Istituto per lo Sviluppo della Formazione Professionale dei Lavoratori

DIPARTIMENTO

Università di Roma "La Sapienza"

Research Program on Labour Market Dynamics

Training, Firing Taxes and Welfare

Andrea Ricci

Discussion Paper n. 5, 2006

ARACNE

This Discussion Paper series collects the contributions coming out from the research partnership between ISFOL and the Dipartimento di Scienze Economiche of the Università di Roma "La Sapienza". Both the research partnership and the discussion paper series are coordinated by Marinella Giovine, Sergio Bruno and Paolo Piacentini.

Questa collana raccoglie i contributi elaborati nell'ambito della convenzione di ricerca tra Isfol ed il Dipartimento di Scienze Economiche dell'Università di Roma "La Sapienza". Sia la convenzione di ricerca che la collana di discussion papers sono coordinati da Marinella Giovine, Sergio Bruno e Paolo Piacentini.

Per ciascuna pubblicazione vengono soddisfatti gli obblighi previsti dall'art.1 del D.L.L. 31.8.1945, n. 660 e successive modifiche. Copie della presente pubblicazione possono essere richieste alla convenzione di ricerca DSE-ISFOL.

ISFOL
Istituto per lo sviluppo della formazione professionale dei lavoratori
Via G.B. Morgagni, 33, 00161 Roma
http://www.isfol.it/

Dipartimento di Scienze Economiche (DSE)
Via Cesalpino 12-14, 00161 Roma
http://dipartimento.dse.uniroma1.it

DSE-ISFOL homepage:
dipartimento.dse.uniroma1.it/DSE-ISFOL/index.htm

www.aracne–editrice.it
info@aracne–editrice.it

00173 Roma
via Raffaele Garofalo, 133 A/B
(06) 72672222 – (06) 93781065
telefax 72672233

ISBN 978–88–7999–1069–3

I edizione: dicembre 2006

Finito di stampare nel mese di dicembre del 2006
dalla tipografia « Grafica Editrice Romana S.r.l. » di Roma
per conto della « Aracne editrice S.r.l. » di Roma
Printed in Italy

TRAINING, FIRING TAXES AND WELFARE

Andrea Ricci
University of Rome “Tor Vergata”, Faculty of Economics

Abstract

This paper shows that the distortion of privately provided training caused by hold up an justify the introduction of positive firing tax in economies populated by risk neutral or perfectly insured agents. More precisely we highlight two results. First, an efficient economic policy, which makes use of a small lay-off tax and hiring subsidies, always increases employment, productivity and the welfare of unemployed workers. Second, with no hiring subsidies, the relationship between firing penalties and welfare depends on the returns to training. In this case, if returns to training are high enough, the introduction of a small firing tax causes an increase in job tenure, training investment and the welfare of unemployed.

1. INTRODUCTION

Employment protection has been analysed intensely in last decade, a period during which a number of European countries have experienced labour market reforms aimed at reducing the rigidity of employment relationships. Most contributions have considered employment protection as a cost incurred by firms, and the focus has been on employment dynamics and unemployment duration.

Among these, Bentolila and Bertola (1990) have established the opposite mechanisms through which firing costs prevent layoffs and discourage hiring, with ambiguous overall impact on employment. Mortensen and Pissarides (1999), also, discuss the implications of firing costs in a search and matching framework, with similar results the level of firing taxes reduces the inflows into unemployment and increases the average duration of unemployment, because of their negative impact on the propensity to hire unemployed workers.

Other papers related to the relationship between labour market flow and jobs security provisions reach the same conclusions (Garibaldi,1997; Hopenhayn and Rogerson,1998; Cauch and Zylberberg, 1999; Garibaldi and Violante, 2002). Employment protection, however, affects human capital accumulation, labour productivity and welfare, not only labour market flows. For instance, firing taxes may stimulate productivity growth because it increases job tenures and favours on-the-job training, as pointed out by Nickell and Layard (1999).

The aim of this paper is to tackle this issue by using a discrete-time matching model *a la* Mortensen-Pissarides' (1994), where the firms finance training of their employees. A search and matching framework seems a useful tool to treat problems involving rent sharing caused by private investment, and allows us to consider alternative wage setting institutions to model both flexible and rigid labour markets.

The crucial assumption is that training cannot be contracted between firms and workers, because of the unverifiable and unenforceable nature of firm specific training. The contractual incompleteness implies, in turn, a moral hazard on the firm's side, as firms maximize their expected profits without considering the share of initial cost sustained by the workers, through a wage cut for newly hired workers.

The consequent under-investment equilibrium causes excessive layoffs and lower job creation, higher unemployment and lower welfare with respect to socially optimal allocation. Wage renegotiation does not eliminate this inefficiency because of the timing of events. The amount of training is chosen in first stage of employment relation, while the returns to investment are realized at second stage.

Thus workers are not able to affect the amount of training, which is chosen unilaterally by firms. In such a context, an increase in job security reduces turnover and induces more training, increasing the privately chosen level towards the social efficient level. This causes increased market tightness and reduced layoff rates when hiring subsidies are introduced in a equilibrium budget constraint regime. Differently, when employment protection is pure waste and there are no hiring subsidies, the impact of firing taxes on welfare depends on the elasticity of training function, even though it continues to favour insider workers by increasing their wages and job durations.

There are a number of contributions related to our model. In the paper of Boone, Belot and van Ours (2002) employment protection strengthens the incentives of workers to invest in human capital in order to reduce the probability of being fired so that, when the firm offers a contract with high separation costs, it commit itself to a stable employment relationship.

Conversely, our discrete dynamic matching model introduces the additional decision margin about training investment on the firms' side, rather than discussing the effort choice of workers. And the emphasis is on hold up problem associated with the firms' provision of training, the amount of which is determined on the basis of future returns, not on the suitability of the workers.

This apparently minor change resolves significant theoretical problems with the model of Boone, Belot and van Ours. In their model, an efficient outcome can be obtained if the worker buys the firm. The firm can achieve higher returns by selling itself, on credit, to the worker resolving any possible problems with liquidity constraints.

The analogous solution in our model would be for the worker to sell himself to the firm, or at least make himself an indentured servant. This is obviously illegal. Thus our model obtains results which are robust to consideration of innovative contractual arrangements due to the inalienability of human capital.

