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Abstract

Political parties compete over income tax functions, and voters vote and decide whether to pay full taxes or to make an effort to modify their tax burden. We show that political parties only propose efficient income tax functions, in a similar manner to the probabilistic voting theory. Regarding the shape of income tax functions, it need not be the case that the majority of voters prefer progressive taxation to regressive taxation as a consequence of the distortions. Nevertheless, we prove that the political appeal for progressivity is restored under mild conditions.

KEYWORDS: Income taxation, Distortions, Efficiency, Progressivity, Political competition.

JEL Classification Numbers: H23; H31; D72; D78.

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1 Introduction

Income taxation and its distortionary effects have been analyzed extensively in the economics literature. A distortion can be broadly defined as a taxpayer's reaction to his level of tax burden (in the sense that his behavior is altered by the existence of the tax). Most of the studies dealing with distortions focus on the inefficiency created by income taxation. They seek to determine, from a normative point of view, properties of the tax system so that these inefficiencies vanish (or similarly to maximize some social welfare function). In contrast with such a strand of the literature, the present article adopts a positive perspective. The starting point of our model is the following observation: as strategic political parties anticipate distortions, they adapt their electoral promises to maximize their probability of victory. When confronted with this observation, the main question one might consider is how do political parties adapt their electoral promises?

In order to provide an answer to this question, we build a simple model in the Downsian tradition: income tax functions are determined by the electoral competition of two office-seeking political parties. Taxes are used to finance a public good. Given the size of the public good, each party makes binding electoral promises to each voter. Voters differ in endowed income and benefit equally from the public good. Taxes are distortionary in the sense that they modify voters' behavior. Specifically, a voter votes for one of the political parties and makes a binary decision. A voter chooses between paying full taxes and making an effort to modify his tax burden[1]. In the latter case, the monetary amount the voter must pay is represented by the cost function. Such a function is common knowledge among both parties and voters. Examples of distortions studied within this model are labor supply and tax avoidance activities[2]. Hence, a voter votes for the party that maximizes his final income (available income once the binary decision has been made).

Building on this simple model, we address two salient issues of the positive approach to income taxation: the efficiency of the electoral competition and the shape of income tax functions in equilibrium.

[1] One can think of the voter as choosing between standard taxation and some broadly defined outside option.

[2] Tax avoidance can be defined as the wide variety of *legal* activities people engage in with the sole purpose of lowering their tax burden. Tax avoidance represents 2 to 7 % of the GDP and 5 to 20 % of the population avoid 10 to 20 % of their official tax payments (for a detailed account, see Andreoni [1]). The model accounts also for tax evasion but does not incorporate the anti-fraud mechanisms.

Vis-à-vis the former issue, the distortions in the model can lead to the emergence of inefficiencies. Even if the level of public good delivered by the different tax functions is constant, voters individually decide whether to make an effort modifying the aggregate level of final income. A tax function is said to be efficient whenever it maximizes the aggregate level of final income, in contrast with an inefficient one that does not. We show that parties *uniquely* propose efficient income tax functions[3]. The reason political competition leads to an efficient outcome is quite simple. Let us consider the case where party 1 promises to implement some inefficient tax function. Now, suppose that his opponent, party 2, tries to defeat him. As the tax function proposed by party 1 is inefficient, it does not maximize the aggregate level of final income. For that reason, party 2 can construct a tax function that delivers for every voter a level of final income which is equal or higher than the final income delivered by the tax function offered by party 1. Roughly speaking, it is *weakly dominated* for a strategic party to offer an inefficient tax function to voters. The idea that competition between political parties leads to the maximization of a Benthamite social welfare function is in accordance with the conclusions of the probabilistic voting models à la Lindbeck and Weibull [9].

The second feature of political competition we analyze is the shape of tax functions in equilibrium. The main objective is to understand whether the political appeal for progressivity is robust with respect to distortions. Such an appeal simply relies on the fact that any progressive tax function[4] is preferred to any regressive tax function under majority voting in a pure endowment economy; this observation is satisfied only when the median income is lower than the mean[5]. In order to make our arguments transparent, we focus on a canonical example: the flat tax game. Voters are restricted to choosing between paying full taxes proposed by parties and spending a proportional amount of their endowed income. We first emphasize how unfounded the usual arguments given in public debates can be in our setting: the poorest voters

[3]Technically, we prove that only the efficient tax functions lie in the uncovered set. As stated by Banks, Duggan and Le Breton [2], the support of any mixed strategy equilibria of a two-player, symmetric, zero-sum game lies in the uncovered set, given some conditions which are satisfied by our game.

[4]We define progressive taxation in the marginal sense: a tax is said to be progressive whenever its marginal tax rates are increasing in income (a convex function). Similarly, a regressive tax function is a concave function.

[5]This condition over the income distribution is not too disturbing, as most OECD countries satisfy it. Even if different authors have argued that this appeal is enough to justify the implementation of progressive taxation, the only work that obtained such a result in an equilibrium framework is the recent article of Carbonell-Nicolau and Ok [3] (see Section 1.1 for a review).

need not prefer progressive income taxation. It is even the case that a regressive tax function is *unanimously* preferred to a progressive tax function. Hence, situations exist in which strategic parties advocate regressive taxation. However, such situations are characterized by constrained policy spaces. In the example provided, parties can only propose two tax functions: an inefficient progressive tax function and an efficient regressive one. When the policy space is rich enough (it is only required that parties can propose efficient progressive and regressive tax functions), the political appeal for progressivity is restored.

This paper is organized as follows: Section 1.1 presents a review of the literature, and Section 2 sets up the model of public good provision with distortionary taxes. Section 3 describes the political game between the two Downsian parties. Section 4 presents the results concerning the efficiency of the electoral competition. Finally, Section 5 deals with the shape of income taxation in equilibrium, and Section 6 gives concluding comments.

1.1 Related literature

This work incorporates political competition into a setting with distortionary taxes to explore the role of the political arena in shaping income taxation. Such an interaction has not deserved much attention due to the well-known problem of the inexistence of pure strategy equilibria as a consequence of political cycles: one can always design a tax function that is preferred by a majority of voters to any other tax function. The most-used solutions to tackle this problem can be classified into two categories: restricting the policy space (linear or quadratic tax functions)[6] or introducing some ideological component into voters' preferences (the so-called probabilistic voting theory[7]). Whereas the first approach seems too artificially constrained (one might wonder which is the *good* restriction), the second one has become a standard approach in political economics.

This paper takes a much more seldom used route: keeping the richness of the policy space and focusing on equilibria in mixed strategies without incorporating ideological concerns. In our model, a voter votes for the party that offers him the highest level of final income. This idea leads to a discontinuity in a party's payoff,

[6] The main examples of this strand of literature are Meltzer and Richard [12], Roberts [17] and Romer [18].

[7] See Chapter 2 in Person and Tabellini [15] and the works of Lindbeck and Weibull[9] and Coughlin [4].

as voters abruptly change their vote from one party to the other in contrast with probabilistic voting models. In such models, if party 1 increases his promise of final income to some group of voters, the share of voters in this group that vote for party 1 increases continuously. Hence, the role of mixed strategies in our model is to smooth the party's payoffs, allowing the existence of equilibrium. Such an approach has been used by Myerson [14], Laslier[8], Lizzeri and Persico [10] and Crutzen and Sahuguet [5]. However, these models assume that voters are ex-ante equal, and hence they do not deal with the shape of income taxation in equilibrium. To our knowledge, only one recent paper (Carbonell-Nicolau and Ok [3]) focuses on the mixed strategy approach to determine the shape of income taxation. Their work focuses on a pure endowment economy where voters differ only by income level. They find that when parties are restricted to propose either progressive or regressive taxation, they only advocate progressive taxation in equilibrium (given that the median income is lower than the mean income). Nevertheless, when this restriction is softened (any shape of income taxation is permitted), the probability that parties advocate progressive taxation is strictly lower than one.

2 The basic setting

We consider an economy with a continuum of voters. Voters only differ in endowed income, $x \in [0,1]$, distributed according to a distribution function F belonging to the set of distribution functions F. The function F is assumed to be an increasing and differentiable continuous function with $F(0) = 0$ and $F(1) = 1$. By definition, p_F denotes the probability measure induced by F on [0,1]. Formally, the share of voters whose endowed income is located in the set $A \subseteq [0,1]$ equals $p_F(A) = \int_0^1 1_A dF$. We consider the set of distribution functions F as a metric space under the sup-metric.

There are two parties in the election denoted by 1 and 2. A pure strategy for a party is a function which assigns to each endowed income an amount of taxation due to the government. Due to the distortions present within the model, an *ex-ante* tax function represents the official tax payments of a voter, whereas an *ex-post* tax function stands for the actual level of expenditures of voters once the distortions have been taken into account.

Definition 1 (Ex-ante tax function)**.** *A function $T \in \mathbf{C}[0,1]$ is an ex-ante tax function if it satisfies the following two properties:*

1. $0 \leq T(x) \leq x$ *for all* $x \in [0,1]$ *,*

2. $x \longmapsto T(x)$ *and* $x \longmapsto x - T(x)$ *are increasing functions on* $[0,1]$.

Whereas Property 1 implies that voters do not face negative taxation or taxation larger than their income, Property 2 implies that the tax burden increases with income and that pre-tax and post-tax income rankings are identical.

