Preference Heterogeneity and Optimal Capital Taxation

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Abstract

We analytically and quantitatively examine a prominent justification for capital income taxation: goods preferred by those with high ability ought to be taxed. We study an environment where commodity taxes are allowed to be nonlinear functions of income and consumption and find that, when ability is positively related to a preference for a good, optimal marginal commodity taxes on this good may be regressive: i.e., declining with income. We derive an analytical expression for optimal commodity taxation, allowing us to study the forces for and against regressivity. We then parameterize the model to evidence on the relationship between skills and preferences and examine the quantitative case for taxes on future consumption (saving). The relationship between skill and time preference delivers quantitatively small, generally regressive capital income taxes and would justify only a fraction of the prevailing level of capital income taxation.

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Introduction

One justification for positive capital income taxation is that the goods preferred by high-ability individuals ought to be taxed because the consumption of these goods provides a signal of individuals' otherwise unobservable ability.[1,2] If individuals' abilities are positively related to preferences for saving, this argument implies that capital income should be taxed. Two prominent expositions of this justification are Saez (2002) and Banks and Diamond (2009). Saez shows that a small linear tax on a commodity preferred by individuals with higher ability generates a smaller efficiency loss than does an increase in the optimal nonlinear income tax that raises the same revenue from each individual. He applies this logic to capital income taxation and concludes "...the discount rate δ is probably negatively correlated with skills. This suggests that interest income ought to be taxed even in the presence of a non-linear optimal earnings tax." Banks and Diamond (2009) is the chapter on direct taxation in the Mirrlees Review. Commissioned by the Institute for Fiscal Studies, the Review is the successor to the influential Meade Report of 1978 and is the authoritative summary of the current state of tax theory as it relates to policy. Their chapter concludes:

> "With the plausible assumption that those with higher earnings abilities discount the future less (and thus save more out of any given income), then taxation of saving helps with the equity-efficiency tradeoff by being a source of indirect evidence about who has higher earnings abilities and thus contributes to more efficient redistributive taxation."

In this paper, we analytically and quantitatively study this justification for taxing goods preferred by those with high ability, in particular future consumption (i.e., saving), when commodity taxes are allowed to be nonlinear functions of both income and consumption.[3]

We first derive analytical expressions that indicate the shape of optimal commodity taxation. We start in a two-type, two-commodity economy and demonstrate that the high ability type faces no distortion to its chosen commodity basket while the low type faces a distortion away from consumption of the good preferred by the high type. In other words, marginal taxes are regressive on the good preferred by those with high ability.[4] We then derive the condition describing optimal commodity taxes in an economy with two goods and a continuum of types where the relative preference for one good rises with ability. The marginal commodity tax on the good preferred by the able is again equal to zero for the highest type, and it is positive for lower-ability types. As is common in Mirrleesian models (e.g., Saez 2001) we then analytically study the forces for and against regressivity in the tax on the good preferred by the able. The intuition for why regressive commodity taxation on such a good may be optimal starts with the realization that the goal of optimal tax

[1] This result originated in Mirrlees (1976) and also appears in Mirrlees (1986). Nearly all comprehensive treatments of modern tax policy contain a section on this result as a deviation from the standard Atkinson-Stiglitz (1976) recommendation of uniform commodity taxation. Tuomala (1991) writes "...the marginal tax rates on commodities that the more able people tend to prefer should be greater;" Salanie (2003) warns "If there is a positive correlation between the taste for fine wines and productivity, then fine wines should be taxed relatively heavily (God Forbid!);" while Kaplow (2008) argues "it tends to be optimal to impose a heavier burden on commodities preferred by the more able and a lighter burden on those preferred by the less able." Enthusiasm for this result may be because, as Mirrlees put it "This prescription is most agreeable to common sense."

[2] A different justification for the positive capital wedge is the New Dynamic Public Finance literature (see e.g., Golosov, Kocherlakota, and Tsyvinski 2003; Golosov, Tsyvinski and Werning 2006; Kocherlakota 2009).

[3] Though most research on this issue has focused on the linear tax problem, Mirrlees (1976, 1986) is clear that his results apply to *nonlinear* marginal commodity tax rates. A few later authors also noted the potential for optimal nonlinear rates: e.g., Kaplow (2008). Banks and Diamond (2009) look for but find no work on the nonlinear problem. They write: "In the context of this issue, how large the tax on capital income should be and how the marginal capital income tax rates should vary with earnings levels has not been explored in the literature that has been examined."

[4] The flip side of this regressive tax on the good preferred by the high ability is a progressive subsidy on the good not preferred by the high ability. Throughout the paper, we describe the optimal policy in terms of its effects on the goods preferred by the high ability, in keeping with the existing literature's emphasis on capital taxation.

policy (in the Mirrleesian framework) is to redistribute from high-ability workers without discouraging their work effort. With this as the goal, the optimal use of commodity taxation is to increase the attractiveness of earning a high income. Commodity taxes that are regressive (i.e., that fall with income) on those goods most valued by high-ability individuals will encourage them to earn more, allowing the tax authority to levy higher income taxes on them and redistribute more resources to those with lower ability.[5]

The second objective of the paper is to examine the quantitative case for capital income taxation. We use data from the National Longitudinal Survey of Youth (NLSY) to show a positive correlation between ability[6] and relative preference for future consumption. Using these data to estimate a mean value for time preference by ability quantile, we find that optimal capital income tax rates are regressive but quantitatively small. For the baseline example, the maximal capital income tax rate in the nonlinear case is everywhere less than 6%, and the constrained-optimal linear capital income tax rate is 3.1%. Moreover, welfare gains from these optimal capital income taxes are negligible.[7] These results suggest that the empirical relationship between ability and time preference justifies only a small fraction of the prevailing level of capital income taxation in developed economies.

This paper also studies the importance of preference normalization in our optimal taxation model. We normalize preferences over commodities in two ways. These normalizations are similar to two assumptions made by Saez (2002) in his analysis of optimal commodity taxes with preference heterogeneity. First, we normalize preferences to eliminate any incentive for the planner to redistribute across agents based simply on their preferences over goods. Specifically, the marginal social value to a Utilitarian planner of allocating resources to an undistorted individual is independent of that individual's preferences over consumption goods. We also normalize preferences in a second way. We model preferences over commodities, including future and current consumption, as having no direct effect on the labor supply decisions of individuals. This normalization contrasts with the approach in recent work by Diamond and Spinnewijn (2009), who model preferences such that more patient individuals are more willing to work. Because the challenge of optimal tax policy is to encourage the high-ability to work despite redistributive taxation, our normalization increases the role for capital taxation as part of the optimal policy.

Finally, we extend our analysis of optimal capital taxation to a stochastic setting in which there is a relationship between ex post ability and preferences over goods consumed within a period. We show that this relationship has ambiguous effects on optimal intertemporal distortions.

The paper proceeds as follows. Section 1 provides an illustrative example of our theoretical results in an economy with two ability types and heterogeneity in preferences over two goods. Section 2 specifies a general model of optimal taxation with heterogeneity in ability and preferences and derives conditions on the optimal policy. In Section 3, we parameterize the model with data from the NLSY on heterogeneous preferences for consumption over time and calculate the optimal taxes for these data. We also test the robustness of our results to variation in individual risk aversion and labor supply elasticity. In Section 4, we compare these results to prevailing capital income tax rates in developed economies and characterize the relationship between ability and time preference that would be required for prevailing rates to be optimal. Section 5 discusses the importance of preference normalization in these models. Section 6 considers the

[5] The standard argument against nonlinear commodity taxation is arbitrage or retrading (see Hammond 1987, Golosov and Tsyvinski 2006). That may be an appropriate restriction for many goods, but important categories of personal expenditure can feasibly be taxed nonlinearly or as a function of income.

[6] We measure ability by the survey respondent's score on the cognitive ability portion of the Armed Forces Qualification Test (AFQT). While it is impossible to measure ability perfectly, the AFQT score is commonly used, such as in the study of the returns to education.

[7] If we took into account variation around mean preference values within ability levels, the optimal taxes and welfare gains are likely to be even smaller.

dynamic, stochastic model. An Appendix contains technical details referred to in the text.

1 A simple example

In this section we provide a simple example that captures the main intuition behind the more general model.[8] We show that, in this setting, the optimal commodity tax is regressive for the good preferred by those with high ability. In particular, the relative tax is positive on this good for the low-ability individual, while the high-ability individual faces no distortion.

Consider an economy populated by a continuum of measure 1 of two types of individuals $i = \{1, 2\}$, where the size of each group is equal to 1/2. These individuals differ in wage w^i, where $w^2 > w^1$. The wage is private information to the agent. Suppose there are two commodities, c_1 and c_2. The utility function for an individual with wage w^i is given by:

$$u\left(c_1^i, c_2^i, \frac{y^i}{w^i}\right).$$

The planner's problem is to specify consumption and income allocations for each individual to maximize a Utilitarian social welfare function.

Problem 1 *Planner's problem in two-type example*

$$\max_{\left\{c_1^i, c_2^i, y^i\right\}_{i=1,2}} \sum_i u\left(c_1^i, c_2^i, \frac{y^i}{w^i}\right) \tag{1}$$

subject to

$$u\left(c_1^2, c_2^2, \frac{y^2}{w^2}\right) \geq u\left(c_1^1, c_2^1, \frac{y^1}{w^2}\right), \tag{2}$$

$$\sum_i y^i - c_1^i - c_2^i \leq 0. \tag{3}$$

Constraint (2) is an incentive compatibility constraint stating that an individual of type $i = 2$ prefers the consumption and income bundle intended for it by the planner $\left\{c_1^2, c_2^2, y^2\right\}$ to a bundle $\left\{c_1^1, c_2^1, y^1\right\}$ allocated to an individual of type $i = 1$.[9] Constraint (3) is feasibility, where we assume that the marginal rate of transformation of commodities is equal to 1.

Let u_n be the partial derivative of $u(c_1, c_2, l)$ with respect to the n^{th} argument. Note that these partial derivatives may depend on the wage rate.[10] Let μ be the multiplier on constraint (2). Using the first order conditions for consumption in the above problem, we obtain the following expressions for an individual of type $i = 2$:

$$\frac{u_1\left(c_1^2, c_2^2, \frac{y^2}{w^2}\right)}{u_2\left(c_1^2, c_2^2, \frac{y^2}{w^2}\right)} = 1, \tag{4}$$

[8] Similar examples are found in Diamond (2007) and Diamond and Spinnewijn (2008). However, as discussed in Section 6, we normalize preferences in important ways that these other examples do not. This normalization has direct effects on the optimal policies we derive.

[9] Writing this constraint we assumed that only an individual of type $i = 2$ can misrepresent his type. This is easy to ensure if the ratio w^2/w^1 is high enough.

