Simple Relations between Elements of the Three-Dimensional Orthogonal Matrix of the Basic Representation and Euler & RPY' Angles

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Abstract: *In this Article transformation angles of the methods of RPY (Roll, Pitch and Yaw) and Euler are found using Spinor method [Milnikov A.A., Prangishvili A.I., Rodonaia I.D. (2005)], which The method is based on the principally new approach-spinor representation of the spatial generalized rotations. And provides easy, reliable and efficient way of solution of inverse kinematics problem of multijoints spherical manipulators.*

Keywords: RPY' angles, Euler's angles, spinors, rotations, and kinematics inverse problem

Introduction

Robot is a system with a mechanical body using computer as its brain. Integrating the sensors and actuators built into the mechanical body, the motions are realized with the computer software to execute the desired tasks such as welding, painting, fulfilling assembly, handling and so on.

All these movements to do such tasks are achieved by means of rotation in 2D or 3D. Rotations cannot be arbitrary they must be from specific start point to end point of which coordinates are known in advance.

Let us simplify the problem, as shown in figure1: We rotate point X to point Y

· Center of rotation is origin

· Point X is initial point

· Point Y is final point

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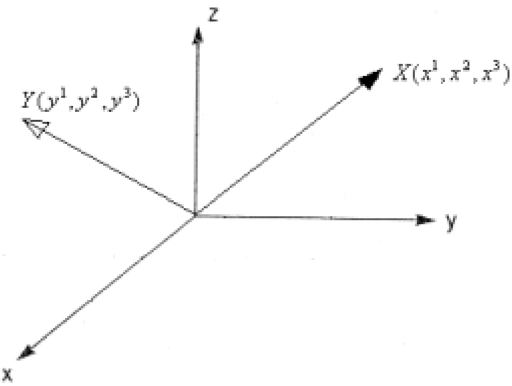
Figure 1: Simple spherical rotation from point X to point Y



To rotate the point X to point Y, we will consider two methods: Roll, Pitch, Yaw and Euler transformation. It is clear and we will see that it is very complex to find angles of rotations in both method.

Roll Pitch and Yaw Angles

If we imagine a ship steaming along the z axis, then roll corresponds to a rotation Φ about the z axis, pitch corresponds to a rotation θ about the y axis, and yaw corresponds to a rotation ψ about the x axis. The rotations applied to a manipulator end effectors are show in Figure2

Figure 2: Roll Pitch and Yaw Angles

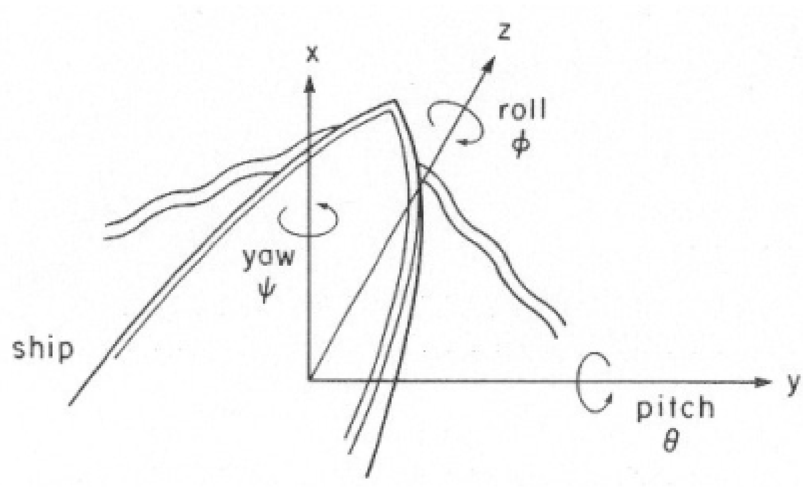
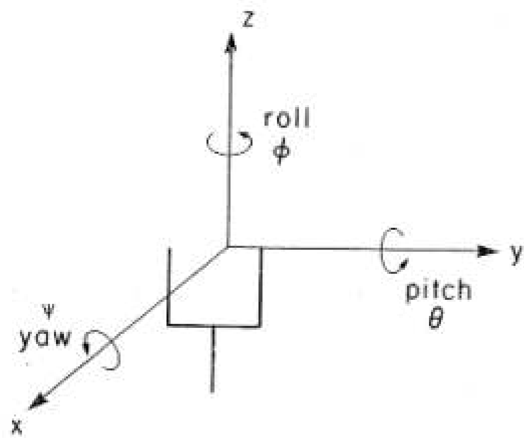