As previously discussed, taxes are distortionary in the sense that they modify voters' behavior. Given an ex-ante tax function T, the voter decides whether to pay full taxes or to make an effort. In the latter case, the monetary amount the voter must pay is represented by the cost function $c(T)$ (which might depend on T). This cost function is common knowledge for both parties and voters.

The voter's decision is simple. For any ex-ante tax function T and its associated function $c(T)$, a voter chooses to pay full taxes if this choice results in a higher final income than making an effort, i.e.,

$$\text{a voter with income } x \text{ pays full taxes iff } x - T(x) \geq x - c(T)(x).$$

The set E_T stands for the set of voters who pay full taxes:

$$E_T = \{x \in [0,1] \mid T(x) \leq c(T)(x)\}.$$

Formally, given an ex-ante tax function T, its associated cost function $c(T)$ has two components: the amount of money collected by the government T_G and the deadweight cost of making an effort T_P.

Definition 2 (Cost function)**.** *Given a function* $T \in \mathbf{C}[0,1]$*, a cost function is a continuous function that satisfies*

$$c(T)(x) = T_G(x) + T_P(x).$$

Besides, a cost function satisfies the following properties for any $S \in \mathbf{C}[0,1]$*:*

1. *(Linearity.)* $c(\varepsilon S + (1-\varepsilon)T)(x) = \varepsilon c(S)(x) + (1-\varepsilon)c(T)(x)$ *for any* $\varepsilon \in [0,1]$*,*

2. *(Monotonicity.) if* $S(x) \leq T(x)$ *for any* $x \in (0,1)$ *then* $c(S)(x) \leq c(T)(x)$ *for any* $x \in [0,1]$*,*

The first cost function property simply states that the cost function of a linear combination of two ex-ante tax functions equals the linear combination of the cost function corresponding to the two ex-ante tax functions. This property implies some linearity with respect to the tax function T[8]. The second property states that if every voter has a tax burden with tax function S which is lower or equal to the one with tax function T, the cost function corresponding to S is lower than or equal to the cost function corresponding to T.

Generally speaking, we consider that a cost function represents tax avoidance activities whenever $T_P(x) > 0$ for any $x \in (0, 1)$; the amount of money which is not collected by the government represents a *cost* for the voter. With such a cost function, the voter faces a trade-off between either paying full taxes or incurring a cost to reduce his personal tax burden.

Example 1: The flat tax. For some $a \in [0, 1]$, $c(T)(x) = ax$. Independently of the ex-ante function T, the voter can simply pay a proportional amount of his income which is not collected by the government to supply the public good. Hence, a voter with endowed income x gets a final income which equals either $x - T(x)$ or $(1 - a)x$. Both properties of the cost function are satisfied as $c(\varepsilon T + (1 - \varepsilon)S)(x) = ax = \varepsilon T(x) + (1 - \varepsilon)S(x)$ and $c(T)(x) = ax$ for any $T \in \mathbf{C}[0, 1]$. This example is particularly appealing in the debate over how should a government modify its tax system when some neighbor country sets up a flat tax. As will be shown in Section 5, progressive taxation is implemented in presence of a flat tax alternative.

Example 2: Tax avoidance. For some $a, b \in [0, 1]$, $c(T)(x) = a + bT(x)$. Voters can avoid declaring a share $(1 - b)T(x)$ of their taxable income through investing a fixed amount a in tax avoidance. For any given *ex-ante* tax function T, the cost function satisfies $c(T)(x) = a + bT(x)$. A voter chooses to invest an amount a in tax avoidance if that choice results in higher utility than paying full taxes; that is if $T(x) > a + bT(x)$. Whenever a voter avoids paying taxes, the amount of money he allocates to the government is equal to $T_G(x) = bT(x)$, and parameter a stands for the amount of money which is not collected by the government T_P. To see that this modeling satisfies the first condition, it suffices to see that for any function

[8] Examples of functions that do not satisfy property 1 are $c(T)(x) = T(x)^2$ or $c(T)(x) = \log(T(x))$.

$S \in \mathbf{C}[0,1]$ and for any $\varepsilon \in [0,1]$:

$$\begin{aligned} c(\varepsilon S + (1-\varepsilon)T)(x) &= a + \varepsilon b S(x) + (1-\varepsilon) b T(x) \\ &= \varepsilon(a + bS(x)) + (1-\varepsilon)(a + bT(x)) \\ &= \varepsilon c(S)(x) + (1-\varepsilon) c(T)(x). \end{aligned}$$

The second condition is satisfied as $c(S)(x) = a + bS(x) \geq c(T)(x) = a + bT(x)$ whenever $S(x) \geq T(x)$ for any $x \in [0,1]$.

Symmetrically to the description of tax avoidance activities, we consider that an cost function represents labor supply activities whenever $T_P(x) < 0$, when $x \in (0,1)$; the amount of money which is not collected by the government represents a *benefit* for the voter. Such a modeling is also appropriate to represent a schooling decision, in which some investment in education leads to a rise in income. The proofs provided are done for the tax avoidance activities case and are symmetric for the case representing labour activities.

Example 3: Labor supply. For some $b \in [0,1]$, $c(T)(x) = -bx + T((1+b)x)$. A voter with income x makes an effort to raise his income. This rise in income gets compensated by a rise in his income tax burden. Given any ex-ante tax function T, its associated cost function satisfies $c(T)(x) = -bx + T((1+b)x)$ with $b > 0$. The rise in income is represented by $-bx$, and $T((1+b)x)$ symbolizes the increase in the tax burden. Therefore, a voter makes an effort if $bx > T((1+b)x) - T(x)$ (his rise in income is higher than his rise in taxes). Besides, in such a problem the amount of money that a voter with income x allocates to the government is equal to $T_G(x) = T((1+b)x)$. To see that the first condition holds with this modeling, it suffices to write that for any function $S \in \mathbf{C}[0,1]$ and for any $\varepsilon \in [0,1]$:

$$\begin{aligned} c(\varepsilon S + (1-\varepsilon)T)(x) &= -bx + [\varepsilon S((1+b)x) + (1-\varepsilon)T((1+b)x)] \\ &= \varepsilon(-bx + S((1+b)x) + (1-\varepsilon)(-bx + T((1+b)x)) \\ &= \varepsilon c(S)(x) + (1-\varepsilon) c(T)(x). \end{aligned}$$

It is simple to see that the second condition is satisfied; as $c(S)(x) = -bx + S((1+b)x) \geq -bx + T((1+b)x)$ whenever $S(x) > T(x)$.

Given an ex-ante tax function T and its corresponding cost function $c(T)$, we define its ex-post tax function t as the real amount of expenditure of voters once the distortion has taken place.

Definition 3 (Ex-post tax function). *Given an ex-ante tax function T and its cost function $c(T)$, we define the ex-post tax function t as a continuous function such that:*

$$t(x) = \min\{\ T(x)\ ,\ c(T)(x)\}.$$

The game proceeds in three stages:

⊸ *Stage 1* Simultaneously, parties 1 and 2 announce their strategies T_1 and T_2 to voters.

⊸ *Stage 2* A voter votes for party 1 if $x - t_1(x) > x - t_2(x)$ and conversely for party 2. If a voter is indifferent, he randomizes over both parties as usual.

⊸ *Stage 3* The winner of the election sets up a tax function, and voters decide whether to pay full taxes or to make an effort.

2.1 The size of the government

The political game is played on the ex-ante tax functions. However, voters vote according to their preferences over the ex-post tax functions. This work assumes that political parties anticipate the effect of the cost function on redistribution. Hence, political parties can only advocate tax functions that are budget-balanced after the voter's binary decision has taken place.

Given the set of voters who pay full taxes E_T, the government revenues are represented by function $G(T)$. Hence,

$$\text{the government collects } G(T)(x) = \begin{cases} T(x) & \text{if } x \in E_T \\ T_G(x) & \text{if not.} \end{cases}$$

Hence, given an ex-ante tax function T, the function $G(T)$ represents the amount of money collected by the government once voters have made their binary decision. Whenever a voter's income x belongs to E_T, the voter pays $T(x)$ to the government (i.e full taxes), and if $x \notin E_T$, the voter decides to make an effort, and therefore the amount of money he pays to the government equals $T_G(x)$. We assume that a party can only advocate budget-balanced ex-ante tax functions that belong to the set R iff the ex-ante tax function collects a predetermined amount of taxes $r > 0$. Formally,

$$T \in R \Longleftrightarrow \int_0^1 G(T)dF \geq r.$$

The set R is viewed as a metric subspace of $\mathbf{C}[0,1]$ and is divided in two subsets: E which stands for the efficient tax functions and I for the inefficient ones, so that

$$R = E \cup I.$$

An efficient tax function $T \in E$ maximizes the aggregate level of final income and can be defined as follows:

$$\begin{aligned} T \in E &\Longleftrightarrow \int_0^1 x - t(x)dF = \max_{S \in R} \int_0^1 x - s(x)dF \\ &\Longleftrightarrow \int_0^1 t(x)dF = \min_{S \in R} \int_0^1 s(x)dF. \end{aligned}$$

An inefficient tax function in the set R belongs to the set I if it does not belong to the set E. We represent by Q an element of Q, the taxation environment which consists of the following set

$$\mathsf{Q} = \{(F, r, c(T)) : F \in \mathsf{F}, r > 0\}.$$

3 The political game

Take any taxation environment $Q \in \mathsf{Q}$, and consider two political parties who want to maximize their vote share. Parties are restricted to pick ex-ante tax functions from the set R. If party 1 proposes the ex-ante tax function S and party 2 proposes T, the share of voters that strictly prefer the ex-post tax function s over the ex-post tax function t is denoted as

$$W(S,T) = p_F(s(x) < t(x)) = \int_0^1 1_{\{s(x)<t(x)\}} dF.$$

Symmetrically, the share of voters who strictly prefer the victory of party 2 is $W(T,S)$. To refer to the pure endowment game (taxes are not distortionary), we use the notation $w(S,T) = p_F(S(x) < T(x))$. We assume that the parties' purpose is to maximize their utility $u_i : R^2 \to [-1,1]$, which is understood as the relative popular support of the proposed ex-ante tax function. That is, we suppose that

$$u_i(S,T) = \begin{cases} W(S,T) - W(T,S) & \text{if } i = 1 \\ W(T,S) - W(S,T) & \text{if } i = 2. \end{cases}$$

We denote the two player zero-sum symmetric game as $G = (R, (u_1, u_2))$.