[10] For example, using the utility function from the general model stated later, (18), $u_1\left(c_1^i, c_2^i, l^i\right) = \frac{\alpha(w^i)}{1+\alpha(w^i)} \frac{1}{c_1^i}$.

and for the individual of type $i = 1$:

$$\frac{u_1\left(c_1^1, c_2^1, \frac{y^1}{w^1}\right)}{u_2\left(c_1^1, c_2^1, \frac{y^1}{w^1}\right)} = \frac{1 - \frac{u_2\left(c_1^1, c_2^1, \frac{y^1}{w^2}\right)}{u_2\left(c_1^1, c_2^1, \frac{y^1}{w^1}\right)}\mu}{1 - \frac{u_1\left(c_1^1, c_2^1, \frac{y^1}{w^2}\right)}{u_1\left(c_1^1, c_2^1, \frac{y^1}{w^1}\right)}\mu}. \tag{5}$$

Equation (4) shows that the consumption choices of the high-ability individual are undistorted. The marginal rate of substitution $\frac{u_1(\cdot)}{u_2(\cdot)}$ is equal to the marginal rate of transformation. Equation (5) shows that if the multiplier μ on the incentive compatibility constraint is not equal to zero, then the consumption choices of the low-ability individual are distorted. In particular, if an individual's ratio $\frac{u_1}{u_2}$ is less than 1, the policy has caused him to consume more of good 1 relative to good 2 than he would have chosen in autarky.

Now, suppose we impose a condition requiring that if all individuals are given the same consumption and income allocation, the marginal utility of good 2 relative to good 1 is higher for the high-ability individual j (type 2) than for the low-ability individual i (type 2).

Assumption 1 *If* $w^j > w^i$:

$$\frac{u_2\left(c_1, c_2, \frac{y}{w^j}\right)}{u_1\left(c_1, c_2, \frac{y}{w^j}\right)} > \frac{u_2\left(c_1, c_2, \frac{y}{w^i}\right)}{u_1\left(c_1, c_2, \frac{y}{w^i}\right)} \tag{6}$$

for any (c_1, c_2, y).

We now can summarize the argument in a proposition characterizing the distortions in the optimal allocation.

Proposition 2 *Suppose that* $\left\{c_1^i, c_2^i, y^i\right\}_{i=1,2}$ *is an optimal allocation solving (1). Then the optimal choice of consumption for the high-ability individual is not distorted. Suppose that Assumption 1 holds. Then the optimal choice of consumption for the low-ability agent is distorted away from good 2 in favor of good 1:*

$$\frac{u_1\left(c_1^1, c_2^1, \frac{y^1}{w^1}\right)}{u_2\left(c_1^1, c_2^1, \frac{y^1}{w^1}\right)} < 1.$$

This Proposition states that if good 2 is particularly enjoyed by high-ability workers, the planner should impose a distortion (i.e., a positive relative tax) on the consumption of good 2 by the low-ability workers (but not on consumption of that good by high-ability workers).[11] The intuition for this result is as follows. The planner wants to discourage a high-ability individual from deviating and claiming that he is a low type. A high-ability agent will find deviating less attractive if doing so will cause him to face a positive relative tax on the good that he values highly. The cost of such a positive relative tax is a distortion in the consumption choices by the low-ability agent. Assumption 1 ensures that the costs of such distortion are smaller than the gain from relaxing the incentive compatibility constraint.

It is important to be clear that this result depends on preferences varying by ability level, not income. In particular, it does not apply to goods with an income elasticity of demand greater than 1 but for which preferences are unrelated to ability. For those goods, the inequality in (6) would be an equality because each type would have the same ratio of marginal utilities given the same consumption and income bundle. Instead, the case for regressive taxes requires the high-ability to prefer good 2 even when at the same income level as the low-ability.

[11] Mirrlees (1986) shows a similar relationship between the marginal rates of substitution by ability level and optimal distortions. He does not characterize the optimal pattern of these distortions, however, our central goal in this paper.

2 Model

In this section, we set up a model with a continuum of ability types, as in the classic Mirrlees (1971) framework. We derive a formula for optimal relative commodity taxes that are allowed to be nonlinear in consumption and to depend on income. To capture preference heterogeneity, we assume that preferences across consumption goods are a function of ability. This simplifies the planner's problem by retaining a single dimension of heterogeneity: two or more dimensions introduce multiple screening problems for which a tractable analytical approach has not been developed.[12]

There is a continuum of measure one of individual agents. We index agents by $i \in [0,1]$. Individuals differ in their abilities, which we measure with their wages, denoted by w^i and distributed according to the density function $f(w)$ over the interval $\{w_{\min}, w_{\max}\}$. Ability is private information to the agent. The utility function of an individual depends on $\alpha\left(w^i\right)$, so that the preference parameter for an individual depends directly on his or her wage.

Each individual maximizes the utility function:

$$U\left(w^i\right) = u\left(c_1^i, c_2^i, l^i, \alpha\left(w^i\right)\right). \tag{7}$$

Note that utility is a function of the consumption of good 1, c_1, and the consumption of good 2, c_2, as well as of labor effort l, and the preference parameter $\alpha\left(w^i\right)$. Superscripts i on consumption and labor denote the values of these variables for the individual of wage w^i.

A social planner maximizes a utilitarian social welfare function. The planner offers incentive compatible triplets of $\left\{c_1^i, c_2^i, y^i\right\}_i$.

Problem 3

$$\max_{\left\{c_1^i, c_2^i, y^i\right\}_i} \int_{w_{\min}}^{w_{\max}} U\left(w^i\right) f\left(w^i\right) dw^i \tag{8}$$

subject to

$$\int_{w_{\min}}^{w_{\max}} \left(y^i - c_1^i - c_2^i\right) f\left(w^i\right) dw^i \geq 0, \tag{9}$$

and

$$u\left(c_1^i, c_2^i, \frac{y^i}{w^i}, \alpha\left(w^i\right)\right) \geq u\left(c_1^j, c_2^j, \frac{y^j}{w^i}, \alpha\left(w^i\right)\right), \tag{10}$$

for all i, j.

Constraint (10) is the incentive compatibility constraint stating that an individual of type i prefers the consumption and income allocation intended for it by the planner to an allocation intended for an individual of type j.

Solving the planner's problem in equations (8) through (10) can yield insights into the wedges that optimal policy drives into private optimization.

It is standard to rewrite the planner's problem with explicit tax functions. In this alternative formalization of the problem, each individual maximizes the utility function (7) subject to the individual's after-tax budget constraint,

$$l^i w^i - T\left(w^i l^i\right) - \left(c_1^i + t^1\left(w^i l^i, c_1^i\right)\right) - \left(c_2^i + t^2\left(w^i l^i, c_2^i\right)\right) \geq 0. \tag{11}$$

[12] See Kleven, Kreiner, and Saez (2009), Tarkiainen and Tuomala (2007), and Judd and Su (2008) for discussions of the approach to optimal taxation with multi-dimensional heterogeneity.

The budget constraint requires careful examination. The nonlinear income tax $T\left(w^i l^i\right)$ is a continuous, differentiable function of income $y^i = w^i l^i$. The two other tax functions, $t^1\left(w^i l^i, c_1^i\right)$ and $t^2\left(w^i l^i, c_2^i\right)$, are commodity tax functions that we also assume to be continuous and differentiable. Importantly, note that we explicitly allow for the taxation of each commodity to be nonlinear in consumption of that good and to depend on income.[13] The budget constraint (11) has the multiplier μ.

To characterize optimal taxes with this formalization of the planner's problem, we follow the formal techniques of the Mirrleesian literature. In particular, we consider the following social planner's problem:

Problem 4 *Planner's Problem*

$$\max_{\left\{c_1^i, c_2^i, l^i\right\}_i} \int_{w_{\min}}^{w_{\max}} U\left(w^i\right) f\left(w^i\right) dw^i \tag{12}$$

subject to feasibility

$$\int_{w_{\min}}^{w_{\max}} \left(w^i l^i - c_1^i - c_2^i\right) f\left(w^i\right) dw^i \geq 0, \tag{13}$$

and incentive compatibility, which is that each individual maximizes (7) *subject to* (11) *given tax policies* $T\left(w^i l^i\right)$, $t^1\left(w^i l^i, c_1^i\right)$, *and* $t^2\left(w^i l^i, c_2^i\right)$.

In words, the social planner chooses a tax system to maximize Utilitarian social welfare subject to a budget constraint that assumes no government spending for simplicity. The government must also take into account that each individual will choose labor supply to maximize his or her utility subject to the specified tax system.

2.1 The optimal commodity choice wedge

We now derive a formula for the optimal commodity wedge, i.e., the wedge distorting commodity choices.[14] We formulate the Hamiltonian from the planner's problem above. The Hamiltonian includes the following differential constraint:

$$\frac{\partial U^i}{\partial w^i} = u_{w^i}\left(c_1^i, c_2^i, l^i, \alpha\left(w^i\right)\right) + \mu\left(l^i\left(1 - T'\left(w^i l^i\right) - t_{y^i}^1\left(w^i l^i, c_1^i\right) - t_{y^i}^2\left(w^i l^i, c_2^i\right)\right)\right), \tag{14}$$

derived using the envelope condition on the individual's utility maximization problem. To remove the tax functions from this expression, we use the individual's first order condition with respect to labor l^i :

$$u_{l^i}\left(c_1^i, c_2^i, l^i, \alpha\left(w^i\right)\right) = -\mu w^i\left(1 - T'\left(w^i l^i\right) - t_{y^i}^1\left(w^i l^i, c_1^i\right) - t_{y^i}^2\left(w^i l^i, c_2^i\right)\right). \tag{15}$$

Substituting (15) into (14) yields:

$$\frac{\partial U^i}{\partial w^i} = u_{w^i}\left(c_1^i, c_2^i, l^i, \alpha\left(w^i\right)\right) - \frac{l^i u_{l^i}\left(c_1^i, c_2^i, l^i, \alpha\left(w^i\right)\right)}{w^i}.$$

The Hamiltonian is then:

$$H\left(w^i\right) = \left(U\left(w^i\right) + \lambda\left(w^i l^i - c_1^i - c_2^i\right)\right)\frac{\pi^i}{dw^i} + \phi\left(u_{w^i}\left(\cdot\right) - \frac{l^i u_{l^i}\left(\cdot\right)}{w^i}\right),$$

[13] These tax instruments are redundant, in that a single tax function of the consumption of one good and income would be sufficient to characterize the full policy. Separating taxes into these functions aids interpretation.