Acemoglu and Shimer (1999), also, analyse the under investment due to hold up in a search environment and examine how markets can internalise the resulting externalities. In particular, Acemoglu and Shimer first show that the equilibrium is always inefficient when firms make ex ante investments before matching with workers and wages are determined by ex post bargaining. Either wages increase with output, creating a hold up problem, or all the bargaining power is vested in the firms, leading to very low wage levels and excessive entry of firms.

Then, an economy where firms post wages and workers direct search towards more capital intensive jobs is examined, establishing that in this case the equilibrium is efficient, a result in striking contrast with a traditional conjecture in the hold-up literature (Williamson, 1985). In our model jobs do not differ in terms of physical capital investment but in human capital accumulation.

This feature allows to discuss the hold up implications on labour market outcomes without assuming that firms' investment are financed before the match is formed. The unverifiable nature of training causes a contractual incompleteness even after the employer and employee meet, which makes the inefficient not dependent on search frictions.

Thus, contractual incompleteness is of a different kind than that considered by Acemoglu and Shimer. Furthermore, we use a search and bargaining model in which equilibrium is efficient for an appropriate bargaining solution (Hosios' condition) and highlight the role of employment protection to ameliorate the inefficient private provision of training, moving its level toward the social optimum. That is, our emphasis is on hold up and how this implies that employment protection can be welfare enhancing, not on search externalities [1].

Finally, the firm specific capital literature also makes the point that training investments are positively related to job tenure (Lazear, 1979). Hence employment protection is supposed to increase the welfare of employed workers in terms of wage growth and lower unemployment incidence, as well to hurt unemployed workers because of higher unemployment duration. Empirical evidence in this

1 A key result of this paper, the under investment in private provision of training, has also a similar intuition to Grout (1984), who pointed out that there will be inefficiently low investment in presence of rent-sharing. In Grout (1984) firms invest directly and choose the type of jobs in term of expected productivity: higher training firms suffer more from rent sharing then low training firms.

perspective are Topel (1991), Abraham and Farber (1987), Antonji and Sakoto, (1997). Nevertheless, none of these papers explicitly investigate the role of job security provisions on returns to tenure, a focus that allows us to obtain no standard conclusion.

This paper is organized as follows. Section 2 describes the basic theoretical framework and Section 3 focuses on the labour market equilibrium and its inefficient properties in terms of training investment and separation rate. Section 4 analyses the role of firing taxes to reduce hold up problem in the private provision of training and to increase the welfare of unemployed workers, when different design of public policy are concerned. Section 5 provides a numerical exercise. Finally section 6 concludes. The appendix contains proofs.

2. THE MODEL

The economy is populated by a continuum of risk-neutral workers and risk-neutral firms. Each group has mass 1 and lives infinitely. All agents discount the future at the exogenous rate r, which is strictly positive, and enjoy the consumption of the only good of this economy. Time is discrete.

Firms post vacancies at a cost *k* per period while workers supply labour inelastically and search with fixed intensity. The number of workers and firms that match is determined by a matching function $m(u,v)$, where u is the number of unemployed workers, v is the number of vacancies and the ratio $\theta = \frac{v}{u}$ is the market tightness. Then the transition rate facing the firms to match with an unemployed worker is $m(1,\theta) = m(\theta)$, while the probability of an unemployed worker to find a job is $m(\theta^{-1},1) = \theta m(\theta)$. The faction $(1 - m(1,\theta))$ of workers that are not matched stay unemployed and do not receive unemployment benefits. Workers are ex-ante homogenous in terms of ability .

Upon matching, the employment relation is formalized through a two stage structure, to highlight firms' incentives to train in the early period of their employees' career. In the first stage, the initial productivity of a matched worker is equal to the expected value of the idiosyncratic productivity ε, distributed according to a continuous and differentiable function $F(\varepsilon)$ over the finite support $[\varepsilon_L, \varepsilon_U]$: $y_0 = E(\varepsilon) = \varepsilon^e$. In addition, firms can train employees, to increase the future productivity of the match. In the model training is firm specific in the sense that the worker cannot use his skills with any other firm so workers can bargain over its returns[2]. The cost of training is linear and is incurred in terms of lower output in the first period of the match: $c(h) = h$ where h is the amount of training.

The returns of training are realized only at second stage of the employment relationship and are defined by a differentiable function, $f(h)$, with positive and decreasing marginal returns, i.e. $f^{'}(h) > 0$ and $f^{''}(h) < 0$. To find a parametric specification of the equilibrium we also assume that training returns equal to $f(h) = ah^{\alpha}$, where $a > 0$ is an efficiency parameter and $0 < \alpha < 1$ guarantees the concavity of the function.

At the second stage the returns of training are realized, if the worker-employer pair is not dissolved. Then the employment relationship enter into a “insider” phase, where the pair’s output is increased by the returns of training, i.e: $y_1 = \varepsilon + ah^{\alpha}$. Additive returns of training simplify analysis without affecting the results; moreover, they highlight that firm-sponsored training does not arise because of complementary between training and match specific productivity (see Acemoglu and Pischke,1999).

At the start of second stage it may be that workers and firms decide to split up if the idiosyncratic productivity of the match is too low. In such a case the firm pays a firing cost to the state and return on the labour market to fill a new vacancy position, while workers become unemployed. When the employment relationship survives until the “insider” phase, the match is no longer subject to productivity shocks, but expires only if worker retires from the labour market, an event which happens with probability s each period, with $0 < s < 1$.

In the economy employment protection is defined as firing restrictions imposed by the government on the firms when matches are

2 As Becker (1964) pointed out, certain investment go into specific capital committing the firm to irreversible expenditure that only have value in the context of the relationship between the employer and employee, who share the benefits of its investments through wage bargain. Training investment analysed in our model are of this type.

dissolved[3]. The tax paid on separation, T, is not a compensation given to the worker but a loss borne entirely by the firm. Firms can also receive an exogenous amount of hiring subsidy each time a worker is hired, σ. In modelling these policy interventions we consider two alternatives. In the first case the amount of hiring subsidies are determined by the equilibrium budget constraint and compensate exactly the cost of separation sustained by firms. Thus no fiscal externalities arise in equilibrium. In the second there are no hiring subsidies, the public financing constraint does not matter and firing costs are interpreted as pure waste from social point of view.