One of the main difficulties to determine the structure of equilibria in this political game is the lack of existence of a pure strategy Nash equilibrium as stated by Myerson [14] and Carbonell-Nicolau and Ok [3], due to the discontinuity of the utility payoffs.[9] Thus, we cannot guarantee the existence of an equilibrium in pure strategies for the game G.

Even if mixed strategy equilibria are considered to be conceptually acceptable in political environments since Downs [6], they have not been extensively used in the class of game which we study. In addition to the classical interpretations of mixed strategies[10], the novel one stated by Laslier [7] seems particularly relevant to our context. Under this approach, parties are ambiguous and this ambiguity is represented by their mixed strategies. Each voter associates one party with one tax function. The probability that a policy alternative is offered by a party equals the fraction of voters identifying a party with this alternative.

A mixed strategy over the set of allowed tax functions R is a Borel probability measure over R. Therefore, party i's expected payoff of the pair of mixed strategies (μ_1, μ_2) is denoted by

$$U_i(\mu_1, \mu_2) = \int_{R \times R} u_i(S, T) d(\mu_1 \times \mu_2), \;\; \mu_i \in B(R), \;\; i = 1, 2,$$

where $B(R)$ denotes the set of all Borel probability measures of R. The expected utility payoff $U_i : B(R)^2 \to [-1, 1]$ is well defined, since any Borel measurable function on R is measurable in the associated product measure space. Then,

the mixed extension of $G = (R, (u_1, u_2))$ is denoted by $\hat{G} = (B(R), (U_1, U_2))$.

The pair $\mu = (\mu_1, \mu_2)$ constitutes a mixed strategy equilibrium of the game $\hat{G} = (B(R), (U_1, U_2))$, if for every mixed strategy $\hat{\mu} \in B(R)$, the expected payoff (to party 1) satisfies

$$U_1(\hat{\mu}, \mu_2) \leq U_1(\mu_1, \mu_2),$$
$$U_1(\mu_1, \hat{\mu}) \geq U_1(\mu_1, \mu_2),$$

[9] To see this, it suffices to build an example where $W(S_n, T) - W(T, S_n)$ is positive for every n, and $W(S, T) - W(T, S) < 0$ with S_n uniformly converging towards S as n goes to infinity. For a more detailed construction, see section 4.2 of Carbonell-Nicolau and Ok [3].

[10] See Rubinstein [19].

by the MinMax Theorem, as the political game is zero-sum. Besides, the expected payoff to both parties satisfies $U_i(\mu_1, \mu_2) = 0$ in equilibrium.

3.1 Existence of equilibrium

Once we have properly defined the game, we address the existence of political equilibrium. As we assume infinite space strategies, the existence of an equilibrium is not ensured[11]. The existence problem with infinite space strategies is not a trivial matter. However, the distortions introduced within this work do not deeply modify the structure of the game (when compared with a pure endowment economy). Thus, we show that the existence problem can be solved by checking that the political game satisfies the conditions stated by Reny [16] (Proposition 5.1 and Corollary 5.2) (in a similar manner to the one used by Carbonell-Nicolau and Ok [3]). To show that the mixed extension of the game verifies payoff-security we use a novel result of Monteiro and Page [13] which simplifies the proof.[12]

Theorem 1. *The political game $(R, (u_1, u_2))$ has at least one mixed strategy equilibrium.*

To prove that the game $G = (R, (u_1, u_2))$ has a mixed strategy equilibrium, we need to show that it satisfies the following properties:

1. the sum of utility functions $u_1 + u_2$ is upper semi continuous in T on R,
2. the strategy space R is a compact subset of $\mathbf{C}[0, 1]$,
3. the utility functions u_i are both bounded and measurable,
4. the mixed extension of the game $\hat{G} = (B(R), (U_1, U_2))$ of G is payoff-secure.

The function $u_1 + u_2$ is continuous and so upper semi continuous, which implies that property 1 is verified. Properties 2, 3 and 4 are proved in the appendix.

[11] To ensure existence of equilibrium in mixed strategies, one can assume that parties dispose from a finite set of tax functions. However, the result concerning efficiency (Theorem 2) will not anymore hold. Indeed in the current setting, the number of tax functions is not arbitrarily bounded. Hence, a party can build a tax function that covers any inefficient tax function as stated by Proposition 1.

[12] The author would like to thank Oriol Carbonell-Nicolau for this useful suggestion.

4 Efficient Political Competition

To prove that the parties uniquely propose efficient tax functions in equilibrium, we show that advocating an inefficient tax function is *weakly dominated*. Proposition 1 shows that whenever a party advocates an inefficient income tax function (which does not maximize the aggregate level of final income), his opponent can build another tax function which is unanimously preferred by the electorate.

Once we have proved that inefficiency is weakly dominated, we show that the support of both parties' strategies does not include inefficient tax functions. To do so, we need to introduce some binary relation between the tax functions. For any pair of tax functions S and T in the set R, we define the covering relation $\mathcal{C}$ as follows:

$$\begin{aligned} S\,\mathcal{C}\,T \iff & u_i(S,T) > 0 \text{ and} \\ & \forall\, Z \in R:\ u_i(S,Z) \geq u_i(T,Z) \text{ and} \\ & \exists\, Z \in R : u_i(S,Z) > u_i(T,Z). \end{aligned}$$

The uncovered set, denoted by U, is a subset of the set of tax functions R and consists of the maximal elements of the covering relation: $S \in U$ if and only if there is no $T \in R$ such that $T\mathcal{C}S$. We let $V = R\backslash U$ denote the set V of covered strategies.

Using the covering relation $\mathcal{C}$, Proposition 1 can be interpreted as follows: any inefficient tax function $T \in I$ belongs to the set of covered strategies. This observation is crucial, because a result of Banks, Duggan and Le Breton [2] (included in the appendix) entails that, under some conditions, the support of any mixed strategy equilibrium is included in the uncovered set U.

Proposition 1. *For any tax function $T \in I$ under which the aggregate final income is not maximized, we can construct one tax function $S \in R$ such that $s(x) \leq t(x)$ for any $x \in [0,1]$ and $p_F(s(x) < t(x)) > 0$.*

Theorem 2 formalizes the idea that, in equilibrium, rational parties advocate efficient tax functions in which the aggregate final income is maximized. The proof is provided in the appendix.

Theorem 2. *In any mixed strategy equilibrium of the game $(R,(u_1,u_2))$, parties solely propose efficient tax functions, i.e. for any equilibrium (μ_1,μ_2) of this game,* *we have for some set $D \subseteq E$*

$$\mu_1(D) = \mu_2(D) = 1.$$

5 On the shape of income taxation

Once we have analyzed the efficiency of the electoral competition, we focus on the shape of income taxation in equilibrium. In order to make the arguments transparent, the analysis is restricted to the flat tax game throughout. Hence, the cost function $c(T)$ satisfies $c(T) = ax$ for some $a \in [0, 1]$. A simple interpretation for the flat tax game is based on the classical economic situation in which two countries are involved. In country A, there is an electoral competition between two parties. Parties make electoral promises to voters over the income tax functions they will apply if elected. However, both parties and voters know that voters have the possibility of not paying taxes in country A by paying some amount of money in country B. A natural question that arises is how should parties react to this possibility of not paying taxes? We first show that this outside option modifies the classical observation according to which the majority of the voters prefer progressive taxation. However, when the set of available tax functions is rich enough, the political appeal for progressivity is restored. Hence, even taking into account the possibility that voters will not pay taxes, a strategic party should advocate progressive taxation to maximize his probability of victory. A discussion on the robustness of our results with different cost functions is provided.

5.1 Regressive taxation can be unanimously preferred

We give an example that shows how unfounded the arguments given in the public debate can be in our setting. Distortions can have a deep impact on the predictions of the model. Our definition of the tax functions eliminates the traditional observation that low income voters prefer progressive tax functions. Such an observation was formalized by Marhuenda and Ortuño Ortín [11] as follows: let S, T be two ex-ante tax functions such that S is non-linear convex on [0,1], T is concave on $[0, 1]$ and such that $\int_0^1 S(x)dF = r \leq \int_0^1 T(x)dF$ for some $0 < r < \int_0^1 xdF$. The intersection of both curves is denoted by $\theta \in [0, 1]$ and is located above the median m of the income distribution F, which in our framework is denoted by $w(S, T) > 1/2$. Their result is conditional on the fact that function F is such that the median income $m = F^{-1}(\frac{1}{2})$ is lower than the mean income $\int_0^1 xdF$. We refer to the previous result as *the political appeal for progressivity.* An equilibrium version of the previous inequality is given by Carbonell-Nicolau [3] in a pure endowment economy. The following example shows that it could be the case that a regressive tax function can be unanimously preferred in equilibrium to a progressive one when tax functions are distortionary.