[14] For a textbook treatment, see Salanie (2003), chapter 5.2.

where subscripts denote partial derivatives and $(\cdot)$ denotes the set of arguments of the utility function, $\left(c_1^i, c_2^i, l^i, \alpha\left(w^i\right)\right)$. The first term of the Hamiltonian is the utility of the individual with wage w^i. The second is government's budget constraint multiplied by a shadow price λ. The third term is the evolution of the state variable $U\left(w^i\right)$ with respect to w^i, as derived above, and is multiplied by the costate variable ϕ.

To solve for the optimal policy, choose l and c_1^i as the control variables, with c_2^i an implicit function defined by the budget constraint. The first order condition with respect to c_1 is:

$$\lambda\left(-1-\frac{dc_2^i}{dc_1^i}\right)\frac{\pi^i}{dw^i}+\phi\left(u_{w^ic_1^i}(\cdot)+u_{w^ic_2^i}(\cdot)\frac{dc_2^i}{dc_1^i}-\frac{l^iu_{l^ic_1^i}(\cdot)}{w^i}-\frac{l^iu_{l^ic_2^i}(\cdot)}{w^i}\frac{dc_2^i}{dc_1^i}\right)=0,$$

or, rearranging

$$\frac{dc_2^i}{dc_1^i}=\frac{-\left(\lambda\frac{\pi^i}{dw^i}-\phi\left(u_{w^ic_1^i}(\cdot)-\frac{l^iu_{l^ic_1^i}(\cdot)}{w^i}\right)\right)}{\left(\lambda\frac{\pi^i}{dw^i}-\phi\left(u_{w^ic_2^i}(\cdot)-\frac{l^iu_{l^ic_2^i}(\cdot)}{w^i}\right)\right)}.$$

Individuals maximizing (7) subject to (11) will allocate their after-tax income so that the following relationships hold:

$$\frac{dc_2^i}{dc_1^i}=-\frac{u_{c_1^i}}{u_{c_2^i}}=-\frac{1+t1_{c_1}\left(w^il^i,c_1^i\right)}{1+t2_{c_2}\left(w^il^i,c_2^i\right)} \tag{16}$$

so we can write:

$$\frac{1+t_{c_1}^1\left(w^il^i,c_1^i\right)}{1+t_{c_2}^2\left(w^il^i,c_2^i\right)}=\frac{\lambda\frac{\pi^i}{dw^i}-\phi\left(w^i\right)\left(u_{w^ic_1^i}(\cdot)-\frac{l^iu_{l^ic_1^i}(\cdot)}{w^i}\right)}{\lambda\frac{\pi^i}{dw^i}-\phi\left(w^i\right)\left(u_{w^ic_2^i}(\cdot)-\frac{l^iu_{l^ic_2^i}(\cdot)}{w^i}\right)}. \tag{17}$$

To fully characterize the optimal distortion to commodity purchases given by (17), we solve for λ and $\phi\left(w^i\right)$ in a specific example.

2.1.1 A specific example

We assume the individual utility function is

$$U^i=u\left(c_1^i,c_2^i,l^i,\alpha\left(w^i\right)\right)=\frac{\alpha\left(w^i\right)}{1+\alpha\left(w^i\right)}\ln c_1^i+\frac{1}{1+\alpha\left(w^i\right)}\ln c_2^i-\frac{1}{\sigma}\left(l^i\right)^{\sigma}. \tag{18}$$

It is important to note that this utility function normalizes preferences over consumption goods in the two ways mentioned in the Introduction. The first normalization, following the techniques of Weinzierl (2009), ensures that the marginal social value to a Utilitarian planner of allocating resources to an undistorted individual is independent of that individual's preference parameter $\alpha\left(w^i\right)$. This prevents preference heterogeneity, which is inherently ordinal, from artificially driving redistribution by making the cardinal utility of consumption higher for an individual depending on his or her preferences. The second normalization separates heterogeneity in commodity preferences from the consumption-leisure choice of individuals. Specifically, it ensures that two individuals of the same ability w^i will choose the same labor effort when undistorted.[15]

The next proposition derives an expression for optimal commodity taxes.

[15] Logarithmic utility of consumption makes is possible to achieve these two normalizations simultaneously. For a more general case, the Appendix to this paper contains the details of both normalizations.

Proposition 5 *Given the individual utility function* (18), *the solution to the Planner's Problem satisfies:*

$$\frac{1+t^1_{c_1}\left(w^i l^i, c^i_1\right)}{1+t^2_{c_2}\left(w^i l^i, c^i_2\right)} = \frac{f\left(w^i\right) + u_{w^i c^i_1}\left(1-F_t\left(w^i\right)\right)\begin{pmatrix} \frac{1}{1-F(w^i)}\int_{w^j=w^i}^{w^j=w_{\max}} \frac{1}{u_{c^j_2}} f\left(w^j\right) dw^j \\ -\frac{1}{1-F(w_{\min})}\int_{w^j=w_{\min}}^{w^j=w_{\max}} \frac{1}{u_{c^j_2}} f\left(w^j\right) dw^j \end{pmatrix}}{f\left(w^i\right) + u_{w^i c^i_2}\left(1-F\left(w^i\right)\right)\begin{pmatrix} \frac{1}{1-F(w^i)}\int_{w^j=w^i}^{w^j=w_{\max}} \frac{1}{u_{c^j_2}} f\left(w^j\right) dw^j \\ -\frac{1}{1-F(w_{\min})}\int_{w^j=w_{\min}}^{w^j=w_{\max}} \frac{1}{u_{c^j_2}} f\left(w^j\right) dw^j \end{pmatrix}} \tag{19}$$

Proof. In the Appendix, we derive the following expressions for λ and $\phi\left(w^i\right)$:

$$\lambda = \frac{1}{\int_{w^j=w_{\min}}^{w^j=w_{\max}} \frac{1}{u_{c^j_2}} f\left(w^j\right) dw^j}$$

$$\phi\left(w^i\right) = \left(1-F\left(w^i\right)\right)\left(1 - \frac{\frac{1}{(1-F(w^i))}\int_{w^j=w^i}^{w^j=w_{\max}} \frac{1}{u_{c^j_2}} f\left(w^j\right) dw^j}{\frac{1}{(1-F(w_{\min}))}\int_{w^j=w_{\min}}^{w^j=w_{\max}} \frac{1}{u_{c^j_2}} f\left(w^j\right) dw^j}\right).$$

Using these results in expression (17), we obtain (19). ■

As with the conditions for optimal marginal income tax rates from, e.g., Saez (2001), concave utility of consumption prevents result (19) from being fully closed-form, instead relying on optimal utility and consumption levels. Nevertheless, we can establish some important lessons from it.

First, on the top type, $(1-F(w_{\max}))$ is zero, and the result reduces to

$$\frac{1+t^1_{c_1}(w_{\max})}{1+t^2_{c_2}(w_{\max})} = 1.$$

so the commodity distortion is zero on the highest ability worker.

Second, the distortion is also zero on the lowest ability worker, as the terms in large parentheses in the numerator and denominator are zero.

In addition, examination of terms in (19) gives detail about the determinants of the optimal distortion.

The parenthetical term common to the numerator and denominator is the difference in the average cost of raising utility for the population with wages above w^i and for the entire population. It is positive, since if it were negative the planner could raise social welfare by incentive-compatible and feasible transfers of c_2 from the overall population to the high-ability. As such, this difference measures the loss in welfare that results from having to satisfy the incentives of the high-ability rather than being able to spread resources across all workers. When this loss is large, the optimal distortion to consumption at wage w^i is larger because that distortion discourages higher-ability workers from working less.

The relationship between $u_{w^i c^i_1}$ and $u_{w^i c^i_2}$ determines whether policy discourages consumption of good 1 or good 2 for intermediate ability levels. With utility function (18) this relationship is determined by the sign of $\alpha'\left(w^i\right)$. If $\alpha'\left(w^i\right) < 0$, then high-ability workers relatively prefer good 2, and $u_{w^i c^i_1} < 0$ while $u_{w^i c^i_2} > 0$. Then, the ratio on the right-hand side of (19) is less than one, and the optimal distortion discourages marginal consumption of good 2. That is, the good preferred by the more able workers ought to be marginally taxed.

The term $f\left(w^i\right)$ provides a measure of the share of the population distorted by a given commodity tax. When this share is high, the optimal consumption distortion is smaller, as the planner wants to avoid distortions on large sub-populations. Mathematically, $f\left(w^i\right)$ enters both the numerator and the denominator, pushing the tax ratio toward unity.

The term $\left(1 - F\left(w^i\right)\right)$ is the share of individuals with higher wages who are encouraged to exert more effort due to the distortion at w^i. The larger this term, the more valuable is the distortion to the planner, all else the same. Mathematically, $\left(1 - F\left(w^i\right)\right)$ multiplies the terms in the numerator and denominator that push the tax ratio away from unity. We know that $\left(1 - F\left(w^i\right)\right)$ falls as the wage rises, so this lowers the optimal distortion as we move up the ability distribution.

Finally, suppose there exists an ability level $\tilde{w}$ such that the distribution of all abilities above that level follows a Pareto form, as in Saez (2001). Then for all such $w^i > \tilde{w}$, $\frac{w^i f\left(w^i\right)}{\left(1-F\left(w^i\right)\right)}$ is constant. Rearrange the expression (19) to obtain

$$\frac{1+t^1_{c_1}\left(w^il^i, c^i_1\right)}{1+t^2_{c_2}\left(w^il^i, c^i_2\right)} = \frac{\frac{w^i f\left(w^i\right)}{1-F\left(w^i\right)} + u_{w^i c^i_1} w^i \left(\frac{1}{1-F\left(w^i\right)}\int_{w^j=w^i}^{w^j=w_{\max}} \frac{1}{u_{c^j_2}} f\left(w^j\right) dw^j - \frac{1}{1-F\left(w_{\min}\right)}\int_{w^j=w_{\min}}^{w^j=w_{\max}} \frac{1}{u_{c^j_2}} f\left(w^j\right) dw^j\right)}{\frac{w^i f\left(w^i\right)}{1-F\left(w^i\right)} + u_{w^i c^i_2} w^i \left(\frac{1}{1-F\left(w^i\right)}\int_{w^j=w^i}^{w^j=w_{\max}} \frac{1}{u_{c^j_2}} f\left(w^j\right) dw^j - \frac{1}{1-F\left(w_{\min}\right)}\int_{w^j=w_{\min}}^{w^j=w_{\max}} \frac{1}{u_{c^j_2}} f\left(w^j\right) dw^j\right)}.$$

From above, we know that the parenthetical terms are positive; they are also increasing in w^i following the same argument. Therefore, assuming $u_{w^i c^i_1} < 0$ and $u_{w^i c^i_2} > 0$, whether the optimal tax on good 2 is regressive or progressive in the upper tail of the income distribution depends on how quickly $u_{w^i c^i_1}$ and $u_{w^i c^i_2}$ converge to zero. If they do not converge quickly enough, the tax on good 2 is progressive in the tail.

