Moderating Factors of Immediate, Dynamic, and Long-run Cross-Price Effects

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Moderating factors of immediate, dynamic, and long-run cross-price effects

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ABSTRACT

In this article the authors describe their comprehensive analysis of moderating factors of cross-brand effects of price changes and contribute to the literature in five major ways. (1) They consider an extensive set of potential variables influencing cross-brand effects of price changes. (2) They examine moderators for the immediate as well as the dynamic cross-price effect. (3) They decompose price into regular and promotional price and study both cross-price effects separately. (4) They compare their findings with previous literature on the moderating factors of own-price effects to understand which factors influence own-price elasticity through affecting brand switching. (5) The authors use an advanced Bayesian estimation technique. The results show evidence of the neighborhood price effect and suggest that it is conditional on whether the promoted brand is priced above or below its competitor. The promoted brand's activities turn out to play a much more important role in determining the cross-price promotional effects than its competitor's similar activities. The authors outline conditions when cross-brand post-promotion dips tend to occur. Finally, they argue that the brand choice portion of the overall own-brand effect of a promotion depends on the brand's marketing strategy and on category-specific characteristics.

Key words: Cross-price elasticity; Asymmetry; Dynamic effects; Hierarchical Bayes;

1 Introduction

For management purposes knowledge of cross-price effects is especially important. First of all, managers need to know which rival brands have the strongest impact on their sales. Second, they want to identify the competing brands that are affected the most by their price changes and consequently from which competitor they should anticipate reactions. As a next step, it is beneficial for them to recognize whether and how the cross-price effects can be moderated by their and their competitors' marketing activities and positioning decisions. Finally, an interesting question is whether general characteristics of the product category can explain the degree at which their sales are affected by other brands' price changes and their influence on others' sales.

Patterns of cross-price effects have also been in the center of marketing research (e.g., Blattberg and Wisniewski, 1989; Bronnenberg and Wathieu, 1996; Sethuraman and Srinivasan, 2002). These patterns help understand brand price competition and market structure, thereby forces guiding pricing strategies (Sethuraman *et al.*, 1999).

In this study we provide a comprehensive investigation of the moderating factors of cross-price brand effects. More specifically, we focus on the moderating factors for the immediate and the dynamic impact of price changes of a brand on its rivals' sales. With the dynamic impact we mean the impact of a current marketing action on future periods. For a temporary price promotions this can be measured by the cumulative effect over the future periods. For a permanent change in price, such as the adjustment of regular price level, we measure the permanent impact, that is the impact on periods far in the future.

We contribute to the literature in five major ways: (i) we extend the list of studied moderating variables for cross-price effects; (ii) we look at immediate and dynamic cross-price effects; (iii) we decompose price into the regular price and the promotional price and consider their cross effects separately; (iv) we compare the moderators of cross-price elasticities with the moderators for the own-price effects found in Fok *et al.* (2006), and (v) we estimate the cross-price effects and the impact of moderating variables simultaneously in an efficient manner using a Hierarchical Bayes model. Below we further elaborate on these contributions.

First, we consider several category-specific factors and separate their effects from the influence of similar brand-specific variables. Furthermore, in addition to providing new empirical evidence on the neighborhood and the asymmetric price effects (e.g., Blattberg and Wisniewski, 1989; Bronnenberg and Wathieu, 1996; Sethuraman and Srinivasan, 2002) we investigate whether the neighborhood effect depends on which of the two brands is priced above. That is, we allow for asymmetry in the neighborhood effect. Regarding the brand-specific covariates, an interesting question is which brand's characteristics play a more important role in shaping the cross-price effects; those of the "victim brand", i.e., the brand whose sales are affected by the price changes of the other brand, or those of the "attacker brand", i.e., the brand that changes its price. Note that although these terms clearly give the direction of the impact, it is not a priori known that the "victim brand" will actually loose sales. The cross-brand effect could be zero or in special cases even negative. Nonetheless, we will use the terms attacker and victim throughout this paper to denote the two brands involved in a specific cross-brand effect.

Second, we examine the moderators for the immediate effects and the dynamic cross-price effects. So far, the literature on cross-brand effects of price promotions focused on the immediate or the long-run impact and did not consider the possibility of temporary dynamic effects. The impact of price promotions of competing brands is probably not limited to the immediate effect. For example, purchase acceleration may result in extra inventory of the promoted brand. This additional inventory preempts future purchases of competing brands (Ailawadi *et al.*, 2007). This will especially hold for the influence of a high-priced brand on a lower-priced brand. If an expensive high quality product is for sale customers who, due to their tight budget constraint, usually buy a cheaper lower quality brand may be inclined to stockpile the product and enjoy its benefits in the coming periods, thereby postponing their usual purchase. So, asymmetry with respect to price may be more important when considering the dynamic effect of a price promotion than in case of the immediate effect. Additionally, deeper discounts provide higher financial gains when switching to the promoted brand and stockpiling it. Such promotions may result in a larger dynamic effect of price promotion. These examples illustrate that certain variables' influence may differ concerning the immediate effect and the dynamic effect. We investigate these differences in this article.

Third, we decompose price into regular and promotional price and analyze the cross-brand effects of both. The literature on own-brand effects has shown that changes in regular price and price promotions have quite different effects on sales (Bijmolt *et al.*, 2005; Bucklin and Gupta, 1999; Fok *et al.*, 2006). This is also likely to be the case for cross-brand effects. For the promotional price and the regular price the immediate effect is clearly of interest. Concerning the dynamic effect, for regular price we focus on the (moderating factors of the) permanent or long-run effect. In this case, the permanent effect is the effect of the price change on competitors' sales far into the future. With respect to the price promotions, we consider the (moderating factors of the) cumulative effect. The cumulative effect equals the sum of the promotional effects on competitors' sales over all periods.

Fourth, we compare our findings with the results of Fok *et al.* (2006) on the moderating factors of own-price effects to understand which factors influence own-price elasticity through affecting brand switching. These results reveal whether the category incidence/brand choice/quantity division of the total own-effects of promotions may depend on the involved brands' marketing strategies or category-specific characteristics. That is, they provide further insights into the decomposition of promotional response (Bell *et al.*, 1999).

Finally, in our investigation we rely on an efficient econometric technique, the Hierarchical Bayes (HB) - Error Correction Model (ECM). This model allows us to directly estimate the potentially differing immediate and dynamic effects of price changes of a brand on its rival brands' sales and to simultaneously relate these effects to characteristics of brands and categories. Within the model we explicitly distinguish between (cross) promotional price elasticities and (cross) regular price elasticities.

In sum, our analysis provides important insights for brand managers of the attacker and the victim brands and for marketing scholars on the determinants that moderate the cross-brand effect of price changes.

The remainder of this paper is as follows. We first present a systematic discussion of the literature on the considered moderating effects and outline our conceptual research framework. Next, we specify a sales response model to measure and explain the differences in the immediate and dynamic cross-brand effects of promotional and regular prices. In Section 4 we present the results of or empirical application based on a scanner database on weekly sales volumes of the largest four brands in 25 product categories of fast moving consumer goods. We end the paper with a discussion and conclusions in Section 5.

2 Moderating factors of the cross-price effects

The conceptual framework that guides our research is depicted in Figure 1. In this figure we relate the cross-price effect to brand variables, category variables and variables describing the relative positioning of the two involved brands. More precisely, we focus on the effect of price promotions and regular price changes of brand A on the sales of the rival brand V in the short- and in the long-run. In the figure letter A refers to the attacker and V to the victim brand.

Below we discuss the literature concerning the variables we consider in our empirical section. For each characteristic we summarize the literature and, if possible, provide a hypothesis for the expected moderating effect. However, in many cases, especially for the dynamic effects, we cannot formulate such hypotheses based on the literature. In these cases we search for empirical evidence on whether and how the considered characteristics are related to the (dynamic) effects of price. To facilitate a direct comparisons with findings on own-price effects, we keep our set of moderating factors as similar as possible to that in Fok *et al.* (2006).

2.1 Relative positioning of brands.

An interesting and well-researched aspect of cross-price effects concern the relative (price) positioning of the attacker versus the victim brand. Examples of this are the asymmetric and the neighborhood cross-price effect (Blattberg *et al.*, 1995; Bronnenberg and Wathieu, 1996; Sethuraman, 1995; Sethuraman *et al.*, 1999; Sethuraman and Srinivasan, 2002).

Asymmetric price effect. Higher priced brands have been found to be perceived as being of better quality. This implies that price promotions by a higher-priced (better quality) brand will be more attractive for the lower-budget segment and therefore affect lower-priced (lower quality) brands more than the other way around (Allenby and Rossi, 1991; Blattberg and Wisniewski, 1989; Sethuraman *et al.*, 1999, etc.). While promotions of high-priced brands draw sales from their own price-tier competitors and from the tier below, lower-quality brands' promotions rarely take sales from tiers above (Blattberg and Wisniewski, 1989; Sethuraman, 1995). Asymmetry can also arise from a difference in the composition of the customer base of the two brands: a high quality (high-priced) brand captures more loyal customers than a low quality (cheap) one. This leads to a higher substitution effect for the better quality brand. Further explanations have been provided in, for example, Blattberg and Neslin (1989) and Hardie *et al.* (1993). At the same time, Bronnenberg and Wathieu (1996) point out that this asymmetry only holds if and only if the quality advantage of the higher priced (higher quality) brand is sufficiently large, in comparison with its price premium.

Neighborhood price effect. According to the neighborhood price effect hypothesis, brands whose prices are closer to each other usually have larger cross-price effects than brands that have more dissimilar prices. For example, Kamakura and Russell (1989) show that consumers tend to switch only among brands within a certain price range. Additional empirical evidence on the neighborhood effect has been found in Rao (1991), Russel (1992), Sethuraman (1995), and Sethuraman *et al.* (1999).

An interesting question is whether the neighborhood or the asymmetric price effect is stronger and whether one amplifies the other. Sethuraman *et al.* (1999) investigate this and find that the neighborhood price effect is stronger than the asymmetric price effect, both when relying on elasticities as on absolute price effects. We furthermore investigate whether the neighborhood effect depends on which of the two brands is priced above.

Overall, we expect that if the attacker is priced above the victim brand and if the two brands are positioned closer to each other, the cross-price effect will be higher. We furthermore anticipate the neighborhood effect to be stronger when the attacker is more expensive than the victim brand.

Asymmetric and neighborhood size effects. Analogous to the relative price positioning, the relative positioning of the brands with respect to size may also influence their cross-price effect. We measure two aspects of the size difference. We investigate whether brands that are similar in size tend to have larger or smaller cross-price effects and whether larger brands tend to be less sensitive to smaller brands' promotions or the other way around. We use similar measures as for the price effects and call the two effects the asymmetric and neighborhood size effect.

Literature shows that larger brands usually have higher brand awareness and better brand salience. Their promotions are therefore easier noticed and identified by customers (e.g., Rao and Miller, 1975). Additionally, according to the well-established "double jeopardy" phenomenon smaller brands tend to attract less "loyalty" among their buyers than large brands do among theirs (Ehrenberg *et al.*, 1990; Martin, 1973). Based on such considerations Kamakura and Russell (1989), Sethuraman and Srinivasan (2002), and Sethuraman (1995) suggest that larger brands have a larger influence in a market, and that their promotions can easily hurt smaller brands. On the other hand, larger brands are also less vulnerable to smaller brands' discounts. This shows the possible existence of asymmetry and the neighborhood size effects. In our empirical study we capture these effects and also include interaction between the two effects to test for non-linearity in the neighborhood size effect.

