q(\theta), \theta) + \frac{F(\theta)(1 + \phi)}{f(\theta)} c_{\theta q}(q(\theta), \theta)$$

which implies that the distortion is maximal when $\theta^* = \overline{\theta}$. Given the second order condition on $q(.)$, it follows that $\frac{dq}{d\phi} < 0$ and consequently $\frac{dq(\theta)}{d\theta^*} < 0$.

4 Does status-seeking behavior yield to more effort?

We consider an extension of the previous model where the agent exerts some effort e that allows to reduce the cost of providing the quantity q. We will consider for simplicity the following popular specification

$$c(q, e, \theta) = (\theta - e)q$$

used extensively by Laffont and Tirole (1991).

Exerting effort is costly and we denote $\psi(e)$ the disutility of effort which is increasing and convex ($\psi' > 0$ and $\psi'' > 0$) with $\psi(0) = 0$. We also assume that the utility of the type-θ agent is given by

$$\begin{aligned} U &= v(\pi, \alpha) - \psi(e) \\ &= \pi + \rho(\pi - \alpha) - \psi(e) \end{aligned}$$

where $\pi = t - c(q, e, \theta)$ and $\alpha = \int_{\Theta} \omega(\theta)\pi(\theta)dF(\theta)$. Facing the allocation (t, q) the agent chooses his effort such that

$$\begin{aligned} \max_e U &= v(\pi, \alpha) - \psi(e) \\ &= (1 + \rho)(t - (\theta - e)q) - \rho\alpha - \psi(e) \end{aligned}$$

with the corresponding first-order condition

$$(1 + \rho)q = \psi'(e) \tag{9}$$

The impact of envy is such that the marginal benefit of effort is higher ($\rho > 0$) for a given production level. Hence, ceteris paribus, envy leads to more effort. However, the production level depends itself on ρ and can be found by solving the principal's problem.

The program of the principal writes

$$\max_{q(.),t(.)} \int_{\Theta} [S(q(\theta)) - t(\theta)]\, dF(\theta)$$

s.t.

$$U(\theta) = (1+\rho)\,(t(\theta) - (\theta - e(\theta))q(\theta)) - \rho\alpha - \psi(e(\theta)) \geq 0$$

$$U(\theta) \geq U(\theta, \tilde{\theta}) \text{ for any } \theta, \tilde{\theta}$$

$$(1+\rho)q(\theta) = \psi'(e(\theta))$$

$$\alpha = \int_{\Theta} \omega(\theta)\pi(\theta)dF(\theta)$$

Solving this program allows to obtain the following proposition.

Proposition 4 *Assuming a separating equilibrium, the optimal allocation is such that*

$$S'(q(\theta)) = \theta - e(\theta) + \frac{1}{f(\theta)} \int_{\underline{\theta}}^{\theta} (1 + \phi\omega(x))\, dx + \phi\omega(\theta)\frac{\psi'(e(\theta))}{\psi''(e(\theta))}$$

with $\phi = \frac{\rho}{1+\rho}\left(1 + \int_{\Theta} \omega(\theta) f(\theta) d\theta\right) \geq 0.$

Proof: See appendix B. ■

In the absence of envy ($\phi = 0$), we obtain the standard equation stipulating the downward distortion on production as a result of the efficiency-rent extraction trade-of:

$$S'(q(\theta)) = \theta - e(\theta) + \frac{F(\theta)}{f(\theta)}.$$

Here, not only the presence of envy yields to distort even more the production level as

$$\frac{1}{f(\theta)} \int_{\underline{\theta}}^{\theta} (1 + \phi\omega(x))\, dx > \frac{F(\theta)}{f(\theta)}$$

but there is also an additional therm that tends increase the cost of production, namely $\phi\omega(\theta)\frac{\psi'(e(\theta))}{\psi''(e(\theta))}$. This term is due to the impact of production on the effort chosen privately by the agent which in turn affects the extent of the externality α. Intuitively, the presence of envy gives more incentives to exert some effort for a given production level but at the same time it also contributes positively to the size of the negative externality. The principal hence designs a contract that reduces the production level in order to internalize the impact of envy over effort.