2.1 Asset values

Consider the asset value equations that characterize firms and workers. Let $J_0(\varepsilon^e, h)$ be the expected present value of the profit from a position filled as an entry level job, $J_1(\varepsilon, h)$ the value of a regular job to the firm, and V value of a vacancy to the firms:

$$(1)\ J_0(\varepsilon^e, h) = \frac{\varepsilon^e - h - w_0(\varepsilon^e, h) + \sigma}{1+r} + \frac{1}{1+r}\int_{\varepsilon_l}^{\varepsilon_u} Max\left[J_1(\varepsilon', h), V - T\right] dF(\varepsilon')$$

$$(2)\qquad J_1(\varepsilon, h) = \frac{\varepsilon + ah^{\alpha} - w_1(\varepsilon, h)}{1+r} + \frac{1}{1+r}\left\{(1-s)J_1(\varepsilon, h) + sV\right\}$$

$$(3)\qquad V = \frac{-k}{1+r} + \frac{1}{1+r}\left\{m(\theta)J_0(\varepsilon^e, h) + (1 - m(\theta))V\right\}$$

Entry level and regular jobs differ in terms of profit flows and capital gains due to the layoff option. The flow profits in (1) are reduced by the cost of training, while in (2) they are augmented by return of investments done at previous stage. Besides in equation (1)

3 In reality employment protection regulation has other components to be considered, such as severance payments and experience rating. Severance payments is a pure transfer in wage bargaining, so that it only reduces the equilibrium wage rate without affecting the unemployment rate (Lazear, 1990; for some different result see Garibaldi and Violante, 2002). This neutrality result explains our choice to focus on the effect of firing costs on labour market outcomes. Exclusion of experience rating analysis is motivated by its limited relevance for European labour market (Cahuc and Malherbet, 2001)

the firm receives a hiring subsidy σ when the worker arrives and the job is created, independently from the training decision. In the second stage the subsidy is zero. The integral term of equation (1) reflects the expected value of the match to the firms when the match survives, while if it is dissolved the firm pays the firing costs and returns to the market with a new vacancy. Regular jobs do not present this option value because, the productivity of the match remains constant, if the workers do not retire.

Analogously, let $E_0(\varepsilon^e, h)$ denotes the expected present value of workers' utility, $E_1(\varepsilon, h)$ the value of being employed in a regular job and U the value of being unemployed. These asset values are described respectively by:

(4) $$E_0(\varepsilon^e, h) = \frac{w_0(\varepsilon^e, h)}{1+r} + \frac{1}{1+r}\int_{\varepsilon_l}^{\varepsilon_u} Max\left[E_1(\varepsilon', h), U\right] dF(\varepsilon')$$

(5) $$E_1(\varepsilon, h) = \frac{w_1(\varepsilon, h)}{1+r} + \frac{(1-s)}{1+r} E_1(\varepsilon, h)$$

(6) $$U = \frac{1}{1+r}\left\{\theta m(\theta) E_0(\varepsilon^e, h) + (1-\theta m(\theta))U\right\}$$

Note that the expected lifetime utility of unemployed workers, equation (6), may be considered a welfare measure if the economy is initially characterized by high unemployment rate (see Blanchard and Lendier, 2002).

2.2. Wage negotiation and Hold up

In the search equilibrium literature wage determination is typically assumed to be perfectly flexible and fixed through a generalized Nash bargaining game (Mortensen, 1982; Mortensen and Pissarides, 1998). A bilateral cooperative bargaining is also assumed in our model. In the first period wage is determined according to a Nash rule in which firing penalties are not binding, hiring subsidies are distributed to the firms and the cost of training is shared with workers. Then, workers and firms agree over the initial wage according to the following sharing rule:

(7) $$w_0(\varepsilon^e, h) = \arg\max[J_0(\varepsilon^e, h) - V + \sigma]^{1-\beta}[E_0(\varepsilon^e, h) - U]^{\beta}$$

where parameter β represents the workers' bargaining power, assumed constant in both stages of employment relationship. Employment protection do not enter the negotiation as the outsider worker is not eligible by law and firms are not constrained to pay firing costs in case of disagreement during negotiation. After having substituted correspondent asset values we derive the equilibrium wage for "outsider" workers :

(7') $$w_0(\varepsilon^e, h) = \beta(\varepsilon^e - h + \sigma) - \beta T + (1-\beta)U$$

Conversely, once the employer-employee pair enter into second stage, job termination policies affect the pairs' threat point because firing costs have to be paid by firms upon separations. The cooperative bargaining game in second stage entails :

(8) $$w_1(\varepsilon, h) = \arg Max[J_1(\varepsilon, h) - V + T]^{1-\beta}[E_1(\varepsilon, h) - U]^{\beta}$$

which implies the equilibrium wage for insider workers:

(8') $$w_1(\varepsilon, h) = \beta(\varepsilon + ah^{\alpha}) + (r+s)\beta T + (r+s)(1-\beta)U$$

It is straightforward to note that the cost of training reduces the initial wage, because it is conditional on agreement to form the match and hence must be shared. But once the job becomes regular such a cost is sunk and does not influence the wage. The firing tax represents an employer liability if the job is destroyed once the match survives at second stage, implying a higher wage. Finally the outside wage increases by a fraction of the hiring subsidy because the payment of the subsidy is conditional on the worker's agreement to accept the job offer; in contrast, the insider wage is independent of the hiring subsidy, since it has already been received. This higher expected wage for trained generates a downward pressure on the initial wage. The wage schedule (7')-(8') implies an hold up problem in the private provision of training, though the perfect flexibility hypothesis for newly hired workers allows the firms to share a fraction of their training costs with employees. Hold up arises because the nature of firm specific human capital impedes the firms to commit to a wage-training contract, i.e. to make a credible commitment to provide training in the amount agreed with the workers at first stage. Such a commitment cannot be verified because training activities taking place inside the firms are in general observable, but not verifiable. This property allows the firms to hire workers at low wage pretending to offer them training, and then employ them as cheap labour force. The firm can always renege on its training promise even if the worker takes a wage cut to finance his training (Malcomson, 1997) [4].