2.2 Category-and Brand-Specific Characteristics

To study the moderating effect of marketing activities we look at the intensity of marketing activities at the category level as well as at such intensity for a brand relative to its category. For example, in some categories promotions may be more frequent or deeper than in others, but within a category there may also be relevant differences in promotional policies.

Frequency of price promotions. According to the theory of price consciousness high frequency of price promotions in a category leads to more price conscious customers and this induces them to purchase on deal (Fok *et al.*, 2006; Kopalle *et al.*, 1999; Mela *et al.*, 1998; Yoo *et al.*, 2000). More price conscious consumers may be more likely to switch between brands. Papatla and Krishnamurthi (1996) similarly find that increased purchases using coupons erode brand loyalty and increase price sensitivity. These results suggest that categories with a high frequency of promotional activities are likely to show substantial brand-switching and therefore a high immediate and dynamic cross-brand price elasticity.

Within a category, other processes may also play a role. Brands with relatively frequent price promotions are often considered of lower quality. Consequently, their price discounts will be less attractive than similar brands with infrequent promotions and will stimulate less brand switching and less stockpiling behavior. Frequent discounts also lower the perceived quality of the victim brand whose share therefore is expected to respond more its competitors' price discounts (see Fok *et al.*, 2006, and the discussion above on the asymmetric price effect). For the price promotion frequency of the victim other effects may also play a role. Frequently promoted brands draw a larger proportion of price-sensitive customers (Zenor *et al.*, 1998) who are likely to switch away when competitive brands are on sale. In contrast, Jedidi *et al.* (1999) point out that frequent promotions of brands make it unnecessary for the (loyal) customers to switch brands since they know that a deal on their favored brand will occur in the near future. Finally, Jedidi *et al.* (1999) mention that frequent discounts may make customers more likely to stockpile their favorite brand because they fulfill a greater portion of their demand in promoted periods.

Overall, the above findings point towards that the frequency of price promotional activity of the attacker brand reduces the immediate cross-brand price elasticity and may have an further moderating impact on the cumulative effect. For the victim we find mixed indications in the literature and hypothese that, on the whole, their promotion frequency increases the cross-price effect.

Average depth of promotions. Mehta *et al.* (2003) find that frequent price promotions with deep discounts lead to large consideration sets. This is confirmed by Teng (2008) who point out that deep discounts may stimulate otherwise loyal customers to try other brands. Positive brand experience may make these customers to also consider the alternative brand in future periods. This indicates a high brand-switching probability in frequently promoted categories and in categories that are characterized by deep discounts. Additionally, deep discounts have been found to increase deal sensitivity (Anderson and Simester, 2004) and the variability of brand sales within a category (Raju, 1992). We therefore expect higher cross-brand elasticities in categories characterized by deeper discounts.

At the brand level, consumers may believe that deep discounts indicate lower quality (Jedidi *et al.*, 1999; Mela *et al.*, 1998) and unjustly high margins. On the other hand, deep price promotions provide a high financial incentive for buying additional items of the discounted brand and thereby stockpiling the product. This may result in an own-brand post-promotion dip and may induce a prolonged dip in the sales of competing brands as well. So, based on the literature we anticipate that general usage of deeper discounts of the attacker brand will moderate the immediate price elasticity. However, the sign of the effect is difficult to predict. Deep discounts by the attacker are likely to amplify the cumulative elasticity. Deep discounts by the victim brands are expected to increase the immediate effect.

Frequency of feature and display activities. Features and displays are often used to increase or maintain brand awareness. High brand awareness may increase the likelihood that the brand will be in the consideration set of customers (Keller, 1993) who will therefore be more inclined to notice price promotions of these brands and eventually buy them. Several studies have shown that feature and display activities can be used to form consideration sets (e.g., Bronnenberg and Wathieu, 1996; Fader and McAlister, 1990; Mehta *et al.*, 2003). Furthermore, retailers and manufacturers often support price promotions with feature and display activities driven by the belief that such combinations will generate larger incremental sales response to the promotion (Zhang, 2006). In sum, price promotions of attacker and the victim brands and categories with more frequent display or feature activities are expected to have higher cross-price promotional elasticity.

2.3 Category-Specific Characteristics

In this section we discuss how additional category-characteristics can influence the cross-brand effect of price changes.

Perishability. The perishable nature of a product category has been found to be an important feature for the own-price effects, especially with respect to the willingness to store and stockpile the product (Narasimhan *et al.*, 1996; Raju, 1992; Fok *et al.*, 2006). In line with these arguments for the own-price effects, we expect that in a category with non-perishable products a part of the larger own-effect may arise from brand switchers who have a high propensity to stockpile the product. The larger inventory of the promoted brand will preempt the purchase of other brands hereby reducing their sales in the near future (Ailawadi *et al.*, 2007) causing a larger direct cross-brand effect as well as a larger dynamic (post-promotion) effect.

Utilitarian nature. Shoppers with a relatively low income constitute a large proportion of the consumer population of necessity goods (Bell *et al.*, 1999; Wakefield and Inman, 2003). This group is more price sensitive and deal-prone. Additionally, in low-involvement categories brand awareness may be a sufficient condition for brand choice (Keller, 1993). These findings point towards more brand-switching and therefore larger cross-brand price effects in more utilitarian categories.

Average budget share. The budget share of a category captures two separate dimensions: the general price level of the category and the average quantity purchased in a period. The customer base of more expensive categories consists of higher income households in general. They are less price sensitive and also less inclined to switch from their favorite brand when a competitor is on sale. At the same time, promotions in categories with high average purchase quantity induce customers to switch between brands and stockpile more (Fok *et al.*, 2006; Macé and Neslin, 2004). It is therefore difficult to make a prediction for the impact of this variable.

Competitive intensity. In the literature on cross-price effects one of the important findings regarding the competitive intensity is that cross-price effects are stronger when there are fewer competing brands in the product category (Sethuraman *et al.*, 1999). In the presence of several similar brands, brand switching is likely to be the dominant source of variability in brand sales in a category (Raju, 1992). This is supported by Narasimhan *et al.* (1996) who point out that brand proliferation can be a source of weakened brand loyalty. Macé and Neslin (2004) and Ailawadi *et al.* (2007) on the other hand show that a large number of competitors may as well imply a lot of differentiation. Product differentiation makes brands less exposed to competitors' actions (Narasimhan *et al.*, 1996) and leads to less brand-switching.

3 Methodology

To describe the dynamic pattern in sales of brands in a number of product categories we specify a set of flexible vector error correction (VEC) models. Within the model, the differences in parameters across categories are explained in a second level model. This hierarchical model is inspired on Fok *et al.* (2006). The important difference is that in this article we explicitly focus on the cross-price effects. So, we define moderating factors for cross-brand price effects in the second level of the model.

Also in this section we use the terminology "attacker" and "victim" brand, that is, when we consider the effect of the price of brand j on the performance of brand i, we will call brand j the "attacker" and brand i the "victim".

3.1 Hierarchical Bayes Analysis

Let S_{ct} denote the I_c-dimensional vector of sales for category c in week t. The I_c-dimensional vectors X_{ckt} contain the k-th marketing mix variables for the brands in category c in week t. We specify the following vector error-correction model for category c:

$$\Delta \log S_{ct} = \mu_c + \sum_{k=1}^{K} A_{ck} \Delta \log X_{ckt} + \Pi_c (\log S_{c,t-1} - \sum_{k=1}^{K} B_{ck} \log X_{ck,t-1}) + \varepsilon_{ct}, \quad (1)$$

with $\varepsilon_{ct} \sim N(0, \Sigma_c)$ for $c = 1, \ldots, C$ and $t = 1, \ldots, T_c$. The matrix Π_c measures the speed of adjustment of the sales to the long-run steady state. The matrices A_{ck} and B_{ck} contain own *and* cross-brand effects. The own effects are on the diagonal, the cross effects correspond to the off-diagonal elements. In general, let $A_{ij,ck}$ denote the element of A_{ck} on row i and column j, this corresponds to the effect of the k-th marketing instrument of brand j on brand i.

It is relatively straightforward to show that $A_{ij,ck}$ can be interpreted as the *immediate effect* of a change in $X_{j,ckt}$ on brand i. More formally, it holds that

$$\frac{\partial S_{ict}}{\partial X_{jct}} \frac{X_{jckt}}{S_{ict}} = \frac{\partial \log S_{ict}}{\partial \log X_{jckt}} = A_{ij,ck}. \tag{2}$$

Note that because of the log-log specification in (1), we can interpret these effects as elasticities. The parameter $B_{ij,ck}$ can be interpreted as either the *cumulative effect* of a temporary change in X_{jckt} on $S_{ic,t+\tau}$, summed over $\tau = 0, 1, \ldots$ or as the *long-run* effect of a permanent shift in X_{jckt} on the sales levels of brand i in market c, see also Fok *et al.* (2006). A temporary change is considered to be a one time change in X_{jckt} only at time t, at time $t+1$ the variable has returned to its previous level. For a permanent change, X_{jckt} increases at time t and stays at that level. In terms of partial effects or elasticities, the cumulative effect equals

$$\sum_{\tau=0}^{\infty} \frac{\partial S_{ict}}{\partial X_{jckt}} \frac{X_{jckt}}{S_{ict}} = \sum_{\tau=0}^{\infty} \frac{\log S_{ict}}{\log X_{jckt}} = B_{ij,ck}, \tag{3}$$

and the long-run effect equals

$$\lim_{\tau\to\infty} \frac{\partial S_{ic,t+\tau}}{\partial X_{jck}} \frac{X_{jck}}{S_{ic,t+\tau}} = \frac{\partial \log S_{ic,t+\tau}}{\partial \log X_{jck}} = B_{ij,ck} \qquad \text{assuming } X_{jck} = X_{jckt} = X_{jck,t+1} = \ldots. \tag{4}$$

Our main interest here is in the cross-effects, for notational efficiency we collect all cross-effects for category c and marketing instrument k in $I_c \cdot (I_c - 1)$-dimensional vectors. The cross-effect elements from A_{ck} are collected in $\tilde{\alpha}_{ck}$, the elements from B_{ck} in $\tilde{\beta}_{ck}$. More precisely,

$$\begin{aligned} \tilde{\alpha}_{ck} &= (A_{12,ck}, A_{13,ck}, \ldots, A_{1I,ck}, A_{21,ck}, A_{23,ck}, \ldots, A_{2I,ck}, \cdots, A_{I1,ck}, A_{I2,ck}, \ldots, A_{I,I-1,ck})' \\ \tilde{\beta}_{ck} &= (B_{12,ck}, B_{13,ck}, \ldots, B_{1I,ck}, B_{21,ck}, B_{23,ck}, \ldots, B_{2I,ck}, \cdots, B_{I1,ck}, B_{I2,ck}, \ldots, B_{I,I-1,ck})'. \end{aligned} \tag{5}$$

Now $\tilde{\alpha}_{ck}$ captures all the immediate effects of a change in the k-th marketing instrument of a brand in category c on another brand's sales in the same category. $\tilde{\beta}_{ck}$ refers to all respective long-run or dynamic effects. Finally, we introduce two functions $i(l)$ and $j(l)$. These two functions map the index of an element of $\tilde{\alpha}_{ck}$ or $\tilde{\beta}_{ck}$ to the index of the victim and attacker brand, respectively. As an example, $i(2) = 1$ and $j(2) = 3$, that is, the second elasticity in $\tilde{\alpha}_{ck}$ corresponds to the cross elasticity of brand 3 on brand 1, see also (5).