Example 4: Let $Q \in \mathsf{Q}$ be a taxation environment such that $r = 0.1$ and the income distribution function F has the following density:

$$f(x) = 2 - 2x, \quad x \in [0, 1].$$

This income distribution is such that the median income m is lower than the mean income ($m = 0.29$ and $\int_0^1 xdF = 1/3$). The cost function $c(T)$ is equal to $c(T)(x) = ax$ with $a = 0.535$. Let us consider the ex-ante tax functions S and T defined as follows:

$$S(x) = \frac{1}{3}(x^2 + x) \text{ and } \quad T(x) = -0.05x^2 + 0.325x \text{ with } x \in [0, 1].$$

The convexity of S and the concavity of T are ensured by the fact that $S'' > 0$ and $T'' < 0$.

Given the ex-ante tax functions S and T, then their respective ex-post tax functions s and t are such that

$$s(x) = \min\{S(x), ax\},$$

and

$$t(x) = \min\{T(x), ax\} = T(x).$$

In this game, the ex-post tax function t coincides with the ex-ante tax function T, as $T(x) < ax$ for every $x \in [0, 1)$. The cost ax of making an effort is too high, and so every voter pays full taxes under the ex-ante tax function T.

The voters who pay full taxes under the tax function S are the ones with an income in the set $E_S = \{x \in [0, 1] \mid S(x) \leq ax\} = \{0 \leq x \leq 0.607\}$. Besides, both ex-ante tax functions S and T satisfy budget balance after the distortions have taken place, as

$$\int_0^1 G(T)\, dF = \int_0^1 T(x)\, dF = 0.1,$$

and

$$\int_0^1 G(S)\, dF = \int_{E_S} S(x)\, dF = \int_{\{0 \le x \le 0.607\}} S(x)\, dF = 0.1.$$

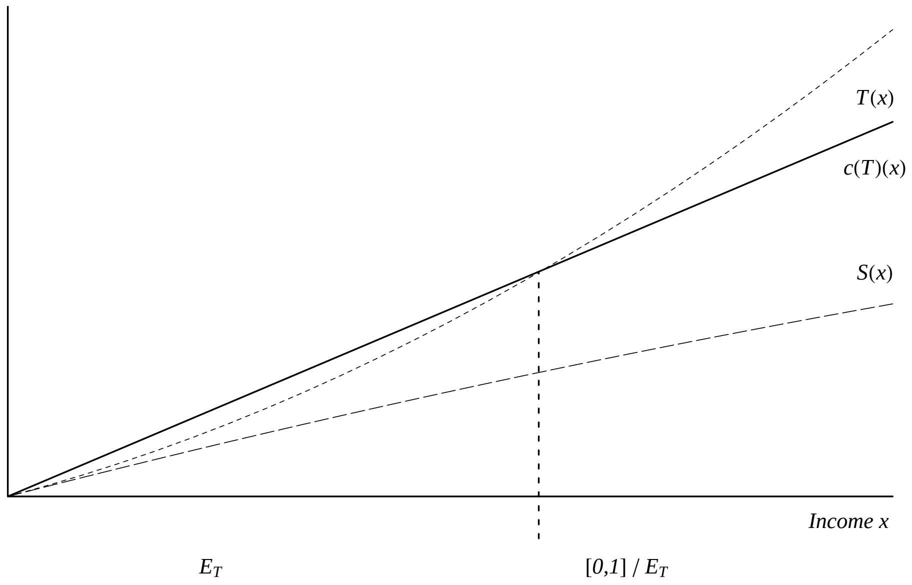

Figure 1: The regressive tax function T is unanimously preferred to the progressive tax function S.

As depicted by Figure 1, the regressive tax function T is unanimously preferred to the progressive tax function S, i.e.

$$W(T, S) = 1.$$

Hence, in the game $G = ((S, T), (u_1, u_2))$, the regressive tax function is unanimously preferred by voters, and consequently it will be the tax function advocated by both political parties in equilibrium. Thus, in our framework it is not anymore true that low income voters prefer progressive income taxation. A progressive tax function in which an important share of voters do not pay full taxes generates an inefficiency in the economy.

5.2 The political appeal for progressivity

One of the aims of the positive theory of income taxation is to understand the shape of income tax functions that will be implemented by self-interested parties in equilibrium. The previous example shows that traditional arguments do not anymore hold within our setting. However, as will be shown, the political appeal for progressivity can be restored when the political parties can choose their electoral promises from a set of taxes rich enough. If we allow parties to advocate progressive and regressive tax functions, only progressive taxation will be implemented.

Let $\hat{R}$ be the restricted policy space in which ex-ante tax functions are either progressive or regressive[13]. Formally, $T \in \hat{R}$ iff T is progressive or regressive, and $T \in R$. Thus, the set $\hat{R}$ is the union of the set of progressive tax functions R_p and the set of regressive tax functions R_r. Among the set of progressive tax functions, the subclass N_p of non-linear members deserves special attention.

Proposition 2. *Any non-linear progressive tax function $S \in N_p$ with which every voter pays full taxes is preferred to any regressive tax function $T \in \hat{R}$ (provided than the median income is lower than the mean one).*

Condition 1 The set $\hat{R}$ contains non-linear progressive tax functions with which every voter pays full taxes.

Theorem 3. *In any mixed strategy of the flat tax game $(\hat{R}, (u_1, u_2))$, parties solely propose progressive tax functions whenever the set $\hat{R}$ satisfies condition 1. Formally, any equilibrium (μ_1, μ_2) of this game satisfies from*

$$\mu_1(N_p) = \mu_2(N_p) = 1.$$

5.3 Discussion

The previous result shows that the political appeal for progressivity is robust to the introduction of distortions on the flat tax game. It is only required that parties can propose progressive tax functions with which every voter pays full taxes. Even if it need not be the case that any progressive tax function is preferred to any regressive tax function (as it is the case in a pure endowment economy), we can still show that

[13]Let us recall that an ex-ante tax function is said to be progressive (regressive) whenever it is convex (concave).

any progressive tax function with which every voter pays full taxes is preferred to any regressive one. Hence, as it is weakly dominated for both parties to propose regressive taxation, the political appeal for progressivity is restored. Our result is hence a generalization of Carbonell-Nicolau and Ok [3]: whenever the cost function $c(T)$ satisfies $c(T)(x) \geq x$ (i.e. $a \geq 1$), every voter pays full taxes. Hence, as the cost of making an effort is too high, we are back in an economy without distortions, a pure endowment economy. Therefore, the political appeal for progressivity (Theorem 3, Carbonell-Nicolau and Ok [3]) is a particular case of our result.

The results are identical when studying the class of smooth cost functions. Formally, the definition of smooth cost function is as follows.

Definition 4. *A cost function is smooth whenever $c(T)$ progressive (resp. regressive) if and only if T progressive (resp. regressive).*

The reason the result is valid for this class of smooth cost functions is simple. Let us pick a convex ex-ante function S with which every voter pays full taxes and a concave ex-ante function T. As every voter pays full taxes, the ex-post tax function s coincides with the ex-ante tax function S which implies that s is convex. Besides, the minimum of two concave functions is concave. Hence, the ex-post tax function t is concave as the cost function is concave. Whenever the deadweight cost of making an effort is positive (i.e. $T_P(x) > 0$ used to represent tax avoidance activities)[14], the ex-post tax function t collects an amount of money strictly higher than the ex-post tax function s. Using Marhuenda and Ortuño-Ortin [11] result, we can prove that the unique intersection between the ex-post tax functions s and t must be located above the median income so that the same result applies.

6 Conclusion

We have built a simple model to study the interaction between the political arena and the determination of income taxation in the presence of distortionary taxes. The mixed strategy approach has allowed us to obtain an equilibrium of the game. Furthermore, in equilibrium, parties only advocate efficient tax functions. Whenever

[14]When the deadweight cost of making an effort is negative ($T_P(x) < 0$ used to represent labour supply decisions), Condition 1 does not ensure that Theorem 3 holds. In this case, a sufficient condition for such a theorem to hold is that the policy space contains a progressive tax function with which every voter provides an effort.

a party does not advocate a tax function that maximizes aggregate final income, the other party can build a unanimously weakly preferred tax function that collects the same amount of taxes. This result shows the existence of a link between the literature of probabilistic voting theory[15] and the positive theory of income taxation. According to our results parties' mixed strategies play the same role as ideological components in the probabilistic voting theory to ensure both the existence and efficiency of equilibrium in these political games.

Finally, as far the shape of income taxation is concerned, we have shown that when taking into account distortions generated by tax functions, it is no longer true that low-income voters always prefer progressive taxation, and it can even be the case that every voter in the society prefers regressive income taxation. However, under mild conditions, parties propose progressive taxation. This result shows that the political appeal for progressivity is robust to the introduction of distortions. When parties anticipate that voters have the possibility of not paying full taxes, they still advocate progressive taxation to maximize their probability of victory. This result is in accordance with observed income tax schemes in most OECD countries.

One of the main limits of our model is an implicit assumption of the modeling strategy. We consider here a one-shot voting game with binding electoral promises, and thus do not take into account commitment problems. Our conclusion applies only to what has been called "pre-electoral politics". Problems associated with commitment are a source of inefficiencies and are not taken into account in our model.