3. LABOUR MARKET EQUILIBRIUM

In this section we define the equilibrium value of labour market tightness, the job destruction rate and the level of training. Then we consider the socially optimal equilibrium and discuss the relationship between hold up and inefficiency of the decentralized equilibrium.

As it is standard in the matching literature, competition among entrant firms will make the ex-ante value of a vacancy equal to zero. The firm and worker separate when the match specific productivity implies a discounted present value of operational losses higher than T, the total firing cost. The amount of training is chosen to maximize the firms' expected discounted profit, after the match is formed and the first period wage is negotiated. Renegotiation is possible in the second stage of the employment relationship when the privately optimal level of training has already been chosen.

3.1 Equilibrium

The equilibrium amount of training and threshold level of productivity below which workers are fired are chosen by the firm to maximize the value of the stream of expected discounted profits

4 Similar conclusions are reached if the firm could write a binding contract about training, but workers cannot take enough of wage cut to finance it. This happens, for example, in presence of liquidity constraints and/or a binding legal minimum wage for outsider workers.

resulting from a new match. Define ε_f^d the threshold value of productivity, the firm's optimal behaviour implies that:

(11) $$\varepsilon^d = (r+s)U - (r+s)T - ah^\alpha$$

The first term on the right hand side of equation (11) shows that the reservation productivity depends on the opportunity cost of employment to the worker. The firing tax causes firms to lower the threshold productivity, and thus to destroy fewer jobs. The returns to training, finally, reduce the threshold value as they increases the overall productivity of the pair[5].

Maximizing the expected profits with respect to training, firms choose the investment level such that that expected marginal return equalizes the private marginal cost, then :

(12) $$h = \left[\frac{a\alpha(1-\beta)[1-F(\varepsilon^d)]}{(r+s)}\right]^{\frac{1}{1-\alpha}}$$

In equation (12) the amount of training depends negatively on the separation rate and the share of labour. What drive this result is that the first period wage is already fixed when the firm chooses the level of training. In contrast the wage paid in later periods increases in training when the Hold-up implies a reduction of the return to investment by the factor $(1-\beta)$.The wage negotiated in first stage adjusts to share the cost of training between firm and employee, but it is not considered a variable within firms' maximizing problem because it is fixed when the firm chooses the level of training. Once the employee has paid a share of the cost of training through wage bargaining in first stage, the firm has no incentive to maintain is training promise. Wage renegotiation happens in the second stage, after training is completed[5].

The job creation equation is derived from the asset value of a vacant job (3) and the value of a new hire to the firm (1) when free entry condition $V=0$ holds:

---

5 Equation (11) can be equivalently derived from: (11') $J_1(\varepsilon^d, h) = -T$. Since the asset value $J_1(\varepsilon, h)$ increases with the idiosyncratic component ε, there is a unique threshold value. Given labour market equilibrium both (11) and (11') mean that firms and employees decide to separate only if the value of the surplus in the second stage of the match is negative

$$(13)\quad \frac{k}{m(\theta)} = (1-\beta)\left\{ \begin{array}{l} \frac{\varepsilon^e - h + \sigma}{1+r} - \frac{F(\varepsilon^d)T}{1+r} - \frac{\left[r + (1 - F(\varepsilon^d))\right]U}{1+r} + \\ + \frac{1}{(1+r)(r+s)} \int_{\varepsilon^d}^{\varepsilon_u} (\varepsilon' + ah^\alpha)\, dF(\varepsilon') \end{array} \right\}$$

This equation indicates that the expected cost of posting a vacant job must equalize the expected profit from a starting job. The left-hand side is the expected cost of a vacancy: it increases with labour market tightness because of matching externalities. The tighter the market, the longer is the expected time to fill a vacancy, and the more costly is posting a vacancy. The right-hand side is the expected profits from employing a newly hired worker.[6] Expected profits are decreasing with respect to labour market tightness, because a tighter labour market increases the exit rate from unemployment and the asset value of being unemployed. Increased training and a lower threshold productivity increase the right hand side of equation (13). That is, firms' expected profits would be higher if they could pre-commit to a high level of training. If such a pre-commitment were possible, workers would accept a lower wage in the first period of the employment relationship. Unfortunately for firms and workers, such a commitment can not be written into an enforceable contract, because no third party can evaluate whether the promised training has actually been provided.

The asset value of being unemployed reduces the right hand side of equation (13) because it exerts an upward wage pressure during bargaining. Its equilibrium value will be :

$$(14)\qquad U = \frac{\theta\beta k}{r(1-\beta)}$$

where is evident a positive linear relationship between the unemployment value and the market tightness. At this point the labour market equilibrium with flexible wages occurs when equations (13)

6 Note that under wage flexibility, the right hand side of equation (13) is the firms' share of total surplus evaluated at the first stage of the match: $J_0(\varepsilon^e, h) = (1-\beta)\left(J_0(\varepsilon^e, h) + E_0(\varepsilon^e, h) - U\right)$

and (14) hold and J_0 assumes the maximum value for a given value of being unemployed.

Proposition 1 *The first order equations for the maximum expected profit, given by equations (11) and (12), the market tightness equation (13) and the value of being unemployed (14) are necessary and sufficient conditions for an internal local maximum. Then there exists an unique internal equilibrium for* $\{h, \theta, \varepsilon^d\}$ (see appendix b)

This proposition derives from Kukutani's fixed point theorem and guarantees that in the labour market always exists an unique equilibrium associated with appositive separation rate and a positive probability to remain unemployed. A corner equilibrium where no lay-offs occur is also possible, but it relies on the high values of elasticity of the training function. Proposition 1 does not imply a clear general equilibrium relationship between the endogenous variables, even though the envelope properties in the expected profit function implies that training always increases with job tenure.

3.3. Hold up and efficiency

The equilibrium equation (12) derives from the assumption that a firm-worker pair cannot write a completely enforceable and binding contract to determine the division of the increased surplus due to training. Workers and firms, upon meeting, cannot write a contract that specifies contingent transfers between themselves, and therefore are not able to agree how to share the present discounted value of the total surplus of the partnership.