The immediate and dynamic elasticities are expected to differ across brands and across categories. Some of these differences can be attributed to observable characteristics of the category, the attacker brand and/or the victim brand. Examples of such characteristics are, depth and frequency of promotion or the perishability of the product, as we have discussed in Section 2. The earlier mentioned neighborhood effects indicate that the difference between the attacker and victim brand on some scale may be important. Another part of the differences across elasticities cannot be explained. In sum, we propose to describe the immediate and dynamic elasticity parameters for the promotional price and the regular price[1] by

$$\tilde{\alpha}_{l,ck} = \theta^1_{0k} + \theta^{1\prime}_{1k} z_{i(l),c} + \theta^{1\prime}_{2k} z_{j(l),c} + \theta^{1\prime}_{3k} g(z_{i(l),c}, z_{j(l),c}) + \theta^{1\prime}_{4k} z_c + \eta^1_{l,ck} \quad (6)$$

$$\tilde{\beta}_{l,ck} = \theta^2_{0k} + \theta^{2\prime}_{1k} z_{i(l),c} + \theta^{2\prime}_{2k} z_{j(l),c} + \theta^{2\prime}_{3k} g(z_{i(l),c}, z_{j(l),c}) + \theta^{2\prime}_{4k} z_c + \eta^2_{l,ck} \quad (7)$$

where $z_{i,c}$ is an vector containing explanatory variables for brand i in category c, like frequency and depth of promotion. The vector z_c contains a number of variables on the category level, like category expensiveness. The neighborhood and asymmetry effect is captured by the function $g()$, which for example may give the distance between the brands. In the empirical analysis below we specify

$$g(z_{i(l),c}, z_{j(l),c}) = \begin{pmatrix} |z_{j(l),c} - z_{i(l),c}| \\ 1_{z_{j(l),c} > z_{i(l),c}} \\ |z_{j(l),c} - z_{i(l),c}| \times 1_{z_{j(l),c} > z_{i(l),c}} \end{pmatrix}, \quad (8)$$

where 1_A denotes an indicator function that equals one if the condition A is true and zero otherwise. In words the function $g()$ contains the absolute difference between the two brands, an indicator whether the attacker is bigger or more expensive than the victim brand, and the interaction between these two variables. Note that we do not include these non-linear effects for all brand characteristics. For notational convenience we have not made this explicit in (6)-(8). The vectors θ^1_{nk} and θ^2_{nk}, $n = 1, \ldots, 4$ describe the effects of the brand characteristics on the immediate and the dynamic elasticities, respectively. For the error terms $\eta^1_{l,ck}$ and $\eta^2_{l,ck}$ we assume multivariate normal distributions. More specifically, we assume these error terms to be uncorrelated across brands, and categories. We

[1] In the empirical section we will also include other marketing instruments. For these instruments we do not have a second layer specification as in (6)-(7). We do allow the elasticities for these variables to differ across brands.

do however allow for correlation in the error terms across the K marketing-mix variables, that is, we assume that

$$\begin{aligned} \eta^1_{l,c} &= (\eta^1_{l,c1}, \ldots, \eta^1_{l,cK})' \sim \mathrm{N}(0, \Sigma^1_\eta) \text{ and} \\ \eta^2_{ij,c} &= (\eta^2_{l,c1}, \ldots, \eta^2_{l,cK})' \sim \mathrm{N}(0, \Sigma^2_\eta). \end{aligned} \tag{9}$$

For the own effects we have a similar specification as in (6) and (7) to link the own effects to brand and category characteristics. Of course in this case only characteristics of the brand itself and the category play a role. As the focus here is not on the own effects we do not discuss this further. More details can be found in Appendix A.

To estimate the parameters of our model, (1) with (6)–(8), we consider a Bayesian approach. In Appendix A we present the full details of the Markov Chain Monte Carlo simulation method that we apply.

4 Empirical results

We apply our model to explain differences in immediate and dynamic cross-brand effects of promotional price and regular price on sales across brands and product categories in an extensive dataset. In Section 4.1 we describe the available data and the product categories we consider in our analysis. Section 4.2 contains the estimation results.

4.1 Data and Variables

We use the Dominick's Finer Foods data set for our empirical analysis. This data has been used and described extensively in, for example, Fok *et al.* (2006) and Srinivasan *et al.* (2004). So, we refer the reader for detailed description of the data to these papers.

We specify 25 error-correction models as in (1) for the 25 FMCG product categories contained in the data set. In each model, the dependent variable S_{ct} consists of the total weekly sales of the four largest brands in product category c. As explanatory variables in the first level of the model we consider the marketing-mix variables, display, feature, regular price and promotional price indexes. These models are linked together through the second level equations describing the price effects. The parameters in all model parts are estimated simultaneously in a Bayesian setting. We consider several variables as moderating factors of the cross-price elasticities (for their theoretical discussion see

Section 2 and for our conceptual model framework Figure 1). A summary and the formal definition of these variables can be found in Appendix B.

The original database only contains the actual price. We use the decomposition of the actual price into regular and promotional price as suggested in Fok *et al.* (2006). Additionally, we account for seasonality and special holidays and run unit-root tests on the seasonally adjusted series according to Fok *et al.* (2006). In line with previous research (e.g., Fok *et al.*, 2006; Horváth *et al.*, 2005; Nijs *et al.*, 2001; Pauwels *et al.*, 2002), our unit-root analysis shows that all sales series are (trend) stationary, after correcting for possible seasonality and possible breaks in the regular price series.

Our approach differs in several important aspects from Fok *et al.* (2006). The main difference is of course in the object that we study. Here we focus on the cross-brand effects of price changes. Therefore, we also have a substantially higher number of elasticities to model. In the considered 25 categories with 4 competing brands there are $(16 - 4) \cdot 25 = 300$ cross-promotional effects and only $4 \cdot 25 = 100$ own-brand effects per instrument. Additionally, when analyzing differences in the cross-brand effects we consider characteristics of both the attacker and the victim brand.

4.2 Estimation results

We use Gibbs sampling as presented in the Appendix A to obtain insight into the parameter values. The posterior results below are based on 200,000 draws of which the first 100,000 are used as burn in. To remove correlation in the chain we only consider every 10th draw for the computation of the posterior results. Unreported plots of the draws of the model parameters of the second layer (6) and (7) show that the Markov Chain has converged.

First, we summarize the posterior means of the cross effects of the (log) promotional price index and the log regular price in graphs. Figure 2 presents the distribution of the posterior means of the immediate cross effects of price promotions and of regular price changes, the cumulative cross effect of price promotions, and the long-run cross effect of a regular price change. These histograms show the distribution of the posterior means of all brand-pairs in the 25 categories. Furthermore, we calculate the number of significant positive and negative cross effects (see Table 1). Here we loosely use the term significant to indicate the zero is not contained in the 95% highest posterior density region of the corresponding parameter.

For the promotional price most of the (short- and cumulative) cross-price effects are positive as expected (84% of the 300 cases) and about half of these positive values are significantly larger than zero. The few negative effects are relatively small in size and mostly not significant. Van Heerde *et al.* (2003) provide a possible explanation for the negative values. A promotion of a brand may remind some customers about the category, but these customers buy their strongly preferred brand that is different from the promoted brand. When turning to the cumulative cross-promotional effects, the number of significant effects drop. We do not find a single significant negative parameter and only 95 (32%) significant positive mean parameters.

Comparing the histograms for the promotional and regular price actions, we observe much more dispersion of the cross-effects of regular price changes than of price promotions, both in the short- and in the long-run. This is similar to the findings of Fok *et al.* (2006) for own elasticities. For the regular price the graphs of Figure 2 show much more dispersion with several positive and negative values. The majority of the posterior means, though, is positive. Table 1 shows that for about 35% of the brand combinations we find a negative impact. However, of all immediate cross-regular price effects only about 15% are significantly different from zero. Interestingly, we find much more (almost 30%) significant mean parameters for the long-run effect, despite the smaller dispersion. This suggests that the reduction of regular price of a brand may have serious consequences on the sales of at least a few of its competing brands. This may be due to a change in brand image or realization of increased customer value, which are important considerations when the customer develops its consideration set and later makes the final purchase decision. It is also important to note that although not many of the effects are significant statistically, they may very well be economically significant. A reason for not finding many significant effects can be the lack of (variance in the) data. The regular prices do not change as frequently as the promotional price index. Therefore, estimating the effects becomes a more complicated task.

Only by looking at the effects over all brands can we make generalizing statements on the patterns and the relevance of the cross regular price effects. So, in order to gain insight into the pattern of the cross-brand dynamic effects we look at the post cross-promotion dip, using results for each brand in (the first layer of) our model. If consumers stockpile when a product is on promotion, this may preempt purchases of other brands at a later point in time. A promotion of a brand will then lead to an immediate disadvantageous impact on other brands, but also a harmful impact on other brands in some periods to come. The cumulative cross-price effect will in this case be larger than the immediate impact. To measure this, we calculate the posterior probability that the immediate effect is smaller than the corresponding cumulative/long-term effect for each cross-elasticity. If we find a large probability, this is a sign that there is a "post cross-promotional dip". This probability is calculated as the average, over all draws in the MCMC sampler, of the percentage of cross effects for which the immediate effect is smaller than the cumulative/long-term effect. For the promotional price this probability equals 0.543, for the regular price it is 0.460. This result provides no clear empirical evidence of the cross-brand post-promotion dip or any systematic dynamic effect. In other words, most of the cross-price effect happens at the moment of the price change. However, when we only compare the immediate and dynamic/long-run cross-brand effects when the attacker brand is priced above the victim brand, this probability increases to 0.596 for the promotional price (and 0.481 for regular price). This supports the premise that when an expensive high quality product is for sale customers, who otherwise buy a cheaper and lower quality alternative, may decide to buy and even to stockpile the discounted brand. As a result they may postpone their usual purchase of lower quality brand whose sales will experience a cross-brand post-promotional dip. We find even stronger indication of cross-brand post promotion dip when focusing on effects of price changes of brands that use deeper than average promotions and are priced higher than the victim. In this case the dynamic cross-brand effect is larger than the immediate effect in about 66 percent of the cases (result not presented in the tables). Deeper discount provides higher financial gains (and higher increase in customer value) that could induce customers to stockpile the promoted better quality brand.

4.2.1 Moderating factors of the cross-brand promotional effects

We now turn to the findings based on the second layer of our model and discuss which of the considered moderating factors explain differences in the cross-effects for promotional price. Table 2 shows the corresponding posterior results. All brand and category characteristics have been standardized. Thus, the intercept can be interpreted as the mean effect across all brand combinations. The intercept estimates are smaller for the immediate than for the cumulative cross-brand effect, providing an additional evidence for the cross-brand post-promotional dip.

We find quite some factors to have a significant influence on the cross promotional price effect. Our results indicate partial evidence for the neighborhood price effect in the short-run. Brands that are closer together in terms of average price, tend to have larger cross-price effects if the attacker brand is cheaper than the victim brand. However, when the attacker is cheaper, asymmetry seems to dominate and the cross-brand response. Cross-brand price elasticity appears to increase with the price (quality) difference between these brands.[2] The cumulative effects are also positively related to these two phenomena, however, the parameter of the cumulative neighborhood effect is not significantly different from zero. This is probably due to the increased uncertainty (and standard deviations) about the effect of brand- and category-specific variables on the cumulative cross-promotional price effects. The increased uncertainty about the moderating factors' influence on the cumulative promotional price effect as compared to the immediate influence seems to be a general phenomenon.

Our results provide evidence for the neighborhood size effect as well, suggesting that sales of brands with similar size tend to react more to each others' price promotional activities. This finding holds irrespectively of whether the attacker or the victim brand is larger; we find no evidence for an asymmetric size effect. The neighborhood size effect is stronger in the long run; the parameters we find are about twice as large and significant. Thus the neighborhood effect not only holds for the impact of a competitor's price on the current period, but also for the future periods.