Besides, the relationship between taxation and the beliefs held by voters about the consequences of their own actions and those of others on the aggregate tax system are very important and not well understood. An example of this lack of understanding is the classic question of why people pay taxes given the low probability of being audited (i.e. why is there not more *tax evasion*). Thus, introducing a system of social norms as has been previously done in the literature of tax evasion could be an interesting extension of this work.

[15]As argued by Persson and Tabellini [15], probabilistic voting theory has become a standard tool in political economy.

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A Appendix: Properties of the sets E_S

Let $Q \in \mathsf{Q}$ be a taxation environment. Let us define for any continuous function $T \in \mathbf{C}[0,1]$, the set E_T as follows:

$$E_T = \{x \in [0,1] \mid T(x) \leq c(T)(x)\}.$$

For any pair of functions S and T in $\mathbf{C}[0,1]$ and any $\varepsilon \in [0,1]$, let $S(\varepsilon) : [0,1] \to [0,1]$ be the function such that $S(\varepsilon)(x) = \varepsilon S(x) + (1-\varepsilon)T(x)$. To simplify the notations, we assume that the cost function associated to the nil function is identically equal to zero: for any $x \in [0,1]$, we write $c(0)(x) = 0$ for any $x \in [0,1]$ so that $E_0 = [0,1]$.

Lemma 1 (Inclusion Lemma)**.** *If the sets E_S and E_T satisfy $E_S \subset E_T$ then $E_S \subset E_{S(\varepsilon)} \subset E_T$.*

Proof. Let T and M be a pair of functions in $\mathbf{C}[0,1]$. For any $\varepsilon \in [0,1]$, let $S(\varepsilon) : [0,1] \to [0,1]$ be the function such $S(\varepsilon)(x) = \varepsilon S(x) + (1-\varepsilon)T(x)$. Then, the sets

E_S, E_T and $E_{S(\varepsilon)}$ are given by :

$$\begin{aligned} E_S &= \{x \in [0,1] \mid S(x) < c(S)(x)\}, \\ E_T &= \{x \in [0,1] \mid T(x) < c(T)(x)\} \text{ and} \\ E_{S(\varepsilon)} &= \{x \in [0,1] \mid S(\varepsilon)(x) < c(S(\varepsilon))(x)\}. \end{aligned}$$

We assume that $E_S \subset E_T$ that is if $x \in E_S$ then $x \in E_T$.
Let us first show that if $x \in E_S$ then $x \in E_{S(\varepsilon)}$. To do so, let us assume that there exists $x \in [0,1]$ such that $x \in E_S$ and $x \notin E_{S(\varepsilon)}$. Given that $x \notin E_{S(\varepsilon)}$, we know that $S(\varepsilon)(x) \geq c(S(\varepsilon))(x)$ which implies that

$$\begin{aligned} \varepsilon S(x) + (1-\varepsilon)T(x) \geq \varepsilon c(S)(x) + (1-\varepsilon)c(T)(x) \iff \\ (1-\varepsilon)[T(x) - c(T)(x)] \geq \varepsilon[c(S)(x) - S(x)]. \end{aligned}$$

As we know that $x \in E_S$, we can write that $S(x) < c(S)(x)$ so that the right part of the inequality is positive. Thus, given that $\varepsilon \in [0,1]$, the left part of the inequality must be strictly positive to satisfy the inequality. However, this would imply $T(x) - c(T)(x) > 0$ and so that $x \notin E_T$. This is a contradiction as we have assumed that if $x \in E_S$ then $x \in E_T$ which implies that $E_S \subset E_{S(\varepsilon)}$.
Let us now show that if $x \in E_{S(\varepsilon)}$ then $x \in E_T$. To do so, let us assume that there exists $x \in [0,1]$ such that $x \in E_{S(\varepsilon)}$ and $x \notin E_T$. Given that $x \in E_{S(\varepsilon)}$, we know that $S(\varepsilon)(x) < c(S(\varepsilon))(x)$ which implies that

$$\begin{aligned} \varepsilon S(x) + (1-\varepsilon)T(x) < \varepsilon c(S)(x) + (1-\varepsilon)c(T)(x) \iff \\ (1-\varepsilon)[T(x) - c(T)(x)] < \varepsilon[c(S)(x) - S(x)]. \end{aligned}$$

As we know that $x \notin E_T$, we can write that $T(x) \geq c(T)(x)$ so that the left part of the inequality is positive. Thus, given that $\varepsilon \in [0,1]$, the right part of the inequality must be strictly positive to satisfy the inequality. However, this would imply that $c(S)(x) > S(x)$ and then $x \in E_S$. This is a contradiction as we have assumed that if $x \in E_S$ then $x \in E_T$, implying that $E_{S(\varepsilon)} \subset E_T$. □

Lemma 2 (Equality lemma)**.** *If the sets E_S and E_T satisfy $E_S = E_T$ then $E_S = E_{S(\varepsilon)} = E_T$.*

Proof. To show $E_S \subseteq E_{S(\varepsilon)}$, let us assume that there exists some $x \in E_S$. As by assumption $E_S = E_T$, if $x \in E_S$ then $S(x) < c(S)(x)$ and $T(x) < c(T)(x)$.

Besides, by definition $S(\varepsilon)(x) = \varepsilon S(x) + (1-\varepsilon)T(x)$. Then, given that $c(S(\varepsilon))(x) = \varepsilon c(S)(x) + (1-\varepsilon)c(T)(x)$, we can write that $S(\varepsilon)(x) > c(S(\varepsilon))(x)$, that is $x \in S(\varepsilon)$.

To prove the other inclusion $E_{S(\varepsilon)} \subseteq E_S$, let us assume that there exists some $x \in E_{S(\varepsilon)}$ and such that $x \notin E_S$. Given that $x \in E_{S(\varepsilon)}$ then $S(\varepsilon)(x) < c(S(\varepsilon))(x)$. Besides, $x \notin E_S$ entails that $S(x) \geq c(S)(x)$ and that $T(x) \geq c(T)(x)$ as $E_S = E_T$. Then, given that $c(S(\varepsilon))(x) = \varepsilon c(S(x)) + (1-\varepsilon)c(T)(x)$, we can write that $c(S(\varepsilon))(x) \geq S(\varepsilon)(x)$, that is $x \notin S(\varepsilon)$ which is a contradiction. □

Lemma 3. *For any ex-ante tax function T there exists an ex-ante tax function S such that $S(x) \leq T(x)$ for any $x \in [0,1]$ with $E_T \subset E_S$ and $p_F(E_T \setminus E_S) > 0$.*

Proof. Let T be an ex-ante tax function with $E_T \subset [0,1]$. Let us pick the function εT for some $\varepsilon > 0$ which satisfies $\varepsilon T(x) \leq T(x)$ for any $x \in [0,1]$. The function εT is a convex combination of T and the nil function N with $N(x) = 0$ for any $x \in [0,1]$. By assumption, we have $E_N = [0,1]$ so that $E_N \subset E_T$ and then by the Inclusion Lemma, we write that $E_N \subset E_{\varepsilon T} \subset E_T$. It suffices to pick some ε low enough such that $p_F(E_T \setminus E_{\varepsilon T}) > 0$ and then to choose $S = \varepsilon T$ to conclude the proof. □

Lemma 4. *Let S and T be a pair of tax functions such that $S(x) \leq T(x)$ for any $x \in [0,1]$. If $E_T \subset E_S$ with $p_F(E_T \setminus E_S) > 0$ then $p_F(s(x) < t(x)) > 0$.*

Proof. Let S and T be a pair of tax functions such that $S(x) \leq T(x)$ for any $x \in [0,1]$ with $E_T \subset E_S$. To prove the claim, let us assume that $p_F(s(x) < t(x)) = 0$. Given the second property of the cost functions, we know that if $S(x) \leq T(x)$ for any $x \in [0,1]$ then $c(S)(x) \leq c(T)(x)$ for any $x \in [0,1]$. Hence, we can write that the respective ex-post tax functions s and t satisfy $s(x) \leq t(x)$ as $s = \min\{S, c(S)\}$ and $t = \min\{T, c(T)\}$. Besides as we have assumed that $p_F(s(x) < t(x)) = 0$, then we can write that $p_F(s(x) = t(x)) = 1$. To do so, we must have that $s(x) = t(x)$ on any subset $A \subset [0,1]$ such that $p_F(A) > 0$.

Given that $E_T \subset E_S$ with $p_F(E_S \setminus E_T) > 0$, we can write that for any $x \in E_S \setminus E_T$, $S(x) = s(x) = t(x) = c(T)(x)$. However, by definition, for any $x \in E_S$ we have $S(x) < c(S)(x)$. Therefore, for any $x \in E_S \setminus E_T$, we have $c(S)(x) > S(x) = s(x) = t(x) = c(T)(x)$ which implies that $c(S)(x) > c(T)(x)$ which is a contradiction as $c(S)(x) \leq c(T)(x)$ for any $x \in [0,1]$.

□

B Appendix: Existence of a mixed strategy equilibrium

Proposition 3. *The strategy space R is a compact subset of* $\mathbf{C}[0,1]$.

Proof. In a metric space, the continuous image of compact space is compact. Hence, we can show that the set of budget balanced ex-post tax functions is compact by proving that such a set is the continuous image of a compact space.

