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Abstract: We generalize the analyses of Hazledine (2006, "Price discrimination in Cournot-Nash oligopoly", Economics Letters) and Kutlu (2009, "Price discrimination in Stackelberg competition", Journal of Industrial Economics) with asymmetric cost firms. We show that the main result of Hazledine, which shows that the average revenue is not dependent on the extent of price discrimination, remains under cost asymmetry but at the industry level. However, the main result of Kutlu, which shows that the Stackelberg leader does not price discriminate at all, does not hold under cost asymmetry. Both the leader and the follower discriminate price under cost asymmetry.

Key Words: Cost asymmetry; Cournot competition; Price discrimination; Stackelberg competition

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1. Introduction

The literature on price discrimination considers different types of price discrmination. Stole (2007) provides a nice of survey of this literature. In a recent paper, Hazledine (2006) examines second degree price discrimination in a Cournot model. Under second degree price discrimination, the firm is able to segment consumer demand by ranges of reservation price. For example, the consumers with reservation price between r_1 and r_2 pay one price, those between r_2 and r_3 pay another, and so on (Kutlu, 2009). The main result of Hazledine (2006) is to show that the average revenue (i.e., the output-weighted revenue) is independent of the extent of price discrimination. In other words, the average revenue under k different prices is the same to the average revenue under $(k+1)$ different prices.

In a framework similar to Hazledine (2006) but with a Stackelberg leader-follower competition, Kutlu (2009) shows that the Stackelberg leader serves only the high-valued customers, and therefore, does not price disciminate at all. The Stackelberg follower does all price discrimination. While Hazledine (2006) and Kutlu (2009) ask different questions under different types of product market competition, both papers focus on symmetric cost firms. Hence, a natural qustion is to ask whether their results hold under cost asymmetry.

We generalize the analyses of Hazledine (2006) and Kutlu (2009) with asymmetric cost firms and show that the result of Hazledine (2009) about the average revenue remains but at the industry level. However, the result of Kutlu (2009), which shows that the leader dos not price discriminate, does not hold under cost asymmetry.

The remainder of the paper is organized as follows. Section 2 shows the equilibrium outcomes under Cournot and Stackelberg competition with no price discrimination. Section 3 focuses on price discrimination. Section 4 concludes.

2. The results under no price discrimination

First, we briefly mention the results of both Cournot competition and Stackelberg competition under no price discrimination.

Assume that there are two firms, firm 1 and firm 2, producing a homogeneous product with the constant marginal costs zero and $c_f = c$ respectively, with $c > 0$. We assume that each consumer buys at most one unit of the good and the consumers differ in terms of valuations. We assume that the price of the product is given by

$$P = 1 - q^1 - q^2, \tag{1}$$

where q^i is the output of firm i, $i = 1, 2$, and P is the price.

2.1. Cournot competition

If firms 1 and 2 produce like Cournot duopolists, straightforward calculation shows that the equilibrium output of firms 1 and 2 are respectively

$$q^{1c} = \frac{1+c}{3} \quad \text{and} \quad q^{2c} = \frac{1-2c}{3}. \tag{2}$$

Both firms produce positive outputs for $c < \frac{1}{2}$.

The industry output, i.e., the total outputs of both firms, is $q^c = \frac{2-c}{3}$ and the uniform price under Cournot competition is

$$p^c = \frac{1+c}{3}. \tag{3}$$

2.2. Stackelberg competition

Now consider Stackelberg competition where firm 1 behaves like a Stackelberg leader and firm 2 behaves like a Stackelberg follower. Straightforward calculations shows that the equilibrium outputs of firms 1 and 2 are respetively

$$q^{1l} = \frac{1+c}{2} \quad \text{and} \quad q^{2f} = \frac{1-3c}{4}. \tag{4}$$

Both firms produce positive outputs for $c < \frac{1}{3}$.

The industry output is $q^s = \frac{3-c}{4}$ and the uniform price under Stackelberg competition is

$$p^s = \frac{1+c}{4}. \tag{5}$$

3. The case of price discimnation

Now we consider the case of price discrimination.

3.1. Cournot competition

We adopt the model under price discrimination from Hazledine (2006) and Kutlu (2009). The firms know the valuations of the consumers and can prevent resale of the good. The firms divide the consumers into different groups accoding to their valuations. Since the main results of Hazledine (2006) and Kutlu (2009), which have been mentioned in the introduction, do not depend on the number of different prices or the number of consumer groups, to show our results in the simplest way, we focus on two groups of consumers, i.e., consider two different prices. We assume that the prices of the product for group 1 and the group 2 are respectively

$$P_1 = 1 - q_1^1 - q_1^2 \tag{6}$$

$$P_2 = 1 - q_1^1 - q_1^2 - q_2^1 - q_2^2, \tag{7}$$

where P_i is the price for group i and q_i^j is the output of firm j for group i, where $i, j = 1,2$ and $i \neq j$. The firms chosse their outputs simultaneously and the profits are realized.