Though these interpretations aid in understanding result (19), we may want to reformulate that result in terms of observable quantities in the spirit of Saez (2001). The Appendix derives the following version of result (19):

$$\frac{1+t^1_{c_1}}{1+t^2_{c_2}} = \frac{w^i f\left(w^i\right) + \varepsilon_{c^i_1 w^i}\frac{1+t^1_{c_1}}{1+t^2_{c_2}}\left(1-F\left(w^i\right)\right)\left(\frac{1}{1-F\left(w^i\right)}\int_{w^j=w^i}^{w^j=w_{\max}} \frac{\hat{y}^j}{\hat{y}^i} f\left(w^j\right) dw^j - \frac{1}{1-F\left(w_{\min}\right)}\int_{w^j=w_{\min}}^{w^j=w_{\max}} \frac{\hat{y}^j}{\hat{y}^i} f\left(w^j\right) dw^j\right)}{w^i f\left(w^i\right) + \varepsilon_{c^i_2 w^i}\left(1-F\left(w^i\right)\right)\left(\frac{1}{1-F\left(w^i\right)}\int_{w^j=w^i}^{w^j=w_{\max}} \frac{\hat{y}^j}{\hat{y}^i} f\left(w^j\right) dw^j - \frac{1}{1-F\left(w_{\min}\right)}\int_{w^j=w_{\min}}^{w^j=w_{\max}} \frac{\hat{y}^j}{\hat{y}^i} f\left(w^j\right) dw^j\right)} \tag{20}$$

where $\varepsilon_{c_m w}$ denotes the Frisch elasticity (holding marginal utility constant) of consumption of good m with respect to the wage, $\hat{y}^i$ is the disposable income individual i would choose to earn in an economy with income taxes only (i.e., before the introduction of optimal commodity taxes, the planner can observe the distribution of $\hat{y}^i$). This alternative representation of the main result on optimal commodity taxes can be more readily applied with observable data.

If we restrict attention to commodity taxes that are a linear function of the consumption of the good, a modification of result (19) confirms the results of the previous literature (e.g., Saez 2002, Salanie 2003) that goods preferred by the highly able ought to be taxed.

The results of Sections 1 and 2 suggest that optimal commodity taxes may be regressive on goods preferred by the high-ability, but the analytical expression (19) makes it clear that the shape of optimal commodity taxes will depend on many details of the economy. In the next two sections, we turn to a quantitative study of optimal commodity taxation when the commodities in the utility function are current and future consumption (savings).

3 Optimal capital income taxes

We begin our study of capital income taxation by examining empirical evidence on the relationship between ability and time preference, or intertemporal discounting. We then simulate optimal capital income taxes justified by this relationship.

Throughout this section, we use a generalized form of the utility function (18) in which the utility from consumption is constant relative risk aversion (CRRA), rather than logarithmic:

$$U = \frac{1}{\varphi^i}\left(\left(\frac{\alpha\left(w^i\right)}{1+\alpha\left(w^i\right)}\right)^{\gamma}\frac{\left(c_1^i\right)^{(1-\gamma)}-1}{(1-\gamma)}+\left(\frac{1}{1+\alpha\left(w^i\right)}\right)^{\gamma}\frac{\left(c_2^i\right)^{(1-\gamma)}-1}{(1-\gamma)}-\frac{1}{\sigma}\left(l^i\right)^{\sigma}\right) \tag{21}$$

where φ^i is a normalization term derived in the Appendix that causes the marginal social welfare of resources allocated to individuals at their private optima to be independent of preferences and to match the values implied by a utilitarian social welfare function as in (8) operating on the utility function in (18). As a baseline case, we assume $\gamma = 2$ and $\sigma = 3$.

3.1 Evidence on ability and time preference

A sizeable literature exists on measuring and explaining differences in saving behavior across income groups. In general, this research has acknowledged the possible role of heterogeneous time preferences but has not found solid evidence of their importance. Hubbard, Skinner, and Zeldes (1995) build a lifecycle model with individual uncertainty, precautionary saving, means-tested social insurance, and homogeneous time preferences. Using data from the Panel Study of Income Dynamics (PSID), they argue that their model can explain the wide variation in saving rates across education (as a proxy for income) levels without heterogeneity in time preferences. Samwick (1998) uses Survey of Consumer Finances data on wealth and income to estimate a lifecycle model and notes that variations in wealth profiles not explained by his model may be due to variation in time preferences, though he does not have direct evidence for them. Dynan, Skinner, and Zeldes (2004) find a "strong, positive relationship between saving rates and lifetime income," using data from the PSID, but they argue that preference differences cannot explain their findings (at least, without a strong bequest motive). Lawrance (1991) calculates annual time preference rates using data on food consumption and finds that implied discount factors rise with income, but Dynan (1993) shows that Lawrance's results are sensitive to the inclusion of controls.

Though this literature casts doubt on the potential for heterogeneous time preferences to justify substantial capital taxation, little if any research exists on whether saving preferences are related to innate *ability*, the relationship of interest for our analysis.

For direct evidence on that question, we perform new analysis using data from the National Longitudinal Survey of Youth (NLSY). The NLSY consists of a nationally representative sample of individuals born between 1957 and 1964, first interviewed in 1979, and interviewed annually or biannually since. The NLSY contains data on individuals' net worth and income over time, allowing us to roughly estimate saving rates as described below.

The key advantage of the NLSY for our purposes is that it includes a standard, direct measure of ability. This allows us to relate a measure of ability, not income, to time preferences. In 1980, the NLSY administered the Armed Forces Qualification Test (AFQT) to 94 percent of its participants. This test measured individuals' aptitudes in a wide range of areas, including some mechanical skills relevant to military service.

We use an aggregation of scores in some of the areas covered by the AFQT as the indicator of ability[16]. This aggregation, the AFQT89, is calculated by the Center for Human Resource Research at Ohio State University, as follows:

> Creation of this revised percentile score, called AFQT89, involves (1) computing a verbal composite score by summing word knowledge and paragraph comprehension raw scores; (2) converting subtest raw scores for verbal, math knowledge, and arithmetic reasoning; (3) multiplying the verbal standard score by two; (4) summing the standard scores for verbal, math knowledge, and arithmetic reasoning; and (5) converting the summed standard score to a percentile.

Our measure of preferences will be the discount factor implied by using NLSY data on income and net worth in a simple model of individual optimization. Suppose individuals live for three periods. In the first two periods, roughly corresponding to ages 20 through 42 and 43 through 65, they work, consume, and perhaps borrow or save. In the third period, they are retired and live for 23 years (for simplicity, as this makes all three periods of similar length). The individual solves the following utility maximization problem:

$$\max_{c_1,c_2,c_3} \left[\ln(c_1) + \delta \ln(c_2) + \delta^2 \ln(c_3) - v(y_1, y_2)\right]$$

subject to

$$\left((y_1 - c_1) R^2 + (y_2 - c_2)\right) R - c_3 = 0.$$

where c_t and y_t are consumption and income in period t, δ is the discount factor across 23-year periods (i.e., if the one-year-ahead discount factor is β, then $\delta = \beta^{23}$), $R = (1.05)^{23}$ is the average return to saving over a 23-year period, and $v(\cdot)$ is an unspecified function for the disutility of earning income.

We make the assumption that an individual's total value of income prior to age 43 is identical to the income it will earn from age 43 until retirement. In the notation of the model, we assume $y_1 = y_2$ for all individuals. The first-order conditions of the individual's problem yield the following expression for δ:

$$1 + \delta + \delta^2 = \frac{y_1}{c_1}\frac{1+R}{R}.$$

or

$$\delta = \frac{1}{2}\left(\left(-3 + 4\frac{y_1}{c_1}\frac{1+R}{R}\right)^{\frac{1}{2}} - 1\right)$$

As expected, the higher is income relative to consumption, the greater the estimated δ for an individual. We drop 37 individuals whose estimated δ is negative or exceeds two, leaving 7,008 observations.

To estimate δ, we need values for y_1 and c_1 for each individual. For y_1, we use the NLSY's observations on income over time for each individual to calculate the "future value" of income earned prior to and including 2004.[17] Formally, $y_1 = \sum_{t=1979}^{2004} R^{\frac{1}{23}(2004-t)} y_t$. Using the full time series of income rather than simply the most

[16] The AFQT most likely measures some combination of innate ability and accumulated achievement. To the extent that more innately patient individuals invest more in human capital and thereby have higher AFQT scores because of achievement, not ability, our analysis will be biased toward finding a stronger relationship between ability and time preferences than that which truly holds.

[17] We do not observe income in all years for each individual. To obtain an income figure comparable to ending net worth for each individual, we calculate the future value of the observed incomes for each individual. Then, we scale that future value by the maximum number of years observable over the number of years observed for each individual. We also do not observe initial Net Worth. However, if we control for net worth in 1985, just six years after the survey began, the coefficient on AFQT is hardly changed.

recent observation of income is important for two reasons. First, it gives a better measure of the individual's likely lifetime or permanent income. Second, to calculate c_1, we assume that any income not accumulated as net worth by 2004 was consumed. Formally, we denote the NLSY variable "family net worth" NW and calculate $c_1 = y_1 - NW$.[18]

In Table 1, we show the mean and standard deviations of δ along with the implied values for $\alpha\left(w^i\right)$, all by AFQT quintile. Note that δ is related to the preference parameter $\alpha\left(w^i\right)$ from the utility function (21) by $\alpha\left(w^i\right) = \left(\delta\left(w^i\right)\right)^{\left(\frac{-1}{23}\right)}$. The variation in δ within AFQT quintiles is large relative to the variation across wage levels. Of course, the data are likely to be very noisy, and our inference of δ is based on a highly simplified model. Nevertheless, our findings are consistent with the findings of the literature cited above that relates saving to income and with Benjamin, Brown, and Shapiro (2006), who find a "positive relationship between AFQT score and the propensity to have positive net assets" in the NLSY.