5 Does status-seeking behavior yields to more fraud? (incomplete)

In this section we investigate whether status-seeking behavior yields to more fraud. For this, we assume that the decision variable q can only be observed at a cost by the Principal. This implies that in general the Principal would want to observe q at random. Facing a contract $\{q(\theta), t(\theta), \mu(\theta), f(\theta, q)\}$ where $\mu(\theta)$ denotes the probability of inspection and $f(\theta, q) \geq 0$ whenever $q \neq q(\theta)$ denotes the penalty to be paid in case of fraud, that is when, given the audit report, the actual decision q does not correspond to the one that should have been taken given the type announced θ. In case where the observed decision q is $q(\theta)$ then the penalty is $f(\theta, q(\theta)) = 0$ (no additional payment in case of non frauding behavior). Then the expected utility can be written as follows:

$$U(\theta, \tilde{\theta}, q, \alpha) = (1 - \mu(\tilde{\theta}))u(q, t(\tilde{\theta}), \theta, \alpha) + \mu(\tilde{\theta})u(q, T(\tilde{\theta}, q), \theta, \alpha)$$

where $T(\tilde{\theta}, q) = t(\tilde{\theta}) - \mu(\tilde{\theta})f(\tilde{\theta}, q)$ is the expected payment in case of proven frauding behavior.

The incentives constraints write as follows:

$$R(\theta) \equiv U(\theta, \theta, q(\theta), \alpha) \geq U(\theta, \tilde{\theta}, q(\tilde{\theta}), \alpha) \quad \text{(IC1)}$$

$$R(\theta) \geq U^\circ(\theta) \equiv \max_{\tilde{\theta}, q} U(\theta, \tilde{\theta}, q, \alpha) \quad \text{(IC2)}$$

and the individual rationality constraints are:

$$R(\theta) \geq 0 \quad \text{(IR)}$$

The program of the Principal is:

$$\max_{q(.), t(.), f(.,.), \mu(.), R(.)} \int_\Theta \{S(q(\theta)) - t(\theta) - k\mu(\theta)\} dF(\theta)$$
$$\text{s.t. (IC1), (IC2), (IR)}, \ f(\theta, q) \leq \bar{f}, \ 0 \leq \mu(\theta) \leq 1$$

where k is the unit cost of inspection and

It is not easy to see where (IC2) is binding given the generality of the model. Note also that from (IC1), we get $\dot{R}(\theta) = u_\theta(q(\theta), t(\theta), \theta, \alpha) < 0$.

To pursue the analysis, we use the specification: $u = \pi + \rho(\pi - \alpha)$ with $\alpha = \int_\Theta \omega(\theta)\pi(\theta)dF(\theta)$. Then, we get:

$$\begin{aligned} U(\theta,\tilde{\theta},q,\alpha) &= (1-\mu(\tilde{\theta}))\left[(1+\rho)\left(t(\tilde{\theta})-c(q,\theta)\right)-\rho\alpha\right]+\mu(\tilde{\theta})\left[(1+\rho)\left(T(\tilde{\theta},q)-c(q,\theta)\right)-\rho\alpha\right] \\ &= (1+\rho)\left[(1-\mu(\tilde{\theta}))t(\tilde{\theta})+\mu(\tilde{\theta})T(\tilde{\theta},q)\right]-(1+\rho)c(q,\theta)-\rho\alpha \\ &= (1+\rho)\left[t(\tilde{\theta})-\mu(\tilde{\theta})f(\tilde{\theta},q)\right]-(1+\rho)c(q,\theta)-\rho\alpha \end{aligned}$$

and

$$R(\theta) \equiv (1+\rho)\left(t(\theta)-c(q(\theta),\theta)\right)-\rho\alpha \geq (1+\rho)\left(t(\tilde{\theta})-c(q(\tilde{\theta}),\theta)\right)-\rho\alpha \qquad \text{(IC1)}$$

$$R(\theta) \geq U^\circ(\theta) \equiv \max_{\tilde{\theta},q} U(\theta,\tilde{\theta},q,\alpha) = (1+\rho)K-(1+\rho)c(q^\circ(\theta),\theta)-\rho\alpha \qquad \text{(IC2)}$$

where $K = \max_{\tilde{\theta}} t(\tilde{\theta}) - \mu(\tilde{\theta})\bar{f}$ and $q^\circ(\theta) = \arg\min_q c(q,\theta)$. Thus the incentives constraints reduce to

$$t(\theta)-c(q(\theta),\theta) \geq t(\tilde{\theta})-c(q(\tilde{\theta}),\theta) \qquad \text{(IC1)}$$

$$t(\theta)-c(q(\theta),\theta) \geq K-c(q^\circ(\theta),\theta) \qquad \text{(IC2)}$$

