Variation Approach in Adaptive Control of Mobile Robots

Cabir ERGUVEN[1]

Abstract

In this article, various problems of movement control of mechanical objects like   problem of guidance, problem of acceleration, problem of control with excluded time and
problem of control as a function of time have been discussed and solved. The simple expressions for relevant optimum phase trajectories and algorithms of adaptive movement control have been obtained, which allow their effective practical realization to be carried out.

Key Words: *Terminal Control, Moving Object, Boundary Value Problem, Adaptive Movement Control.*

1. Introduction

Problems related to the control of moving mechanical objects belong to the class of sufficiently well studied problems, many of which for a long time have been regarded as classical ones (Bellman, 1957 and Lee, 1967). First of all, they include such methods as the principle of maximum, dynamic programming, the momentum method and others directly connected with the classical methods of variation calculus. These methods are rather difficult for application, since the eventual control algorithms obtained with their aid are actually of programming character, i.e. explicitly depending on time. Therefore, it is impossible to carry out the current correction of a phase trajectory, though such a correction is absolutely necessary because a moving object is influenced by perturbing environmental factors (both systematic and random).

Below, we will consider only one-dimensional problems of adaptive control which, from the standpoint of application, are important to solve control problems for mobile robots. Since a mobile machine (MM) has a specific design, it makes sense to consider only movements along rectilinear segments. Thus, a plane problem actually reduces to an one-dimensional problem with the

[1] MSc, International Black Sea University, Agmasheneblis Kheivani 13km No2,
cabir@ibsu.edu.ge

two-dimensional phase space $(\psi, \dot{\psi})$, where ψ is MM movement along the straight line, and $\dot{\psi}$ is velocity.

2. Principal Part

2.1. General Problem.

Let us assume that the acceleration $\ddot{\psi}$ and the control force F are related as follows:

$$\ddot{\psi} = kF,$$
(1)

where F is the moment of force acting on the controlled wheels, and k is the proportionality coefficient.

The control algorithm synthesis can be reduced to the following variational problem: Given two points with coordinates $(\psi_0, \dot{\psi}_0)$ and $(\psi_f, \dot{\psi}_f)$ in a two-dimensional phase space, it is required to find equations for the curve which lies in the phase plane and connects these points and on which the functional

$$J_F = \frac{1}{T} \int_0^T F^2(t)dt,$$

(A)

where T the object control time interval, takes a minimal value. The curve defined in this manner is obviously optimal in the sense of minimum (A), i.e. in the sense of control forces energy.

Acceleration along the optimal curve is a function of phase coordinates

$$\ddot{\psi} = \varphi(\psi, \dot{\psi}).$$
(2)

Comparing (2) with (1), we obtain the equality

$$kF = \varphi(\psi, \dot{\psi}).$$
(3)

The substitution of (1) into (A) gives

$$J = \frac{1}{T}\int_0^T k_1[\varphi(\psi,\dot{\psi})]^2 dt = \frac{1}{T}\int_0^T [k_1\ddot{\psi}]^2 dt$$

(4)

where $\quad k_1 = \frac{1}{k}$.

Since functional (4) belongs to the type of functional containing second order derivatives, the Euler equation takes the following form (Gelfand, 1961 and Hestenes, 1966).

$$\frac{d^2\ddot{\psi}}{dt^2} = 0.$$

(5)

The solution of this equation is written as the third order polynomial

$$\psi = C_0 + C_1 t + C_2 \frac{t^2}{2} + C_3 \frac{t^3}{6}.$$

(6)

The boundary conditions have the form

t=0; $\psi = \psi_0$; $\dot{\psi} = \dot{\psi}_0$,   (7)

t=T; $\psi = \psi_f$; $\dot{\psi} = \dot{\psi}_f$.   (8)

These four conditions are sufficient for defining four constants C_i (i=0,1,2,3) contained in (6).

Thus the optimal trajectory is defined completely.

Let us first consider two particular problems.

2.2. Guidance Problem.

The guidance problem is defined by the following boundary conditions:

t=0; $\psi = \psi_0$; $\dot{\psi} = \dot{\psi}_0$,   (9)

t=T; $\psi = \psi_f$;   (10)

Conditions (9) and (10) require that the states $\psi = \psi_0$; and $\dot\psi = \dot\psi_0$ of the object must be reduced to the state $\psi = \psi_f$ under the assumption that the guidance velocity is arbitrary. In terms of variation calculus, this is the problem with moving ends.

For such problems, to the available conditions (recall that we have four constants, while the boundary conditions are only three) we add the missing transversal condition (Gelfand , 1961 and Hestenes, 1966) which ,in our case, is written as

$$\dot G_\psi - \frac{d}{dt}\dot G_{\ddot\psi} = 0 \tag{11}$$

where G denotes the sub integrand in (4).

It is obvious that

$$\dot G_{\dot\psi} = 0 \tag{12}$$

and

$$\dot G_{\ddot\psi} = 2\dddot\psi \tag{13}$$

Thus, equation (11) can be reduced to

$$2\dddot\psi = 0 \tag{14}$$

Using the triple differentiation of (6), initial conditions (9), (1) and transversal condition (14), we define C_i (i=0,1,2,3) as follows:

$$C_3 = 0; C_2 = \frac{2(\psi_0 - \psi_f)}{T^2} - \frac{2\psi_f}{T}; C_1 = \dot\psi_0; C_0 = \psi_0 \tag{15}$$

Equality (15) enables us to write the following expressions for an optimal trajectory in the phase space:

$$\psi = (\frac{(\psi_0 - \psi_f)}{T^2} - \frac{\psi_f}{T})t^2 + \dot\psi_0 t + \psi_0 ,$$

$$\dot\psi = (\frac{2(\psi_0 - \psi_f)}{T^2} - \frac{2\psi_f}{T})t + \dot\psi_0 \tag{16}$$

Phase acceleration turns out to be the constant value

$$\ddot{\psi} = \frac{2(\psi_0 - \psi_f)}{T^2} - \frac{2\psi_f}{T} \quad . \tag{17}$$

For the reduction problem this is the law of control.