By definition, an ex-post tax function s is budget balanced if and only if there exists an ex-ante tax function T such that $s(x) = \min\{T(x), c(T)(x)\}$ and $\int_0^1 G(T)dF \geq r$. We define a continuous mapping $\psi : \mathbb{T} \to \mathbb{T} \times C(\mathbb{T})$ from the set of ex-ante tax functions $\mathbb{T}$ to the set $\mathbb{T} \times C(\mathbb{T})$, the product set of $\mathbb{T}$ and the set of cost functions $C(\mathbb{T})$ with

$$\psi(T) = (T, C(T)).$$

Similarly, we define a continuous mapping $\phi : \mathbb{T} \times C(\mathbb{T}) \to R$ with R standing for the set of budget balanced ex-post tax functions,

$$\phi(T, S) = \min\{T, S\}.$$

Hence, any ex-post tax function $s \in R$ satisfies $s \in (\phi \circ \psi)(\mathbb{T})$. In other words, for every ex-post tax function s there exists an ex-ante tax function $T \in \mathbb{T}$ such that $s = \phi(\psi(T))$ with both ϕ and ψ continuous.

Therefore, we have proven that the set of budget-balanced ex-post tax functions is the continuous image of the set of ex-ante tax functions with $\int_0^1 TdF \geq r$[16]. It remains to be shown that the set of ex-ante tax functions $\mathbb{T}$ such that $\int_0^1 TdF \geq r$ is a compact set.

Given that the set $\mathbf{C}[0,1]$ is Hausdorff measurable, the Arzelà-Ascoli theorem states that any of its subsets which is bounded, closed and equicontinuous is a compact subset.

[16] Let us recall that the proof is done in the case in which the deadweight cost of making an effort is positive. Thus, as we focus on tax sheltering activities the real amount of taxes collected by the government is lower or equal to the statutory amount. Hence, the set of ex-ante tax functions satisfies $\int_0^1 TdF \geq r$. In the case, in which the deadweight cost is positive, the set of ex-ante functions must satisfy $\int_0^1 TdF \leq r$ as the amount of money collected by the government is higher or equal than the statutory amount.

To show that $\mathbb{T}$ is bounded, take any two ex-ante tax functions S and T of the space $\mathbb{T}$. We can write $||S-T||_\infty \leq k$ for some positive k as the range of both functions is located between 0 and 1. Hence, the space $\mathbb{T}$ is bounded.

Let us now show that $\mathbb{T}$ is closed in $\mathbf{C}[0,1]$. Take any sequence of T_n in $\mathbb{T}$ such that $||T_n - T||_\infty \to 0$ for some function $T \in \mathbf{C}[0,1]$. Then, the sequence T_n uniformly converges to T and this guarantees that T is an ex-ante tax function.

As any $T_n \in \mathbb{T}$, we know that $\int_0^1 T_n dF \geq r$. Thus, due to uniform convergence of T_n, T satisfies

$$\int_0^1 T dF = \int_0^1 \lim T_n dF = \lim \int_0^1 T_n dF \geq r.$$

due to Lebesgue's dominated convergence theorem. Thus, the ex-ante tax function T satisfies the budget balanced constraint and thus $T \in \mathbb{T}$ which entails that the set $\mathbb{T}$ is closed in $\mathbf{C}[0,1]$.

Finally, it needs to be shown that $\mathbb{T}$ is equicontinuous. To do so, let us pick some ex-ante tax function T and $0 \leq y < x \leq 1$. By definition, we have $x - T(x) \leq y - T(y)$ which implies that $T(x) - T(y) \geq x - y$. Similarly, if $x < y$, we can write that $T(y) - T(x) \geq y - x$ which implies that $|T(x) - T(y)| \leq |x - y|$ for all $0 \leq x, y \leq 1$. So, for any $x \in [0,1]$ and any $\varepsilon > 0$, we have $|t(x) - t(y)| < \varepsilon$ whenever $|x - y| < \varepsilon$. Hence, we can conclude that the set $\mathbb{T}$ is equicontinuous and hence that $\mathbb{T}$ is a compact subset of $\mathbf{C}[0,1]$. □

The function $u_i(S,T) = W(S,T) - W(S,T)$ is obviously bounded for any two given tax functions S and T which entails the first part of property 3.

Proposition 4. *The utility function $u_i : R^2 \to [-1,1]$ is measurable for any $i = 1,2$.*

Proof. To show the measurability of the utility function, it suffices to show that both $W(S,T)$ and $W(T,S)$ are lower semi continuous for any ex-ante tax functions (S,T) (and thus measurable). Indeed, the sum of two measurable functions is measurable and thus $u_i(S,T) = W(S,T) - W(T,S)$ is measurable.

A function $W : R^2 \to [-1,1]$ is lower semi continuous if for any sequence (S_n, T_n) converging to (S,T), the function W verifies $\liminf W(S_n, T_n) \geq W(S,T)$. To prove the lower semi continuity of W, we take a sequence (S_n, T_n) converging to (S,T).

By Fatou's lemma,

$$\liminf W(S_n,T_n) = \liminf \int_0^1 1_{\{s_n<t_n\}} dF \geq \int_0^1 \liminf 1_{\{s_n<t_n\}} dF$$
$$\geq \int_0^1 1_{\{\lim s_n < \lim t_n\}} dF = W(S,T),$$

in which the second inequality comes from the observation that $\liminf 1_{\{s_n<t_n\}}(x) \geq 1_{\{\lim s_n<\lim t_n\}}(x)$[17] for any $x \in [0,1]$. Then, $W(S,T)$ is lower semi continuous and whence measurable which proves the claim. □

Prior to proceeding with the proof of payoff security, we introduce a definition from Monteiro and Page [13] that will be necessary throughout.

Definition 5. *The game $G = (R, (u_1, u_2))$ is uniformly payoff secure if for any $S \in R$ and every $\varepsilon > 0$, there exists a tax function $S^l \in R$ such that for every $T \in R$, there exists an open neighborhood $\mathcal{N}(T)$ of T in R such that*

$$F \in \mathcal{N}(T) \Longrightarrow u_i(S^l, F) \geq u_i(S,T) - \varepsilon.$$

Besides, if a compact game G is uniformly payoff secure, then its mixed extension $\hat{G} = (B(R), (U_1, U_2))$ is payoff secure.

Proposition 5. *The game $G = (R, (u_1, u_2))$ is uniformly payoff-secure and hence its mixed extension $\hat{G}$ is payoff-secure.*

Proof. Let us pick some ex-ante tax function $S \in R$ and some $\varepsilon > 0$. We define its ex-post tax function s as $s = \min\{S, c(S)\}$. Let us choose a tax function S^l such that for some $l \in (0,1)$ and some λ that verifies

$$0 < \lambda < \min\{l, 1-l, \frac{\varepsilon}{12}\},$$

for some $\varepsilon > 0$ and such the ex-post tax function $s^l = \min\{S^l, c(S^l)\}$ satisfies

$$s^l > s \text{ on } [l-\lambda, l+\lambda] \text{ and}$$
$$s^l < s \text{ on } [0,1] \setminus [l-\lambda, l+\lambda]$$

[17] If the left-hand side of the inequality equals 0 for some x, then $s_n(x) \geq t_n(x)$ for infinitely many n, and this means that the right-hand side must be equal to zero.

with $p_F(\{s^l = s\}) = 0$[18]. We choose a number η with

$$\lambda < \eta < \min\{l, 1-l, \frac{\varepsilon}{12}\}.$$

Let D denote the interval $[l-\eta, l+\eta]$ and D^c its complement on $[0,1]$.
Define

$$\tau = \min_{x \in [s_0+\eta, 1] \cap D^c} |s(x) - s^l(x)|.$$

For any $T \in R$, let us take some $F \in N_\tau(T)$ such that

$$|f - t| < \tau \text{ on } [0,1],$$

in which both f and t stand for the respective ex-post tax functions of F and T. Taking into account previous definitions, we can write

$$\begin{aligned} W(S,T) - W(T,S) &= 2W(S,T) + p_F(s=t) - 1 \\ &= 2[p_F(\{s<t\} \cap D) + p_F(\{s<t\} \cap D^c)] \\ &\quad + p_F(\{s=t\} \cap D) + p_F(\{s=t\} \cap D^c) - 1. \end{aligned}$$

As the measure of D is bounded by 2η and $\eta < \frac{\varepsilon}{12}$, the measure of D is less than $\frac{\varepsilon}{6}$. Therefore,

$$2p_F(\{s<t\} \cap D) + p_F(\{s=t\} \cap D) < \varepsilon/2 < \varepsilon.$$

Then it follows that

$$W(S,T) - W(T,S) < \varepsilon + 2p_F(\{s<t\} \cap D^c) + p_F(\{s=t\} \cap D^c) - 1.$$

[18] In the case in which we cannot pick a function s^l such that $p_F(\{s^l = s\}) > 0$, a similar proof applies.