Simple AFQT quintile means of δ are likely to be misleading, however, as they fail to control for variables correlated with both ability and saving behavior. Table 2 shows the results of a regression that controls for some observable characteristics:

$$\ln\delta = \beta_1 age + \beta_2 age^2 + \beta_3 gender + \beta_4 \ln\left(income\right) + \beta_5 \ln\left(AFQT\right).$$

where "*income*" is the future value of income, y_1, described above. This regression yields a highly significant estimate for β_5 of 0.026 (standard error of 0.004).[19,20] Exponentiating this equation and using the estimate for β_5 yields

$$\delta = 0.356\left(AFQT\right)^{0.026}, \tag{22}$$

where the constant multiplying the right-hand-side is determined by matching the value of δ for the middle AFQT quintile. Expression (22) allows us to calculate, from the average AFQT score by quintile, a "regression-based δ" for each quintile that can be compared to the simple means in Table 1. The results are shown in Table 3, along with the implied values of $\alpha\left(w^i\right)$ and the mean reported wages by AFQT quintile.[21] We can use the $\alpha\left(w^i\right)$ and w^i values in Table 3 to estimate the following functional relationship:

$$\alpha\left(w^i\right) = 1.0529\left(w^i\right)^{-0.0037}. \tag{23}$$

Expression (23) allows us to predict $\alpha\left(w^i\right)$ for a wide range of wages.

To make clearer the difference between the values of δ by AFQT quintile shown in Tables 1 and 3, Figure 1 shows both the "Mean δ" and the "Regression-based δ" plotted against the mean wage by AFQT quintile.

[18] Our data do not include components of individuals' expected future income, such as Social Security payments or other social transfers. To the extent that these omissions bias down the estimate of net worth, we will understate saving rates. Therefore, if these transfers are progressive, we will be overestimating the slope of discount factors versus ability. In a similar way, expected future gifts and inheritances are not taken into account in the data. To the extent that these are increasing in recipient income, we are underestimating the slope of discount factors versus ability.

[19] We also have run simulations controlling for the slope of income during the 1979-2004 period and over the past ten years for each individual. These controls reduce the coefficient on AFQT to 0.021 and 0.014, but it remains significant at the 1% level. Note that the diminution of the coefficient implies an even weaker relationship between ability and time preferences.

[20] Measurement error likely affects both our estimates of ability and discounting, though bias would be introduced only by error in the former. While AFQT is an imperfect measure of ability, its retest reliability is very high. Moreover, if AFQT mismeasures ability, it is unclear whether that biases our results down or up. It may be that AFQT measures those parts of ability that are particularly highly correlated with preferences (i.e., ability to delay gratification, cognitive alacrity), and a more accurate measure of ability would show less relationship with preferences.

[21] We compute wages from the total wage and salary income divided by the total hours worked in 1992, as reported in 1993. We calculate mean wages by AFQT quintile limiting the sample to workers who reported more than 1,000 hours worked. Using all workers does not change the pattern, but all wage levels rise because some workers with low reported hours have high imputed hourly wages.

The flatter slope of the latter line reflects the variation explained by the control variables in the regression summarized in Table 2.

3.2 Optimal capital income taxes

To simulate optimal capital income taxes using the estimated form for $\alpha\left(w^i\right)$ in expression (23), we specify a wage (w^i) distribution, calculate the implied values for $\alpha\left(w^i\right)$, and numerically simulate the planner's problem in (8), (9), and (10). We also simulate an augmented planner's problem that limits the planner to a constant rate of capital income taxation. Formally, we add to the planner's problem the constraint that the intertemporal wedge be constant:

$$\frac{u_{c_1^i}}{u_{c_2^i}} = \frac{u_{c_1^j}}{u_{c_2^j}} \tag{24}$$

for all types i, j. Recall that the assumed individual utility function is shown in (21).

We use a wage distribution that runs from \$4 to \$100 with 25 equally-spaced discrete values. Based on Saez (2001), we assume that the distribution of the population across these wages is lognormal up to \$62.50 and Pareto with a parameter value of two for higher wages. We calibrate the lognormal distribution with the 2007 wage distribution for full-time workers in the United States as reported in the Current Population Survey.

To measure the intertemporal wedge we use the expression:

$$\tau = 1 - \frac{\frac{u_{c_1^i}}{u_{c_2^i}} - 1}{r} \tag{25}$$

where r is the annual rate of return to savings.[22] The variable τ measures the relative distortion toward good 1 and away from good 2 at a given income level. Under the capital income tax interpretation, τ is the implicit tax on the interest income earned on good 2, i.e., capital. If this expression is positive, the tax policy is discouraging future consumption relative to current consumption. More informally, it is taxing the return to saving, so we will refer to it as the implied capital income tax.

Figure 2 shows the optimal nonlinear capital income tax rates that are justified by the relationship we estimate from the NLSY data. In the baseline case, where $\gamma = 2$ and $\sigma = 3$, optimal capital income tax rates are less than 6 percent everywhere, flat over much of the distribution, and declining at higher incomes. The constrained-optimal linear capital income tax rate is 3.1% (not shown). The welfare gains from optimal capital income taxation given the empirical $\alpha\left(w^i\right)$ are negligible.

This result is robust to alternative values of the parameters of the utility function and form for the wage distribution. Figure 2 also shows optimal capital income taxes for a range of values for γ and σ, as well as one in which we use a lognormal distribution of wages rather than a combination lognormal-Pareto distribution. Tax rates are consistent with the baseline case except when we raise risk aversion (γ) or the elasticity of labor supply $\left(\frac{1}{\sigma-1}\right)$ to high levels. Even in these cases, however, justified rates are small relative to prevailing rates (as will be discussed in detail below). Specifically, when $\gamma = 4$ (rather than the baseline value of 2), the average of the optimal nonlinear capital income taxes is 6.9%, which is also the best linear capital tax rate. When $\sigma = 1.5$ (rather than the baseline value of 3), the average of the optimal nonlinear capital income taxes is also 6.9%, while the best linear capital tax rate is 6.0%. Welfare gains from optimal

[22] In the simulations, we assume that $1 + r = \frac{1}{\int \alpha(w^i) f(w^i)}$. The implicit tax τ is on net capital income, i.e., the implicit after-tax return to saving is $(1 + r(1-\tau))$.

capital taxation are negligible in all cases.

One variation to the model that has the potential to generate large optimal capital income taxes is to make the social welfare function more concave. In the extreme case of a Rawlsian social welfare function, for example, the optimal capital income tax rate averages 37.1% across the population, and the constrained-optimal linear capital income tax rate is 32.9%.

4 Comparing optimal to existing capital income taxes

In this section, we compare the empirical relationship between time preferences and ability to that which would be required to justify prevailing levels of capital income taxes in developed economies, using our baseline model specification.

One natural measure of the taxation of capital income is the level of statutory tax rates on various forms of capital income. The Organization for Economic Cooperation and Development (OECD 2008) reports tax rates on corporate profits and capital income earned by individuals. Figure 3 shows the range of the combined impact of these statutory rates across OECD member countries for 2007. The average combined corporate and personal statutory rate on distributed corporate profits was 42.4 percent in 2007, down from 50 percent in 2000.

An alternative measure is the "tax ratio" of capital income tax revenue to total capital income. This measure has weaknesses–for example, it is backward-looking–but it has the virtue of implicitly controlling for the complicated exemptions, definitional variations, and tax incentives that cause the economic extent of capital taxation to differ from that implied by statutory rates. Carey and Rabesona (2004) calculate the tax ratio for capital income across sixteen OECD countries in 2000 to be 46.3.

Taken together, these measures suggest that tax rates on capital income in developed economies today are over 40 percent. These rates are generally flat or somewhat progressive, as some countries use corporate tax rates that rise with corporate profits and the personal taxes paid on corporate income are in some cases tied to progressive tax rates on labor income.

To find the $\alpha\left(w^i\right)$ functions that yield constrained-optimal linear[23] intertemporal wedges corresponding to prevailing capital income tax rates, we continue to model the function $\alpha\left(w^i\right)$ as a two-parameter power function

$$\alpha\left(w^i\right) = \psi\left(w^i\right)^{\varepsilon}, \tag{26}$$

where ψ and ε are scalars. We fix $\alpha\left(w^i = \$28\right) = 1.0413$, the value implied by our analysis of the NLSY data, to ensure comparability of these preferences to our empirical estimates. Then, we use the wage (w^i) distribution and utility function (21) from Section 3 with $\gamma = 2$ and $\sigma = 3$, and we vary the values of ψ and ε in (26) while simulating the planner's problem in (8), (9), (10), and (24).

Figure 4 plots the $\alpha\left(w^i\right)$ required for the best linear intertemporal wedge to imply capital income tax rates of 40%, 20%, and 10% as well as the values for $\alpha\left(w^i\right)$ from our analysis of the NLSY data. To aid intuition, Figure 5 plots the conventional annual discount factor $\beta\left(w^i\right)$ implied by these $\alpha\left(w^i\right)$. As these Figures make clear, the empirical relationship between time preferences and ability is far weaker than that which would justify the capital income tax rates prevailing in developing economies today.

[23] Existing capital tax rates are usually constant, while optimal wedges are generally not constant, so we in fact match the constrained-optimal linear intertemporal wedge to prevailing rates.

5 Role of preference normalization

In this section, we explore the role of preference normalization in the study of optimal commodity taxation. In this paper, we normalize preferences in two ways: to neutralize the role of preferences over goods in how much the planner values individuals; and to neutralize the effect of preferences over goods on the labor supply choices of individuals.

First, we normalize so as to eliminate any incentive for the planner to redistribute across agents based simply on their preferences over goods. Consider the following two representations of the same preferences over consumption goods:

$$U = \frac{\alpha\left(w^i\right)}{1+\alpha\left(w^i\right)} \ln c_1^i + \frac{1}{1+\alpha\left(w^i\right)} \ln c_2^i - \frac{1}{\sigma}\left(l^i\right)^{\sigma} \tag{27}$$

and

$$U = \ln c_1^i + \frac{1}{\alpha\left(w^i\right)} \ln c_2^i - \frac{1}{\sigma}\left(l^i\right)^{\sigma}, \tag{28}$$

Expression (27) normalizes preferences as in our main analysis, whereas (28) does not. Specifically, starting at individuals' undistorted optimal allocations, a planner using (27) has no desire to redistribute across preference types (conditional on the wage) because the marginal social value of resources is equalized across preference types. In contrast, a planner using (28) obtains a larger increase in social welfare from allocating a marginal unit of resources to the individual with lower $\alpha\left(w^i\right)$. Therefore, optimal tax policy will favor individuals with lower $\alpha\left(w^i\right)$. Importantly, if we were to multiply the first two terms of expression (28) by $\alpha\left(w^i\right)$, the impact of preferences over consumption goods would reverse even though we would be using an observationally equivalent representation of them. In that case, lower $\alpha\left(w^i\right)$ types would yield smaller increases in social welfare to the planner. For utility that is of the constant relative risk aversion form (rather than logarithmic) over consumption, the Appendix shows the derivation of a normalizing factor that generates the same marginal social welfare for each type as does (27) for a utilitarian planner.