Note that as $\dot{R}(\theta) = -(1+\rho)c_\theta(q(\theta),\theta) < 0$ then (IR) reduces to $R(\overline{\theta}) \geq 0$. Also note that

$$\frac{d}{d\theta}\left[R(\theta)-U^\circ(\theta)\right] = (1+\rho)(-c_\theta(q(\theta),\theta)+c_\theta(q^\circ(\theta),\theta)) = (1+\rho)\int_{q(\theta)}^{q^\circ(\theta)} c_{\theta q}(x,\theta)dx \leq 0$$

as $c_{\theta q} > 0$ and $q^\circ(\theta) \leq q(\theta)$. This implies that (IC2) reduces to[6]

$$R(\overline{\theta}) \geq (1+\rho)\left(K-c(q^\circ(\overline{\theta}),\overline{\theta})\right)-\rho\alpha.$$

Moreover, the definition of K implies that for any θ,

$$K \geq t(\theta)-\mu(\theta)\bar{f}$$

[6] If on the contrary we make the opposite assumption w.r.t the single crossing condition, i.e. $c_{\theta q} < 0$, then (IC2) would reduce to $R(\underline{\theta}) \geq (1+\rho)\left(K-c(q^\circ(\underline{\theta}),\underline{\theta})\right)-\rho\alpha$.

which in turn implies that

$$\begin{aligned}\mu(\theta) &\geq \frac{t(\theta)-K}{\bar{f}} \\ &\geq \frac{1}{\bar{f}}\left[\frac{R(\theta)+\rho\alpha}{1+\rho}+c(q(\theta),\theta)-K\right]\end{aligned}$$

The program of the Principal then rewrites as follows, assuming that $q(\theta) \geq q^{\circ}(\theta)$ for any θ and neglecting the monotonicity condition on the decision $q(.)$:

$$\max_{q(.),\mu(.),R(.),K} \int_{\Theta} \{S(q(\theta)) - \frac{R(\theta)+\rho\alpha}{1+\rho} - c(q(\theta),\theta) - k\mu(\theta)\}dF(\theta)$$

s.t.

$$\dot{R}(\theta) = -(1+\rho)c_{\theta}(q(\theta),\theta)$$

$$R(\overline{\theta}) \geq (1+\rho)\left(K - c(q^{\circ}(\overline{\theta}),\overline{\theta})\right) - \rho\alpha$$

$$R(\overline{\theta}) \geq 0$$

$$\mu(\theta) \geq \frac{1}{\bar{f}}\left[\frac{R(\theta)+\rho\alpha}{1+\rho}+c(q(\theta),\theta)-K\right]$$

$$0 \leq \mu(\theta) \leq 1$$

$$\alpha = \int_{\Theta} \omega(\theta)R(\theta)dF(\theta)$$

where the last constraint comes from the definition of α together with $\pi(\theta) = \frac{R(\theta)+\rho\alpha}{1+\rho}$.

Note that the constraint on μ should be binding as μ is costly from the Principal's viewpoint. Replacing in the objective gives

$$\begin{aligned}&\int_{\Theta} \{S(q(\theta)) - \frac{R(\theta)+\rho\alpha}{1+\rho} - c(q(\theta),\theta) - k\mu(\theta)\}dF(\theta) \\ = &\int_{\Theta} \{S(q(\theta)) - \left(1+\frac{k}{\bar{f}}\right)\left(\frac{R(\theta)+\rho\alpha}{1+\rho}+c(q(\theta),\theta)\right) + \frac{k}{\bar{f}}K\}dF(\theta)\end{aligned}$$