Concerning the use of promotions, the attacker brand's activities play an important role in determining the cross-price promotional elasticities, while the victim brand's activities do not seem to influence the cross-brand promotional effects. The attacker's promotion frequency reduces the impact of its price cuts on competing brands' sales. As discussed before, an explanation can be that brands that are promoted frequently are often considered to have lower quality, than similar brands that are rarely promoted. At the same time, while frequent usage of promotions lowers immediate cross-brand elasticity, it does not affect the cumulative effect.

Brands with deeper promotions in general seem to have a stronger immediate and

[2] We test the significance of the neighborhood price effect when the attacker is more expensive than the victim brand by checking whether zero is contained in the highest posterior density region of the sum of parameters of |Attacker Price Index − Victim Price Index| and |Attacker Price Index − Victim Price Index| $\times$ I[Attacker Price Index $>$ Victim Price Index], the posterior mean of this effect size is $-0.041 + 0.061 = 0.020$.

cumulative impact on other brands. These results suggest that the additional financial incentive exceeds the effect of lower perceived quality due to deeper discounts. Not only does it induce more immediate brand switching but also stronger dynamic effects.

A frequent use of displays by the attacker is positively related to the size of the cross promotional price effect. This result also holds for the cumulative effect. This, as outlined in Section 2 may be due to the fact that overall displays tend to rise brand awareness that increases the probability that the brand will be in the consideration set of customers (Keller, 1993). Notably the effect size is almost the same for the immediate as the cumulative effect, this implies that the display usage has a very limited impact on the effect of a price promotion on future sales of other brands.

We find that several category-specific variables shape the cross promotional price effect, however, some only in the short-run. In markets with a high market concentration, we find smaller short-term cross promotional price effects. In the long-term it is the other competitiveness measure, price differentiation, that appears to reduce brand switching. Indeed, in markets without fierce competition between brands, we would expect less switching among brands. In line with Mehta *et al.* (2003), Mela *et al.* (1998), and Zenor *et al.* (1998), we find that markets with more frequent use of price promotions or relatively deep price promotions tend to have stronger cross promotional price effects. For the frequency we only find an effect in the short run, for the depth of price promotions we find a short run and a long run effect. Additionally, average depth of price promotions within the category seems to be more influential with respect to the cross-brand effect than the attacker's depth of price promotion. Another interesting finding is the impact on the cumulative effect that is about twice as large as on the immediate effect. This may suggest that deep promotions seriously erode brand-loyalty; people tend to stay with the brand they bought on sale till a new one is promoted.

Finally, we find that cross promotional price effects are larger in more utilitarian (lower involvement) categories. In such categories consumers tend to be less brand loyal and more likely to switch brands.

4.2.2 Moderating factors of the cross-brand effects of changes in regular price

While we find quite some significant moderating variables for the cross-regular price effect in the short-run, there are none for its long-run effect. An explanation may be that these moderators mainly affect the speed of realizing the permanent price change and the pace at which customers act upon it. Although there are quite some significant long-run effects, 29% see Table 1, it turns out to be very difficult to explain the differences in these long-run effects. In some cases the entire category will profit from a regular price decrease, leading to negative cross-price effects. In other cases, we get positive cross price effects indicating that competitors loose sales if a brand reduces its regular price.

For the immediate effect, we find that a small brand is more affected by a change in regular price of a large brand, than the other way around. In accordance with our findings on the promotional effects, we find that mainly the attacker brand's activities influence the cross-brand effect of regular price changes. However, this time we find that the attacker's promotional frequency increases the cross-price effect. A possible explanation is that if a frequently promoted brand increases its regular price, more people will be switching away searching for alternative brands that are of better price-quality ratio. While the attacker's feature frequency did not affect price promotional elasticity, it increases the regular price effect. On the other hand, feature frequency of the category turns out to reduce the effect of a regular price change.

An interesting finding is that category characteristics that are mostly independent from any marketing activities play the most important moderating role in the case of regular price. In utilitarian and perishable categories changes in regular price induce higher brand switching, but again, only in the short-run.

4.2.3 Relation between the first and the second layer of the model

In Table 3 we present the (co)variances of the error terms for the cross price effects (9). These results show that while we are able to explain a relatively large part of the variation in the cross promotional price effects, for the cross regular price effects this is not the case. Especially for the long run effect the variance of the unexplained part is large. This of course could also be noted from the fact that none of the explanatory variables turned out to be significant for the long-run cross regular price effect. Note that the size of the variance should be judged relative to the observed variation in the price effects, see Figure 2. Furthermore, we find that there are no significant correlations between the unobserved components for the cross-promotional and cross-regular price effects.

4.2.4 Comparison of findings for own- and cross-brand effects

We compare our findings with the results of Fok *et al.* (2006) on the moderating factors of own-price effects to understand which factors influence own-price elasticity through affecting brand switching.

Many of the brand specific variables that influence the own effects of a price change found in Fok *et al.* (2006) also explain the cross effects. Average depth of promotion and display frequency of the brand amplifies own-brand effects and the cross-brand effects in the short, and in the long-run as well. So, some of the additional own-brand effect due to deeper promotions or more frequent use of displays of the attacker brand arises from higher brand-switching. Feature frequency neither influences own-brand nor cross-brand effects. While for the own effect promotion frequency only influences the cumulative effect, for the cross-elasticity it only moderates the immediate effect.

At the category level, we find, similar to the brand-specific counterpart, that the use of deeper promotions in a category increases the own-effect and induces higher brand switching as well. At the same time, more frequent use of displays, with strong influence at the brand level, does not have any significant influence at the category level. With respect to competition we find that less concentrated categories with low price differentiation are characterized by relatively large own-price promotional effects possibly due to the high brand-switching.

For utilitarian nature and perishability of categories the results match the least with respect to the own- and cross-brand promotional effect. While a price promotion in utilitarian categories results in the same amount of incremental sales as in non-utilitarian ones, a higher portion of the incremental sales will be due to brand switching for utilitarian products. Moreover, more perishable categories are characterized by low immediate own-effects and average dynamic own effect, possibly due to lack of incentives to stockpile. This phenomenon, however, does not affect the cross-brand elasticities.

Now we turn to the comparison with respect to the moderation for own- and cross-brand effects of regular price changes. The most intriguing finding is that whereas in Fok *et al.* (2006) no brand-specific variables had significant influence on the own-effects, we find some brand-specific variables that moderate the cross-brand effects. The attacker's strategy about the rate of price promotion and feature usage amplifies the immediate cross-brand price effect of its regular price changes. Among the category-specific variables perishability and average depth of promotions influence immediate cross-effect, but do not affect own-brand elasticity. At the same time, price dispersion only seems to influence the effect of regular price changes on own sales. The utilitarian nature of the category amplifies both the own- and the cross-brand effects in the short-run, however only influences the own-effect in the long-run.

These results suggest that the category incidence/brand choice/quantity division of the total own effects of promotions is not unconditional; it's brand choice portion depends on the brands' strategy about its marketing instruments and mostly on category-specific characteristics. A full study of such a conditional decomposition is beyond the scope of the present paper. However, our results do provide further insights into the decomposition of promotional response (Bell *et al.*, 1999).

5 Conclusions

Our study provides new insights into the variability of cross-price effects and addresses a couple of questions raised in previous literature on cross-price elasticities. Through a hierarchical Bayes Error Correction model for 25 different product categories, we obtained a number a generalizing findings.

First of all, we find support for the neighborhood cross price promotional effect. However, we show that this effect only holds if the attacker is in general cheaper (and therefore probably of lower quality) than the victim brand. In this case if the attacker's price is closer to the victim's price, it's price promotion induces more customers of the victim brand to switch. However, when the attacker is more expensive, we do not find evidence for the neighborhood effect. So, the asymmetry plays a moderating role for the neighborhood effect of price promotions. We also find evidence of neighborhood-size effects in the short- and in the long-run as well.

Secondly, an interesting finding is that, among the brand-specific variables, activities of the attacker brand significantly influence the cross-brand promotional price and cross-brand regular price effects. The variables associated with the victim brand turn out not to influence the cross-brand price effects. Most of the significant attacker characteristics coincide with the own brand-specific variables that have been found to influence own-brand elasticities significantly in Fok *et al.* (2006). Apparently, some of these earlier established effects arise from higher brand-switching.

Furthermore, we find that an important factor influencing the cross-brand price effect is whether the product category is utilitarian. Both the cross-brand promotional price and the cross-brand effects of regular changes are significantly larger in more utilitarian (lower involvement) categories. Additionally, the utilitarian nature seems to be the most important moderator for the regular price, followed by whether the products in the category are perishable or not.

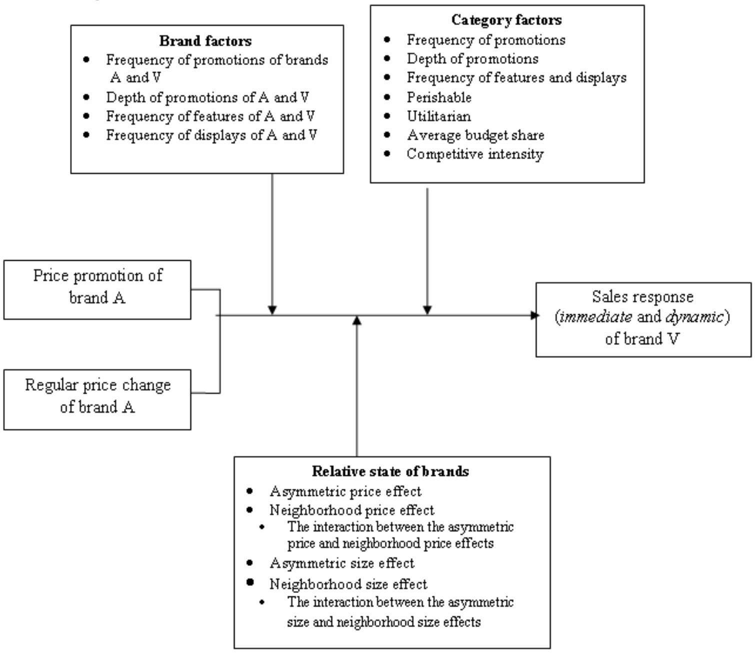
Our study can be extended in several ways. First of all, we restricted our analysis to the cross-price effects within a category. Analysis of possible moderating factors for cross-price effects across categories, and hence considering the possibility of complementarity in addition to substitutability, would bring new insight to the literature of cross-price elasticities that would be useful for researchers and practitioners as well. However, one should not expect to find many significant moderating effects in this case. The cross-effects across categories will very likely be even smaller than within a category.

The investigation of cross-brand post-promotional effects seems to require further investigation as well. We showed that if the attacker brand is priced above the victim brand and if it uses deep promotions, situation that indicates cross-brand post-promotion dip is likely to occur. There may be situations when such dynamic cross-brand effects arise. The investigation of these conditions would be interesting for marketing theory and practice. An interesting line of research would be to reveal the conditions under which such cross-brand price promotion effects arise.

As we discussed in our section about the empirical analysis, the insignificant cross-brand effects of the immediate and long-run effects of regular price changes, and therefore the insignificant effect of the moderating factors on these elasticities, may be due to the low volatility of the regular price variables in our dataset. Data with a larger longitudinal dimension and with more regular price changes would facilitate the understanding of cross-brand effects of changes in regular price.