To evaluate such an inefficiency we calculate the optimal training from social point of view, that is the amount of training that would be invested if the employer-employee pair maximizes the joint surplus. Then differentiating the joint surplus of the match, i.e. $S_0 = J_0(\varepsilon^e, h) + E_0(\varepsilon^e, h)$, with respect to h yields :

(15) $$h^* = \left[\frac{a\alpha[1 - F(\varepsilon^d)]}{(r+s)}\right]^{\frac{1}{1-\alpha}}$$

A simple comparison of equation (15) with equation (12) shows the under-investment that characterizes both the flexible wage equilibrium when the workers' bargaining power is positive, $\beta > 0$. The reason for such an inefficient outcome is that the firm bears all the cost of training during the first stage of the match, but gets only a fraction of the gains of the additional output in the second stage, when the productivity of trained employees is increased. When the level of investment is set to maximize the joint surplus the wage rule does not matter, because it only affects the division of the surplus not its magnitude.

It is worth noting that inefficiency in the private provision of training is due to hold-up, not to search externalities. From equation (12), labour market tightness does not affect training, because the optimal amount is chosen after a match is formed and it is possible to revise such a choice. Hold-up causes under-investment and excessive lay-offs irrespective of the relationship between workers' bargaining share and the properties of the matching function. So a welfare improving policy may simply aim at increasing training without being targeted to reduce search frictions. For this reason, hereafter, we assume that Hosios condition holds, that is $\beta = \eta(\theta)$, where $\eta(\theta) = -\theta m'(\theta)/m(\theta)$ is the elasticity of the matching function with respect to unemployment.[7].

4. COMPARATIVE STATICS

In this section the impact of firing penalties and hiring subsidies on labour market outcomes is analysed both with flexible wages and rigid wages for newly hired. In particular, two types of policy intervention are implemented: the first is an economic policy which makes use of hiring subsidies to minimize the negative impact of firing taxes on job creation incentives, the second is a policy where hiring subsidies play no role and firing taxes are pure administrative costs.

7 This approach to design a policy can be justified by arguing that since in general search externalities have an ambiguous impact on welfare and β is unobservable, in practice, policy may not succeed in improving this aspect of resource allocation (Pissarides, 2001).

4.1. Economic policy with $\sigma > 0$.

The aim of hiring subsidies, given to firms which hire workers, is to offset direct costs of firing taxes. It is assumed that the public budget is balanced, so that the amount of job subsidies is determined by the revenues from firing taxes:

$$\sigma = \frac{F(\bar{\varepsilon}^d)T}{(1+r)} \tag{16}$$

Equation (16) specifies that the amount of the subsidy depends on the average threshold level of productivity, in a symmetric equilibrium where all firms optimally choose the same separation rate, $\bar{\varepsilon}^d = \varepsilon^d$. Thus the amount σ is given to the firms which hire workers. Under these hypotheses, the welfare of unemployed workers always increases if a small firing tax is introduced in a *laisser-faire* economy. In particular the following proposition holds:

Proposition 2 *With hiring subsidies, the introduction of a small firing penalty increases training investment,* $\left.\frac{dh}{dT}\right|_{T=0} > 0$, *and job creation,* $\left.\frac{d\theta}{dT}\right|_{T=0} > 0$, *while it decreases job destruction,* $\left.\frac{d\varepsilon^d}{dT}\right|_{T=0} < 0$ *.Thus, a small positive firing penalty causes increased employment, productivity and welfare of unemployed workers* (see appendix c).

Employment protection has standard effects on firms' maximization behaviour once the match is formed. The introduction of firing taxes prolongs the expected duration of a match, along which the returns to training are realized. This tends to augment the present value of an employment relationship and leads to more training and higher productivity. Specifically, a positive firing tax has a positive impact on productivity because it delays separations and leads to higher training, so that it increases the welfare of the employed. On the other hand, a rise of T increases the expected cost of labour services by augmenting the direct cost at separation and, indirectly, causes increased bargained wages by improving workers' threat point. These negative effects on firms' expected profit discourage job creation and hurt the welfare of unemployed.

With an efficient economic policy, however, the direct negative effect of T on firms' profit is neutralized by hiring subsidies while its impact on the wages of trained workers will be offset by a reduction in the wage of newly hired workers. Moreover, the firing taxes have a first order effect on welfare because the social returns to training are higher than the private returns, even though the firms do not consider the social gains associated to their choice. That is, the envelope property does not hold when T is introduced and the optimal investment increases for a proportion equal to β .

4.2. *Economic policy with* $\sigma = 0$.

When there are no hiring subsidies and employment protection is pure waste from social point of view, firing taxes maintain the same effect on training and job tenure whatever wage setting institution, i.e. it always prolongs job tenure and motivates firms to train more. The main difference with respect to an efficient policy design regards the impact on market tightness and, hence, on the welfare of unemployed workers. The next proposition defines the overall impact of firing cost on the equilibrium value of job creation, job destruction and training :

Proposition 4 *With no hiring subsidies, the introduction of a firing tax causes higher training,* $\left.\frac{dh}{dT}\right|_{T=0} > 0$*, lower job destruction* $\left.\frac{d\varepsilon^d}{dT}\right|_{T=0} < 0$*, while its impact on equilibrium market tightness and unemployed value depends on the elasticity of the training function and other parameter values .*

Firing costs generate more training and longer job tenure, as in the case of efficient policy with hiring subsidies. Equations (11)-(12) react with the expected signs to the introduction of firing costs, once the match is formed and wage at entry is paid, while the impact on job creation differs in terms of sign and magnitude.

With flexible wages, the introduction of a small $T > 0$ has a positive effect on the welfare of the unemployed, because higher training is associated with higher surplus to be divided, a surplus that firms do not take into account when they maximize profits. On other hand, the higher costs of labour services are not neutralized by hiring subsidies, see equation (13). Thus the overall impact on the market tightness and the value of being unemployed will depend on the returns to training and on the other parameter values.

5. NUMERICAL EXERCISES

We have just shown that with hiring subsidies the introduction of firing taxes leads to clear analytical results in terms of welfare and labour market outcomes in a flexible wage equilibrium. In absence of hiring subsidies, instead, the relationship between firing tax and the value of being unemployed is ambiguous. In this section a computational exercise is carried out to provide an example where the introduction of a firing tax acts as a Pareto improving policy even with no hiring subsidies. The baseline parameters were chosen to guarantee that the system of equations (11)-(12)-(13)-(14) converges to a fixed point so that emphasis is on the sign rather than the magnitude of the results.