Therefore, applying a simple decomposition, we write

$$\begin{aligned}
&W(S,T) - W(T,S)\\
&\quad < \varepsilon + 2[p_F(\{s^l < s < t\} \cap D^c) + p_F(\{s^l \geq s < t\} \cap D^c)]\\
&\qquad + p_F(\{s^l < s = t\} \cap D^c) + p_F(\{s^l \geq s = t\} \cap D^c) - 1\\
&\quad = \varepsilon + 2[p_F(\{f \leq s^l < s < t\} \cap D^c) + p_F(\{f > s^l < s < t\} \cap D^c)\\
&\qquad + p_F(\{f \leq s^l \geq s < t\} \cap D^c) + p_F(\{f > s^l \geq s < t\} \cap D^c)]\\
&\qquad + p_F(\{f \leq s^l < s = t\} \cap D^c) + p_F(\{f > s^l < s = t\} \cap D^c)\\
&\qquad + p_F(\{f \leq s^l \geq s = t\} \cap D^c) + p_F(\{f > s^l \geq s = t\} \cap D^c) - 1.
\end{aligned}$$

Given the previous inequalities, if we can show that the following inequalities hold

$$p_F(\{f \leq s^l < s < t\} \cap D^c) = 0, \quad (a)$$

$$p_F(\{f \leq s^l < s = t\} \cap D^c) = 0, \quad (b)$$

$$p_F(\{f \leq s^l \geq s < t\} \cap D^c) = 0, \quad (c)$$

$$\text{and } p_F(\{f \leq s^l \geq s = t\} \cap D^c) = 0 \leq p_F(f = s^l), \ (d)$$

then

$$\begin{aligned}
&W(S,T) - W(T,S)\\
&\quad < \varepsilon + 2[p_F(\{f > s^l < s < t\} \cap D^c) + p_F(\{f > s^l \geq s < t\} \cap D^c)]\\
&\qquad + p_F(\{f > s^l < s = t\} \cap D^c) + p_F(\{f > s^l \geq s = t\} \cap D^c)\\
&\qquad + p_F(f = s^l) - 1\\
&\quad \leq \varepsilon + 2W(S^l, F) + p_F(s^l = f) - 1\\
&\quad = \varepsilon + W(S^l, F) - W(F, S^l),
\end{aligned}$$

as desired which proves that the game $G = (R, (u_1, u_2))$ is uniformly payoff-secure. Furthermore, as the game is compact as shown by Proposition 3, its mixed extension $\hat{G}$ is payoff secure.

To conclude the proof it remains to be shown that inequalities (a), (b), (c) and

(d) hold. Any ex-post tax function $f \in \mathcal{N}_\tau(t)$ satisfies

$$t(x) - f(x) < \tau \text{ on } [s_0 + \eta, 1] \cap D^c.$$

Besides, s^l is lower than s on the interval $[s_0 + \eta, 1] \cap D^c$ by at least τ.

$$s(x) - s^l(x) \geq \tau \text{ on } [s_0 + \eta, 1] \cap D^c,$$
$$\text{and } s(x) > s^l(x) \text{ on } [s_0 + \eta, 1] \cap D^c.$$

Combining previous inequalities entails (a) and (b). To see why (c) and (d) hold, it suffices to see that $p_F(\{s^l \geq s\} \cap D^c) = 0$ and hence

$$p_F(\{f \leq s^l \geq s < t\} \cap D^c) = 0, \quad (c)$$

and

$$p_F(\{f \leq s^l \geq s = t\} \cap D^c) = 0 \leq p_F(f \leq s^l). \ (d)$$

□

This section has proved the existence of a mixed strategy equilibrium of the game $(R, (u_1, u_2))$. Section 5 is devoted to the study of the restricted game $(\hat{R}, (u_1, u_2))$ in which the set $\hat{R}$ is defined as follows:

$$T \in \hat{R} \Longleftrightarrow T \in R \text{ and } T \text{ is convex or concave}$$

The proof of existence is similar to the one presented with the whole strategy space R and hence is omitted.

C Appendix: Proof of Theorem 2

Proof of Proposition 1.

Proof. We focus on the case in which the deadweight cost associated included in the cost function is positive: $T_P(x) > 0 \, \forall \, x \in (0, 1)$. Showing the claim whenever $T_P(x) < 0 \, \forall \, x \in (0, 1)$ is similar and hence its proof is omitted. Let us recall that for any ex-ante tax function T, the government revenues function $G(T)$ satisfies

$$G(T)(x) = \begin{cases} T(x) & \text{if } x \in E_S \\ T_G(x) & \text{if not} \end{cases}$$

Let us choose some inefficient ex-ante tax function $T \in I$. As function T is budget-balanced, we can write that $\int_0^1 G(T)dF = r$.

Let us pick some ex-ante tax function M such that $M(x) \leq T(x)$ for any $x \in [0,1]$ with $E_T \subset E_M$ with $p_F(E_T \setminus E_M) > 0$ and $\int_0^1 G(M)dF < r$. Given M, the ex-post tax function m is defined as $m = \min\{M\ ,\ c(M)\}$. As we know that $M(x) \leq T(x)$ whenever $x \in (0,1)$, then we can also write that $m(x) \leq t(x)$ for any $x \in (0,1)$ by applying the second property of the cost functions.

We define for any $\varepsilon \in [0,1]$, the function $U(\varepsilon) = \varepsilon T + (1-\varepsilon)M$ and its associated set $E_{U(\varepsilon)} = \{x \in [0,1] \mid U(\varepsilon)(x) < c(U(\varepsilon))(x)\}$. Therefore $U(\varepsilon)(x) \leq T(x)$ for any $x \in [0,1]$ which implies that $c(U(\varepsilon))(x) \leq c(T)(x)$ and hence that $u(x) \leq t(x)$ for any $x \in [0,1]$. Given that the sets E_T and E_M are such that $E_T \subset E_M$, the Lemma 1 entails that $E_T \subset E_{U(\varepsilon)} \subset E_M$. Furthermore, given that $p_F(E_{U(\varepsilon)} \setminus E_T) > 0$, Lemma 4 ensures that $p_F(u(x) < t(x)) > 0$.

Then, given the first property of the cost functions, we can write that the integral of function $G(U(\varepsilon))$ is such that

$$\begin{aligned} \int_0^1 G(U(\varepsilon))dF &= \int_{E_{U(\varepsilon)}} \varepsilon T(x) + (1-\varepsilon)M(x)dF + \int_{[0,1]\setminus E_{U(\varepsilon)}} \varepsilon T_G(x) + (1-\varepsilon)M_G(x)dF \\ &= \varepsilon[\int_{E_{U(\varepsilon)}} T(x)dF + \int_{[0,1]\setminus E_{U(\varepsilon)}} T_G(x)dF] \\ &\qquad + (1-\varepsilon)[\int_{E_{U(\varepsilon)}} M(x)dF + \int_{[0,1]\setminus E_{U(\varepsilon)}} M_G(x)dF]. \end{aligned}$$

Given that $E_T \subset E_{U(\varepsilon)}$ and that $T(x) > T_G(x)$ whenever $x \notin E_T$[19]

$$\begin{aligned}\int_{E_{U(\varepsilon)}} T(x)dF + \int_{[0,1]\setminus E_{U(\varepsilon)}} T_G(x)dF &= \int_{E_T} T(x)dF + \int_{E_{U(\varepsilon)}\setminus E_T} T(x)dF + \int_{[0,1]\setminus E_{U(\varepsilon)}} T_G(x)dF \\ &> \int_{E_T} T(x)dF + \int_{E_{U(\varepsilon)}\setminus E_T} T_G(x)dF + \int_{[0,1]\setminus E_{U(\varepsilon)}} T_G(x)dF \\ &= \int_{E_T} T(x)dF + \int_{[0,1]\setminus E_T} T_G(x)dF \\ &= \int_0^1 G(T)dF = r.\end{aligned}$$

Similarly as $E_{U(\varepsilon)} \subset E_M$ and $M(x) > M_G(x)$, we can write that

$$\int_{E_{U(\varepsilon)}} M(x)dF + \int_{[0,1]\setminus E_{U(\varepsilon)}} M_G(x)dF < \int_{E_M} M(x)dF + \int_{[0,1]\setminus E_M} M_G(x)dF < r.$$

Then if we can choose an ε' such that $U(\varepsilon')$ satisfies $\int_0^1 G(U)dF = r$ then the proof is finished.

If not, let us choose an $\varepsilon' > 0$ such that $U = U(\varepsilon')$ with $\int_0^1 G(U)dF > r$. Let us now pick some ex-ante tax function N such that $N(x) \leq M(x)$ for any $x \in [0,1]$ and such that $E_M \subset E_N$ with $p_F(E_N \setminus E_M) > 0$. Then, we define for any $\varepsilon \in [0,1]$, the function $V(\varepsilon) = \varepsilon T + (1-\varepsilon)N$ and its associated set $E_{V(\varepsilon)} = \{x \in [0,1] \mid V(\varepsilon)(x) < c(V(\varepsilon))(x)\}$. By similar reasonings to the ones previously detailed with function U, we choose an $\varepsilon^* > 0$ such that $V = V(\varepsilon^*)$ satisfies $\int_0^1 G(V)dF < r$ and such that $E_U = E_V$.