Our analysis concerned FMCGs. An important question would be whether our findings could be generalized over other categories, to durable products and/or to categories that induce higher involvement of the customers.

Figure 1: Our conceptual research framework



A refers to the Attacker and V to the Victim brand

Figure 2: Histogram of cross price effects

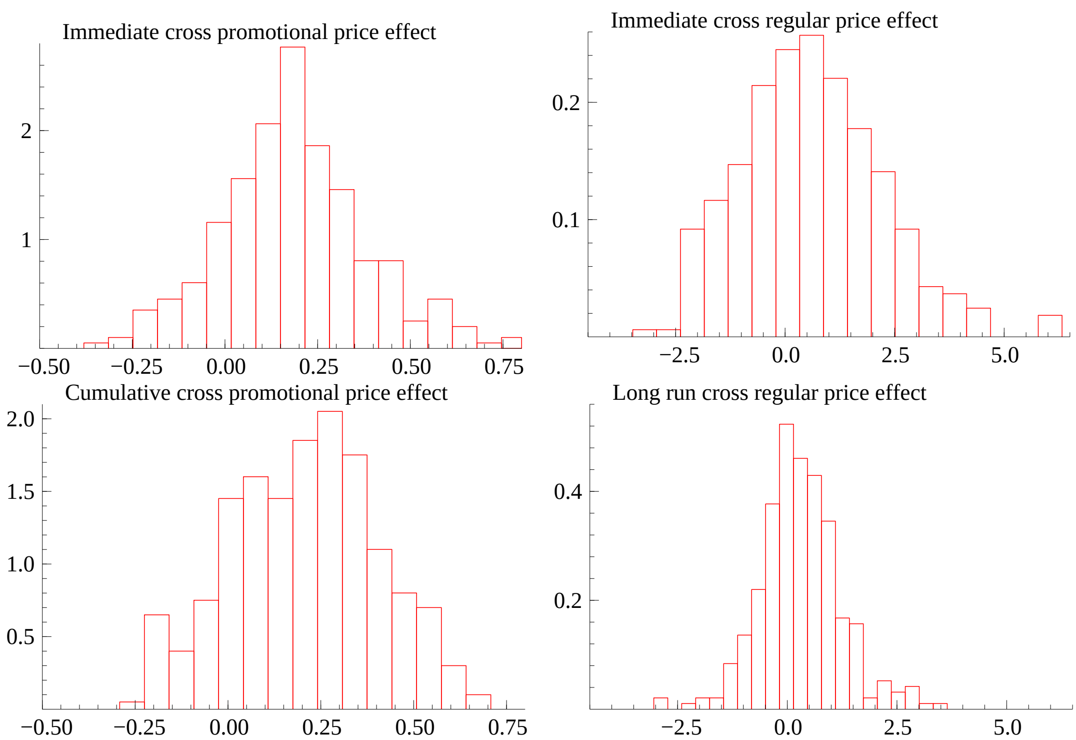


Table 1: Counts related to the sign and significance of the cross-price effects per brand combination. In total there are $25 \times 4 \times 3 = 300$ cross-price effects

	Promotional price	Regular price
	Immediate effect	
Significant and negative	10	2
Not significant and negative	39	102
Not significant and positive	119	157
Significant and positive	132	39
	Cumulative/Long run effect	
Significant and negative	0	24
Not significant and negative	52	85
Not significant and positive	153	128
Significant and positive	95	63
	Cumulative/Long run effect − Immediate effect	
Significant and negative	3	33
Not significant and negative	134	127
Not significant and positive	155	124
Significant and positive	8	16

Note: "Significant" should be read as: zero is not contained in the 95% HPD region.

Table 2: Posterior means (and standard deviations) for cross price effect moderators

Variable[1]	Cross Promotional Price Effect				Cross Regular Price Effect			
	Immediate effect		Cumulative effect		Immediate effect		Long Run	
Intercept	0.166	(0.043)***	0.252	(0.061)***	0.142	(0.464)	0.103	(0.235)
|Attacker Price Index – Victim Price Index|	-0.041	(0.021)**	-0.048	(0.030)	0.106	(0.247)	-0.005	(0.115)
|Attacker Brand Size – Victim Brand Size|	-0.029	(0.018)*	-0.060	(0.026)**	0.231	(0.171)	0.057	(0.106)
I[Attacker Price Index > Victim Price Index]	0.035	(0.042)	0.061	(0.060)	-0.228	(0.442)	0.029	(0.236)
I[Attacker Brand Size > Victim Brand Size]	0.061	(0.041)	-0.046	(0.057)	0.848	(0.439)*	-0.116	(0.231)
|Attacker Price Index – Victim Price Index| $\times I$[Attacker Price Index > Victim Price Index]	0.061	(0.027)**	0.088	(0.041)**	-0.034	(0.362)	0.061	(0.157)
|Attacker Brand Size – Victim Brand Size| $\times I$[Attacker Brand Size > Victim Brand Size]	0.029	(0.025)	0.043	(0.038)	-0.398	(0.310)	0.230	(0.145)
Victim relative price promotion frequency	0.023	(0.018)	0.024	(0.028)	0.154	(0.210)	0.019	(0.106)
Victim relative depth of price promotions	0.004	(0.012)	-0.005	(0.017)	0.071	(0.222)	-0.046	(0.073)
Victim relative feature frequency	-0.010	(0.015)	0.013	(0.025)	0.002	(0.170)	0.026	(0.097)
Victim relative display frequency	0.019	(0.013)	0.014	(0.019)	0.016	(0.159)	-0.048	(0.084)
Attacker relative price promotion frequency	-0.042	(0.019)**	0.011	(0.029)	0.607	(0.331)*	-0.028	(0.111)
Attacker relative depth of price promotions	0.021	(0.012)*	0.026	(0.015)*	0.327	(0.277)	0.075	(0.077)
Attacker relative feature frequency	-0.009	(0.017)	-0.026	(0.023)	0.540	(0.308)*	-0.060	(0.096)
Attacker relative display frequency	0.055	(0.017)***	0.056	(0.025)**	-0.243	(0.249)	-0.039	(0.092)
Category price dispersion	-0.018	(0.020)	-0.060	(0.029)**	-0.113	(0.372)	-0.077	(0.096)
Market concentration index	-0.061	(0.018)***	-0.016	(0.027)	-0.539	(0.334)	-0.073	(0.099)
Category expensiveness	0.020	(0.020)	0.044	(0.028)	0.121	(0.525)	0.013	(0.113)
Category price promotion frequency	0.058	(0.023)**	0.011	(0.033)	-0.303	(0.418)	0.144	(0.114)
Category depth of price promotions	0.049	(0.017)***	0.071	(0.022)***	-0.029	(0.361)	0.043	(0.085)
Category feature frequency	0.022	(0.023)	0.047	(0.030)	-0.653	(0.367)*	-0.058	(0.108)
Category display frequency	0.000	(0.019)	-0.035	(0.029)	-0.129	(0.444)	-0.061	(0.111)
Utilitarian	0.080	(0.020)***	0.072	(0.028)**	1.179	(0.415)***	-0.043	(0.098)
Perishability	0.014	(0.019)	0.026	(0.026)	0.708	(0.361)*	-0.096	(0.096)

***,**,*: 99%,95%,90% highest posterior density regions do not contain zero, respectively.

[1]: I[Attacker Price Index > Victim Price Index] is an indicator that equals one if the attacker brand is more expensive than the victim brand.

Table 3: Error variances cross price effects (Σ_η^1 and Σ_η^2), posterior means with posterior standard errors in parentheses

	Immediate cross promotional price	Immediate cross regular price
Immediate cross promotional price effect	0.0272	-0.0006
	(0.0036)	(0.0131)
Immediate cross regular price effect	-0.0006	0.2753
	(0.0131)	(0.2100)

	Cumulative cross promotional price	Long run cross regular price
Cumulative cross promotional price	0.0301	0.0139
	(0.0051)	(0.0237)
Long run cross regular price	0.0139	1.1954
	(0.0237)	(0.1567)

A Bayesian estimation

In this appendix we discuss the details of the Bayesian analysis of our model. We consider some technical details and we present all used conditional distributions. For the model analysis we explicitly model the first observation, that is for the analysis we consider the exact likelihood function. As for the first observation lags are not available, we put this observation equal to the long-run equilibrium level, that is,

$$\log S_{c1} = -\Pi_c^{-1}\mu_c + \sum_{k=1}^{K} B_{ck} \log X_{ck1} + \varepsilon_{c1} \tag{10}$$

with $\varepsilon_{1c} \sim N(0, V_c)$, where V_c is the long-run variance. This variance can be obtained from solving the system by repeated substitution and is given by $V_c = \sum_{j=0}^{\infty} \Gamma_c^j \Sigma_c (\Gamma_c')^j$, where $\Gamma_c = \mathbf{I} + \Pi_c$. The variance is finite if the eigenvalues of Γ_c are within the unit circle, that is, in case of stationarity.

To derive the likelihood function, we summarize the elements of A_k and B_k which we relate to the own effects, in the K-dimensional row vectors $\alpha_{ic} = [A_{ii,ck}]_{k=1}^K$ and $\beta_{ic} = [B_{ii,ck}]_{k=1}^K$. The equations we impose for the own effects can be written as

$$\alpha_{ic} = \Lambda_1' h_{ic} + \nu_{ic}^1 \tag{11}$$

$$\beta_{ic} = \Lambda_2' h_{ic} + \nu_{ic}^2 \tag{12}$$

for $i = 1, \ldots, I_c$, where h_{ic} denotes an $L \times 1$ vector of explanatory variables. We specify $\nu_{ic}^1 \sim N(0, \Sigma_\nu^1)$ and $\nu_{ic}^2 \sim N(0, \Sigma_\nu^2)$. We collect the cross effects over different marketing instruments in the vectors $\tilde{\alpha}_{lc} = (\tilde{\alpha}_{lc1}, \ldots, \tilde{\alpha}_{lcK})'$ and $\tilde{\beta}_{lc} = (\tilde{\beta}_{lc1}, \ldots, \tilde{\beta}_{lcK})'$. We compactly write the second stage equations (6) and (7) as

$$\tilde{\alpha}_{lc} = \Theta_1' Z_{lc} + \eta_{lc}^1 \tag{13}$$

$$\tilde{\beta}_{lc} = \Theta_2' Z_{lc} + \eta_{lc}^2 \tag{14}$$

where Z_{lc} collects all variables in (6) and (7), and $\eta_{lc}^n = (\eta_{lc1}^n, \ldots, \eta_{lcK}^n)'$, $n = 1, 2$.

The likelihood function of the model is given by

$$\prod_{c=1}^{C} \int_{\alpha_c,\beta_c} \int_{\tilde{\alpha}_c,\tilde{\beta}_c} \phi(\varepsilon_{c1}; 0, V_c) \prod_{t=2}^{T_c} \phi(\varepsilon_{ct}; 0, \Sigma_c) \prod_{i=1}^{I_c} \phi(\alpha_{ic}; \Lambda_1' h_{ic}, \Sigma_\nu^1) \phi(\beta_{ic}; \Lambda_2' h_{ic}, \Sigma_\nu^2) \times$$
$$\prod_{l=1}^{I_c(I_c-1)} \phi(\tilde{\alpha}_{lc}; \Theta_1' Z_{ic}, \Sigma_\eta^1) \phi(\tilde{\beta}_{lc}; \Theta_2' Z_{lc}, \Sigma_\eta^2) d\tilde{\alpha}_c d\tilde{\beta}_c d\alpha_c d\beta_c, \tag{15}$$

where $\phi(x; \mu, \Sigma)$ is the density function of the multivariate normal distribution with mean μ and variance Σ evaluated at x, and where $\alpha_c = (\alpha_{1c}', \ldots, \alpha_{I_c c}')'$, $\beta_c = (\beta_{1c}', \ldots, \beta_{I_c c}')'$, $\tilde{\alpha}_c = (\tilde{\alpha}_{1c}', \ldots, \tilde{\alpha}_{I_c(I_c-1),c}')'$, and $\tilde{\beta}_c = (\tilde{\beta}_{1c}', \ldots, \tilde{\beta}_{I_c(I_c-1),c}')'$.