In particular we assume a log-linear specification of the matching function, $q(\theta) = A\theta^{-\eta}$, where the scale parameter $A > 0$ indicates the efficiency of the matching process and η is the constant elasticity of the matching function with respect to unemployment. The distribution of idiosyncratic productivity is uniform over a finite support $[0, \gamma]$, i.e. $\varepsilon \sim unif(0, \gamma)$. The initial values of endogenous variables and the baseline parameters are reported in Table 1, where the time period is a year.

The real interest rate and the probability of exogenous separation are equal to 0.02 and 0.06 respectively. The labour's share are fixed to 0.5, while search externalities are ruled out by imposing the condition $\beta = \eta$. The critical value of parameter α is such that the second order condition on the Hessian matrix holds. Setting $\alpha = 0.6$ we also guarantee that the interior local maximum is the global maximum in the expected profit function, so that the analysis will concern an equilibrium where $\varepsilon^d > 0$ and $h < h_{max}$.

With regard to the endogenous variables, the market tightness is fixed to 0.35 , implying an initial value of being unemployed equal to 11,75 (see equation 14). The initial amount of training and threshold productivity are determined by equations (11) and (12) when the upper bound of the idiosyncratic productivity distribution γ is chosen so that separation rate at the first stage of the match $\frac{\varepsilon^d}{\gamma}$ is equal to 0.25. It is worth to noting that the value of being unemployed and average productivity for newly hired workers causes, in turn, a parametric restriction on the efficiency of the matching technology, the parameter *A*, and on the parameter *a* of the training function, $f(h) = ah^{\alpha}$. Similarly the recruitment cost of posting a vacancy, k, is determined endogenously by the value of a filled job and the exit rate from unemployment, taken to be equal to 0.3 . Finally there is neither employment protection, *T=0* , nor hiring subsidies, $\sigma = 0$.

Table 1:

interest rate r	0.02
separation rate s	0.06
upper productivity support γ	3
bargaining power β	0.5
creation cost k	1
training *h*	1.82
average productivity ε^e	0.75
market tightness θ	0.35
unemployment value U	11.75

* Baseline parameters and endogenous variables

In the search environment described above, we consider the introduction of a firing tax equal to T=0.1, without any compensation given to the firms in terms of hiring subsidies. The impact of such a policy is shown in Table 2, where the new equilibrium values of training , separation rate, market tightness and the welfare of unemployed are reported. In particular the value of being unemployed represents the target variable to evaluate the welfare improving implications of the firing taxes. Effectively Table 2 confirms that a small firing tax could have beneficial effects both for insider workers and outsider workers.

Table 2:

training h	1.85
separation rate $\frac{\varepsilon^d}{\gamma}$	0.24
market tightness θ	0.3501
unemployment value U	17.503

In our numerical example the increase of the welfare of employees is driven by the increase of job tenure and training, while the increase of the market tightness reduce the unemployment duration and the cost of being unemployed. In other terms, the productivity effect of trained employees offset the higher cost of labour services on the firms profit function . This leads the firms to open more vacancies and, then, to increase the welfare of unemployed.

It is worth to noting, again, that these results depend on the parameterized structure of the model and , in particular, on the chosen value of α to perform the numerical analysis. On the other hand, in the range $0<\alpha<0.6$ the productivity of training investment are not high enough to offset the higher expected wage due to fining tax, so that the introduction of $T=0.1$ reduces the market tightness and, consequently, the welfare of outsider workers.

6. CONCLUDING REMARKS

This paper shows that inefficiency induced by hold up in the private provision of training can justify the introduction of positive amount of firing tax in economies populated by risk neutral or perfectly insured agents. Two main results are illustrated. First, an efficient economic policy, which makes use of a combination of a small lay-off taxes and hiring subsidies, always increases employment, productivity and welfare. Second, there is not a clear relationship between employment protection and welfare when there are no hiring subsidies and firing penalties are pure administrative costs,. The relationship depends on the concavity of the returns to training and on the parametric specification of the model. In the case that the returns to training are high enough, the introduction of firing taxes lead to an increase of the firms ‘ expected profit, higher market tightness and, as a consequence, higher welfare of being unemployed. That is, employment protection acts as a Pareto improving policy. Conversely, when the returns to training are low, our analysis confirms the standard result of a trade off between adjustment costs and productivity gains related to employment protection.

References

Abraham, K. and H. Farber (1987), "Job Duration, Seniority and Earnings", *American Economic Review*, 77(3), 278-297

Acemoglu, D. (2001), "Good Jobs versus Bad Jobs", *Journal of Labor Economics*, 19, 1-22

Acemoglu, D. and S. Pischke (1999a) " Beyond Becker: Training in Imperfect labor market", *Economic Journal*, 109, F112-142

Acemoglu, D. and R. Shimer (1999c)," Holdups and Efficiency with Search Frictions", *International Economic Review*, 40, 827-850

Altonji, J. and R. Williams (1997), "Do Wages Rise with Job Seniority? A Reassessment", *NBER Working Paper* n. 6010

Arulampalam, W. and A. Booth (1998) "Training and Labour Market Flexibility: Is There a Trade-off ?" , *The British Journal of Industrial Relations*, 36(4), 521-536

Belot, M., Boone, J. and van Ours, J. (2002), "Welfare Effects of Employment Protection", IZA Discussion paper

Bentolila, S. and G. Bertola (1990), "Firing Costs and Labor Demand: How Bad is Eurosclerosis? " , *Review of Economic Studies*, 57, 381-402

Bertola, G. and R. Rogerson (1997), "Institutions and Labor Reallocation", *European Economic review*, 41,1147-1171

Bertola, G. (1990), " Job Security, Employment and Wages", *European Economic Review*, 34, 851-886

Blanchard, O. and A. Landier (2002) " The perverse effecs of partial labour market reform: Fixed term contracts in France", *The Economic Journal*, 112, F214-244

Binmore K., A. Rubinstein and A. Wolinsky (1986), "The Nash Bargaining Solution in Economic Modelling", *Rand Journal of Economics*, 17,176-188