The justification for such a setting follows from Hazledine (2006) and Kutlu (2009). Consider the airline industry, where the airline tickets are purchased in unit quantity. Consumers come to market at different times and their valuations differ. The airlines charge different prices to consumers with different valuations.

Given the demand cost functions, firms 1 and 2 determine their outputs simultaneously to maximize the following expressions respectively:

$$\underset{q_1^1, q_2^1}{Max}(1 - q_1^1 - q_1^2)q_1^1 + (1 - q_1^1 - q_1^2 - q_2^1 - q_2^2)q_2^1 \tag{8}$$

$$\underset{q_1^2, q_2^2}{Max}(1 - q_1^1 - q_1^2 - c)q_1^2 + (1 - q_1^1 - q_1^2 - q_2^1 - q_2^2 - c)q_2^2. \tag{9}$$

Straightforward calculation gives:

$$q_1^1 = \frac{2-c}{7} \quad \text{and} \quad q_2^1 = \frac{1+3c}{7} \tag{10a}$$

$$q_1^2 = \frac{2-c}{7} \quad \text{and} \quad q_2^2 = \frac{1-4c}{7}. \tag{10b}$$

Both firms produce positive outputs for both groups if $c < \frac{1}{4}$. The comparison with no price discrimination under Cournot competition shows that if $\frac{1}{4} < c < \frac{1}{2}$, firm 2 (which is more cost inefficient) serves only the high-valued consumers.

We concentrate on those values of cost asymmetries such that the outputs of both firms are positive for both groups of consumers. The equilibrium prices are then:

$$P_1 = \frac{3+2c}{7} \quad \text{and} \quad P_2 = \frac{1+3c}{7}. \tag{11}$$

The output-weighted revenue or the average revenue for firms 1 and 2 are respectively

$$P_{c,av}^{1}=\frac{P_1q_1^1+P_2q_2^1}{q_1^1+q_2^1}=\frac{1+c+c^2}{3+2c} \text{ and } P_{c,av}^{2}=\frac{P_1q_1^2+P_2q_2^2}{q_1^2+q_2^2}=\frac{1-2c^2}{3-5c}. \quad (12)$$

The output-weighted industry revenue or average industry revenue is

$$P_{c,av}=\frac{P_1(q_1^1+q_1^2)+P_2(q_2^1+q_2^2)}{q_1^1+q_1^2+q_2^1+q_2^2}=\frac{1+c}{3}. \quad (13)$$

Proposition 1: *(a) Under Cournot competition, the average revenue for firm 1 (firm 2) is lower (higher) under price discrimination than under no price discrimination, for any cost difference between firms 1 and 2.*

(b) Under Cournot competition, the average industry revenue is the same under no price discrimination and under price discrimination, irrespective of the cost difference between firms 1 and 2.

Proof: The result (a) follows from the comparison between (3) and (12) and the result (b) follows from the comparison between (3) and (13). ■

It is interesting to see from the above result that, under cost asymmetry, the average revenue of firm 1 (which is more cost efficient) is lower but that is of firm 2 (which is more cost inefficient) is higher under price discrimination compared to no price discrimination. This is due to the effect of cost asymmetry on the equilibrium prices and outputs.

The average revenue of firms 1 and 2 are the same under price discrimination and no price discrimination if there is no cost asymmetry, i.e., $c = 0$. However, as the cost asymmetry increases, i.e., c increases, under price discrimination, the prices for both groups increase, which tend to increase the average revenue of both firms 1 and

2. On the other hand, as c increases, it reduces firm 1's relative share for the high-valued consumers, while it increases its relative share for the low-valued consumers. This output composition effect tends to reduce the average revenue of firm 1. On the balance, the output composition effect dominates the price effect and reduces firm 1's average revenue under price discrimination compared to no price discrimination.

In contrast, as c increases, firm 2's relative share for high-valued consumers increases, while its relative share for low-valued consumers reduces. As a result, firm 2's average revenue under price discrimination is higher compared to no price discrimination.

While looking at the average industry revenue, the price effect balances with the output composition effect, and the average industry revenue is the same under price discrimination and no price discrimination.

It must be noted that, under cost asymmetry, even if the output-weighted revenue of firm 1 is lower under price discrimination compared to no price discrimination, the total revenue of firm 1 and that of firm 2 are higher under price discrimination than under no price discrimination for any $c \in (0, \frac{1}{4})$. Further, the total industry revenue is also higher under price discrimination than under no price discrimination.

3.2. Stackelberg competition

Now we consider price discrimination under Stackelberg competition, where firm 1 behaves like a Stackelberg leader and firm 2 behaves like a Stackelberg follower. Again, we consider two groups of consumers, i.e., two different prices.