To obtain posterior results, we use the Gibbs sampling technique of Geman and Geman (1984) with data augmentation, see Tanner and Wong (1987). An introduction into the Gibbs sampler can be found in Casella and George (1992), see also Smith and Roberts (1993) and Tierney (1994). Latent variables are sampled alongside the model parameters. The Bayesian analysis is largely based on uninformative priors for the model parameters. To improve convergence of the MCMC sampler we impose inverted Wishart priors on the Σ_η^n and Σ_ν^n, $n = 1, 2$ parameters with scale parameter $\kappa_1 \mathbf{I}_K$ and degrees of freedom κ_2. We set the value of κ_1 to 1 and κ_2 equal to $K + 3$ such that the influence of the prior on the posterior distribution is marginal, see Hobert and Casella (1996) for a discussion.

Below we derive the full conditional posterior distributions of the model parameters and all latent variables. In deriving the sampling distributions we build on the general results in Zellner (1971, Chapter VIII), and those in Fok et al. (2006). Altough we use a slightly different notation here, the sampling distributions for some parameters are actually exactly the same as in Fok et al. (2006). For completeness, we however present all results here.

Sampling of Π_c

To sample Π_c we follow the same approach as in Fok et al. (2006), that is we rely on a Metropolis-Hastings [MH] sampler (Metropolis *et al.*, 1953; Hastings, 1970). As the candidate we consider the distribution that would result if we ignore the first observation, that is, we write (1) as

$$\Delta \log S_{ct} - \mu_c - \sum_{k=1}^{K} A_{ck} \Delta \log X_{ckt} = \Pi_c (\log S_{c,t-1} - \sum_{k=1}^{K} B_{ck} \Delta \log X_{ck,t-1}) + \varepsilon_{ct}. \tag{16}$$

We summarize this equation as $Y_{ct} = \Pi_c W_{ct} + \varepsilon_{ct}$, where Y_{ct} and W_{ct} denote the corresponding terms in (16). Ignoring the observation for $t = 1$ we would obtain a normal full conditional posterior distribution with mean

$$\hat{\Pi}'_c = \left(\sum_{t=2}^{T_c} W_{ct} W'_{ct} \right)^{-1} \left(\sum_{t=2}^{T_c} W_{ct} Y'_{ct} \right) \tag{17}$$

and variance

$$\hat{\Sigma}_{\Pi'_c} = \left(\Sigma_c \otimes \left(\sum_{t=2}^{T_c} W_{ct} W'_{ct} \right)^{-1} \right). \tag{18}$$

This distribution is expected to be similar to the true posterior distribution, therefore it will work well as a candidate for the M-H sampler. The sampled candidate is denoted by Π_c^{cand}. The difference in the candidate and the target density is only in the first observation. This allows us to write the acceptance-rejection probability as

$$\frac{\phi(\varepsilon_{1c}; 0, V_c)|_{\Pi_c = \Pi_c^{\text{cand}}} \phi(\Pi_c^{\text{cand}\prime}; \hat{\Pi}'_c, \hat{\Sigma}_{\Pi'_c}) \phi(\Pi_c^{\text{old}\prime}; \hat{\Pi}'_c, \hat{\Sigma}_{\Pi'_c})}{\phi(\varepsilon_{1c}; 0, V_c)|_{\Pi_c = \Pi_c^{\text{old}}} \phi(\Pi_c^{\text{old}\prime}; \hat{\Pi}'_c, \hat{\Sigma}_{\Pi'_c}) \phi(\Pi_c^{\text{cand}\prime}; \hat{\Pi}'_c, \hat{\Sigma}_{\Pi'_c})} = \frac{\phi(\varepsilon_{1c}; 0, V_c)|_{\Pi_c = \Pi_c^{\text{cand}}}}{\phi(\varepsilon_{1c}; 0, V_c)|_{\Pi_c = \Pi_c^{\text{old}}}}, \tag{19}$$

where Π_c^{old} denotes the previous draw and $\varepsilon_{c1} = \log S_{c1} + \Pi_c^{-1}\mu_c - \sum_{k=1}^{K} B_{ck} \log X_{ck1}$, see Chib and Greenberg (1995) for a similar approach in an exact likelihood analysis of an autoregressive model.

Sampling of Σ_c

Our treatment of the first observations also leads to a non standard sampling distribution for Σ_c. We use the same procedure as for Π_c, that is, we use a Metropolis-Hastings sampler where we construct the candidate ignoring the first observations. The candidate Σ_c^{cand} is therefore sampled from an inverted Wishart distribution with scale parameter $\sum_{t=2}^{T_c} \varepsilon_{ct}\varepsilon_{ct}'$ and $T_c - 1$ degrees of freedom, where $\varepsilon_{ct} = \Delta \log S_{ct} - \mu_c - \sum_{k=1}^{K} A_k \Delta \log X_{ckt} - \Pi_c(\log S_{c,t-1} - \sum_{k=1}^{K} B_{ck} \Delta \log X_{ck,t-1})$. The acceptance-rejection probability equals

$$\frac{\phi(\varepsilon_{1c}; 0, V_c)|_{\Sigma_c = \Sigma_c^{\text{cand}}}}{\phi(\varepsilon_{1c}; 0, V_c)|_{\Sigma_c = \Sigma_c^{\text{old}}}}, \tag{20}$$

where Σ_c^{old} denotes the previous draw of Σ_c.

Sampling of Λ_n and Θ_n, $n = 1, 2$

The full conditional distributions of Λ_n and Θ_n are all multivariate normal. Below we present the derivation for Λ_1. Next we summarize the parameters of the multivariate normal for Λ_2 and Θ_n, $n = 1, 2$ in a table. To sample Λ_1, we note that we can write (11) as

$$\alpha_{ic}' = h_{ic}'\Lambda_1 + \nu_{ic}^{1\,\prime}, \tag{21}$$

and hence it is a multivariate regression model with regression matrix Λ_1. Hence, the full conditional posterior distribution of Λ_1 is a matrix normal distribution with mean

$$\left(\sum_{c=1}^{C}\sum_{i=1}^{I_c} h_{ic}h_{ic}'\right)^{-1}\left(\sum_{c=1}^{C}\sum_{i=1}^{I_c} h_{ic}\alpha_{ic}\right), \tag{22}$$

and covariance matrix

$$\left(\Sigma_\nu^1 \otimes \left(\sum_{c=1}^{C}\sum_{i=1}^{I_c} h_{ic}h_{ic}'\right)^{-1}\right). \tag{23}$$

The corresponding means and covariance matrices for the other full conditional distributions are given below

Parameter	Mean	Covariance matrix
Λ_2	$\left(\sum_{c=1}^{C}\sum_{i=1}^{I_c} h_{ic}h_{ic}'\right)^{-1}\left(\sum_{c=1}^{C}\sum_{i=1}^{I_c} h_{ic}\beta_{ic}\right)$	$\left(\Sigma_\nu^2 \otimes \left(\sum_{c=1}^{C}\sum_{i=1}^{I_c} h_{ic}h_{ic}'\right)^{-1}\right)$
Θ_1	$\left(\sum_{c=1}^{C}\sum_{l=1}^{I_c(I_c-1)} Z_{lc}Z_{lc}'\right)^{-1}\left(\sum_{c=1}^{C}\sum_{l=1}^{I_c(I_c-1)} Z_{lc}\tilde{\alpha}_{lc}\right)$	$\left(\Sigma_\eta^1 \otimes \left(\sum_{c=1}^{C}\sum_{l=1}^{I_c(I_c-1)} Z_{lc}Z_{lc}'\right)^{-1}\right)$
Θ_2	$\left(\sum_{c=1}^{C}\sum_{l=1}^{I_c(I_c-1)} Z_{lc}Z_{lc}'\right)^{-1}\left(\sum_{c=1}^{C}\sum_{l=1}^{I_c(I_c-1)} Z_{lc}\tilde{\beta}_{lc}\right)$	$\left(\Sigma_\eta^2 \otimes \left(\sum_{c=1}^{C}\sum_{l=1}^{I_c(I_c-1)} Z_{lc}Z_{lc}'\right)^{-1}\right)$

Sampling of Σ_ν^n and Σ_η^n, $n = 1, 2$

To sample these covariance matrices we note that (11) to (14) are a multivariate regression models. Hence the full conditional posterior distribution of are inverted Wishart distribution with scale parameter and degrees of freedom as defined in the overview below.

Covariance	Scale parameter	Degrees of freedom
Σ_ν^1	$\kappa_1 \mathbf{I}_K + \sum_{c=1}^C \sum_{i=1}^{I_c} (\alpha_{ic} - \Lambda_1' z_{ic})(\alpha_{ic} - \Lambda_1' h_{ic})'$	$\kappa_2 + \sum_{c=1}^C I_c$
Σ_ν^2	$\kappa_1 \mathbf{I}_K + \sum_{c=1}^C \sum_{i=1}^{I_c} (\beta_{ic} - \Lambda_2' h_{ic})(\beta_{ic} - \Lambda_2' g_{ic})'$	$\kappa_2 + \sum_{c=1}^C I_c$
Σ_η^1	$\kappa_1 \mathbf{I}_K + \sum_{c=1}^C \sum_{l=1}^{I_c(I_c-1)} (\tilde{\alpha}_{lc} - \Theta_1' Z_{lc})(\tilde{\alpha}_{lc} - \Theta_1' Z_{lc})'$	$\kappa_2 + \sum_{c=1}^C I_c(I_c - 1)$
Σ_η^2	$\kappa_1 \mathbf{I}_K + \sum_{c=1}^C \sum_{l=1}^{I_c(I_c-1)} (\tilde{\beta}_{lc} - \Theta_2' Z_{lc})(\tilde{\beta}_{lc} - \Theta_2' Z_{lc})'$	$\kappa_2 + \sum_{c=1}^C I_c(I_c - 1)$

The κ terms results from the inverted Wishart priors which are used to improve convergence of our Gibbs sampler (Hobert and Casella, 1996).

Sampling of α_c and β_c

First we need to define some additional notation. We split up $X_{ckt} = (X_{ck1t}, \ldots, X_{ckI_ct})'$ for $k = 1, \ldots, K$ into two parts $X_{cit}^{\text{own}} = [X_{ckit}]_{k=1}^K$ and $X_{cit}^{\text{cross}} = [[X_{ckjt}]_{k=1}^K]_{j=1\neq i}^{I_c}$ to disentangle the own effects from the cross effects. Note that X_{cit}^{own} and X_{cit}^{cross} are both row vectors. Now further define $X_{ct}^{\text{own}} = \text{diag}(X_{c1t}^{\text{own}}, \ldots, X_{cI_ct}^{\text{own}})$ and $X_{ct}^{\text{cross}} = \text{diag}(X_{c1t}^{\text{cross}}, \ldots, X_{cI_ct}^{\text{cross}})$.