Second, we use a representation of preferences that implies no relationship between preferences across goods and the willingness to work. For utility that is of the constant relative risk aversion form (rather than logarithmic) over consumption, this representation is:

$$U = \left(\left(\frac{\alpha\left(w^i\right)}{1+\alpha\left(w^i\right)}\right)^{\gamma} \frac{\left(c_1^i\right)^{(1-\gamma)}-1}{(1-\gamma)} + \left(\frac{1}{1+\alpha\left(w^i\right)}\right)^{\gamma} \frac{\left(c_2^i\right)^{(1-\gamma)}-1}{(1-\gamma)} - \frac{1}{\sigma}\left(\frac{y^i}{w^i}\right)^{\sigma}\right)$$

To see that chosen income is independent of preferences with this representation, solve the individual's maximization problem with respect to the budget constraint:

$$y^i - c_1^i - c_2^i = 0.$$

The individual's first order conditions yield:

$$y^i = \left(w^i\right)^{\frac{\sigma}{\sigma-1+\gamma}},$$

so that income earned does not depend on preferences $\alpha\left(w^i\right)$. Without this normalization, preferences over goods would be related to preferences between leisure and consumption.

Diamond and Spinnewijn (2009) use a representation of preferences similar to (28) to study optimal capital income taxation. Their social welfare function puts a small enough weight on the high-ability to ensure redistribution away from them, so they address the issues raised by our first normalization in an alternative way. Related to our second normalization, their preferred functional form reflects a greater willingness to work among the highly-able. If we simulate the optimal policy model of Section 2 with that functional form (i.e., without our second normalization), the average optimal capital tax rate falls from 3.1% to 1.6%. Intuitively, the relationship between patience and willingness to work reduces the distortionary impact of labor income taxes on the patient and, therefore, the high income earners. This makes the benefits of capital taxes smaller, so the optimal policy avoids them. Therefore, our second normalization causes us to overestimate optimal capital taxes relative to the most natural alternative.

These normalizations are similar to two assumptions Saez (2002) states in his analysis of this topic. His Assumption 1 is that the planner's marginal social welfare weights on individuals are independent of their tastes for goods, conditional on their incomes. Our first normalization pursues the same neutrality of marginal social welfare weights, though we use the laissez-faire allocations rather than the optimal allocations as the starting point for the normalization. This normalization captures the idea that the government does not want to redistribute resources across individuals simply because they will spend them on different consumption baskets. Our second normalization parallels Saez's Assumption 2, which states that, conditional their income, individuals' labor supply responses to tax changes are unaffected by their preferences. Though our normalization focuses on isolating from preferences the chosen level of labor supply, rather than its response to tax changes, the idea of the two approaches is similar. Intuitively, this normalization means that individuals choose how much to work without regard to how they plan to spend their disposable income. Saez notes that both of his Assumptions seem like reasonable ones in the context of capital income taxation. We take a similar perspective, believing that our normalizations provide a natural, and neutral, starting point for modeling preference heterogeneity and its effects on optimal commodity taxation.

6 Optimal Capital Taxation when Stochastic Abilities are Related to Preferences

In this section, we extend our study of optimal capital taxation when preferences vary with ability to a stochastic setting. Agents start with a common wage w_1 and consume one good c_1 in the first period. In the second period, their wage can take one of two values w_2^i or $w_2^{i'}$, and they can consume two goods c_2 and x_2. Agents with the low second-period wage have a relative preference for x_2 over c_2.

6.1 Individual's problem

An individual's problem in this setting with no policy is:

$$\max_{c_t, y_t} \left[u\left(c_1\right) - v\left(\frac{y_1}{w_1}\right) + \sum \pi^i \beta \left[\frac{1}{1+\alpha\left(w^i\right)} \ln c_2^i + \frac{\alpha\left(w^i\right)}{1+\alpha\left(w^i\right)} \ln x_2^i - v\left(\frac{y_2^i}{w_2^i}\right) \right] \right]$$

s.t. feasibility

$$R\left(y_1 - c_1\right) + \left(y_2^i - c_2^i - x_2^i\right) \geq 0$$

for all i, where π^i is the probability of wage w_2^i : $\sum \pi^i = 1$, and $\alpha\left(w^i\right)$ measures the preference for good x_2 in period two.

The individual chooses to satisfy a standard stochastic Euler:

$$u'(c_1) = R\sum \pi^i \beta \frac{1}{2}\left[\frac{1}{1+\alpha(w^i)}u'(c_2^i) + \frac{\alpha(w^i)}{1+\alpha(w^i)}u'(x_2^i)\right]$$

6.2 Planner's problem

The social planner's problem is analogous to the static problem in Section 2:

Problem 6

$$\max_{c_t, y_t}\left[u(c_1) - v\left(\frac{y_1}{w_1}\right) + \sum_{i=1}^{I}\pi^i\beta\left[\frac{1}{1+\alpha(w^i)}\ln c_2^i + \frac{\alpha(w^i)}{1+\alpha(w^i)}\ln x_2^i - v\left(\frac{y_2^i}{w_2^i}\right)\right]\right] \tag{29}$$

subject to feasibility

$$R(y_1 - c_1) + \sum \pi^i\left(y_2^i - c_2^i - x_2^i\right) \geq 0, \tag{30}$$

and incentive compatibility in period 2

$$\frac{1}{1+\alpha(w^i)}\ln c_2^i + \frac{\alpha(w^i)}{1+\alpha(w^i)}\ln x_2^i - v\left(\frac{y_2^i}{w_2^i}\right) \geq \frac{1}{1+\alpha(w^i)}\ln c_2^{i'} + \frac{\alpha(w^i)}{1+\alpha(w^i)}\ln x_2^{i'} - v\left(\frac{y_2^{i'}}{w_2^i}\right), \tag{31}$$

for all i, i'.

Without loss of generality, suppose that $w_2^i > w_2^{i'}$ so that individuals with second-period ability w_2^i are tempted to claim ability $w_2^{i'}$, but not vice versa. Then $\mu^{i'|i} > 0$ and $\mu^{i|i'} = 0$, and the first-order conditions of the planner's problem yield:

$$u'(c_1) = R\left[\begin{array}{c}\sum \pi^i\beta\frac{1}{2}\left[\frac{1}{1+\alpha(w^i)}u'(c_2^i) + \frac{\alpha(w^i)}{1+\alpha(w^i)}u'(x_2^i)\right] - \\ \mu^{i'|i}\frac{1}{2}\left(\frac{1}{1+\alpha(w^i)}\left(u'\left(c_2^{i'}\right) - u'(c_2^i)\right) + \frac{\alpha(w^i)}{1+\alpha(w^i)}\left(u'\left(x_2^{i'}\right) - u'(x_2^i)\right)\right)\end{array}\right]. \tag{32}$$

To interpret this condition, note that the first row on the right-hand side is the right-hand side of the standard stochastic Euler that the individual chooses to satisfy if left undistorted. Therefore, the second row on the right-hand side determines the treatment of the intertemporal margin.

The second row on the right-hand side of (32) is positive, but it is subtracted from the first row. This lowers the marginal utility of first-period consumption relative to the individual's undistorted optimum. In other words, an intertemporal distortion that discourages saving is optimal in this setting. This does not tell us, however, whether the ability to tax commodities in a nonlinear, income-dependent way increases or decreases the optimal level of capital income taxes. While an analytical answer to this question is unavailable, simple numerical examples show that the effect of this sophisticated commodity taxation is ambiguous.

In the numerical simulations, we consider two planner's problems. The first is the social planner's problem shown in equations (29), (30), and (31): we call this the optimal model. The second constrains this planner to tax commodities in the second period at the same rate for individuals with both wage realizations, so that:

$$\frac{1}{1+\alpha(w^i)}u'(c_2^i) = \frac{\alpha(w^i)}{1+\alpha(w^i)}u'(x_2^i)$$

for $i = \{i, i'\}$. We call this the restricted model. For either problem, define the intertemporal wedge (the implicit capital tax) as θ in:

$$u'(c_1) = R(1-\theta)\sum \pi^i \beta \frac{1}{2}\left[\frac{1}{1+\alpha(w^i)}u'\left(c_2^i\right) + \frac{\alpha(w^i)}{1+\alpha(w^i)}u'\left(x_2^i\right)\right],$$

which means that θ is the distortion to the individuals' private intertemporal optimization.[24] Assume $u(c) = \ln c$, $R = 1.05$, and $\beta R = 1$.

For the first numerical example, suppose wages are

$$\begin{aligned} w_1 &= 4 \\ w_2^i &= \{2, 6\} \end{aligned}$$

and

$$\alpha\left(w^i\right) = \{0.75, 0.25\}.$$

Note that the individual with the low wage relatively prefers x_2.

The optimal values of θ in these two cases are:

$$\begin{aligned} \theta^{optimal} &= 0.061 \\ \theta^{restricted} &= 0.051 \end{aligned}$$

For the second example, assume the same economy except that preferences are more similar across wage types in the second period.

$$\alpha\left(w^i\right) = \{0.75, 0.50\}.$$

Simulating the same planners problems yields the following results:

$$\begin{aligned} \theta^{optimal} &= 0.0861 \\ \theta^{restricted} &= 0.0866. \end{aligned}$$

In the first case, allowing for optimal commodity taxation increased the size of the optimal intertemporal distortion. In the second case, it decreased it.

Therefore, optimal capital taxes may be more useful or less useful to a tax authority who confronts an economy in which stochastic abilities are associated with preferences for particular goods.

7 Conclusion

Among others, Mirrlees (1976) and Saez (2002) have argued that goods preferred by the high-ability ought to be taxed as part of an optimal tax policy that seeks to redistribute toward the (unobservably) low-ability. We show that, in contrast to these previous results, optimal commodity taxation when preferences vary with ability may be regressive in income on those goods preferred by those who are more able. We obtain this result by allowing taxes on goods to be nonlinear functions of income and the consumption of the good, which is plausible for many important categories of consumption such as education, health, housing, and

[24] Note that τ is defined here as on the capital stock, not on the income from capital (as in the previous sections).

future consumption.

The logic for taxing goods preferred by those with high ability has been used to argue for positive capital income taxation, for example by Banks and Diamond (2008). We examine data on preferences for current relative to future consumption and find that the relationship between ability and time discounting is unlikely to justify more than a small fraction of the substantial capital income taxation prevailing in developed economies today.