Figure 3: Roll, Pitch, and Yaw Coordinates for a Manipulator



We will specify the order of rotation as

$$RPY(\phi,\theta,\psi)=Rot(z,\phi)Rot(y,\theta)Rot(x,\psi) \quad (1)$$

That is a rotation of ψ about x, followed by a rotation θ about y, and finally, a rotation Φ about station z. The transformation is as follows

$$RPY(\phi,\theta,\psi)=Rot(z,\phi)\begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\psi & -\sin\psi & 0 \\ 0 & \sin\psi & \cos\psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

$$RPY(\phi,\theta,\psi)=\begin{bmatrix} \cos\phi & -\sin\phi & 0 & 0 \\ \sin\phi & \cos\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta\sin\psi & \sin\theta\cos\psi & 0 \\ 0 & \cos\psi & -\sin\psi & 0 \\ -\sin\theta & \cos\theta\sin\psi & \cos\theta\cos\psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

$$RPY(\phi,\theta,\psi)=\begin{bmatrix} \cos\phi\cos\theta & \cos\phi\sin\theta\sin\psi-\sin\phi\cos\psi & \cos\phi\sin\theta\cos\psi+\sin\phi\sin\psi & 0 \\ \sin\phi\cos\theta & \sin\phi\sin\theta\sin\psi+\cos\phi\cos\psi & \sin\phi\sin\theta\cos\psi-\cos\phi\sin\psi & 0 \\ -\sin\theta & \cos\theta\sin\psi & \cos\theta\cos\psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

Euler Angles

Orientation is more frequently specified by a sequence of rotations about the *x*, *y*, and *z* axes. Euler angles describe any possible orientation in terms of a rotation Φ about the *z* axis, then a rotation θ about the new *y* axis, $y^{/}$, and finally, a rotation about the new z axis, z^{n}, of ψ. (See Figure 4).

Figure 4: Euler Angles

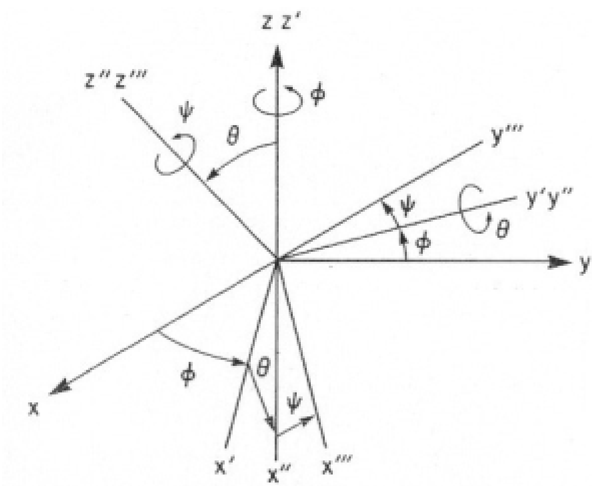
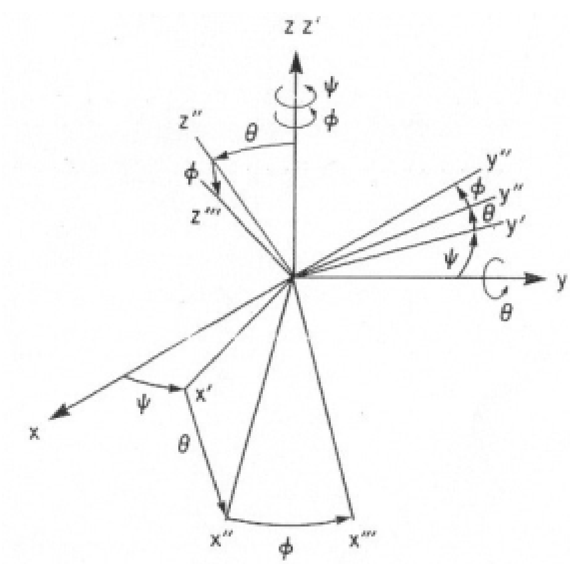


Figure 5: Euler angles interpreted in base coordinates



$$\mathrm{Euler}(\Phi,\theta,\psi) = \mathrm{Rot}(z,\Phi)\,\mathrm{Rot}(y,\theta)\,\mathrm{Rot}(z,\psi) \tag{5}$$

As in every case of a sequence of rotations, the order in which the rotations are made is important. Notice that this sequence of rotations can be interpreted in the reverse order as rotations in base coordinates: A rotation ψ about the z axis, followed by a rotation θ about the base y axis, and finally a rotation Φ, once again about the base z axis.