Given the demand functions (6) and (7), and the cost functions, firm 2 determines its outputs to maximize the following expression:

$$\underset{q_1^2, q_2^2}{Max}(1-q_1^1-q_1^2-c)q_1^2+(1-q_1^1-q_1^2-q_2^1-q_2^2-c)q_2^2 . \tag{14}$$

Straightforward calculation gives:

$$q_1^2=\frac{(1-c-q_1^1)+q_2^1}{3} \tag{15a}$$

$$q_2^2=\frac{1-c-q_1^1-2q_2^1}{3} . \tag{15b}$$

Firm 1 maximizes the following expression to determine its outputs:

$$\underset{q_1^1, q_2^1}{Max}(1-q_1^1-q_1^2)q_1^1+(1-q_1^1-q_1^2-q_2^1-q_2^2)q_2^1 \tag{16}$$

subject to conditions (15a) and (15b).

The equilibrium outputs of firm 1 can be found as

$$q_1^1=\frac{1}{2} \text{ and } \quad q_2^1=\frac{c}{2} . \tag{17}$$

The equilibrium outputs of firm 2 are:

$$q_1^2=\frac{(1-c)}{6} \quad \text{and} \quad q_2^2=\frac{1-4c}{6} . \tag{18}$$

Both firms produce positive outputs for $c<\frac{1}{4}$. The comparison with no price discrimination under Stackelberg competition shows that if $\frac{1}{4}<c<\frac{1}{3}$, firm 2 (which is more cost inefficient) serves only the high-valued consumers.

The following results are immediate from the above discussion.

Proposition 2: *(a) If* $0<c<\frac{1}{4}$*, both firms produce positive outputs for both groups, i.e., both firms discriminate price for both groups. In this situation, both firms produce more for group 1, i.e.,* $q_1^1>q_2^1$ *and* $q_1^2>q_2^2$.

Proposition 2(a) is in contrast to Kutlu (2009), which shows that the Stackelberg leader sells only to the high-valued concumers. However, we show that it is not the case with cost asymmetries, i.e., for $c > 0$.

The reason for Proposition 2(a) follows from Kutlu (2009). As in Kutlu (2009), while choosing the output for the low-valued consumers, firm 1 considers the implication it has on the output of firm 2 for the high-valued consumers. It is clear from (15a) and (15b) that a higher output of firm 1 for the low-valued consumers, i.e., q_2^1, will reduce firm 2's output for the low-valued consumers, i.e., q_2^2, but it will increase firm 2's output for the high-valued consumers, i.e., q_1^2. This reaction of firm 2 eliminates firm 1's incentive for serving the low-valued consumers in Kutlu (2009). However, if firm 2's cost inefficiency compared to firm 1 increases, firm 1's loss of profit in the high-valued consumer group reduces following firm 2's higher output for the high-valued consumers, thus inducing firm 1 to serve also the low-valued consumers. However, firm 1 prefers to sell more outputs to the high-valued consumers.

If there is no cost asymmetry between the firms, firm 2 sells the same amount to both groups of consumers. However, cost asymmetry encourages firm 1 to sell to the low-valued consumers also, which induces firm 2 to reduce its output for the low-valued consumers and to produce more for the high-valued consumers. As a result, in the case of cost asymmetry, firm 2 also sells higher outputs to the high-valued consumers.

Now consider the average revenue under Stackelberg competition. Under price discrimination, the average revenue for firms 1 and 2 are respectively

$$P_{s,av}^1 = \frac{P_1 q_1^1 + P_2 q_2^1}{q_1^1 + q_2^1} = \frac{1 + c + c^2}{3(1+c)} \text{ and } P_{s,av}^2 = \frac{P_1 q_1^2 + P_2 q_2^2}{q_1^2 + q_2^2} = \frac{1 - 2c^2}{3 - 5c}. \tag{19}$$

The average industry revenue under Stackelberg competition with price discrimination is

$$P_{s,av} = \frac{P_1(q_1^1 + q_1^2) + P_2(q_2^1 + q_2^2)}{q_1^1 + q_1^2 + q_2^1 + q_2^2} = \frac{3 + c - c^2}{2(5 - 2c)}. \tag{20}$$

Proposition 3: *Under Stackelberg competition, the average revenues of firm 1, firm 2 and the industry are higher under price discrimination than under no price discrimination, for any cost asymmetry between firms 1 and 2.*

Proof: The result follows from the comparison of the expressions in (5) and (19) and (20). ■

4. Conclusion

We generalize the analyses of Hazledine (2006) and Kutlu (2009) to show the implications of cost asymmetries between the firms. We show that, under Cournot competition, the industry average revenue is the same under price discrimiation and no price discrimination, for any cost asymmetries between the firms. However, the average revenue of firm 1 (firm 2) is lower (higher) under price discrimination than under no price discrimination. Under Stackelberg competition, the average revenues of both firms and the industry are higher under price discrimination than under no price discrimination. We also show that both the Stackelberg leader and the Stackelberg follower price discriminate for any cost difference between the firms.

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