Also, as K is valued positively by the Principal, the constraint on $R(\overline{\theta})$ should be binding and hence

$$K = \frac{R(\overline{\theta})+\rho\alpha}{1+\rho} + c(q^{\circ}(\overline{\theta}),\overline{\theta})$$

Furthermore, as leaving rents is costly too, the participation constraint on $R(\overline{\theta})$ should be binding:

$$R(\overline{\theta}) = 0.$$

Hence the program reduces to (neglecting the constraint on μ, to be checked later)

$$\max_{q(.),R(.)} \int_\Theta \{S(q(\theta)) - \left(1 + \frac{k}{\overline{f}}\right) \left(\frac{R(\theta) + \rho\alpha}{1+\rho} + c(q(\theta), \theta)\right) + \frac{k}{\overline{f}} K\} dF(\theta)$$

s.t.

$$\dot{R}(\theta) = -(1+\rho) c_\theta(q(\theta), \theta)$$

$$R(\overline{\theta}) = 0$$

$$K = \frac{\rho\alpha}{1+\rho} + c(q^\circ(\overline{\theta}), \overline{\theta})$$

$$\alpha = \int_\Theta \omega(\theta) R(\theta) dF(\theta)$$

The Lagrangean writes

$$\begin{aligned}\mathcal{L} &= \int_\Theta \{S(q(\theta)) - \left(1 + \frac{k}{\overline{f}}\right)\left(\frac{R(\theta)+\rho\alpha}{1+\rho} + c(q(\theta),\theta)\right) + \frac{k}{\overline{f}}\left(\frac{\rho\alpha}{1+\rho} + c(q^\circ(\overline{\theta}), \overline{\theta})\right)\} dF(\theta) \\ &\quad + \int_\Theta \phi(\alpha - \omega(\theta) R(\theta)) dF(\theta) + \int_\Theta \lambda(\theta)\left(\dot{R}(\theta) + (1+\rho) c_\theta(q(\theta),\theta)\right) d\theta\end{aligned}$$

Integrating by parts the last term, we get

$$\begin{aligned}\mathcal{L} &= \int_\Theta \{S(q(\theta)) - \left(1 + \frac{k}{\overline{f}}\right)\left(\frac{R(\theta)+\rho\alpha}{1+\rho} + c(q(\theta),\theta)\right) + \frac{k}{\overline{f}}\left(\frac{\rho\alpha}{1+\rho} + c(q^\circ(\overline{\theta}), \overline{\theta})\right)\} dF(\theta) \\ &\quad + \int_\Theta \phi(\alpha - \omega(\theta) R(\theta)) dF(\theta) + [\lambda(\theta) R(\theta)]_{\underline{\theta}}^{\overline{\theta}} - \int_\Theta \dot{\lambda}(\theta) R(\theta) d\theta + \int_\Theta \lambda(\theta)(1+\rho) c_\theta(q(\theta),\theta) d\theta\end{aligned}$$

Deriving the Lagrangean and dropping arguments for clarity, we obtain the following necessary conditions:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial q} &= S' f - \left(1 + \frac{k}{\overline{f}}\right) c_q f + \lambda(\theta)(1+\rho) c_{\theta q} = 0 \\ \frac{\partial \mathcal{L}}{\partial R} &= -\left(1 + \frac{k}{\overline{f}}\right) \frac{f}{1+\rho} - \phi\omega(\theta) f - \dot{\lambda}(\theta) = 0 \\ \frac{\partial \mathcal{L}}{\partial \alpha} &= \int_\Theta \left[-\left(1 + \frac{k}{\overline{f}}\right)\frac{\rho}{1+\rho} + \frac{k}{\overline{f}}\frac{\rho}{1+\rho} + \phi\right] dF(\theta) = 0\end{aligned}$$

together with $\lambda(\underline{\theta}) = 0$ as $R(\underline{\theta})$ is free.