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Table 1. Summary of δ by AFQT quintile

	AFQT89 quintile				
	Bottom	2	3	4	Top
Mean δ	0.336	0.374	0.394	0.418	0.466
Std. Dev. of δ	0.16	0.18	0.18	0.21	0.25
Implied $\alpha(w^i)$	1.0486	1.0437	1.0413	1.0387	1.0338

Table 2. Results of regression of log of discount factor, ln(delta), on ability and controls

Variable	Coefficient	Standard Error	T-statistic
age	-2.62E-02	2.97E-02	-0.88
agesq	8.80E-04	8.36E-04	1.05
sex	1.16E-02	8.15E-03	1.42
ln(fvincome)**	1.69E-01	7.61E-03	22.15
ln(afqt)**	2.60E-02	4.46E-03	5.82
Note: ** indicates significance at the 1% level or lower			
Observations	7,008		
F-statistic	203.98		
R-squared	0.127		

Table 3. Regression-based δ by AFQT quintile

	AFQT89 quintile				
	Bottom	2	3	4	Top
Regression-based δ	0.377	0.389	0.394	0.398	0.400
Implied $\alpha(w^i)$	1.0435	1.0418	1.0413	1.0410	1.0406
w^i	$12.27	16.21	19.20	21.62	25.73

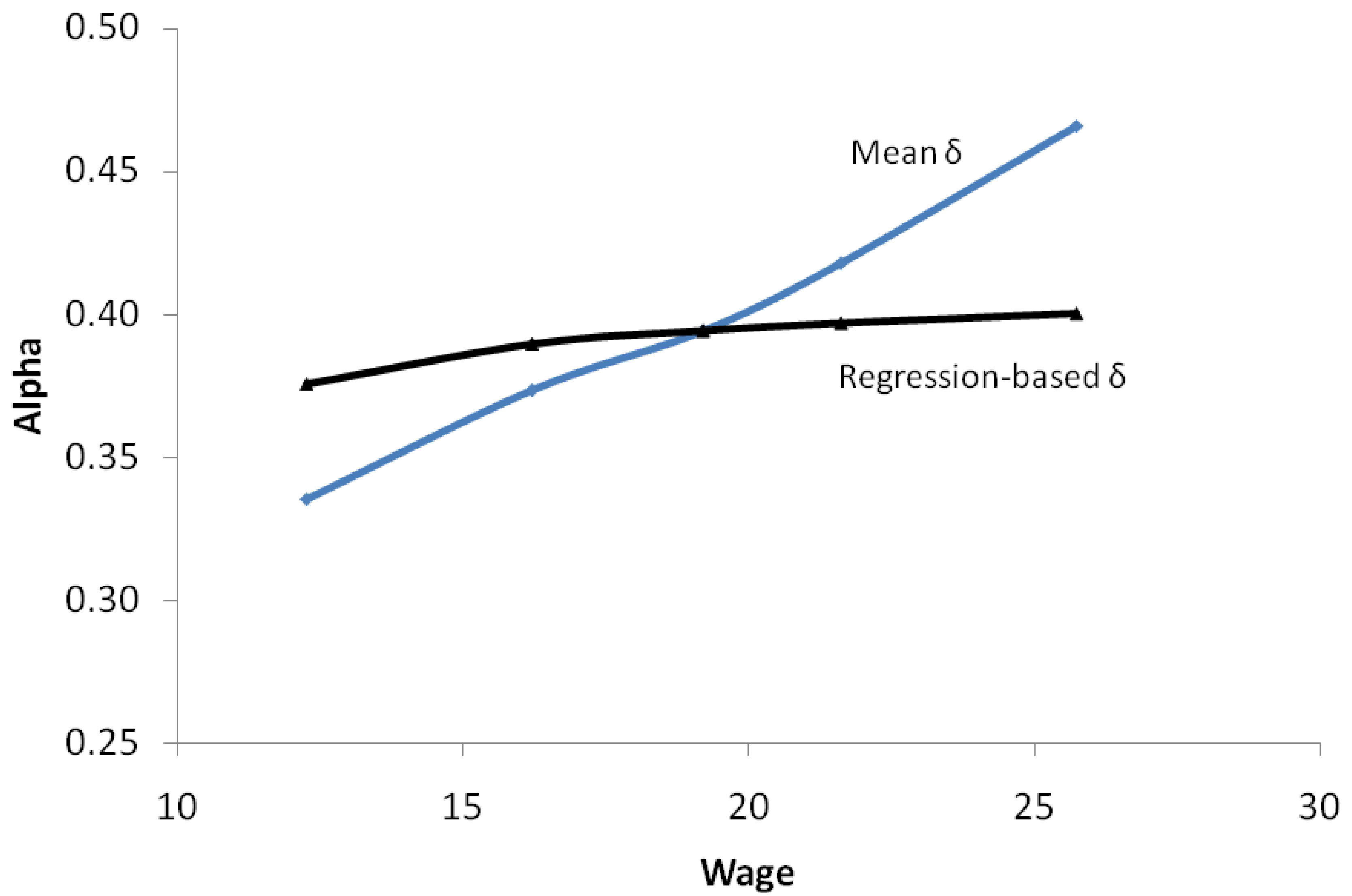


Figure 1: "Mean δ" and "Regression-based δ"

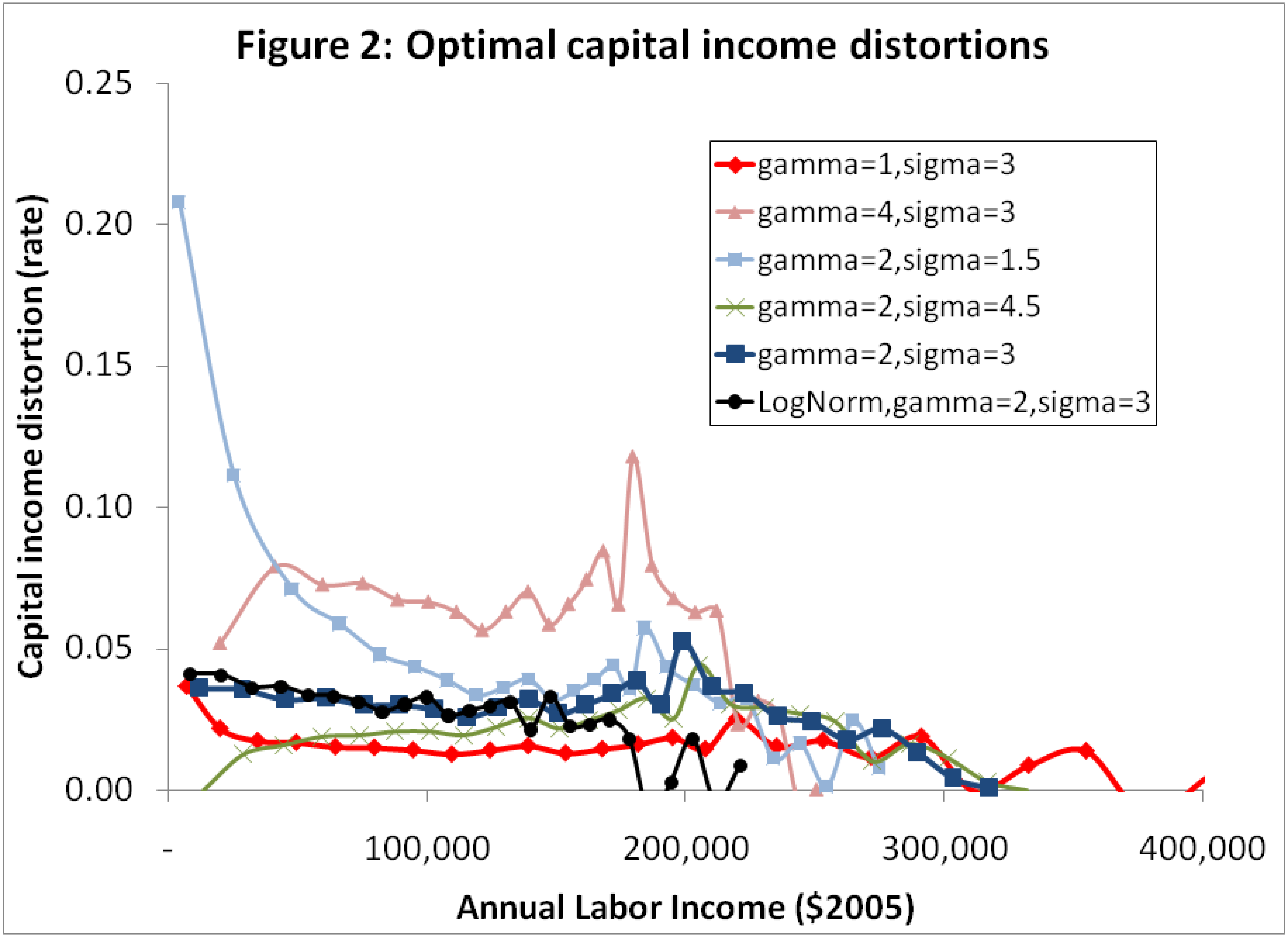
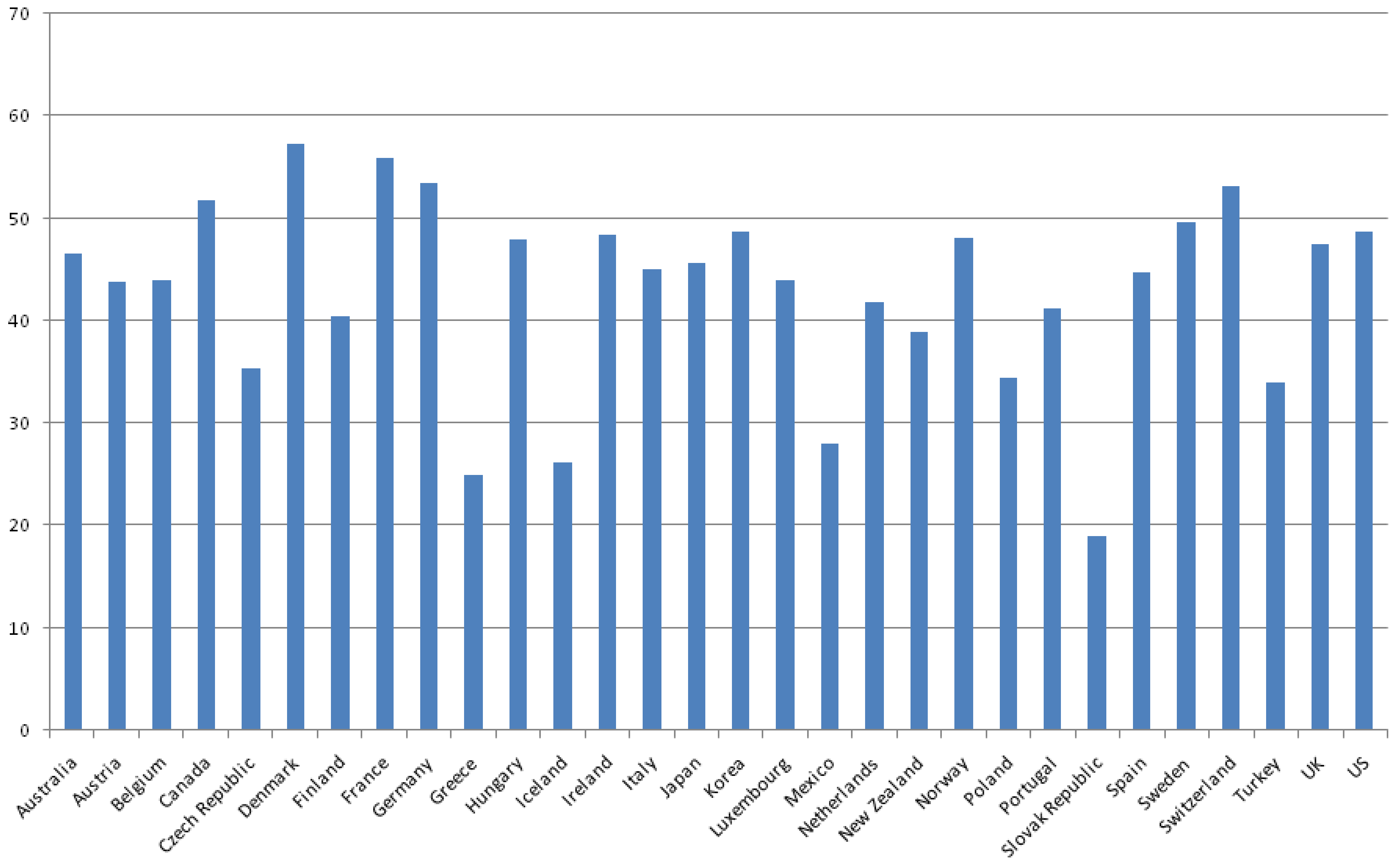