To sample α_c and β_c jointly we rewrite the second equation of (30) as

$$\begin{aligned} \log S_{c1} - \log X_{c1}^{\text{cross}} \tilde{\beta}_c + \Pi_c^{-1} \mu_c &= \begin{pmatrix} 0 & \log X_{c1}^{\text{own}} \end{pmatrix} \begin{pmatrix} \alpha_c \\ \beta_c \end{pmatrix} + \varepsilon_{c1} \\ \Delta \log S_{ct} - \mu_c - \Delta \log X_{ct}^{\text{cross}} \tilde{\alpha}_c - \Pi_c (\log S_{c,t-1} - \log X_{c,t-1}^{\text{cross}} \tilde{\beta}_c) &= \begin{pmatrix} \Delta \log X_{ct}^{\text{own}} & -\Pi_c \log X_{c,t-1}^{\text{own}} \end{pmatrix} \begin{pmatrix} \alpha_c \\ \beta_c \end{pmatrix} + \varepsilon_{ct}, \end{aligned} \tag{24}$$

which we write in matrix notation as

$$Y_{ct} = W_{ct} \begin{pmatrix} \alpha_c \\ \beta_c \end{pmatrix} + \varepsilon_{ct}, \tag{25}$$

with the obvious definitions for Y_{ct} as the left-hand side of the equation and W_{ct} the matrix appearing on the right-hand side. The variance of ε_{c1} is V_c and the variance of ε_{ct}, $t > 1$ equals Σ_c. Note that we use different definitions for W_{ct} and Y_{ct} for $t = 1$ versus $t > 1$. Next we write the I_c equations of (11) and (12) as

$$\begin{aligned} -U_c^1 &= -\mathbf{I}_{KI_c} \alpha_c + \nu_c^1 \\ -U_c^2 &= -\mathbf{I}_{KI_c} \beta_c + \nu_c^2, \end{aligned} \tag{26}$$

where U_c^n is a (KI_c)-dimensional vector containing the terms $\Lambda_n' h_{ic}$, $i = 1, \ldots, I_c$, for $n = 1, 2$ and where $\mathbf{I}_{KI_c}$ is a (KI_c)-dimensional identity matrix. The error term ν_c^n is normal distributed with mean 0 and covariance matrix $(\mathbf{I}_{I_c} \otimes \Sigma_\nu^n)$, $n = 1, 2$. To sample α_c and β_c, we combine and standardize (25) and (26)

$$
\begin{aligned}
{V_c}^{-\frac{1}{2}} Y_{c1} &= {V_c}^{-\frac{1}{2}} W_{c1} \begin{pmatrix} \alpha_c \\ \beta_c \end{pmatrix} + {V_c}^{-\frac{1}{2}} \varepsilon_{c1} \\
{\Sigma_c}^{-\frac{1}{2}} Y_{ct} &= {\Sigma_c}^{-\frac{1}{2}} W_{ct} \begin{pmatrix} \alpha_c \\ \beta_c \end{pmatrix} + {\Sigma_c}^{-\frac{1}{2}} \varepsilon_{ct} \qquad (27) \\
\begin{pmatrix} -(\mathbf{I}_{I_c} \otimes \Sigma_\nu^{1-\frac{1}{2}}) U_c^1 \\ -(\mathbf{I}_{I_c} \otimes \Sigma_\nu^{2-\frac{1}{2}}) U_c^2 \end{pmatrix} &= \begin{pmatrix} -(\mathbf{I}_{I_c} \otimes \Sigma_\nu^{1-\frac{1}{2}}) & 0 \\ 0 & -(\mathbf{I}_{I_c} \otimes \Sigma_\nu^{2-\frac{1}{2}}) \end{pmatrix} \begin{pmatrix} \alpha_c \\ \beta_c \end{pmatrix} + \begin{pmatrix} (\mathbf{I}_{I_c} \otimes \Sigma_\nu^{1-\frac{1}{2}}) \nu_c^1 \\ (\mathbf{I}_{I_c} \otimes \Sigma_\nu^{2-\frac{1}{2}}) \nu_c^2 \end{pmatrix},
\end{aligned}
$$

where $A^{-\frac{1}{2}}$ denotes the inverse of the Choleski decomposition of the matrix A, that is $A^{-\frac{1}{2}} = (A^{\frac{1}{2}})^{-1}$ with $A^{\frac{1}{2}} A^{\frac{1}{2}\prime} = A$. Hence, the full conditional posterior distribution of $(\alpha_c', \beta_c')'$ is normal with mean

$$
\left(\begin{pmatrix} (\mathbf{I}_{I_c} \otimes \Sigma_\nu^{1^{-1}}) & 0 \\ 0 & (\mathbf{I}_{I_c} \otimes \Sigma_\nu^{2^{-1}}) \end{pmatrix} + W_{c1}' V_c^{-1} W_{c1} + \sum_{t=2}^{T_c} (W_{ct}' \Sigma_c^{-1} W_{ct}) \right)^{-1}
\left(\begin{pmatrix} (\mathbf{I}_{I_c} \otimes \Sigma_\nu^{1^{-1}}) U_c^1 \\ (\mathbf{I}_{I_c} \otimes \Sigma_\nu^{2^{-1}}) U_c^2 \end{pmatrix} + (W_{c1}' V_c^{-1} Y_{c1}) + \sum_{t=2}^{T_c} (W_{ct}' \Sigma_c^{-1} Y_{ct}) \right), \qquad (28)
$$

and covariance matrix

$$
\left(\begin{pmatrix} (\mathbf{I}_{I_c} \otimes \Sigma_\nu^{1^{-1}}) & 0 \\ 0 & (\mathbf{I}_{I_c} \otimes \Sigma_\nu^{2^{-1}}) \end{pmatrix} + W_{c1}' V_c^{-1} W_{c1} + \sum_{t=2}^{T_c} (W_{ct}' \Sigma_c^{-1} W_{ct}) \right)^{-1} \qquad (29)
$$

Sampling of μ_c, $\tilde{\alpha}_c$ and $\tilde{\beta}_c$

Equation (10) and (1) can now be written as

$$
\begin{aligned}
\log S_{c1} - \log X_{c1}^{\text{own}} \beta_c &= \begin{pmatrix} -\Pi_c^{-1} & 0 & \log X_{c1}^{\text{cross}} \end{pmatrix} \begin{pmatrix} \mu_c \\ \tilde{\alpha}_c \\ \tilde{\beta}_c \end{pmatrix} + \varepsilon_{c1} \\
\Delta \log S_{ct} - \Delta \log X_{ct}^{\text{own}} \alpha_c & \qquad (30) \\
-\Pi_c(\log S_{c,t-1} - \log X_{c,t-1}^{\text{own}} \beta_c) &= \begin{pmatrix} \mathbf{I}_{I_c} & \Delta \log X_{ct}^{\text{cross}} & -\Pi_c \log X_{c,t-1}^{\text{cross}} \end{pmatrix} \begin{pmatrix} \mu_c \\ \tilde{\alpha}_c \\ \tilde{\beta}_c \end{pmatrix} + \varepsilon_{ct},
\end{aligned}
$$

where $\tilde{\alpha}_c$ and $\tilde{\beta}_c$ capture the cross-effects in the matrices A_{ck} and B_{ck} for $k = 1, \ldots, K$. This system can be written in a multivariate regression model

$$Y_{ct} = W_{ct}\gamma + \varepsilon_{ct}, \tag{31}$$

where Y_{ct} contains the left-hand side of (30), W_{ct} contains $(-\Pi_c^{-1}\vdots 0 \vdots \log X_{c1}^{\text{cross}})$ for the first observation and $(\mathbf{I}_c \vdots \Delta \log X_{ct}^{\text{cross}} \vdots -\Pi_c \log X_{c,t-1}^{\text{cross}})$ for the remaining observations, and where $\gamma = (\mu_c', \tilde{\alpha}_c', \tilde{\beta}_c')'$. The error term is normal distributed with mean 0 and covariance matrix Σ_c (and V_c for the first observation).

In the same manner as before we collect and standardize the information from the second layer of the hierarchical model.

$$\begin{aligned} -\tilde{U}_c^1 &= -\mathbf{I}_{KI_c(I_c-1)}\tilde{\alpha}_c + \eta_c^1 \\ -\tilde{U}_c^2 &= -\mathbf{I}_{KI_c(I_c-1)}\tilde{\beta}_c + \eta_c^2, \end{aligned} \tag{32}$$

where $\tilde{U}_c^n$ stacks the $I_c(I_c - 1)$ vectors $\Theta_n' Z_{lc}$, $n = 1, 2$.

$$\begin{aligned} V_c^{-\frac{1}{2}} Y_{c1} &= V_c^{-\frac{1}{2}} W_{c1}\gamma + V_c^{-\frac{1}{2}}\varepsilon_{c1} \\ \Sigma_c^{-\frac{1}{2}} Y_{ct} &= \Sigma_c^{-\frac{1}{2}} W_{ct}\gamma + \Sigma_c^{-\frac{1}{2}}\varepsilon_{ct} \\ \begin{pmatrix} -(\mathbf{I}_{I_c(I_c-1)} \otimes \Sigma_\eta^{1-\frac{1}{2}})\tilde{U}_c^1 \\ -(\mathbf{I}_{I_c(I_c-1)} \otimes \Sigma_\eta^{2-\frac{1}{2}})\tilde{U}_c^2 \end{pmatrix} &= \begin{pmatrix} 0 & -(\mathbf{I}_{I_c(I_c-1)} \otimes \Sigma_\eta^{1-\frac{1}{2}}) & 0 \\ 0 & 0 & -(\mathbf{I}_{I_c(I_c-1)} \otimes \Sigma_\eta^{2-\frac{1}{2}}) \end{pmatrix}\gamma + \\ &\quad + \begin{pmatrix} (\mathbf{I}_{I_c(I_c-1)} \otimes \Sigma_\eta^{1-\frac{1}{2}})\eta_c^1 \\ (\mathbf{I}_{I_c(I_c-1)} \otimes \Sigma_\eta^{2-\frac{1}{2}})\eta_c^2 \end{pmatrix}. \end{aligned} \tag{33}$$

Hence, the full conditional distribution of γ is normal with mean

$$\left(\begin{pmatrix} 0 & 0 & 0 \\ 0 & (\mathbf{I}_{I_c(I_c-1)} \otimes \Sigma_\eta^{1^{-1}}) & 0 \\ 0 & 0 & (\mathbf{I}_{I_c(I_c-1)} \otimes \Sigma_\eta^{2^{-1}}) \end{pmatrix} + W_{c1}' V_c^{-1} W_{c1} + \sum_{t=2}^{T_c} (W_{ct}' \Sigma_c^{-1} W_{ct}) \right)^{-1} \left(\begin{pmatrix} 0 \\ (\mathbf{I}_{I_c(I_c-1)} \otimes \Sigma_\eta^{1^{-1}})\tilde{U}_c^1 \\ (\mathbf{I}_{I_c(I_c-1)} \otimes \Sigma_\eta^{2^{-1}})\tilde{U}_c^2 \end{pmatrix} + (W_{c1}' V_c^{-1} Y_{c1}) + \sum_{t=2}^{T_c} (W_{ct}' \Sigma_c^{-1} Y_{ct}) \right), \tag{34}$$

and covariance matrix

$$\left(\begin{pmatrix} 0 & 0 & 0 \\ 0 & (\mathbf{I}_{I_c(I_c-1)} \otimes \Sigma_\eta^{1^{-1}}) & 0 \\ 0 & 0 & (\mathbf{I}_{I_c(I_c-1)} \otimes \Sigma_\eta^{2^{-1}}) \end{pmatrix} + W_{c1}' V_c^{-1} W_{c1} + \sum_{t=2}^{T_c} (W_{ct}' \Sigma_c^{-1} W_{ct}) \right)^{-1}. \tag{35}$$

B The definition of explanatory variables ($z_{ij,c}$)

In this appendix we describe the characteristics that we use to explain the cross-price effects. If necessary we give a formal mathematical definition. We organize the characteristics based on the level at which they are defined (category level, brand level or both) and the concept they measure (eg. competitive intensity). For the sake of comparion we closely follow Fok *et al.* (2006) for the choice and definitions of the variables.