We deduce that $\phi = \frac{\rho}{1+\rho}$ and consequently that

$$-\left(1 + \frac{k}{\overline{f}}\right)\frac{f}{1+\rho} - \frac{\rho}{1+\rho}\omega(\theta) f = \dot{\lambda}(\theta)$$

which yields to

$$\lambda(\theta) = -\frac{1}{1+\rho}\int_{\underline{\theta}}^{\theta}\left(1+\frac{k}{\bar{f}}+\rho\omega(x)\right)dF(x)$$

Replacing in the first order condition for q, we have

$$S'(q(\theta)) = \left(1+\frac{k}{\bar{f}}\right)c_q(q(\theta),\theta) + \frac{c_{\theta q}(q(\theta),\theta)}{f(\theta)}\int_{\underline{\theta}}^{\theta}\left(1+\frac{k}{\bar{f}}+\rho\omega(x)\right)dF(x)$$

Furthermore, the probability of inspection is given by

$$\mu(\theta) = \frac{1}{\bar{f}}\left[\frac{R(\theta)}{1+\rho} + c(q(\theta),\theta) - c(q^{\circ}(\overline{\theta}),\overline{\theta})\right] > 0$$

as long as $q(\theta) > q^{\circ}(\overline{\theta})$ and it does not depend directly on α.

We are in a position to show that when ρ increases then the probability of inspection μ decreases. Indeed, the direct effect of raisng ρ yields to decrease μ, but in addition both R and q are decreasing in ρ.

[To be completed]

6 Conclusion

We have explored in this note the consequences of status seeking preferences among agents contracting with a principal in the context of production. We have examined in particular the case of envy and we have shown that in general envy entails augmented distortions due to asymmetric information in optimal contracts. Furthermore if the principal neglects the preferences of the agents with respect to status, then potentially there is under-participation to the contract. We also showed that if the principal is free to choose who can participate to the contract, then under some conditions the principal may prefer to contract with only a subset of potentially "profitable" agents (that is where his utility is strictly positive). We then asked whether contracting with agents seeking status would yield to more incentives to exert unobservable effort. We actually show that the principal has incentives to discourage effort. In the last part of the paper, we considered the case of a public principal that seeks to reduce negative externalities from production under a budget constraint.

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Appendix

A Proof of proposition 1

As usual, incentive compatibility constraints reduce to the following first-order condition

$$u_q q' + u_t t' = 0$$

together with the second-order condition $q' \leq 0$. Note also that $U'(\theta) = u_\theta < 0$ so that the participation constraints reduce to $U(\overline{\theta}) \geq 0$. Then the Lagrangean writes as follows:

$$\mathcal{L} = \int_\Theta V(q(\theta), t(\theta))dF(\theta) + \int_\Theta \phi(\alpha - H(q(\theta), t(\theta), \theta))dF(\theta) + \int_\Theta \lambda(\theta)(u_q q' + u_t t')d\theta$$

Integrating by parts the last term, we obtain:

$$\begin{aligned} \mathcal{L} &= \int_\Theta V(q(\theta), t(\theta))dF(\theta) + \int_\Theta \phi(\alpha - H(q(\theta), t(\theta), \theta))dF(\theta) \\ &\quad + [\lambda(\theta)u(q(\theta), t(\theta), \theta, \alpha)]_{\underline{\theta}}^{\overline{\theta}} - \int_\Theta \left[\lambda'(\theta)u(q(\theta), t(\theta), \theta, \alpha) + \lambda(\theta)u_\theta(q(\theta), t(\theta), \theta, \alpha)\right] d\theta \end{aligned}$$

Deriving the Lagrangean and dropping arguments for clarity, we obtain the following necessary conditions:

$$\frac{\partial \mathcal{L}}{\partial q} = V_q f - \phi H_q f - \lambda' u_q - \lambda u_{\theta q} = 0 \quad (10)$$

$$\frac{\partial \mathcal{L}}{\partial t} = V_t f - \phi H_t f - \lambda' u_t - \lambda u_{\theta t} = 0 \quad (11)$$

$$\frac{\partial \mathcal{L}}{\partial \alpha} = \int_\Theta \left[\phi f - \lambda' u_\alpha - \lambda u_{\theta\alpha}\right] d\theta = 0 \quad (12)$$

together with $\lambda(\underline{\theta}) = 0$ as $U(\underline{\theta})$ is free and $\lambda(\overline{\theta})U(\overline{\theta}) = 0$ with $\lambda(\overline{\theta}) \geq 0$, $U(\overline{\theta}) \geq 0$.