Figure 2: Optimal capital income distortions
0.25
0.20
0.15
0.10
0.05
0.00
Capital income distortion (rate)
gamma=1,sigma=3
gamma=4,sigma=3
gamma=2,sigma=1.5
gamma=2,sigma=4.5
gamma=2,sigma=3
LogNorm,gamma=2,sigma=3
-
100,000
200,000
300,000
400,000
Annual Labor Income ($2005)

Figure 3: Combined Corporate and Personal Tax Rate on Distributed Profits in 2007 (in percent)



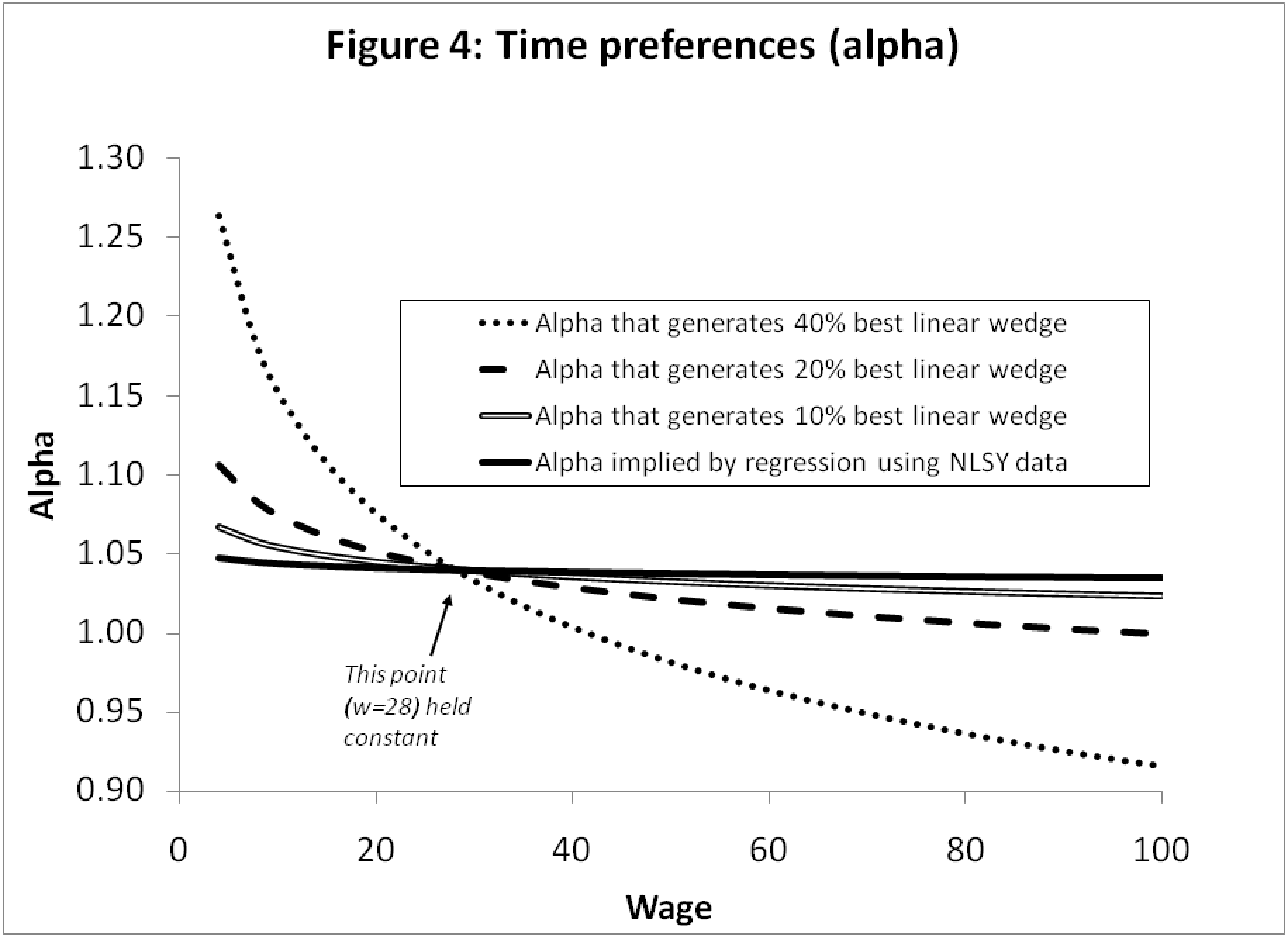


Figure 4: Time preferences (alpha)
Alpha
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0.95
0.90
0
20
40
60
80
100
Wage
Alpha that generates 40% best linear wedge
Alpha that generates 20% best linear wedge
Alpha that generates 10% best linear wedge
Alpha implied by regression using NLSY data
This point (w=28) held constant

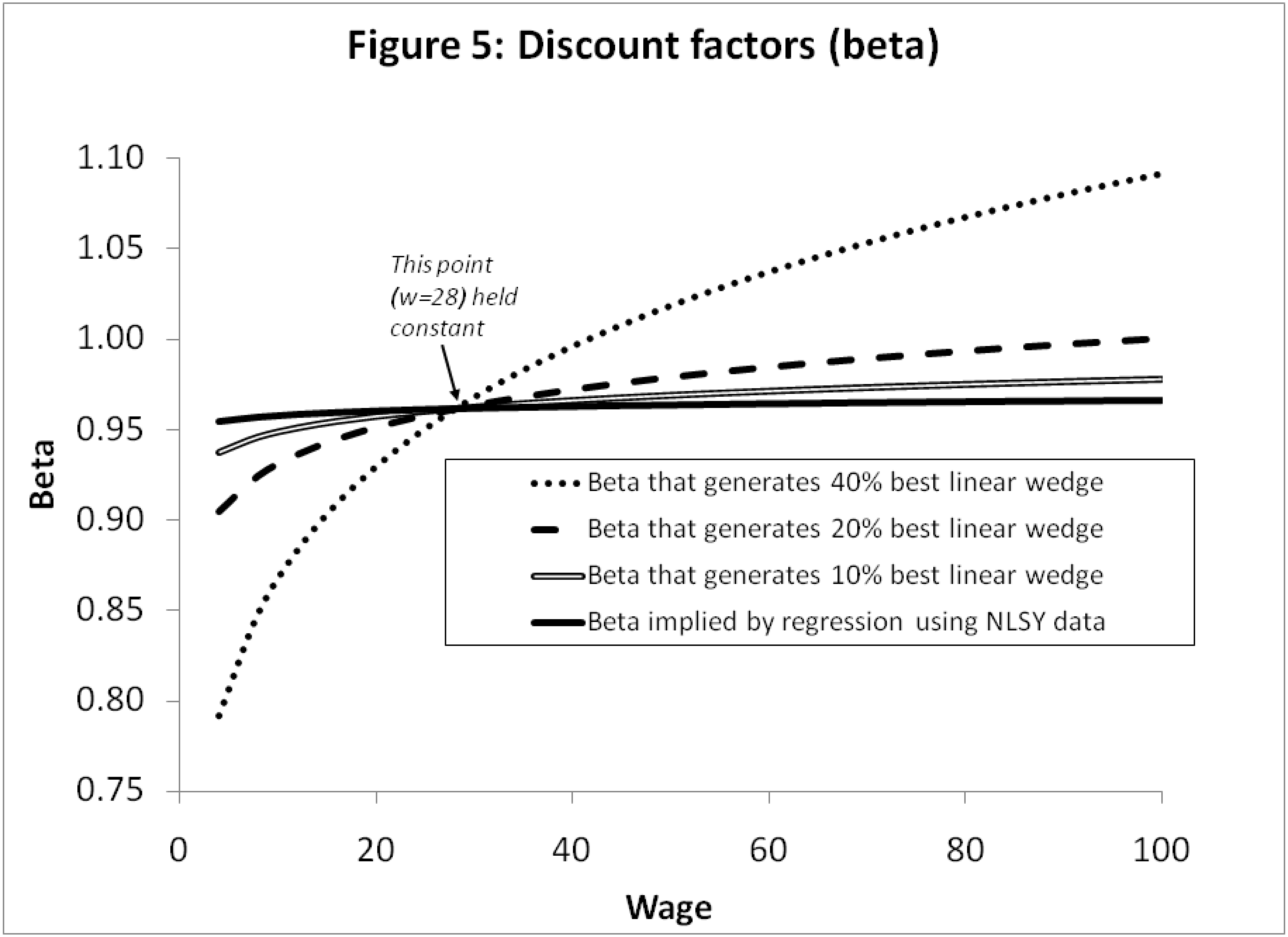


Figure 5: Discount factors (beta)
1.10
1.05
1.00
0.95
0.90
0.85
0.80
0.75
Beta
0
20
40
60
80
100
Wage
This point (w=28) held constant
Beta that generates 40% best linear wedge
Beta that generates 20% best linear wedge
Beta that generates 10% best linear wedge
Beta implied by regression using NLSY data