In this appendix we use the following notation:

S_{ict}	Sales volume of brand i in category c at time t
$M_{ict} = S_{ict}/\sum_{i=1}^{I_c} S_{ict}$	Market share of brand i at time t
$\overline{M}_{ic} = \frac{1}{T_c}\sum_{t=1}^{T_c} M_{ict}$	(Time) average market share
P_{ict}	(Actual) price of brand i in category c at time t
RP_{ict}	Regular price of brand i in category c at time t
$\overline{RP}_c = \frac{1}{I_c T_c}\sum_{t=1}^{T_c}\sum_{i=1}^{I_c} RP_{ict}$	Average regular price in category c
$PI_{ict} = P_{it}/RP_{it}$	(Promotional) Price index

The explanatory variables are defined as follows

Category-Specific Characteristics

Average budget share	$\frac{1}{T_c}\sum_{t=1}^{T_c}\sum_{i=1}^{I_c} S_{ict}P_{ict}$
Utilitarian	low (0), middle (0.5), high (1) (defined in Fok *et al.* (2006))
Perishability	low (0), middle (0.5), high (1) (defined in Fok *et al.* (2006))
Market concentration	$\sum_{i=1}^{I_c} \overline{M}_{ic} \log \overline{M}_{ic}$, see Raju (1992)
Price dispersion	$\sum_{t=1}^{T_c} \left(\max_i \left(RP_{ict}\right) - \min_i \left(RP_{ict}\right)\right) / \left(T_c \cdot \overline{RP}_c\right)$

Category- and Brand-Specific Characteristics

Price promotion frequency	percentage of observations where price index is below 0.95.
Depth of price promotions	$\frac{\sum_{t=1}^{T_c} \log(PI_{it})}{FREQ_{ic}}$, where $FREQ_{ic}$ denotes the price promotion frequency of brand i in category c.
Feature/Display frequency (brand level)	average of the percentage of SKUs promoted by the brand over time.
Feature/Display frequency (category level)	$\frac{\sum_{t=1}^{T_c} 1-\prod_{i=1}^{I_c}(1-x_{it})}{T_c}$, where x_{ict} denotes the percentage of SKUs promoted by brand i in category c at time t.

Brand-Specific Characteristics

Similarity of price	Absolute distance between attacker and victim brand's average regular price relative to the average category regular price
Similarity of size	absolute distance between attacker and victim average market share
Asymmetric price effect	indicator variable that equals one if the attacker brand is more expensive than the victim brand, zero otherwise.
Asymmetric size effect	indicator variable that equals one if the attacker brand has a higher average market share than the victim brand, zero otherwise.

References

Ailawadi KL, Gedenk K, Lutzky C, and Neslin SA. 2007. Decomposition of the Sales Impact of Promotion-induced Stockpiling. *Journal of Marketing Research* **44**: 450–467.

Allenby GM and Rossi PE. 1991. Quality Perceptions and Asymmetric Switching Between Brands. *Marketing Science* **10**: 185–204.

Anderson ET and Simester DI. 2004. Long-Run Effects of Promotion Depth on New Versus Established Customers: Three Field Studies. *Marketing Science* **23**: 4–20.

Bell DR, Chiang J, and Padmanabhan V. 1999. The Decomposition of Promotional Response: An Empirical Generalization. *Marketing Science* **18**: 504–526.

Bijmolt TH, Van Heerde HJ, and Pieters RG. 2005. New Empirical Generalizations on the Determinants of Price Elasticity. *Journal of Marketing Research* **42**: 141–156.

Blattberg RC, Briesch R, and Fox EJ. 1995. How Promotions Work. *Marketing Science* **14**: G122–G132.

Blattberg RC and Neslin SA. 1989. Sales Promotion: The Long and the Short of it. *Marketing Letters* **1**: 81–97.

Blattberg RC and Wisniewski KJ. 1989. Price-Induced Patterns of Competition. *Marketing Science* **8**: 291–309.

Bronnenberg BJ and Wathieu L. 1996. Asymmetric Promotion Effects and Brand Positioning. *Marketing Science* **15**: 379–394.

Bucklin RE and Gupta S. 1999. Commercial Use of UPC Scanner Data: Industry and Academic Perspectives. *Marketing Science* **18**: 247–273.

Casella G and George EI. 1992. Explaining the Gibbs Sampler. *American Statistician* **46**: 167–174.

Chib S and Greenberg E. 1995. Understanding the Metropolis-Hastings Algorithm. *American Statistician* **49**: 327–335.

Ehrenberg ASC, Goodhardt GJ, and Barwise TP. 1990. Double Jeopardy Revised. *Journal of Marketing* **54**: 82–91.

Fader PS and McAlister L. 1990. An Elimination by Aspects Model of Consumer Response to Promotion Calibrated on UPC Scanner Data. *Journal of Marketing Research* **27**: 322–332.

Fok D, Horváth C, Paap R, and Franses P. 2006. A Hierarchical Bayes Error Correction Model to Explain Dynamic Effects of Price Changes. *Journal of Marketing Research* **43**: 443–461.

Geman S and Geman D. 1984. Stochastic Relaxations, Gibbs Distributions, and the Bayesian Restoration of Images. *IEEE Transaction on Pattern Analysis and Machine Intelligence* **6**: 721–741.

Hardie B, Johnson E, and Fader P. 1993. Modeling Loss Aversion and Reference Dependence Effects on Brand Choice. *Marketing Science* **12**: 378–394.

Hastings WK. 1970. Monte Carlo Sampling Methods Using Markov Chains and Their Applications. *Biometrika* **57**: 97–109.

Hobert JP and Casella G. 1996. The Effect of Improper Priors on Gibbs Sampling in Hierarchical Linear Mixed Models. *Journal of the American Statistical Association* **91**: 1461–1473.

Horváth C, Leeflang PSH, Wieringa JE, and Wittink DR. 2005. Competitive reaction- and feedback effects based on VARX models of pooled store data. *International Journal of Research in Marketing* **22**: 415–426.

Jedidi K, Mela CF, and Gupta S. 1999. Managing Advertising and Promotion for Long-Run Profitability. *Marketing Science* **18**: 1–22.

Kamakura W and Russell G. 1989. A probabilistic Choice Model for Market Segmentation and Elasticity Structure. *Journal of Marketing Research* **26**: 379–390.

Keller KL. 1993. Conceptualizing, Measuring, and Managing Customer-Based Brand Equity. *Journal of Marketing* **57**: 1–22.

Kopalle PK, Mela CF, and Marsh L. 1999. Dynamic Effect of Discounting on Sales: Empirical Analysis and Normative Pricing Implications. *Marketing Science* **18**: 317–332.

Macé S and Neslin SA. 2004. The Determinants of Pre- and Postpromotion Dips in Sales of Frequently Purchased Goods. *Journal of Marketing Research* **41**: 339–350.

Martin GR. 1973. The Theory of Double Jeopardy. *Journal of the Academy of Marketing Science* **1**: 148–155.

Mehta N, Rajuv S, and Srinivasan K. 2003. Price Uncertainty and Consumer Search: A Structural Model for Consideration Set Formation. *Marketing Science* **22**: 58–54.

Mela CF, Gupta S, and Jedidi K. 1998. Assessing Long-Term Promotional Influences on Market Structure. *International Journal of Research in Marketing* **15**: 89–107.

Metropolis N, Rosenbluth AW, Rosenbluth MN, Teller AH, and Teller E. 1953. Equation of State Calculations by Fast Computing Machines. *Journal of Chemical Physics* **21**: 1087–1092.

Narasimhan C, Williams JD, and Leone RP. 1996. Promotional Elasticities and Category Characteristics. *Journal of Marketing* **60**: 427–446.

Nijs VR, Dekimpe MG, Steenkamp JBEM, and Hanssens DM. 2001. The Category- Demand Effects of Price Promotions. *Marketing Science* **20**: 1–22.

Papatla P and Krishnamurthi L. 1996. Measuring Dynamic Effects of Promotions on Brand Choice. *Journal of Marketing Research* **33**: 20–35.

Pauwels K, Hanssens DM, and Siddarth S. 2002. The Long-Term Effect of Price Promotions on Category Incidence, Brand Choice, and Purchase Quality. *Journal of Marketing Research* **39**: 421–439.

Raju JS. 1992. The Effect of Price Promotions on Variability in Product Category Sales. *Marketing Science* **11**: 207–220.

Rao AG and Miller PB. 1975. Advertising/Sales Response Functions. *Journal of Advertising Research* **15**: 7–15.

Rao RC. 1991. Pricing and Promotions in Asymmetric Duopolies. *Marketing Science* **10**: 131–144.

Russel G. 1992. A Model of Latent Symmetry in Cross-Price Elasticities. *Marketing Letters* **3**: 157–169.

Sethuraman R. 1995. A Meta-Analysis of National Brand and Store Brand Cross-Promotional Pprice Elasticities. *Marketing Letters* **6**: 275–286.

Sethuraman R and Srinivasan V. 2002. The Asymmetric Share Effect: An Empirical Generalization on Cross-Price Effects. *Journal of Marketing Research* **39**: 379–386.

Sethuraman R, Srinivasan V, and Kim D. 1999. Asymmetric and Neighborhood Cross-Price Effects: Some Empirical Generalizations. *Marketing Science* **18**: 23–41.

Smith AFM and Roberts GO. 1993. Bayesian Computing via the Gibbs Sampler and Related Markov Chain Monte Carlo Methods. *Journal of the Royal Statistical Society B* **55**: 3–23.

Srinivasan S, Pauwels K, Hanssens DM, and Dekimpe MG. 2004. Do Promotions Benefit Manufacturers, Retailers, or Both? *Management Science* **50**: 617–629.

Tanner MA and Wong WH. 1987. The Calculation of Posterior Distributions by Data Augmentation. *Journal of the American Statistical Association* **82**: 528–550.

Teng L. 2008. A Comparison of Two Types of Price Discounts in Shifting Consumers' Attitudes and Purchase Intentions. *Journal of Business Research* .

Tierney L. 1994. Markov Chains for Exploring Posterior Distributions. *Annals of Statistics* **22**: 1701–1762.

Van Heerde HJ, Gupta S, and Wittink DR. 2003. Is 75% of the Sales Promotion Dump Due to Brand Switching? No, only 33% Is. *Journal of Marketing Research* **40**: 481–491.

Wakefield KL and Inman JJ. 2003. Situational price sensitivity: the role of consumption occasion, social context and income. *Journal of Retailing* **79**: 199–212.

Yoo B, Donthu N, and Lee S. 2000. An Examination of Selected Marketing Mix Elements and Brand Equity. *Journal of the Academy of Marketing Science* **28**: 195–211.

Zellner A. 1971. *An Introduction to Bayesian Inference in Econometrics*. Wiley, New York.

Zenor MJ, Bronnenberg BJ, and McAlister L. 1998. The Impact of Marketing Policy on Promotional Price Elasticities and Baseline Sales. *Journal of Retailing and Consumer Services* **5**: 25–32.

Zhang J. 2006. An Integrated Choice Model Incorporating Alternative Mechanisms for Consumers'Reactions to In-Store Display and Feature Advertising. *Marketing Science* **25**: 278–290.

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