From equation (10), we get

$$\lambda' = \frac{1}{u_q}\left[V_q f - \phi H_q f - \lambda u_{\theta q}\right]$$

which is a linear differential equation in λ. The solution is given by

$$\begin{aligned} \lambda(\theta) &= \lambda(\underline{\theta})\left(\exp \int_{\underline{\theta}}^{\theta} -\frac{u_{\theta q}}{u_q} dx\right) + \int_{\underline{\theta}}^{\theta} \frac{1}{u_q}\left[V_q - \phi H_q\right]\left(\exp \int_x^\theta -\frac{u_{\theta q}}{u_q} dy\right) dF(x) \\ &= \int_{\underline{\theta}}^{\theta} \frac{1}{u_q}\left[V_q - \phi H_q\right]\left(\exp \int_x^\theta -\frac{u_{\theta q}}{u_q} dy\right) dF(x) \end{aligned}$$

recalling that $\lambda(\underline{\theta}) = 0$. Let us denote

$$\begin{aligned} A(\theta) &= \int_{\underline{\theta}}^{\theta} \frac{V_q}{u_q} \left(\exp \int_x^{\theta} -\frac{u_{\theta q}}{u_q} dy \right) dF(x) \\ B(\theta) &= -\int_{\underline{\theta}}^{\theta} \frac{H_q}{u_q} \left(\exp \int_x^{\theta} -\frac{u_{\theta q}}{u_q} dy \right) dF(x) \end{aligned}$$

then we get $\lambda(\theta) = A(\theta) + \phi B(\theta)$. Plugging this into equation (12), we obtain an expression for ϕ :

$$\begin{aligned} \phi &= \int_{\Theta} \left[\lambda' u_{\alpha} + \lambda u_{\theta\alpha} \right] d\theta \\ \phi &= \int_{\Theta} \left[\left(\frac{1}{u_q} \left[V_q f - \phi H_q f - (A + \phi B)\, u_{\theta q} \right] \right) u_{\alpha} + (A + \phi B)\, u_{\theta\alpha} \right] d\theta \end{aligned}$$

Rearranging, we get

$$\phi = \frac{\int_{\Theta} \left[\left(\frac{1}{u_q} \left[V_q f - A u_{\theta q} \right] \right) u_{\alpha} + A u_{\theta\alpha} \right] d\theta}{1 + \int_{\Theta} \left[\frac{u_{\alpha}}{u_q} \left[H_q f + B u_{\theta q} \right] - B u_{\theta\alpha} \right] d\theta}.$$

Finally, eliminating λ' in equations (10) and (11) and rearranging, we obtain:

$$V_q - \frac{u_q}{u_t} V_t = \frac{\lambda}{f} \left[u_{\theta q} - \frac{u_q}{u_t} u_{\theta t} \right] + \phi \left[H_q - H_t \frac{u_q}{u_t} \right]$$

which concludes the proof.

B Proof of Proposition 4

The program of the principal is as follows:

$$\max_{t(.),q(.)} \int_{\Theta} \{ S(q(\theta)) - t(\theta) \} \, dF(\theta)$$

s.t.

$$U(\theta) = (1 + \rho)\, (t(\theta) - (\theta - e(\theta)) q(\theta)) - \rho\alpha - \psi(e(\theta)) \geq 0$$

$$\psi'(e(\theta)) = (1 + \rho) q(\theta)$$

$$U(\theta) \geq U(\theta, \tilde{\theta}) \text{ for any } \theta, \tilde{\theta}$$

$$\alpha = \int_{\Theta} \omega(\theta)\, (t(\theta) - (\theta - e(\theta)) q(\theta))\, dF(\theta)$$