Booth, A. and G. Zoega (1999) " Do Quits Cause Under-Training? ", *Oxford Economic Papers*, 51,374-386

Burda, M. (1992), "A Note on Firing Costs and Severance Benefits in Equilibrium Unemployment", *Scandinavian Journal of Economics*, 94, 479-489

Farber, H. (1999), " Mobility and Stability: The Dynamics of Job Change in Labor Market". In O. Ashenfelter and D. Card (Eds.), *Handbook of Labor Economics*, Amsterdam. North Holland

Garibaldi, P. and G. L. Violante (2002), "Firing Tax and Severance Payment in Search Economies: A Comparison", CEPR *Discussion Paper*, n. 3636

Garibaldi, P. (1997), "Job Flow Dynamics and Firing Restrictions", *European Economic Review*, 42, 245-275

Grout, P. (1984), "Investment and Wages in the Absence of Binding Contracts: A Nash Bargaining Approach", *Econometrica*, 52(2), 449-460

Hart, O. and J. Moore (1999), " Foundations of Incomplete Contracts", *Review of Economic Studies*, 66, 115-138

Hart, O. and J. Moore (1988), "Incomplete Contracts and Renegotiations", *Econometrica*, 56(4), 755-785

Hosios, A. (1990) On the Efficiency of Matching and Related Models of Search and Unemployment. *Review of Economic Studies*, 57, 279-298

Jovanovic, B. (1979), "Job Matching and the Theory of Turnover", *Journal of Political Economy*, 69, 972-990

Layard, R. and S. Nickell (1999) " Labor market institutions and economic performances", in Ashenfelter, O. and D. Card (eds.), *Handbook of Labor Economics*, Amsterdam, North Holland

Lazear, E. P. (1990), "Job Security Provisions and Employment", *Quarterly Journal of Economics* 105, 699-726

Lindbeck, A. and D. Snower (1988) *The Insider-Outsider Theory of Employment and Unemployment,* Cambridge, MIT University Press

Malcomson, J. (1999), "Individual Employment Contracts". In O. Ashenfelter and D. Card (Eds.), *Handbook of Labor Economics*, Amsterdam. North Holland

Mortensen, D. (1978), "Specific Capital and Labor Turnover", *Bell Journal of Economics* 9, 572-586

Mortensen, D. and C. Pissarides (1994)" Job Creation and Job Destruction in the Theory of Unemployment", *Review of Economic Studies*, 61, 397-415

Pissarides C. (2001), "Employment Protection", *Labor Economics* 8, 131-159

Topel, R. (1991), " Specific Capital, Mobility and Wages: Wages Rise with Job Seniority", *Journal of Political Economy*, 99,1, 145-176

Appendix A: Local maximum and firm's expected profit

Firm maximize their expected profit with respect to training and threshold productivity, i.e. $\underset{\varepsilon^{d},h}{Max} J_0(\varepsilon^{e},h)$. The equilibrium wage at entry does not vary when firms make their optimal choices, so that the expected profit is:

(i)

$$J_0(\varepsilon^{e},h)=\frac{\varepsilon^{e}-h-w_0+\sigma}{1+r}-\frac{[F(\varepsilon^{d})-\beta(1-F(\varepsilon^{d})]\cdot T}{1+r}-\frac{(1-\beta)[1-F(\varepsilon^{d})]\cdot U}{1+r}+\frac{(1-\beta)}{(r+s)(1+r)}\int_{\varepsilon^{d}}^{\varepsilon_u}(\varepsilon^{'}+ah^{\alpha})dF(\varepsilon^{'})$$

First order conditions are given by equations (11) and (12) . To find a local maximum the Hessian matrix must be semi definite negative so that :

$$\frac{(1-\beta)^2\alpha(1-\alpha)h^{\alpha-2}[1-F(\varepsilon^{d})]f(\varepsilon^{d})-(1-\beta)[f(\varepsilon^{d})]^2\alpha^2a^2h^{2\alpha-2}}{(r+s)^2(1+r)^2}>0$$

that is:

(ii) $$\frac{(1-\alpha)\cdot[1-F(\varepsilon^{d})]}{a\cdot\alpha\cdot f(\varepsilon^{d})}>h^{\alpha}$$

Appendix B: Existence and Uniqueness

1.b) Existence. *Define the four component vector* $z\equiv(h,\varepsilon^{d},\theta,U)$. *An equilibrium is a fixed point of the correspondence* Ψ *from* z *to* z, *with* $\Psi(z)=(h(z),\varepsilon^{d}(z),\theta(z),U(z))$, *where* $\theta_f(z)$ *is defined implicitly by equation (13),* $U_f(z)$ *is defined by equation (14) and* $h(z)$ *and* $\varepsilon^{d}(z)$ *are chosen to maximize* $J_0(\varepsilon^{e},h)$. *The correspondence* Ψ *is the product of four continuous functions on a best response relation. Thus it is convex valued and has a closed graph.* Ψ *maps a convex compact set into itself, so*

Kukutani's fixed point theorem implies that Ψ has a fixed point, which is an equilibrium.

Proof. To prove the continuity of functions in vector z is straightforward. The optimal training choice is a continuous function and takes values in the range $0 \leq h \leq \left[\frac{\alpha a(1-\beta)}{(r+s)}\right]^{\frac{1}{1-\alpha}} \equiv \bar{h}$; then the set of possible values of training is compact. Market tightness is also continuous and assumes values in the range: $0 \leq m^{-1}(\theta) < \left[\frac{(1-\beta)\varepsilon^{e}}{k(1+r)} + \frac{(1-\beta)(\varepsilon + a\bar{h}^{\alpha})}{k(1+r)(r+s)}\right]$. The value of being unemployed can be written as $U = \left(h, \varepsilon, \frac{r(1-\beta)}{\beta k} U\right)$, with $\theta = \frac{r(1-\beta)}{\beta k} U$, so that $0 < U \leq U(\bar{h}, \varepsilon_U) \equiv \bar{U}$. Finally $\underline{\varepsilon} \equiv \varepsilon(0, \bar{h}) < \varepsilon^{d} < \varepsilon(\bar{U}, 0) \equiv \bar{\varepsilon}$, where $[\varepsilon_L, \varepsilon_U]$ is the finite support of the distribution of the idiosyncratic productivity and $h_{\max}$ is the maximum amount of training.