2.3. Acceleration Problem.

Let us replace the boundary condition (10) by

$$t=T; \quad \dot{\psi} = \dot{\psi}_f \quad . \tag{18}$$

From conditions (18) it obviously follows that the phase velocity of the controlled object should be changed so that at the time moment $t = T$ it become equal to the given value ($\dot{\psi} = \dot{\psi}_f$) for any phase coordinate value.

This is again the problem with moving ends. The transversal equation (14) remains the same. Analogously to the case considered in Subsection 2.2, we obtain the following values for the constants C_i (i=0,1,2,3):

$$C_3 = 0; C_2 = \frac{\dot{\psi}_f - \dot{\psi}_0}{T}; C_1 = \dot{\psi}_0; C_0 = \psi_0 \quad . \tag{19}$$

Hence

$$\psi = (\frac{\dot{\psi}_f - \dot{\psi}_0}{2T})t^2 + \dot{\psi}_0 t + \psi_0 \quad , \tag{20}$$

$$\dot{\psi} = (\frac{\dot{\psi}_f - \dot{\psi}_0}{T})t + \dot{\psi}_0 \quad . \tag{21}$$

Phase acceleration is again a constant value equal to

$$\ddot{\psi} = \frac{\dot{\psi}_f - \dot{\psi}_0}{T} \quad . \tag{22}$$

Formula (22) is the law of control for the acceleration problem.

2.4. Control with Excluded Time.

Combining the two problems considered above, we can solve a general problem of defining the control law for the movement of the phase point $(\psi_0, \dot{\psi}_0)$ to the phase point $(\psi_f, \dot{\psi}_f)$ with excluded time. From (22) we have

$$T = \frac{\dot{\psi}_f - \dot{\psi}_0}{\ddot{\psi}}$$

$$(23)$$

For $t = T$ the first equation from (16) can be rewritten as

$$\psi_f = \ddot{\psi}\frac{T^2}{2} + \dot{\psi}_0 T + \psi_0 ,$$

$$(24)$$

where $\quad \ddot{\psi} = 2(\frac{(\psi_0 - \psi_f)}{T^2} - \frac{\psi_f}{T})$.

The substitution of (23) into (24) gives

$$\ddot{\psi} = \frac{\dot{\psi}_f^2 - \dot{\psi}_0^{\,2}}{2(\psi_f - \psi_0)}$$

$$(25)$$

If in (25) we replace the initial values ψ_0 and $\dot{\psi}_0$ by the current ones

$$\ddot{\psi} = \frac{\dot{\psi}_f^2 - \dot{\psi}^{\,2}}{2(\psi_f - \psi\ )} ,$$

$$(26)$$

then (25) gives solution of the posed problem on bringing the controlled object from the state $(\psi_0, \dot{\psi}_0)$ to the state $(\psi_f, \dot{\psi}_f)$. The procedure is as follows: the current position and velocity of the moving object are measured and both values (ψ and $\dot{\psi}$) are entered into the computing unit of the automatic control system, where expression (26) is calculated and multiplied by the proportionality coefficient k_1 in order to define the control force F (3). The process stops at $\psi = \psi_f$ and $\dot{\psi} = \dot{\psi}_f$.

Because of the operation of replacement, in (26), of the initial point $(\psi_0, \dot{\psi}_0)$ by the current point $(\psi, \dot{\psi})$, functional (26) is calculated not on the entire trajectory, but on its individual segments. Hence, a question arises whether the trajectory defined in this manner is optimal, i.e. whether it corresponds to the trajectory defined by minimum (A). The answer is provided by the following statement which underlies the dynamic programming method: The end of an optimal trajectory is always optimal (Bellman, 1957). The operation of replacement, in (26), of the initial point $(\psi_0, \dot{\psi}_0)$ by the current

point ($\Psi, \overset{.}{\Psi}$) results in the minimization of the phase trajectory end, i.e. this operation realizes implicitly the dynamic programming principle, which may serve as a proof that the trajectory defined by (26) is optimal.

2.5. Control as a Function of *T*.

Let us return to the problem with four boundary conditions. After defining the constants C_i (i=0,1,2,3) by the initial conditions (7), (8), we obtain the phase acceleration value

$$\ddot{\psi} = \frac{6(\psi_0 - \psi_f)}{T^2} + \frac{2(\psi_0 + 2\psi_f)}{T}.$$ (27)

By virtue of the arguments given at the end of the preceding subsection, we can rewrite (27) as

$$\ddot{\psi} = \frac{6(\psi - \psi_f)}{(T-t)^2} + \frac{2(\psi + 2\psi_f)}{T-t}.$$ (28)

Formula (28) is the law of feedback control of putting, at a given time T, the controlled object to the reference trajectory.

The control law (28) can be called as "strict" control. A change of the phase acceleration value in the course of control brings about a change of the impact force (3), which results in the movement of the object. Moreover, a change occurs in the object acceleration and, accordingly, in all uncontrolled forces acting on the object. Control has to compensate (in the sense of energy) for a change of uncontrolled forces, since otherwise the process might become uncontrollable, for instance, if T has been chosen rather small.

The above reasoning explains the meaning of the term "strict". As different from this, the law of control with excluded time T can be called adaptive or self-adapting. In this case the approach of the phase point to the terminal point is of asymptotic character and therefore the controlling force always compensates uncontrollable forces.

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