We define for any $\alpha \in [0,1]$, the function $W(\alpha) = \alpha U + (1-\alpha)V$ and its associated set $E_{W(\alpha)} = \{x \in [0,1] \mid W(\alpha)(x) < c(W(\alpha))(x)\}$. Then,

$$\begin{aligned}\int_0^1 G(W(\alpha))dF = \alpha[&\int_{E_{W(\alpha)}} U(x)dF + \int_{[0,1]\setminus E_{W(\alpha)}} U_G(x)dF] \\ &+ (1-\alpha)[\int_{E_{W(\alpha)}} V(x)dF + \int_{[0,1]\setminus E_{W(\alpha)}} V_G(x)dF].\end{aligned}$$

Given Lemma 2, we can write that $E_{W(\alpha)} = E_U$ as $E_U = E_V$. Therefore, given

[19] Indeed, whenever $x \notin E_T$, we know that $T(x) \geq c(T)(x)$ which implies that $T(x) > T_G(x)$. To see this, let us assume that $T(x) \leq T_G(x)$ for some $x \notin E_T$. If this was true then for some $x \notin E_T$, $T(x) \leq T_G(x) < T_G + T_P(x) = c(T)(x)$ as an cost function that represents tax avoidance activities satisfies $T_P(x) > 0$ for any $x \in (0,1)$.

both tax functions U and V belong to the set R, we can write that

$$\int_{E_{W(\alpha)}} U(x)dF + \int_{[0,1]\setminus E_{W(\alpha)}} U_G(x)dF = \int_{E_U} U(x)dF + \int_{[0,1]\setminus E_U} U_G(x)dF > r,$$

and

$$\int_{E_{W(\alpha)}} V(x)dF + \int_{[0,1]\setminus E_{W(\alpha)}} V_G(x)dF = \int_{E_V} V(x)dF + \int_{[0,1]\setminus E_V} V_G(x)dF < r.$$

It is important to emphasize that neither $\int_{E_{W(\alpha)}} U(x)dF + \int_{[0,1]\setminus E_{W(\alpha)}} U_G(x)dF$ nor $\int_{E_{W(\alpha)}} V(x)dF + \int_{[0,1]\setminus E_{W(\alpha)}} V_G(x)dF$ depend on α as $E_U = E_V = E_{W(\alpha)}$. Hence, we have constructed a function $W(\alpha)$ in $\mathbf{C}[0,1]$ such that $\int_0^1 G(W(\alpha))dF = \alpha p + (1-\alpha)q$ with $p > r$ and $q < r$ and p and q independent from α.

Thus, we can choose an α' such that $W = W(\alpha')$ satisfies $\int_0^1 G(W)dF = r$ and hence $W \in R$. Furthermore, its corresponding ex-post tax function $w = \min\{W, c(W)\}$ satisfies $w(x) \leq t(x)$ for any $x \in [0,1]$ and so Lemma 4 implies that $p_F(w(x) < t(x)) > 0$ as $E_V \subset E_M$ with $p_F(E_M \setminus E_V) > 0$ which concludes the proof. □

Proposition 6 (Banks, Duggan and Le Breton [2]). *The support of any mixed strategy equilibrium* (μ_1, μ_2) *in the game* $G = (R, (u_1, u_2))$ *satisfies* $\mu_1(\hat{U}) = \mu_2(\hat{U})$ *for some set* $\hat{U} \subseteq U$ *given that:*

1. *the set* R *of tax functions is a complete and separable metric space and*

2. *for any given tax function* $T \in R$*, the set* $P(T) = \{(S,T) \in R \times R \,|\, u_i(S,T) > 0\}$ *is open.*

The rest of this section shows that the game $G = (R, (u_1, u_2))$ studied within this work fulfills the conditions stated by the previous proposition.

Lemma 5. *The set of tax functions* R *is a complete and separable metric space.*

Proof. By definition, the set R of tax functions is viewed as a metric subspace of $\mathbf{C}[0,1]$. Furthermore, as Proposition 3 shows, the set R is compact. This proves the claim as any compact metric space is complete and separable. □

Lemma 6. *In the game* $G = (R, (u_1, u_2))$*, for any tax function* $T \in R$*, the set* $P(T)$ *is open.*

Proof. The proof is done for party 1 and the same claim remains true for party 2 as G is a symmetric game. Let us choose a pair of ex-ante tax functions S and T in the set R such that $E_S = E_T$. Furthermore, let us assume that the tax S belongs to $P(T)$ which implies that $u_1(S,T) = w(S,T) - w(T,S) > 0$ and such that $||S - T||_\infty < \delta$ for some positive $\delta > 0$.

To prove that the set $P(T)$ is open, we have to show that there exists a tax function Z in some neighborhood of S such that $u_1(Z,T) > 0$. To do so, it suffices to define $S(\varepsilon) = \varepsilon S + (1-\varepsilon)T$ for any $\varepsilon \in [0,1]$. Let us recall that the cost functions corresponding to tax functions S and T are respectively given by $c(S)(x) = S_G(x) + S_P(x)$ and $c(T)(x) = T_G(x) + T_P(x)$. To verify that $S(\varepsilon)$ satisfies the budget balanced constraint, it suffices to write:

$$\int_0^1 G(S(\varepsilon))dF = \varepsilon[\int_{E_{S(\varepsilon)}} S(x)dF + \int_{[0,1]\setminus E_{S(\varepsilon)}} S_G(x)dF] \\ + (1-\varepsilon)[\int_{E_{S(\varepsilon)}} T(x)dF + \int_{[0,1]\setminus E_{S(\varepsilon)}} T_G(x)dF].$$

Given the Equality lemma (Lemma 2), we can write that $E_{S(\varepsilon)} = E_T$ as $E_S = E_T$. Therefore, given both tax functions S and T belong to the set R, we can write that

$$\int_{E_{S(\varepsilon)}} S(x)dF + \int_{[0,1]\setminus E_{S(\varepsilon)}} S_G(x)dF = \int_{E_S} S(x)dF + \int_{[0,1]\setminus E_S} S_G(x)dF = r,$$

and

$$\int_{E_{S(\varepsilon)}} T(x)dF + \int_{[0,1]\setminus E_{S(\varepsilon)}} T_G(x)dF = \int_{E_T} T(x)dF + \int_{[0,1]\setminus E_T} T_G(x)dF = r$$

which implies that for any $0 \leq \varepsilon \leq 1$, $\int_0^1 G(S(\varepsilon))dF = r$.

Finally, it remains to be shown that if $u_1(S,T) > 0$ then $u_1(S(\varepsilon),T) > 0$. As $u_1(S,T) > 0$, we can write that $p_F(s(x) < t(x)) > 1/2$. Given that $S(\varepsilon) \in R$, we define its corresponding ex-post tax function s_ε as $s_\varepsilon = \min\{S(\varepsilon), c(S(\varepsilon))\}$.

Furthermore, as the construction of $S(\varepsilon)$ implies that $E_S = E_{S(\varepsilon)}$, we can write that

$$s_\varepsilon(x) = \begin{cases} S(\varepsilon)(x) & \text{if } x \in E_S \\ c(S(\varepsilon))(x) & \text{if } x \notin E_S. \end{cases}$$

Therefore, if $x \in E_S$, we can see that whenever $S(x) > T(x)$ (resp. $S(x) < T(x)$) $S(\varepsilon)(x) = \varepsilon S(x) + (1-\varepsilon)T(x) > T(x)$ (resp. $S(\varepsilon)(x) < T(x)$). Similarly, if $x \notin E_S$, we can see that whenever $c(S)(x) > c(T)(x)$ (resp. $c(S)(x) < c(T)(x)$), we can write that $c(S(\varepsilon))(x) = \varepsilon c(S)(x) + (1-\varepsilon)c(T)(x) > c(T)(x)$ (resp. $c(S(\varepsilon))(x) < c(T)(x)$).

Previous inequalities imply that $p_F(s_\varepsilon(x) < t(x)) = p_F(s(x) < t(x)) > 1/2$, which implies that $S(\varepsilon) \in P(T)$ and by definition $S(\varepsilon)$ is in a δ-neighborhood of S as $||S(\varepsilon) - T||_\infty \leq ||S - T||_\infty < \delta$.

Therefore, if we pick $Z = S(\varepsilon)$, we have proven that for any $S \in P(T)$, there exists a tax function Z in some neighborhood of S such that $Z \in P(T)$, showing that $P(T)$ is an open set. □

D Appendix: On the shape of income tax functions

Proof of Proposition 2.

Proof. Let us pick a convex S with $E_S = [0,1]$ and a concave T in with $E_T \neq [0,1]$. Their respective ex-post tax functions are denoted by $s = \min\{S, c(S)\} = S$ and $t = \min\{T, c(T)\}$. By definition, both S and T are budget-balanced. Hence, we can write

$$\int_0^1 SdF = \int_{E_T} TdF + \int_{[0,1]\setminus E_T} T_G dF = r.$$

Hence, we can write that

$$\int_0^1 tdF > r,$$

with $t = \min\{T, c(T)\}$ as $T_P(x) > 0$ which implies that $c(T)(x) > T_G(x)$ with $x \in (0,1)$. Besides, t is the minimum of two concave functions (as the cost function is smooth) and then t is concave. Hence, we know that the unique intersection θ between $s = S$ and t is located above the median (as stated by Marhuenda and Ortuño-Ortin [11]). This concludes the proof as we have proved that $W(S,T) > 1/2$. □

Proof of Theorem 3.

Proof. Proposition 2 states that any progressive tax function $S \in \hat{R}$ with $E_S = [0,1]$ is preferred to any regressive tax function $T \in \hat{R}$. Hence, if party 1 proposes a mixed strategy μ_1 that puts some positive weight over the regressive tax functions ($\mu_1(\hat{R}_{reg}) > 0$), his opponent party 2 can beat him by advocating a mixed strategy

μ_2 that replicates party 1's strategy over the set of progressive tax functions and that puts the same weight on the progressive tax functions S with $E_S = [0, 1]$ as μ_1 on the set of regressive tax functions. As the game is zero-sum, both parties are expected to have a zero payoff at equilibrium which shows that advocating μ_1 is not a best response for party 1. As the game is symmetric, repeating the same argument for party 2 completes the proof. $\square$