2.b) Uniqueness To prove that the equilibrium is unique, consider the firm's maximization problem with respect to training and threshold productivity. When condition (ii) is met, the maximized expected profit can be represented as function of the equilibrium value of being unemployed: $J_0(U) = \underset{\varepsilon^{d}, h}{Max} J_0(\varepsilon^{e}(U), h(U), U)$. $J_0(U)$ is continuous and decreasing with respect to the value of being unemployed, $\frac{dJ_0}{dU} < 0$, so that $\frac{d\theta(J_0)}{dU} < 0$. Considering the positive monotonic relation between the equilibrium market tightness and workers' outside option , equation (14), and the continuous positive relation between θ and $J_0(U)$, equation (13), it is verified that: $\frac{dU(J_0, \theta(J_0))}{dU} < 0$. Thus there can be only one equilibrium value of U and, as a consequence, there is only an equilibrium value of θ. Two different solutions to (h, ε^{d}) would have to give the same maximal $J_0(\varepsilon^e, h)$ and the same U. However this result is not possible since the sum $J_0 + E_0$ is increasing in h at the equilibrium.

3.b) Corner solution. A corner equilibrium where no layoff decision is taken and the amount of training is maximum is also possible in the model. When $\varepsilon_f^d = \varepsilon_L$, the equilibrium values of training and market tightness are given by equations (*12)-(13)*:

$$h_{\max} = \left[\frac{\alpha a(1-\beta)}{r+s}\right]$$

$$m^{-1}(\theta) = \frac{(1-\beta)}{k(1+r)}\left[\left((\varepsilon^e - h_{\max}) - rU + (\varepsilon^e + ah^{\alpha})\right)\right]$$

while the value of being unemployed is determined by equation (14). We note that in equation (14), for $\theta = 0$, $U = 0$, and for $\theta \to +\infty$, $U = U_{\max}$. In equation (13) for $U = 0$, $\theta > 0$, and for $U > 0$, $\frac{\partial\theta}{\partial U} < 0$. This implies a decreasing relationship between job creation condition and workers' outside option when $\varepsilon^d = \varepsilon_L$ and $h = h_{\max}$. Thus, there exists a candidate equilibrium $\{\varepsilon_L, h_{\max}, \theta, U\}$.

Appendix C: Comparative Statics

1.c) Hiring subsidy, $\sigma > 0$

Differentiating totally equations (11) (12) (13), once substituted equilibrium budget constraint, $\sigma = F(\overline{\varepsilon}^d)T$, one obtains:

$$\frac{d\theta}{dT} = \frac{1}{\Theta}\left[\frac{(1-\beta)}{(1+r)}\beta\frac{dh}{dT} + \frac{(1-\beta)}{(1+r)}\left[F(\overline{\varepsilon}^d) - F(\varepsilon^d)\right] + \frac{(1-\beta)\, f(\varepsilon^d)T}{(1+r)}\right]$$

where: $\Theta = \frac{\eta \cdot \theta}{m(\theta)} + \frac{r+[1-F(\varepsilon_f^d)]}{1+r} \cdot \frac{\beta k}{r(1-\beta)}$. Given that $\bar{\varepsilon}^d = \varepsilon^d$ and evaluating job creation at T=0, it reduces to: $\left.\frac{d\theta}{dT}\right|_{T=0} = \frac{1}{\Theta}\frac{(1-\beta)}{(1+r)}\beta\frac{dh}{dT}$. From equation (14) we have $\frac{dU}{dT} = \frac{\beta^2 k}{\Theta \cdot r(1+r)}\frac{dh}{dT}$ so that, on the job destruction condition, will be:

$$\left.\frac{d\varepsilon^d}{dT}\right|_{T=0} = \frac{-(r+s)}{\left[\underbrace{1-\frac{f(\varepsilon_f^d)a^2\alpha^2(1-\beta)h^{2\alpha-1}}{(1-\alpha)(r+s)}}_{>0} + (r+s)\cdot\frac{\beta^2 k}{\Theta \cdot r(1+r)}\cdot\frac{f(\varepsilon^d)a\alpha(1-\beta)h^{\alpha}}{(1-\alpha)(r+s)}\right]} < 0$$

Training is inversely related to the threshold productivity, when firing cost varies, i.e. $\left.\frac{dh}{dT}\right|_{T=0} = -\frac{f(\varepsilon^d)}{(1-\alpha)[1-F(\varepsilon^d)]h^{\alpha-2}}\cdot\frac{d\varepsilon^d}{dT} > 0$ and , as a consequence, $\left.\frac{d\theta}{dT}\right|_{T=0} > 0$ and $\left.\frac{dU}{dT}\right|_{T=0} > 0$.

2.c) No hiring subsidy, $\sigma = 0$

With no hiring subsidies, the introduction of firing costs implies on job creation condition:

$$\frac{d\theta}{dT} = \frac{(1-\beta)}{\Theta(1+r)}\left[\beta\frac{dh}{dT} - F(\varepsilon^d)\right]$$

Following the same procedure as in the case of efficient policy, we have:

$$\left.\frac{d\varepsilon^d}{dT}\right|_{T=0} = \frac{-(r+s)\left[\dfrac{\Theta\cdot r(1+r)+(r+s)\beta kF(\varepsilon^d)}{\Theta\cdot r(1+r)}\right]}{\left[\underbrace{1-\dfrac{f(\varepsilon^d)a^2\alpha^2(1-\beta)h^{2\alpha-1}}{(1-\alpha)(r+s)}}_{>0}+(r+s)\cdot\dfrac{\beta^2 k}{\Theta\cdot r(1+r)}\cdot\dfrac{f(\varepsilon^d)a\alpha(1-\beta)h^{\alpha}}{(1-\alpha)(r+s)}\right]}<0$$

and $\left.\dfrac{dh}{dT}\right|_{T=0} = -\dfrac{f(\varepsilon^d)}{(1-\alpha)[1-F(\varepsilon^d)]h^{\alpha-2}}\cdot\dfrac{d\varepsilon^d}{dT}>0$. The sign of the market tightness variation is not clear *a priori*.