The Euler transformation, $\mathrm{Euler}(\Phi,\theta,\psi)$, can be evaluated by multiplying the three rotation matrices together

$$Euler(\phi,\theta,\psi) = Rot(z,\phi)\,Rot(y,\theta)\,Rot(z,\psi) \tag{6}$$

$$Euler(\phi,\theta,\psi) = \begin{bmatrix} \cos\phi\cos\theta\cos\psi - \sin\phi\sin\psi & -\cos\phi\cos\theta\sin\psi - \sin\phi\cos\psi & \cos\phi\sin\theta & 0 \\ \sin\phi\cos\theta\cos\psi + \cos\phi\sin\psi & -\sin\phi\cos\theta\sin\psi + \cos\phi\cos\psi & \sin\phi\sin\theta & 0 \\ -\sin\theta\cos\psi & \sin\theta\sin\psi & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

(Shahinpoor M., (1988))

Defining angles with spinor method

So, to define the rotation, two transformation matrixes (2.4, 3.3) can be used but the problem is how to find the numerical value of the matrixes. In order to get the numerical value of the matrixes we must have in advance equivalent transformation matrix to make them equal and get the value of the angles. By using angles we define the transformation matrixes.

From this formula: AX = Y

$$\begin{vmatrix} a_1^1 \; a_2^1 \; a_3^1 \\ a_1^2 \; a_2^2 \; a_3^2 \\ a_1^3 \; a_2^3 \; a_3^3 \end{vmatrix} \times \begin{vmatrix} x^1 \\ x^2 \\ x^3 \end{vmatrix} = \begin{vmatrix} y^1 \\ y^2 \\ y^3 \end{vmatrix}$$

Actually it is impossible to find elements of matrix A. Because, after multiplication we get tree equation and nine unknowns. But by using spinor method we obtain the elements of matrix A (let us call spinor transformation matrix) (Milnikov A., Onal H., Partskhaladze R., Rodonaia I. (2004)).

That's:

$$A=\begin{vmatrix} a_1^1\, a_2^1\, a_3^1 \\ a_1^2\, a_2^2\, a_3^2 \\ a_1^3\, a_2^3\, a_3^3 \end{vmatrix} = \begin{bmatrix} \cos\phi\cos\theta & \cos\phi\sin\theta\sin\psi-\sin\phi\cos\psi & \cos\phi\sin\theta\cos\psi+\sin\phi\sin\psi & 0 \\ \sin\phi\cos\theta & \sin\phi\sin\theta\sin\psi+\cos\phi\cos\psi & \sin\phi\sin\theta\cos\psi-\cos\phi\sin\psi & 0 \\ -\sin\theta & \cos\theta\sin\psi & \cos\theta\cos\psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$a_1^3=-\sin\theta \qquad a_2^3=\cos\theta\sin\psi \qquad a_3^3=\cos\theta\cos\psi \qquad (8)$$

$$A=\begin{vmatrix} a_1^1\, a_2^1\, a_3^1 \\ a_1^2\, a_2^2\, a_3^2 \\ a_1^3\, a_2^3\, a_3^3 \end{vmatrix} = \begin{Vmatrix} \cos\varphi\cos\psi-\cos\theta\sin\varphi\sin\psi & -\cos\varphi\sin\psi-\cos\theta\sin\varphi\sin\psi & \sin\varphi\sin\theta \\ \sin\varphi\cos\psi+\cos\theta\cos\varphi\sin\psi & -\sin\varphi\sin\psi+\cos\theta\cos\varphi\cos\psi & -\cos\varphi\sin\theta \\ \sin\psi\sin\theta & \cos\psi\sin\theta & \cos\theta \end{Vmatrix}$$

$$\cos\theta=a_{33} \qquad \sin\varphi\sin\theta=a_{31} \qquad \sin\psi\sin\theta=a_{13} \qquad (9)$$

Conclusion

Simple relations are obtained between elements of the three-dimensional orthogonal matrix of the basic representation and Euler angles, RPY angles.

It is shown that spinor method can be used for the solution of the inverse kinematic problem for spatial mechanisms.

References

1. Milnikov A.A., Prangishvili A.I., Rodonaia I.D.(2005) Spinor Model of Generalized Three-dimensional Rotations: Automation and remote Control, Vol.66, No 6, p.p. 876-872. Moscow

2. Milnikov A., Onal H., Partskhaladze R., Rodonaia I. (2004), Spinor evaluation of Euler's angles: Jornal of applied mechanics, No2, p.p. 48-53. Tbilisi