As usual, (IC) constraints reduce to

$$\begin{aligned} \dot{U}(\theta) &= -(1+\rho)q(\theta) < 0 \\ \dot{q} &\leq 0 \end{aligned}$$

and we deduce that (IR) constraints can be reduced to $U(\overline{\theta}) \geq 0$. Replacing t and forgetting for the moment the second order condition, the program of the principal reduces to

$$\max_{\pi(.),q(.)} \int_\Theta \left\{ S(q(\theta)) - \left(\frac{U(\theta) + \rho\alpha + \psi(e(\theta))}{1+\rho} + (\theta - e(\theta))q(\theta) \right) \right\} dF(\theta)$$

s.t.

$$U(\overline{\theta}) = 0$$

$$\dot{U}(\theta) = -(1+\rho)q(\theta)$$

$$\alpha = \int_\Theta \omega(\theta) \left(\frac{U(\theta) + \rho\alpha + \psi(e(\theta))}{1+\rho} \right) dF(\theta)$$

The Lagrangean writes as

$$\begin{aligned} \mathcal{L} &= \int_\Theta \left\{ S(q(\theta)) - \left(\frac{U(\theta) + \rho\alpha + \psi(e(\theta))}{1+\rho} + (\theta - e(\theta))q(\theta) \right) \right\} dF(\theta) \\ &+ \int_\Theta \lambda(\theta)(-(1+\rho)q(\theta) - \dot{U}(\theta))d\theta \\ &+ \int_\Theta \phi(\alpha - \omega(\theta) \left(\frac{U(\theta) + \rho\alpha + \psi(e(\theta))}{1+\rho} \right))f(\theta)d\theta \end{aligned}$$

Integrating by parts, we obtain

$$\begin{aligned} \mathcal{L} &= \int_\Theta \left\{ S(q(\theta)) - \left(\frac{U(\theta) + \rho\alpha + \psi(e(\theta))}{1+\rho} + (\theta - e(\theta))q(\theta) \right) \right\} dF(\theta) - \int_\Theta \lambda(\theta)(1+\rho)q(\theta)d\theta \\ &+ \int_\Theta \dot{\lambda}(\theta)U(\theta)d\theta + \int_\Theta \phi(\alpha - \omega(\theta) \left(\frac{U(\theta) + \rho\alpha + \psi(e(\theta))}{1+\rho} \right))f(\theta)d\theta \end{aligned}$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial q} = (S'(q) - (\theta - e(\theta)))f(\theta) - (1+\rho)\lambda(\theta) - \frac{1}{1+\rho}\phi\omega(\theta)f(\theta)\psi'(e(\theta))\frac{de}{dq} = 0 \quad (13)$$

$$\frac{\partial \mathcal{L}}{\partial \alpha} = -\frac{\rho}{1+\rho} + \phi - \frac{\rho}{1+\rho}\int_\Theta \omega(\theta)f(\theta)d\theta = 0 \quad (14)$$

$$\frac{\partial \mathcal{L}}{\partial U} = -\frac{1}{1+\rho}f(\theta) + \dot{\lambda}(\theta) - \frac{1}{1+\rho}\phi\omega(\theta)f(\theta) = 0 \quad (15)$$

together with the transversality condition $\lambda(\underline{\theta}) = 0$ as $U(\underline{\theta})$ is free and $\lambda(\overline{\theta}) \geq 0$ with $\lambda(\overline{\theta})U(\overline{\theta}) = 0$.

From (15), we get $\lambda(\theta) = \int_{\underline{\theta}}^{\theta} \frac{1+\phi\omega(x)}{1+\rho} dx$. From (14) we get

$$\phi = \frac{\rho}{1+\rho}\left(1 + \int_{\Theta} \omega(\theta) f(\theta) d\theta\right) \geq 0$$

And for the production level, we obtain

$$\begin{aligned} S'(q(\theta)) &= \theta - e(\theta) + \frac{1}{f(\theta)} \int_{\underline{\theta}}^{\theta} 1 + \phi\omega(x)dx + \frac{1}{1+\rho}\phi\omega(\theta)\psi'(e(\theta))\frac{de}{dq} \\ S'(q(\theta)) &= \theta - e(\theta) + \frac{1}{f(\theta)} \int_{\underline{\theta}}^{\theta} 1 + \phi\omega(x)dx + \phi\omega(\theta)\frac{\psi'(e(\theta))}{\psi''(e(\theta))} \end{aligned}$$

This concludes the proof.